Explaining Distortions in Utility Elicitation through the Rank-dependent Model for Risky Choices

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The standard-gamble (SG) method has been accepted as the "gold standard" for the elicitation of utility when risk or uncertainty is involved in decisions, and thus for the measurement of utility in medical decisions. It is based on the assumptions of expected-utility theory. Unfortunately, there is now abundant evidence that expected utility is not empirically valid, and that the SG method overestimates risk aversion and the utilities of impaired health states. This paper shows how rank-dependent utility theory, a newly developed theory in decision science, can explain the main violations of expected utility. Thus it provides a means for correcting the SG method and for improving the assessments of quality-adjusted life years for medical decisions in which there is uncertainty about outcomes. *Key words* health-state utility; risk aversion; rank dependence: medical decision making; standard gamble.
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Risk is a central aspect in medical decision making, as one usually faces uncertainty about the outcomes of a medical decision. Hence this paper examines the measurement of utility as an index for health outcomes that can be used to evaluate decisions involving risk or uncertainty. The three most common methods for utility elicitation in medical decision making are the time-tradeoff (TTO) method, the visual-analog scale, and the standard gamble. The first two do not explicitly involve risk; hence they lack validity for applications in which uncertainty about outcomes is relevant. Risk and uncertainty are the topics of this paper, therefore we concentrate on the third method, the standard gamble (SG).

If a person behaves perfectly well in agreement with expected utility theory, then the SG method yields perfect utility values. Hence the method has been accepted as the "gold standard" for utility elicitation in the medical field. However, critical tests of expected utility in the decision-theory literature have shown that expected utility is not empirically valid,[1-10] and, as is discussed further below, anomalies for the SG method have commonly been demonstrated. No systematic test of the empirical validity of expected utility has yet been reported in the medical-decision-making literature; some indirect evidence is discussed.",'"

It has been observed before in the medical field that people exhibit a systematic dislike for risk that seems to be based on more than the characteristics of outcomes. The term "gambling effect" has sometimes been used to describe that systematic dislike for risk."-'" The classic description through risk aversion in expected utility does not suffice to explain all aspects of the gambling effect, and as a result, cannot explain the commonly found anomalies of the SG method. Explanations have been proposed for the gambling effect,[14,16] but they have not been extended to coherent theories that describe general decision making and incorporate more complex and realistic choice situations than those occurring in the utility-elicitation experiments. This paper presents a new explanation of the gambling effect that does result from a generally applicable and coherent theory.

The basic assumption of the theory discussed in this paper is that subjects pay attention to outcomes in a way that is not proportional to the associated probabilities; that is, probabilities are *transformed* into decision weights. Thus an explanation can be provided for the dimension of risk aversion that cannot be described in terms of valuations of outcomes.

While the idea of probability transformation is old, it has only recently become possible to build coherent theories on this idea, by the rank-dependent-utility approach. This approach has become popular in decision theory in recent years.[5,7-9,17-24] We show how transformed probabilities can explain the commonly found anomalies of the SG method. On the basis of rank-dependent utility, an alternative way is proposed for deriving utility values from SG questions. As an illustration, this alternative way is used to correct the utilities elicited from two testicular cancer patients" for distortions of probability. Finally, a general explanation of rank-dependent utility theory is given in the appendix.

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Distorting Factors for the Standard Gamble

Expected utility (EU) is the standard normative model for decision under risk and uncertainty. In EU one determines quantitative values, "utilities," for outcomes, and probabilities for uncertainties. To evaluate a medical treatment, one multiplies all outcome utilities by their probabilities of occurrence and takes the sum of all these products as the EU value of the treatment. The treatment with the maximum EU is chosen. Axioms to justify the model have been proposed.[25-27]

An example of the SG method is illustrated in figure 1A. In medical decision making, the SG method conventionally adopts the variable-probability-equivalent method: the probability in the gamble is varied until indifference is obtained between the gamble and the certain outcome. Alternatively, one can use a variable-certainty-equivalent method, where an indifference between a certain outcome and a risky gamble is obtained by varying the certain outcome. In medical decision making this method has been used for the elicitation of utility for life years. Henceforth this paper uses the term SG exclusively for the probability-equivalent method; the term certainty-equivalent method is used for the other method.

People generally exhibit a dislike for gambles and seem to value risky outcomes relatively lower than certain outcomes. For instance, people usually prefer ten years of life in good health to a 50-50 gamble resulting in either 20 years of good health or immediate death. That is, people disprefer a gamble for life years over expected length of life. This phenomenon which has been called "risk aversion," leads, given a utility value of 0 for the worst outcome and a utility value of 1 for the best outcome, to a relatively higher valuation of the intermediate outcomes,[28] i.e., to a concave utility.

The above explanation of risk aversion by means of concavity of utility remains within the realm of EU theory, and has been one of the classic EU results. Thus, risk aversion seems to pose no problem for EU theory, and is by itself no challenge to the SG method. However, several problems have been found in relation to the SG method that do question its validity. For instance, if the EU model were descriptively accurate, then the elicitation of utilities through the SG method and the certainty-equivalent method should yield the same utility values, and the utility curves derived from the certainty-equivalent method should be independent of the particular probability used. However, this is not the empirical finding; the SG method usually gives higher utility values than the certainty-equivalent method, and utility curves derived from the certainty-equivalent method are higher as the probability used is closer to 1. This finding has been extensively documented.[21,29-36]

No systematic test of the empirical validity of expected utility has yet been reported in the medical decision-making literature. An indirect test was obtained by Llewellyn-Thomas et al." There, patients' utilities for health states were elicited by means of SGs, first with perfect health as best outcome and death as worst outcome, but next also with other best and worst outcomes. Under EU, the latter SGs should give consistent results, but this was not found. Llewellyn-Thomas et al. did suggest that alternative theories than EU might explain their finding, but their study was not designed to investigate that possibility. An indirect test of EU can also be derived from the utility measurements for life duration described by Stiggelbout et al.[12] Here, in addition to the SG method, an adjusted TT0 measurement was used. In the latter measurement, the "certainty-equivalent method" was used to adjust TTO results and make them agree with expected-utility theory; this method had been introduced by Pliskin et al."' Under expected utility the SG and the adjusted TT0 method should indeed, up to random error, provide identical utility values. The differences that were found were, however, systematic: the SG method yielded higher utility values and thus suggested higher risk aversion; this agrees with the common finding in the decision literature cited above.

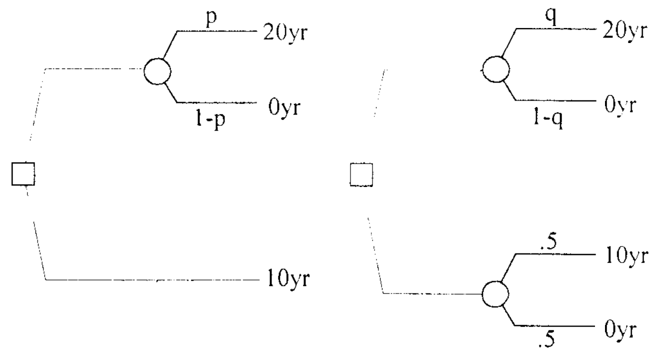

FIGURE 1. (standard gamble). A(left), Find p to give indifference. Under EU. $U(10yr) = p$. B(right), Find q to give indifference. Under ELI, $U(10yr) = 2q$.

In the present paper we argue that both methods give utility values that are too ***high,*** thereby suggesting a degree of risk aversion that is too strong. In general, the dislike for gambles has dimensions that cannot be described by EU with concave utility. Therefore, we search for explanations that relax the restrictions of EU. First we present an example to demonstrate that the classic EU theory does not suffice to explain all aspects of choice under risk.

Let us compare the value p, to give indifference in figure IA, with the value q to give indifference in figure 1B. Notice that, under EU, not only p but also $2q$ should be identical to the utility of ten years in perfect health (setting the utility of 20 years in perfect health equal to 1 and the utility of immediate death equal to 0). The EU of the upper branch in figure 1B is $q \cdot U(20yr) = q$. The utility of the lower branch is $0.5 \cdot U(10yr)$.

Equality results if $2q$ = U (1Oyr). The empirical finding is, however[21,29,30,34]

The value p in figure 1A is usually higher than the value 29 in figure 1B.

This finding obviously falsifies EU; it provides an example of aversion to risk that lies outside the realm of ELJ. Because of aversion to risk, the "riskless gamble" (10 years, perfect health)* is overvalued in comparison with the three remaining risky gambles, and p must be additionally high to make up for this. The utility measurement of figure 1B has been used in the lottery-equivalent method for eliciting utilities, introduced by McCord and de Neufville.[33] McCord and de Neufville did not provide an elaborate theory to justify their method.

In the literature on utility measurement for risky medical decisions, no elaborated models that describe deviations from EU have hitherto been introduced. We briefly discuss two explanations that have been proposed for phenomena such as that in the above example. For the first explanation,'" suppose an individual compares a 50-50 gamble giving either ten healthy years or immediate death with a riskless gamble giving three certain healthy years. The riskless gamble receives utility U (3yr), where U denotes the utility. Under EU theory, the risky gamble is valued by 0.5. U(lOyr) + 0.5 $\cdot U(0\text{yr})$. One way to explain the gambling effect is by subtracting a term Us, to incorporate the disutility of gambling; that is, the gamble is valued by $0.5 \cdot U(10\text{yr})$ + 0.5 $\cdot U(0\text{yr}) - U_g$. For a general theory along these lines, however, it must be specified how the term U_g is determined and in which manner it depends on the gamble g with which it is associated. We are not aware of a general coherent theory that can be developed along these lines.

Let us now turn to a second explanation. Gafni and Torrance'" propose the following utility function for life duration x:

$$[1 - \exp(-\lambda \cdot x)]/k$$

The parameter k determines the scale unit of measurement and is chosen so that the utility of the maximal length of life is 1. The utility of 0 years is 0, independent of the choices of k and A. The constant of interest is A; it is assumed to consist of three terms, i.e., $\lambda = b + r$ + g. Here b is a parameter for the "quantity effect," r for the "time effect," and g for the "gambling effect." The larger A is, the more concave (often interpreted as risk-averse) the utility is. It will be obvious that this utility function, if used in an expected-utility criterion, cannot explain the phenomena of interest to this paper, i.e., violations of expected utility. In particular, the gambling parameter then cannot account for a dislike of risk that extends beyond the valuation of outcomes. Gafni and Torrance do not present a theory in which their utility function could be used that would deviate from expected utility.

Therefore, a theory is needed that explains deviations from expected utility, and solves problems encountered in the SG measurements. Such a theory is presented in the next section.

Non-expected Utllily to Explain the Gambling Effect and to Improve Utility Elicitations

In the decision-theory literature, many models have recently been developed to explain empirical deviations from EU. The models studied most are the rank-dependent models, which formalize transformed probabilities.[17] That idea has a much older history, but the previous theories were hindered by theoretical problems. Further comments are provided in the appendix. For simplicity we describe only the theory for simple choices in the main body of the paper, where the ideas of the theory are easy to understand. A reader interested in the general theory is referred to the appendix, where rank-dependent utility is described in detail. The appendix also demonstrates that rank-dependent utility is a coherent theory that can describe general decision making.

The basic idea of the theory is that people do not treat probabilities in a linear manner, as EU theory supposes, but that people transform probabilities into decision weights. This can of course happen because of psychological misconceptions of numerical probabilities, but it can also be a conscious and deliberate choice of the decision maker. For instance, it could be a deliberate decision to pay relatively more attention to worst outcomes of a decision than to better outcomes. The decision weight for the worst outcome is then larger than its probability. The general dislike of gambling that extends beyond the valuation of outcomes can thus be modeled through transformed probabilities: if one lets the decision weights for bad outcomes be relatively higher than those for good outcomes, then this will systematically decrease the value of risky gambles. An intuitive motivation for rank-dependent utility is that the change in decision attitude, caused by the presence of uncertainty and probabilities, may be more naturally modeled through a freedom in the processing of ***probabilities*** than in the processing of utilities.

We use below a notation such as (p, x) to denote a gamble that with probability p results in x years of life and with probability $1-p$ in immediate death. For instance $(0.5, 20\,\text{yr})$ denotes the gamble that with probability 0.5 results in 20 years of life and with probability 0.5 in immediate death. We start by denoting the prob-

*A gamble for which it is certain what the resulting outcome will be is called a riskless gamble.

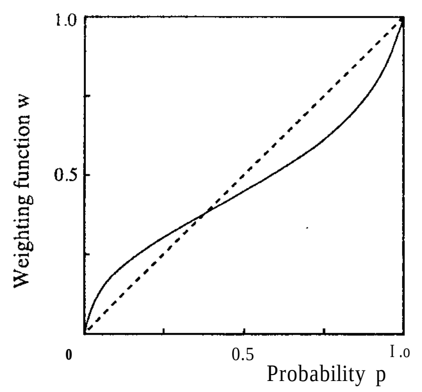

FIGURE 2. Probability transformation. The most commonly found probability transformation.

ability transformation of a person by the symbol w; so for each probability p, w(p) is the transformed value. Natural assumptions are $w(0) = 0$ (no weight for impossibility), $w(1) = 1$ (all weight to certainty), and $w(p) > w(q)$ for $p > q$ (higher weight for higher probability). Now the value of a gamble (p, x) is $w(p) \cdot U(x)$ instead of $p \cdot U(x)$ as it was in EU theory.

We explained the model above for outcomes describing life durations. Rank-dependent utility can similarly be applied to other outcomes. For instance, consider a SG experiment where a patient is indifferent between living 20 years in an impaired health state Q or a gamble that with probability p gives 20 years of life in perfect health, and with probability $1 - p$ gives immediate death. Suppose we set the utility U(20yr, perfect health) equal to 1, and the utility of immediate death equal to 0. Under rank-dependent utility, the value of the gamble is not $p \cdot U$(20yr, perfect health) = p, as it would be under expected utility, but it is $w(p) \cdot U$(20yr, perfect health) = $w(p)$. The value of the "riskless gamble" of living 20 years in impaired health is $w(1) \cdot U$(20yr, impaired health) = $1 \cdot U$(20yr, impaired health) = U(20yr, impaired health), which does not deviate from expected utility. The indifference expressed by the patient implies that the utility U(20yr, impaired health) is given by

$$U(20 \text{ yr, impaired health}) = w(p)$$

which is different from the value p that would be assigned under expected utility.

The form of a probability transformation that is usually found is presented in figure 2; the graph is S-shaped. This shape has been confirmed in many studies.[7,9,18,21,34,38-42] Small probabilities are overestimated, high probabilities are underestimated, and the graph intersects the diagonal around 0.35; the value at 0.5 is approximately 0.42. The largest deviation from the diagonal is around 0.85, and the graph is fairly linear, in accordance with EU, in the middle region. It deviates considerably from EU for small and large probabilities only. This agrees with the common finding that a small probability that changes impossible into possible ($p = 0.1$ instead of $p = 0$), or possible into certainty ($p = 1$ instead of $p = 0.91$ receives more attention than a probability that merely changes possibility into a higher possibility ($p = 0.6$ instead of $p = 0.5$).

Of course, probability transformation will vary from one individual to another, and its elicitation is no easy task.[20,39] It is, however, clear from the decision-theory literature that for the average individual the probability-transformation function is as depicted in figure 2, rather than the identity function as assumed in expected utility. Hence, in general, without further information about an individual's personal probability-transformation function, it will be a definite improvement for the utility elicitation of outcomes if the transformation as depicted in figure 2 is used instead of linear probabilities, as has been the case hitherto in SG experiments. Let us assume now that the transformation in figure 2, denoted w, is the true transformation for a specific patient. We consider below the implications for utility elicitation, first for the SG, then for the certainty-equivalent method, and, finally, for the lottery-equivalent method.

Assume that in a SG a patient chooses a probability p such that (20yr, impaired health) is indifferent to a gamble that with probability p yields (20yr, perfect health) and with probability $1 - p$ immediate death. Then, given the convention U(20yr, perfect health) = 1 and U(immediate death) = 0, the utility U(20yr, impaired health) should be assessed as $w(p)$, rather than as p. For most impaired health states, p will usually be higher than 0.5, so $w(p)$ is lower than p. Then also the true utility $w(p)$ is lower than p, the utility value that is assigned in traditional applications of the SG method. This explains our claim that the traditional SG gives utility values that are too high.

Next we turn to the implications for the certainty-equivalent elicitation method. Here one usually takes a 50-50 gamble, say (0.5, 20yr), and finds a certain outcome that is indifferent to it. Say the outcome (7yr) is indifferent to it. It is custom to derive from this that U(7yr) = 0.5. In absence of further information, however, we suggest using U(7yr) = 0.42, given that the average value for $w(0.5)$ found in the decision literature is 0.42. Notice that this does not deviate much from the customary result of the certainty-equivalent method, because $w(0.5)$ is not very different from 0.5. The probabilities occurring in SG questions are usually higher, and their w-values thus deviate more. This gives a theoretical explanation for the higher utility values that are usually found by SG elicitations than by certainty-equivalent elicitations.

Next, let us consider the indifference (0.5, 10yr) $\sim$ (q, 20yr) as used in the lottery-equivalent method. The rank-dependent model gives $w(0.5) \cdot U(10\text{yr}) = w(q) \cdot 1$,

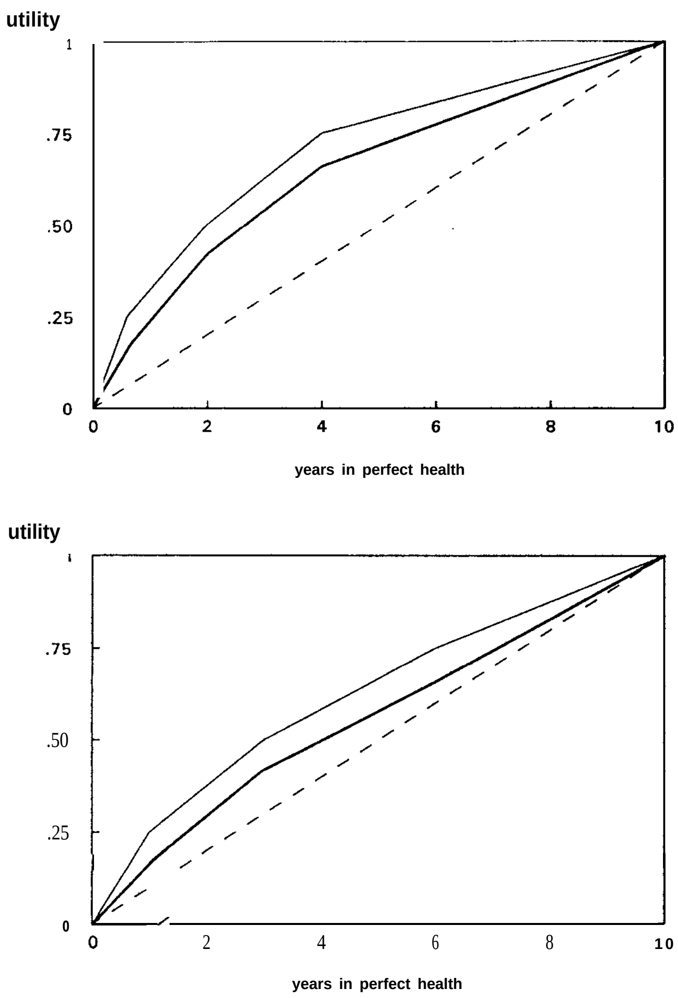

FIGURE 3. **Old** and new **elicited utilities.** The thin curves represent utilities as traditionally elicited. The bold curves describe the utilities after correction for probability distortion.

so U(10yr) = w(q)/w(0.5). Here q will be a value around 0.3, so that w(q) may even be larger than q. As **w(0.5)** is usually somewhat smaller than 0.5, it may be expected that in this range of values the lottery-equivalent method, which sets U(10yr) = 2 q, may even give utility values that are too low.

Mostly, as can be seen from figure 2, the transformed probabilities are lower than the original probabilities, and gambles are therefore valued systematically lower than EU suggests. For small probabilities, the transformed values are higher and a higher preference for lotteries than EU would suggest can be expected. Thus, for short life durations that involve small probabilities in SG questions, one might expect that traditional utility measurement analyses suggest a risk-seeking utility. Verhoef et al.“‘:’ did indeed find risk seeking for short durations of life, although the method of analysis employed there was different from ours.

Figure 3 illustrates two characteristic utility curves for life duration, elicited from testicular cancer patients’” using the certainty-equivalent method. The thin curves in the graphs illustrate the utilities as assessed, with no transformation of probabilities assumed. One of the more risk-averse and one of the less risk-averse patients were chosen for this illustration. The bold curves illustrate the utilities that result when we assume rank-dependent utility with w(0.5) = 0.42. These lead to a lower degree of risk aversion in the utilities, because the risk aversion is now partly incorporated in terms of the processing of probabilities. Given the evidence described above, the bold utility curves are more realistic and hence more appropriate for use in normative decisions than the thin ones.

Our method is obviously ad hoc because no individual probability transformations were elicited from the patients. Still, in the absence of such information, the general average value w(0.5) = 0.42 is preferable to 0.5 as commonly used in utility elicitations, because the latter value is systematically biased upwards! Further arguments for the use of general average values in individual elicitations are given by Camerer and Ho,” who state, regarding elicitations of probability transformations: “and the parameter estimates were remarkably stable across studies.” For these reasons, we recommend the use of curves such as the bold ones in figure 3, based on rank-dependent utility, in utility elicitation, rather than the (thin) conventional curves.

Let us finally summarize the implications of rank-dependent utility for utility elicitation, and the immediate improvement that it makes possible:

- People do not behave according to expected utility, but deviate systematically, and their choices are better described by rank-dependent utility. This means that one should incorporate probability transformations, modeled by a function w(p).

- The function w(p) varies from individual to individual, and its elicitation is no easy task. Abundant evidence has been gathered in the decision literature, however, that the average w function is as depicted in figure 2, rather than the identity function used in expected-utility theory. Therefore, in the absence of further information about the w function of an arbitrary individual, inference based on the w function of figure 2 will, on the average, give better utility assessments than traditional assessments based on expected utility.

- A topic for future research is the shape of the w function in specific medical contexts.

Conclusion

Rank-dependent utility is a coherent theoretical model that does satisfy natural conditions for decision making; these conditions have been established in axiomatizations.“” The theory explains anomalies that have been found in applications of the SG method, in whidh health measures are to be obtained that can be used in evaluations of risky medical decisions. A first improvement for utility elicitation that is immediately suggested by this theory is as follows: An elicited certainty equivalent of a 50-50 gamble does not have a utility value that is the midpoint of the utility values of the two risky'outcomes, but the utility value is 0.42 times the highest utility plus **0.58** times the lowest utility. This leads to a lower risk aversion, thus to decisions where long life durations and good health states are valued relatively higher than in traditional analyses. Further refinements can be obtained by investigations of the probability transformations of individual subjects, and their dependency on the subjects' characteristics and other variables and circumstances. Thus, measurements of quality-adjusted life years and other multiattribute utility measures in the health area can be improved.

We conclude that rank-dependent utility gives a promising new explanation for risk behavior and can lead to improved utility elicitations.

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APPENDIX

General Explanation of Rank-dependent Utility

In the main body of the text we have, for accessibility of the exposition, restricted attention to the simplest possible gambles. In this appendix we briefly describe rank-dependent utility in general. It is presently the model most often used in decision theory to explain deviations from expected-utility (EU) theory, and is based on the idea of expressing risk attitudes not only by the valuation of outcomes by the utility, but also by a valuation of the probabilities through a probability transformation.

The idea of expressing risk attitudes through transformed probabilities is old; the earliest reference that we are aware of is Preston and Baratta (1948).[40] However, the early approaches would transform probabilities for fixed outcomes and use versions of the following formula to value gambles; here we denote by $(p_1, x_1; ; p_n, x_n)$ the gamble giving outcome x_1 with probability p_1, , and outcome x_n with probability p_n:

$$(p_1, x_1; .; p_n, x_n) \rightarrow \sum_{j=1}^{n} w(p_j)U(x_j)$$

This theory never became very popular. Remarkably, only quite recently[19,44] it was discovered that this formula leads to a violation of "stochastic dominance." That is, there exists a gamble such that an increase of one of its payments leads to a decrease of its value. This is a counterintuitive implication, and therefore the formula has been abandoned in the recent decision-theory literature.

The idea of expressing risk attitude by transformed probabilities was, however, kept alive, and was revived by Quiggin," who invented rank-dependent utility. The basic idea is to transform not the probabilities for fixed outcomes, but so-called "cumulative probabilities." As an example, suppose that in the gamble $(p_1, x_1; ; p_n, x_n)$ the outcomes are rank-ordered from most to least preferred, e.g., $x_1 > > x_n$ if outcomes describe life duration. Then $p_1, (p_1 + p_2), , (p_1 + + p_n)$ are the cumulative probabilities, i.e., probabilities of receipt of a value x_j or more. The rank-dependent utility of the gamble now is

$$(p_1, x_1; . . ; p_n, x_n) \rightarrow \sum_{j=1}^{n} \pi_j U(x_j)$$

where $\pi_1 = w(p_1)$, $\pi_2 = w(p_1 + p_2) - w(p_1)$, $\pi_3 = w(p_1 + p_2 + p_3) - w(p_1 + p_2)$, ... $\pi_n = w(p_1 + + p_n) - w(p_1 + + p_{n-1})$. *So,* the "decision weight" π_j associated with outcome x_j is a difference of two transformed cumulative probabilities, first the cumulative probability $p_1 + + p_j$ of receiving x_j or more, second the cumulative probability $p_1 + + p_{j-1}$ of receiving strictly more than x_j. Notice that for the calculation of the decision weights the rank-order of the outcomes is important; this explains the name of the rank-dependent utility theory.

For gambles $(p, x; 1 - p, 0)$ with $U(0\text{yr}) = 0$, the rank-dependent utility reduces to the simple $w(p)U(x)$, and this formula was used throughout the main body of the text. For understanding the formula of rank-dependent utility it is useful to verify that, if w is the identity function, the formula reduces to the classic *EU*. The theory has recently been used to modify prospect theory.[7,20] An extensive discussion is beyond the scope of this