Electronic Companion to
The Midweight Method to Measure Attitudes toward Risk and Ambiguity

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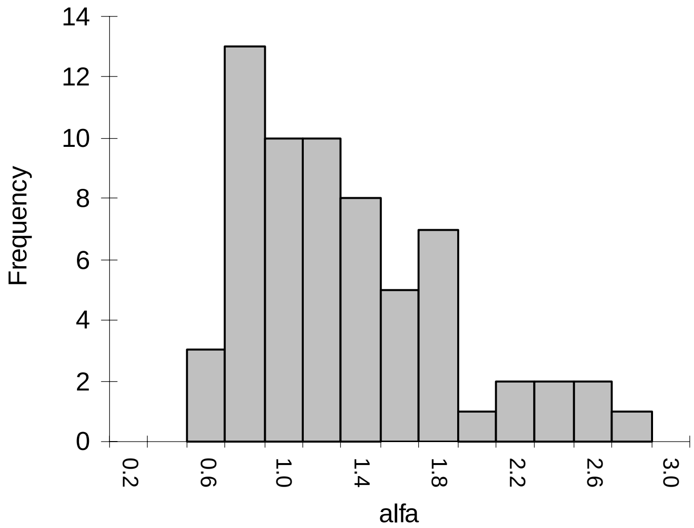
The references in this appendix (Appendixes C-F) can be found in the paper, with the exception of:

Goldstein, William M. and Hillel J. Einhorn (1987), "Expression Theory and the Preference Reversal Phenomena," *Psychological Review* 94, 236–254.

Appendix C. Histograms of Parameter Estimates

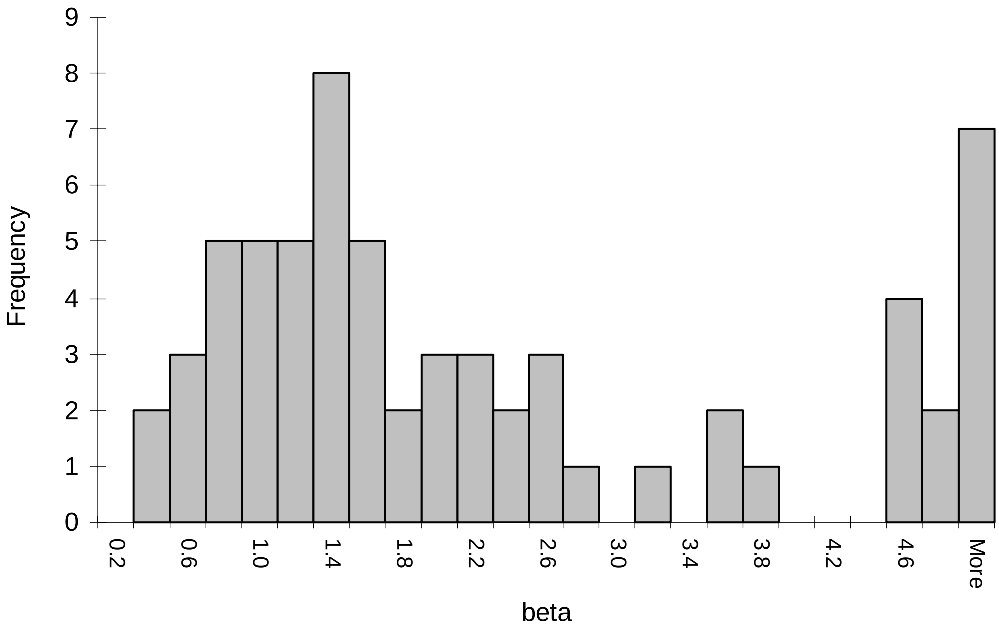
HISTOGRAM C.1. α in risk experiment (§4)

(mean = 1.239; median = 1.145)



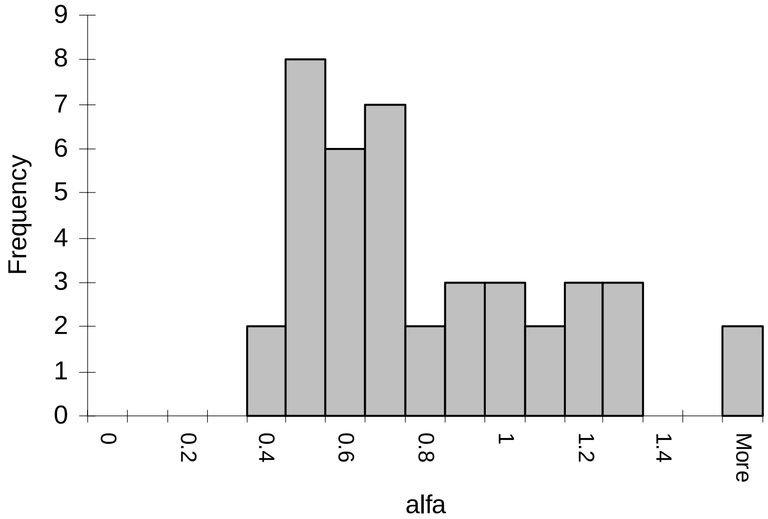
HISTOGRAM C.2. β in risk experiment (§4)

(mean = 3.071; median = 1.578)



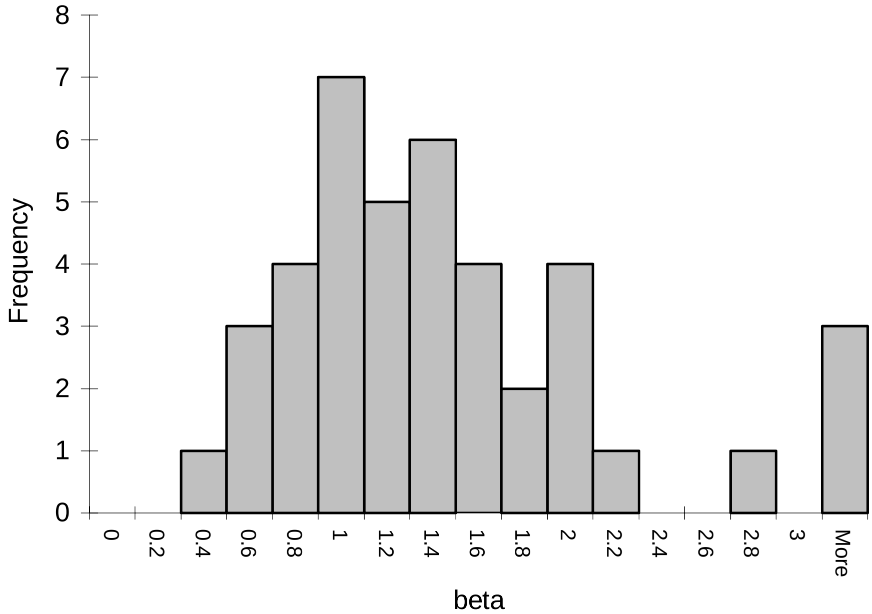
HISTOGRAM C.3. α in uncertainty experiment (§6)

(mean = 0.835; median = 0.684)



HISTOGRAM C.4. β in uncertainty experiment (§6)

(mean = 1.594; median = 1.208)



Appendix D. Further Results on Probability Weighting for Risk (§4)

TABLE D.1. Summary statistics for the elicitation of probability

$w^{-1}(p)$	Mean	Median	Standard deviation
1/8 (=0.125)	0.330	0.285	0.228
2/8 (=0.250)	0.441	0.430	0.223
4/8 (=0.500)	0.608	0.620	0.193
6/8 (=0.750)	0.793	0.820	0.150
7/8 (=0.875)	0.872	0.910	0.132

Parametric Analysis. Several functional forms of the probability weighting function have been proposed in the literature. The most popular one-parameter specifications are the ones proposed by Tversky and Kahneman (1992) and Prelec (1998). The most popular two-parameter functional forms are the ones proposed by Goldstein and Einhorn (1987) and Prelec (1998). The power family, as used by Hey and Orme (1994), has not often been used; a recent application is in Qiu and Steiger (2011). The second column of Table D.2 lists the parametric specifications proposed by the aforementioned authors. The following results will illustrate clearly that patterns found can be driven more by the parametric family chosen than by the actual data.

TABLE D.2. Parameter Estimates

Study	w(p)	Median estimate	Individual distance	Distance from median data
Hey and Orme (1994)	p^{γ}	$\gamma = 1.5041$ (0.28)	0.0515	0.0019
Tversky and Kahneman (1992)	$\frac{p^{\gamma}}{(p^{\gamma}+(1-p)^{\gamma})^{1/\gamma}}$	$\gamma = 2.0521$ (1.24)	0.0556	0.0037
Goldstein and Einhorn (1987)	$\frac{\delta p^{\gamma}}{\delta p^{\gamma}+(1-p)^{\gamma}}$	$\gamma = 1.3054$ (0.10) $\delta = 0.5119$ (0.12)	0.0153	0.0031
Prelec(1) (1998)	$e^{-(-\ln p)^{\alpha}}$	$\alpha = 0.5943$ (0.31)	0.1174	0.0752
Prelec(2) (1998)	$e^{-\beta(-\ln p)^{\alpha}}$	$\alpha = 1.1454$ (0.07) $\beta = 1.5781$ (1.54)	0.0161	0.0022

Standard errors are in parentheses.

For each subject and each parametric specification, we estimated the optimal parameter values by minimizing sums of squared distances:

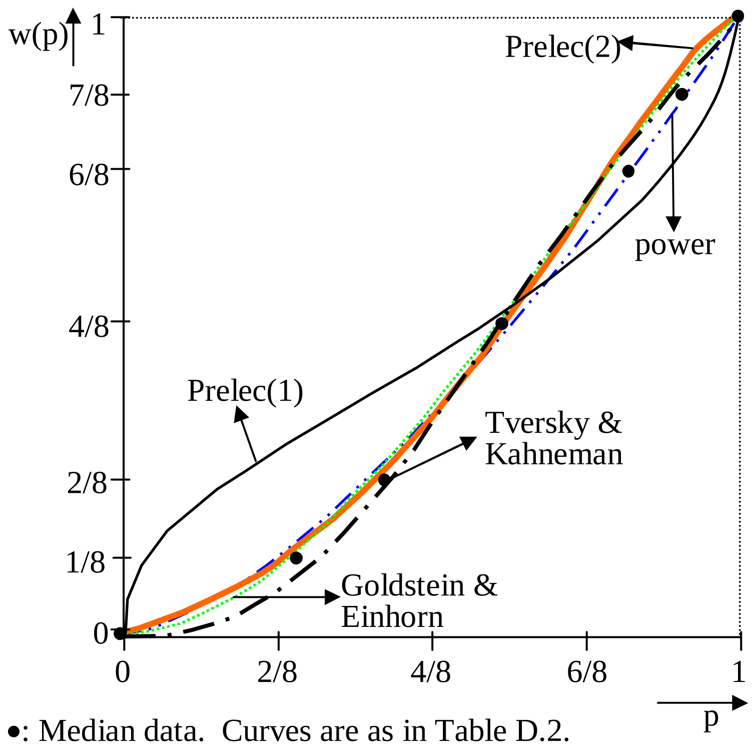
$$\sum_{i=1}^{5}(w_i - \hat{w}_i)^2 . \tag{D.1}$$

Here w_i is the i-th element of the sequence of the probability weights for which the probabilities (w-inverses) were elicited and $\hat{w}_i$ is the i-th element of the estimated sequence of probability weights under the various parametric specifications. For example, for $w_1 = 1/8$, we define p_1 as the probability elicited in the experiment as $w^{-1}(1/8)$ for this subject, and then $\hat{w}_1$ is the value that the probability weighting function of the parametric family assigns to p_1. Its distance from 1/8 indicates how far the family is from the data.

To avoid convergence to local minima we used a wide variety of starting points. The medians of the individual estimators as well as the standard errors are reported in the third column of Table D.2 and the corresponding weighting functions are plotted in Figure D.1. Table D.2 also gives the average sum of squared distances if for each individual the optimal parameters are determined, as well as the squared distances of the optimal fits from the median data.

Figure D.1 displays a remarkable variety of patterns, which should caution against the unqualified use of parametric fitting. The one-parameter family advocated by Prelec (1998), denoted Prelec(1), displays the most commonly found shape, the inverse-S shape. It is, however, clear that the function is completely off, and its distances from the data in Table D.2 are extreme. This family necessarily imposes the inverse-S shape and intersects the diagonal at (1/e,1/e), which explains part of its bad performance. The one-parameter family by Tversky and Kahneman (1992) and the two-parameter families by Prelec (1998) and Goldstein and Einhorn (1987) show a similar pattern: mostly convex but, being oriented to inverse-S like phenomena, they go the other way and to some extent display the opposite pattern, being a slight S shape. Finally, the power family yields a convex function.

FIGURE D.1. Parametric fittings of median



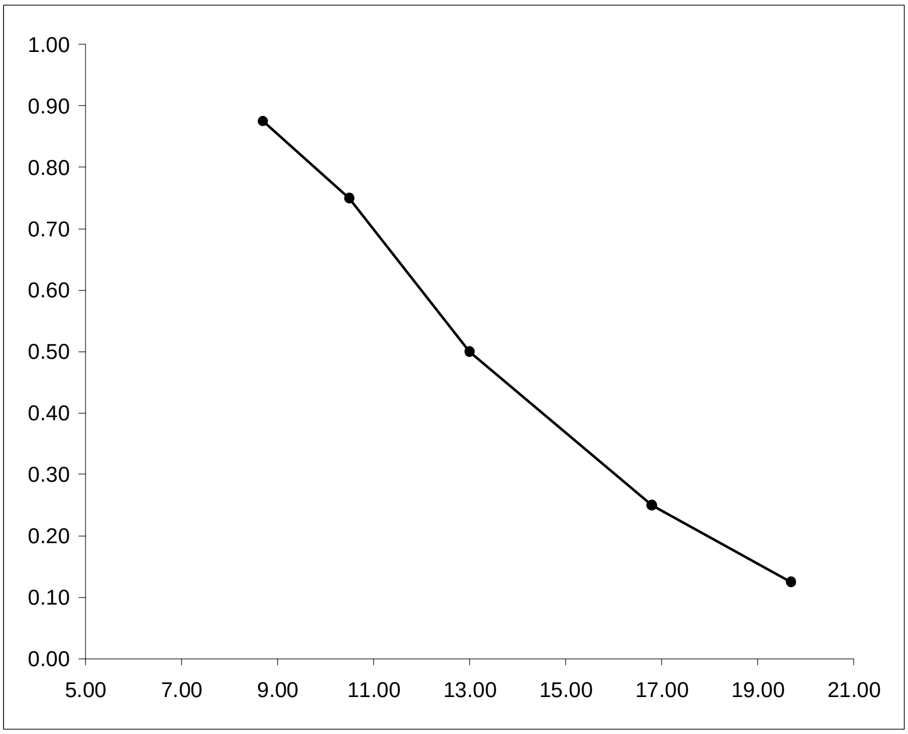
•: Median data. Curves are as in Table D.2.

The average sum of squared distances reported in the 4^{th} column of Table D.2 suggests that of the one-parameter families, the family of Tversky and Kahneman performs considerably better than Prelec's, but the power family performs best. The two two-parameter families perform very similarly, and have a smaller distance than the one-parameter families which is no surprise given their larger number of free parameters. The optimal fits for the median data give similar results with one exception: the power-family now yields the best fit, even better than the families with an extra parameter.

Appendix E. Values t_j Measured for W (§5)

Table E.1: Summary statistics

	Mean	Median	Standard deviation
t_1	19.06	19.75	2.21
t_2	16.75	16.85	2.41
t_4	13.22	13.00	2.81
t_6	10.96	10.25	2.97
t_7	9.70	8.55	2.53



Appendix F. Further Comments and Statistics

Removing subjects for risk (§4). In the risk experiment we removed all subjects for whom $w^{-1}(4/8) > 0.99$, $w^{-1}(6/8) > 0.99$, $w^{-1}(4/8) < 0.01$, or $w^{-1}(6/8) < 0.01$. These subjects, in all five questions of the bisection measurement, chose left or right. They did not pay attention to the stimuli. It also made no sense for these subjects to measure $w^{-1}(p)$ for some other probabilities p. For example, if $w^{-1}(6/8) > 0.99$ then surely $w^{-1}(7/8) > 0.99$ and no more measurement of $w^{-1}(7/8)$ can or should be made.

Bisection method to measure x_1 and x_2 for risk (§4). The particular bisection method that we used is similar to the one used by Abdellaoui (2000), and is explained next. To obtain x_1 in $x1_{0.25}30 \sim x0_{0.25}40$, we iteratively narrowed down what we call indifference intervals containing the indifference value of x_1 as follows. Based on extensive pilots, we assumed that x_1 would not exceed $x_0 + 96$ and took $[x_0, x_0 + 96)$ as the first indifference interval, denoted $[\ell^1,u^1)$. To construct the j+1th indifference interval from the jth indifference interval $[\ell^j,u^j)$, we observed the choice between $(\ell^j + u^j)/20_{.25}30$ and $x0_{0.25}40$. A left choice meant that the midpoint $(\ell^j + u^j)/2$ exceeded x_1, so that x_1 was contained in $[\ell^j,\frac{\ell^j + u^j}{2})$, which was then defined as the j+1th indifference interval $[\ell^{j+1},u^{j+1})$. After a right choice we similarly took $[\frac{\ell^j + u^j}{2},u^j)$ as the j+1th indifference interval $[\ell^{j+1},u^{j+1})$. We did five iteration steps, ending up with $[\ell^6,u^6)$ (of length $96 \times 2^{-5} = 3$), and took its midpoint as the elicited indifference value x_1. We similarly elicited x_2 (substitute x_2 for x_1 and x_1 for x_0 above).

Removing subjects for uncertainty (§5). In the uncertainty experiment we removed 2 subjects who always chose left or right in the elicitation of $W^{-1}(2/8)$, suggesting that their weight for the event of temperature exceeding 7.4^0C exceeded 2/8, whereas they had been informed that the temperature has never been below 7.4^0C. For these subjects, $W^{-1}(1/8)$ can and should no more be asked. We also removed a subject who always chose left (or right) when measuring matching probabilities. His answers would imply that a temperature between 15.8^0C and 16^0C was as likely as a temperature exceeding 16^0C.

Comparisons between subjective and objective probabilities (§§5-6). Further comparisons between the event E, used to measure utilities under uncertainty, (see footnote 2) and objective probability 0.25 used for this purpose under risk are here:

Risk: mean w(0.25) = 0.163; median w(0.25) = 0.108; median x_1 = 92.25; median x_2 = 123.

Uncertainty: mean P(E) = 0.406; median P(E) = 0.375; mean W(E) = 0.358; median W(E) = 0.310; median x_1 = 77.25; median x_2 = 91.50.

We tested the mean subjective probabilities found in the uncertainty experiment against the historical probabilities (Figure 12), and the subjective probabilities all significantly exceeded the historical probabilities:

p = 1/8: historical 17.44^0C, subjective 18.1^0C; Z = 3.04, $p = 0.002$;
p = 2/8: historical 15.5^0C, subjective 16.8^0C; Z = 5.25, $p < 0.001$;
p = 4/8: historical 13.120C, subjective 14.8^0C; Z = 5.03, $p < 0.001$;
p = 6/8: historical 11.56^0C, subjective 12.7^0C; Z = 3.60, $p < 0.001$;
p = 7/8: historical 10.71^0C, subjective 11.1^0C; Z = 2.55, $p = 0.01$.

P-values in Table 3 for one-sided Wilcoxon signed-rank tests: W = 1/8: p = 0.0442; W = 2/8: p = 0.0689; W = 4/8: p = 0.00085; W = 6/8: p = 0.0005; W = 7/8: p = 0.0001.

Error propagation in chained measurement. §9 presents some results on a simulation test of error propagation. The following table gives details.

Variable	#Obs[§]	Mean	Std.Dev.	Min	Max
x_0	1000	60	0	60	60
x_1	1000	86.35	1.35	82.08	90.25
x_2	1000	113.76	1.97	107.32	119.43
$w^{-1}(0.125)$	1000	0.077	0.016	0.039	0.138
$w^{-1}(0.25)$	1000	0.202	0.027	0.130	0.289
$w^{-1}(0.50)$	1000	0.494	0.036	0.397	0.625
$w^{-1}(0.75)$	1000	0.776	0.046	0.592	0.907
$w^{-1}(0.875)$	993	0.902	0.045	0.741	0.997

Appendix G. References Finding Inverse-S

{% **inverse-S**: this is indeed found for 62.5%. 30% had convex prob transformation, rest linear. P. 1507: bounded SA is confirmed %}

Abdellaoui, Mohammed (2000), "Parameter-Free Elicitation of Utility and Probability Weighting Functions," *Management Science* 46, 1497–1512.

{% **probability elicitation** ; **inverse-S; ambiguity seeking for unlikely** %}

Abdellaoui, Mohammed, Aurélien Baillon, Laetitia Placido, & Peter P. Wakker (2010), "The Rich Domain of Uncertainty: Source Funcions and Their Experimental Implementation," *American Economic Review,* forthcoming.

{% **Risk averse for gains, risk seeking for losses**: they find this.

inverse-S: find it, both for gains and losses, fully in agreement with the predictions of PT.

Use a measurement method where utility is measured through parametric fitting, assuming power utility. %}

Abdellaoui, Mohammed, Han Bleichrodt, & Olivier L'Haridon (2008), "A Tractable Method to Measure Utility and Loss Aversion under Prospect Theory," *Journal of Risk and Uncertainty* 36, 245–266.

{% **inverse-S** & **uncertainty amplifies risk**: confirm less sensitivity to uncertainty than to risk. This implies: **ambiguity seeking for unlikely**

(**Concave utility for gains, convex utility for losses**:) gives concave utility for gains (power-fitting gives power of about 0.88 on average) and some convex, but close to linear, utility for losses.

W satisfies bounded SA (= inverse-S extended to uncertainty) for almost all subjects. Bounded SA is similar for gains and losses, but elevation is larger for losses. Bounded SA also holds for the factor B (p. 1395 bottom of first column), and for w, so that all common hypotheses of diminishing sensitivity of Fox & Tversky (1998), Tversky & Fox (1995), Wakker (2004), and others are confirmed. One small deviation is that for losses they find overweighting of unlikely events but no significant underweighting of likely events (§5.4, p. 1394). P. 1398: "The similarity of the properties of judged probabilities and choice-based probabilities comes as good news for the link between the psychological concept of judged probabilities and the more standard economic concept of choice-based probabilities." Pp. 1398-1399 top has nice texts on status of source preference, as comparative phenomenon that may not be part of transitive individual choice.

TO method's error propagation: do so on p. 1394, §5.3 end. %}

Abdellaoui, Mohammed, Frank Vossmann, & Martin Weber (2005), "Choice-Based Elicitation and Decomposition of Decision Weights for Gains and Losses under Uncertainty," *Management Science* 51, 1384–1399.

{% **inverse-S** is found. Bettor's subjective probs are estimated from portion of money bet on a horse. Objective probs are estimated from percentage of times that some horse (say favorite, or no. 5-favorite, etc.) wins. Thus, bettors overestimate small probs of winning and understimate large probs. of winning.

Uses power family to estimate utility and find that bettors are risk seeking (P.s.: no wonder, for horse race bettors!%}

Ali, Mukhtar M. (1977), "Probability and Utility Estimates for Racetrack Betting," *Journal of Political Economy* 85, 803–815.

{% three out of four participants show **inverse-S** prob. weighting%}

Allais, Maurice (1988), "The General Theory of Random Choices in Relation to the Invariant Cardinal Utility Function and the Specific Probability Function." *In* Bertrand R. Munier (Ed.), *Risk, Decision and Rationality*, 233–289, Reidel, Dordrecht, the Netherlands.

{% **inverse-S**: Cites literature that find inverse-S shape. Does a first experiment in which participants' behavior confirms that they relatively overvalue longshot lotteries (so small prob. for gain). Payments was in "points" (not explained more). Unfortunately, the gambles always seem to deal with both gains and losses so loss aversion plays a role. Then comes the second experiment. Participants are first asked for estimations of prob. and it seems that they !under!estimate small probs and they !over!estimate bigger ones. However, not much explanation is given about experimental details there seem to be many complicating factors. For instance, probs are measured by having participants indicate percentages of occurrences of events when repeated 100 times. First they are asked to calculate the mathematical answer, then they are asked what they think will really be the percentage. They also choose between gambles but it is repeated choices

and they seem to play for totals of points. In this second experiment, no clear relation between gambling behavior and estimated probabilities was found. %}

Attneave, Fred (1959), “A Priori Probabilities in Gambling,” *Nature* 183, 842–843.

{% **inverse-S**: Seem to find it, with overestimation of low probabilities and underestimation of high. %}

Beach, Lee R. and Lawrence D. Phillips (1967), “Subjective Probabilities Inferred from Estimates and Bets,” *Journal of Experimental Psychology* 75, 354–359.

{% “Squiggle Hypothesis” for **probability triangle** supports **inverse-S** weighting functions; intersection point, however, seems to be below .16 iso .33. That is, at .16 their observations already suggest convex probability transformation; leads him to question RDU. %}

Bernasconi, Michele (1994), “Nonlinear Preference and Two-Stage Lotteries: Theories and Evidence,” *Economic Journal* 104, 54–70.

{% **inverse-S**: find that (Fig. 11, p. 341). %}

Birnbaum, Michael H., Gregory Coffey, Barbara A. Mellers, and Robin Weiss (1992), “Utility Measurement: Configural-Weight Theory and the Judge’s Point of View,” *Journal of Experimental Psychology: Human Perception and Performance* 18, 331–346.

{% They found preference for equality in sense of overweighting of the worst-off, but also: **inverse-S**: people overweight the richest and poorest, suggesting insensitivity to groupsize. Insensitivity dominated pessimism, so that the typical inverse-S shape resulted. The authors then advance an interesting argument: Insensitivity is a cognitive limitation at the level of numerical misperception, so that it is reasonable to correct for it. They present the equity weighting that results after doing so, which is, obviously, convex and pessimistic. %}

Bleichrodt, Han, Jasan Doctor, and Elly Stolk (2005), “A Nonparametric Elicitation of the Equity-Efficiency Tradeoff in Cost-Utility,” *Journal of Health Econonomics* 24, 655–678.

{% **inverse-S**: they find that, doing it for health outcomes instead of monetary. The curve is more elevated/curved than for money. Table 1, p. 1488, gives a convenient listing of studies of prob. weighting. They clearly find inverse-S, more than for monetary experiments. P. 1492 bottom of 2nd column: They find more bounded S.A (so lower and

upper SA) than monetary experiments did. Strangely enough, p. 1493/1494 finds slightly more lower SA than upper SA in one analysis, slightly less in another. So, roughly, it looks equal.

P. 1494 1st column: The find approximately linear prob. weighting in the middle region.

p. 1495: compares fit of different parametric weighting function families.

Weighting function for health is both more elevated (abstract, p. 1495; higher δ in Table 4) and more inverse-S (p. 1492 bottom; lower γ in Table 4) than commonly found for money. %}

Bleichrodt, Han and José Luis Pinto (2000), “A Parameter-Free Elicitation of the Probability Weighting Function in Medical Decision Analysis,” *Management Science* 46, 1485–1496.

{% **inverse-S**: find that because incorporating inverse-S probability weighting improves utility measurement. %}

Bleichrodt, Han, Jaco van Rijn, and Magnus Johannesson (1999), “Probability Weighting and Utility Curvature in QALY-Based Decision Making,” *Journal of Mathematical Psychology* 43, 238–260.

{% **inverse-S**: confirm it. In exp. 3 elicited certainty equivalents for some gambles (hypothetical only) using ping-pong à la Tversky and Fox (1995), only for one nonzero outcome. Assume that utility is $x^{0.88}$ and then find inverse-S w confirmed. Do not say whether or not they used real incentives. %}

Brandstätter, Eduard, Anton Kühberger, and Friedrich Schneider (2002), “A Cognitive-Emotional Account of the Shape of the Probability Weighting Function,” *Journal of Behavioral Decision Making* 15, 79–100.

{% Uses real incentives for gains; **losses from prior endowment**;

Determine CEs from choice lists, and fit PT. Do mixture models. Optimal result is with 2 groups, one (20%) doing EV and the other doing PT with all the patterns of T&K 92 confirmed:

Concave utility for gains, convex utility for losses;

inverse-S; find it using Einhorn&Hogarth family.

Risk averse for gains, risk seeking for losses

Have no mixed prospects and, hence, model and measure no loss aversion.

For gains, Chinese students are more optimistic and more likelihood insensitive than Swiss students. They also have more concave utility and, because CE data may not

separate utility well from probability weighting (colinearity), it was not clear to me to what extent the higher concavity of utility drives the lower probability weighting.
The authors are happy about each subject clearly falling in one of the two categories (w, probability weighting, linear or nonlinear). I did not understand what else could happen than these two. There are few subjects of the "ambiguous type" (between the two categories, with p = 0.4 of being one catefory and p = 0.6 of being the other, as an example they give) but I don't know if there probabilistic models give much space to such types in, say, randomly generated choices for instance. %}

Bruhin, Adrian, Helga Fehr-Duda, and Thomas Epper (2010), "Risk and Rationality: Uncovering Heterogeneity in Probability Distortion," *Econometrica* 78, 1375–1412.

{% **inverse-S**: Pp. C33-C34, Section 3.3, refers to Hsu et al. (2005) for neuroeconomic evidence supporting inverse-S probability weighting. P. C34 also explains the "three-valued logic" of probability weighting.
%}

Camerer, Colin F. (2007), "Neuroeconomics: using Neuroscience to Make Economic Predictions," *The Economic Journal* 117, C26–C42.

{% p. 188: **inverse-S**; (on (parameter)-estimation of weighting functions: "These estimates are remarkably close to the estimate ... for CPT," and Figure 7 (plotting the Tversky and Kahneman (92) function for the parameter found by Camerer and Ho)
p. 191: "and the similarity of the probability weighting estimates across eight studies suggest ..." %}

Camerer, Colin F. and Teck-Hua Ho (1994), "Violations of the Betweenness Axiom and Nonlinearity in Probability," *Journal of Risk and Uncertainty* 8, 167–196.

{% **inverse-S**%}

Cohen, Michèle and Jean-Yves Jaffray (1988), "Certainty Effect versus Probability Distortion: An Experimental Analysis of Decision Making under Risk," *Journal of Experimental Psychology: Human Perception and Performance* 14, 554–560.

{% **uncertainty amplifies risk**: seem to say that;
An impressive paper on ambiguity. Probably the first to seriously put forward the concept of likelihood insensitivity/inverse-S, although empirical studies such as Preston & Baratta (1948) had found the phenomenon before. Those empirical studies did not

discuss the concepts though.

They use an anchoring-and-adjustment model for ambiguity. First there is an anchor probability p_A of event A, presented in their stimuli. The decision weight is a transform $S(p_A)$, where the transform reflects ambiguity about p_A and the decision-maker's attitude towards this. This anchoring-and-adjustment makes sense for the stimuli that the authors use, where always an anchoring probability is salient; and it can be put on the x-axis for graphs. It does not hold for ambiguity in general, because in many situations of ambiguity there is no particular anchor probability.

To discuss attitudes towards an ambiguous probability, it is useful to specify things about the outcome associated with the outcome (is it a favorable outcome or an unfavorable one?). Remarkably, however, in their initial discussion on pp. 436-437 the authors don't specify the associated outcome, taking ambiguity as if something in its own right and independent of decisions or outcomes.

P. 437 clumn 1 ℓ. 12-13: "Attitude toward ambiguity is denoted by β, ..." (where β reflects elevation and not inverse-S). Thus, only β reflects attitude and not θ (θ reflects inverse-S). Indeed, inverse-S is perceptual/cognitive and not motivational, as confirmed by Hogarth (personal communication, March 9, 2007, 11:55 AM, in Barcelona).

The authors take $S(p_A) = (1-\theta)p_A + \theta(1-p_A^{\beta})$ (Eq. 6b, p. 437). The parameter θ reflects degree of inverse-S (for β=1 a large $\theta \le 0.5$ move the weight towards 0.5; the authors assume $\theta \le 1$ but $\theta > 0.5$ does not make much sense, leading to weights decreasing in p_A for β=1), and β reflects source preference.

inverse-S is found; **ambiguity seeking for unlikely**: p. 435 cites Ellsberg on it and p. 439 Gärdenfors & Sahlin (1982); their model has it also (e.g., Fig. 2). Their data "confirm" their model, though they don't discuss the issue of ambiguity seeking for unlikely events explicitly in the results and discussion. That is, the paper does not make clear if there is ambiguity seeking for unlikely. p. 453: Judged probs show inverse-S shape, and choices suggest transformation downwards of judged prob.

When they use the term "source" they mean something like an expert, being a source of information about the uncertain states of nature. So source does not have the same meaning as in the works initiated by Tversky in the early 1990s.

Most of their tests are on non-choice-based data. Experiment 3 tests predictions of their model for prospect choices, but uses a very weak test (whether their model is better than completely random choice).

%}

Einhorn, Hillel J. and Robin M. Hogarth (1985), "Ambiguity and Uncertainty in Probabilistic Inference," *Psychological Review* 92, 433–461.

{% **inverse-S** is found for losses, both large and small; also upper and lower subadditivity are. %}

Etchart, Nathalie (2004), "Is Probability Weighting Sensitive to the Magnitude of Consequences? An Experimental Investigation on Losses," *Journal of Risk and Uncertainty* 28, 217–235.

{% **TO method**; Uses the method of Abdellaoui (2000) to measure probability weighting. N=30 subjects. Flat payment. Section 3.1 suggests that shallow probability weighting in the middle can be strategically, in cases the distinction does not matter for decisions.

inverse-S: is confirmed.

Section 3.2, around Table 2, retrospectively gives another interpretation for a deviating finding in her 2004 paper. Investigate influence of level of outcomes (all high or all low) and spacing (big if some outcomes are low and others are high) only for losses. Find that for moderate and high probabilities there is some influence, with more pessimism for high spacing.

%}

Etchart-Vincent, Nathalie (2009), "Probability Weighting and the 'Level' and 'Spacing' of Outcomes: An Experimental Study over Losses," *Journal of Risk and Uncertainty* 39, 45–63.

{% **inverse-S**: confirm it both for gains and for losses, using Einhorn and Hogarth two-parameter family.

Risk averse for gains, risk seeking for losses: find it well confirmed.

Experiment in China with real incentives for Chinese students (N = 153), using a finite mixture regression model. Stakes were like 1-hour wage (low-stake) versus 40-hour wages (high-stake). Always choice between sure outcome and 2-outcome prospect in choice lists to get CEs. Use the Golstein and Einhorn (1987) two-parameter family for probability weighting, and power-utility.

Unfortunately, they implemented two choices for real for each subject, being one for high-stake and one for low-stake (the high-low stake comparison is within-subject). It will, unfortunately, amplify a contrast effect with subjects simply taking low-stakes not very seriously. Not much can be done about this (other than do between-subject).

P. 154 footnote 5 properly points out that loss aversion does not affect choices between losses under PT; this paper only considers nonmixed prospects.

Point out that measurements of utility and risk aversion, and investigations of whether risk aversion is decreasing or increasing and whether concavity of utility is decreasing or increasing, cannot be settled properly if there is no correction for probability weighting and other things. Find increase in relative risk aversion for gains, but find that this is primarily driven by different probability weighting for high outcomes than for low. The latter entails a violation of prospect theory. No increase or decrease but constant attitude is found for losses.

Losses with real incentives are implemented in an unconventional way: For each gain-choice there was a corresponding loss-choice that consisted of first a (choice-situation-dependent!) prior endowment and then the losses-choice, such that after integration of the endowment with the loss-choice the loss-choice was the same as the gain-choice. So differences between gains and losses are a matter of framing, and this is how the authors often refer to it. Discussion of it on p. 170.

P. 151 top references several studies showing that heterogenous models can be really off. They find 1/4 subjects doing EV, and 3/4 PT. %}

Fehr-Duda, Helga, Adrian Bruhin, Thomas Epper and Renate Schubert (2010), "Rationality on the Rise: Why Relative Risk Aversion Increases with Stake Size," *Journal of Risk and Uncertainty* 40, 147–180.

{% **inverse-S**: find it, and more pronounced for women than for men. %}

Fehr-Duda, Helga, Manuele de Gennaro, and Renate Schubert (2006), "Gender, Financial Risk, and Probability Weights," *Theory and Decision* 60, 283–313.

{% **ambiguity seeking for unlikely**; **inverse-S** Option traders do EV for given probs, and subadditivity for unknown probabilities; ascribe it to subadditivity in judged probability. P. 7: "Note that risk can be viewed as a special case of uncertainty where probability is defined via a standard chance device so that the probabilities of outcomes are known," The value function is elicited by asking for equivalences. %}

Fox, Craig R., Brett A. Rogers, and Amos Tversky (1996), "Options Traders Exhibit Subadditive Decision Weights," *Journal of Risk and Uncertainty* 13, 5–17.

{% **inverse-S**: argue that nonadditive models can describe source sensitivity but not so easily source preference because the latter may be a comparative effect, see P. 601: "This suggests that models based on decision weights or nonadditive probabilities (e.g., Quiggin [1982]; Gilboa [1987]; Schmeidler [1989]; Tversky and Wakker [forthcoming]) can accommodate source sensitivity, but they do not provide a satisfactory account of source preference because they do not distinguish between comparative and noncomparative evaluation." %}

Fox, Craig R. and Amos Tversky (1995), "Ambiguity Aversion and Comparative Ignorance," *Quarterly Journal of Economics* 110, 585–603.

{%;**CPT: data on probability weighting**; **inverse-S**; **ambiguity seeking for unlikely**; %}

Fox, Craig R. and Amos Tversky (1998), "A Belief-Based Account of Decision under Uncertainty," *Management Science* 44, 879–895.

Reprinted with minor changes *in* Daniel Kahneman and Amos Tversky (2000, Eds), *Choices, Values and Frames*, Ch. 6, pp. 118–142, Cambridge University Press, New York.

{% **CPT: data on probability weighting**; **inverse-S**: Finds inverse-S for all 10 participants! They tested the lower- and upper SA conditions of Tversky and Wakker (1995) and found them well confirmed. %}

Gonzalez, Richard and George Wu (1999), "On the Shape of the Probability Weighting Function," *Cognitive Psychology* 38, 129–166.

{% **inverse-S**: racetrack betting finds nonlinear probability inverse-S weights. These data from a different domain do corroborate Preston and Baratta (1948) with intersection of diagonal around .18. Main drawback of horse racing data is that the population is more risk seeking than average people are. %}

Griffith, Richard M. (1949), "Odds Adjustments by American Horse Race Bettors," *American Journal of Psychology* 62, 290–294.

{% **uncertainty amplifies risk**: although I found no place where this was stated explicitly, it is throughout their model and theory.

inverse-S; **Risk averse for gains, risk seeking for losses**: Table 2 on p. 792 suggests some more risk aversion for gains than risk seeking for losses. Table 4 on p. 795 suggests the same for large outcomes, but the opposite for small outcomes. %}

Hogarth, Robin M. and Hillel J. Einhorn (1990), "Venture Theory: A Model of Decision Weights," *Management Science* 36, 780–803.

{% **inverse-S**: They find that for losses, i.e. ambiguity aversion for unlikely losses and seeking for likely losses. They find more inverse-S for ambiguity than for chance. So also: **ambiguity seeking for losses**;
ambiguity seeking for unlikely: they study losses and there they find the reflection, in accordance with what CPT predicts, see above.
They asked what is a reasonable premium for p-prob at losing $100,000, for various probs. They also cite market evidence (earth-quake insurance, flood-insurance, etc.) suggesting much ambiguity aversion for small-prob losses.%}

Hogarth Robin M. and Howard C. Kunreuther (1985), "Ambiguity and Insurance Decisions," *American Economic Review, Papers and Proceedings* 75, 386–390.

{% **CPT: ambiguity seeking for losses** & **ambiguity seeking for unlikely**: They consider losses and there the data confirm all the hypotheses of Tversky and Wakker (1995) perfectly well.
inverse-S: There is risk aversion for **small probabilities** and risk seeking for high (not stated explicitly in the paper I think, but visible in Table 2, Fig. 2, Tables 4 and 5)
(**uncertainty amplifies risk**) These phenomena are amplified for ambiguity, by ambiguity aversion for small probabilities and ambiguity seeking for high. (Note that only the consumer data are relevant. The "firm" data consider selling of insurance which means both gains and losses, and loss aversion being relevant. Indeed, as expected by CPT, there more risk aversion etc. is found.) Unfortunately, the data for ambiguous probabilities may be prone to distortion by regression to the mean, which can be an alternative explanation of the overestimation of small ambiguous probs and understimation of high ambiguous probs. I do not understand the analysis in §3.4, in particular why M(P) + M(1−p) = 1 on page 18. If p and 1−p are ambiguous and participant to 2nd order distributions, they may, as mentioned by the authors, differ from their "anchor values." The participants, however, need not know that these referred to complementary events and may distort both downwards. %}

Hogarth, Robin M. and Howard C. Kunreuther (1989), "Risk, Ambiguity, and Insurance," *Journal of Risk and Uncertainty* 2, 5–35.

{% **inverse-S**; discussion of it on p. 1121; %}

Kachelmeier, Steven J. and Mohamed Shehata (1992), “Examining Risk Preferences under High Monetary Incentives: Experimental Evidence from the People’s Republic of China,” *American Economic Review* 82, 1120–1141; for comment see Steven J. Kachelmeier and Mohammed Shehata (1994), *American Economic Review* 84, 1104–1106.

{% **inverse-S; uncertainty amplifies risk** (for inverse-S probability weighting): p. 281, lines -6/-5: inverse-S: “In many real-life situations, overestimation and overweighting may both operate to increase the impact of rare events.” This relates to the preference condition in my 2004-Psych. Rev. paper! Similarly, p. 289 l. 5-6: “Consequently, subcertainty should be more pronounced for vague than for clear probabilities.” %}

Kahneman, Daniel and Amos Tversky (1979), “Prospect Theory: An Analysis of Decision under Risk,” *Econometrica* 47, 263–291.

{% **inverse-S**: they find it for risk, and more pronounced for uncertainty; latter also concerns: **uncertainty amplifies risk**. %}

Kilka, Michael and Martin Weber (2001), “What Determines the Shape of the Probability Weighting Function under Uncertainty,” *Management Science* 47, 1712–1726.

{% **inverse-S**: §5.2 finds support for inverse-S weighting function and EU for nonextreme probabilities.

risk-u = strength.pr.v: §4.2.5 discusses idea of transformation between value function v and risky utility fion u and says that transformation idea does not seem to be correct.

p. 253: influence formulation for str. of pr.%}

Krzysztofowicz, Roman (1994), “Generic Utility Theory: Explanatory Model, Behavioral Hypotheses, Empirical Evidence.” *In* Maurice Allais and Ole Hagen (eds.), “C*ardinalism;* A Fundamental Approach,” 249–288, Kluwer Academic Publishers, Dordrecht, the Netherlands.

{% **inverse-S**; use the two-parameter extension of Karmarkar, as Goldstein and Einhorn, 1987) also did, and find inverse S for both gains and, as it seems, losses. %}

Lattimore, Pamela M., Joanna R. Baker, and Ann D. Witte (1992), “The Influence of Probability on Risky Choice,” *Journal of Economic Behavior and Organization* 17, 377–400.

{% **inverse-S**: p. 289 says that insurance was accepted mostly for small-prob-high-losses. P. 295 finds inverse-S for RDU which is the special case of CPT where weighting for gains is dual to weighting for losses %}

Loehman, Edna (1998), “Testing Risk Aversion and Nonexpected Utility Theories,” *Journal of Economic Behavior and Organization* 33, 285–302.

{% **inverse-S**: p. 207 gives many citations to extent to which people pay attention to good and bad outcomes. %}

Lopes, Lola L. (1995), “Algebra and Process in the Modeling of Risky Choice,” *The Psychology of Learning and Motivation* 32, 177–220.

{% **inverse-S**: §4.3 reviews the literature up to that point on probability transformation, finding inverse-S as the prevailing pattern. %}

Luce, R. Duncan and Patrick Suppes (1965), “Preference, Utility, and Subjective Probability.” *In* R. Duncan Luce, Robert R. Bush, and Eugene Galanter (eds.), *Handbook of Mathematical Psychology*, Vol. III, 249–410, Wiley, New York.

{% **inverse-S**: Confirmed; finds risk seeking for low prob. high gains, risk neutrality for prob, of gain between .15 and .22, and risk aversion for higher probs, from data on betting behavior in horse races (mostly from 1947-1953).%}

McGlothlin, William H. (1956), “Stability of Choices among Uncertain Alternatives,” *American Journal of Psychology* 69, 604–615.

{% **inverse-S**, intersecting diagonal at about .2 (for utility linear). Prob. transformation seems to be .42 at .50!

Likelihood-sensitivity (inverse-S) ordering: Unsophisticated men exhibit least, then sophisticated subjects, then women, in the sense that the first category has least overweighting of small probabilities and least underweighting of high probabilities (see Table II). %}

Preston, Malcolm G. and Philip Baratta (1948), “An Experimental Study of the Auction Value of an Uncertain Outcome,” *American Journal of Psychology* 61, 183–193.

{% **Principle of Complete Ignorance**: p. 11

inverse-S: this paper discusses in much detail the psychology of being more or less sensitive to numerical scales, and the ability to more or less discriminate between options, and maybe taking numbers only as categories. I did not understand all experimental details though; for example on p. 38, isn't a 1/3 prob. to save "some" people trivially inferior to a certainty of saving "some" people? %}

Reyna, Valerie F. and Charles J. Brainerd (1995), "Fuzzy-Trace Theory: An Interim Synthesis," *Learning and Individual Differences* 7, 1–75.

{%**inverse-S**: Finds over-betting on small-prob. gain horses (p. 604: for p < .03)

Rosett, Richard N. (1965), "Gambling and Rationality," *Journal of Political Economy* 73, 595–607.

{% **inverse-S**: Data support finding of Yaari which suggests inverse-S probability weighting: Sets of lotteries preferred to status quo is convex suggesting concave utility but decision weights, inferrable from tangent of convex set of lotteries, differ from objective probabilities and suggest overweighting of low probabilities. %}

Rosett, Richard N. (1971), "Weak Experimental Verification of the Expected Utility Hypothesis," *Review of Economic Studies* 38, 481–492.

{% **inverse-S**?; argues so on the basis of French, Spanish, and Mexican lotteries%}

Sprowls, R. Clay (1953), "Psychological-Mathematical Probability in Relationships of Lottery Gambles," *American Journal of Psychology* 66, 126–130.

{% **inverse-S**: they find that probability weighting is inverse-S. %}

Stalmeier, Peep F.M. and Thom G. G. Bezembinder (1999), "The Discrepancy between Risky and Riskless Utilities: A Matter of Framing?," *Medical Decision Making*, 19, 435–447.

{% **inverse-S**: Found for both risk and uncertainty
ambiguity seeking for unlikely: is found here; they have gain outcomes only. %}

Tversky, Amos and Craig R. Fox (1995), "Weighing Risk and Uncertainty," *Psychological Review* 102, 269–283.

{% P. 316, §3: That coexistence of gambling and insurance is explained by overweighting of small probabilities. Find **inverse-S** %}

Tversky, Amos and Daniel Kahneman (1992), "Advances in Prospect Theory: Cumulative Representation of Uncertainty," *Journal of Risk and Uncertainty* 5, 297–323.

{% **inverse-S**: Reanalyze data of their 1990 paper on chemical workers' risk perceptions and decisions. Analyzed judged probabilities but also decision weights derived from

decisions, finding that the decision weights depended on the stated probabilities through the usual inverse-S relationship. Their curve fit found decision weights never below 0.10 and never above 0.49, so that the inverse-S is very strong. They jointly fit decision weights and utility, with utility results plausible. %}

Viscusi, W. Kip and William N. Evans (2006), "Behavioral Probabilities," *Journal of Risk and Uncertainty* 32, 5–15.

{% **inverse-S** of weighting fion; §5 does estimations; use preference ladders, which means choices that differ only regarding their common outcome. %}

Wu, George and Richard Gonzalez (1996), "Curvature of the Probability Weighting Function," *Management Science* 42, 1676–1690.

{% **inverse-S** of weighting fion%}

Wu, George and Richard Gonzalez (1998), "Common Consequence Conditions in Decision Making under Risk," *Journal of Risk and Uncertainty* 16, 115–139.

{% **inverse-S**: End of §IV finds longshot effect, and explains it by overestimation of small probability rather than by EU.

inverse-S: Yaari posits this on p. 290: "one finds that some subjects tend to overstate low probabilities and to understate high probabilities" and refers to Preston and Baratta (1948) and Mosteller and Nogee (1951) for related findings.%}

Yaari, Menahem E. (1965), "Convexity in the Theory of Choice under Risk," *Quarterly Journal of Economics* 79, 278–290.

Appendix H. References Finding Evidence against Inverse-S

{% Measure beliefs through subjective probabilities in first-price auctions. Measure it by introspective judgment, quadratic scoring rule, and prediction (rewarding those whose probability estimates are closest to true objective probability). Argue that the third method is a good compromise between being incentive-compatible (which it is only partly) and understandable.

inverse-S: They find that subjects throughout underestimate their probability of winning, going some against inverse-S. They find that probability weighting better explains data than utility curvature (which they call risk aversion), which supports the importance of probability weighting and prospect theory. %}

Armantier, Olivier and Nicolas Treich (2009), "Subjective Probabilities in Games: An Application to the Overbidding Puzzle," *International Economic Review* 50, 1013–1041.

{% **PT falsified**: Subjects have to do common-ratio choices, and others, not once, but repeatedly, say 200 times. They don't get any info about probs. etc., only can push one of two buttons and from experience find out what probability distribution can be. They don't even know that it is one fixed probability distribution. Real incentives: They are paid in points, and in end sum total of points is converted to money. Loss aversion is confirmed. Other than that, all phenomena are opposite to prospect theory, with underweighting of **small probabilities**, anti-certainty effect, more risk seeking with gains than with losses, etc. A very remarkable and original finding. The authors' explanation is that the subjects in their experiment experience the gambles rather than get descriptions thereof. It is surprising to me that subjects do not get close to expected value maximization.

My explanation (ex post indeed): The subjects put the question "which button would give the best outcome" central, and not "which button would give the best probability distribution over outcomes." They get to see which button gave best outcomes in most of the cases, with recency effect reinforcing it. Thus, subjects experience only the likelihood aspect, whether or not events with good/better outcomes obtain or not. The subjects do not experience the outcomes, because these are just abstract numbers to be experienced only after the experiment. This procedure leads to likelihood-oversensitivity, and S-shaped rather than inverse S-shaped nonlinear measures. Example of recency effect: If subjects, for instance, remember only which option gave the best result on the last trial, then they choose the event that with highest probability gives the

best outcome (a heuristic advanced by Blavatskyy). Outcomes will be perceived as ordinal more than as cardinal. The authors themselves may have alluded to this explanation on p. 221 just above Experiments 3a and 3b, when they refer to MacDonald, Kagel, and Battalio (1991, EJ) who found the opposite of what they found in an experiment with animals:
"For example, Macdonald et al. used a within-subject design and allowed the decision makers to immediately consume their rewards." %}

Barron, Greg and Ido Erev (2003), "Small Feedback-Based Decisions and Their Limited Correspondence to Description-Based Decisions," *Journal of Behavioral Decision Making* 16, 215–233.

{% **error theory for risky choice**: Shows, with data, theoretical analysis, and simulation, that inverse-S probability estimates can be generated by errors. %}

Bearden, J. Neil, Thomas S. Wallsten, and Craig R. Fox (2007), "Contrasting Stochastic and Support Theory Accounts of Subadditivity," *Journal of Mathematical Psychology* 51, 229–241.

{% An interesting decomposition of some things going on in the Allais paradox.
Finds violations of the s.th.pr. like Birnbaum and McIntosh (1996), falsifying the **inverse-S** prob weighting of CPT. %}

Birnbaum, Michael H. (2004), "Causes of Allais Common Consequence Paradoxes: An Experimental Dissection," *Journal of Mathematical Psychology* 48, 87–106.

{% Pp. 484-486 present the evidence against inverse-S initiated by Birnbaum and McIntosh (1996) where in three-outcome-prospect choices with one common outcome increasing the common outcome does not increase risk aversion as Pt would predict, but decreases it in the spirit somewhat of risk aversion decreasing with increasing wealth.
P. 493, 2nd column, 3rd para argues that evidence favoring inverse-S is confounded by framing effects. The author, however, only cites his, in itself valid, counterevidence against one particular implication of inverse-S and not much other evidence favoring it. %}

Birnbaum, Michael H. (2008), "New Paradoxes of Risky Decision Making," *Psychological Review* 115, 463–501.

{% branch independence is the sure-thing principle for events for which prob. is also given.
PT falsified: evidence against **inverse-S**

Finds violations of the s.th.pr. like Birnbaum and McIntosh (1996), falsifying the inverse-S prob weighting of CPT, %}

Birnbaum, Michael H. and Darin Beeghley (1997), "Violations of Branch Independence in Judgments of the Value of Gambles," *Psychological Science* 8, 87–94.

{% **PT falsified**: evidence against **inverse-S**

Finds violations of the s.th.pr. like Birnbaum and McIntosh (1996), falsifying the inverse-S prob weighting of CPT, also for four-outcome gambles distribution-independence is something of that kind, shifting probability mass from one common outcome to the other. Humphrey and Verschoor (2004) independently found the same.%}

Birnbaum, Michael H. and Alfredo Chavez (1997), "Tests of Theories of Decision Making: Violations of Branch Independence and Distribution Independence," *Organizational Behavior and Human Decision Processes* 71, 161–194.

{% **PT falsified**: evidence against **inverse-S**

Considers choices (R_1, R_2, C) versus (S_1, S_2, C), $R_1 > S_1 > S_2 > R_2$. CPT with inverse-S predicts that there will be fewer risky choices as C increases. (If C increases from worst ($< R_2$) to intermediate (between S_1 and S_2) then inverse-S would have the decision weight of S_2 and R_2 increase, enhancing safe choice. If C increases from intermediate to highest ($> R_1$) then inverse-S would have the decision weight of S_1 and R_1 *decrease,* which *again* enhances risk aversion.) It is found, however, that there are more risky choices (in agreement, in fact, with Machina's fanning out). As the lotteries get better because of C increasing, people get more risk seeking rather than risk averse. See Table 1 where the percentage of safe choices decreases rather than increases as we move to the right. So the extreme outcomes seem to be underweighted rather than overweighted. The paper gives an extensive theoretical analysis. The most extensive tests are in Birnbaum and Navarrete (1998) (the main topic of which, by the way, is another), which also describes the other preceding evidence. In particular, The B&M paper considers only three equally likely outcomes, B&N considers richer probability triples.%}

Birnbaum, Michael H. and William R. McIntosh (1996), "Violations of Branch Independence in Choices between Gambles," *Organizational Behavior and Human Decision Processes* 67, 91–110.

{% **PT falsified**: evidence against **inverse-S**

Real incentives: it was all hypothetical choice;

evidence against inverse-S prob. weighting, especially Table 4, see the comments in Birnbaum and McIntosh (1996). %}

Birnbaum, Michael H. and Juan B. Navarrete (1998), "Testing Descriptive Utility Theories: Violations of Stochastic Dominance and Cumulative Independence," *Journal of Risk and Uncertainty* 17, 49–78.

{% Compares utilities measured through chained SG to utilities measured through unchained SG, all with two-outcome gambles. Under classical elicitation assumption (doing calculations assuming EU descriptively), discrepancies arise, falsifying EU. If a correction is carried out for probability weighting using **inverse-S** within RDU, the discrepancies only increase. This is counterevidence against RDU. Earlier counterevidence, by Wakker, Erev, and Weber (1994), Birnbaum and McIntosh (1996), and Birnbaum and Navarrete (1998), always concerned three-outcome gambles, this paper has two-outcome gambles. The author suggests that loss-aversion and framing can explain the findings. %}

Bleichrodt, Han (2001), "Probability Weighting in Choice under Risk: An Empirical Test," *Journal of Risk and Uncertainty* 23, 185–198.

{% (**PT falsified**:) When they do rank-dependent utility with linear utility, and Prelec's two-parameter family, they find convex and not **inverse-S** weighting functions. This puts the ball in the court of the inverse-S advocates. To maintain their hypothesis, they have to find other explanations for the strategic behavior of subjects than put forward in this paper.%}

Goeree, Jacob K., Charles A. Holt, and Thomas R. Palfrey (2002), "Quantal Response Equilibrium and Overbidding in Private-Value Auctions," *Journal of Economic Theory* 104, 247–272.

{% **PT falsified**: find S-shaped rather than **inverse-S** shaped probabilit weighting. %}

Goeree, Jacob K., Charles A. Holt, and Thomas R. Palfrey (2003), "Risk Averse Behavior in Generalized Matching Pennies Games," *Games and Economic Behavior* 45, 97–113.

{% **inverse-S**: Assumes it to predict future choices, but finds bad results.

Measured utilities/probability weighting (a parameter for every outcome/probabiliity), I think by best-fitting, on three consecutive weeks, to find that they were not stable over time.%}

Hartinger, Armin (1999), “Do Generalized Expected Utility Theories Capture Persisting Properties of Individual Decision Makers?,” *Acta Psychologica* 102, 21–42.

{% The undergrads were risk averse for 0.05 and 0.20, risk neutral for 0.80, and very risk seeking for 0.50 [**risk seeking for (symmetric) fifty-fifty gambles**]. When asked about latter, undergrads said things such as “It’s a good chance” or “it’s fair.” These data go against the fourfold pattern of **inverse-S**. %}

Henrich, Joseph and Richard Mcelreat (2002), “Are Peasants Risk-Averse Decision Makers?,” *Current Anthropology* 43, 172–181.

{% **PT falsified** & **inverse-S**: They test the common consequence effect and find risk aversion increasing and not decreasing, which is the exact opposite of inverse S. This independently replicates the same finding as by Birnbaum, for instance in Birnbaum and Chavez (1997).

More elaborate results, with error theories added, are in Humphrey and Verschoor (2004, *Journal of African Economies*). Unfortunately, the papers have no cross references to explain their overlap and priority. %}

Humphrey, Stephen J. and Arjan Verschoor (2004), “The Probability Weighting Function: Experimental Evidence from Uganda, India and Ethiopia,” *Economics Letters* 84, 419–425.

{% The data do not suggest **inverse-S**. CPT estimations suggest convex (pessimistic) w for gains, concave for losses (also pessimistic, because of dual integration for losses that CPT does). For losses they seem to find risk aversion, for gains a little risk seeking. This is contrary to the common empirical findings although their footnote 17 suggests that it is in agreement with common findings. This population of betters can obviously not be expected to agree with general findings. %}

Jullien, Bruno and Bernard Salanié (2000), “Estimating Preferences under Risk: The Case of Racetrack Bettors,” *Journal of Political Economy* 108, 503–530.

{% **inverse-S**: find pessimism iso inverse-S. %}

Li, Li-Bo, Shu-Hong He, Shu Li, Jie-Hong Xu, and Li-Lin Rao (2009), “A Closer Look at the Russian Roulette Problem: A Re-Examination of the Nonlinearity of the Prospect Theory’s Decision Weight π,” *International Journal of Approximate Reasoning* 50, 515–520.

{% **PT falsified**: regarding **inverse-S**: For RDU, his evidence cannot be reconciled with an inverse-S weighting function (p. 104) but it can neither be with a convex (p. 1-3). %}

Loomes, Graham (1991), “Evidence of a new Violation of the Independence Axiom,” *Journal of Risk and Uncertainty* 4, 92–109.

{% Watch out: they do old-fashioned bottom-up RDU integration, with w around 0 relevant to worst outcomes and w around 1 relevant to best outcomes.

inverse-S & **risk seeking for small-prob. gains**: they find and model overweighting of the best outcome (called “bottom-edge effect”) and, remarkably, not of the worst (see their p. 115 last para, and p. 116 between Eq. 11b and 12a); (**EU+a*sup+b*inf**). It implied that the Prelec one-parameter family performed worst than the simple overweighting of best outcome.

Unfortunately, in their writing they often equate utility with risk attitude, which is not correct for rank-dependent utility. %}

Loomes, Graham, Peter G. Moffat, and Robert Sugden (2002), “A Microeconometric Test of Alternative Stochastic Theories of Risky Choice,” *Journal of Risk and Uncertainty* 24, 103–130.

{% **inverse-S**: P. 306 considers case of two participants, one with $p^{0.5}$, other with $p^{1.5}$, as prob. transformation function. Their average then gives inverse-S shape prob. transformation. Nice example! Estes (1956) seems to give general viewpoints on curves derived from group data.%}

Luce, R. Duncan (1996), “When Four Distinct Ways to Measure Utility Are the Same,” *Journal of Mathematical Psychology* 40, 297–317.

{% **inverse-S**: Suggest that their data for prob. transf. agree with Preston and Baratta’s but this is not much so. Sprowls (1953) says they are more variable. P. 397: For Preston and Baratta prob. transformation (assuming linear utility) intersects the diagonal at about 0.2, in this experiment at 0.5 for guardsmen, and not for the students (they are always risk averse). Domain: [–0.05, 5.50].

Mosteller, Frederick and Philip Nogee (1951), “An Experimental Measurement of Utility,” *Journal of Political Economy* 59, 371–404.

{% P. 217: **risk seeking for small-prob. gains**: not found, only weak risk aversion.

P. 217: Risk aversion for small-prob. losses: neither found, only weak risk seeking. %}

Kühberger, Anton, Michael Schulte-Mecklenbeck, and Josef Perner (1999), “The Effects of Framing, Reflection, Probability, and Payoff on Risk Preference in Choice Tasks,” *Organizational Behavior and Human Decision Processes* 78, 204–231.

{% **small probabilities**
risk seeking for small-prob. gains: nice example that small probs. are often ignored. Give bounded-rationality arguments: for very small probability, even if the catastrophe is small, it is not worth the time to think and have transaction costs about. %}
Kunreuther, Howard C. and Mark Pauly (2003), “Neglecting Disaster: Why Don’t People Insure against Large Losses,” *Journal of Risk and Uncertainty* 28, 5–21.

{% **inverse-S**: find convex w more than inverse-S. %}
Qiu, Jianying and Eva-Maria Steiger (2011), “Understanding the Two Components of Risk Attitudes: An Experimental Analysis,” *Management Science* 57, forthcoming.

{% **inverse-S**: almost not found, Prelec’s one-parameter family fits best with parameter 0.94, which is very close to linear and has almost no inverse S. (Utility $x^{0.19}$ is very concave.) %}
Stott, Henry P. (2006), “Cumulative Prospect Theory’s Functional Menagerie,” *The Journal of Risk and Uncertainty* 32, 101–130.