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The Rich Domain of Uncertainty: Source Functions and Their Experimental Implementation

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We often deal with uncertain events for which no probabilities are known. Several normative models have been proposed. Descriptive studies have usually been qualitative, or they estimated ambiguity aversion through one single number. This paper introduces the source method, a tractable method for quantitatively analyzing uncertainty empirically. The theoretical key is the distinction between different sources of uncertainty, within which subjective (choice-based) probabilities can still be defined. Source functions convert those subjective probabilities into willingness to bet. We apply our method in an experiment, where we do not commit to particular ambiguity attitudes but let the data speak. (*JEL* D81)

An individual … can always assign relative likelihoods to the states of nature. But how does he act in the presence of uncertainty? The answer to that may depend on another judgment, about the reliability, credibility, or adequacy of his information.

—Daniel Ellsberg (1961, p. 659)

In many situations we do not know the probabilities of uncertain events that are relevant for the outcomes of our decisions. The importance of finding tools to analyze such situations has been understood since Frank Knight (1921). In some situations we can still assign subjective probabilities to the relevant events and use expected utility (Leonard J. Savage 1954) or, more generally, nonexpected utility ("probabilistic sophistication"; Mark J. Machina and David Schmeidler 1992). In a fundamental contribution, Ellsberg (1961) showed that it is often impossible to use subjective probabilities, implying that probabilistic sophistication cannot be applied. We therefore have to develop more general models ("ambiguity"). Whereas the importance of developing such models had been understood for a long time, it was not until the end of the 1980s that such models were discovered (multiple priors: Itzhak Gilboa and Schmeidler 1989; rank dependent utility: Gilboa 1987; Schmeidler 1989).

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Ambiguity has subsequently become a central topic of research, and several new models have been developed (David Ahn et al. 2009; Gilboa 2004), most of which were normatively oriented. They usually assumed expected utility for given probabilities (Gilboa and Schmeidler 1989; Peter Klibanoff, Massimo Marinacci, and Sujoy Mukerji 2005; Fabio Maccheroni, Marinacci, and Aldo Rustichini 2006; Schmeidler 1989). Our aim, however, is purely descriptive. Hence, we assume nonexpected utility throughout.

Several recent empirical studies compared ambiguous events with unambiguous events as in Ellsberg's (1961) paradox, and fitted the α-maxmin model (Yan Chen, Peter Katuščák, and Emre Ozdenoren 2007; see also Ahn et al. 2009; for a survey of neurostudies, see Soo Hong Chew et al. 2008). The ambiguity attitude of an individual was then captured by one number, the α parameter, taken as a general degree of ambiguity aversion. Our paper considers richer domains of uncertainty with various events and various levels of likelihood involved, both for experimental Ellsberg-type events and for natural events. The data that we obtain reveal rich patterns of ambiguity attitudes. Besides aversion, insensitivity to ambiguity turns out also to be an important component. Further, within one individual, the two components vary widely between different sources of uncertainty. Hence it is desirable to develop flexible and rich tools to analyze ambiguity, and this is the purpose of the present paper. The richness of ambiguity, with no probabilities of events specified, can be compared to the richness of outcomes with no monetary values specified. In the same way as one cannot expect there to be one index of risk aversion applicable to all nonmonetary outcomes, one cannot expect there to be one index of ambiguity aversion applicable to all ambiguous events. We show that, despite its richness, ambiguity can be quantitatively analyzed in a tractable manner by means of what we call the source method. We can make exact quantitative predictions about future behavior, and we can calculate ambiguity premiums.

A central concept in our analysis will concern sources of uncertainty, as first advanced by Amos Tversky in the early 1990s (Tversky and Craig Fox 1995; Tversky and Daniel Kahneman 1992). Sources of uncertainty are groups of events that are generated by the same mechanism of uncertainty, which implies that they have similar characteristics. Following Chew and Jacob S. Sagi (2008), we can define choice-based probabilities within particular (uniform) sources even when Machina and Schmeidler's (1992) probabilistic sophistication does not hold between sources. Source functions then map choice-based probabilities into willingness to bet. In this way, the richness of attitudes to uncertainty and ambiguity can be captured by graphs on the probability interval rather than by general functions on algebras of events. This considerably increases the tractability of the analysis.

To sum up, we use three components to describe decision under uncertainty: (i) the utility of outcomes; (ii) choice-based probabilities for each source of uncertainty; (iii) source functions. Component (iii) captures the deviations from Bayesianism[1] in a tractable manner. Those deviations include the Allais and Ellsberg paradoxes, the home bias, and ambiguity aversion. Attitudes towards ambiguity are measured by comparing component (iii) for known and unknown probabilities. In the Bayesian

[1] The term Bayesian refers to expected-utility based components.

view, (i) reflects tastes, (ii) reflects beliefs, and (iii) reflects deviations from rational behavior.

The paper proceeds as follows. Section I presents preliminaries, including the decision model assumed. Our model includes most nonexpected utility models used today. Section IIA introduces the source method, with ways to model different attitudes towards uncertainty and ambiguity in Section IIB. Section IIC presents indexes of aversion to uncertainty and of insensitivity to uncertainty. Section III tests our new concepts in the often-studied Ellsberg paradox, and Section IV tests them for natural uncertainties from daily life. In both cases, a rich variety of ambiguity attitudes is found, not only between persons, but also within them. The patterns found confirm, for revealed-preference data, findings in the psychological literature that were based on introspective judgments (Tversky and Fox 1995; Hillel Einhorn and Robin Hogarth 1985). Section V contains a discussion, and Section VI concludes. A Web Appendix gives experimental details, in particular discussing the incentives used. It further gives all parameter estimations at the individual level, all details of statistical tests discussed later, and several other results at the individual level. Examples A.1 and A.2 in this appendix illustrate how our method can give exact quantitative predictions for the home bias (Kenneth R. French and James M. Poterba 1991). The home bias entails that investors systematically prefer domestic stocks to foreign stocks beyond beliefs (subjective probabilities) or tastes (utilities). The bias is accommodated by the different source functions for the different stocks.

I. Preliminaries

This section introduces basic concepts and notation. $(E_1{:}x_1,\ldots,E_n{:}x_n)$ denotes a *prospect* yielding *outcome* x_j if *event* E_j happens. Outcomes are nonnegative real numbers designating money. Exactly one of the events $E_1,\ldots,E_n$ will happen, and it is uncertain which one. Thus it is uncertain which outcome will result from a chosen prospect. $\succcurlyeq$ denotes the *preference relation* of a decision maker over the prospects. We assume *weak ordering* throughout; i.e., $\succcurlyeq$ is complete and transitive. Strict preference $(\succ)$ and indifference or equivalence $(\sim)$ are defined as usual. For each prospect, the *certainty equivalent* is the sure amount that is indifferent to the prospect. *Expected utility* holds if a prospect $(E_1{:}x_1,\ldots,E_n{:}x_n)$ is evaluated by its *expected utility* $\sum_{j=1}^{n} P(E_j)\,u(x_j)$, with u, the *utility function*, continuous and strictly increasing and $P(E_j)$ the subjective probability of event E_j.

In our measurements we will need only two-outcome prospects. The notation $x_E y$ is shorthand for $(E{:}x$, not–$E{:}y)$. It is implicitly assumed in this notation that $x \geq y$. For such binary prospects, most static and transitive nonexpected utility theories[2] use the same evaluation. Since these theories diverge only for prospects with three or more outcomes, the results of this paper apply to all of them. This convenient feature of binary prospects was put forward by Ghirardato and Marinacci (2001), Duncan Luce (1991), and John M. Miyamoto (1988).

[2] See Ahn et al. (2009), Syngjoo Choi et al. (2007), Thibault Gajdos et al. (2008), Gilboa (1987), Gilboa and Schmeidler (1989), Paolo Ghirardato, Maccheroni, and Marinacci (2004), Schmeidler (1989), and Tversky and Kahneman (1992).

We first define our basic model for uncertainty, where no probabilities need to be given. A *weighting function* W assigns a number $W(E)$ between 0 and 1 to each event E, such that:

(i) $W(\varnothing) = 0$;

(ii) W is 1 at the universal event;

(iii) $E \supset F$ implies $W(E) \geq W(F)$.

Binary rank-dependent utility (*RDU*) holds for binary prospects if there exist a strictly increasing *utility function* $u : \mathbb{R} \to \mathbb{R}$ and a weighting function W such that preferences maximize

$$x_E y \mapsto W(E)u(x) + (1 - W(E))u(y). \tag{1}$$

This model generalizes expected utility by allowing W to be nonadditive. W can be interpreted as willingness to bet.

For calibrations of likelihoods of events, we fix a "good" and a "bad" outcome. Let us assume that these are 1,000 and 0, the values used in the second experiment reported later. A *bet on event E* designates the prospect $1{,}000_E 0$. E and F are *revealed equally likely*, denoted $E \sim F$, if $1{,}000_E 0 \sim 1{,}000_F 0$. We next define an exchangeability condition that is stronger than revealed equal likelihood.

DEFINITION 1. Two disjoint events E_1 and E_2 are *exchangeable* if exchanging the outcomes under the events E_1 and E_2 never affects the preference for a prospect; i.e., always $(E_1{:}x_1, E_2{:}x_2, \ldots, E_n{:}x_n) \sim (E_1{:}x_2, E_2{:}x_1, \ldots, E_n{:}x_n)$. A partition $(E_1, \ldots, E_n)$ is *exchangeable* if all of its elements are mutually exchangeable.

Exchangeability of events implies that they are equally likely. Exchangeable partitions were called uniform by Savage (1954), and they played a central role in his analysis. We will use Savage's term uniform for a slightly different and more general concept, for which the following definition prepares.

Probabilistic sophistication holds if there exists a probability measure P such that for each prospect $(E_1{:}x_1, \ldots, E_n{:}x_n)$ the only relevant aspect for its preference is the probability distribution $(p_1{:}x_1, \ldots, p_n{:}x_n)$ that it generates over the outcomes, where $p_j = P(E_j)$ for all j. That is, two different prospects that generate the same probability distribution over outcomes are equivalent in terms of $\succcurlyeq$. Probabilistic sophistication maintains the probability measure P from expected utility but allows for more general (nonexpected-utility) evaluations over probability distributions. Under probabilistic sophistication, revealed equal likelihood is not only necessary, but also sufficient for exchangeability. The special case of known objective probabilities (risk) will be discussed after we have introduced sources in the next section.

II. Sources of Uncertainty

The first step in our analysis is to distinguish between different sources of uncertainty. A source of uncertainty concerns a group of events that is generated by a common mechanism of uncertainty. In Ellsberg's (1961) classical two-color paradox, one source of uncertainty concerns the color of a ball drawn randomly from an urn containing 50 black balls and 50 red ones (the known urn). Another source concerns the color of a ball drawn randomly from an urn with 100 black and red balls in unknown proportion (the unknown urn). People are willing to exchange a bet on black for a bet on red from the known urn, with similar willingness to exchange colors for the unknown urn. They are, however, not willing to exchange a bet on a color from the known urn for a bet on a color from the unknown urn. This willingness to exchange within but not between urns suggests that the events pertaining to the same urn share features and constitute one source of uncertainty, but events concerning different urns belong to different sources. The Ellsberg paradox concerns one special case of the different treatment of different sources (Chew et al. 2008; Tversky and Fox 1995). Alternatively, one source of uncertainty can concern the Dow Jones index, and another source the Nikkei index, as in the home bias. Whereas probabilistic sophistication is usually violated between sources, as first demonstrated by the Ellsberg paradoxes, within single sources it is often still satisfied.

A. Uniform Sources

For convenience, we will assume that *sources* are *algebras*, which means that they contain the universal event (certain to happen), the vacuous event (certain not to happen), the complement of each of their elements, and the union of each pair of their elements. Thus they also contain every finite union and intersection of their elements. Extensions to domains other than algebras are left to future studies.

We call a source S *uniform* if probabilistic sophistication holds with respect to S. Formally, this means that there exists a probability measure P on the events of S such that the preference for each prospect $(E_1{:}x_1, \ldots, E_n{:}x_n)$ with all outcome-relevant events E_j in S depends only on: (a) the source S; (b) the probability distribution $(p_1{:}x_1,\ldots,p_n{:}x_n)$ generated over outcomes, with p_j the probability $P(E_j)$. Under uniformity, P will usually denote the relevant probability measure on the source without further mention. Uniformity is an endogenous concept. Chew and Sagi (2008) emphasized the interest of considering probabilistic sophistication within sources without imposing it between sources. Wakker (2008) pointed out that probabilistic sophistication within a source entails a uniform degree of ambiguity for that source, which is why we call such sources uniform.

If a finite partition $(E_1,\ldots,E_n)$ is exchangeable then the generated source (consisting of unions of events from that partition) is uniform. Chew and Sagi (2006) showed that, under some regularity and richness conditions,[3] a source is uniform if and only if the following conditions hold for the events of the source:

[3] Their richness is satisfied under the common assumption that the probability measure is atomless and countably additive on a sigma algebra. It can also be accommodated for finite equally-likely-state spaces as in our Section III (equation (4) can then be dropped).

(2) $E \sim F$ (E and F are revealed equally likely) implies that E and F are exchangeable (holds for all uniform partitions).

(3) For each pair of disjoint events, one contains a subset that is exchangeable with the other (imposing richness).

(4) For each n there exists an exchangeable n-fold partition (imposing richness).

This result shows that uniformity is a natural extension of exchangeability from finite sources to rich (continuum) structures.

For a rich uniform source, we can elicit probabilities to any desired degree of precision using a bisection method and equation (3) (see Section IVA). We can, for example, partition the universal event into two equally likely events E_2^1 and E_2^2 that then must each have probability 0.5. We next partition E_2^1 into two equally likely events E_4^1 and E_4^2 that must each have probability ¼, and we partition E_2^2 into two equally likely events E_4^3 and E_4^4 that also each have probability ¼. We continue likewise. This method will also be used in the experiments described later. We will then test some implications of the equations (2–4), similar to Baillon (2008).

We next consider an implication of probabilistic sophistication (with probability measure P) on S that will be useful for the analysis of ambiguity for uniform sources in the next subsection. Under probabilistic sophistication on S, there exists a function w_S such that for any event E from S we have[4]

$$W(E) = w_S(P(E)). \tag{5}$$

After substitution in equation (1), we obtain the following evaluation of binary prospects:

$$x_E y \mapsto w_S(P(E))u(x) + (1 - w_S(P(E)))u(y). \tag{6}$$

The function w_S, carrying subjective probabilities to decision weights, is called the *source function*. Probabilistic sophistication on S generalizes the probabilistic sophistication of Machina and Schmeidler (1992) because w_S can now depend on the source. That is, whereas probabilistic sophistication holds within some sources, it need not hold between sources. The *source method* uses source functions to analyze uncertainty.

Vernon L. Smith (1969, p. 325) and Robert L. Winkler (1991, giving several more references) argued for maintaining probabilistic beliefs in the Ellsberg paradox. They preferred to accommodate this paradox using the utility function. We will maintain probabilities but will also leave utility unaffected (the latter was argued for by Robin M. Hogarth and Hillel J. Einhorn 1990, p. 708). Instead, we use source functions as the third component, rather than modifying probabilities (beliefs) or

[4] The implication can be derived as follows. If $P(A)=P(B)$, then $1{,}000_A0 \sim 1{,}000_B0$. Substituting equation (1) shows that then $W(A) = W(B)$. Thus, equality of P implies equality of W. It is well known that equation (5) then follows.

utilities ("tastes"), to capture ambiguity attitudes. Source functions reflect interactions between beliefs and tastes that are typical of nonexpected utility and that are deemed irrational in the Bayesian normative approach.

Under usual regularity conditions, $w_S(0) = 0$, $w_S(1) = 1$, and w_S is continuous and strictly increasing. As is usual, we assume that events with known probabilities constitute one uniform source of uncertainty (Fox, Brett A. Rogers, and Tversky 1996, p. 7), and we write the corresponding source function without subscript. For an event E with known probability $P(E) = p$, we equate $x_E y$ with the probability distribution $x_p y$, yielding x with probability p and y with probability $1 - p$. The convention $x \geq y$ is maintained in this notation. The evaluation is, accordingly:

$$x_p y \mapsto w(p)u(x) + (1 - w(p))u(y). \tag{7}$$

We call probability distributions over outcomes *risky prospects*, or just *prospects* if no confusion will arise. As for uncertainty, also for risk most nonexpected utility theories used today agree for binary prospects and evaluate them by equation (7).

B. Uncertainty Attitudes

Figure 1 depicts the main properties of source functions w_S (cf. Hogarth and Einhorn 1990, Figure 1). The x-axis designates probabilities p, which are choice-based and need not be objective. The y-axis designates weights $w_S(p)$, that is, transformed probabilities. Figure 1A displays expected utility with a linear source function. Figure 1B displays a convex source function, leading to low weights for good outcomes and enhancing risk aversion or *pessimism*. Figure 1C displays an inverse S–shaped source function w_S. The convex part near 1 explains the risk aversion and pessimism found for unfavorable events that happen with a small probability (so that the complementary, favorable, event E weighted by w_S has a high probability). The concave part near 0 explains the risk seeking and *optimism* found for favorable events E that happen with a small probability (the long shot effect). Thus the inverse S shape explains the coexistence of gambling and insurance (Tversky and Kahneman 1992, p. 316).

The inverse S–shaped source functions reflect a lack of sensitivity to intermediate changes in likelihood, so that all intermediate likelihoods are moved in the direction of 50-50. The jumps from certainty to uncertainty are then overweighted. Hence, this phenomenon is also called *likelihood insensitivity*. It suggests that decisions will not be influenced much by the updating of probabilities after receipt of new information. Likelihood insensitivity resembles regression to the mean. It is, however, not a statistical artifact, but a perceptual phenomenon that occurs in actual decisions. Figure 1D, the most common shape, combines the two deviations from expected utility, pessimism and likelihood insensitivity.

Comparative versions of the above concepts can be defined. This can be done between persons (Mr. A is more averse to investing in Dutch stocks than Mr. W). Ellsberg's paradox shows that such comparisons can also be done within persons (this person is more pessimistic about investing in foreign stocks than in domestic stocks; cf. Fox and Tversky 1995, p. 162). Formal definitions and results are in

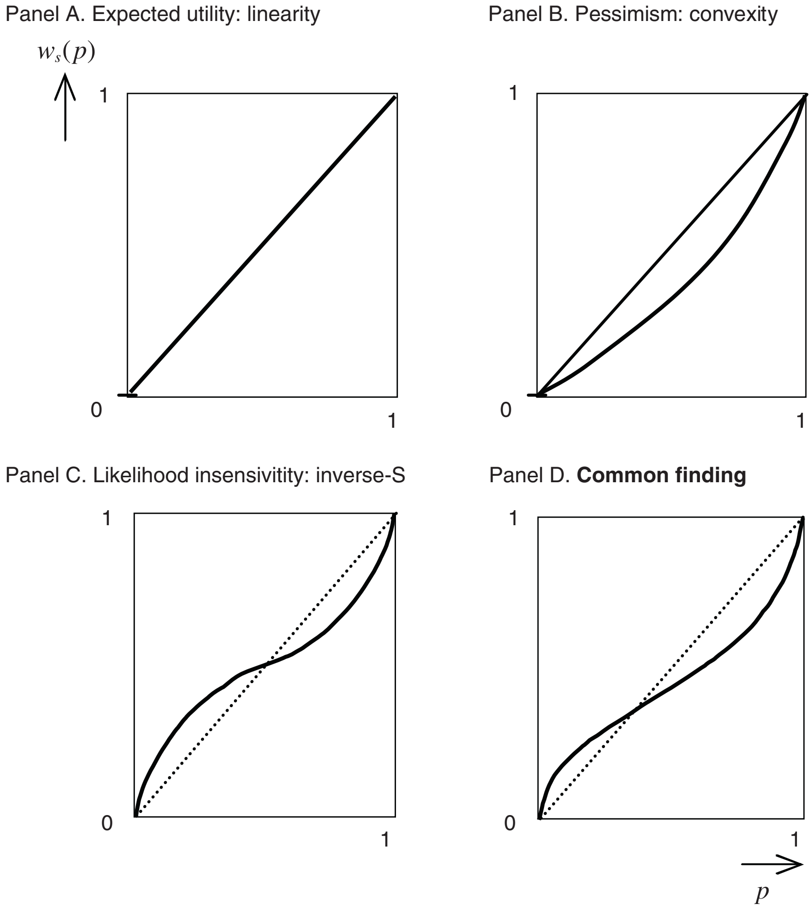

FIGURE 1. SHAPES OF SOURCE FUNCTIONS

Fox and Tversky (1998), Michael Kilka and Martin Weber (2001), Drazen Prelec (1998), Tversky and Fox (1995), Tversky and Wakker (1995), and George Wu and Richard Gonzalez (1999).

Ambiguity reflects what uncertainty comprises beyond risk. That is, it concerns the differences between decisions and beliefs for unknown probabilities versus those for known probabilities. Ambiguity attitudes can be examined by comparing source functions for ambiguous sources to those for sources with known probabilities. More general comparisons, between different sources that are all ambiguous, are possible (Section IV).

C. *Indexes of Uncertainty Aversion and Insensitivity*

The graphs of source functions capture attitudes towards uncertainty. For reasons of parsimony, it is sometimes convenient to summarize ambiguity attitudes in terms of one index number, or two as we will propose. Our proposed indexes are based on neoadditive weighting functions (Alain Chateauneuf, Jürgen Eichberger, and Simon Grant 2007). They were suggested to us by Fox (1995, personal communication). The first index summarizes the degree of pessimism, with optimism as its counterpart.

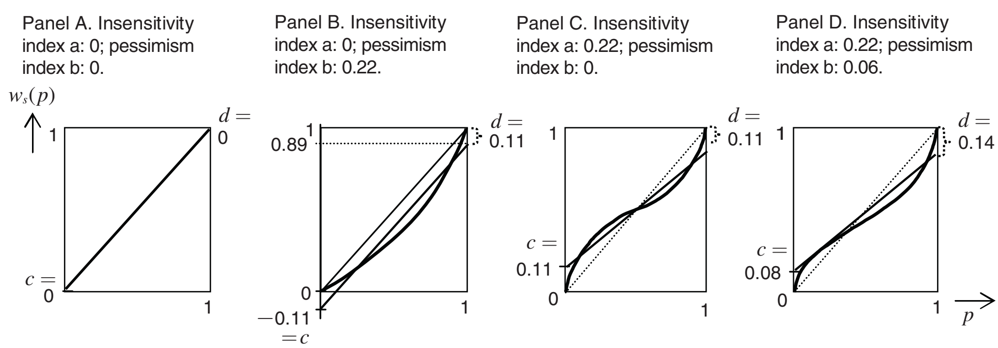

FIGURE 2. QUANTITATIVE INDEXES OF PESSIMISM AND LIKELIHOOD INSENSITIVITY

This index of pessimism will be larger in Figure 1B than in Figure 1A. The difference in pessimism between uncertainty and risk then reflects ambiguity aversion. The data of our experiments show that a second kind of deviation from expected utility, orthogonal to pessimism, is relevant: the degree of likelihood insensitivity. Figure 1C exhibits more likelihood insensitivity than Figure 1A. We use linear regression, illustrated in Figure 2, to define the two indexes. Choosing our indexes corresponds to choosing the neoadditive weighting function that best fits the data.

Assume that the regression line of the source function on the open interval (0,1) is $p \mapsto c + sp$, with c the intercept and s the slope. Let $d = 1 - c - s$ be the distance from 1 of the regression line at $p = 1$; i.e., the "dual intercept." We define

(8) $a = c + d\ (= 1 - \mathrm{s})$ as an index of (likelihood) insensitivity,

and

(9) $b = d - c\ (= 1 - s - 2c)$ as an index of pessimism.

These indexes can be interpreted as simplified versions of indexes used by Kilka and Weber (2001) and Tversky and Fox (1995). Craig Webb and Horst Zank (2008) considered their measurement and preference axiomatizations. An elaborate discussion and theoretical analysis of these measures, as well as of general properties and comparisons of source functions, are left to future research.

III. The Source Method for Ellsberg-like Uncertainties

This section shows how source functions capture attitudes towards uncertainty and ambiguity for the classical two-color Ellsberg paradox. This paradox concerns artificial events in a laboratory setup, but it is the most studied case of ambiguity. Hence, it serves well as a first test of new concepts.

A. *Experimental Design*

$N = 67$ students faced two Ellsberg-like urns. The known urn K contained eight balls of different colors: red, blue, yellow, black, green, purple, brown, cyan. The unknown urn contained eight balls with the same eight colors, but the composition was unknown in the sense that some colors might appear several times and others might be absent. As explained in Section II, the two urns concern two different sources. For both urns, each ball was equally likely to be drawn. In what follows, elementary events of a single color drawn are denoted (with $S = K$ or $S = U$) by R_S, B_S, Y_S, A_S (A for black), G_S, P_S, N_S (N for brown), and C_S. Subjects faced 26 series of choice tasks. Each series involved a choice between a prospect and an ascending range of sure payments, with the midpoint between the switching values taken as certainly equivalent. At the beginning of the experiment, each subject was told that one of his choices would be randomly drawn and then played for real.

B. *Analysis*

Testing Uniformity.—According to uniformity, certainty equivalents of the prospect 25_E0 should be the same for different events E with an equal number of colors (exchangeability). We tested this equality for the unknown urn for three one-color events, randomly chosen per subject, for the two-color events $\{Y_U, A_U\}$, $\{G_U, P_U\}$, $\{N_U, C_U\}$, and $\{R_U, B_U\}$, and for the four-color events $\{R_U, B_U, Y_U, A_U\}$ and $\{G_U, P_U, N_U, C_U\}$.

Elicitation of Utility.—Utility was elicited using the semiparametric method of Abdellaoui, Han Bleichrodt, and Olivier L'Haridon (2008). For each urn we elicited certainty equivalents for seven prospects with outcomes between €0 and €25, and the outcome-relevant event always being $\{R_S, B_S, Y_S, K_S,\}$, with $S = K$ or $S = U$. This event has a subjective probability of 0.5 for both S. We fitted equation (7) assuming a power utility function $u(x) = (x/25)^{\rho_S}$ ($S = K$ or $S = U$), and taking the weight $w_S(0.5)$ of the outcome-relevant event as extra parameter. We used nonlinear least-square estimation with the certainty equivalent as dependent variable. Choi et al. (2007) used a similar model with power utility and the same distance measure to fit multiple-choice data.

Source Functions.—To measure $w_S(p)$ for $p \neq 0.5$, we elicited certainty equivalents CE of prospects 25_E0 with E containing j colors for all $j \neq 4$. Substituting equation (7) then gives $w_S(j/8) = (CE/25)^{\rho_S}$ as a nonparametric estimation. Once the values $w_S(j/8)$ have been elicited, we can do parametric fitting using Prelec's (1998) two-parametric compound invariance family while minimizing quadratic distance:

$$w_S(p) = (\exp(-(-\ln(p))^{\alpha}))^{\beta}. \tag{10}$$

Parameter α has a meaning similar to our index a reflecting insensitivity, and parameter β has a meaning similar to our index b reflecting pessimism. The statistical tests of α and β gave results similar to those for a and b. For brevity, we report only the latter.[5] We also analyzed our data using probabilistic choice-error

[5] The parameters a and b have clearer interpretations, primarily because α impacts both likelihood insensitivity and pessimism. Indeed, α and β were more strongly correlated than a and b, here and also in the natural-event experiment reported later.

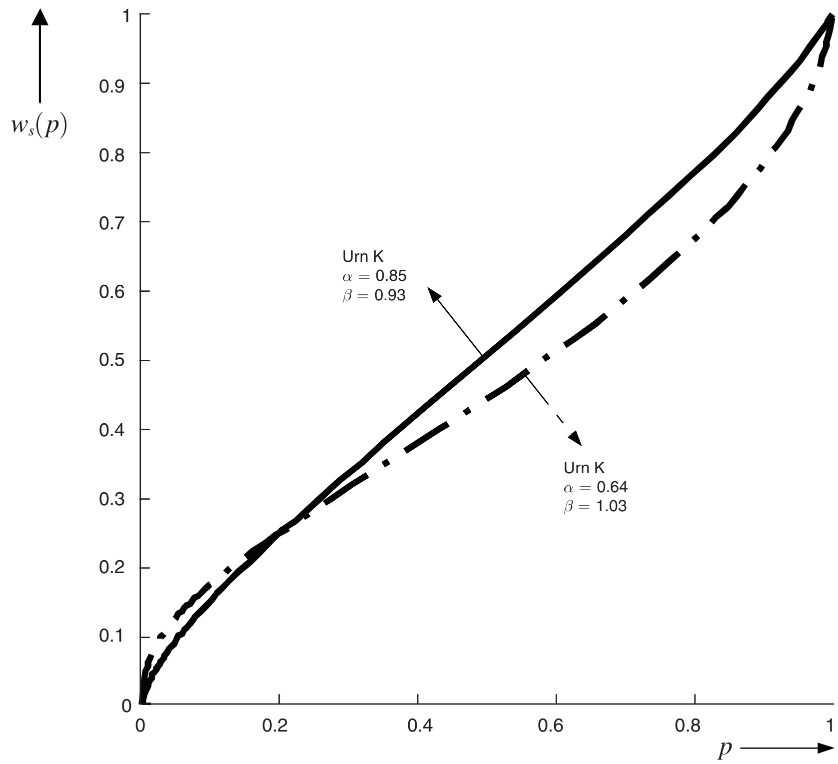

Figure 3. Median Source Functions in K and in U

theories and econometric maximum likelihood estimations. The results are in the Web Appendix, and they all agree with the results reported here. All estimations of utilities and weighting functions were done at the individual level.

C. *Results*

Unless stated otherwise, all statistical tests concern two-sided *t*-tests with 0.05 as level of significance.

Uniformity.—ANOVAs with repeated measures show that uniformity is not rejected ($p = 0.335$ for the single-color events; $p = 0.245$ for the two-color events; $p = 0.824$ for the four-color events). Hence, we will assume the uniform subjective probability distribution. Because of the central role of this assumption in our analysis, we inspected it also at the individual level, rejecting it only for subject 52, who was accordingly removed from the analysis. No conclusion was affected by this removal.

Utility.—The median utility parameters ($\rho_K = 1.05$ and $\rho_U = 1.09$) are not significantly different from 1 for both urns (sign-tests;[6] $p = 0.539$ for K; $p = 0.175$ for U). Utility is the same for the two urns (sign-test; $p = 0.902$).

[6] The parameters had outliers and were skewed, resulting in medians and sign tests (having more power) being more appropriate than means and *t*-tests.

Table 1—Source Functions for K and U

p		Median	Mean	Interquartile range	t-tests $w_S(p) = p$	t-tests $w_U = w_K$
1/8	K	0.19	0.26	[0.12, 0.37]	0.000	0.339
	U	0.19	0.23	[0.06, 0.32]	0.000	
2/8	K	0.31	0.34	[0.22, 0.44]	0.001	0.231
	U	0.27	0.30	[0.11, 0.45]	0.068	
3/8	K	0.44	0.44	[0.30, 0.57]	0.012	0.187
	U	0.40	0.40	[0.22, 0.58]	0.294	
4/8	K	0.50	0.50	[0.36, 0.63]	0.851	0.066
	U	0.48	0.46	[0.34, 0.56]	0.145	
5/8	K	0.64	0.63	[0.50, 0.79]	0.849	0.007
	U	0.58	0.56	[0.42, 0.68]	0.023	
6/8	K	0.75	0.73	[0.63, 0.86]	0.409	0.001
	U	0.68	0.65	[0.51, 0.81]	0.000	
7/8	K	0.94	0.87	[0.80, 0.99]	0.911	0.000
	U	0.82	0.75	[0.63, 0.93]	0.000	

Note: The median value of $w_K(1/8)$ is 0.19, the mean value of $w_U(4/8) = 0.46$, and so on.

Source Functions.—Figure 3 represents the source functions in the two urns (based on median parameters of the Prelec family). The difference between the two curves reflects ambiguity attitudes. The dashed curve, reflecting a general uncertainty attitude towards the unknown urn, then consists of the risk attitude component plus the ambiguity attitude component.

The source functions are significantly different from the identity function $(w_S(p) = p)$ at $p = 1/8, 2/8$, and $3/8$ for urn K, and at $p = 1/8, 5/8, 6/8$, and $7/8$ for urn U (Table 1). Consequently, EU cannot accommodate our data, in agreement with common findings (Gilboa 2004). For large probabilities $(p > 0.5)$, source functions are significantly lower for urn U than for urn K. For small probabilities $(p \leq 0.5)$ there is no significant difference.

The likelihood insensitivity indexes (0.19 for K and 0.31 for U) significantly exceed 0. There is also significantly more insensitivity in urn U than in urn K, which is natural given that U has unknown probabilities, whereas K has known probabilities. The pessimism index is positive in urn U (0.04) and negative (meaning optimism) in urn K (-0.08). None of them is significantly different from 0. Pessimism in the unknown urn, however, significantly exceeds that in the known urn.

Individual Behavior.—There is much variation between subjects. The following figure shows the source functions of three subjects. The values corresponding to observations are represented by black (K) and white (U) circles, and the fitted source functions by a continuous line for K and a dash-dot line for U.

For urn K, subject 2 is mostly pessimistic, subject 44 is likelihood insensitive, and subject 66 combines both. Subject 44 is ambiguity averse, subject 66 is ambiguity seeking, and subject 2 is more likelihood insensitive for urn U than for urn K. The Web Appendix gives the graphs for six more subjects, discussing several other phenomena. It also provides histograms of the shapes of individual source functions and of the number of subjects who are ambiguity averse or ambiguity seeking, plus some scatter plots explained later.

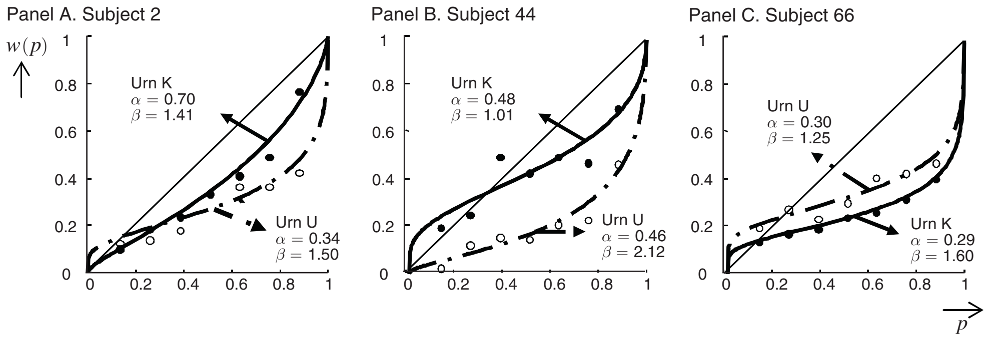

Figure 4. Individual Results

D. *Conclusions from the Ellsberg Experiment*

Given the symmetry of the colors, it is not surprising that uniformity of the sources was verified because the events were exchangeable. Our finding that different sources of uncertainty generate different source functions but not different utility functions is plausible. The source functions directly pertain to uncertainty, whereas the utility functions concern something of a different nature, being the value of outcomes. This is corroborated by Abdellaoui, L'Haridon, and Corina Paraschiv (2009) who also measured utility for risk and ambiguity within binary RDU and also found no difference.

The source functions display natural properties. They deviate from EU (linearity). There is more willingness to bet for risk than for ambiguity if the choice-based probabilities on the x-axis exceed 0.5, as predicted both by greater aversion to uncertainty than to risk, and by greater insensitivity to uncertainty than to risk. Willingness to bet is the same for risk and for uncertainty if the choice-based probabilities are below 0.5. This also agrees with both more aversion to, and more insensitivity to ambiguity, because these effects neutralize each other for such probabilities.

Whereas general ambiguity aversion would predict that all curves for the unknown urn are below those for the known urn, Figures 3 and 4 display more complex patterns. There is also some ambiguity seeking, and considerable insensitivity to ambiguity displayed by inverse S–shaped curves. These findings illustrate the richness of the domain of uncertainty.

IV. The Source Method for Natural Uncertainties

Ellsberg's urns were constructed such that uniformity is automatically satisfied within the sources. Uniformity is less trivial for natural sources of uncertainty. Such sources are the topic of the second experiment, presented in this section.

A. *Experimental Design*

Subjects and stimuli (*sources*).—$N = 62$ students were presented prospects for three sources of uncertainty with unknown probabilities concerning: (a) The French Stock Index (CAC40) (how much it would *change* on a given day); (b) the

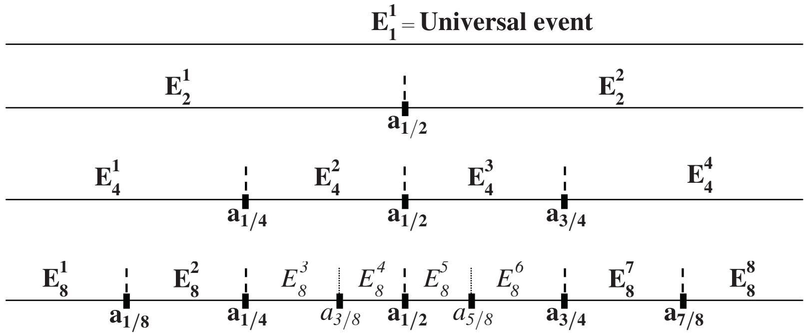

Figure 5. Decomposition of the Universal Event

temperature in Paris; (c) the temperature in a randomly drawn remote country, different for each subject. All events concerned a fixed day (May 31, 2006) about three months after the experiment. All indifferences were obtained using repeated choice and bisection.

Stimuli: exchangeable events.— Figure 5 depicts the partitioning of events that we used. This design for testing exchangeability was first used by Baillon (2008). We elicited values $a_{1/8}$, $a_{1/4}$, $a_{1/2}$, $a_{3/4}$, $a_{7/8}$ using indifferences between (bets on) events that partition events into two equally likely subevents, in the following order of elicitation: (a) $(-\infty, a_{1/2}] \sim (a_{1/2}, \infty)$ generating the exchangeable partition $\{E_2^1, E_2^2\}$ at the second level in the figure; (*b*) $(-\infty, a_{1/4}] \sim (a_{1/4}, a_{1/2}]$ and $(a_{1/2}, a_{3/4}] \sim (a_{3/4}, \infty)$ generating the exchangeable partition $\{E_4^1, E_4^2, E_4^3, E_4^4\}$, at the third level in the figure; (c) $(-\infty, a_{1/8}] \sim (a_{1/8}, a_{1/4}]$, and $(a_{3/4}, a_{7/8}] \sim (a_{7/8}, \infty)$ generating part of the exchangeable partition at the fourth level. To illustrate our measurement, if a subject preferred to bet on (= receiving 1,000 conditional on) event (ℓ, ∞) rather than on event $(-\infty, \ell]$, but preferred to bet on $(-\infty, h]$ rather than on (h, ∞), then we inferred that $a_{1/2}$ was between $\ell < h$. A subsequent preference for betting on $(-\infty, (h+\ell)/2]$ rather than on $((h+\ell)/2, \infty)$ then shows that $a_{1/2}$ is actually between ℓ and $(h+1)/2$; and so on. Next, with $a_{1/2}$ elicited, a preference for betting on $(\ell', a_{1/2}]$ rather than on $(-\infty, \ell']$ then shows that $a_{1/4}$ is between ℓ' and $a_{1/2}$. And so on.

In the notation $E_j^i = (a_{(i-1)/j}, a_{i/j}]$, subscript j indicates the level (number of events in the partition), and superscript i indicates the number of the event in a left-to-right reading. Thus events E_{2j}^{2i-1} and E_{2j}^{2i} partition event E_j^i. In the notation $a_{i/j}$, the subscript i/j designates the probability of not exceeding $a_{i/j}$ under uniformity. We did not measure $a_{3/8}$ and $a_{5/8}$ so as to reduce the burden of the subjects, and because the literature on risk and uncertainty suggests that the most interesting phenomena occur at extreme values. In other words, we did not determine the middle events of the exchangeable partition $\{E_8^1, \ldots, E_8^8\}$. Using the values measured, we carried out several tests of uniformity (= exchangeability for our stimuli).

Stimuli: certainty equivalents.—We measured certainty equivalents of the risky prospects $1{,}000_{1/8}0$, $1{,}000_{1/4}0$, $1{,}000_{1/2}0$, $1{,}000_{3/4}0$, and $1{,}000_{7/8}0$, and of the risky 50-50 prospects $500_{1/2}0$, $1{,}000_{1/2}500$, $500_{1/2}250$, $750_{1/2}500$, and $1{,}000_{1/2}750$. We also measured the certainty equivalents of the prospect $1{,}000_E0$, with E being $(-\infty, a_{1/8}]$, $(-\infty, a_{1/4}]$, $(-\infty, a_{1/2}]$, $(-\infty, a_{3/4}]$, $(-\infty, a_{7/8}]$, and $(a_{1/2}, \infty)$ for each source.

Motivating subjects.—All subjects received a flat payment of €20. For the *hypothetical treatment* ($n = 31$), all choices were hypothetical. For the *real treatment* ($n = 31$), real incentives were implemented by the random incentive system in addition to the flat payment. That is, one of the 31 subjects was randomly selected at the end, and one of his choices was randomly selected to be played for real. The money earned could be collected about three months later, after the uncertainty had been resolved. The subjects in the hypothetical treatment did not know that a real-incentive treatment would follow later for other subjects.

Analysis.—Unless stated otherwise, all statistical tests concern two-sided t-tests with 0.05 as level of significance. We fitted the data similarly as in the first experiment. We first used the certainty equivalents of the (risky) 50-50 prospects $500_{1/2}0$, $1{,}000_{1/2}500$, $500_{1/2}250$, $750_{1/2}500$, and $1{,}000_{1/2}750$, respectively, to optimally fit equation (7) with power utility $u(x) = x^{\rho}$. With the utility function thus determined, we used the certainty equivalents of the prospects $1{,}000_{1/8}0$, $1{,}000_{1/4}0$, $1{,}000_{1/2}0$, $1{,}000_{3/4}0$, $1{,}000_{7/8}0$ to determine the source function $w(p)$ for risk at the probabilities concerned. Then, by equation (7), $\text{CE} \sim 1{,}000_p0$ implies $w(p) = \text{CE}^{\rho}/1{,}000^{\rho}$ for all $p = 1/8$, $1/4$, $1/2$, $3/4$, and $7/8$.

Because the first experiment found no difference in utility between risk and uncertainty, and because there is no prior reason to expect such a difference, we measured utility for risk only in the second experiment and used that utility for uncertainty too. Thus we did not have to measure utility for uncertainty separately and were able to reduce the burden for the subjects. For each uncertain source, we used the certainty equivalents of bets on the events $(-\infty, a_{1/8}]$, $(-\infty, a_{1/4}]$, $(-\infty, a_{1/2}]$, $(-\infty, a_{3/4}]$, and $(-\infty, a_{7/8}]$, and the power utility function to determine the W values of these events, with $\text{CE} \sim 1{,}000_E0$ implying the equality $W(E) = \text{CE}^{\rho}/1{,}000^{\rho}$ by equation (1).

To test exchangeability as implied by uniformity, we first measured, for each source, the value $a'_{1/2}$ such that $(a_{1/4}, a'_{1/2}] \sim (a'_{1/2}, a_{3/4}]$. Exchangeability requires $a'_{1/2} = a_{1/2}$. Next we measured $a''_{1/2}$ such that $(-\infty, a''_{1/2}] \sim (a''_{1/2}, \infty)$, which is simply an exact replication of the measurement of $\mathrm{a}_{1/2}$ as done before. It serves to test for consistency ($a''_{1/2} = a_{1/2}$). The value $a_{1/2}$ is important because the other measurements of events are derived from it, which is why we measured it extensively. We measured preferences between bets on different intervals that should be indifferent under exchangeability: $(-\infty, a_{1/8}]$ versus $(a_{7/8}, \infty]$ and $(a_{1/8}, a_{1/4}]$ versus $(a_{3/4}, a_{7/8}]$.

B. *Results on Uniformity, Subjective Probability, and Utility*

Uniformity.—We use the term *case* to specify both the source of unknown probability (CAC40, Paris temperature, or foreign temperature) and the treatment (real incentives or hypothetical choice). Thus there are six cases. Since there were no irregularities in the answers that subjects supplied, we used the whole sample. The third measurement of $a_{1/2}$ (as midpoint of $(-\infty, \infty)$) was identical to the first measurement and served as a reliability test. Pairwise t-tests never rejected the null hypothesis of equal values (for neither treatment nor for the whole group), and the correlations exceeded 0.85 for all three sources and both treatments. These results suggest that the measurements were reliable.

The most refined level of partitioning for which we obtained observations concerned the eightfold partition of the events E_8^i, which we observed for $i = 1, 2, 7, 8$. The equivalences $E_8^1 \sim E_8^2$ and $E_8^7 \sim E_8^8$ hold by definition. Assuming transitivity of indifference, it suffices to verify the equivalence $E_8^2 \sim E_8^7$ to obtain equivalence of all E_8^i available. For no case did a binomial test reject the null hypothesis of indifference between bets on E_8^2 and E_8^7. The choices between E_8^1 and E_8^8 serve as an extra test of uniformity joint with transitivity of indifference. Again, a binomial test never rejected indifference.

We made no observations of the eightfold partition $\{E_8^i\}$ between $a_{1/4}$ and $a_{3/4}$, but in this region we can test exchangeability (implied by uniformity) for the fourfold partition $\{E_4^i\}$. Given the equivalences $E_4^1 \sim E_4^2$ and $E_4^3 \sim E_4^4$ that hold by definition, and transitivity of indifference, it suffices to verify the indifference $E_4^2 \sim E_4^3$. Although we did not directly test choices between bets on E_4^2 and E_4^3, our second measurement of $a_{1/2}$, as midpoint of $(a_{1/4}, a_{3/4}]$, entails a test of the equivalence $E_4^2 \sim E_4^3$. The correlations between the first and second measurement of $a_{1/2}$ exceeded 0.75 for all three sources and both treatments as well as the whole group, exceeding 0.90 in all but one case. Pairwise t-tests never rejected the null hypothesis of equal values of $a_{1/2}$ (for neither treatment nor for the whole group) with one exception: For the hypothetical group and foreign temperature the difference was significant ($t_{30} = 2.10, p = 0.04$).

Another test of exchangeability can be derived from comparing the certainty equivalents of bets on events E_2^1 to those on events E_2^2. Under exchangeability, these should all be the same. Pairwise t-tests never rejected the null hypothesis of equal values (for neither treatment nor for the whole group), with Pearson correlations of approximately 0.5 and more. Hence these tests do not reject exchangeability.

The tests suggest that uniformity is least satisfied for foreign temperature with hypothetical choice, with no violations found for the other five cases. Because the source method has been developed for uniform sources, we will report our analyses of risk and ambiguity attitudes for only the five remaining cases in what follows.

Subjective Probabilities.—Figures 6 and 7 display median subjective probability estimations for the real and hypothetical treatments contrasted with historical frequencies. The medians are always derived from the medians of the $a_{i/j}$. Figure 6 displays the median subjective probability distribution functions for CAC40. Both curves show that our subjects were optimistic in the sense that they considered increases of the index to be more probable than decreases. The figure also displays the observed frequency distribution over the year 2006. Our subjects expected extreme, primarily positive, changes to be more likely than they actually were.

Figure 7 displays the median subjective distribution function for Paris temperature. The historical distribution for the time considered (May 31, 1 PM) has been added too. The curves are well calibrated. Our subjects are apparently better acquainted with temperature volatility than with stock volatility. The data also suggest that subjects did not expect higher temperatures than the historical distribution over the past century. We do not report the subjective probabilities for foreign cities

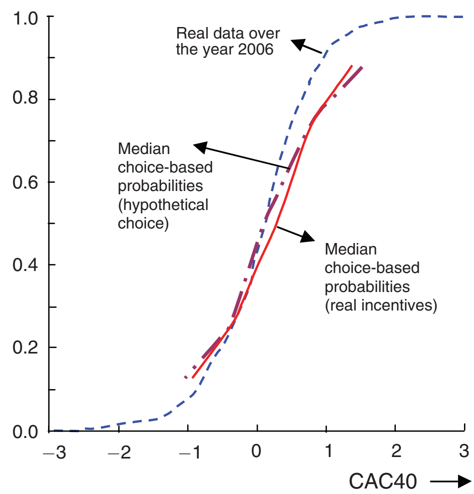

Figure 6. Probability Distributions for CAC40

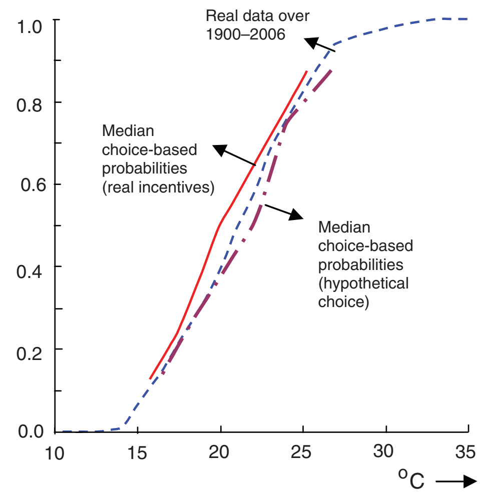

Figure 7. Probability Distributions for Paris Temperature

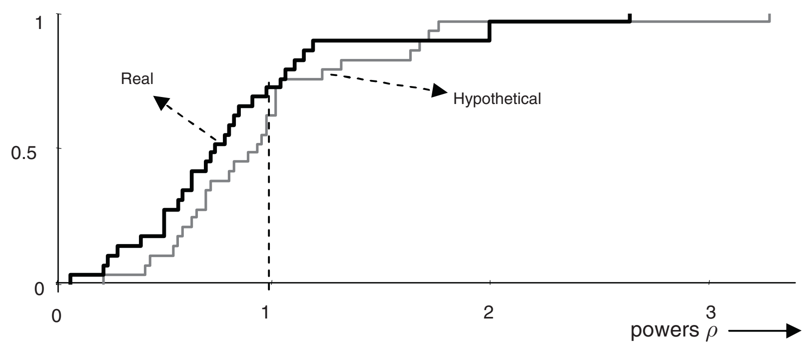

Figure 8. Cumulative Distribution of Powers

because the cities were different for different subjects so that this distribution did not concern the same random event for all subjects. Although it was unlikely that the subjects, who were recruited individually, might know each other, we wanted to avoid any possibility of their learning anything about the city, which is why we changed it for each subject.

Utility.—The certainty equivalents (statistics not reported) suggest risk seeking for low probabilities and risk aversion for moderate and high probabilities, with more risk aversion for the real treatment than for the hypothetical treatment. All these findings agree with common findings in the literature (Colin Camerer and Hogarth 1999; Gonzalez and Wu 1999), and are confirmed by the parametric estimations given in the Web Appendix. Figure 8 displays the empirical distribution of the individual powers of utility. The majority of powers is below 1, suggesting moderate concavity (61.2 percent for the hypothetical treatment and 72.4 percent for the real treatment). Median, mean, and standard deviations are 0.92, 1.01, and 0.59 for the hypothetical treatment and 0.75, 0.85, and 0.56 for the real treatment. The powers of utility were lower for the real-incentive group than for the hypothetical group, but the difference was not significant. A lower power entails more concavity, which will generate more risk aversion (given a fixed weighting function), in agreement with the common findings in the literature of more risk aversion for real incentives.

C. *Overall Results for Source Functions*

This section reports results on source functions, describing the decision attitudes found. It does so only for the five cases where uniformity is satisfied. Figure 9 displays source functions. Panel A displays source functions obtained from the raw data and linear interpolation, and panel B displays the best-fitting function from Prelec's (1998) compound invariance family (equation 10). The statistical results for Prelec's parameters α and β were similar to those for a and b. As with Experiment 1, we report only the latter.

The indexes a and b were calculated for each subject and each source. The parameters displayed are calculated to fit the group averages and will not be used in statistical analyses. Their orderings agree with all qualitative findings made below. Note how Figure 9 compactly and completely presents all components of the decision attitude beyond Bayesian expected utility. Together with the Bayesian components of utility and subjective probabilities, the figure completely captures the decision attitude, exactly quantified, for four sources at the same time. This makes it possible to immediately and visually compare these non-Bayesian components. In particular, by comparisons with the graphs for risk, the figure immediately reveals attitudes towards ambiguity.

The hypothetical-treatment curves (Figure A.13 in the Web Appendix) are similar to those of the real-payment treatment (Figure 9), but hypothetical choices were subject to more noise. All curves display the common inverse S shape of Figure 1D with low probabilities overweighted and high probabilities underweighted. Most observed points $w_S(p)$ deviate significantly from linearity. In other words, the null hypothesis of EU is usually rejected, except at $p = 0.5$, in agreement with inverse S. The insensitivity parameter a is significantly higher for real incentives than for hypothetical choice for CAC40 and foreign temperature, and marginally so for Paris temperature ($p = 0.053$). The pessimism parameter b is not different for the two treatments.

Regarding source functions under hypothetical payment, no significant differences are found between the source functions for different sources. We therefore focus on real payment. We first consider source functions $w_S(p)$ at single probabilities p. With risk included, a repeated-measures analysis ANOVA (corrected by the Huynh-Feldt ε) finds significant source dependence for $w_S(p)$ and real payment except at $p = 0.5$. Figure 9 shows that there is source preference (higher curve, so less pessimism) for risk over all other sources. Indeed, paired t-tests for risk against each of the three sources indicate that the values $w_S(p)$ are significantly higher for risk than for foreign temperature at all probabilities (i.e., ambiguity aversion at all probabilities), for CAC40 at $p = 0.125$ and $p > 0.50$ and for Paris temperature at $p > 0.5$ (i.e., ambiguity aversion for high probabilities). If we exclude risk, then the ANOVA finds significant source dependence for $p = 0.25$. The figure suggests source preference for Paris temperature over CAC40 and foreign temperature, and more pronounced inverse-S for CAC40 than for foreign temperature, but the differences between the curves at the various probabilities are not significant except for Paris against foreign temperature at $p < 0.5$.

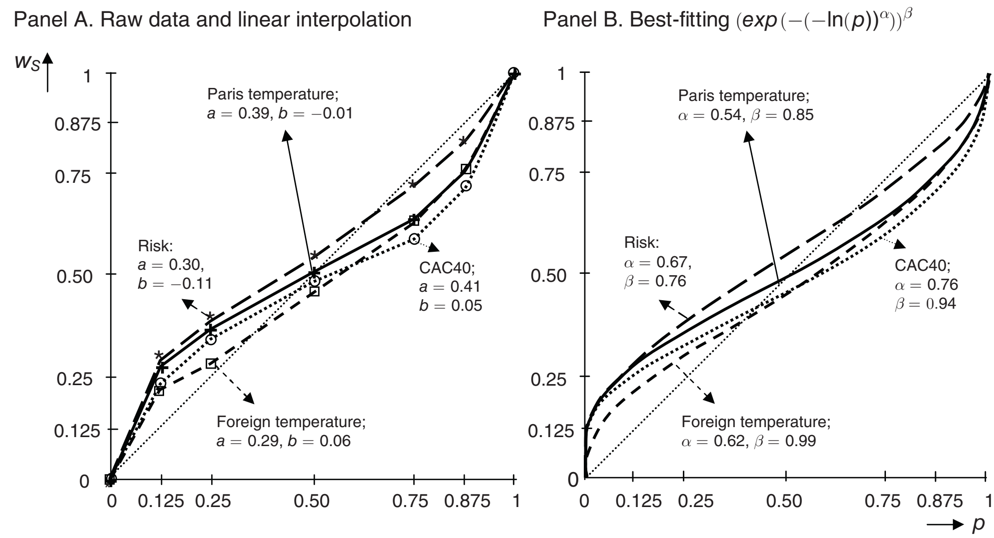

Figure 9. Average Source Functions for Real Payment

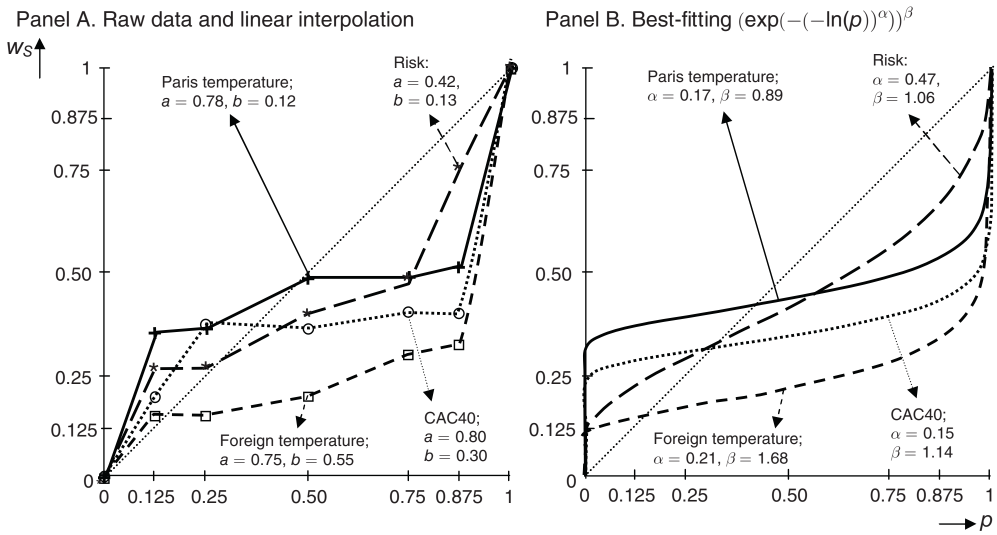

Figure 10. Source Functions for Subject 2 for Real Payment

We next consider tests of pessimism and likelihood insensitivity based on the global parameters a and b. A repeated-measures ANOVA (corrected by the Huynh-Feldt ε) reveals a clear source dependence of the pessimism index b. The insensitivity parameter is not significantly different across sources at 5 percent once the Huynh-Feldt correction is applied.

D. *Results at the Individual Level for Source Functions*

To illustrate that the source method can be used at the individual level, Figure 10 displays the curves for the four sources of one subject, subject 2 from the

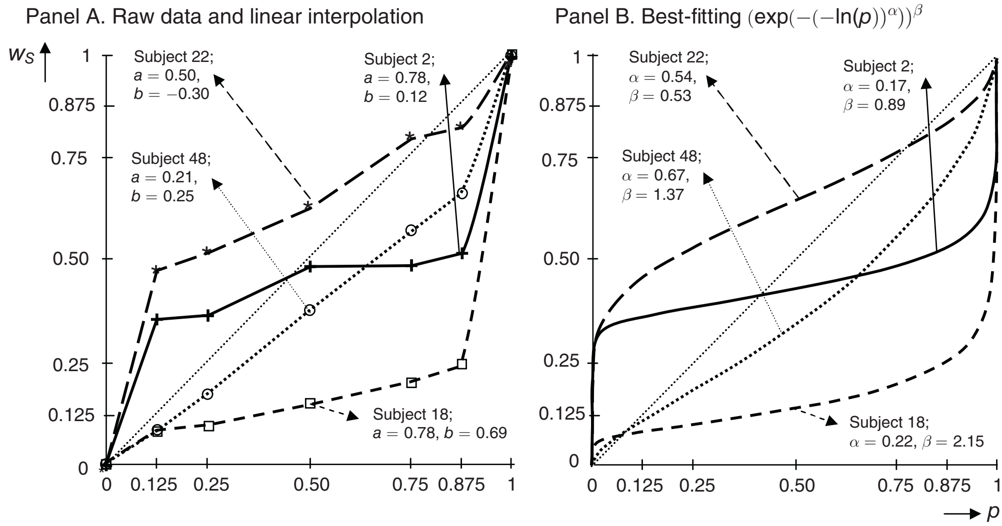

Figure 11. Source Functions for Paris Temperature and 4 Subjects for Real Payment

real-payment treatment. This subject thought long and seriously about each question, and the interview took almost two hours. He exhibits source preference for all sources over foreign temperature. Further, risk is less likelihood insensitive than CAC40 and Paris temperature. In the raw data, the subject slightly violates monotonicity for CAC40, showing that there is noise in the data.

Behavioral implications are that the subject will be more prudent, invest less, and take out more insurance for foreign temperature events than for the other events. The subject will be more open to long shots for Paris temperature and CAC40 than for risk but, on the other hand, will also rather insure for Paris temperature and CAC40 than for risk. An updating of (subjective) probabilities after receipt of new information will affect the subject less for Paris temperature and CAC40 than for risk.

Figures 9 (for a representative agent) and 10 (for subject 2) concerned a within-person comparison of different attitudes towards uncertainty for different sources, which we take as the main novelty initiated by the Ellsberg paradoxes. We can also use source functions and the above indexes of pessimism and likelihood insensitivity for the—more traditional—between-person comparisons of uncertainty attitudes. Figure 11 displays some comparisons. We selected four subjects with clearly distinct curves for the purpose of illustration. All curves concern the same source, namely Paris temperature. The lowest curve (subject 18) is more pessimistic than all other subjects. This subject will buy more insurance, for instance. The dark middle curve (subject 2) clearly displays more pronounced likelihood insensitivity than the dashed curve that is close to linear (subject 48). Hence, simultaneous gambling and insurance is more likely to be found for subject 2 than for subject 48, and subject 2's decisions will be influenced less by new information (updating probabilities) than those of subject 48 (cf. Larry G. Epstein 2008).

In general, there was more variation in the individual parameter estimates for the ambiguous sources than for risk. It is not surprising, indeed, that risk is perceived more homogeneously across individuals than ambiguity.

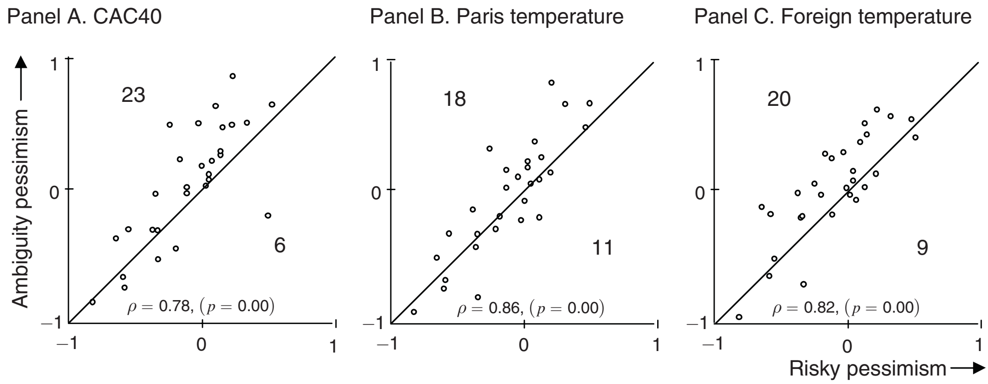

FIGURE 12. PESSIMISM INDEXES IN THE AMBIGUOUS SOURCES WITH RESPECT TO RISK (REAL PAYMENT)

Note: Numbers of observations above and below the diagonal have been indicated.

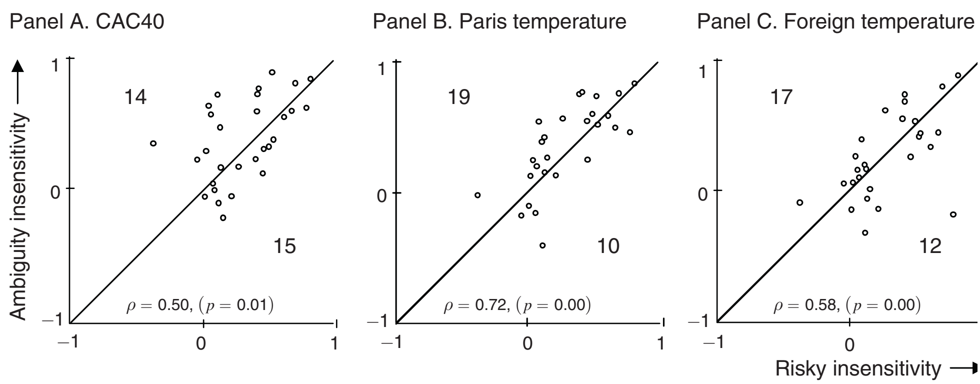

FIGURE 13. INSENSITIVITY INDEXES IN THE AMBIGUOUS SOURCES WITH RESPECT TO RISK (REAL PAYMENT)

Note: Numbers of observations above and below the diagonal have been indicated.

Figures 12 and 13 display scatter plots of the pessimism indexes and the insensitivity indexes, respectively, of the 29 subjects of the real payment group in each ambiguous source (y-axis) as a function of the pessimism indexes under risk (x-axis). The correlations between the pessimism index in each of the three ambiguous sources and the one under risk (the ρs in each graph) are positive and highly significant, as are the corresponding correlations between the insensitivity indexes. Thus some subjects are likely to be more pessimistic (or more insensitive) for all sources than other subjects, showing that there is systematic between-subject heterogeneity. Further, the subjects are significantly more pessimistic in each of the three ambiguous sources than under risk (paired t-tests), showing that there is systematic between-source heterogeneity. The insensitivity indexes were not significantly different between the different sources. Similar scatter plots for the hypothetical choices and for the Ellsberg experiment of Section III, with the same findings, are in the Web Appendix. This Web Appendix also provides histograms of the shapes of individual source functions and of the number of subjects who are ambiguity averse or ambiguity seeking.

E. *Results Regarding Ambiguity*

Ambiguity attitudes are usually taken to reflect the differences between sources with unknown probabilities and sources with known probabilities. We can infer those differences from comparing the curves for risk with the other curves, in Figures 9 and 10. These comparisons have been discussed above, with the risk curves typically dominating the other curves, confirming ambiguity aversion.

F. *Conclusions from Natural-Event Experiment*

In natural sources of uncertainty, uniformity is a nontrivial restriction. We found it violated in one of the six cases considered. Again, the source functions display natural properties, with more willingness to bet for risk than for the other sources when the choice-based probabilities exceed 0.5. There is considerable variation not only between subjects but also within subjects between sources.

Behavioral implications of our findings are that people will be more prudent, invest less, and take out more insurance for unknown probabilities than for known probabilities, confirming ambiguity aversion. As regards the three sources with unknown probabilities in the second experiment, people will be more open to both insurance and long shots, and will update less, for foreign temperature than for CAC40 and Paris temperature.

V. Discussion

We have analyzed decision attitudes using three components, namely utility of outcomes, choice-based probabilities for each source, and source functions. We first discuss some details of the measurements of these components, and then other issues.

Measuring Utility.—Our utility measurements are valid for most of the existing models. In particular, they are not distorted by violations of expected utility, unlike traditional methods based on the latter theory. In the absence of such distortions, we found utility to be close to linear, in agreement with claims by Matthew Rabin (2000), Frank P. Ramsey (1931, p. 176), and others.

Testing Uniformity.—For both experiments, there was no prior reason to expect violations of exchangeability (the implication of uniformity relevant here). Unlike in the three-color Ellsberg paradox (discussed later), our subjects will not perceive different mechanisms of uncertainty underlying the sources considered here. Because uniformity is central to the source method, we nevertheless carried out several tests, and we chose to be on the safe side by rejecting the one case in the natural-event experiment in which one of the several tests gave a violation.

We have restricted attention to two outcomes so as to focus on the likelihood aspects of decision making. We also restricted attention to single-interval events. More elaborate tests, for instance regarding unions of interval events and more general outcomes, are planned for future research. Empirical violations of uniformity can then be expected that are not based on intrinsic nonuniformity, but on perceptual biases. For example, convex unions of intervals may be underestimated relative to nonconvex unions because, in the terminology of Tversky and Derek J. Koehler (1994), the former may be perceived as implicit unions and the latter as explicit unions. This point was tested and confirmed by Baillon (2008). Similarly, events related to extreme values (such as E_2^1) may be perceived differently than events related to intermediate values (such as $E_4^2 \cup E_4^3$).

Within-Source Uniformity versus Between-Source Uniformity.—For the cases where uniformity was satisfied, our tests found that, given a source, the willingness to bet on an event (through its certainty equivalent) depended only on the subjective probability of the event and not on where within the source the event came from otherwise. This implies a uniform degree of ambiguity throughout the source. The violations of probabilistic sophistication occurred only for comparisons of willingness to bet *between* different sources, and not *within* them. Such a phenomenon first occurred in Ellsberg's (1961) examples. We found the phenomenon also for natural events, and showed (Web Appendix) how it can accommodate the home bias. The source method exploits within-source uniformity while allowing between-source heterogeneity.

Other Measurements of Decision Weights in the Literature.—A measurement of decision weights using proper scoring rules, generalizing the latter to binary RDU, appeared in Theo Offerman et al. (2009). They obtained decision weights under uncertainty as functions of decision weights under risk, where the latter need not be additive, unlike our choice-based probabilities, so that they comprise part of the (nonexpected-utility) uncertainty attitude. Steffen Andersen et al. (2009b) measured subjective beliefs using a global maximum likelihood fitting technique. Here all decision components, utility, probability weighting, and subjective probabilities are fitted in one blow. Such a technique is powerful but needs extensive data (hence, all subjects with the same characteristics were treated as one subject). Andersen et al. (2009b) assumed global probabilistic sophistication, so that within-subject between-source heterogeneity and ambiguity aversion as in Ellsberg's paradoxes cannot be handled. If their technique is generalized to allow for source dependence of probability weighting, then it provides a useful alternative to our method for measuring source functions and reckoning with ambiguity attitudes.

Abdellaoui, Frank Vossmann, and Weber (2005) also analyzed general decision weights under uncertainty as functions of decision weights under risk. They used the term choice-based probability to refer to such functions that, again, did not have to be additive. They quantified attitudes towards uncertainty and ambiguity but in a general and complex manner, inheriting the dimensionality of general nonadditive weighting functions (with the same cardinality as the powerset of the state space), so that they do not achieve the tractability and reduction of dimension of our source functions. Unlike Offerman et al. (2009), Abdellaoui et al. (2005) did not use proper scoring rules but carried out a full decision analysis to elicit the required values. Enrico Diecidue, Wakker, and Marcel Zeelenberg (2007) similarly measured general weighting functions assuming linear utility.

Einhorn and Hogarth (1985) used transformations of judged probabilities to empirically investigate ambiguity attitudes. Judged probabilities were obtained through introspection by the subjects so that this approach was not based on revealed preference or on a decision theory for source dependence. Our study can be interpreted as providing a revealed-preference basis for Einhorn and Hogarth's ideas.

Heterogeneity or Noise?—Heterogeneity of risk and ambiguity attitudes between individuals has been widely documented (Ahn et al. 2009; Gilboa 2004; Yoram Halevy 2007), and our findings agree. Heterogeneity between sources within one individual has received less attention as yet, usually being restricted to the known-unknown probability dichotomy. The many significant differences that we found between different sources, and the natural directions of these differences—confirming earlier findings by Tversky and Fox (1995) and others for uncertainty and of numerous studies for risk—show that this heterogeneity is not noise. A maximum likelihood analysis with a choice error theory incorporated (see the Web Appendix) further supports this claim, giving the same results as the analysis reported here.

Precursors.—Several studies compared more refined gradations of ambiguity than the dichotomous known versus unknown probabilities, and they can be considered precursors of the between-source heterogeneity that we have argued for. Such studies include Baillon, Laure Cabantous, and Wakker (2011), Chew et al. (2008), Shawn Curley and J. Frank Yates (1989), Einhorn and Hogarth (1985), Halevy (2007), Ming Hsu et al. (2005), and Tversky and Fox (1995). In the same spirit, Tversky and Kahneman (1981 p. 454) wrote: "The major qualitative properties of decision weights can be extended to cases in which the probabilities of outcomes are subjectively assessed rather than explicitly given. In these situations, however, decision weights may also be affected by other characteristics of an event, such as ambiguity or vagueness." For these authors, subjective probabilities are derived from direct introspective judgments and are nonadditive, unlike those of the source method. Their method was called the two-stage model, was suggested by William Fellner (1961, p. 672), and was also discussed by Tversky and Kahneman (1992, p. 317). It was analyzed further by Abdellaoui, Vossmann, and Weber (2005), Fox, Rogers, and Tversky (1996), Fox and Tversky (1998), Kilka and Weber (2001), Tversky and Fox (1995), and Wu and Gonzalez (1999).

Cases where the Source Method Cannot be Applied.—The most well-known example of a nonuniform source is Ellsberg's (1961) three-color urn. This example can be remodeled as the intersections of events from two different uniform sources (Chew and Sagi 2008, Haluk Ergin and Faruk Gul 2009; Machina 2009a). Machina (2009b) introduced some paradoxes for rank-dependent utility. These are also paradoxes for most other ambiguity models popular today (Baillon, L'Haridon, and Placido, forthcoming). They, however, only concern prospects of three or more outcomes and do not concern our domain of binary prospects.

The Term Uniformity.—We chose Savage's (1954) term uniform rather than exchangeable for two reasons. First, uniformity is slightly more general than exchangeability when imposed on finite sources, not requiring all elementary events to be equally likely. Hence, a different term than exchangeability had to be chosen. Second, the condition suggests a uniform ambiguity of the source where, once two events have been revealed equally likely, they become completely substitutable in every aspect relevant for choice. It is immaterial what their precise location and configuration is relative to other events.[7]

Source Comparisons.—The Ellsberg paradoxes have mostly been interpreted as evidence showing that people are more averse to unknown probabilities than to known probabilities (ambiguity aversion). Our paper contributes to a line of research that extends this interpretation: People behave differently toward different sources of uncertainty, also if none of these sources concerns known probabilities (Chew et al. 2008; Chew and Sagi 2008; Curley and Yates 1989; Carmela Di Mauro and Anna Maffioletti 2001; Fox and Tversky 1998; Tversky and Fox 1995; Wu and Gonzalez 1999). An important example concerns the home bias (Web Appendix, Example A.1). The phenomena that we observed in the data confirm descriptive theories of ambiguity put forward in the psychological literature (Einhorn and Hogarth 1985; Tversky and Fox 1995), with not only a role for ambiguity aversion but also for likelihood insensitivity.

Ambiguity or Different Risk Attitudes?—A Terminological Issue.—It could be argued that the difference between known and unknown probabilities that we found in our natural-event experiment does not reflect ambiguity, but that instead it simply reflects a difference in risk attitude between the sources. It could then be argued in the same way, however, that the classical Ellsberg paradox does not reflect ambiguity either, but instead also reflects a difference in risk attitude between the known and the unknown urn. This point is, in fact, terminological. Risk attitude is *defined* as the attitude towards given probabilities, which is taken as one source. Following Ellsberg, the literature has *defined* ambiguity as the difference between unknown and known probabilities. We follow this terminology. It implies that source functions reflect a general uncertainty attitude that, by definition, consists of the risk attitude component plus an ambiguity attitude component.

Reducing Complexity.—For general weighting functions, a weight has to be chosen for every event separately, the complexity of which becomes intractable for large state spaces Ω[8]. The source method greatly simplifies the complexity of general weighting functions, reducing the number of parameters. We identify uniform sources and, for each such source, have to measure one more function, the source function, in addition to what is required for Bayesian analyses (utilities and probabilities). This procedure is simple enough to be implementable for large state spaces, as we have demonstrated in the experiments.

Predicting Choices between Multioutcome Prospects.—Under the rank-dependent models (Gilboa 1987; Schmeidler 1989) which include prospect theory (Tversky

[7] There are some formal differences between our concept of uniformity and Chew and Sagi's (2008) concept of homogeneity. The main difference is that our sources are a special case of theirs in the sense that our sources always span the universal event so that we never use conditioning on subevents. We prefer to separate the static concept of uniformity from dynamic issues regarding conditioning. Chew and Sagi incorporated conditioning to handle Ellsberg's three-color example, but we prefer to model this example as an intersection of different sources.

[8] The dimension of the set of weighting functions, $(2^{\|\Omega\|}-2)$, grows exponentially.

and Kahneman 1992), source functions as measured by the source method completely determine the choices between all prospects, including those with many outcomes. Under the maxmin and α maxmin multiple priors models, further data about multioutcome prospects is required to predict choices between other multioutcome prospects.

Multistage Recursive Models of Uncertainty.—Klibanoff, Marinacci, and Mukerji (2005), Robert F. Nau (2006), William S. Neilson (2010), and Kyoungwon Seo (2009) considered multistage setups with backward induction and a violation of the reduction of compound lotteries (the multiplication rule for conditional probability) assumed. With the events at each stage taken as a separate source, these authors assumed expected utility (and hence probabilistic sophistication) within each source. It implies that different attitudes toward different sources, including ambiguity attitudes, should be captured by different utility functions. The latter, in our interpretation, then are source-dependent utility functions. This interpretation was explicitly used in a single-stage based experiment by Chew et al. (2008) and Andersen et al. (2009a). Ahn et al. (2009) found that the ("single-stage") binary RDU fitted their data better than the multistage model.

Ergin and Gul (2009) generalized the above multistage approaches by allowing for probabilistic sophistication, rather than expected utility, in each stage, and in that sense are closer to our method. They still committed to the same multistage arrangement of sources and the same dynamic decision principles as the above authors did.[9] They used the term issue instead of Tversky's term source (that we used). Halevy and Ozdenoren (2008) introduced a calibration technique for such models. Chew and Sagi (2008) introduced small worlds that, apart from some formal differences, play a role similar to our sources.

VI. Conclusion

We introduced the source method with source functions as a refined tool to quantitatively capture the full richness of ambiguity. The source method exploits within-source uniformity while allowing between-source heterogeneity. In two experiments, attitudes towards uncertainty and ambiguity depended not only on the person but also on the source of uncertainty. These findings show that uncertainty is a rich domain that can yet be analyzed in a tractable manner.

REFERENCES

Abdellaoui, Mohammed, Han Bleichrodt, and Olivier L'Haridon. 2008. "A Tractable Method to Measure Utility and Loss Aversion under Prospect Theory." *Journal of Risk and Uncertainty*, 36(3): 245–66.

Abdellaoui, Mohammed, Olivier L'Haridon, and Corina Paraschiv. 2009. "Experienced-Based vs. Description-Based Uncertainty: Do We Need Two Different Prospect Theory Specifications?" Unpublished.

[9] This follows primarily from their Axiom 5b that, when restricted to empty events B_2, amounts to the weak separability needed for backward induction.

Abdellaoui, Mohammed, Frank Vossmann, and Martin Weber. 2005. "Choice-Based Elicitation and Decomposition of Decision Weights for Gains and Losses under Uncertainty." *Management Science,* 51(9): 1384–99.

Ahn, David, Syngjoo Choi, Douglas Gale, and Shachar Kariv. 2009. "Estimating Ambiguity Aversion in a Portfolio Choice Experiment." http://www.nyu.edu/econ/user/galed/papers/paper09-03-02.pdf

Andersen, Steffen, John Fountain, Glenn W. Harrison, and E. Elisabet Rutström. 2009a. "Estimating Aversion to Uncertainty." Unpublished.

Andersen, Steffen, John Fountain, Glenn W. Harrison, and E. Elisabet Rutström. 2009b. "Estimating Subjective Probabilities." Unpublished.

Baillon, Aurélien. 2008. "Eliciting Subjective Probabilities through Exchangeable Events: An Advantage and a Limitation." *Decision Analysis,* 5(2): 76–87.

Baillon, Aurélien, Laure Cabantous, and Peter P. Wakker. 2011. "Ambiguity and Expert Aggregation: Subjective Beliefs under Imprecise or Conflicting Sources of Information." Unpublished.

Baillon, Aurélien, Olivier L'Haridon, and Laetitia Placido. Forthcoming. "Ambiguity Models and the Machina Paradoxes." *American Economic Review.*

Camerer, Colin F., and Robin M. Hogarth. 1999. "The Effects of Financial Incentives in Experiments: A Review and Capital-Labor-Production Framework." *Journal of Risk and Uncertainty*, 19(1–3): 7–42.

Chateauneuf, Alain, Jürgen Eichberger, and Simon Grant. 2007. "Choice under Uncertainty with the Best and Worst in Mind: NEO-Additive Capacities." *Journal of Economic Theory*, 137(1): 538–67.

Chen, Yan, Peter Katušcák, and Emre Ozdenoren. 2007. "Sealed Bid Auctions with Ambiguity: Theory and Experiments." *Journal of Economic Theory*, 136(1): 513–35.

Chew Soo Hong, and Jacob S. Sagi. 2006. "Event Exchangeability: Probabilistic Sophistication without Continuity or Monotonicity." *Econometrica*, 74(3): 771–86.

Chew, Soo Hong, and Jacob S. Sagi. 2008. "Small Worlds: Modeling Attitudes toward Sources of Uncertainty." *Journal of Economic Theory*, 139(1): 1–24.

Chew, Soo Hong, King King Li, Robin Chark, and Songfa Zhong. 2008. "Source Preference and Ambiguity Aversion: Models and Evidence from Behavioral and Neuroimaging Experiments." In *Neuroeconomics.* Advances in Health Economics and Health Services Research, Vol. 20, ed. Daniel Houser and Kevin McCabe, 179–201. Bingley, UK: JAI Press.

Choi, Syngjoo, Raymond Fisman, Douglas Gale, and Shachar Kariv. 2007. "Consistency and Heterogeneity of Individual Behavior under Uncertainty." *American Economic Review*, 97(5): 1921–38.

Curley, Shawn P., and J. Frank Yates. 1989. "An Empirical Evaluation of Descriptive Models of Ambiguity Reactions in Choice Situations." *Journal of Mathematical Psychology,* 33(4): 397–427.

Di Mauro, Carmela, and Anna Maffioletti. 2001. "The Valuation of Insurance under Uncertainty: Does Information about Probability Matter?" *Geneva Papers on Risk and Insurance Theory*, 26(3): 195–224.

Diecidue, Enrico, Peter P. Wakker, and Marcel Zeelenberg. 2007. "Eliciting Decision Weights by Adapting de Finetti's Betting-Odds Method to Prospect Theory." *Journal of Risk and Uncertainty*, 34(3): 179–99.

Einhorn, Hillel J., and Robin M. Hogarth. 1985. "Ambiguity and Uncertainty in Probabilistic Inference." *Psychological Review,* 92(4): 433–61.

Ellsberg, Daniel. 1961. "Risk, Ambiguity, and the Savage Axioms." *Quarterly Journal of Economics,* 75(4): 643–69.

Epstein, Larry G. 2008. "Living with Risk." *Review of Economic Studies*, 75(4): 1121–41.

Ergin, Haluk, and Faruk Gul. 2009. "A Theory of Subjective Compound Lotteries." *Journal of Economic Theory*, 144(3): 899–929.

Fellner, William. 1961. "Distortion of Subjective Probabilities as a Reaction to Uncertainty." *Quarterly Journal of Economics,* 75(4): 670–89.

Fox, Craig R., and Amos Tversky. 1995. "Ambiguity Aversion and Comparative Ignorance." *Quarterly Journal of Economics*, 110(3): 585–603.

Fox, Craig R., and Amos Tversky. 1998. "A Belief-Based Account of Decision under Uncertainty." *Management Science,* 44(7): 879–95.

Fox, Craig R., Brett A. Rogers, and Amos Tversky. 1996. "Options Traders Exhibit Subadditive Decision Weights." *Journal of Risk and Uncertainty*, 13(1): 5–17.

French, Kenneth R., and James M. Poterba. 1991. "Investor Diversification and International Equity Markets." *American Economic Review*, 81(2): 222–26.

Gajdos, Thibault, Takashi Hayashi, Jean-Marc Tallon, and Jean-Christophe Vergnaud. 2008. "Attitude toward Imprecise Information." *Journal of Economic Theory*, 140(1): 27–65.

Ghirardato, Paolo, and Massimo Marinacci. 2001. "Risk, Ambiguity, and the Separation of Utility and Beliefs." *Mathematics of Operations Research,* 26(4): 864–90.

Ghirardato, Paolo, Fabio Maccheroni, and Massimo Marinacci. 2004. "Differentiating Ambiguity and Ambiguity Attitude." *Journal of Economic Theory*, 118(2): 133–73.
Gilboa, Itzhak. 1987. "Expected Utility with Purely Subjective Non-Additive Probabilities." *Journal of Mathematical Economics*, 16(1): 65–88.
Gilboa, Itzhak, ed. 2004. *Uncertainty in Economic Theory: Essays in Honor of David Schmeidler's 65th Birthday.* New York: Routledge.
Gilboa, Itzhak, and David Schmeidler. 1989. "Maxmin Expected Utility with Non-Unique Prior." *Journal of Mathematical Economics*, 18(2): 141–53.
Gonzalez, Richard, and George Wu. 1999. "On the Shape of the Probability Weighting Function." *Cognitive Psychology,* 38(1): 129–66.
Halevy, Yoram. 2007. "Ellsberg Revisited: An Experimental Study." *Econometrica*, 75(2): 503–36.
Halevy, Yoram, and Emre Ozdenoren. 2008. "Uncertainty and Compound Lotteries: Calibration." http://faculty.arts.ubc.ca/yhalevy/2SPS.pdf.
Hogarth, Robin M. and Hillel J. Einhorn. 1990. "Venture Theory: A Model of Decision Weights." *Management Science,* 36(7): 780–803.
Hsu, Ming, Meghana Bhatt, Ralph Adolphs, Daniel Tranel, and Colin F. Camerer. 2005. "Neural Systems Responding to Degrees of Uncertainty in Human Decision-Making." *Science,* 310(5754): 1680–83.
Kilka, Michael, and Martin Weber. 2001. "What Determines the Shape of the Probability Weighting Function under Uncertainty?" *Management Science,* 47(12): 1712–26.
Klibanoff, Peter, Massimo Marinacci, and Sujoy Mukerji. 2005. "A Smooth Model of Decision Making under Ambiguity." *Econometrica*, 73(6): 1849–92.
Knight, Frank H. 1921. *Risk, Uncertainty and Profit.* Boston: Houghton Mifflin.
Luce, R. Duncan. 1991. "Rank- and Sign-Dependent Linear Utility Models for Binary Gambles." *Journal of Economic Theory*, 53(1): 75–100.
Maccheroni, Fabio, Massimo Marinacci, and Aldo Rustichini. 2006. "Ambiguity Aversion, Robustness, and the Variational Representation of Preferences." *Econometrica*, 74(6): 1447–98.
Machina, Mark J. 2009a. "Betting on Ellsberg's Urn." Paper presented at Decisions Theory, Experiments and Applications (D-TEA) Workshop, HEC Paris, France, April 5.
Machina, Mark J. 2009b. "Risk, Ambiguity, and the Rank-Dependence Axioms." *American Economic Review*, 99(1): 385–92.
Machina, Mark J., and David Schmeidler. 1992. "A More Robust Definition of Subjective Probability." *Econometrica*, 60(4): 745–80.
Miyamoto, John M. 1988. "Generic Utility Theory: Measurement Foundations and Applications in Multiattribute Utility Theory." *Journal of Mathematical Psychology,* 32(4): 357–404.
Nau, Robert F. 2006. "Uncertainty Aversion with Second-Order Utilities and Probabilities." *Management Science,* 52(1): 136–45.
Neilson, William S. 2010. "A Simplified Axiomatic Approach to Ambiguity Aversion." *Journal of Risk and Uncertainty*, 41: 133–24.
Offerman, Theo, Joep Sonnemans, Gijs Van De Kuilen, and Peter P. Wakker. 2009. "A Truth Serum for Non-Bayesians: Correcting Proper Scoring Rules for Risk Attitudes." *Review of Economic Studies*, 76(4): 1461–89.
Prelec, Drazen. 1998. "The Probability Weighting Function." *Econometrica*, 66(3): 497–527.
Rabin, Matthew. 2000. "Risk Aversion and Expected-Utility Theory: A Calibration Theorem." *Econometrica*, 68(5): 1281–92.
Ramsey, Frank P. 1931. "Truth and Probability." In *The Foundations of Mathematics and Other Logical Essays,* 156–98. London: Routledge.
Savage, Leonard J. 1954. *The Foundations of Statistics.* New York: Wiley.
Schmeidler, David. 1989. "Subjective Probability and Expected Utility without Additivity." *Econometrica*, 57(3): 571–87.
Seo, Kyoungwon. 2009. "Ambiguity and Second-Order Belief." *Econometrica*, 77(5): 1575–1605.
Smith, Vernon L. 1969. "Measuring Nonmonetary Utilities in Uncertain Choices: The Ellsberg Urn." *Quarterly Journal of Economics,* 83(2): 324–29.
Tversky, Amos, and Craig R. Fox. 1995. "Weighing Risk and Uncertainty." *Psychological Review,* 102(2): 269–83.
Tversky, Amos, and Daniel Kahneman. 1981. "The Framing of Decisions and the Psychology of Choice." *Science,* 211(4481): 453–58.
Tversky, Amos, and Daniel Kahneman. 1992. "Advances in Prospect Theory: Cumulative Representation of Uncertainty." *Journal of Risk and Uncertainty*, 5(4): 297–323.
Tversky, Amos, and Derek J. Koehler. 1994. "Support Theory: A Nonextensional Representation of Subjective Probability." *Psychological Review,* 101(4): 547–67.

Tversky, Amos, and Peter Wakker. 1995. "Risk Attitudes and Decision Weights." *Econometrica*, 63(6): 1255–80.

Wakker, Peter P. 2008. "Uncertainty." In *The New Palgrave: A Dictionary of Economics,* Vol. 8, ed. Lawrence Blume and Steven N. Durlauf, 428–39. London: Palgrave MacMillan.

Webb, Craig, and Horst Zank. 2008. "Expected Utility with Certainty and Impossibility Effects." Unpublished.

Winkler, Robert L. 1991. "Ambiguity, Probability, Preference, and Decision Analysis." *Journal of Risk and Uncertainty*, 4(3): 285–97.

Wu, George, and Richard Gonzalez. 1999. "Nonlinear Decision Weights in Choice under Uncertainty." *Management Science,* 45(1): 74–85.