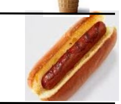
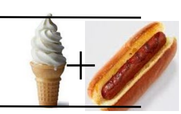
Risk and Ambiguity

Peter P. Wakker

§1.1; 1st meeting

Example 1.1.1 (Street vendor)

Table 1.1.1. Net profits for street vendor

	no rain (s_1)	*some rain* (s_2)	*all rain* (s_3)	
x (“ice cream”)	400	100	-400	
y (“hot dogs”)	-400	100	400	
0 (“neither”)	0	0	0	
$x + y$ (“both”)	0	200	0	

More realistic:

Example 1.1.2 (Finance)

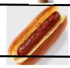
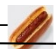
Speculate on copper price next month

K: €1000

Table 1.1.2. Net profits

	price ≥ 2.53	$2.53 >$ *price* ≥ 2.47	$2.47 >$ *price*
x	$50K$	$-30K$	$-30K$
y	$-30K$	$-30K$	$50K$
0 ("neither")	0	0	0
$x+y$ ("both")	$20K$	$-60K$	$20K$

We continue with street vendor:

		no rain (s_1)	*some rain* (s_2)	*all rain* (s_3)
	x	400	100	-400
	y	-400	100	400
	0	0	0	0
 + 	$x+y$	0	200	0

What would you do? …

Notation: (x_1, x_2, x_3): €x_j if s_j obtains;
ice cream = $(400, 100, -400)$

Proposal: do expected value (EV) maximization.
"Determine" p_1, p_2, p_3:
p_j = subjective probability of s_j (…).

Example. $p_1 = 0.40; p_2 = p_3 = 0.30.$
$EV(x) = 0.40 \times 400 + 0.30 \times 100 + 0.30 \times (-400) = 70;$
$EV(y) = -10; EV(0) = 0; EV(x+y) = 60;$
x is chosen!

Story change (now descriptive).
Street vendor $\neq$ you. EV holds.
What are p_j?

Elicitation method

Exercise 1.3.3. $(300, 0, 0) \succcurlyeq (0,0,300)$.
Show $p_1 \geq p_3$.

Solution: $p_1 \times 300 \geq p_3 \times 300 \Rightarrow p_1 \geq p_3$.

Also: $x \succ y \Rightarrow p_1 > p_3$.

Want to find p_1, p_2, p_3 exactly.

Experimental heaven:

can exactly observe preference between any (x_1, x_2, x_3) and (y_1, y_2, y_3).

For any (x_1, x_2, x_3), can find CE α:

$\alpha \sim (x_1, x_2, x_3)$.

Notation: $\alpha = (\alpha, \alpha, \alpha)$.

How find p_1, p_2, p_3?

Find CE of $(1,0,0)$: $\alpha \sim (1,0,0)$

EV: $1 \times \alpha = p_1 \times 1 + p_2 \times 0 + p_3 \times 0$;

$\alpha = p_1$!

p_2 and p_3 similarly.

Measure-predict

Exercise 1.3.5. Assume
$(100,0,0) \sim 50$
$(0,100,0) \sim 25$.
What are p_1, p_2, p_3?
What is $CE(0,0,100)$?
What is preference between $(0,100,0)$ and $(0,0,100)$?

Solution:
$p_1 100 + (1 - p_1)0 = 50 \Rightarrow p_1 = 0.5$
$p_2 100 + (1 - p_2)0 = 25 \Rightarrow p_2 = 0.25$
$p_3 = 1 - p_1 - p_2 x = 0.25$
$CE(0,0,100) = 0.25 \times 100 = 25$
$(0,100,0) \sim (0,0,100)$ (EV = 25)

§1.4-§1.5

Story change (normative again).
Street vendor = you.

You hire expensive advisor.

His advice: “Maximize EV!”

Are you convinced?

Not really!
- Ad hoc.
- p_j’s how?
- Additions/multiplications in EV are ad hoc.

Can you one more time think of justification?

Law of large numbers!?

Not bad. Weak points:
- Needs repeated decisions.
- Sumtotal of wealth over lifetime not always relevant.

Need different story.

Advisor apologizes. Asks for 2nd chance.
Recommends:

1. Monotonicity
2. Transitivity
3. Additivity: $x \succcurlyeq y \Rightarrow x + z \succcurlyeq y + z$

Examples of additivity:

Table 1.5.2 (Finance)

K: €1000

		E_1	E_2	E_3			E_1	E_2	E_3
If	x	$50K$	$-30K$	$-30K$	$\succcurlyeq$	y	$-30K$	$-30K$	$50K$
	z	$30K$	$30K$	$30K$		z	$30K$	$30K$	$30K$
then	$x+z$	$80K$	0	0	$\succcurlyeq$	$y+z$	0	0	$80K$

Table 1.5.3 Central Example

		no rain	some rain	all rain
If	x	400	100	-400
	z	150	100	50
then	$x+z$	550	200	-350

$\succcurlyeq$

$\succcurlyeq$

	no rain	some rain	all rain
y	-400	100	400
z	150	100	50
$y+z$	-250	200	450

Table 1.5.4 (Finance)

K: €1000

		E_1	E_2	E_3			E_1	E_2	E_3
If	x	$50K$	$-30K$	$-30K$	$\succcurlyeq$	y	$-30K$	$-30K$	$50K$
	z	$40K$	0	$-40K$		z	$40K$	0	$-40K$
then	$x + z$	$90K$	$-30K$	$-70K$	$\succcurlyeq$	$y + z$	$10K$	$-30K$	$10K$

Additivity modified: only for moderate amounts. Only if z does not change your life situation (the extra happiness from a €).

We assume moderate amounts henceforth.

We next explore implications of the advice.

Notation:

In this course, subscripts always indicate states of nature.

Superscripts can distinguish different choice options.

For street vendor, x^1 and x^2 indicate different merchandises.

E.g., $x^1 = (x_1^1, x_2^1, x_3^1)$; $x^2 = (x_1^2, x_2^2, x_3^2)$.

Theorem.
If $x^1 \succcurlyeq y^1$ and $x^2 \succcurlyeq y^2$,
then $x^1 + x^2 \succcurlyeq y^1 + y^2$.

Proof.
By additivity (x^2 in role of added z)
$x^1 \succcurlyeq y^1 \Rightarrow$
$x^1 + x^2 \succcurlyeq y^1 + x^2$.
$x^2 \succcurlyeq y^2 \Rightarrow$
$x^2 + y^1 \succcurlyeq y^2 + y^1$, i.e.,
$y^1 + x^2 \succcurlyeq y^1 + y^2$.
Transitivity:
$x^1 + x^2 \succcurlyeq y^1 + y^2$.

Theorem.

If $x^1 \succcurlyeq y^1, x^2 \succcurlyeq y^2, \ldots, x^{20} \succcurlyeq y^{20}$,

then $x^1 + \cdots + x^{20} \succcurlyeq y^1 + \cdots + y^{20}$.

Proof.

By previous theorem,

$$
\begin{array}{l}
x^1 + x^2 \succcurlyeq y^1 + y^2 \ . \\
\qquad\quad x^3 \succcurlyeq y^3 \\
\hline
x^1 + x^2 + x^3 \succcurlyeq y^1 + y^2 + y^3 \\
\qquad\qquad\qquad x^4 \succcurlyeq y^4 \\
\hline
\end{array}
\;+
$$

… etc.

Holds for any number of preferences, also if more/less than 20.

Now imagine you have well-contemplated preferences:
$x^1 \succcurlyeq y^1$
$x^2 \succcurlyeq y^2$

.

.

.

$x^m \succcurlyeq y^m$

Imagine you calculate and find:
s_1: $x_1^1 + \cdots + x_1^m < y_1^1 + \cdots + y_1^m$
Mm …
s_2: $x_2^1 + \cdots + x_2^m < y_2^1 + \cdots + y_2^m$
Mmm …
s_3: $x_3^1 + \cdots + x_3^m < y_3^1 + \cdots + y_3^m$.
Ouch!!!
You violate advice of advisor!
Do you see why?
Is called arbitrage/Dutch book.

§1.6

Following theorem ignores technicalities (see book)

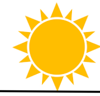
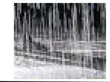
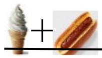
Theorem 1.6.1 [de Finetti's surprise] The following three statements are equivalent:

(i) Expected value holds.

(ii) Monotonicity, transitivity, & additivity hold.

(iii) No arbitrage.

§2.2; 2nd meeting

				
	x	400	100	-400
	y	-400	100	400
	0	0	0	0
	$x+y$	0	200	0

Objective: $p_1 = \frac{1}{4}, p_2 = \frac{1}{2}, p_3 = \frac{1}{4}$.

$$x = (\tfrac{1}{4}: 400, \tfrac{1}{2}: 100, \tfrac{1}{4}: -400)$$ 

$$y = \left(\tfrac{1}{4}: -400, \tfrac{1}{2}: 100, \tfrac{1}{4}: 400\right)$$ 

$$0 = (1: 0)$$

$$x + y = (\tfrac{1}{2}: 200, \tfrac{1}{2}: 0)$$

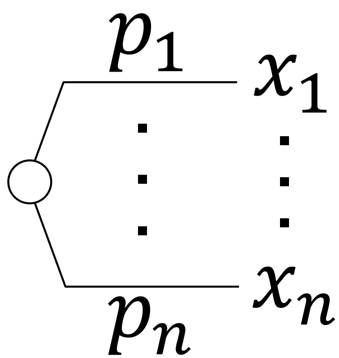
DUR: only prob^s matter 

Prospects: prob^ty distr^s over money

Note something about x and y? $x = y$!!

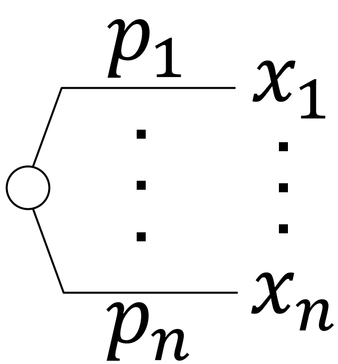
Notation

$(p_1 : x_1, \ldots, p_n : x_n)$



$$(p : \alpha, 1 - p : \beta) = \alpha_p \beta$$
$$(1 : \alpha) = \alpha$$

Risk aversion:

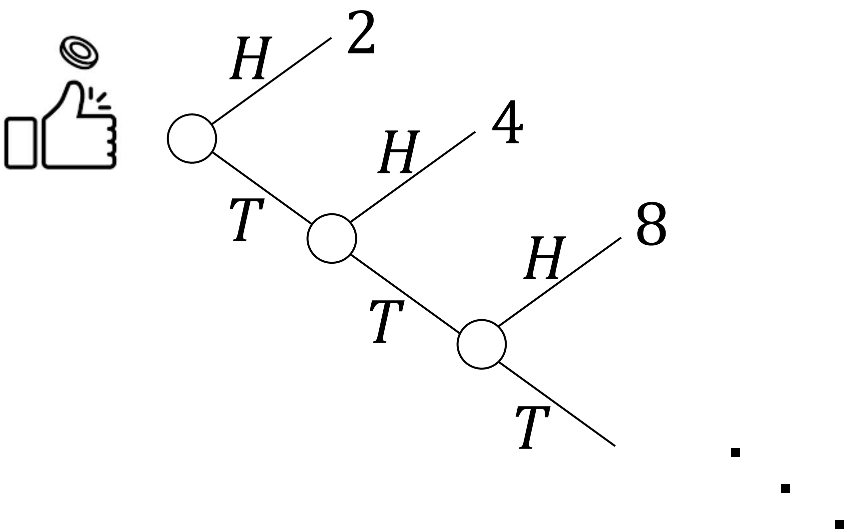


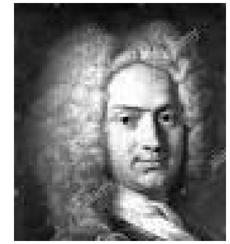
$$\preccurlyeq \; p_1 x_1 + \cdots + p_n x_n$$

Risk neutrality: $\sim$

Risk seeking: $\succcurlyeq$

St. Petersburg paradox





Nicolas Bernoulli
(1713)

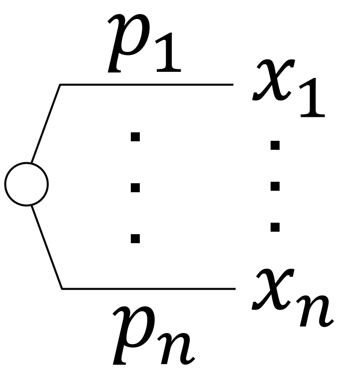
How much you pay to play it once?

$$EV = \frac{1}{2}.2 + \frac{1}{4}.4 + \frac{1}{8}.8 + \cdots$$

$$\begin{array}{ccccccc} \frac{1}{2}.2 & + & \frac{1}{4}.4 & + & \frac{1}{8}.8 & + & \cdots \\ \| & & \| & & \| & & \\ 1 & + & 1 & + & 1 & + & \cdots \end{array} = \infty$$

Empirically: $CE = €4.$ $\between EV!$

Daniel Bernoulli (1738):
Expected utility (EU)



$$\rightarrow \quad p_1U(x_1) + \cdots + p_nU(x_n)$$

"€10M ≠ 10 × €1M"
(M : million)

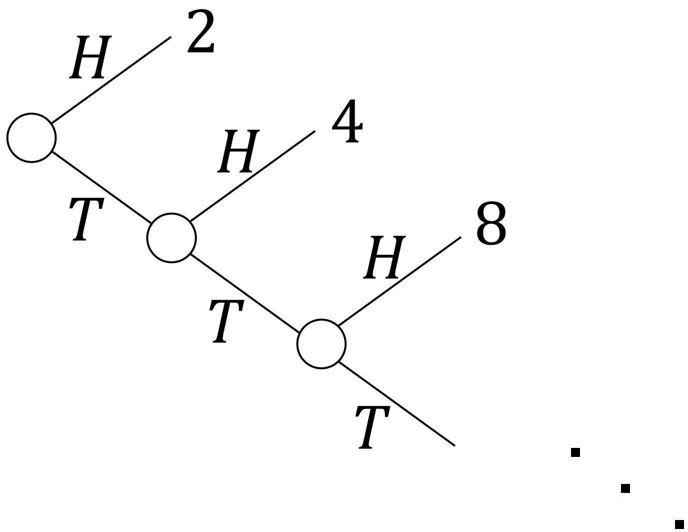
Exercise 2.5.3: Assume EU, with
$U(0) = 0, U(100) = 1$, and $50 \sim 100_{0.58}0$.
(a) What is $U(50)$?
(b) $(0.40{:}\,100, 0.20{:}\,50, 0.40{:}\,0)$
versus
$(0.33{:}\,100, 0.33{:}\,50, 0.34{:}\,0)$;
which is preferred?

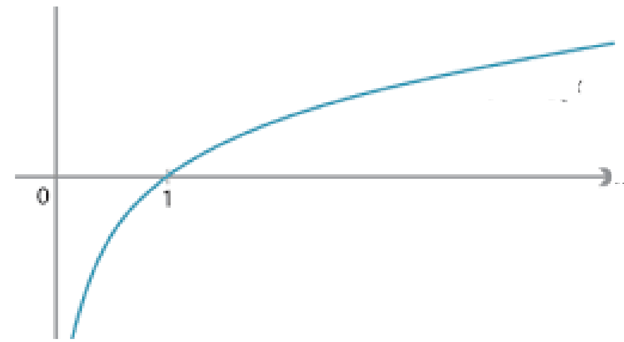
Solution.

(a) $\mathrm{EU}(50) = \mathrm{EU}(100_{0.58}0) = 0.58 \times 1 = 0.58.$
$U(50) = EU(50) = 0.58.$

(b) $\mathrm{EU}(0.40{:}\,100, 0.20{:}\,50, 0.40{:}\,0) = 0.40 \times 1 + 0.20 \times 0.58 = 0.516.$
$\mathrm{EU}(0.33{:}\,100, 0.33{:}\,50, 0.34{:}\,0) = 0.33 \times 1 + 0.33 \times 0.58 = 0.5214.$
$(0.33{:}\,100, 0.33{:}\,50, 0.34{:}\,0)$ is preferred.

St. Petersburg paradox reconsidered





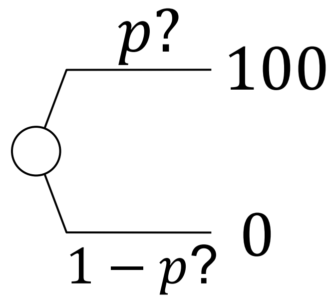
EU with $U(\alpha) = \ln(\alpha)$

$$EU = \frac{1}{2} ln2 + \frac{1}{4} ln4 + \frac{1}{8} ln8 + \cdots = 1.39\ .$$

$$= ln4\ !$$

So, CE (certainty equivalent) now is €4.
Agrees with empirical findings.

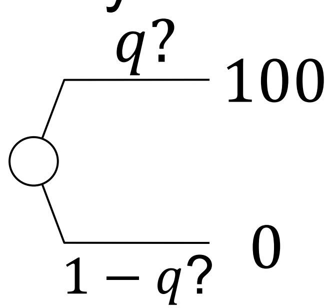
Question. Assume $U(0) = 0$ & $U(100) = 1$. How measure $U(30)$?

$30 \sim$ 

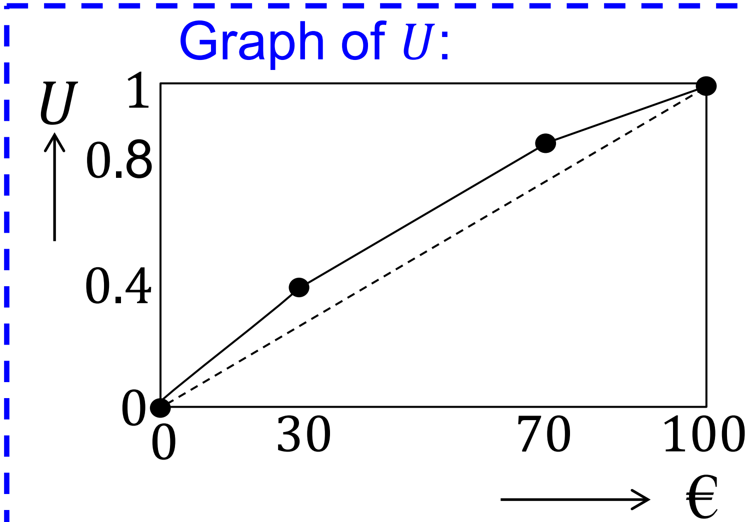
Say $p = 0.4$.

$$U(30) = 0.4 \times 1 + 0.6 \times 0 = 0.4.$$

We similarly measure, say, $U(70)$:

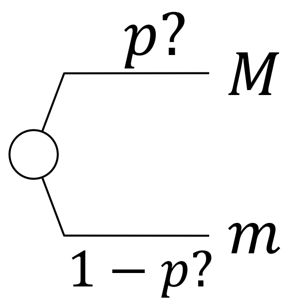
$70 \sim$ 

Say $q = 0.8$. $U(70) = 0.8$.



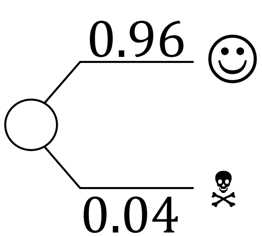
General (standard gamble) method for measuring utility

Assume best conceivable outcome M and worst m.
Set $U(M) = 1$ and $U(m) = 0$.

α ~ 

$\Rightarrow U(\alpha) = p$

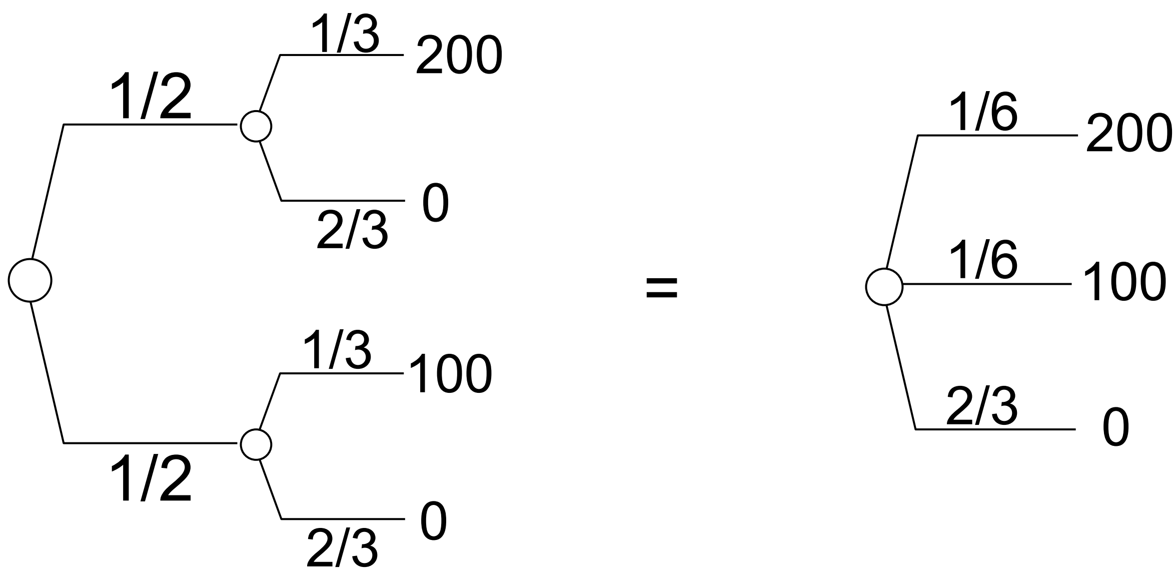
In health domain: quality of life ≈ utility.
Perfect health (☺) is best ($U = 1$).
Death (☠) is worst ($U = 0$).

Blind ~ 

⇒ quality of life of blindness is 0.96.

Blindness brings loss of 4% quality of life.

Preparatory notation: multistage lotteries



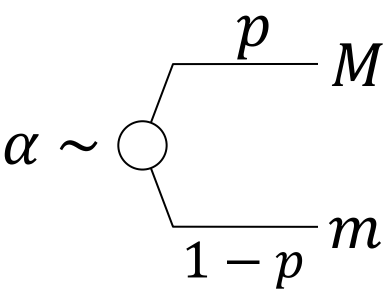
Rule: consecutive probabilities should be multiplied.

McCord & de Neufville (1986):

we are not in experimental heaven!

Standard gamble measurement

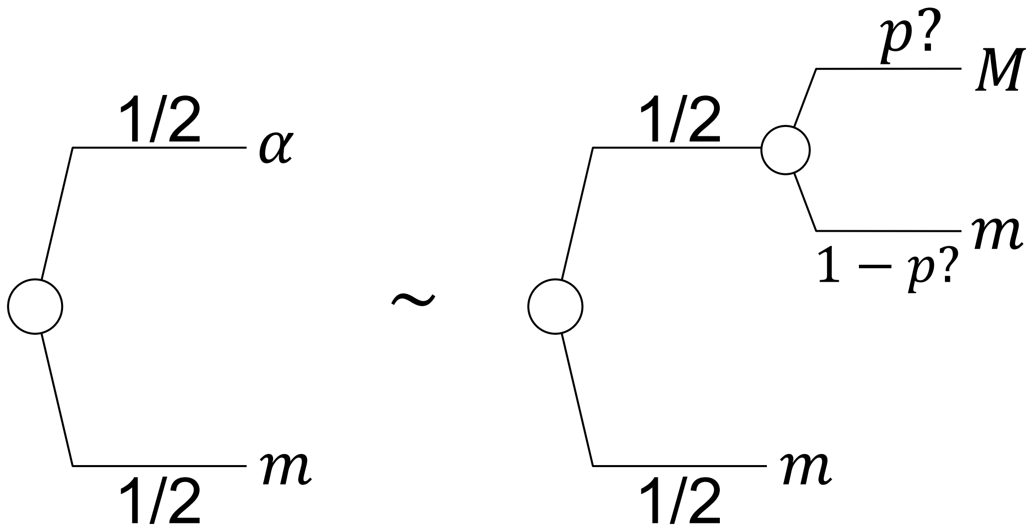


has "certainty effect." 

Violates EU much!

Better take stimuli where EU less violated.

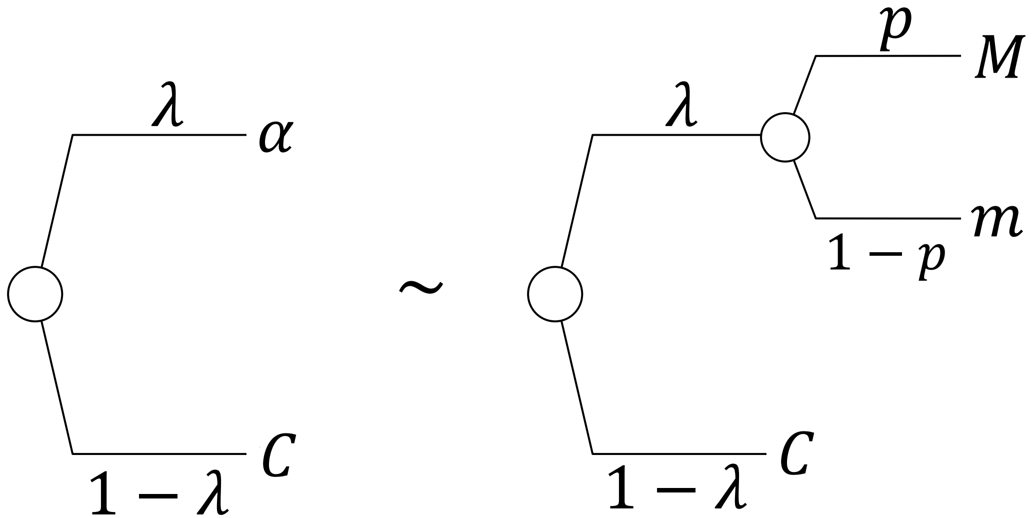
McCord & de Neufville (1986) proposed:



Claim: under EU, $U(\alpha) = p$.

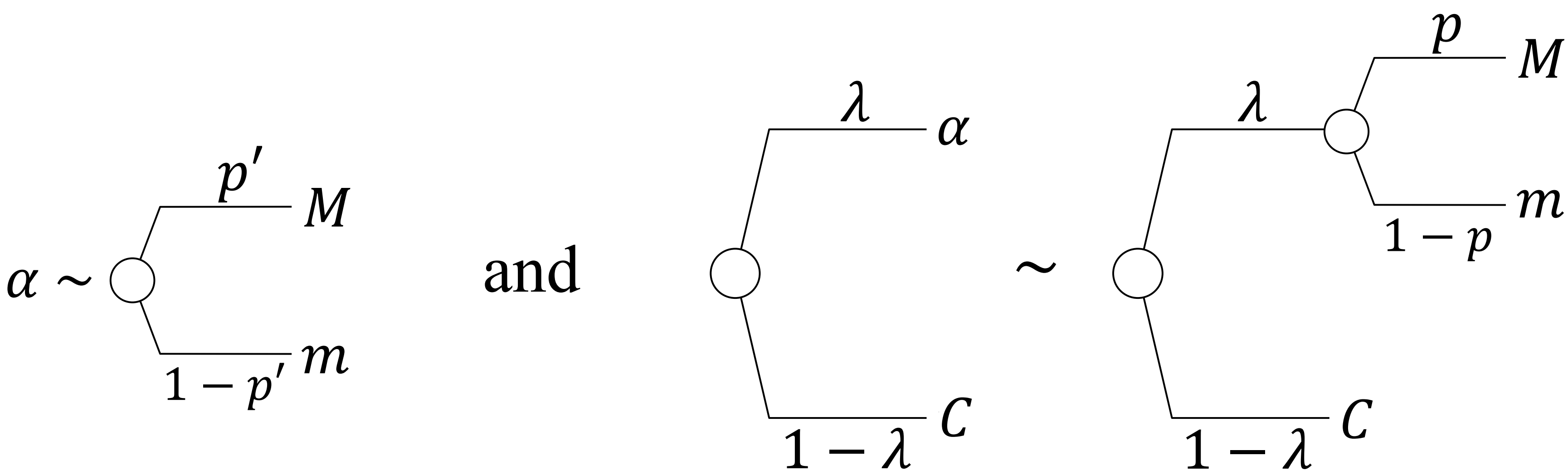
Here no certainty effect.

More general:



Claim: then still $U(\alpha) = p$.

Now imagine both

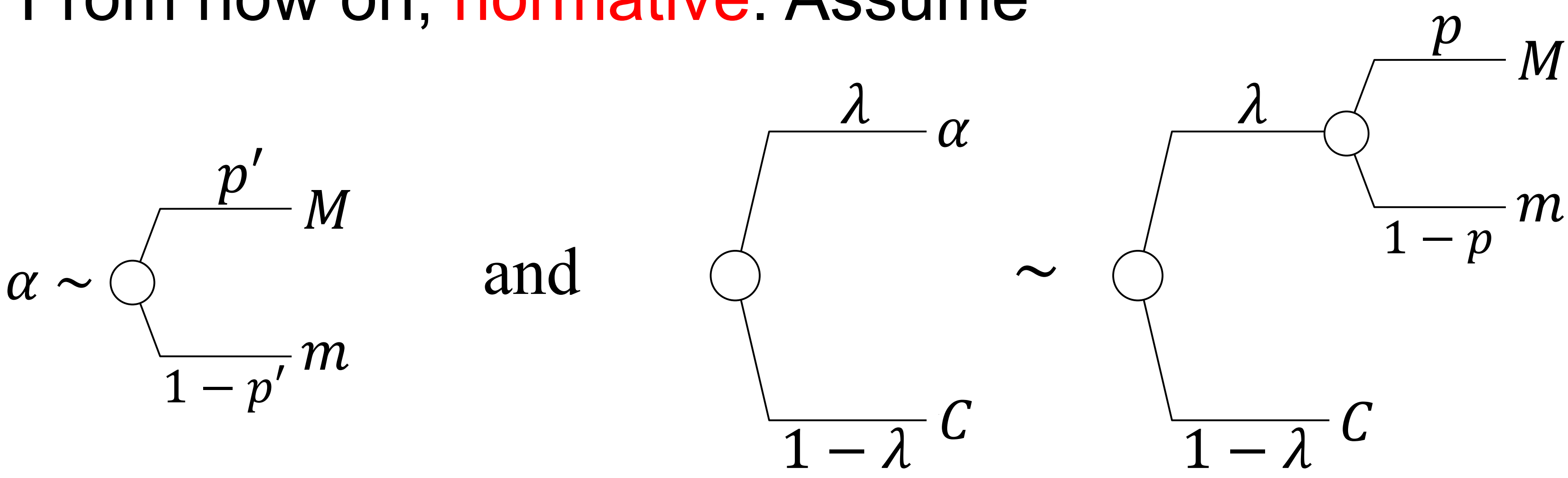


but $p' \neq p$. What to think?

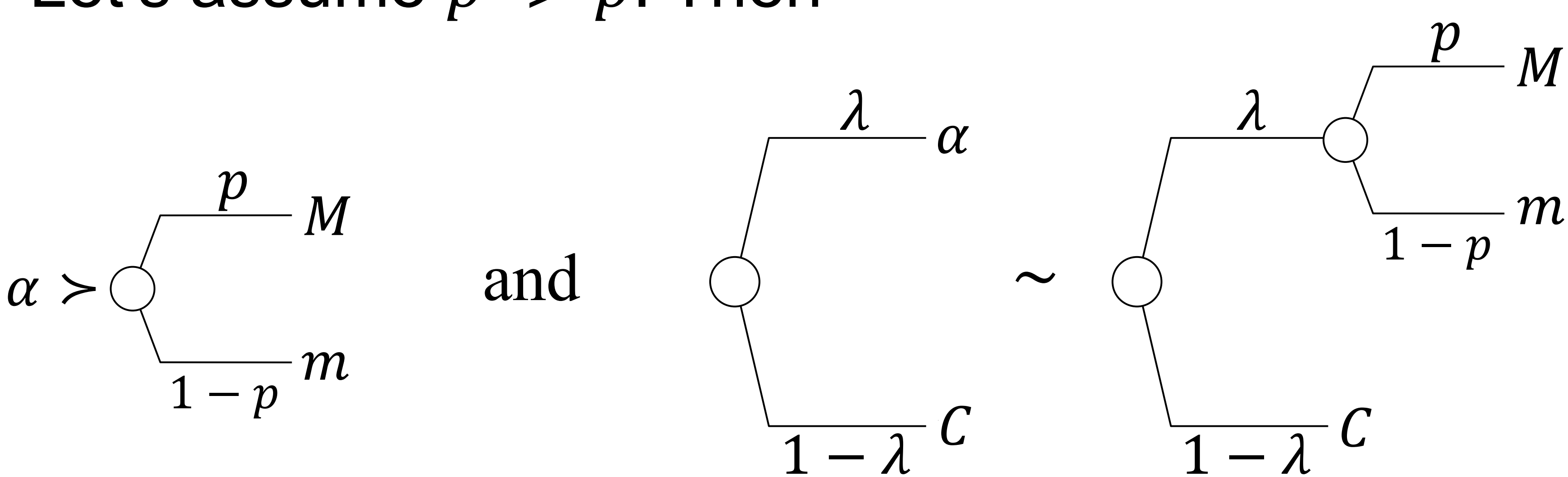
1. EU ✗. (No surprise.)

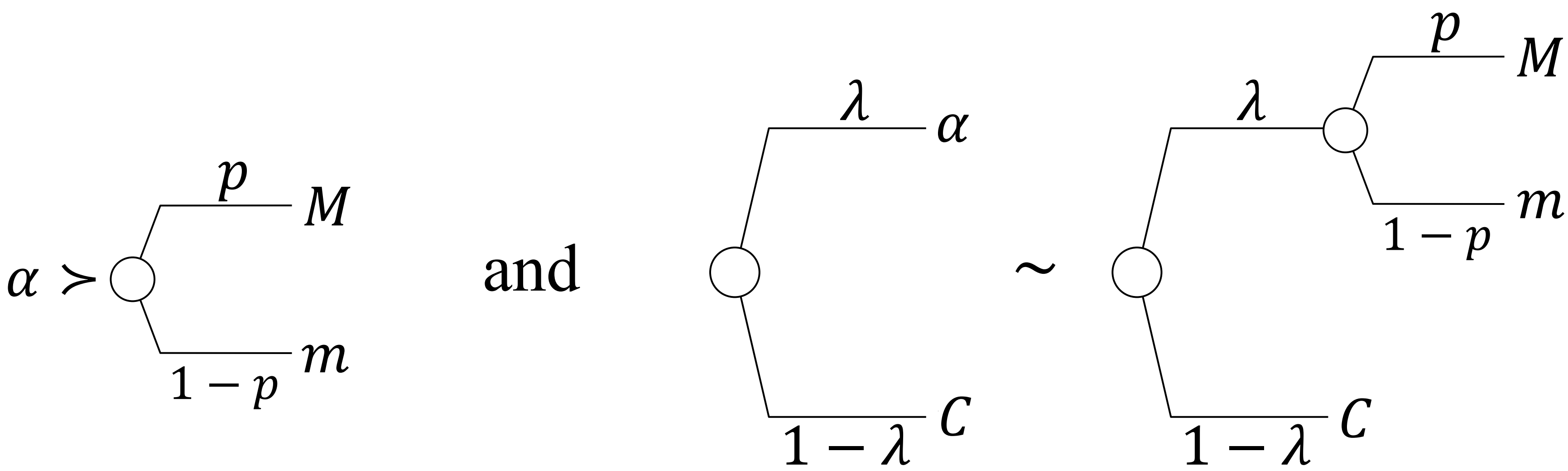
2. McCord & de Neufville (1986):
   "This is exactly our point!
   p is better than p'."

Change of story: so far perspective was descriptive.
From now on, normative. Assume



but $p' \neq p$. What to think normatively?
Let's assume $p' > p$. Then

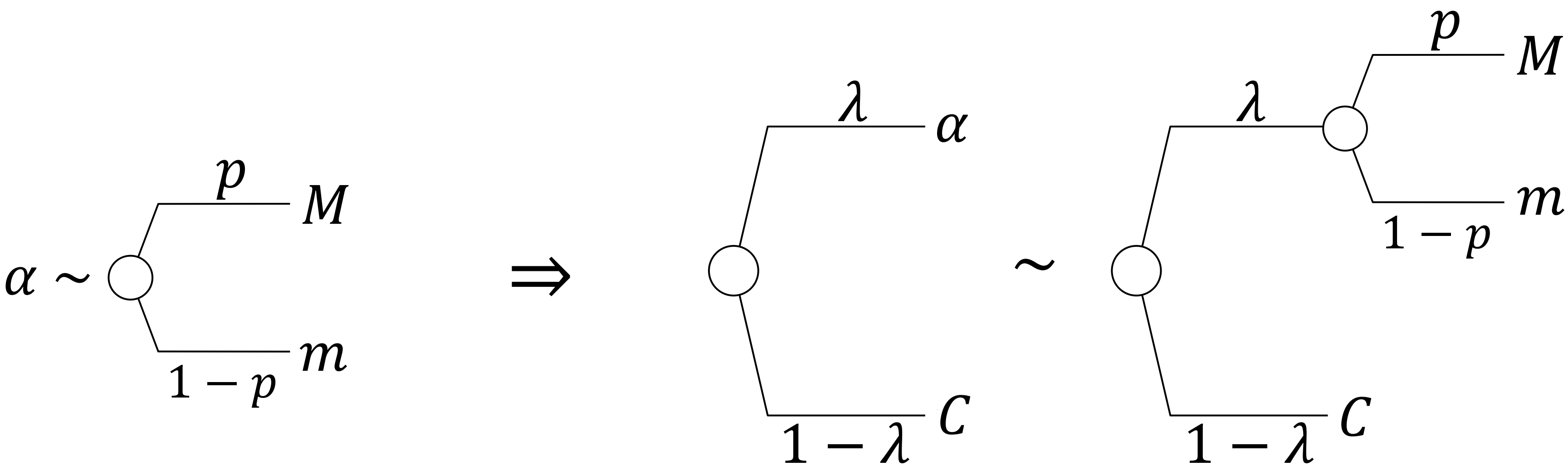




My claim: irrational!

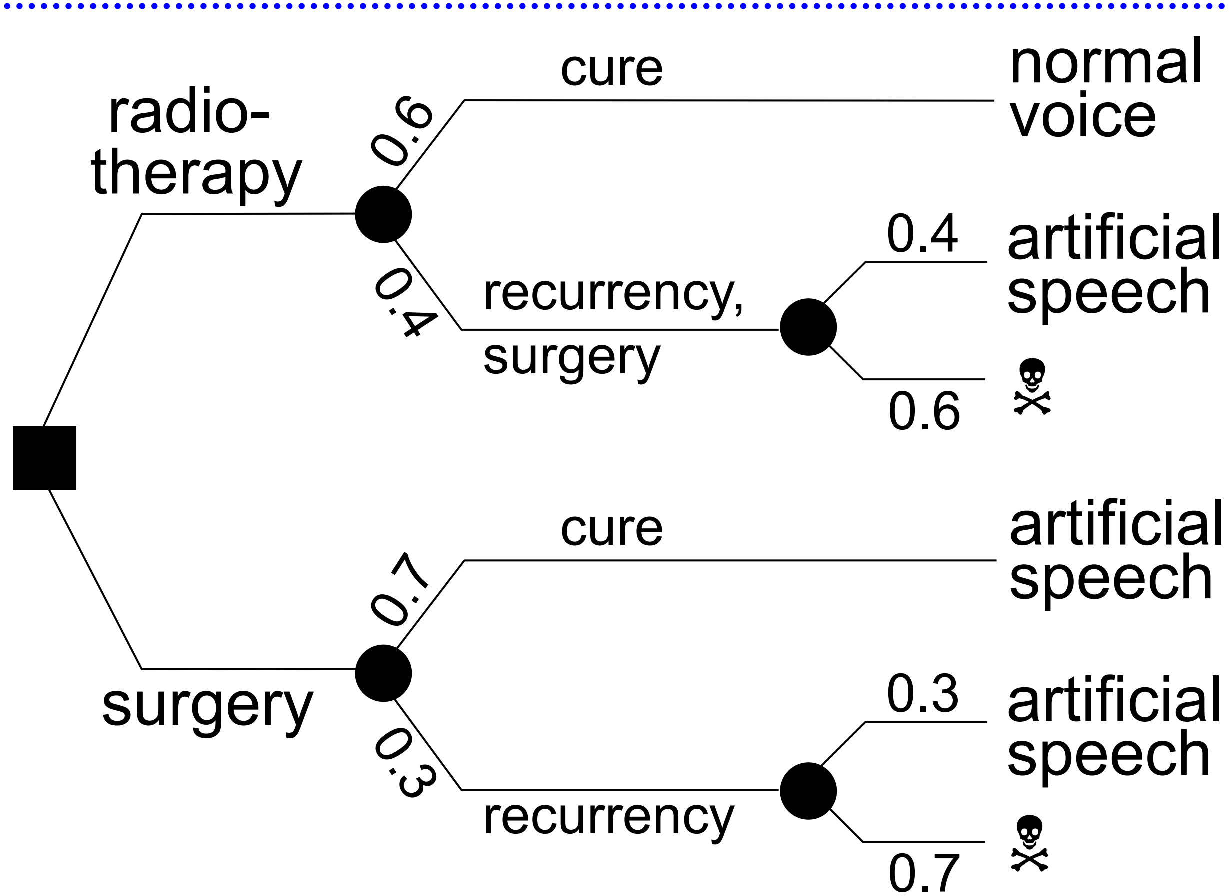
…

Standard gamble consistency:



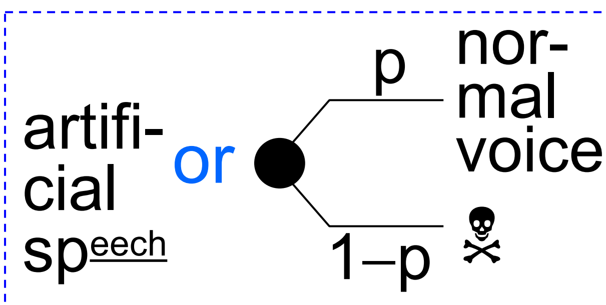
THEOREM (von Neumann-Morgenstern's surprise).
The following two statements are logically equivalent for your preferences:

i) Expected utility holds;
ii) Transitivity, completeness, SG dominance, SG solvability, & SG consistency hold.



U	p	U×p	EU
1	.60	.60	
.9	.16	.144	.744
0	.24	0	+
		.744	
.9	.70	.63	
.9	.09	.081	.711
0	.21	0	+
		.711	

Hypothetical standard gamble question:



For which p equivalence?

Patient answers: p = 0.9.

Expected utility: U(☠) = 0; U(normal voice) = 1; U(artificial speech) = 0.9 × 1 + 0.1 × 0 = 0.9.

Patient with larynx-cancer (stage T3).
Radio-therapy or surgery?

Answer: r.th!

Theorem 3.2.1. Assume EU.
Risk aversion $\Leftrightarrow$ U concave.
Risk neutrality $\Leftrightarrow$ U linear.
Risk seeking $\Leftrightarrow$ U convex.

Comparisons:
$\alpha \sim_1 x \Rightarrow \alpha \succcurlyeq_2 x$: $\succcurlyeq_2$ MRA than $\succcurlyeq_1$

Theorem 3.4.1. Under EU,
$\succcurlyeq_2$ MRA than $\succcurlyeq_1$ $\Leftrightarrow$ $U_2 = \varphi U_1$ for a concave φ.

Holds iff $\frac{-U_2''}{U_2'} \geq \frac{-U_1''}{U_1'}$.
Holds iff $\succcurlyeq_2$ has bigger risk premiums than $\succcurlyeq_1$.

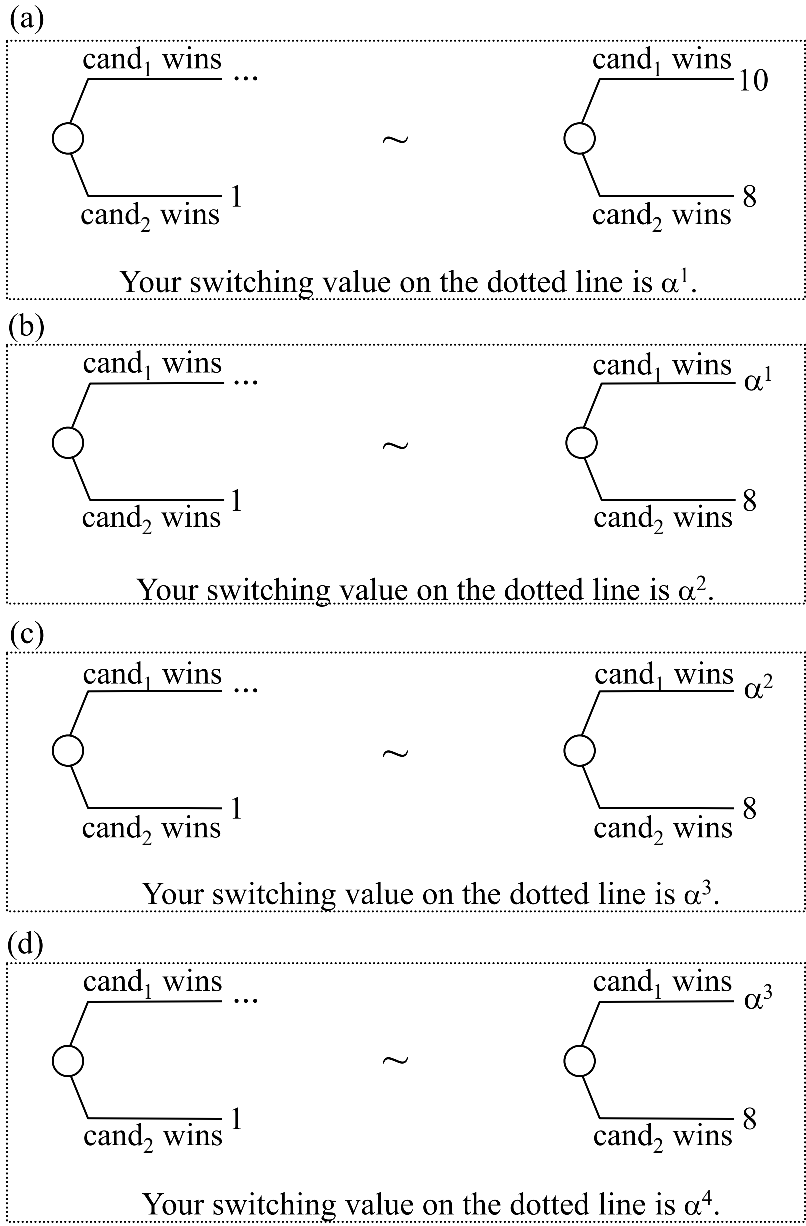
Decreasing absolute risk aversion:
$\alpha \sim x \Rightarrow \alpha + \varepsilon \preccurlyeq x + \varepsilon$ for $\varepsilon \geq 0$.

(S)EU:

There exists P on events AND U on outcomes, s.t.

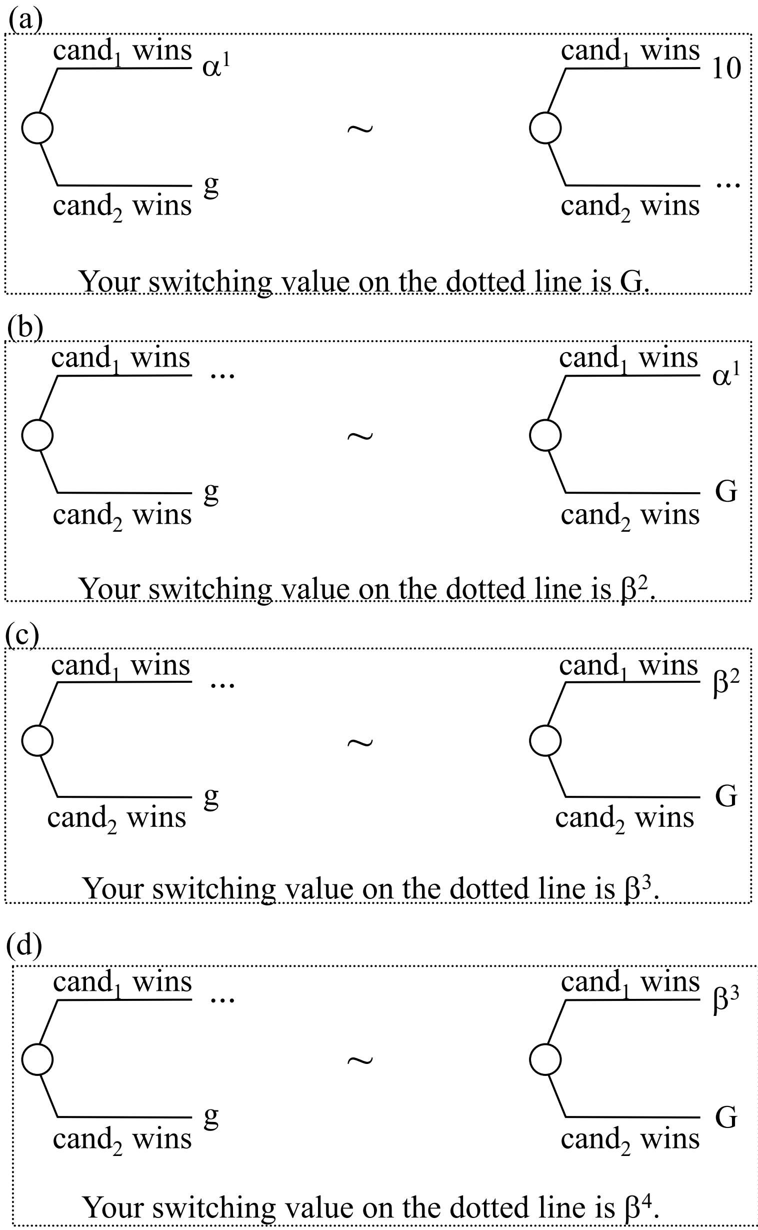
$$(E_1{:}\,x_1, \cdots, E_n{:}\,x_n) \mapsto P(E_1)U(x_1) + \cdots + P(E_n)U(x_n)$$

represents $\succcurlyeq$.



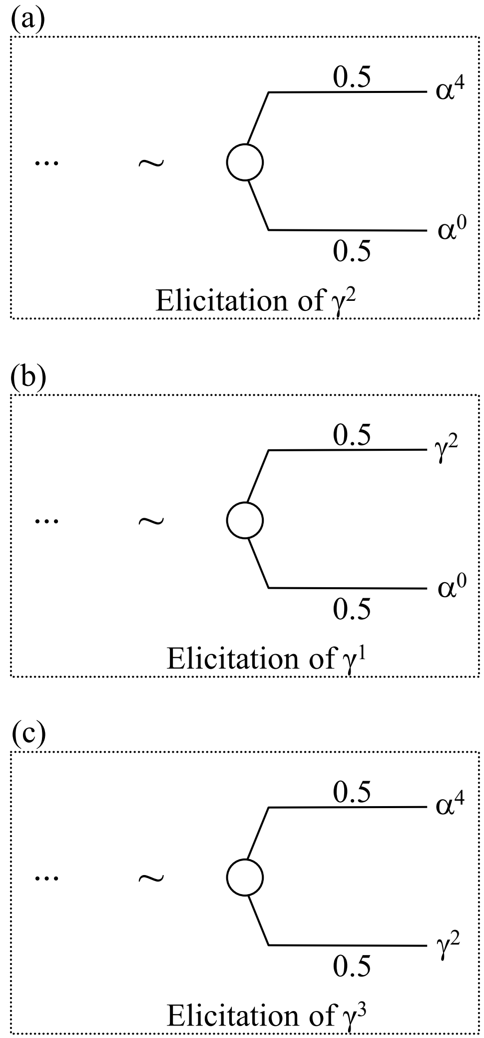
Indicate in each Fig. which outcome on the dotted line ...
makes the two prospects indifferent (the switching value).

Figure 4.1.1 [*TO Upwards*]. Eliciting $\alpha^1 \ldots \alpha^4$ for unknown probabilities



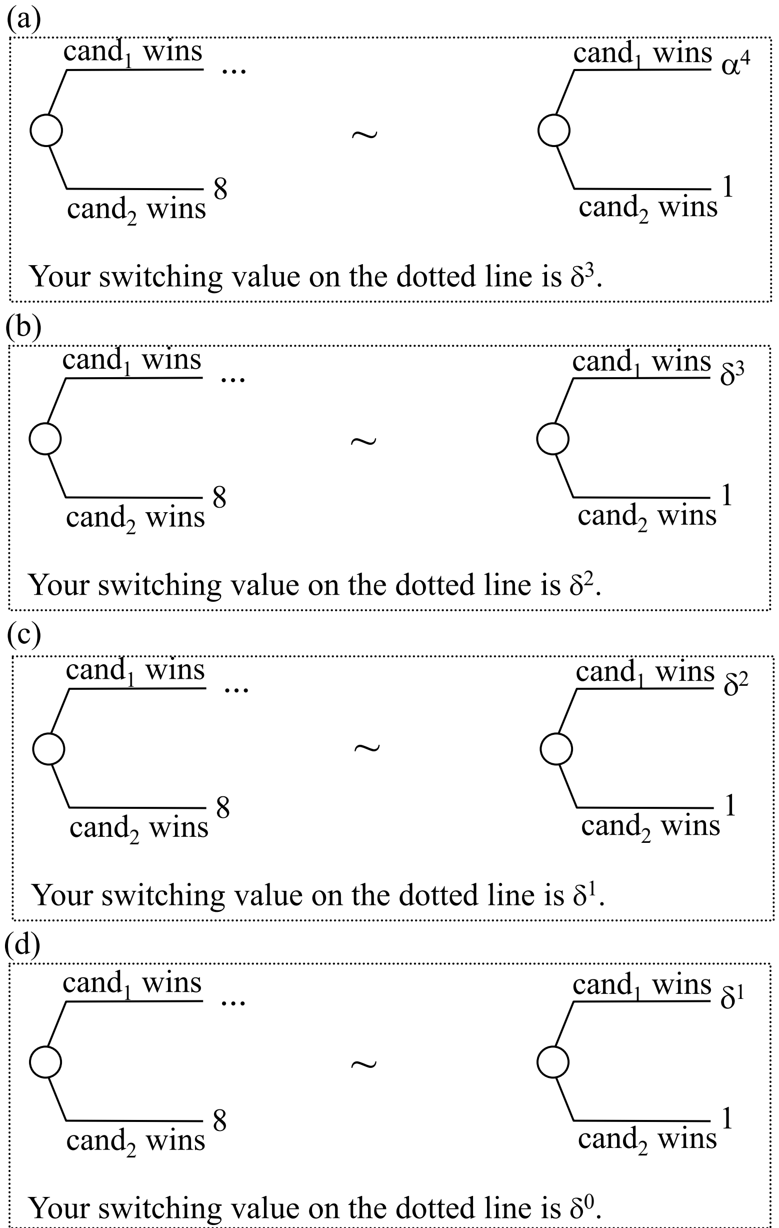
Indicate in each fig. which outcome on the dotted line ...
makes the two prospects indifferent (the switching value).

Figure 4.1.2 [*2nd TO Upwards*]. Eliciting β^2, β^3, β^4



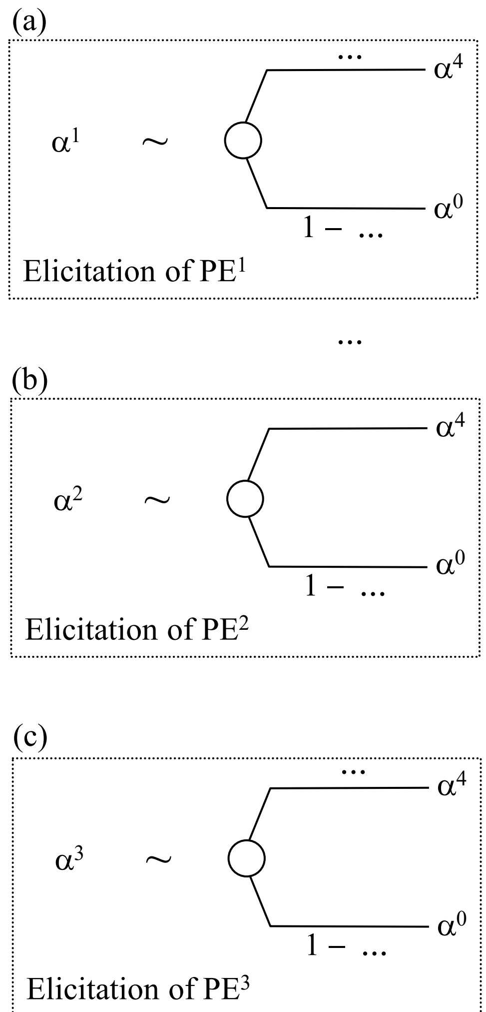
Indicate in each Fig. which outcome on the dotted line ..., if received with certainty, is indifferent to the prospect.

Figure 4.1.3 [*CEs*]. Eliciting γ^2,γ^1,γ^3



Indicate in each fig. which outcome on the dotted line ...
makes the two prospects indifferent (the switching value).

Figure 4.1.4 [*TO Downwards*]. Eliciting δ^3 … δ^0



Indicate in each Fig. which probability on the dotted lines … makes the prospect indifferent to receiving the sure amount to the left.

Figure 4.1.5 [*PEs*]. Eliciting PE^1, PE^2, PE^3

EXERCISE 1 Experiment. Consider Figure 4.1.1 (TO upwards; do not consider the other figures). Assume that both candidates have a nonzero probability of winning. Show that, under EU (with $\alpha^0 = 10$, p_1 for the (subjective) probability of cand_1 winning, and $p_2 = 1 - p_1$):

$$U(\alpha^4) - U(\alpha^3) = U(\alpha^3) - U(\alpha^2) = U(\alpha^2) - U(\alpha^1) = U(\alpha^1) - U(\alpha^0)\,. \qquad (1)$$

First derive the last equality using only Figs. 4.1.1a and b.

EXERCISE 2 Experiment. Assume EU for Figures 4.1.1 and 4.1.2, with nonzero probabilities of winning for both candidates.

a) Show that $U(\beta^4) - U(\beta^3) = U(\beta^3) - U(\beta^2) = U(\beta^2) - U(\alpha^1) = U(\alpha^1) - U(\alpha^0)$.

b) Show that $\beta^j = \alpha^j$ for all j.

EXERCISE 3 Experiment. Assume EU for Figures 4.1.1 and 4.1.3, with nonzero probabilities of winning for both candidates. Show that $\gamma^j = \alpha^j$ for all j.

EXERCISE 4 Experiment. Do not assume EU. Assume only weak ordering of your preference relation. Further assume *strong monotonicity*, which means that any prospect becomes strictly better as soon as one of its outcomes is strictly improved. Under EU, not assumed here, the latter assumption would amount to all outcome events being nonnull. Show that $\delta^j = \alpha^j$ for all j in Figures 4.1.1 and 4.1.4.

EXERCISE 5 Experiment. Assume EU for Figure 4.1.5. Throughout, we normalize $U(\alpha^0) = 0$ and $U(\alpha^4) = 1$. Assume the data of Figure 4.1.1, and the implications of EU there. Do not consider your own answers PE^j in Figure 4.1.5. Instead, consider the answers PE^j that EU predicts given $U(\alpha^j) = j/4$ for all j. Show that EU predicts $PE^j = j/4$ for all j. In other words, your answers in Figures 4.1.1 and 4.1.5 violate EU unless $PE^j = j/4$ for all j.

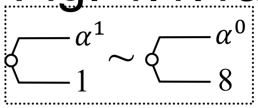
Solution to Exercise 1:

Subjective probabilities

$P(cand_1) = p_1$;

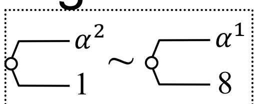
$P(cand_2) = p_2$.

Fig. 4.1.1a: $$p_1 U(\alpha^1) + p_2 U(1) = p_1 \overset{10}{\overset{\parallel}{U(\alpha^0)}} + p_2 U(8)$$



$$p_1 \times \left(U(\alpha^1) - U(\alpha^0)\right) = p_2 \times \left(U(8) - U(1)\right)$$

Fig. 4.1.1b: $$p_1 U(\alpha^2) + p_2 U(1) = p_1 U(\alpha^1) + \mathrm{p}_2 \mathrm{U}(8)$$



$$p_1 \times \left(U(\alpha^2) - U(\alpha^1)\right) = p_2 \times \left(U(8) - U(1)\right)$$

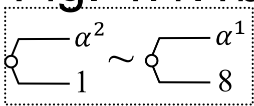
$$U(\alpha^2) - U(\alpha^1) = U(\alpha^1) - U(\alpha^0)$$

We continue and next investigate $U(\alpha^3)$ and $U(\alpha^4)$. As above:

Figs 4.1.1b & 4.1.1c: $U(\alpha^3) - U(\alpha^2) = U(\alpha^2) - U(\alpha^1)$:

Similarly:

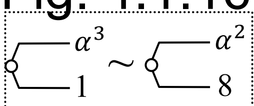
Fig. 4.1.1b:



$$p_1 U(\alpha^2) + p_2 U(1) = p_1 U(\alpha^1) + p_2 U(8)$$

$$p_1 \times \left(U(\alpha^2) - U(\alpha^1)\right) = p_2 \times \left(U(8) - U(1)\right)$$

Fig. 4.1.1c:



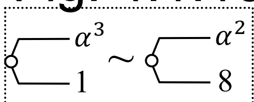
$$p_1 U(\alpha^3) + p_2 U(1) = p_1 U(\alpha^2) + p_2 U(8)$$

$$p_1 \times \left(U(\alpha^3) - U(\alpha^2)\right) = p_2 \times \left(U(8) - U(1)\right)$$

$$U(\alpha^3) - U(\alpha^2) = U(\alpha^2) - U(\alpha^1)$$

Similarly:

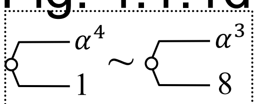
Fig. 4.1.1c:



$$p_1U(\alpha^3) + p_2U(1) = p_1U(\alpha^2) + p_2U(8)$$

$$p_1 \times \left(U(\alpha^3) - U(\alpha^2)\right) = p_2 \times \left(U(8) - U(1)\right)$$

Fig. 4.1.1d:



$$p_1U(\alpha^4) + p_2U(1) = p_1U(\alpha^3) + p_2U(8)$$

$$p_1 \times \left(U(\alpha^4) - U(\alpha^3)\right) = p_2 \times \left(U(8) - U(1)\right)$$

$$U(\alpha^4) - U(\alpha^3) = U(\alpha^3) - U(\alpha^2)$$

Taking all together:

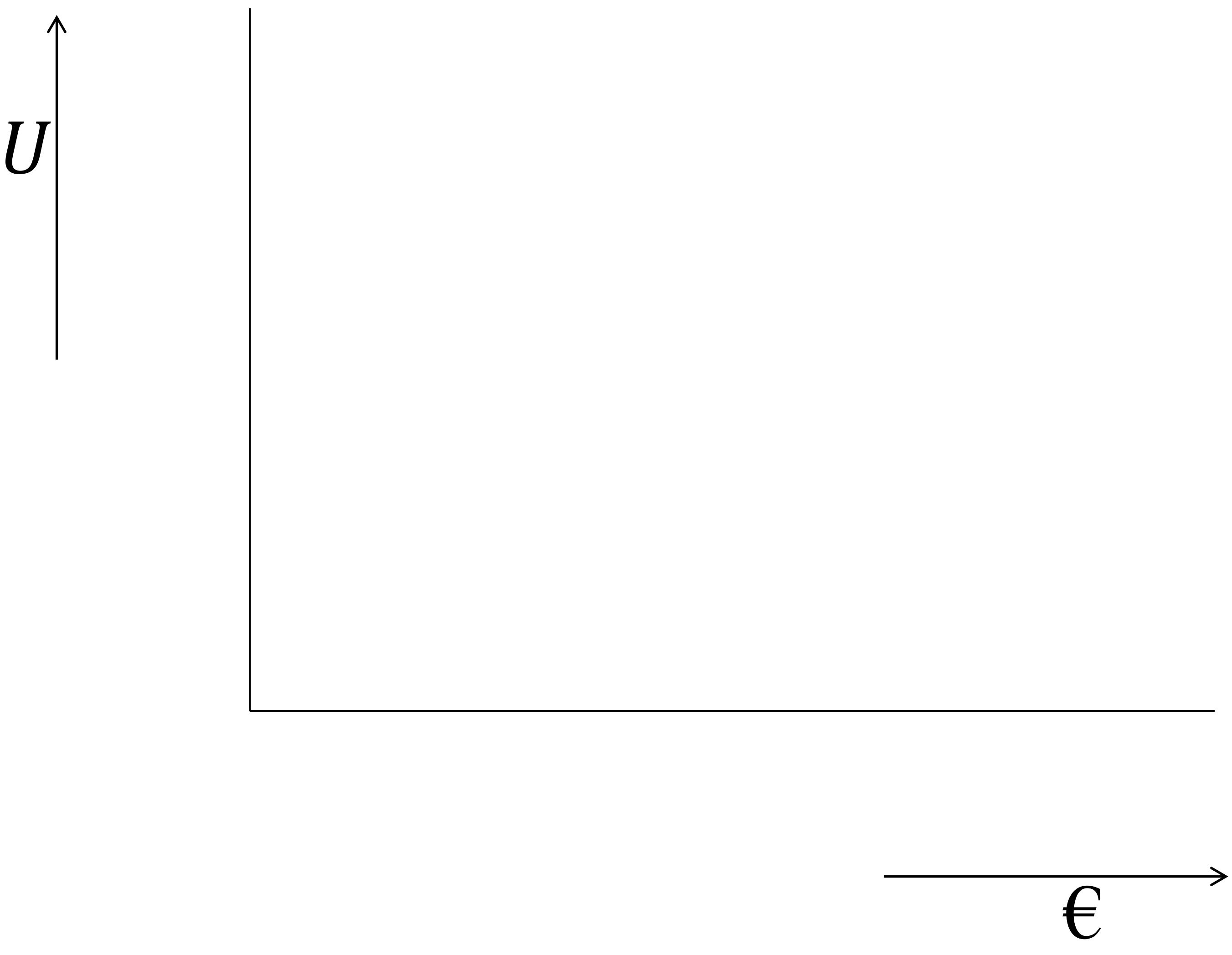
$$U(\alpha^4) - U(\alpha^3) = U(\alpha^3) - U(\alpha^2) = U(\alpha^2) - U(\alpha^1) = U(\alpha^1) - U(\alpha^0)$$

If $U(\alpha^0) = 0, U(\alpha^4) = 1$, then: $U(\alpha^1) = \frac{1}{4}$, $U(\alpha^2) = \frac{2}{4}$, $U(\alpha^3) = \frac{3}{4}$.

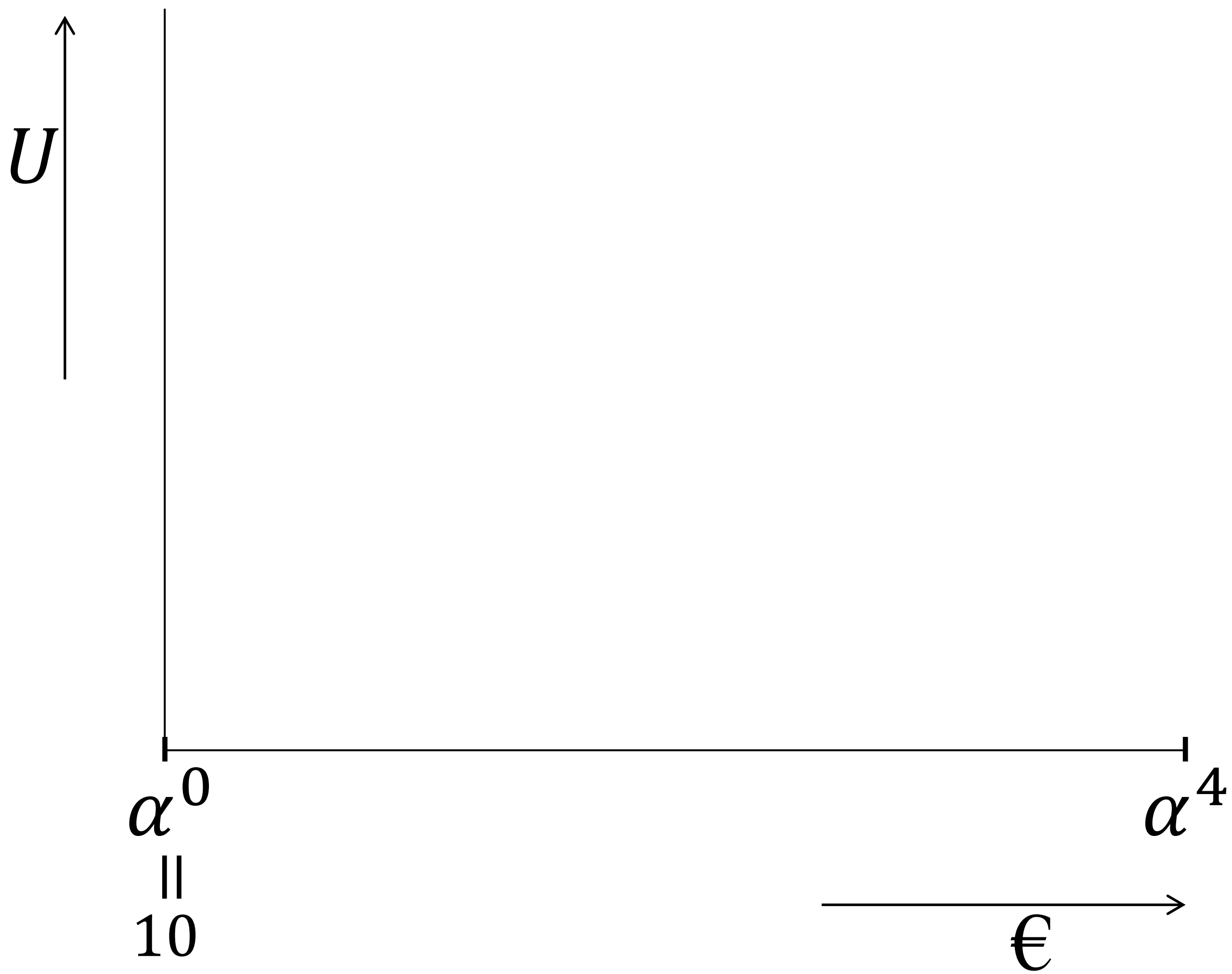
The α's are "equally spaced in utility units."

Now you draw the graph of your U, as follows.
1. Take paper and pencil.

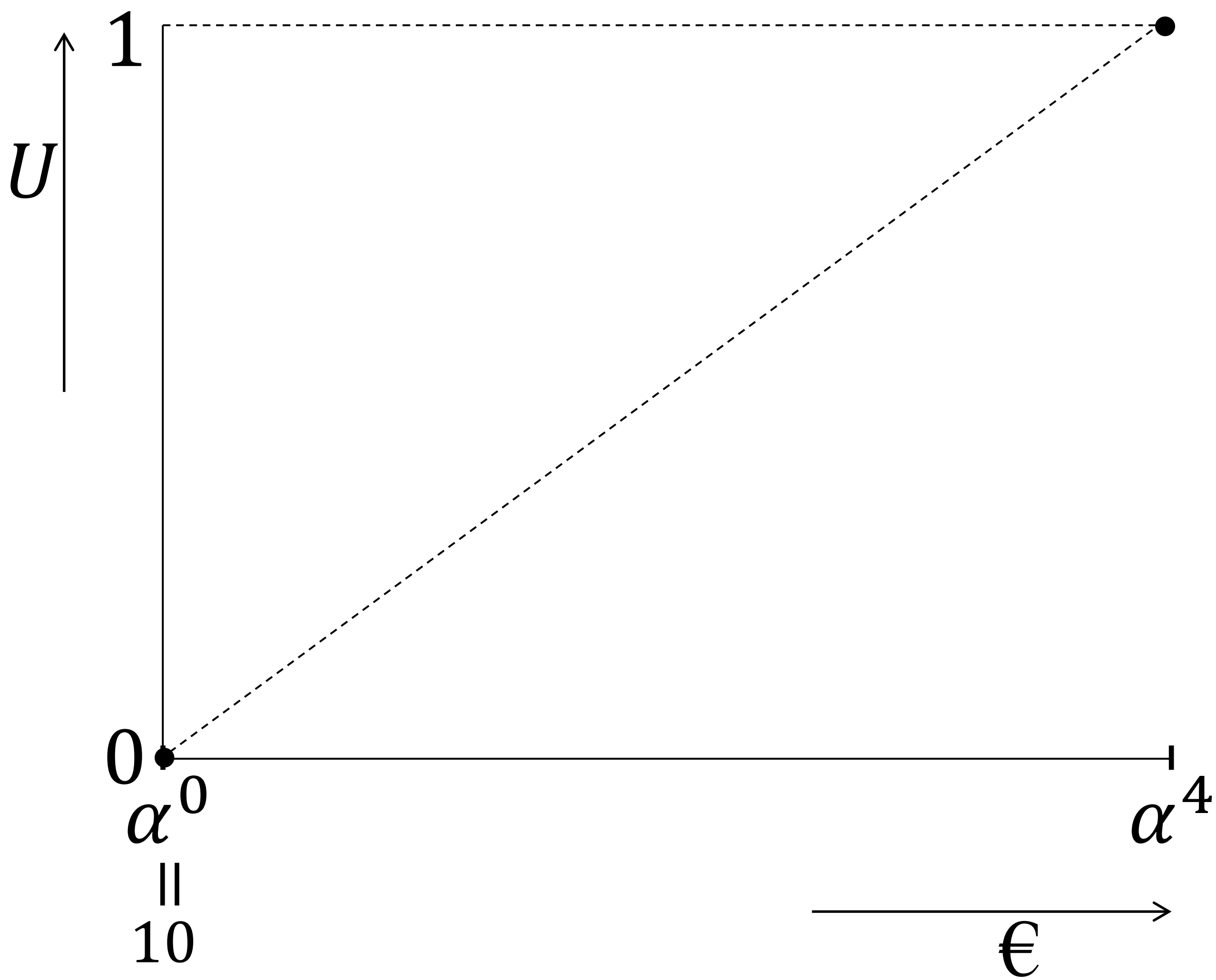
2. Draw axes



3. Write the symbols as below

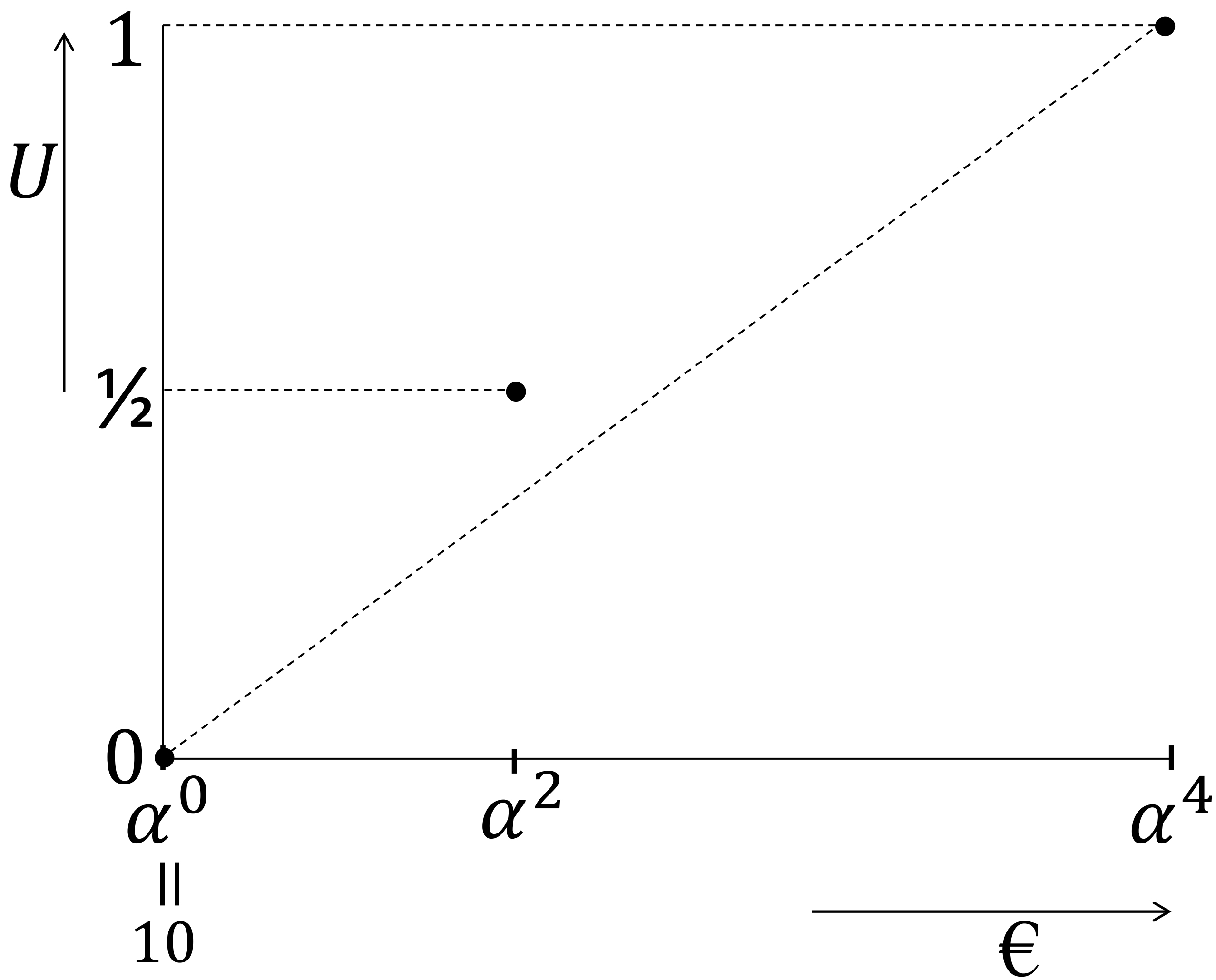


4. Scaling utility



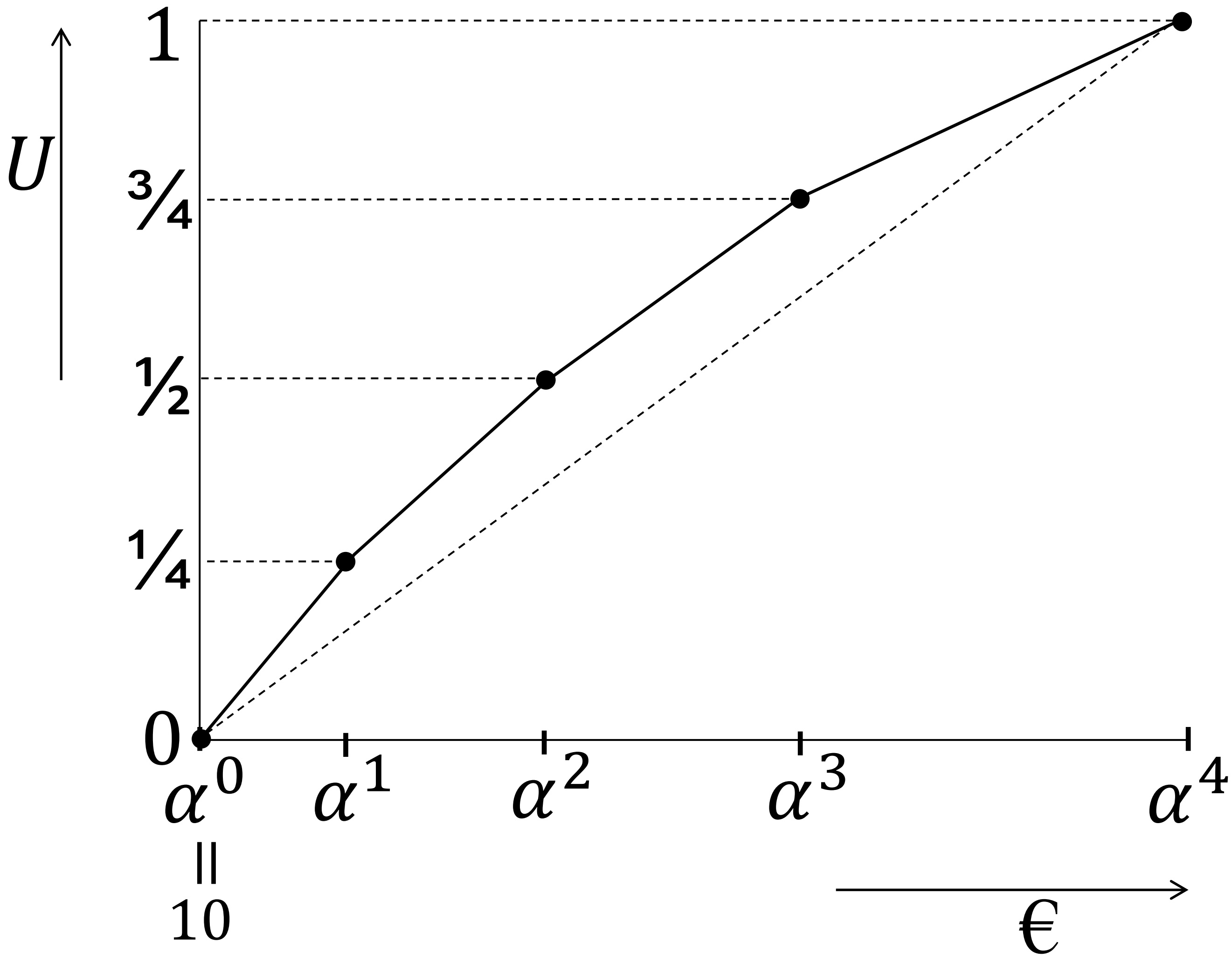
I add the diagonal.

5. Write the symbol α^2 where your own α^2 is.



6. There U is $\frac{1}{2}$.

6. Add α^1 and α^3.

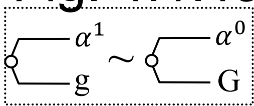


Your utility graph!

Solution to Exercise 2:

Similarly:

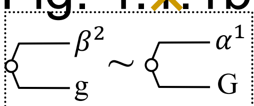
Fig. 4.$\overset{2}{\cancel{1}}$.1a:



$$p_1 U(\alpha^1) + p_2 U(\overset{g}{\cancel{1}}) = p_1 U(\alpha^0) + p_2 U(\overset{G}{\cancel{8}})$$

$$p_1 \times \left(U(\alpha^1) - U(\alpha^o)\right) = p_2 \times \left(U(\overset{G}{\cancel{8}}) - U(\overset{g}{\cancel{1}})\right)$$

Fig. 4.$\overset{2}{\cancel{1}}$.1b:



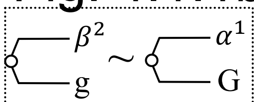
$$p_1 U(\overset{\beta^2}{\cancel{\alpha^2}}) + p_2 U(\overset{g}{\cancel{1}}) = p_1 U(\alpha^1) + p_2 U(\overset{G}{\cancel{8}})$$

$$p_1 \times \left(U(\overset{\beta^2}{\cancel{\alpha^2}}) - U(\alpha^1)\right) = p_2 \times \left(U(\overset{G}{\cancel{8}}) - U(\overset{g}{\cancel{1}})\right)$$

$$U(\overset{\beta^2}{\cancel{\alpha^2}}) - U(\alpha^1) = U(\alpha^1) - U(\alpha^0)$$

Similarly:

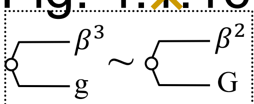
Fig. 4.$\overset{2}{\cancel{1}}$.1b:



$$p_1 U(\overset{\beta^2}{\cancel{\alpha^2}}) + p_2 U(\overset{g}{\cancel{1}}) = p_1 U(\alpha^1) + p_2 U(\overset{G}{\cancel{8}})$$

$$p_1 \times \left(U(\overset{\beta^2}{\cancel{\alpha^2}}) - U(\alpha^1)\right) = p_2 \times \left(U(\overset{G}{\cancel{8}}) - U(\overset{g}{\cancel{1}})\right)$$

Fig. 4.$\overset{2}{\cancel{1}}$.1c:



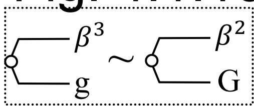
$$p_1 U(\overset{\beta^3}{\cancel{\alpha^3}}) + p_2 U(\overset{g}{\cancel{1}}) = p_1 U(\overset{\beta^2}{\cancel{\alpha^2}}) + p_2 U(\overset{G}{\cancel{8}})$$

$$p_1 \times \left(U(\overset{\beta^3}{\cancel{\alpha^3}}) - U(\overset{\beta^2}{\cancel{\alpha^2}})\right) = p_2 \times \left(U(\overset{G}{\cancel{8}}) - U(\overset{g}{\cancel{1}})\right)$$

$$U(\overset{\beta^3}{\cancel{\alpha^3}}) - U(\overset{\beta^2}{\cancel{\alpha^2}}) = U(\overset{\beta^2}{\cancel{\alpha^2}}) - U(\alpha^1)$$

Similarly:

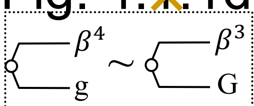
Fig. 4.$\overset{2}{\cancel{1}}$.1c:



$$p_1 U(\overset{\beta^3}{\cancel{\alpha^3}}) + p_2 U(\overset{g}{\cancel{1}}) = p_1 U(\overset{\beta^2}{\cancel{\alpha^2}}) + p_2 U(\overset{G}{\cancel{8}})$$

$$p_1 \times \left(U(\overset{\beta^3}{\cancel{\alpha^3}}) - U(\overset{\beta^2}{\cancel{\alpha^2}})\right) = p_2 \times \left(U(\overset{G}{\cancel{8}}) - U(\overset{g}{\cancel{1}})\right)$$

Fig. 4.$\overset{2}{\cancel{1}}$.1d:



$$p_1 U(\overset{\beta^4}{\cancel{\alpha^4}}) + p_2 U(\overset{g}{\cancel{1}}) = p_1 U(\overset{\beta^3}{\cancel{\alpha^3}}) + p_2 U(\overset{G}{\cancel{8}})$$

$$p_1 \times \left(U(\overset{\beta^4}{\cancel{\alpha^4}}) - U(\overset{\beta^3}{\cancel{\alpha^3}})\right) = p_2 \times \left(U(\overset{G}{\cancel{8}}) - U(\overset{g}{\cancel{1}})\right)$$

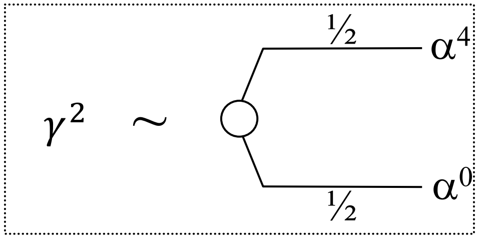
$$U(\overset{\beta^4}{\cancel{\alpha^4}}) - U(\overset{\beta^3}{\cancel{\alpha^3}}) = U(\overset{\beta^3}{\cancel{\alpha^3}}) - U(\overset{\beta^2}{\cancel{\alpha^2}})$$

Solution to Exercise 3.

$$U(\gamma^2) = \frac{1}{2}U(\alpha^4) + \frac{1}{2}U(\alpha^0) = U(\alpha^2);$$

$$\gamma^2 = \alpha^2$$

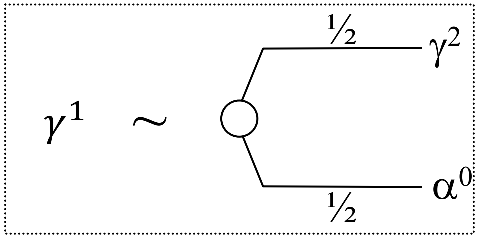
FIG. 4.1.3a.



$$U(\gamma^1) = \frac{1}{2}U(\gamma^2) + \frac{1}{2}U(\alpha^0) = \frac{1}{2}U(\alpha^2) + \frac{1}{2}U(\alpha^0) = U(\alpha^1);$$

$$\gamma^1 = \alpha^1$$

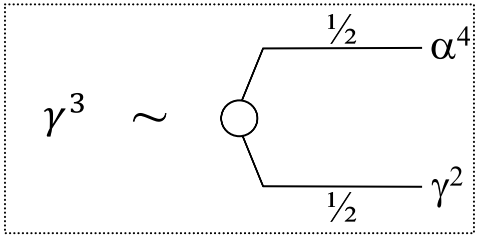
FIG. 4.1.3b.



$$U(\gamma^3) = \frac{1}{2}U(\alpha^4) + \frac{1}{2}U(\gamma^2) = \frac{1}{2}U(\alpha^4) + \frac{1}{2}U(\alpha^2) = U(\alpha^3);$$

$$\gamma^3 = \alpha^3$$

FIG. 4.1.3c.



The γ's are just another way of measuring the same as the α's.

Solution to Exercise 4.

Now the δ's, from Figure 4.1.4.

Below: Figure 4.1.1 with:
upper/lower panels interchanged
left/right prospects interchanged.

FIG. 4.1.4a

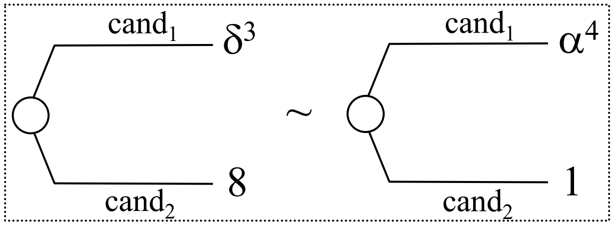


Fig. 4.1.4a: compare Fig. 4.1.1d.
All the same except δ^3.
Hence, $\delta^3 = \alpha^3$ must be.

FIG. 4.1.1d

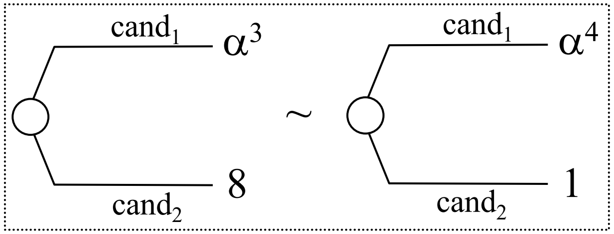


FIG. 4.1.4b

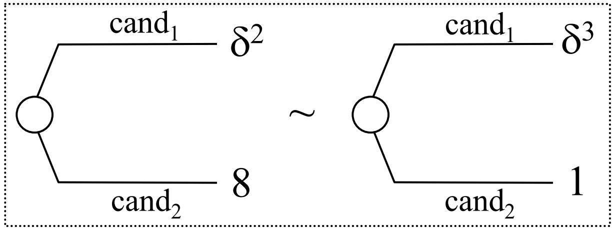


Fig. 4.1.4b: compare Fig. 4.1.1c.
Given that $\delta^3 = \alpha^3$, we must have $\delta^2 = \alpha^2$.

FIG. 4.1.1c

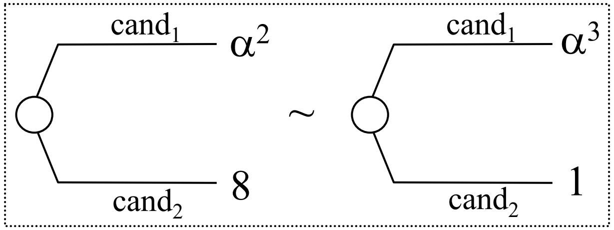


FIG. 4.1.4c

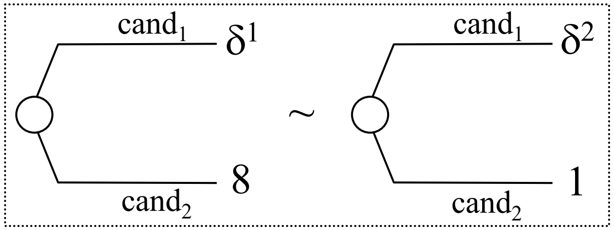


Fig. 4.1.4c: compare Fig. 4.1.1b.
Given that $\delta^2 = \alpha^2$, we must have $\delta^1 = \alpha^1$.

FIG. 4.1.1b

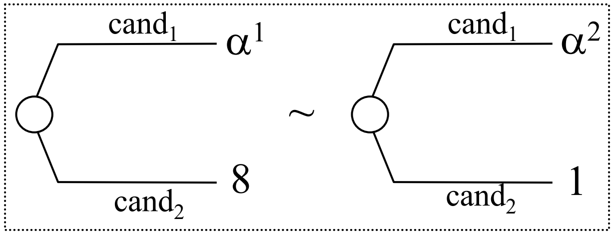


FIG. 4.1.4d

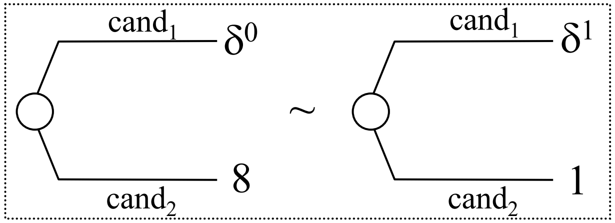
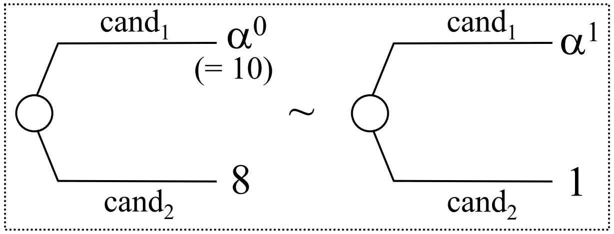


Fig. 4.1.4d: compare Fig. 4.1.1a.
Given that $\delta^1 = \alpha^1$, we must have $\delta^0 = \alpha^0$.

FIG. 4.1.1a



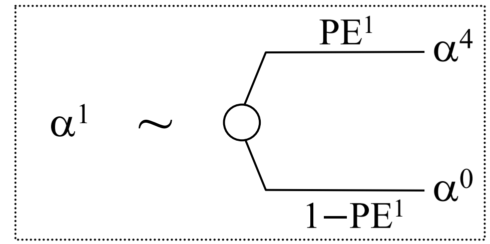
The δ's are just another way of measuring the same as the α's.

Solution to Exercise 5

We may set $U(\alpha^0\,) = 0, U\left(\alpha^j\right) = \frac{j}{4}$. Then:

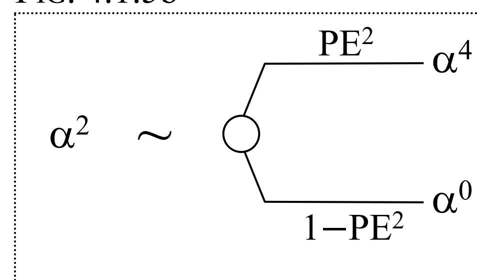
$$U(\alpha^1) = PE^1 U(\alpha^4) + (1 - PE^1)U(\alpha^0);$$
$$\frac{1}{4} = PE^1 \times 1 + (1 - PE^1) \times 0 = PE^1.$$

FIG. 4.1.5a



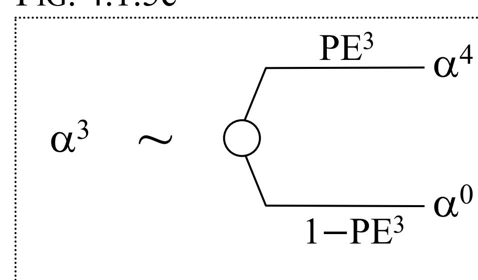
$$U(\alpha^2) = PE^2 U(\alpha^4) + (1 - PE^2)U(\alpha^0);$$
$$\frac{2}{4} = PE^2 \times 1 + (1 - PE^2) \times 0 = PE^2.$$

FIG. 4.1.5b



$$U(\alpha^3) = PE^3 U(\alpha^4) + (1 - PE^3)U(\alpha^0);$$
$$\frac{3}{4} = PE^3 \times 1 + (1 - PE^3) \times 0 = PE^3.$$

FIG. 4.1.5c



Common findings

Table 4.11.2. Statistical tests of equalities

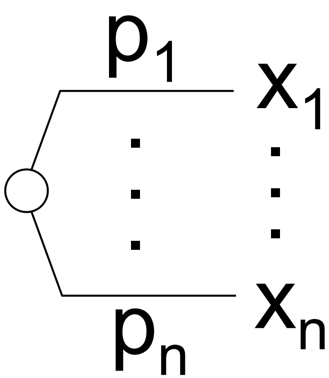
	α^0	α^1	β^2	β^3	β^4	α^0	γ^1	γ^2	γ^3	α^4	δ^0	δ^1	δ^2	δ^3	α^4	PE^1	PE^2	PE^3
α^0	=					=					*							
α^1		=					*					*				ns		
α^2			ns					*					ms				*	
α^3				ns					*					ns				*
α^4					ns					=					=			

* significant

Ch. 5 (reorganized relative to book, with different division over sections)

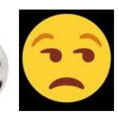
Simplest way to evaluate risky prospects:

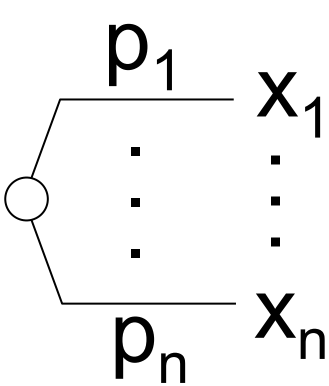
Expected value (C. Huygens 1657)



$$\to \quad p_1x_1 + \cdots + p_nx_n$$

Violated by risk aversion:

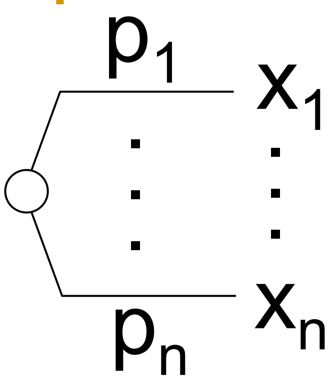




$$\prec \quad p_1x_1 + \cdots + p_nx_n$$

Bernoulli (1738):

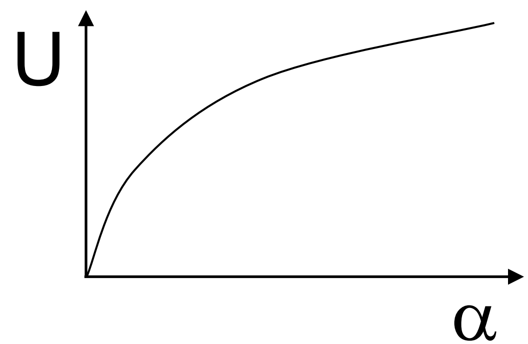
Expected utility (EU)



$$\rightarrow \quad p_1U(x_1) + \cdots + p_nU(x_n)$$

Theorem. EU: Risk aversion $\Leftrightarrow$ U concave

U concave:



Measure of risk aversion: $-U''/U'$ (Pratt & Arrow).

Other often-used index of risk aversion: $-\alpha U''/U'$.

Intuitive problem:

Risk aversion $\Leftrightarrow$ U concave:
travel back in time/memory when first heard.
U reflects value of money;
not risk !?

U determined by specific nature of outcomes.
Different for
 # years to live;
 # hours of listening to music;
 # liters of wine;
 …
nonquantitative outcomes (health states) … ?

Lopes (1987):

"Risk attitude is more than the psychophysics of money."

Criticisms of the EU formula can be found way earlier:

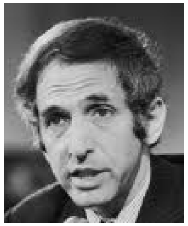
D'Alembert (1768) "*Opuscules Mathématiques*," vol. iv., (extraits de lettres).

"Il me sembloit [in reading Bernoulli's Ars Conjectandi] que cette matière avoit besoin d'être traitée d'une manière plus claire; je voyois bien que l'espérance étoit plus grande, 1^0 que la somme espérée étoit plus grande, 2^0 que la probabilité de gagner l'étoit aussi. Mais je ne voyois pas avec la même évidence, et je ne le vois pas encore, 1^0 que la probabilité soit estimée exactement par les méthodes utilisées; 2^0 que quand elle le seroit, l'espérance doive être proportionnelle à cette probabilité simple, plutôt qu'à une puissance ou même à une fonction de cette probabilité; 3^0 que quand il y a plusieurs combinaisons qui donnent différens avantages ou différens risques (qu'on regarde comme des avantages négatifs) il faille se contenter d'*ajouter* simplement ensemble toutes les espérances pour avoir l'espérance totale." [italics from the original]

Tversky (1975): 

"In utility theory [EU], risk aversion is explained by the concavity of the utility function for money. Once the monetary scale is properly transformed—no risk aversion remains. (In this respect it is somewhat misleading to refer to the measurement of the utility for money as 'the measurement or attitudes towards risk'. One's utility function reflects one's attitude towards money, not towards risk. Risk aversion is an epiphenomenon in utility theory.)"

Empirical problems:



Plentiful (Allais, Ellsberg)

Inconsistencies in utility measurements.

One more (Rabin 2000):
For small amounts EU ≈ EV.
However, empirically not so!

Outline:

§1. Expected Utility and Risk Aversion;

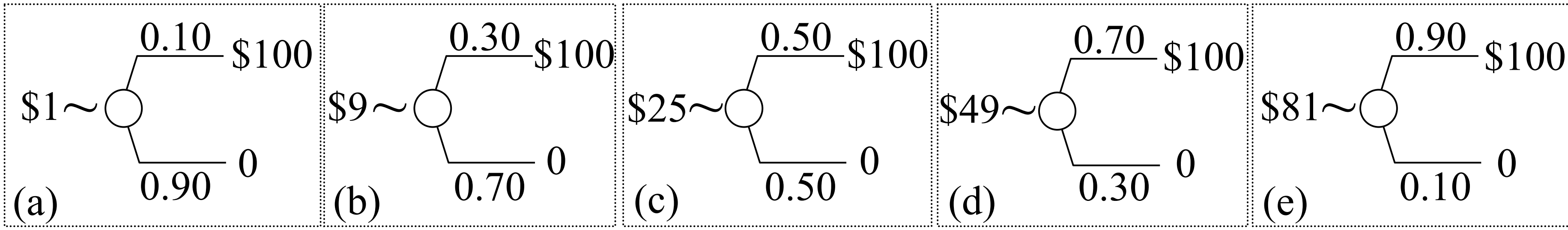
§2. The Valuable Intuition of Probabilistic Sensitivity (Deviating from EU);

§3. The Old Theory on Probabilistic Sensitivity for Multiple Outcomes and why It Is Wrong;

§4. Quiggin/Schmeidler Rank-Dependent Theory on Probabilistic Sensitivity for Multiple Outcomes and why It Is Natural;

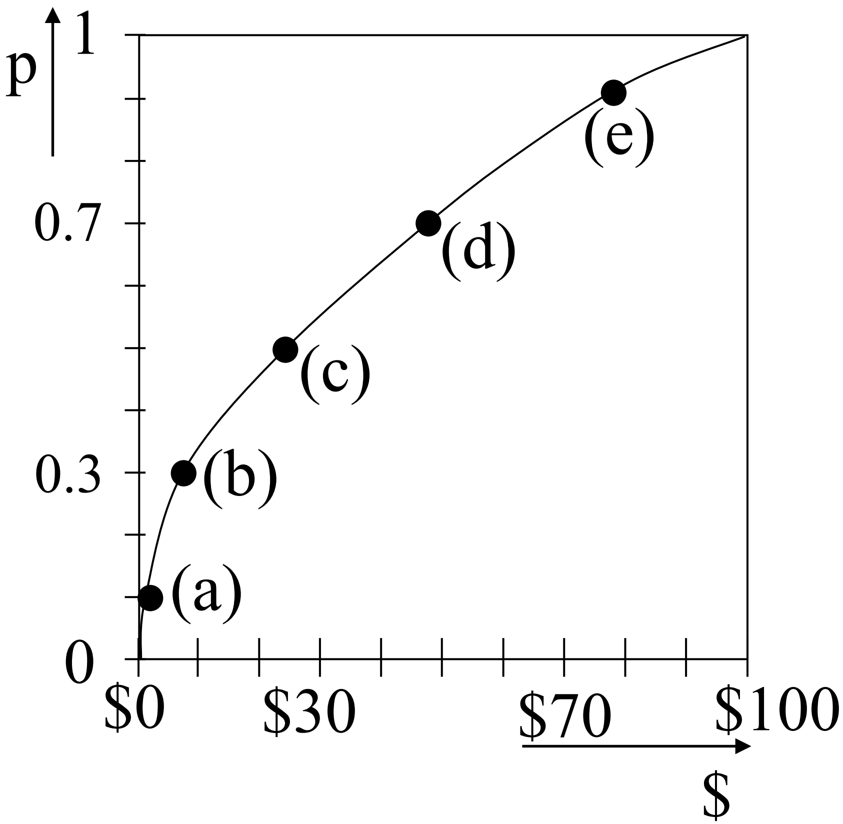
§5. Applications to Modeling Risk.

Assume following data:



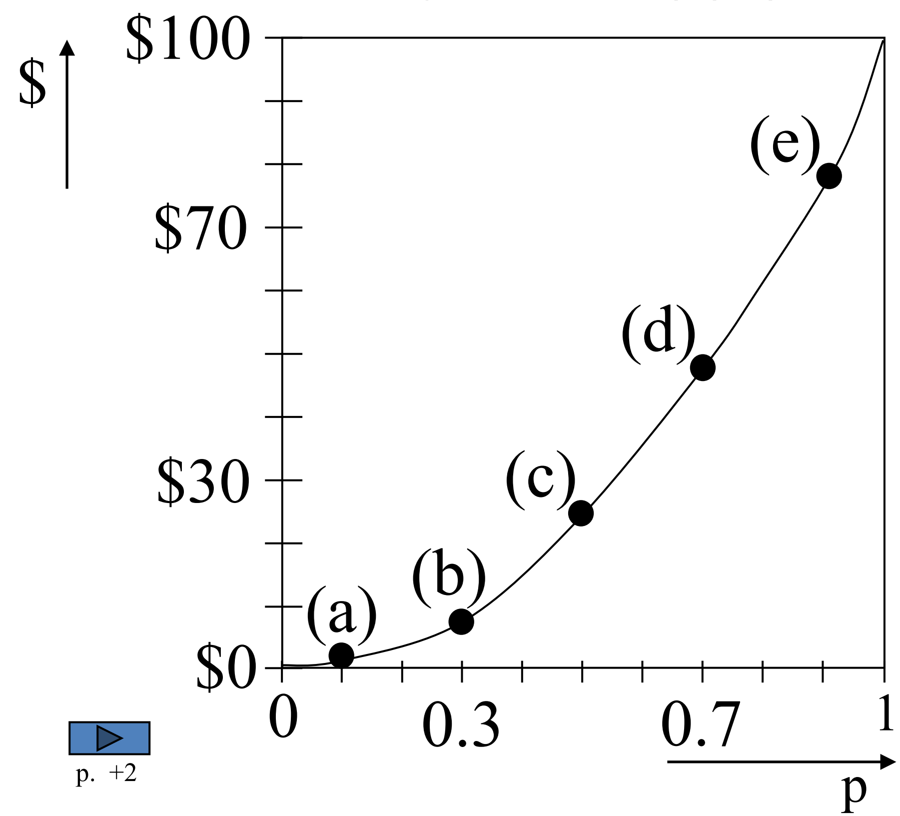
EU: U(α) = pU(100) = p.

Below: is graph of U!



Psychology: α = w(p)100

Below is graph of w(p)! ($\times$ 100)



p. +2

next p.

Psychologists (Lopes etc.): What economists do with money, is better done with probabilities! Risk attitude has more to do with probabilities.  

Economists

p: α; $1-p$: 0 $\rightarrow pU(\alpha)$

Psychologists

p: α; $1-p$: 0 $\rightarrow w(p)\alpha$

w increasing,
$w(0) = 0$,
$w(1) = 1$.

Joint

p: α; $1-p$: 0 $\rightarrow w(p)U(\alpha)$

At first, for simplicity, we consider linear U for Ψs. Is OK for moderate money amounts.

p. U/w graph

Outline:

§1. Expected Utility and Risk Aversion;

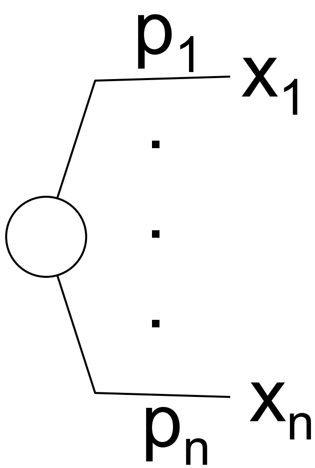
§2. The Valuable Intuition of Probabilistic Sensitivity (Deviating from EU);

§3. The Old Theory on Probabilistic Sensitivity for Multiple Outcomes and why It Is Wrong;

§4. Quiggin/Schmeidler Rank-Dependent Theory on Probabilistic Sensitivity for Multiple Outcomes and why It Is Natural;

§5. Applications to Modeling Risk.

More general, more outcomes:



$$\rightarrow \quad w(p_1)U(x_1) + \cdots + w(p_n)U(x_n)!?$$

To explain what follows in a simple way, we assume U linear.
(What follows holds in fact for general U.)
Above formula is old (Edwards 1954).

Revived by prospect theory (Kahneman & Tversky 1979).

However, problem:

To explain,

Say:

w is not identity,

so <u>not</u> $w(p) = p$ for all p.

Then w is nonlinear.

Then, for some p_1, p_2

$$w(p_1 + p_2) \neq w(p_1) + w(p_2)$$

(take my word).

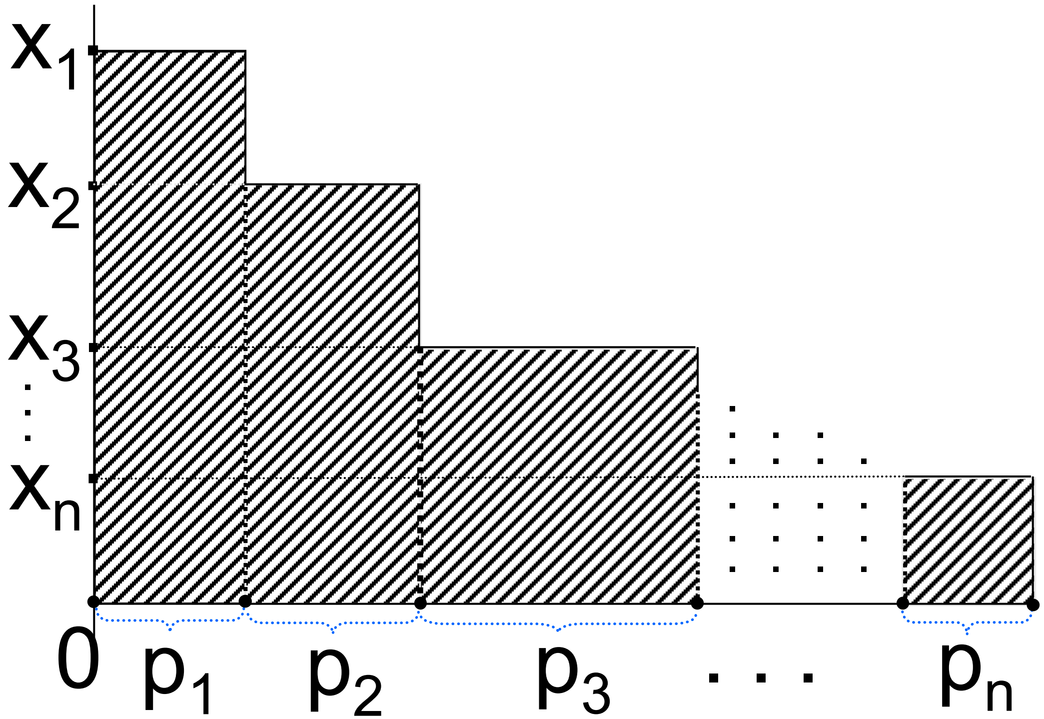
Case 1. $w(p_1 + p_2) > w(p_1) + w(p_2)$.

Something will go wrong (similarly later for Case 2 with <).

- We will consider the theoretical value of a prospect;
- Will change an outcome to see what happens;
- An anomaly will result.

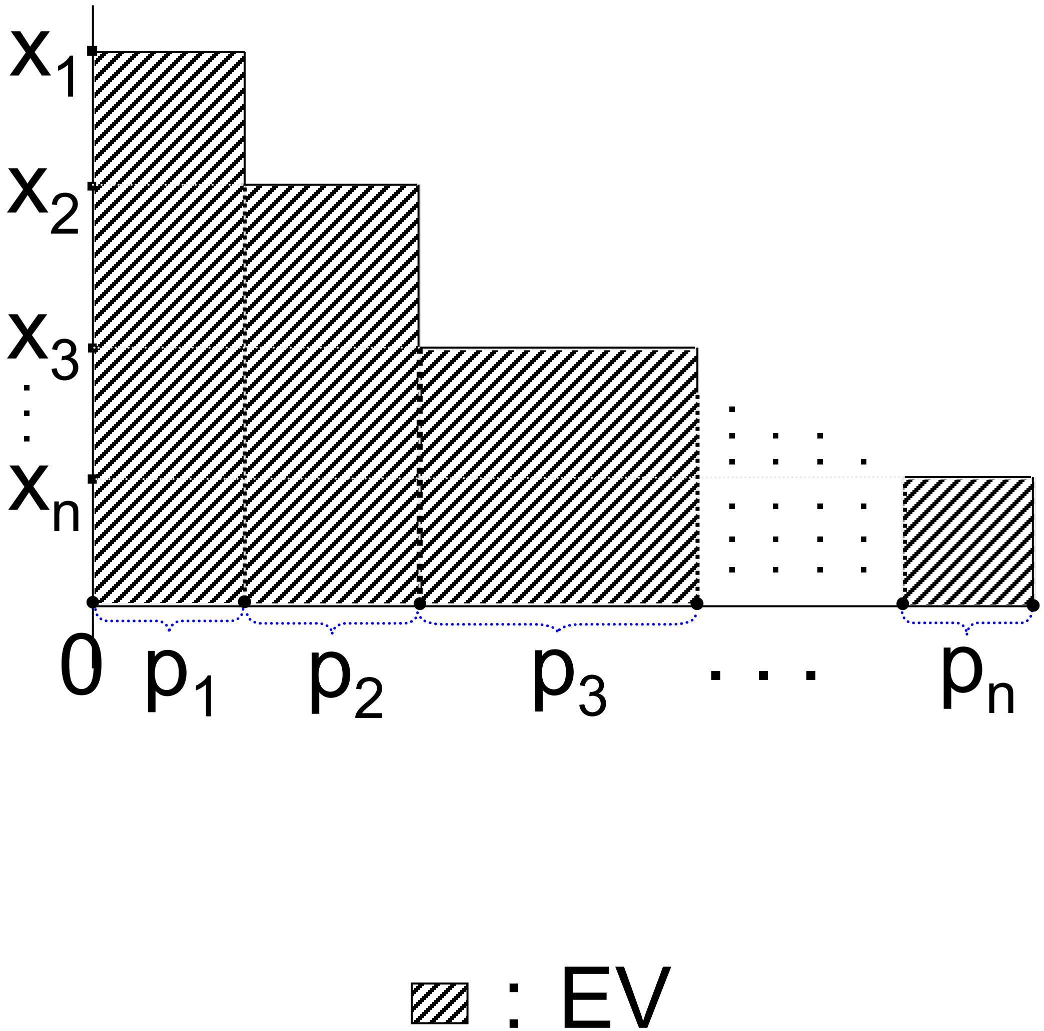
First we repeat well-known things from expected value (EV) and expected utility (EU).

Figure * (for $x_1 > \cdots > x_n \geq 0$;
with p_1 and p_2 as above).

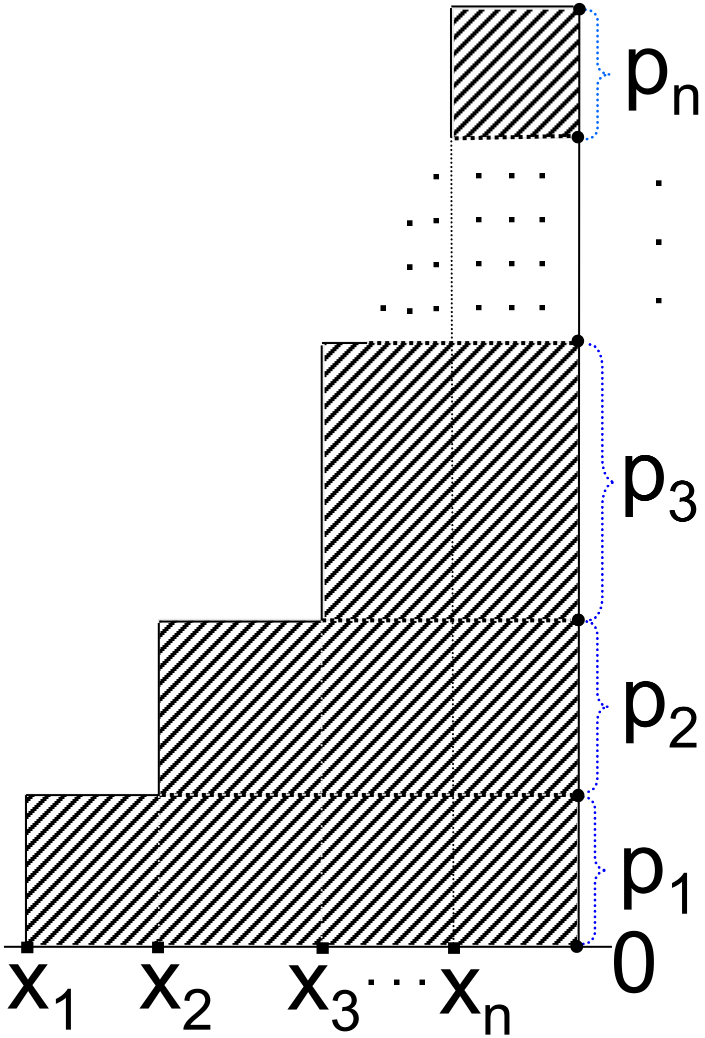


EV, $p_1x_1 + \cdots + p_nx_n$ is area .

Rotating and flipping



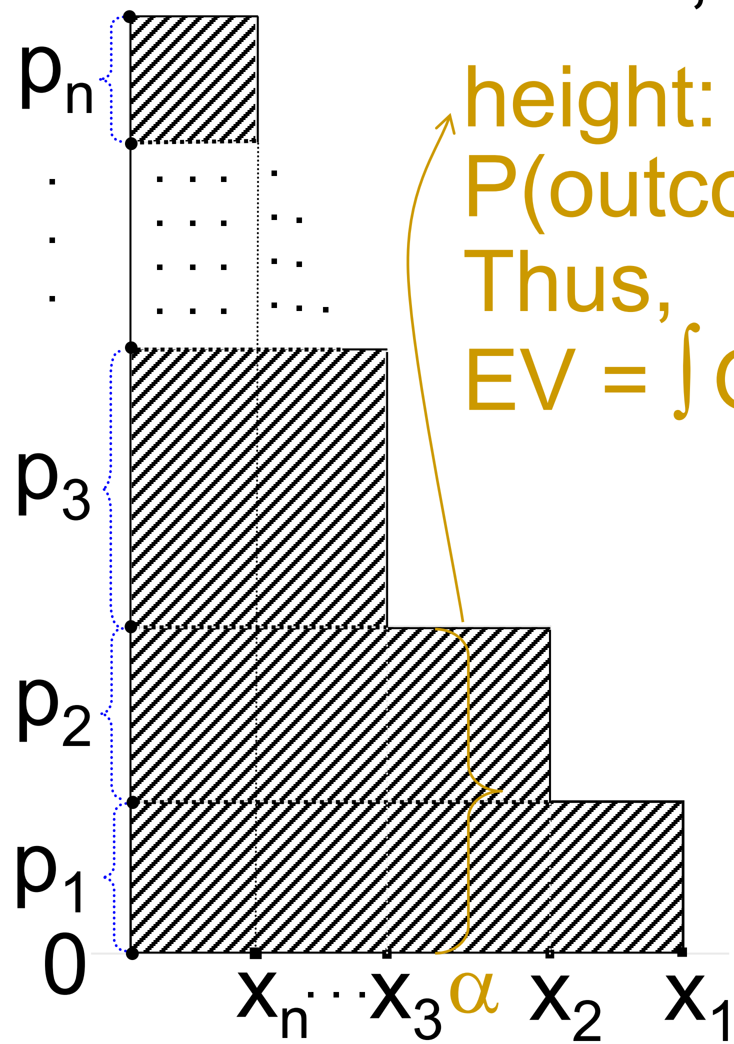
Rotating left



Flipping horizontally

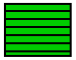
called **rank** of α, $r(\alpha)$

height: $G(\alpha)$ = $P(\text{outcome} > \alpha)$.
Thus,
$EV = \int G(\alpha)d\alpha$.

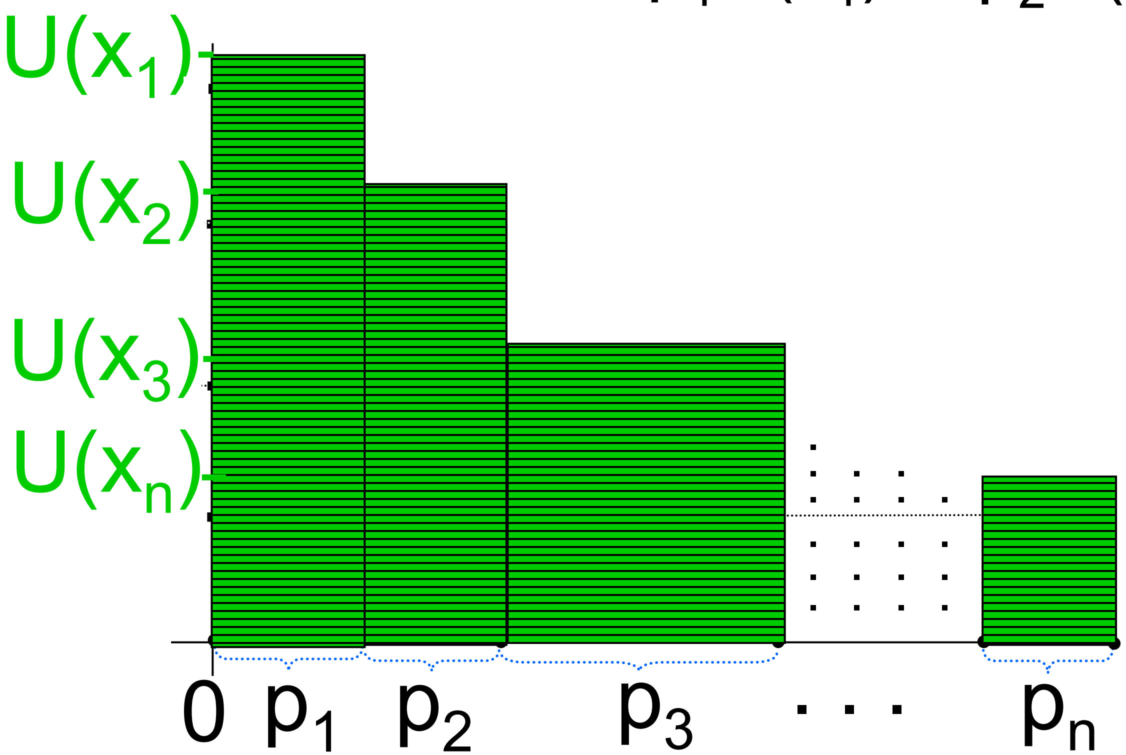


▨ : EV

Figure ** (Preparatory illustration of EU)

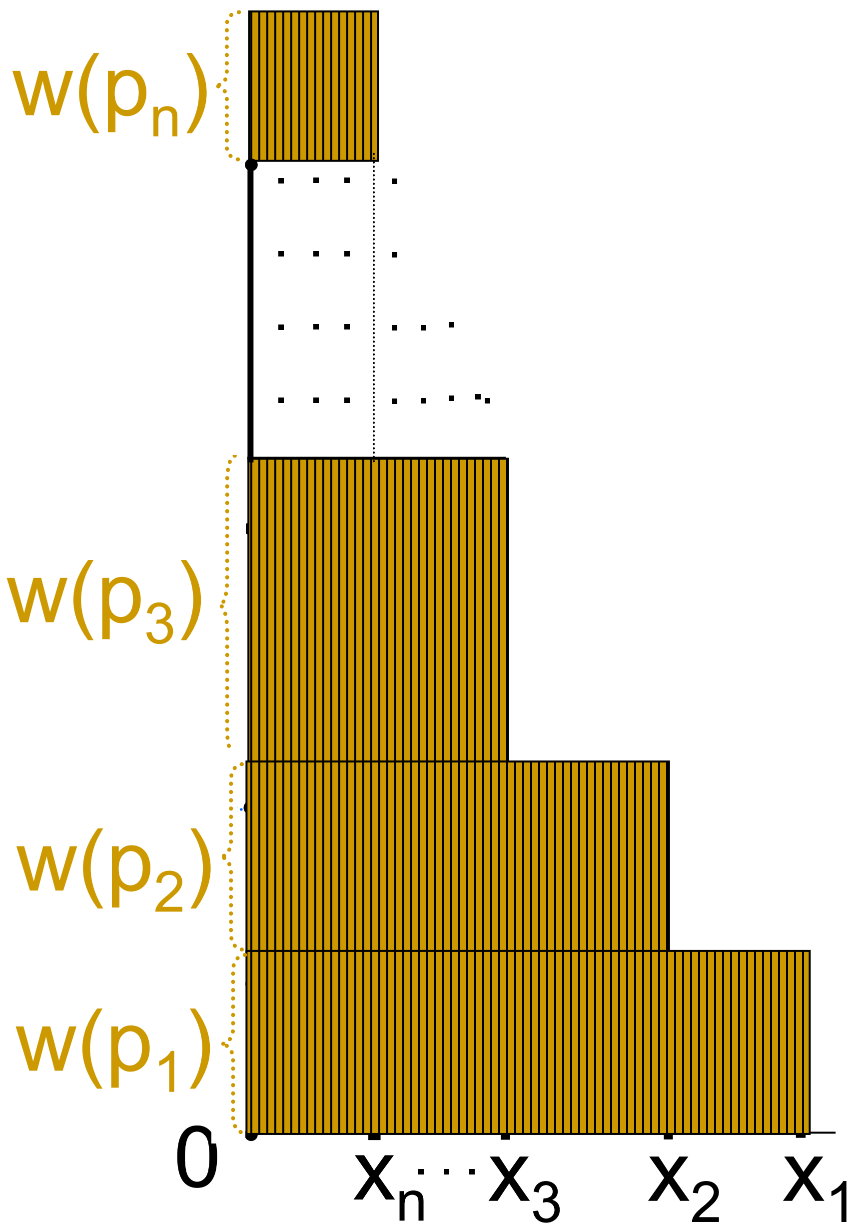
EU is area :

$p_1U(x_1) + p_2U(x_2) + \cdots + p_nU(x_n)$



EU: we transform heights of columns (distances from x_j "all the way down" to the x-axis).

Figure ***



Transforming probabi-lities of fixed outcomes (the old—wrong—way).

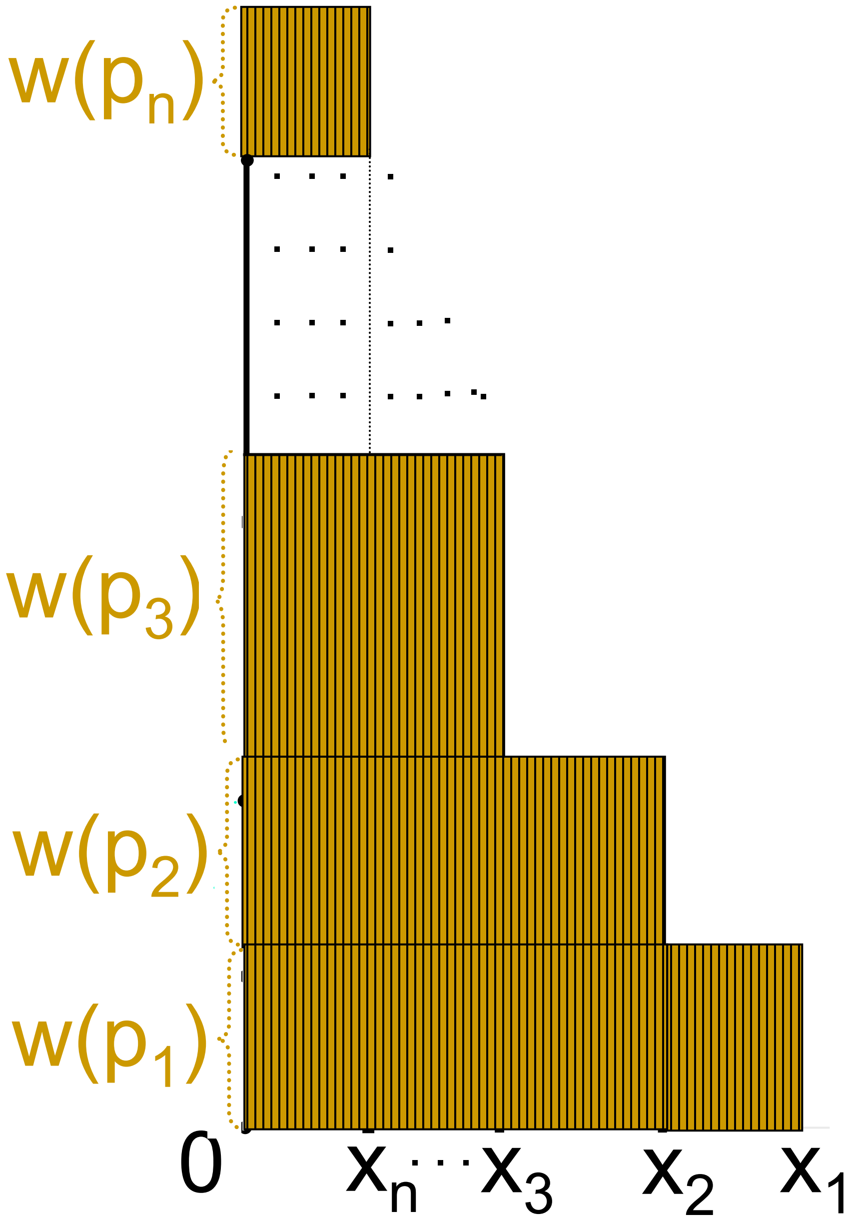
Value of prospect is

$w(p_1)x_1 \;+\; w(p_2)x_2$
$+\; w(p_3)x_3 + \cdots + w(p_n)x_n$
is area :

We have transformed height of each row/layer (distance from endpoint down to its lower neighbor).

Now we “play” with x_1 and see if the old evaluation behaves well. We reduce x_1.

Figure ****



Transforming probabilities of fixed outcomes (the old—wrong—way).

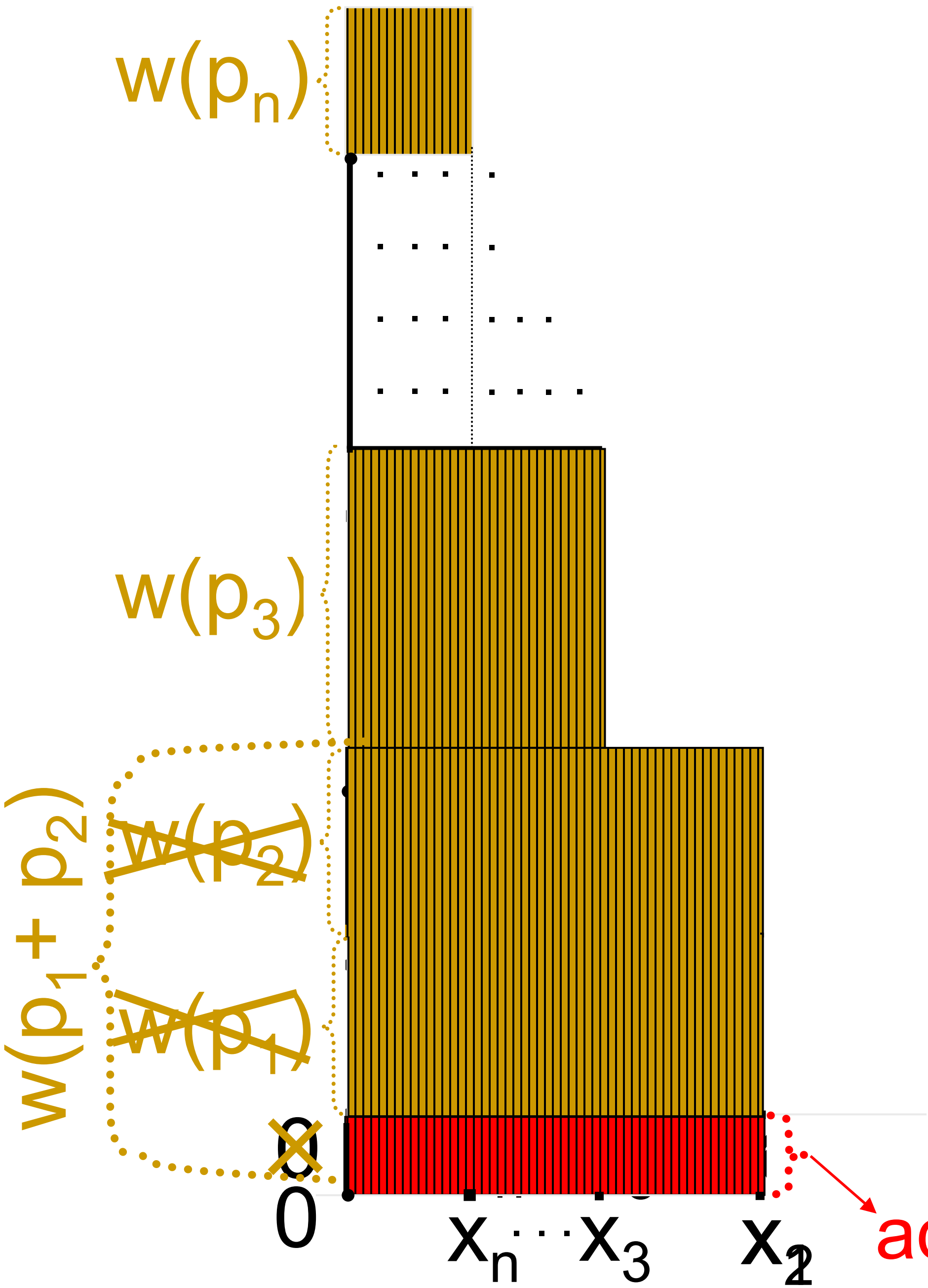
Value of prospect is

$$w(p_1)x_1 + w(p_2)x_2 + w(p_3)x_3 + \cdots + w(p_n)x_n$$

is area :

We have transformed height of each row/layer (distance from endpoint down to its lower neighbor).

Figure ****



Transforming probabilities of fixed outcomes (the old—wrong—way).

Value of prospect is

$$w(p_1+p_2)x_1$$

$$\cancel{w(p_1)x_1 + w(p_2)x_2}$$

$$+ \; w(p_3)x_3 + \cdots + w(p_n)x_n$$

is area :

We have transformed height of each row/layer (distance from endpoint down to its lower neighbor).

additional area!!!

Case 2. $w(p_1 + p_2) < w(p_1) + w(p_2)$.

Similar problems.
Now move x_2 up towards x_1, until it hits x_1:
a sudden implosion of area, with
- discontinuity;
- increasing x_2 may decrease value.

Conclusion. Transforming probabilities in old way is unsound.

Old way does not work.
First discovered: Fishburn (1978). 
Also by: Kahneman & Tversky (1979).

Taking stock of end 1970s:

1. Good psychological intuition that
   risk attitude ⇔ probabilistic sensitivity.

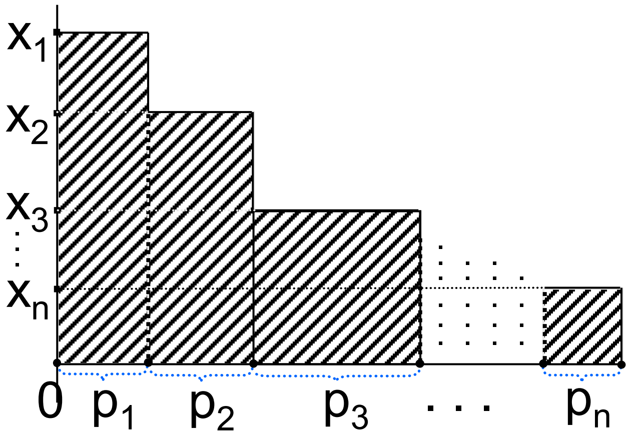
But

2. No theory to do it.  

Then came the Quiggin & Schmeidler
"rank-dependent" idea:

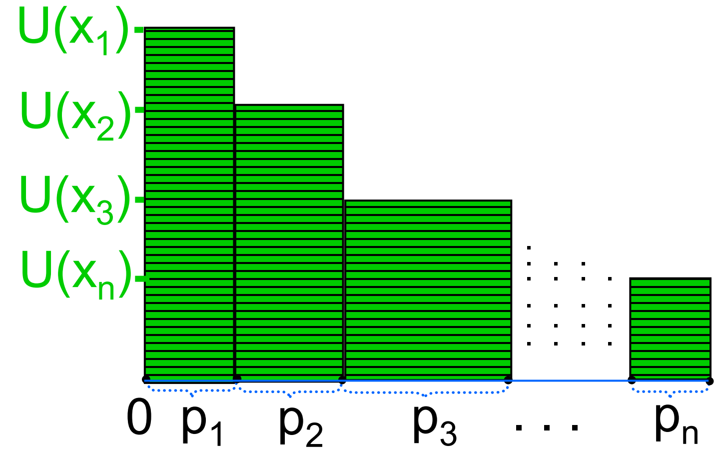
Outline:

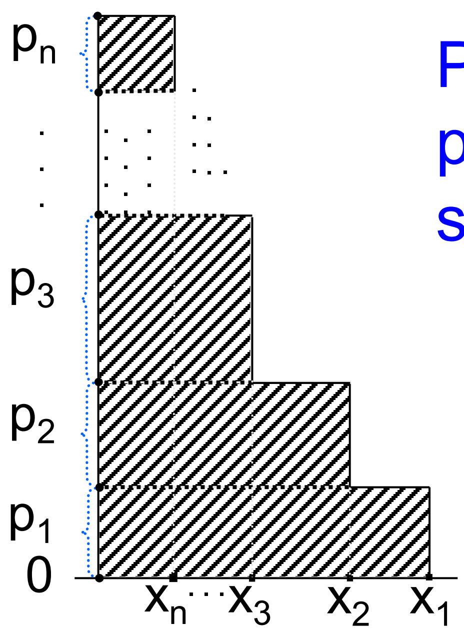
§1. Expected Utility and Risk Aversion;
§2. The Valuable Intuition of Probabilistic Sensitivity (Deviating from EU);
§3. The Old Theory on Probabilistic Sensitivity for Multiple Outcomes and why It Is Wrong;
§4. Quiggin/Schmeidler Rank-Dependent Theory on Probabilistic Sensitivity for Multiple Outcomes and why It Is Natural;
§5. Applications to Modeling Risk.



Economists' outcome sensitivity (EU)

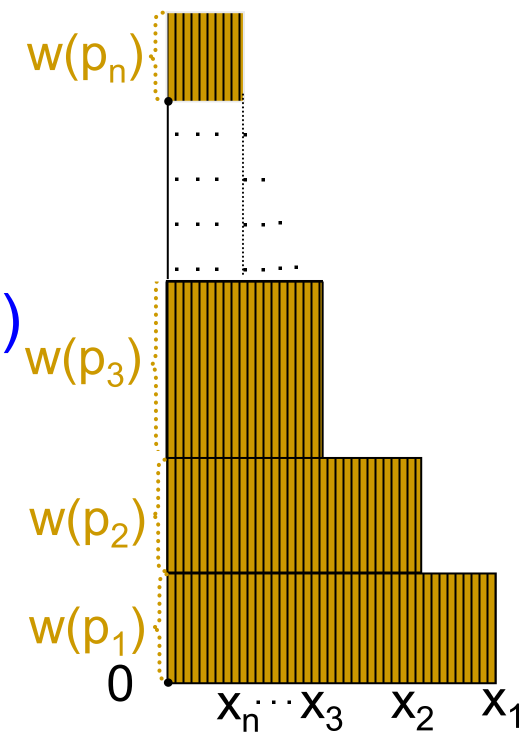
$\Rightarrow$





Psychologists' probabilistic sensitivity ("OPT")

$\Rightarrow$



Quiggin (1982) & Schmeidler (1989): Why not do the same in the probability dimension as in the outcome dimension?

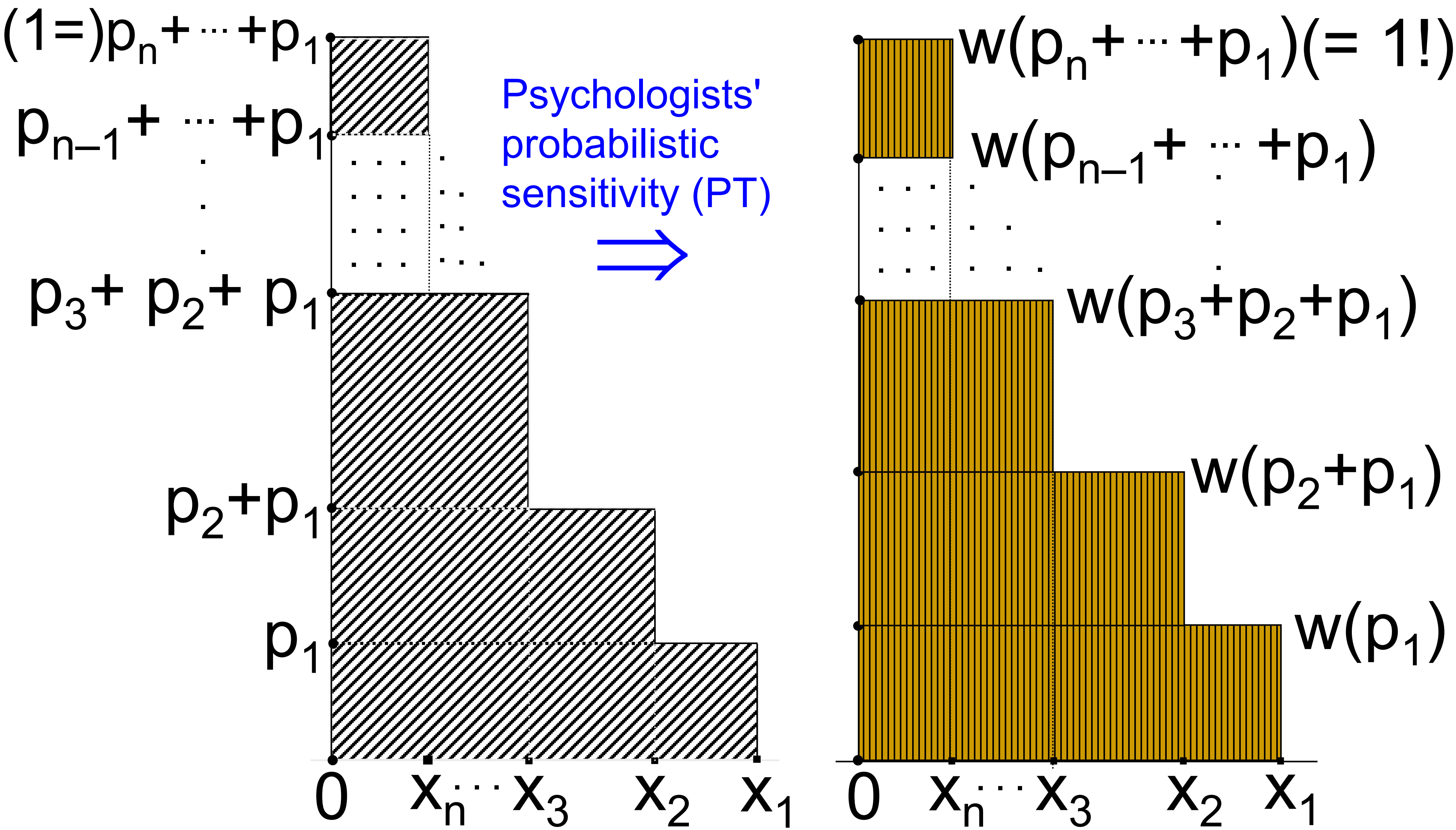
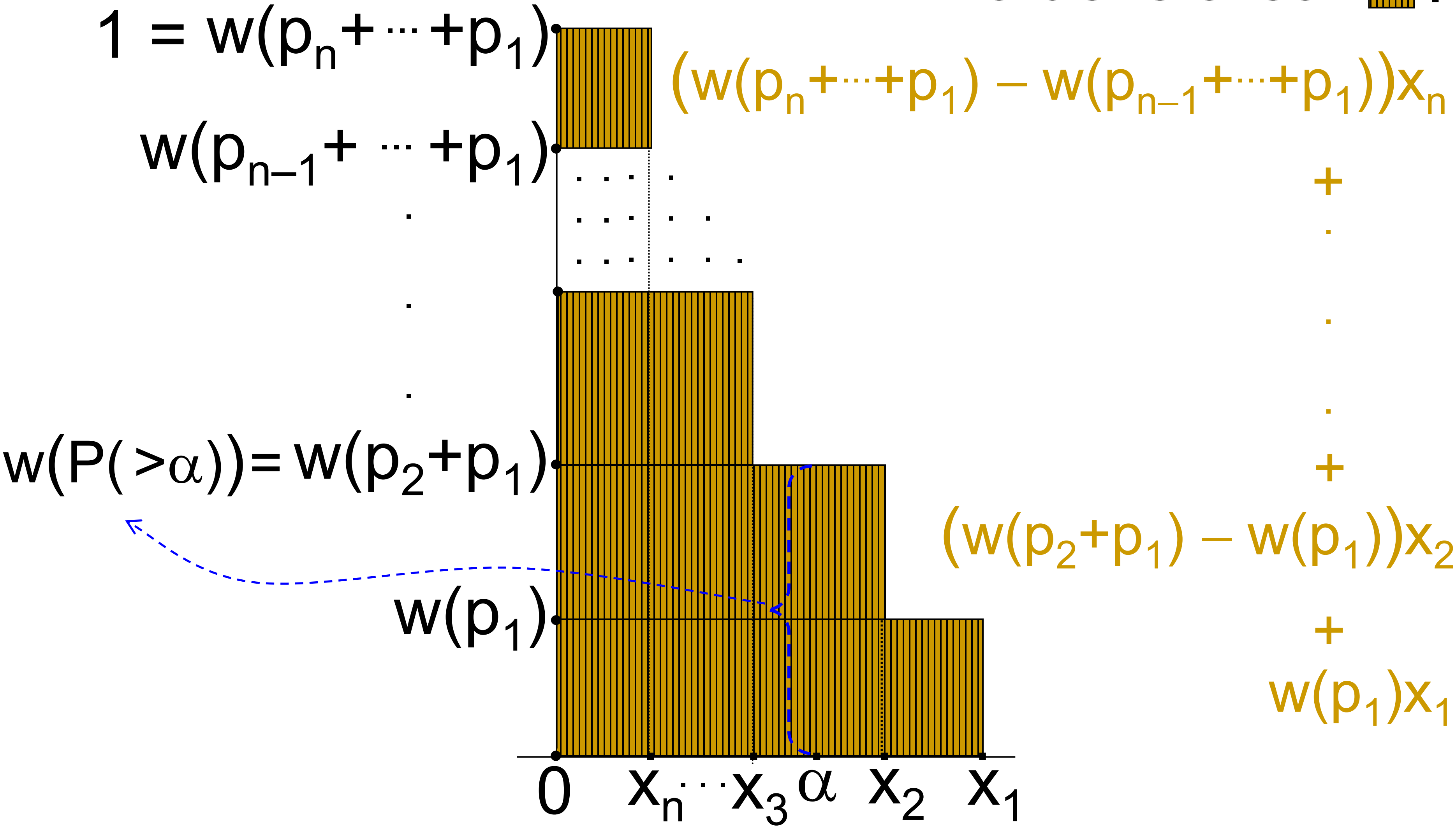

(1=)$p_n+\cdots+p_1$
$p_{n-1}+\cdots+p_1$
$p_3+p_2+p_1$
p_2+p_1
p_1
0 $x_n \cdots x_3$ x_2 x_1
Psychologists' probabilistic sensitivity (PT)
$\Rightarrow$
$w(p_n+\cdots+p_1)$(= 1!)
$w(p_{n-1}+\cdots+p_1)$
$w(p_3+p_2+p_1)$
$w(p_2+p_1)$
$w(p_1)$
0 $x_n \cdots x_3$ x_2 x_1

Figure *****

Value is area:



Using rank-notation simplifies. Then value is

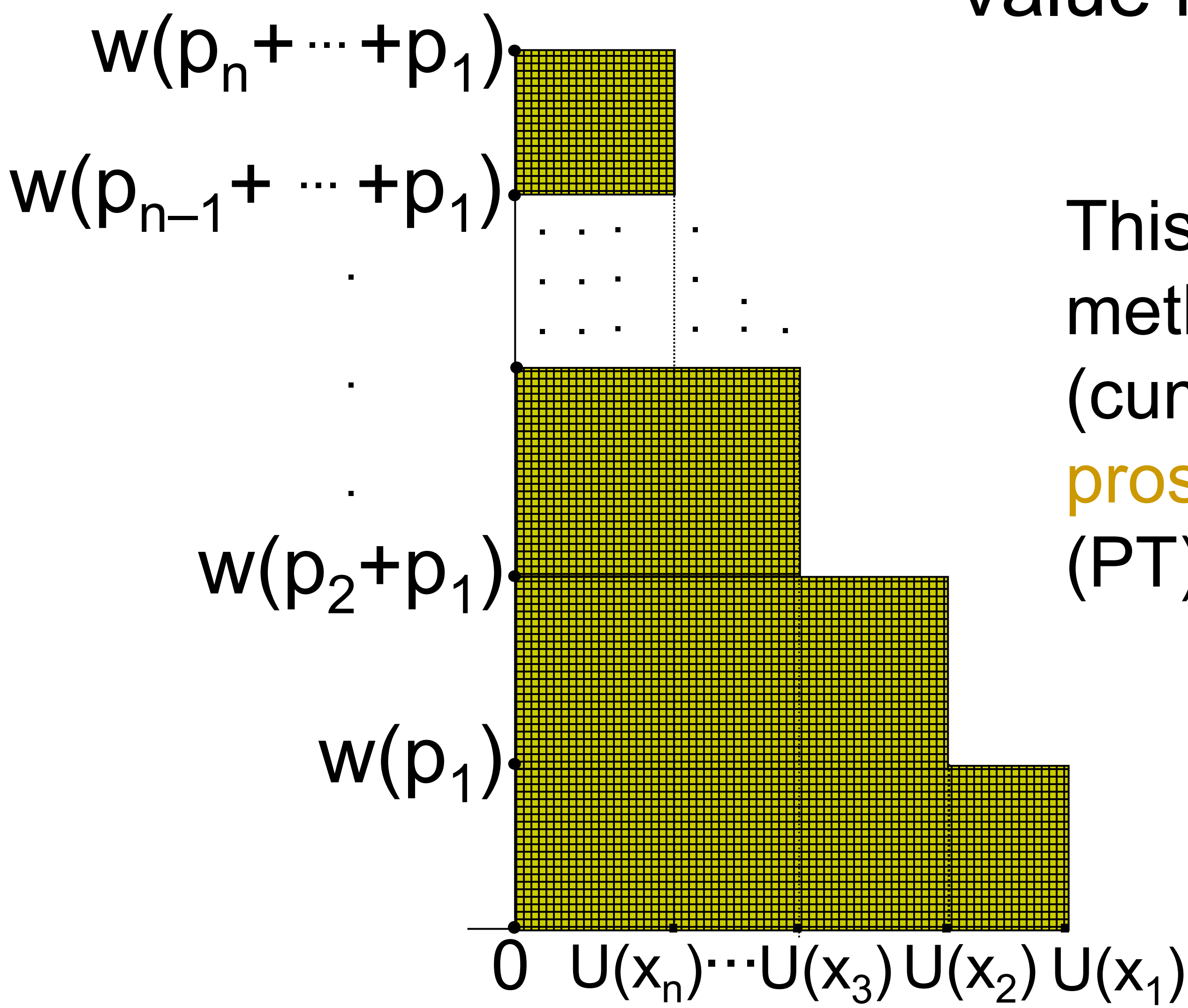
$(w(p_n + r_n) - w(r_n))x_n + \cdots + (w(p_2 + r_2) - w(r_2))x_2 + (w(p_1 + r_1) - w(r_1))x_1$

Preceding formula really did with probabilities what EU did with outcomes. Now for the first time we have the right analog of EU for probabilistic sensitivity!

Best is to combine the two,
with both $w(p)$ and $U(\alpha)$,
resulting in (new) prospect theory:

Figure ******

Value is area :



This valuation method is called (cumulative) prospect theory (PT), or RDU.

Value is
$(w(p_n+r_n)-w(r_n))U(x_n)+\cdots+(w(p_2+r_2)-w(r_2))U(x_2)+(w(p_1+r_1)-w(r_1))U(x_1)$

Ch. 5 finished

(Probability) weighting function w:
$w:[0,1] \to [0,1]$, $w(0) = 0, w(1) = 1$,
strictly increasing.
Utility function: as usual.

Consider
$(p_1\!:x_1, \ldots, p_n\!:x_n)$
$x_1 \geq \cdots \geq x_n$
Example: $\left(\frac{1}{6}\!:9, \frac{1}{3}\!:9, \frac{1}{2}\!:3\right) = \left(\frac{1}{2}\!:9, \frac{1}{2}\!:3\right)$

Now preparatory concepts.

For $(p_1: x_1, \ldots, p_n: x_n)$; $x_1 \geq \cdots \geq x_n$

Rank of x_i: $p_{i-1} + \cdots + p_1$

Ranked probability:

$p^{\backslash r}$ or just p^r with $p \geq 0, r \geq 0, p + r \leq 1$.

Decision weight:

$\pi(p^r) = w(p + r) - w(r)$

For x_i with outcome probability p_i and rank r_i:

decision weight is $\pi(p_i^{r_i})$; is

marginal w contribution of outcome-probability to rank.

Assume DUR.

RDU holds if

∃ weighting function w

∃ utility U

s.t.:

$(p_1 : x_1, \ldots, p_n : x_n) \mapsto \sum_{j=1}^{n} \pi_j U(x_j)$

represents preference

where:

$x_1 \geq \cdots \geq x_n$

$\pi_j = \pi\left(p_j^{r_j}\right) = w(p_j + \cdots + p_1) - w(p_{j-1} + \cdots + p_1)$

Notation. $\pi(x_j) := \pi_j$

p^0: best rank 0; also denoted p^b;

p^{1-p}: worst rank $1 - p$; also denoted p^w.

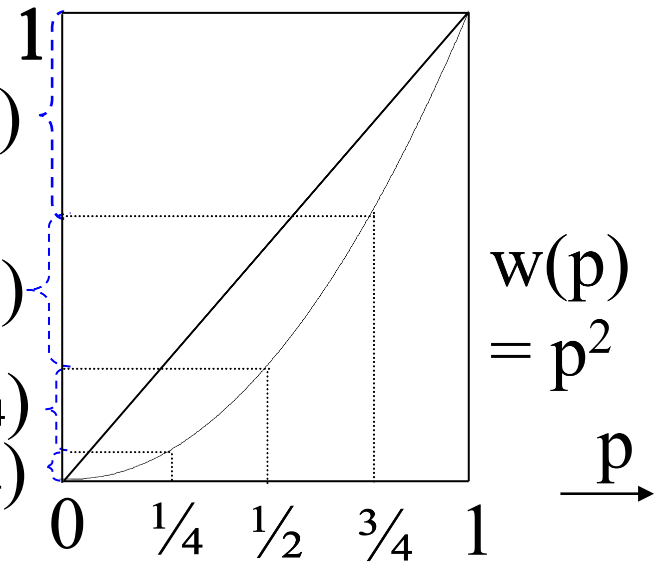
§6.3 Decision weights $\pi(\alpha)$ of outcomes α; consider $(\frac{1}{4}{:}80, \frac{1}{4}{:}60, \frac{1}{4}{:}40, \frac{1}{4}{:}20)$

$\pi(20) = 1 - w(\frac{3}{4})$

$\pi(40) = w(\frac{3}{4}) - w(\frac{1}{2})$

$\pi(60) = w(\frac{1}{2}) - w(\frac{1}{4})$

$\pi(80) = w(\frac{1}{4})$



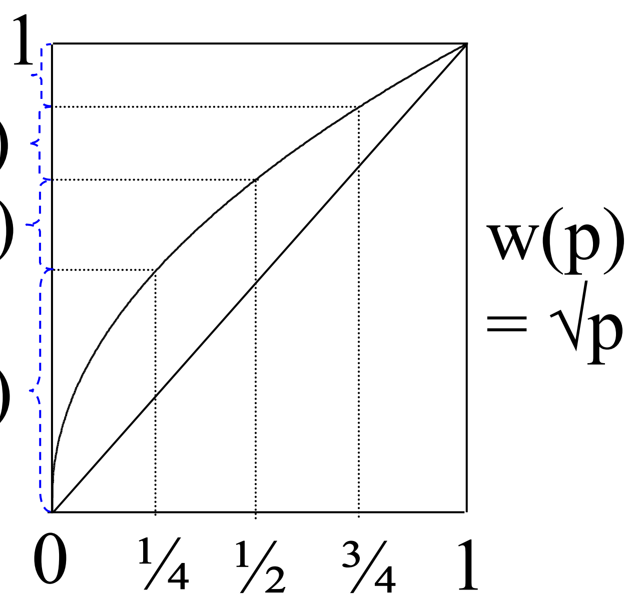
will give pessimism

$\pi(20) = 1 - w(\frac{3}{4})$

$\pi(40) = w(\frac{3}{4}) - w(\frac{1}{2})$

$\pi(60) = w(\frac{1}{2}) - w(\frac{1}{4})$

$\pi(80) = w(\frac{1}{4})$



will give optimism

Define pessimism through
$\pi(p^r)$:
How?

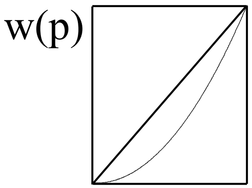
Hint …

Answer: Increasing in r!

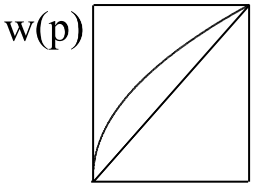
Define optimism through
$\pi(p^r)$:
How?

Decreasing in r.

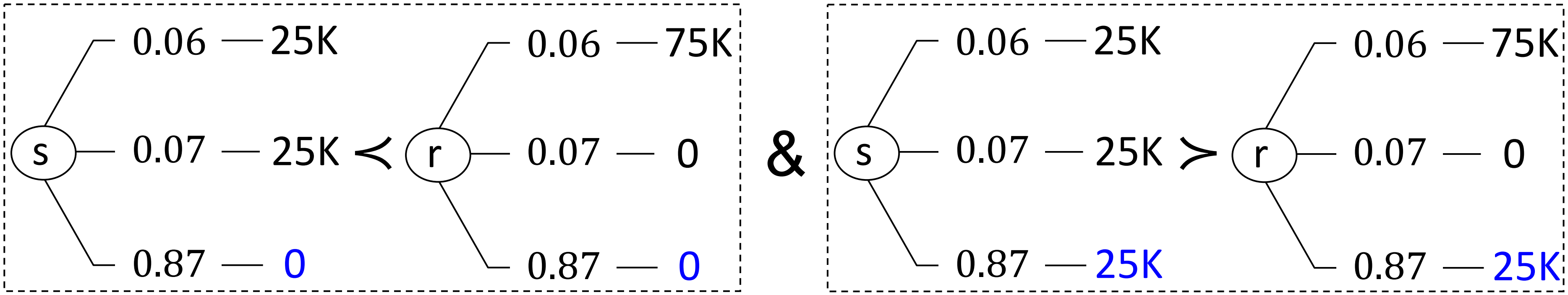
$\pi(p^{r)}$ increasing in r (pessimism):
$w(p+r) - w(r)$ increasing in r:
w convex.
$w'(r)$ increasing in r.



Optimism: w concave.



§6.4



$$\pi(0.07^{0.06})\big(U(25K) - U(0)\big) < \pi(0.07^{w})\big(U(25K) - U(0)\big)$$

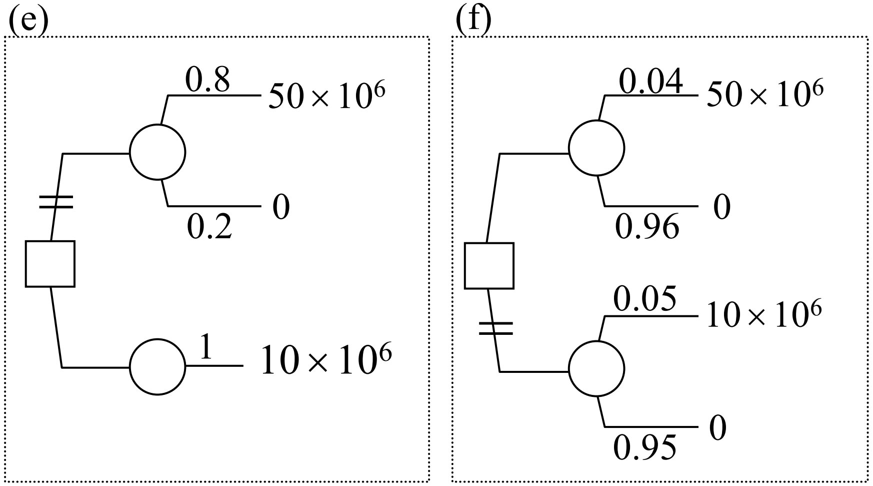
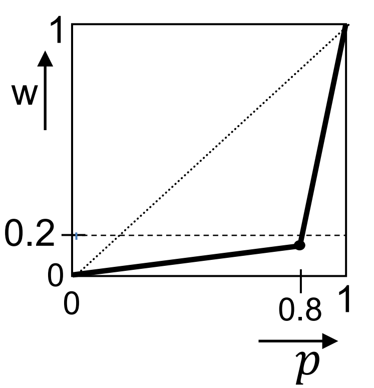


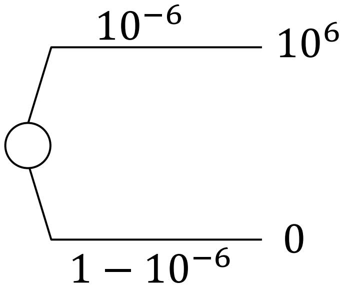
Figure 2.4.1

Quantitatively accommodate using RDU; take U linear.

Fig. (e): $w(0.8) \times 50 < 1 \times 10; w(0.8) < \frac{1}{5}$.

Fig. (f): $w(0.04) \times 50 > w(0.05) \times 10;\ \frac{w(0.04)}{w(0.05)} > \frac{1}{5}$

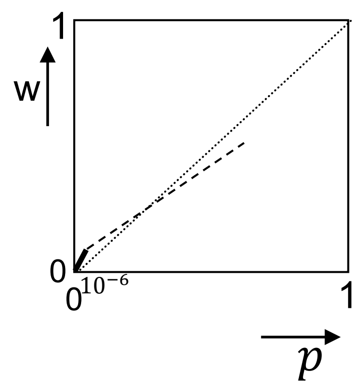




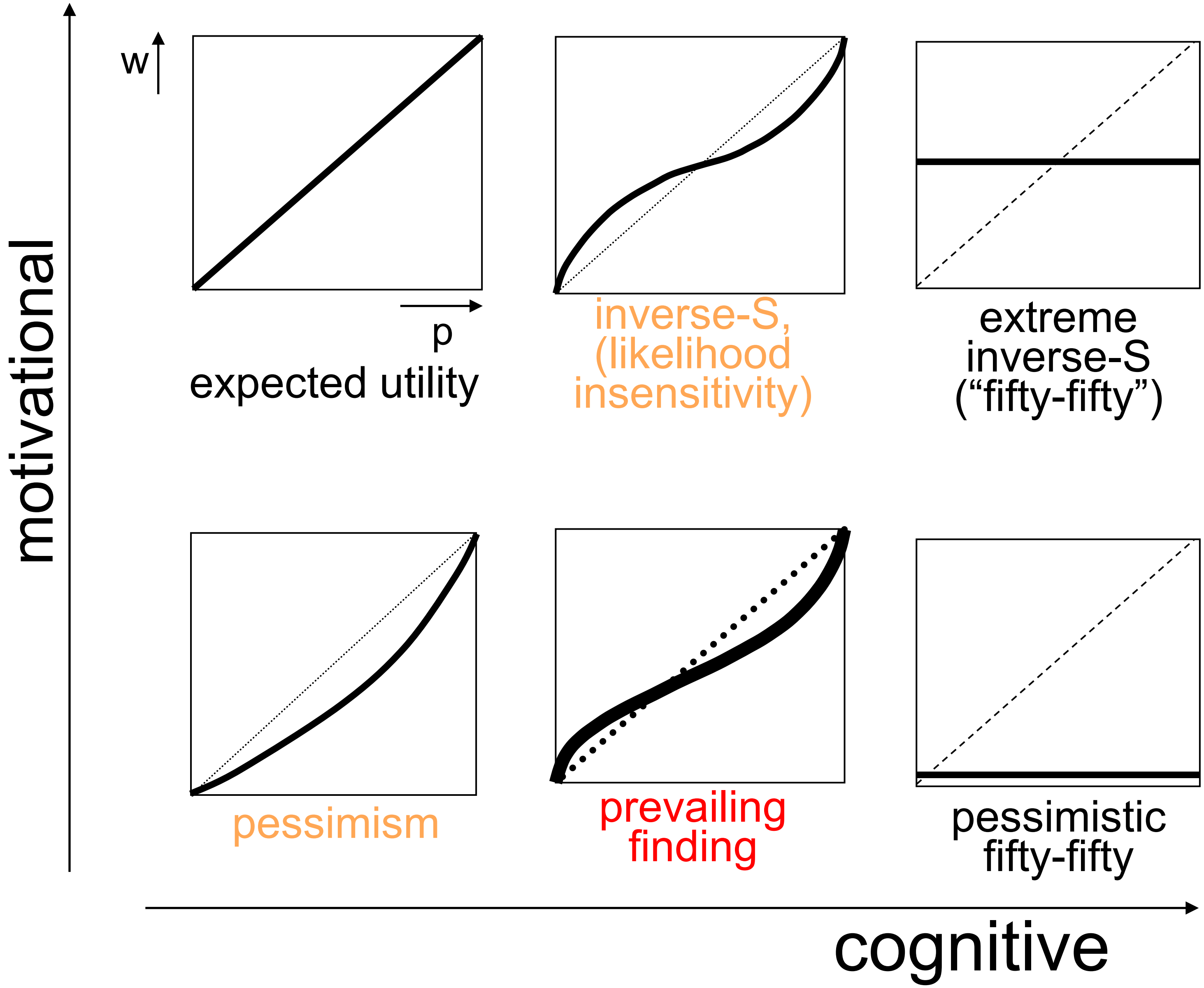
Your certainty equivalent?

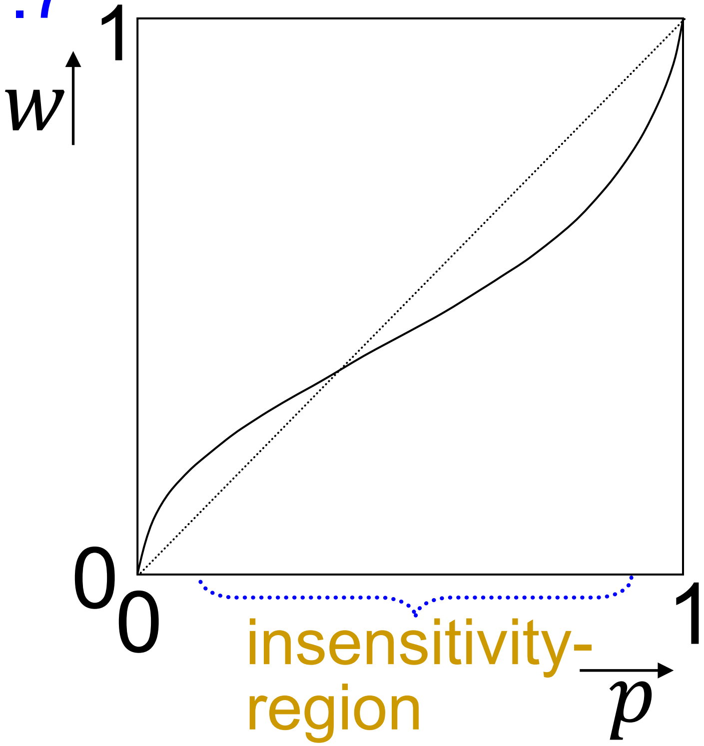
Common: $CE > 1$.
Accommodate this with RDU. Again U linear.

$$w(10^{-6}) > 10^{-6}$$



Typical shapes of probability weighting





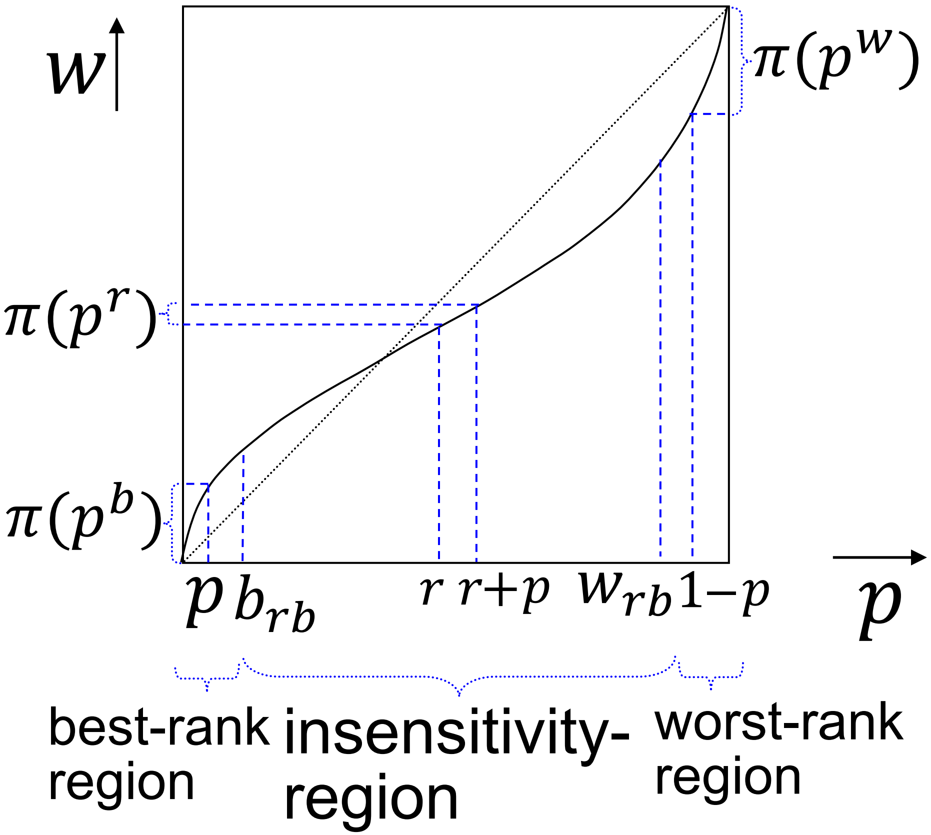
Inverse-S:

How define formally?

"Specify inflection point b, then require concavity to the left, convexity to the right?"

No! We different.

1. Specify insensitivity region in the middle.
2. Specify through inequalities that w differences are smaller there than at extremes.
3. Avoid comparisons between two extremes (by restricting domains of inequalities).



Inverse-S, or likelihood insensitivity, holds on insensitivity region $[b_{rb}, w_{rb}]$ if (i) and (ii) below hold.

In insensitivity region, w differences smaller than at extremes.

best-rank overweighting

(i) $\pi(p^b) \geq \pi(p^r)$ on* $[0, w_{rb}]$ $(r + p \leq w_{rb})$

(ii) $\pi(p^w) \geq \pi(p^r)$ on* $[b_{rb}, 1]$ $(r \geq b_{rb})$

*: restricting domains to avoid comparisons between two extremes.

worst-rank overweighting

§6.4-6.5 (5th meeting)

FIG. 4.1.1a

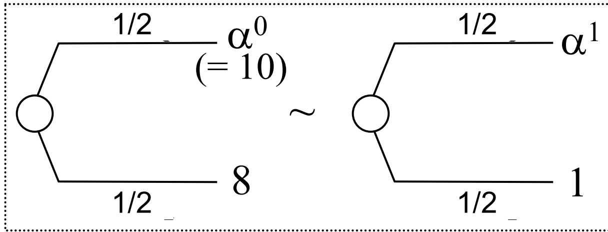
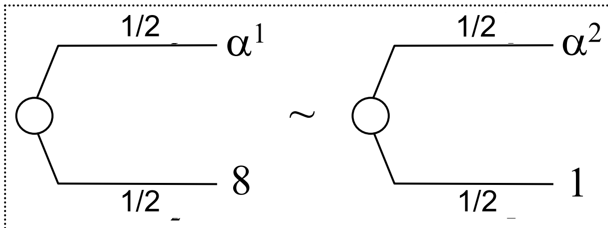


FIG. 4.1.1b



Exercise: Assume objective $P(cand_1) = 0.5 = P(cand_2)$, DUR, and RDU. Set $U(\alpha^0) = 0$, $U(\alpha^1) = 1/4$. What is $U(\alpha^2)$?

FIG. 4.1.1c

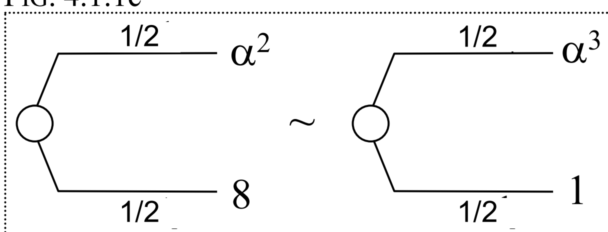
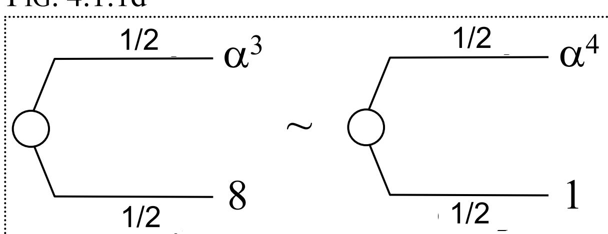


FIG. 4.1.1d



Exercise continued: What are $U(\alpha^3)$ and $U(\alpha^4)$?

You work …

Solution:

Fig. 4.1.1a:

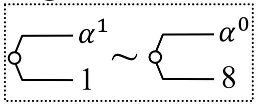
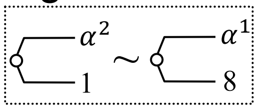


Fig. 4.1.1b:



Calculations under ~~EU~~ RDU (with probs ½)

Fig.a:

$$w(\tfrac{1}{2})U(\alpha^1) + (1-w(\tfrac{1}{2}))U(1) = w(\tfrac{1}{2})U(\alpha^0) + (1-w(\tfrac{1}{2}))U(8)$$

$$w(\tfrac{1}{2})\big(U(\alpha^1) - U(\alpha^0)\big) = (1-w(\tfrac{1}{2}))\big(U(8) - U(1)\big)$$

Fig.b:

$$w(\tfrac{1}{2})U(\alpha^2) + (1-w(\tfrac{1}{2}))U(1) = w(\tfrac{1}{2})U(\alpha^1) + (1-w(\tfrac{1}{2}))U(8)$$

$$w(\tfrac{1}{2})\big(U(\alpha^2) - U(\alpha^1)\big) = (1-w(\tfrac{1}{2}))\big(U(8) - U(1)\big)$$

$$U(\alpha^2) - U(\alpha^1) = U(\alpha^1) - U(\alpha^0)$$

$$U(\alpha^2) = 2/4!$$

Fig. 4.1.1b:

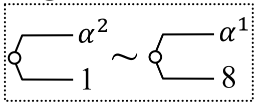
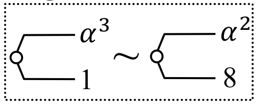


Fig. 4.1.1c:



Similarly:

Fig.b: $$w(\tfrac{1}{2})U(\alpha^2) + (1-w(\tfrac{1}{2}))U(1) = w(\tfrac{1}{2})U(\alpha^1) + (1-w(\tfrac{1}{2}))U(8)$$

$$w(\tfrac{1}{2})\big(U(\alpha^2) - U(\alpha^1)\big) = (1-w(\tfrac{1}{2}))\big(U(8) - U(1)\big)$$

$$\parallel \qquad\qquad\qquad\qquad \parallel$$

Fig.c: $$w(\tfrac{1}{2})U(\alpha^3) + (1-w(\tfrac{1}{2}))U(1) = w(\tfrac{1}{2})U(\alpha^2) + (1-w(\tfrac{1}{2}))U(8)$$

$$w(\tfrac{1}{2})\big(U(\alpha^3) - U(\alpha^2)\big) = (1-w(\tfrac{1}{2}))\big(U(8) - U(1)\big)$$

$$U(\alpha^3) - U(\alpha^2) = U(\alpha^2) - U(\alpha^1) \qquad U(\alpha^3) = 3/4.$$

Fig. 4.1.1c:

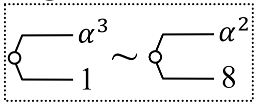
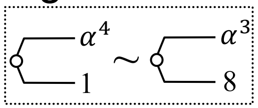


Fig. 4.1.1d:



Similarly:

Fig.b: $$w(\tfrac{1}{2})U(\alpha^3) + (1-w(\tfrac{1}{2}))U(1) = w(\tfrac{1}{2})U(\alpha^2) + (1-w(\tfrac{1}{2}))U(8)$$

$$w(\tfrac{1}{2})\big(U(\alpha^3) - U(\alpha^2)\big) = (1-w(\tfrac{1}{2}))\big(U(8) - U(1)\big)$$

Fig.c: $$w(\tfrac{1}{2})U(\alpha^4) + (1-w(\tfrac{1}{2}))U(1) = w(\tfrac{1}{2})U(\alpha^3) + (1-w(\tfrac{1}{2}))U(8)$$

$$w(\tfrac{1}{2})\big(U(\alpha^4) - U(\alpha^3)\big) = (1-w(\tfrac{1}{2}))\big(U(8) - U(1)\big)$$

$$U(\alpha^4) - U(\alpha^3) = U(\alpha^3) - U(\alpha^2) \qquad U(\alpha^4) = 4/4.$$

RDU:

$U(\alpha^4) - U(\alpha^3) = U(\alpha^3) - U(\alpha^2) = U(\alpha^2) - U(\alpha^1) = U(\alpha^1) - U(\alpha^0)$
Can set $U\left(\alpha^j\right) = j/4$.

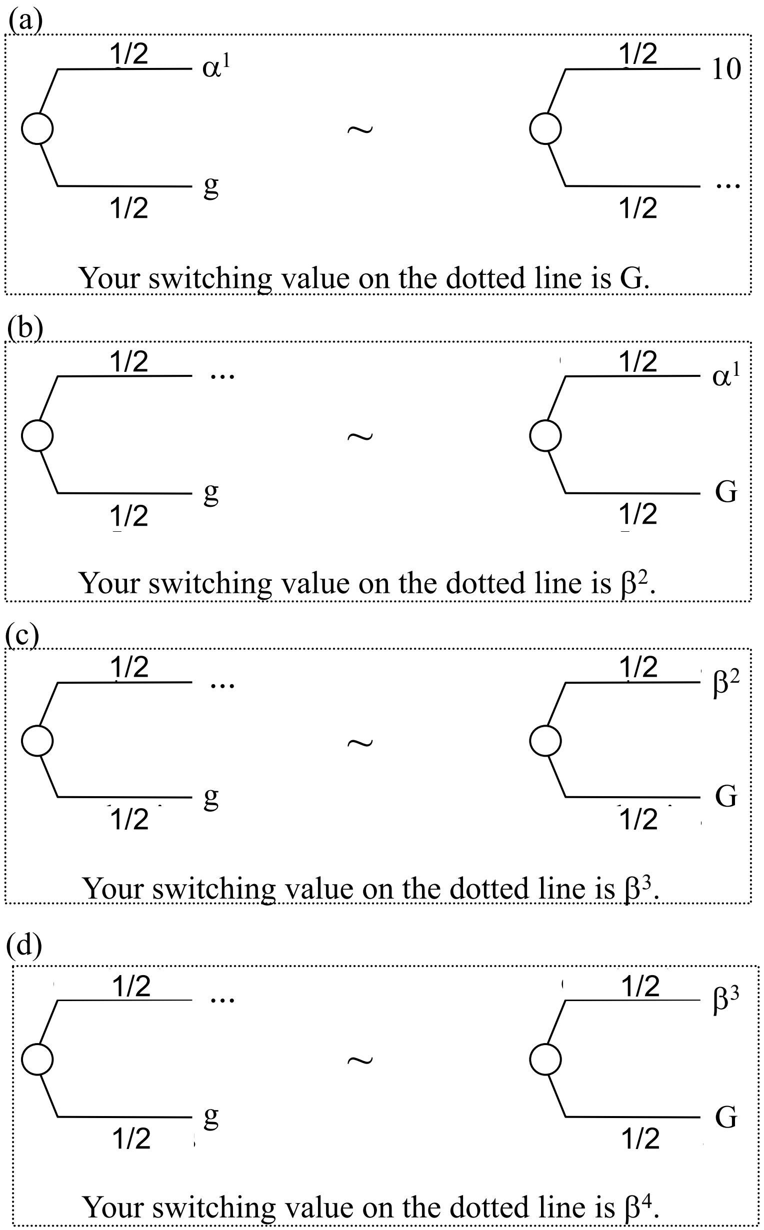


Figure 4.1.2 [*2nd TO Upwards*]. Eliciting β^2, β^3, β^4

Now to βs.

What can you say about their U-values under RDU?

You work …

Fig. 4.2.1a:

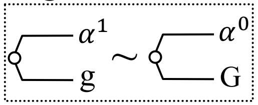
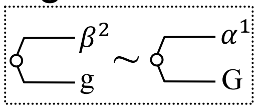


Fig. 4.2.1b:



Calculations under ~~EU~~ RDU (with probs ½)

Fig.a: $\cancel{w(}\frac{1}{2}\cancel{)}U(\alpha^1) + \cancel{(1-w(}\frac{1}{2}\cancel{))}U(g) = \cancel{w(}\frac{1}{2}\cancel{)}U(\alpha^0) + \cancel{(1-w(}\frac{1}{2}\cancel{))}U(G)$

$$\cancel{w(}\frac{1}{2}\cancel{)}\big(U(\alpha^1) - U(\alpha^0)\big) = \cancel{(1-w(}\frac{1}{2}\cancel{))}\big(U(G) - U(g)\big)$$

||

Fig.b: $\cancel{w(}\frac{1}{2}\cancel{)}U(\beta^2) + \cancel{(1-w(}\frac{1}{2}\cancel{))}U(g) = \cancel{w(}\frac{1}{2}\cancel{)}U(\alpha^1) + \cancel{(1-w(}\frac{1}{2}\cancel{))}U(G)$

||

$$\cancel{w(}\frac{1}{2}\cancel{)}\big(U(\beta^2) - U(\alpha^1)\big) = \cancel{(1-w(}\frac{1}{2}\cancel{))}\big(U(G) - U(g)\big)$$

OK!?

$$U(\beta^2) - U(\alpha^1) = U(\alpha^1) - U(\alpha^0)$$

No!

Fig. 4.2.1a:

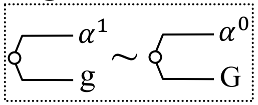
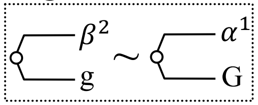


Fig. 4.2.1b:



β's still "quite similar" to α's!

Fig.a:
$$\left(1-w\left(\tfrac{1}{2}\right)\right)U(\alpha^1) + w\left(\tfrac{1}{2}\right)U(\mathrm{g}) = \left(1-w\left(\tfrac{1}{2}\right)\right)U(\alpha^0) + w\left(\tfrac{1}{2}\right)U(\mathrm{G})$$
$$\left(1-w\left(\tfrac{1}{2}\right)\right)\left(U(\alpha^1) - U(\alpha^0)\right) = w\left(\tfrac{1}{2}\right)\left(U(\mathrm{G}) - U(\mathrm{g})\right)$$

Fig.b:
$$\left(1-w\left(\tfrac{1}{2}\right)\right)U(\beta^2) + w\left(\tfrac{1}{2}\right)U(\mathrm{g}) = \left(1-w\left(\tfrac{1}{2}\right)\right)U(\alpha^1) + w\left(\tfrac{1}{2}\right)U(\mathrm{G})$$
$$\left(1-w\left(\tfrac{1}{2}\right)\right)\left(U(\beta^2) - U(\alpha^1)\right) = w\left(\tfrac{1}{2}\right)\left(U(\mathrm{G}) - U(\mathrm{g})\right)$$

$$U(\beta^2) - U(\alpha^1) = U(\alpha^1) - U(\alpha^0)$$ We still get this!

Fig. 4.2.1b:

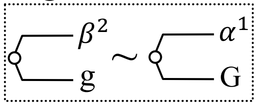
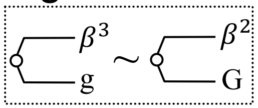


Fig. 4.2.1c:



Similarly:

Fig.b:

$$\left(1-w\left(\tfrac{1}{2}\right)\right)U(\beta^2)+w\left(\tfrac{1}{2}\right)U(\mathrm{g}) = \left(1-w\left(\tfrac{1}{2}\right)\right)U(\alpha^1)+w\left(\tfrac{1}{2}\right)U(\mathrm{G})$$

$$\left(1-w\left(\tfrac{1}{2}\right)\right)\left(U(\beta^2)-U(\alpha^1)\right) = w\left(\tfrac{1}{2}\right)\left(U(\mathrm{G})-U(\mathrm{g})\right)$$

Fig.c:

$$\left(1-w\left(\tfrac{1}{2}\right)\right)U(\beta^3)+w\left(\tfrac{1}{2}\right)U(\mathrm{g}) = \left(1-w\left(\tfrac{1}{2}\right)\right)U(\beta^2)+w\left(\tfrac{1}{2}\right)U(\mathrm{G})$$

$$\left(1-w\left(\tfrac{1}{2}\right)\right)\left(U(\beta^3)-U(\beta^2)\right) = w\left(\tfrac{1}{2}\right)\left(U(\mathrm{G})-U(\mathrm{g})\right)$$

$$U(\beta^3)-U(\beta^2)=U(\beta^2)-U(\alpha^1)$$

Fig. 4.2.1c:

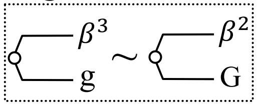
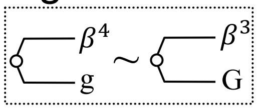


Fig. 4.2.1d:



And similarly:

Fig.c:
$$\left(1-w\left(\tfrac{1}{2}\right)\right)U(\beta^3)+w\left(\tfrac{1}{2}\right)U(\mathrm{g}) = \left(1-w\left(\tfrac{1}{2}\right)\right)U(\beta^2)+w\left(\tfrac{1}{2}\right)U(\mathrm{G})$$
$$\left(1-w\left(\tfrac{1}{2}\right)\right)\left(U(\beta^3)-U(\beta^2)\right) = w\left(\tfrac{1}{2}\right)\left(U(\mathrm{G})-U(\mathrm{g})\right)$$

Fig.d:
$$\left(1-w\left(\tfrac{1}{2}\right)\right)U(\beta^4)+w\left(\tfrac{1}{2}\right)U(\mathrm{g}) = \left(1-w\left(\tfrac{1}{2}\right)\right)U(\beta^3)+w\left(\tfrac{1}{2}\right)U(\mathrm{G})$$
$$\left(1-w\left(\tfrac{1}{2}\right)\right)\left(U(\beta^4)-U(\beta^3)\right) = w\left(\tfrac{1}{2}\right)\left(U(\mathrm{G})-U(\mathrm{g})\right)$$

$$U(\beta^4)-U(\beta^3) = U(\beta^3)-U(\beta^2)$$

Taking all together:

$U(\beta^4) - U(\beta^3) = U(\beta^3) - U(\beta^2) = U(\beta^2) - U(\alpha^1) = U(\alpha^1) - U(\alpha^0)$

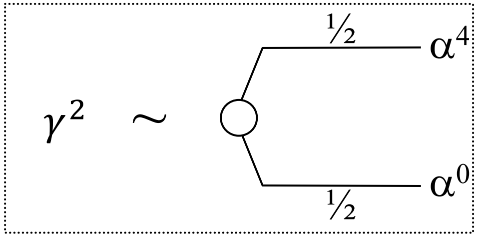
We may set $U(\alpha^0) = 0, U(\alpha^1) = \frac{1}{4}$. Then:
$U(\beta^2) = \frac{2}{4} = U(\alpha^2)$, $U(\beta^3) = \frac{3}{4} = U(\alpha^3)$, $U(\beta^4) = \frac{4}{4} = U(\alpha^4)$;
The β's have the same utility, so are the same, as the α's:
$\beta^2 = \alpha^2, \beta^3 = \alpha^3, \beta^4 = \alpha^4$.

The β's are just another way for measuring the same as the α's, also under RDU.

RDU to accommodate γ's

Exercise. Assume $U(\alpha^j) = \frac{j}{4}, j = 0, \ldots, 4$.
Can RDU accommodate $\gamma^2 < \alpha^2$? (And, similarly, $\gamma^1 < \alpha^1$ and $\gamma^3 < \alpha^3$?)

FIG. 4.1.4a.



Solution.

$U(\gamma^2) = w\left(\frac{1}{2}\right) U(\alpha^4) + \left(1 - w\left(\frac{1}{2}\right)\right) U(\alpha^0)$

The smaller $w(\frac{1}{2})$, the smaller γ^2.

If $w(\frac{1}{2})$ = ½, then $\gamma^2 = \alpha^2$. So,

if $w(\frac{1}{2})$ < ½, then $\gamma^2 < \alpha^2$.

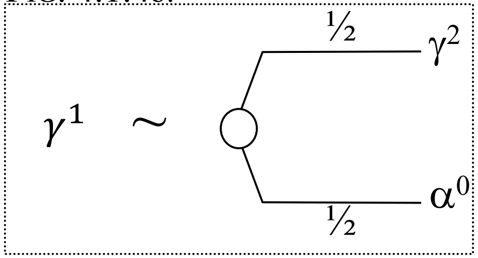
Indeed, then:

$U(\gamma^2) = w\left(\frac{1}{2}\right) U(\alpha^4) + \left(1 - w\left(\frac{1}{2}\right)\right) U(\alpha^0)$

$< \frac{1}{2} U(\alpha^4) + \left(1 - \frac{1}{2}\right) U(\alpha^0) = U(\alpha^2)$;

$U(\gamma^2) < U(\alpha^2)$ implies $\gamma^2 < \alpha^2$.

FIG. 4.1.4b.



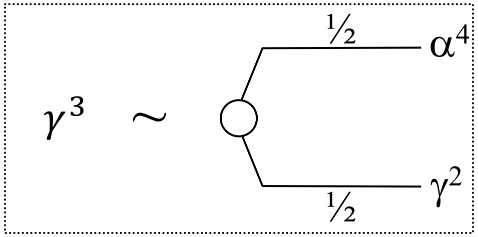
Similarly, $\gamma^1 < \alpha^1$: $U(\gamma^1) = w\left(\frac{1}{2}\right) U(\gamma^2) + \left(1 - w\left(\frac{1}{2}\right)\right) U(\alpha^0)$

$< w\left(\frac{1}{2}\right) U(\alpha^2) + \left(1 - w\left(\frac{1}{2}\right)\right) U(\alpha^0)$

(if $w(\frac{1}{2})$ < ½) $< \frac{1}{2} U(\alpha^2) + \left(1 - \frac{1}{2}\right) U(\alpha^0) = U(\alpha^1)$;

$U(\gamma^1) < U(\alpha^1)$ implies $\gamma^1 < \alpha^1$.

FIG. 4.1.4c.



Similarly, $\gamma^3 < \alpha^3$: $U(\gamma^3) = w\left(\frac{1}{2}\right) U(\alpha^4) + \left(1 - w\left(\frac{1}{2}\right)\right) U(\gamma^2)$

$< w\left(\frac{1}{2}\right) U(\alpha^4) + \left(1 - w\left(\frac{1}{2}\right)\right) U(\alpha^2)$

(if $w(\frac{1}{2})$ < ½) $< \frac{1}{2} U(\alpha^4) + \left(1 - \frac{1}{2}\right) U(\alpha^2) = U(\alpha^3)$;

$U(\gamma^3) < U(\alpha^3)$ implies $\gamma^3 < \alpha^3$.

RDU cannot accommodate δ's

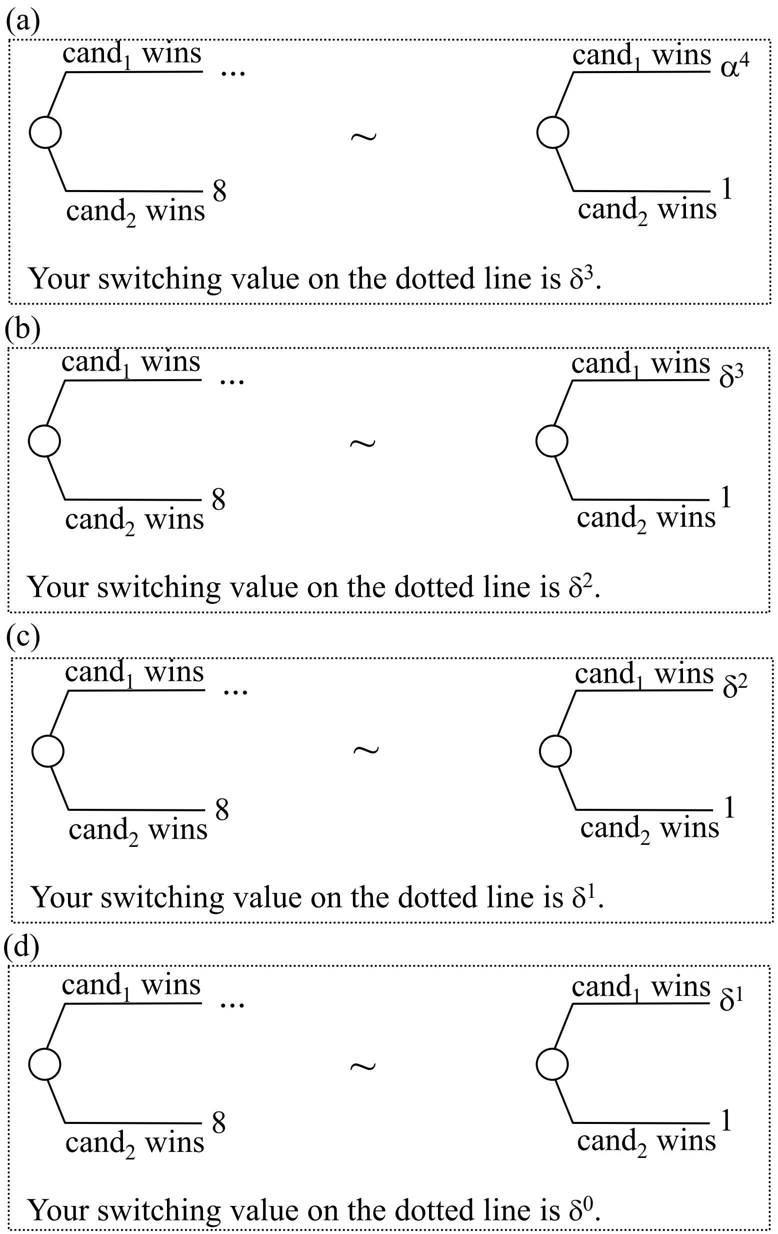


Figure 4.1.4 [*TO Downwards*]. Eliciting $\delta^3 \ldots \delta^0$

(Although some insights will be offered later.)

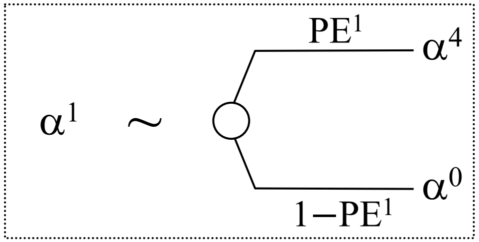
RDU to accommodate PE^j's

Exercise. Assume again $U(\alpha^j) = \frac{j}{4}, j = 0, \ldots, 4$.

Can RDU accommodate $PE^1 > \frac{1}{4}$?

(And, also, $PE^2 > \frac{2}{4}$ and $PE^3 > \frac{3}{4}$.)

FIG. 4.1.5a



You work …

Set $U(\alpha^j) = \frac{j}{4}$:

$U(\alpha^1) = w(PE^1)U(\alpha^4) + (1 - w(PE^1))U(\alpha^0)$;

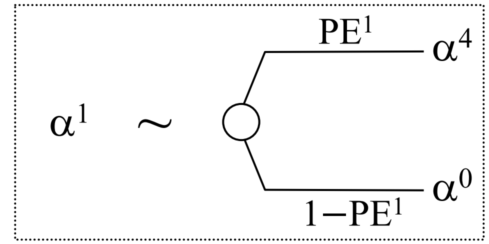
$\mathrm{w}(PE^1) = \frac{1}{4}$.

$PE^1 > \frac{1}{4}$ is well possible: if w “pushes PE^1 down.”

E.g., if $PE^1 = \frac{1}{3}$ but $w\left(\frac{1}{3}\right) = \frac{1}{4}$.

$PE^1 < \frac{1}{4}$ is also possible: if w “pushes PE^1 up.”

FIG. 4.1.5a

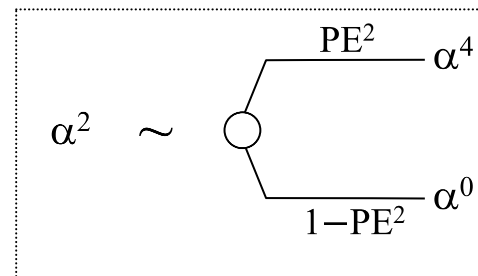


$U(\alpha^2) = w(PE^2)U(\alpha^4) + (1 - w(PE^2))U(\alpha^0)$;

$\mathrm{w}(PE^2) = \frac{2}{4}$.

$PE^2 > \frac{2}{4}$ is well possible: if w “pushes PE^2 down.”

FIG. 4.1.5b

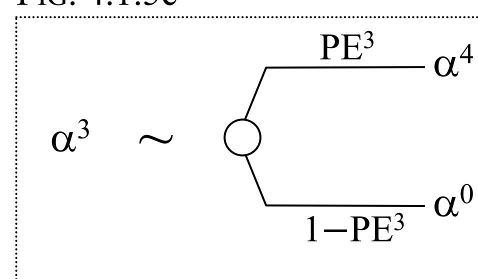


$U(\alpha^3) = w(PE^3)U(\alpha^4) + (1 - w(PE^3))U(\alpha^0)$;

$\mathrm{w}(PE^3) = \frac{3}{4}$.

$PE^3 > \frac{3}{4}$ is well possible: if w “pushes PE^3 down.”

FIG. 4.1.5c



Can you see something “very nice”?

Set $U(\alpha^j) = \frac{j}{4}$:

$U(\alpha^1) = w(PE^1)U(\alpha^4) + (1 - w(PE^1))U(\alpha^0)$;

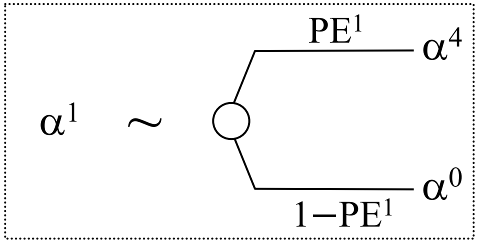
$\boxed{w(PE^1) = \frac{1}{4}.}$

$PE^1 > \frac{1}{4}$ is well possible: if w "pushes PE^1 down."

E.g., if $PE^1 = \frac{1}{3}$ but $w\left(\frac{1}{3}\right) = \frac{1}{4}$.

$PE^1 < \frac{1}{4}$ is also possible: if w "pushes PE^1 up."

FIG. 4.1.5a

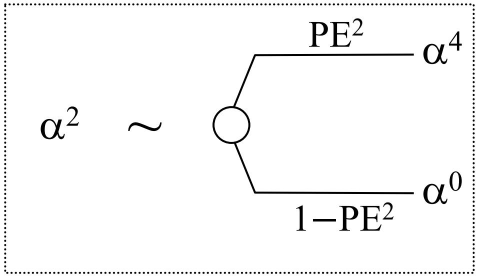


$U(\alpha^2) = w(PE^2)U(\alpha^4) + (1 - w(PE^2))U(\alpha^0)$;

$\boxed{w(PE^2) = \frac{2}{4}.}$

$PE^2 > \frac{2}{4}$ is well possible: if w "pushes PE^2 down."

FIG. 4.1.5b

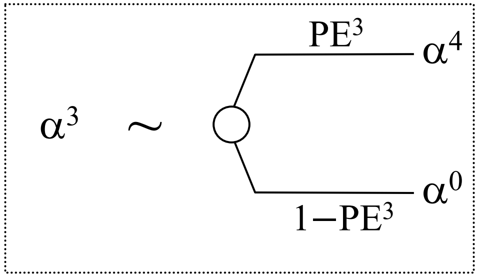


$U(\alpha^3) = w(PE^3)U(\alpha^4) + (1 - w(PE^3))U(\alpha^0)$;

$\boxed{w(PE^3) = \frac{3}{4}.}$

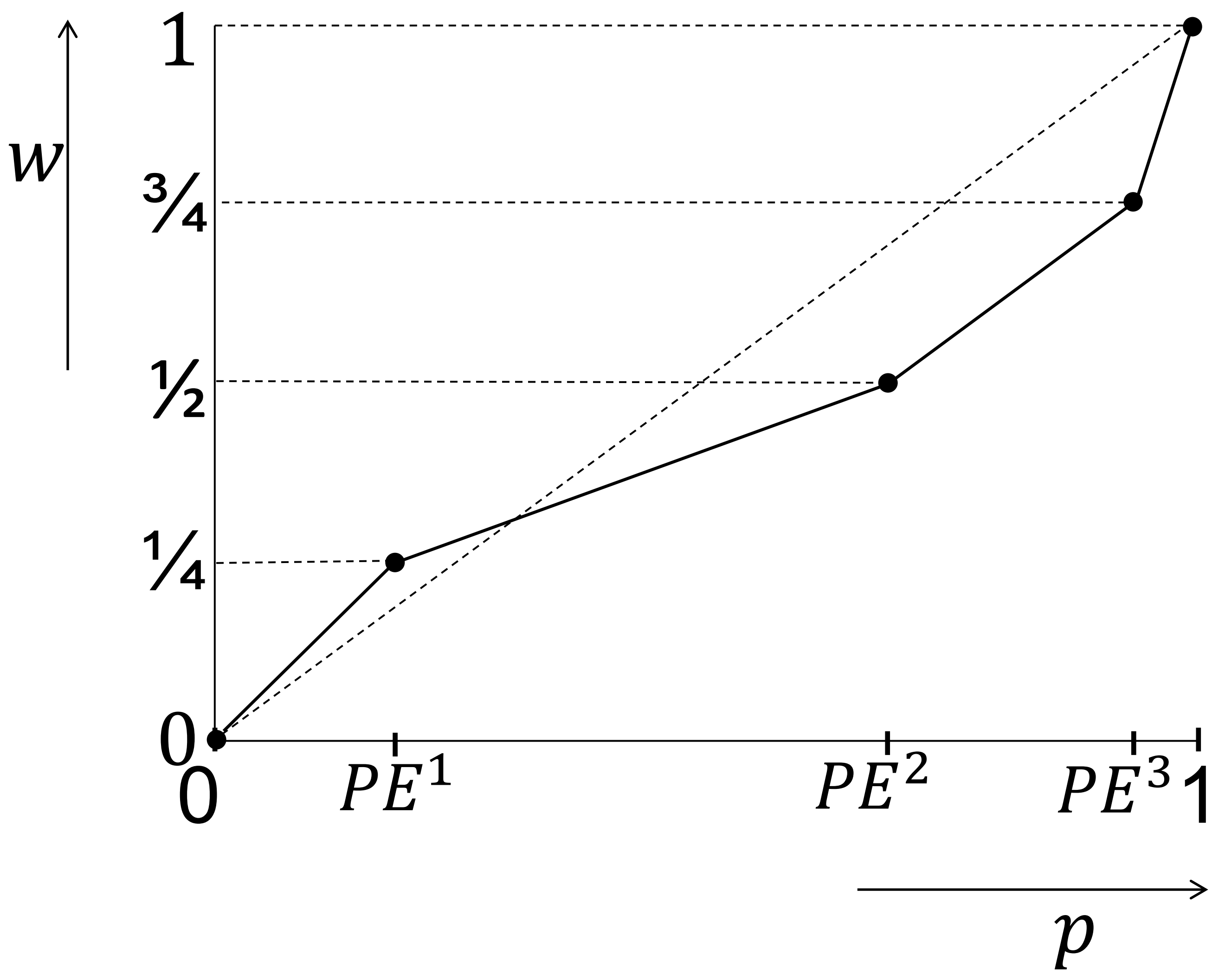
$PE^3 > \frac{3}{4}$ is well possible: if w "pushes PE^3 down."

FIG. 4.1.5c



We measured w!

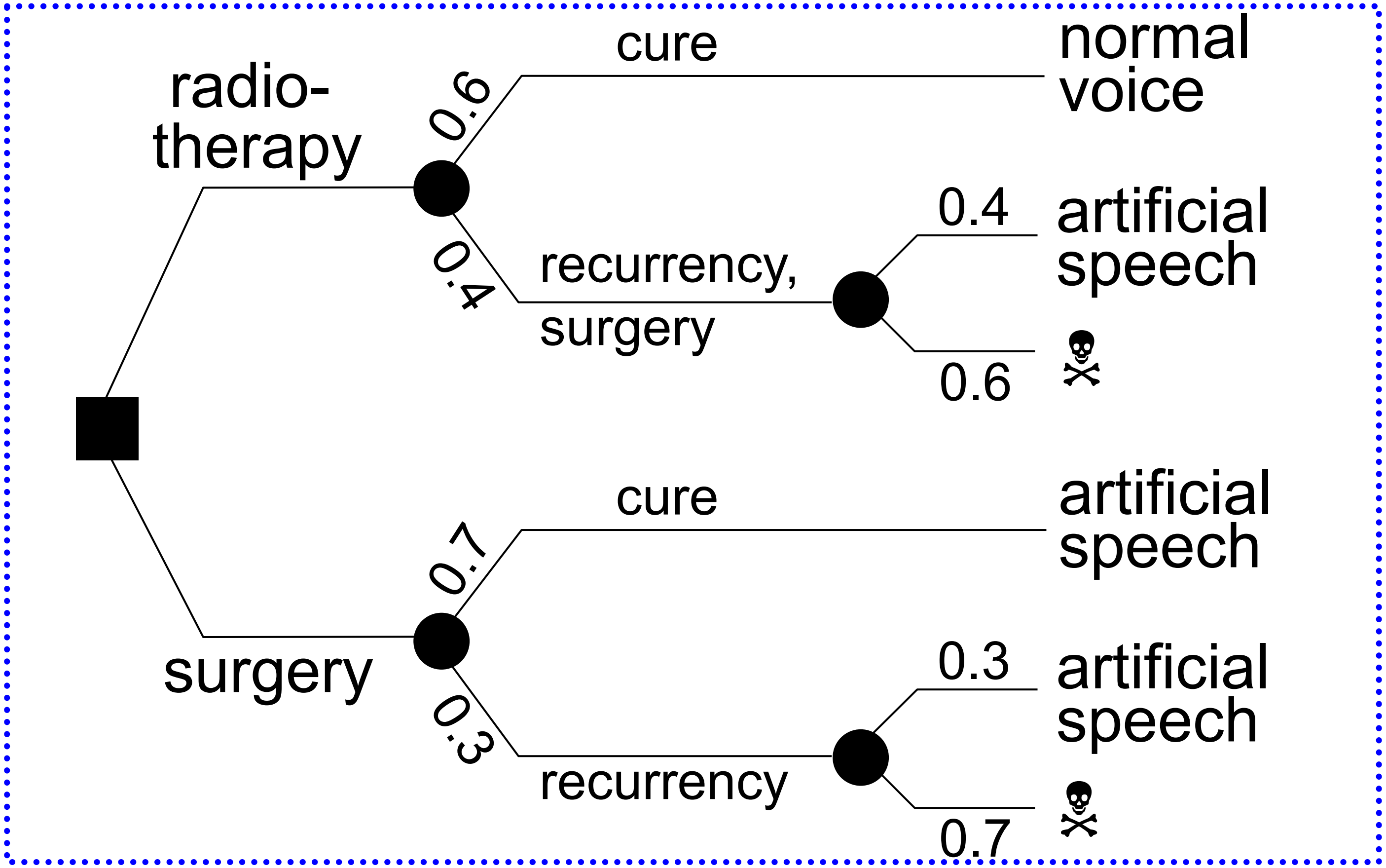
Your w graph:

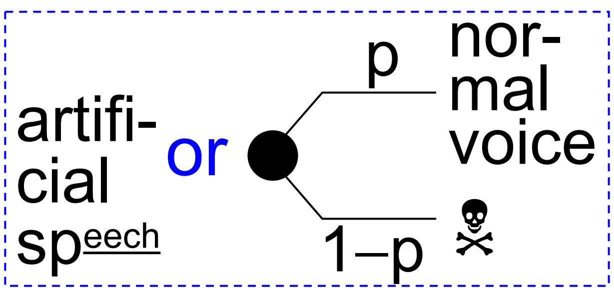


Our experiment measured both U and w;
all of RDU. Easily!
Is Abdellaoui's (2000) method.

Term “accommodate” …

Prescriptive implication of RDU.
New insights?





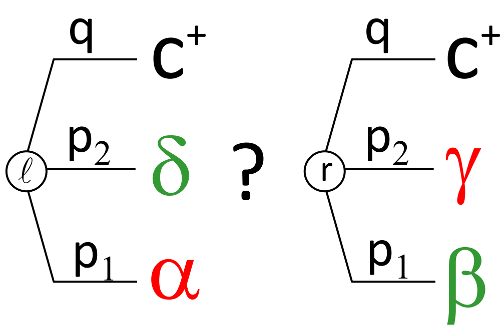
For which p equivalence?

§7.4

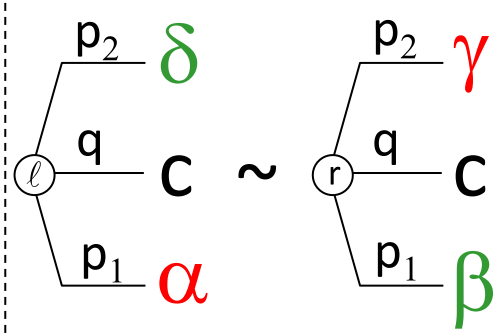
Violations of the sure-thing principle give direct insights into optimism/pessimism

Green: direction you want to go

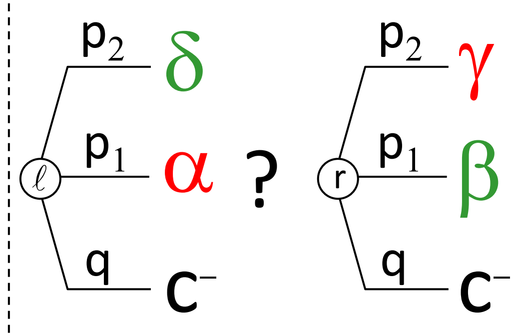
$\delta > \gamma > \beta > \alpha$



Left case:
c replaced by c^+;
$c^+ > \delta$



Reference case:
$\gamma > c > \beta$



Right case:
$\alpha > c^-$

Question: what is preference under *pessimism*?
Answer:
$\succcurlyeq$
Optimism: $\preccurlyeq$
Question: what under *inverse-S*?
Answer:
$\preccurlyeq$

Question: what is preference under *pessimism*?
Answer:
$\succcurlyeq$
Optimism: $\preccurlyeq$
Question: what under *inverse-S*?
Answer:
$\succcurlyeq$

§7.2

§6.9 General way to calculate RDU

General way to calculate EU:

1. Distribution function F_x: $F_x(\alpha) = P(x \leq \alpha)$.
2. Distribution function $F_{x,U}$: $F_{x,U}(\alpha) = P\left(x \leq U^{-1}(\alpha)\right)$.
3. Dual $G_{x,U} = 1 - F_{x,U}$
4. $EU(x) = \int_{\Re^+} G_{x,U}(t)dt - \int_{\Re^-}(1 - G_{x,U}(t))dt$

$$RDU(x) = \int_{\Re^+} w \circ G_{x,U}(t)dt - \int_{\Re^-}(1 - w \circ G_{x,U}(t))dt$$

Moral: get distribution function! (Handles ranking of outcomes.)

§7.6

Loss ranks

Rank: $P(Outcome\ ranked\ better)$,
"goodnews probability."
Note: w always transforms ranks.

Critic:
why not use loss-rank ℓ ("badnews probs") =
$P(Outcome\ ranked\ worse)$?
Use loss-ranked prob $p_{\backslash\ell} = p_\ell$ instead of p^r?
Take nonadditive z: $[0,1] \to [0,1]$
$(p_1: x_1, \ldots, p_n: x_n)$ with $x_1 \geq \cdots \geq x_n$ evaluated by

$$\sum_{j=1}^{n} \pi_j U(x_j)$$

with $\pi_j = \mathrm{z}(p_j + \cdots + p_n) - \mathrm{z}(p_{j+1} + \cdots + p_n)$

Why not??

Answer: Does not matter!

Define $w(p) = 1 - z(1-p)$

(I.e., $z(p) = 1 - w(1-p)$)

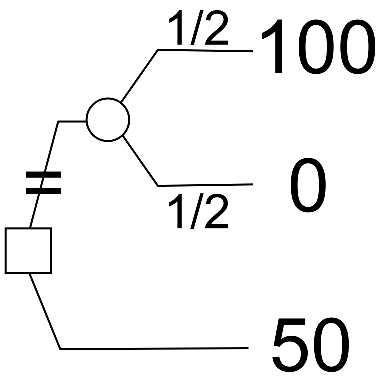
$\pi_j = w\left(p_j + \cdots + p_1\right) - w\left(p_{j-1} + \cdots + p_1\right) =$

$1 - z\left(p_{j+1} + \cdots + p_n\right) - \left(1 - z\left(p_j + \cdots + p_n\right)\right) =$

$z(p_j + \cdots + p_n) - z\left(p_{j+1} + \cdots + p_n\right).$

Is the same!

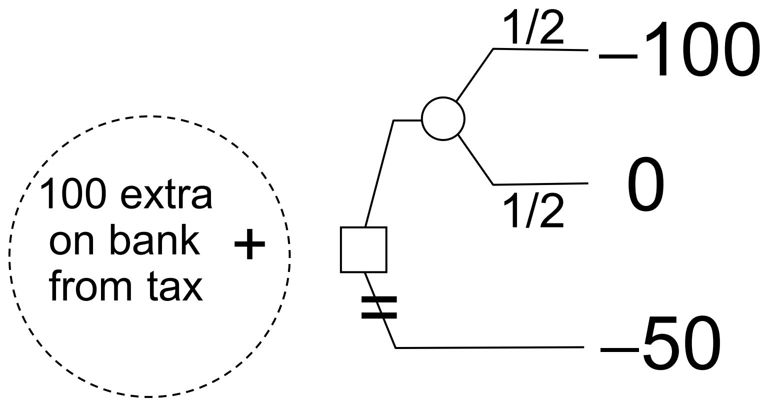
§8.1-8.2 (6th meeting)



Situation left

⇔

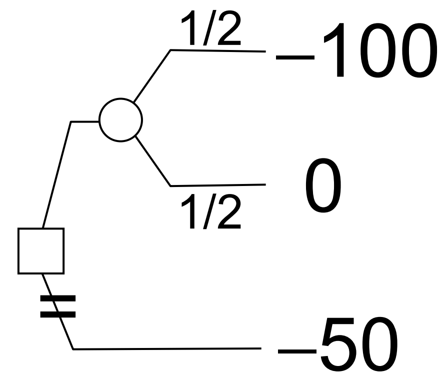
monetary calculus (called "reference independence")



Situation middle

⇔

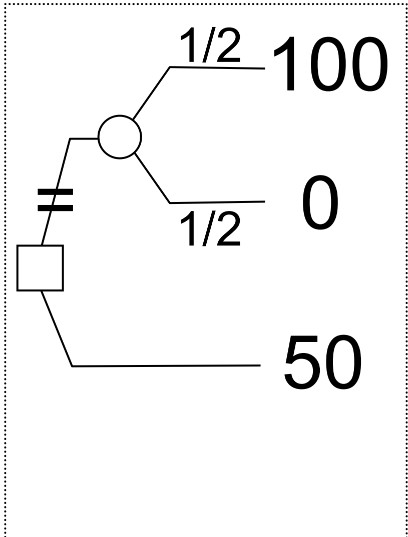
plausible (called "additivity")



Situation right

Common prefs must violate one of the two principles.

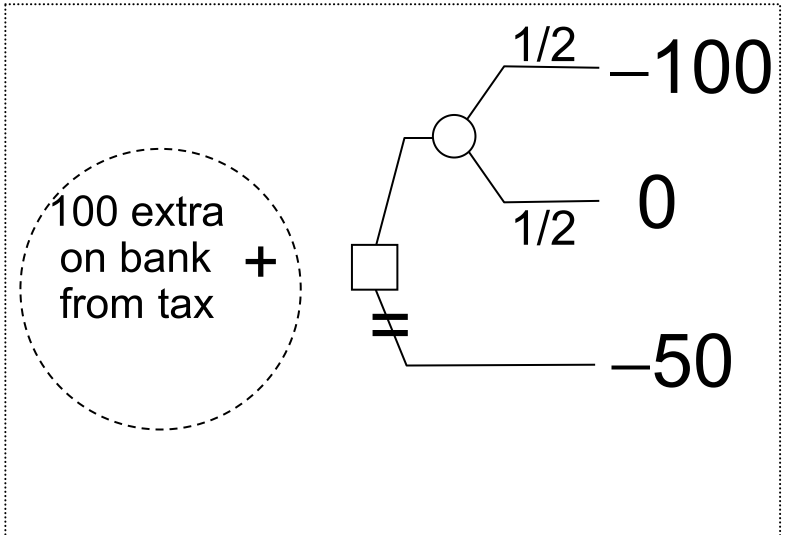
§8.1-8.2 (6th meeting)



Situation left

⇔

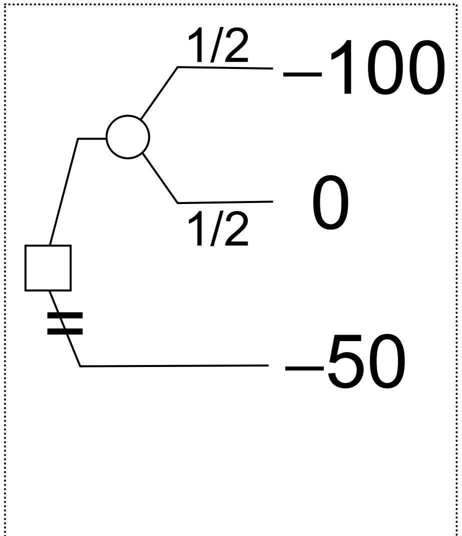
monetary calculus (called "reference independence")



Situation middle

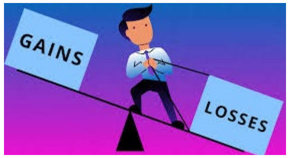
⇔

plausible (called "additivity")



Situation right

Common prefs must violate one of the two principles.



Reference point θ. We scale $\theta = 0$.

$u: \Re \to \Re$ as before.
Basic utility

Scaling: $u(0) = 0$

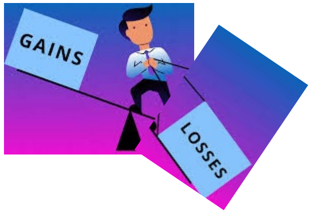
$\lambda > 0$: loss aversion factor

$U: \Re \to \Re$: overall utility

For $\alpha \geq 0$: $U(\alpha) = u(\alpha)$

For $\alpha \leq 0$: $U(\alpha) = \lambda \times u(\alpha)$

Loss aversion: $\lambda \geq 1$



Gain seeking: $\lambda \leq 1$

FIGURE 8.4.1. Loss aversion

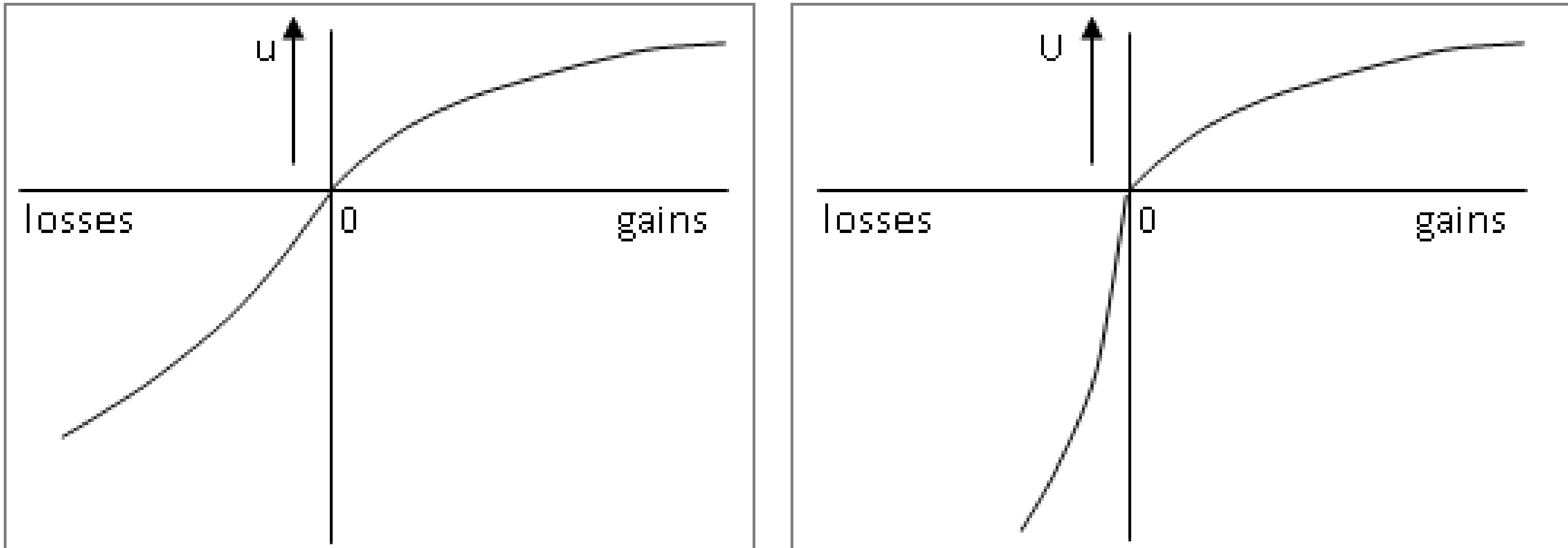


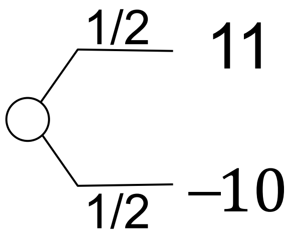
FIG. a. The basic utility u, differentiable at x = 0.

FIG. b. Utility U, obtained by "pulling u down" by a factor λ > 1 for losses.

§8.6

Rabin's (2000) paradox

Assume



$\prec$ 0 (*)

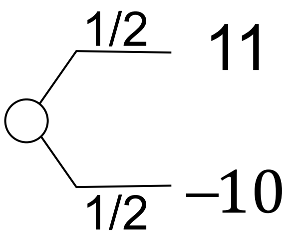
at every wealth level m.

(*) $\Rightarrow$ $\frac{U_m(11)+U_m(-10)}{2} < U_m(0)$, i.e.,

$U_m(11) - U_m(0) \ < \ U_m(0) - U_m(-10)$ for all m.

All U_m a bit concave at 0.
Bit of loss aversion.

Now classical analysis



$\prec$ 0 (*)

$$\Rightarrow \frac{U(11)+U(-10)}{2} < U(0)$$

At every wealth level m:

$$\Rightarrow \frac{U(m+11) + U(m-10)}{2} < U(m)$$

$U(m+11) - U(m) < U(m) - U(m-10)$ for all m.

$$U(m+11) - U(m) < U(m) - U(m-10)$$

$$\frac{U(m+11)-U(m)}{11} < \frac{U(m)-U(m-10)}{11}$$

$$\begin{array}{ccc} \dfrac{U(m+11)-U(m)}{11} & < \dfrac{10}{11} & \boxed{\dfrac{U(m)-U(m-10)}{10}} \\ \rotatebox{90}{$\leq$} & & \rotatebox{90}{$\geq$} \\ U'(m+11) & & U'(m-10) \end{array}$$

$U'(m+11) < \frac{10}{11} U'(m-10)$ for all m.

Increase wealth by 21: U' drops by factor over $\frac{10}{11}$.

Increase wealth by 2100: U' drops > $\left(\frac{10}{11}\right)^{100} \approx 0.00001$.

Decline $M_{0.5}(-100)$ for all M.

?????????

§9.2

Def. of PT

There exist u, U, λ, w^+, w^- s.t.:

For $x = (p_1 : x_1, \ldots, p_n : x_n)$ with
$x_1 \geq \cdots \geq x_k \geq 0 \geq x_{k+1} \geq \cdots \geq x_n$

$PT(x) = \sum_{i=1}^{n} \pi_i U(x_i)$;

if $i \leq k$ $(x_i \geq 0)$:
$\pi_i = \pi\left(p_i^{p_{i-1}+\cdots+p_1}\right) =$
$w^+(p_i + \cdots + p_1) - w^+(p_{i-1} + \cdots + p_1)$;

if j $> k$ $(x_j \leq 0)$:
$\pi_j = \pi\left(p_{j_{p_{j+1}+\cdots+p_n}}\right) =$
$w^-\left(p_j + \cdots + p_n\right) - w^-\left(p_{j+1} + \cdots + p_n\right)$.

Measurement …

Pragmatic measurement of λ:

$\alpha_{0.5}(-1) \sim 0$: then $\lambda \approx \alpha$

Explanation:

$\cancel{w^{+}(0.5)}U(\alpha) + \cancel{w^{-}(0.5)}U(-1) = 0$

$\mathrm{u}(\alpha) + \lambda u(-1) \approx 0$

$u \approx$ linear near 0

$\alpha - \lambda \approx 0$;

$\lambda \approx \alpha$.

$U(1020) \ - \ U(1010) \ < \ U(20) \ - \ U(10)$

How about

$U(-1010) - U(-1020)$? $U(-10) - U(-20)$

Economists: >

Ψs: <

Empirical: Ψs are right.

U is convex on $(-\infty, 0)$!

§9.5 Empirical findings

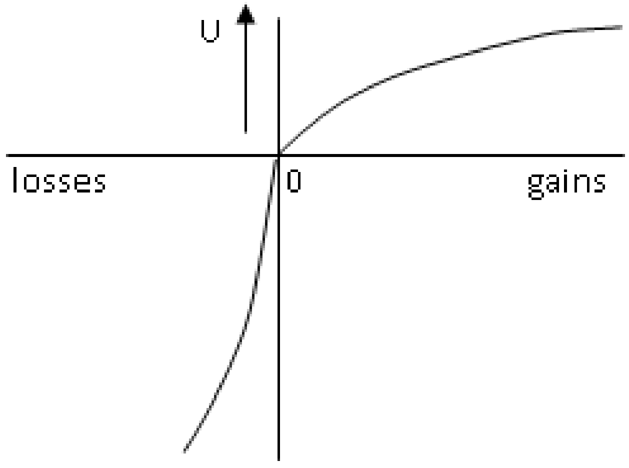
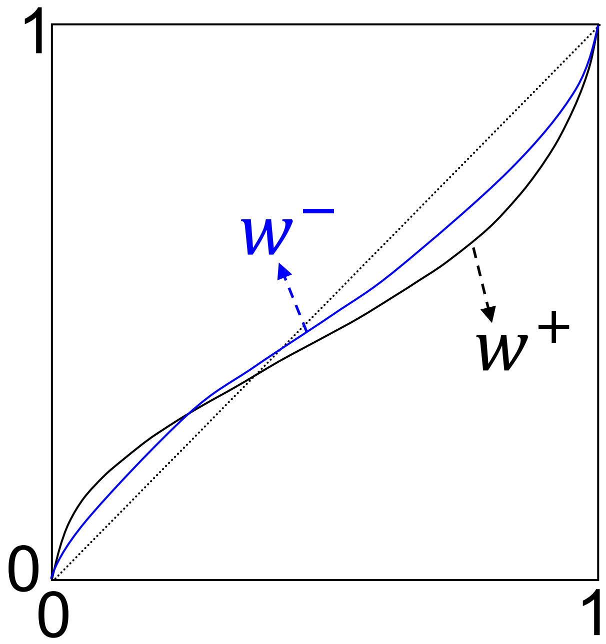
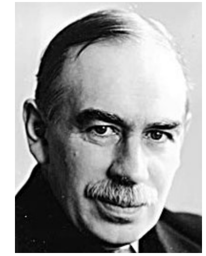


FIG. b. Utility U, obtained by "pulling u down" by a factor $\lambda > 1$ for losses.





Keynes (1921) & Knight (1921):

real uncertainty if new risks.
Unique events.

Financial crises: unforeseen, new events. No hedges.

The Ex Governor of the European Central Bank

- When the crisis came, the serious limitations of existing economic and financial models immediately became apparent. Arbitrage broke down in many market segments, as markets froze and market participants were gripped by panic. Macro models failed to predict the crisis and seemed incapable of explaining what was happening to the economy in a convincing manner. As a policy-maker during the crisis, I found the available models of limited help. In fact, I would go further: in the face of the crisis, we felt abandoned by conventional tools. In the absence of clear guidance from existing analytical frameworks, policy-makers had to place particular reliance on our experience. Judgement and experience inevitably played a key role. Trichet (2010)

Ambiguity ubiquitous in economics/business.
No repeatable experiments with market.
Samuelson & Nordhaus (1985 p. 8) :
"Economists cannot perform the controlled experiments of chemists or biologists because they cannot easily control other important factors."

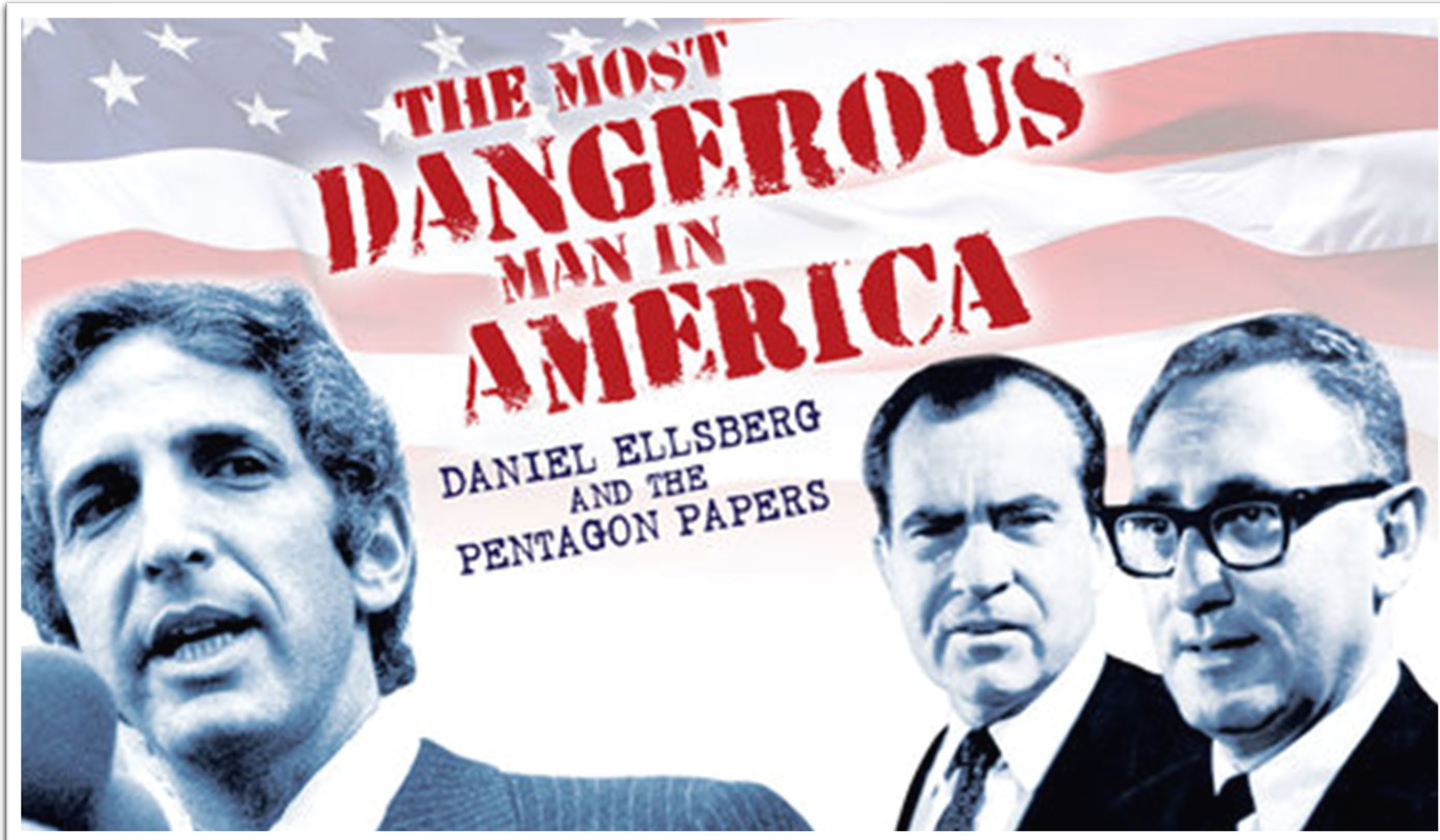
First answer to “how handle ambiguity?”
by Ramsey’31, de Finetti’31, Savage’54:



a) Use probabilities still: subjective ones!
b) Maximize EU

Now you invent a nonEU theory for uncertainty …

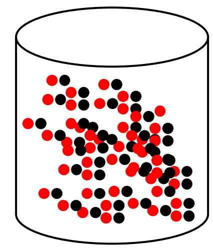
Ellsberg’s (1961) paradox:
Prob^s don’t work.


THE MOST
DANGEROUS
MAN IN
AMERICA
DANIEL ELLSBERG
AND THE
PENTAGON PAPERS

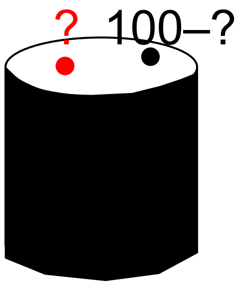
Ellsberg paradox

Known urn K | **Ambiguous urn A**

50 R
50 B



100 R&B
in unknown
proportion



$$(R_K: €20) \succ (R_A: €20)$$
$$(B_K: €20) \succ (B_A: €20)$$

$$P(R_K) > P(R_A)$$
$$P(B_K) > P(B_A)$$
$$\overline{}^{\,+} \qquad \overline{}^{\,+}$$
$$1 > 1$$

This violates subjective probabilities:

Violates subjective probabilities.

Important practical example: homebias

For ambiguity, we need something fundamentally new! Beyond probability.

Hard to invent!

Decision models without probabilities only late 1980s:
Gilboa & Schmeidler ('87, '89)

Hence:
- ambiguity, even though always important, took off only late 1980s
- much to catch up with
- ambiguity popular today
- young researchers may want to work on it!?

W is (event) weighting function if
$W: 2^S \to [0,1]$, $W(\emptyset) = 0$, $W(S) = 1$, $A \supset B \Rightarrow W(A) \geq W(B)$.

For DUU, RDU holds if
there exist weighting function W and utility U s.t.:
If $x_1 \geq \cdots \geq x_n$, then

$$(E_1: x_1, \ldots, E_n: x_n) \to \sum_{j=1}^{n} \pi_j U(x_j)$$

represents $\succcurlyeq$. Here

$$\pi_j = W(E_j \cup \cdots \cup E_1) - W(\overbrace{E_{j-1} \cup \cdots \cup E_1}^{\text{rank of } x_j}). \quad (\pi_1 = W(E_1))$$

π_j also denoted $\pi\left(E_j^{E_{j-1} \cup \cdots \cup E_1}\right)$.
E^R is ranked event $(E \cap R = \emptyset)$
$\pi(E^R) = W(E \cup R) - W(R)$

Repeating

$(E_1 : x_1, \ldots, E_n : x_n) \to \sum_{j=1}^{n} \pi_j U(x_j)$

$\pi_j = W\left(E_j \cup \cdots \cup E_1\right) - W\left(E_{j-1} \cup \cdots \cup E_1\right),\ \left(\pi_1 = W(E_1)\right)$

Exercise. Assume U linear ($U(\alpha) = \alpha$)

How elicit $W(E)$ in experimental heaven?

You work …

Solution: $\alpha \sim 1_E 0$. Then

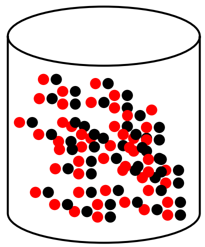
$\alpha = W(E) \times 1 + \left(1 - W(E)\right) \times 0 = W(E)$

Done.

Accommodate Ellsberg paradox

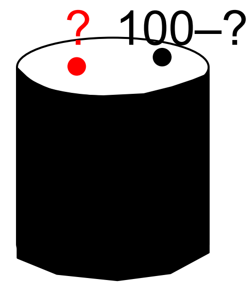
Known urn K

50 R
50 B



Ambiguous urn A

100 R&B
in unknown
proportion



$(R_K: €20) \succ (R_A: €20)$

$(B_K: €20) \succ (B_A: €20)$

$W(R_K) > W(R_A)$

$W(B_K) > W(B_A)$

E.g.,

$W(B_K) = W(R_K) = 0.4;$

$W(B_A) = W(R_A) = 0.3.$

Pessimism: $\pi(E^R)$ ...?

$$R' \supset R \Rightarrow \pi\left(E^{R'}\right) \geq \pi(E^R)$$

Optimism:

$$R' \supset R \Rightarrow \pi\left(E^{R'}\right) \leq \pi(E^R)$$

W convex: pessimism

W concave: optimism

Likelihood insensitivity

$\pi(E^b) \geq \pi(E^R)$ on $[\emptyset, Wrb]$

$[\emptyset, Wrb] = \{E \subset S : \emptyset \preccurlyeq E \preccurlyeq Wrb\}$

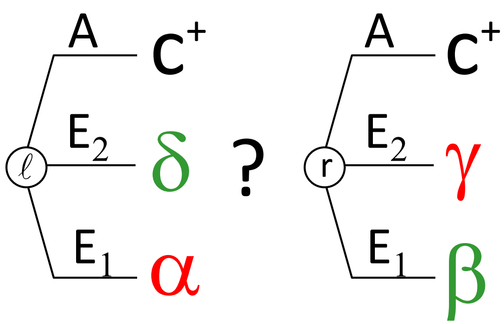
and

$\pi(E^w) \geq \pi(E^R)$ on $[Brb, S]$

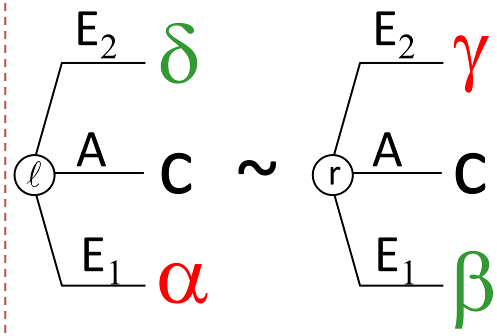
Insensitivity region: $[Brb, Wrb]$

Green: direction you want to go

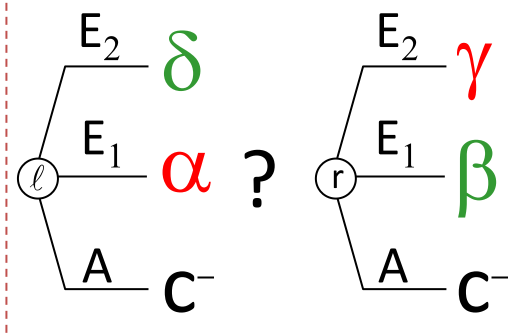
$\delta > \gamma > \beta > \alpha$



Left case:
c replaced by c^+;
$c^+ > \delta$



Reference case:
$\gamma > c > \beta$



Right case:
$\alpha > c^-$

Question: what is preference under *pessimism*?

Answer:

$\succcurlyeq$

Optimism: $\preccurlyeq$

Question: what under *inverse-S*?

Answer:

$\preccurlyeq$

Question: what is preference under *pessimism*?

Answer:

$\succcurlyeq$

Optimism: $\preccurlyeq$

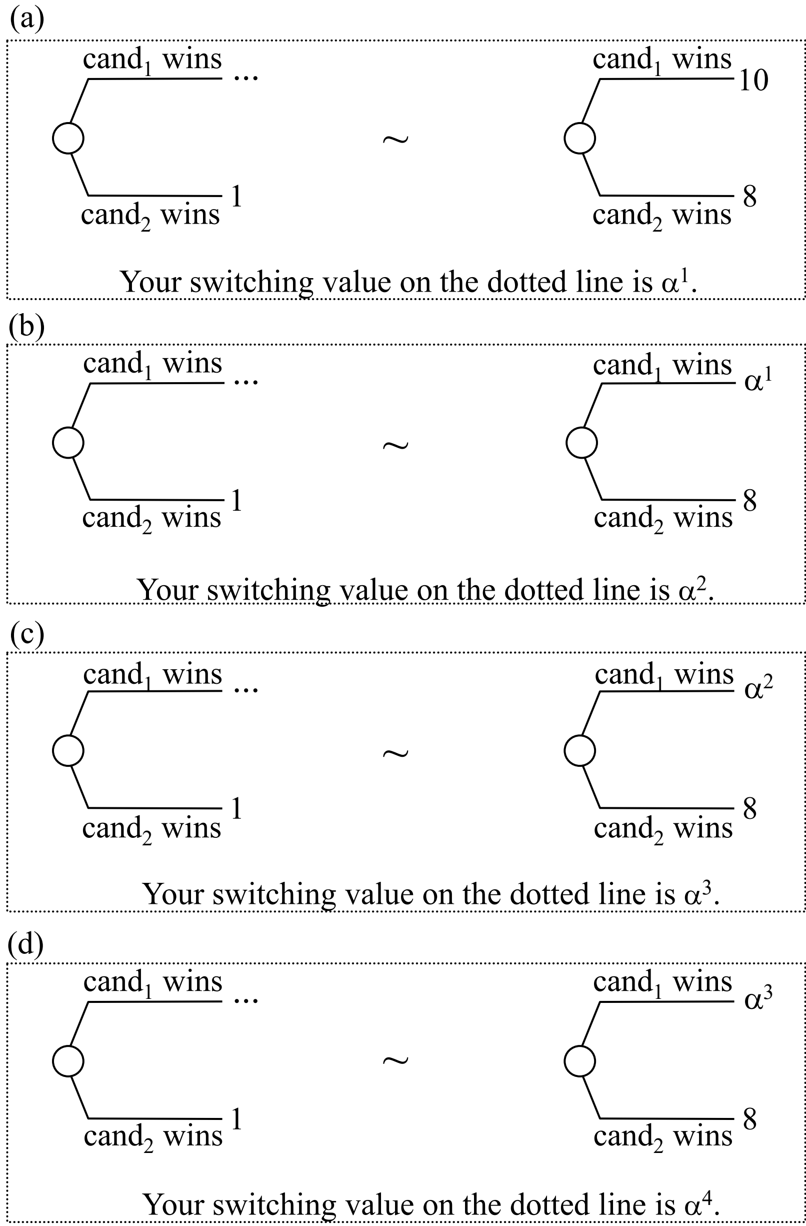
Question: what under *inverse-S*?

Answer:

$\succcurlyeq$

How measure U, W?

Our experiment …



Indicate in each Fig. which outcome on the dotted line ...
makes the two prospects indifferent (the switching value).

Figure 4.1.1 [*TO Upwards*]. Eliciting $\alpha^1 \ldots \alpha^4$ for unknown probabilities

Fig. 4.1.1a:

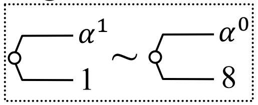
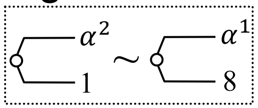


Fig. 4.1.1b:



Set $U(\alpha^0) = 0, U(\alpha^1) = 1/4$.

Replace $w(\frac{1}{2})$ by $W(cand_1)$. Etc.

Calculations under ~~EU~~ RDU ~~(with probs ½)~~

Fig.a: $w(\frac{1}{2})U(\alpha^1) + (1-w(\frac{1}{2}))U(1) = w(\frac{1}{2})U(\alpha^0) + (1-w(\frac{1}{2}))U(8)$

$$w(\tfrac{1}{2})\big(U(\alpha^1) - U(\alpha^0)\big) = (1-w(\tfrac{1}{2}))\big(U(8) - U(1)\big)$$

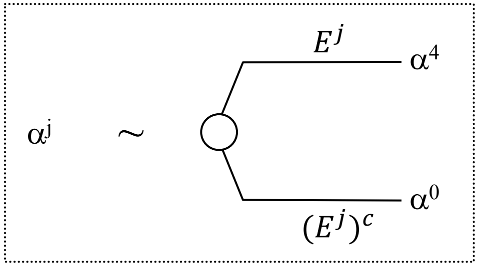
$\|$ $\qquad\qquad\qquad\qquad$ $\|$

Fig.b: $w(\frac{1}{2})U(\alpha^2) + (1-w(\frac{1}{2}))U(1) = w(\frac{1}{2})U(\alpha^1) + (1-w(\frac{1}{2}))U(8)$

$$w(\tfrac{1}{2})\big(U(\alpha^2) - U(\alpha^1)\big) = (1-w(\tfrac{1}{2}))\big(U(8) - U(1)\big)$$

$$U(\alpha^2) - U(\alpha^1) = U(\alpha^1) - U(\alpha^0) \qquad U(\alpha^2) = 2/4!$$

Measurement of W



then $W(E^j) = j/4$

Source: group of events

Sources $\mathcal{A}$ and $\mathcal{B}$

Imagine
$B \succcurlyeq A$.
Systematic preference for $\mathcal{B}$ over $\mathcal{A}$?
Need not be.
Imagine
$B \succcurlyeq A$ & $B^c \succcurlyeq A^c$ (*)
Suggests so.
Imagine
(*) sometimes happens, but
never $A \succcurlyeq B$ & $A^c \succ B^c$ (**)
(**): source preference for $\mathcal{B}$ over $\mathcal{A}$:

$$W(A) \geq W(B) \Rightarrow W(A^c) \leq W(B^c)$$

$\mathcal{A}$ uniform: $W(A) = w_{\mathcal{A}}\big(P(A)\big)$

Accommodate Ellsberg:

Urn $\mathcal{K}$;

$P(B_k) = P(R_k) = 1/2$

$w_{\mathcal{K}}(0.5) = 0.4$

Urn $\mathcal{A}$;

$P(B_a) = P(R_a) = 1/2$

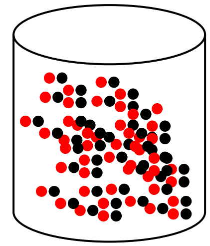
$w_{\mathcal{A}}(0.5) = 0.3$

Revives probability in Ellsberg/ambiguity! (Chew & Sagi'08)

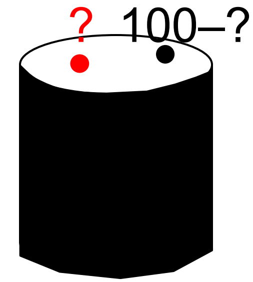
Ellsberg paradox

Known urn K | **Ambiguous urn A**

50 R
50 B



100 R&B
in unknown
proportion



$(R_K: €20) \succ (R_A: €20)$

$(B_K: €20) \succ (B_A: €20)$

$\cancel{P(R_K) > P(R_A)}$

$\cancel{P(B_K) > P(B_A)}$

$\cancel{\overline{}^{+} \;\; 1 > 1} \;\; \overline{}^{+}$

~~This violates subjective probabilities:~~

~~Violates subjective probabilities.~~

Important practical example: homebias

Source function

$\mathcal{A}$ uniform: $W(A) = w_{\mathcal{A}}\big(P(A)\big)$

Experiment

Abdellaoui, Baillon, Placido, & Wakker (2011)
"The Rich Domain of Uncertainty: Source Functions and Their Experimental Implementation,"
American Economic Review 101, 695–723.

Prospect theory for ambiguity:

There exist U (u, λ), W^+, W^- s.t.
for $x_1 \geq \cdots \geq x_k \geq 0 \geq x_{k+1} \geq \cdots \geq x_n$
$(E_1 \colon x_1, \ldots, E_n \colon x_n) \rightarrow \sum_{j=1}^{n} \pi_j U(x_j)$ represents $\succcurlyeq$
$j \leq k \colon \pi_j = \pi^+({E_j}^{R_j})$
$j > k \colon \pi_j = \pi^-\left({E_j}_{L_j}\right)$