The Data of Levy and Levy (2002) "Prospect Theory: Much Ado About Nothing?" Actually Support Prospect Theory

Peter P. Wakker
Department of Economics, University of Amsterdam, Roetérsstraat II, Amsterdam 1018 WB, The Netherlands
p.p.wakker@uva.nl

Levy and Levy (*Management Science* 2002) present data that, according to their claims, violate prospect theory. They suggest that prospect theory's hypothesis of an S-shaped value function, concave for gains and convex for losses, is incorrect. However, all the data of Levy and Levy are perfectly consistent with the predictions of prospect theory, as can be verified by simply applying prospect theory formulas. The mistake of Levy and Levy is that they, incorrectly, thought that probability weighting could be ignored.
(*Prospect Theory; Stochastic Dominance; Utility; Probability Weighting; Inverses*)

Levy and Levy (*Management Science* 2002, henceforth LL) present data that, according to their claims, violate the S-shaped value function posited by prospect theory. This comment will show, however, that LL's data are in perfect agreement with prospect theory. Following LL, we will throughout restrict attention to outcomes that are not very extreme, say between \$6,000 and $-\$6,000$.

The classical views on risk attitudes assumed universal risk aversion. Empirical studies have revealed a more complex, fourfold pattern in behavior. For gains, people are mostly risk averse; but for specific prospects, yielding a best outcome with a low probability (below 1/3), we often find risk-seeking behavior, as observed in gambling for instance. The pattern for losses is less clear but seems to be reversed. People are mostly risk seeking, but for prospects yielding a worst outcome with a low probability (below 1/3), risk aversion can occur, as observed for instance in insurance. This fourfold pattern is based on extensive empirical evidence (reviewed in Starmer 2000 and Luce 2000) and entails extremity-orientedness whereby the best and worst outcomes of a prospect are overweighted and middle outcomes are underweighted.

Prospect theory models the fourfold pattern. We focus on the most recent, cumulative version of prospect theory (Tversky and Kahneman 1992). It assumes a *utility* or *value function* $v(x)$ that is S-shaped: increasing for all amounts, concave for gains, and convex for losses. This assumed shape reflects the psychological phenomenon of diminishing sensitivity as one moves away from the "reference point." The reference point divides gains from losses, and is taken to be zero in this comment. Consider a prospect (or gamble) with outcomes $x_1 \leq \cdots \leq x_k \leq 0 \leq x_{k+1} \leq \cdots \leq x_n$ having probabilities $p_1, \ldots, p_n$. Prospect theory predicts that people will choose prospects according to the value given by

$$\sum_{i=1}^{k} \pi_i \lambda v(x_i) + \sum_{j=k+1}^{n} \pi_j v(x_j), \tag{1}$$

where $\lambda > 0$ is a *loss-aversion parameter*, and the πs are *decision weights* that are calculated based on the "cumulative" probabilities associated with the

0025-1909/03/4907/0979\$05.00
1526-5501 electronic ISSN

outcomes. The details of this model are provided in the appendix.

Following Markowitz (1952), LL posit that the value function is convex for gains and concave for losses, implying a reverse S-shape—the opposite of that assumed by prospect theory. In their theoretical analysis, LL assume expected utility theory and, thus, assume that the decision weights in Equation (1) are simply equal to the probabilities associated with the outcomes. This coupled with their assumption about the form of the value function implies risk aversion (and, more strongly, second-order stochastic dominance) for losses and risk seeking (and reversed second-order stochastic dominance) for gains, in contrast to the more complex fourfold pattern of observed behavior. LL develop a stochastic dominance rule that they call *Markowitz stochastic dominance*. Given expected utility theory, this rule allows them to show that some gambles are preferred to others for all value functions having the reverse S-shape.

LL test their hypothesis about the shape of the value function through three choice exercises that they refer to as head-to-head competitions, and in which they claim that an S-shaped value function would lead to one choice and a reverse S-shaped value function to another. In all three experiments, the majority choice corresponds to the choice that supposedly supports the reverse S-shape, and LL interpret this as evidence contradicting prospect theory. Simple calculations show, however, that prospect theory with the functional forms and parameter estimates of Tversky and Kahneman (1992) *correctly* predicts the majority choice in all head-to-head competitions; these calculations are displayed in Table 1, with explanations given in the appendix. This finding is contrary to LL's claims (p. 1344, "Thus, we can state that *at least* 62% of the choices are inconsistent with prospect theory"). We conclude that LL's data actually support prospect theory.

The error in LL's analysis is that they neglect the probability weighting function of prospect theory. They argue, "All probabilities given in the experiments are relatively large ($p \geq 0.25$), hence it is unlikely that subjective probability distortion plays an important role in the decision-making process" (p. 1341; a similar statement appears on p. 1346).[1] As Table 1b shows, the extreme outcomes of F and G in Experiment 2 have decision weights about twice as much as the intermediate outcomes

Table 1 Prospects Yield Outcome x with Probability p.

Prospect	p	x	π	v	PT	Choice
(a) LL's Experiment 1, Task 1						
F	0.50	−3,000	0.45	−1,148	−483	71%
	0.50	4,500	0.42	1,640		
G	0.25	−6,000	0.29	−2,112	−743	27%
	0.75	3,000	0.57	1,148		
(b) LL's Experiment 2						
F	0.25	−1,600	0.29	−660	−216	38%
	0.25	−200	0.16	−106		
	0.25	1,200	0.13	512		
	0.25	1,600	0.29	660		
G	0.25	−1,000	0.29	−437	−138	62%
	0.25	−800	0.16	−359		
	0.25	800	0.13	359		
	0.25	2,000	0.29	803		
(c) LL's Experiment 3, Task 3						
F	0.50	−1,500	0.45	−624	53	76%
	0.50	4,500	0.42	1,640		
G	0.25	−3,000	0.29	−1,148	−106	23%
	0.75	3,000	0.57	1,148		

Note: Participants in Levy and Levy (2002) chose between the head-to-head prospect pairs F and G. The PT column gives the values of the prospects according to prospect theory, using the parameters estimated by Tversky and Kahneman (1992). Each π is the decision weight of outcome x, and v its utility/value. Bold printing indicates the majority choice, which is always the option preferred according to prospect theory because it has the higher value under that theory.

[1] In addition to this argument, LL suggest (p. 1344) that taking the outcomes to be equally likely in their experiment 2 "makes any subjective probability distortion very unlikely." In their Footnote 15, they expand on this and say "This point was made by Quiggin (1982). In addition, any subjective transformation performed directly on the probabilities (as in prospect theory) will still attach an equal probability weight to each outcome." While in the original version of prospect theory (Kahneman and Tversky 1979) the transformation was performed directly on the probabilities alone, this is not so in the current version and equally likely outcomes can be weighted differently. Contrary to what LL claim, Quiggin (1982) also argued that equally likely outcomes may be weighed differently (end of §1).

under prospect theory (weights 0.29 versus 0.13 or 0.16), which deviates considerably from the equal weighting assumed by LL. While these calculations are based on the specific parameter assumptions suggested by Tversky and Kahneman (1992), LL's experimental results are also qualitatively consistent with the extremity-orientedness predicted by prospect theory. In each of LL's three head-to-head competitions, the majority chose the gamble that had both the best maximal outcome and the best minimal outcome, as would be done if only the extreme outcomes mattered.

In conclusion, the data of LL support the predictions of Tversky and Kahneman's (1992) prospect theory. The incorrect claims of LL are mostly due to their overlooking the crucial role of probability weighting in prospect theory. While the data could also be consistent with other theories, it is extremely misleading to interpret this as evidence against prospect theory or to suggest that prospect theory is "much ado about nothing." In particular, the results of LL do not provide new insights into the shape of the value/utility function. Their hypothesis of convex utility for gains is contrary to the diminishing marginal utility assumed in classical analyses, the diminishing sensitivity assumed in prospect theory, and virtually all empirical findings of the vast literature on this topic.

Acknowledgments

The author thanks John Payne, Thierry Post, Pim van Vliet, George Wu, and three anonymous referees for helpful comments, and the department editor for numerous thoughtful and constructive suggestions.

Appendix: Prospect Theory

Prospect theory assumes, besides the utility or value function and the loss-aversion parameter, a *probability weighting function* w^+: $[0,1] \to [0,1]$ for gains, and a *probability weighting function* w^-: $[0,1] \to [0,1]$ for losses. The decision weights π in Equation (1) are defined as follows. If $k \geq 1$ then $\pi_1 = w^-(p_1)$, and $\pi_i = w^-(p_1 + \cdots + p_i) - w^-(p_1 + \cdots + p_{i-1})$ for $2 \leq i \leq k$. If $k < n$ then $\pi_n = w^+(p_n)$ and $\pi_j = w^+(p_n + \cdots + p_j) - w^+(p_n + \cdots + p_{j+1})$ for $n-1 \geq j > k$.

Tversky and Kahneman (1992) estimated the following parametric form: $v(x) = x^{0.88}$ for $x \geq 0$, $v(x) = -(-x)^{0.88}$ for $x \leq 0$, $\lambda = 2.25$, $w^+(p) = p^{0.61}/(p^{0.61} + (1-p)^{0.61})^{1/0.61})$, $w^-(p) = p^{0.69}/(p^{0.69} + (1-p)^{0.69})^{1/0.69})$. For prospect F in Table 1b, the decision weights are $\pi_1 = w^-(0.25) = 0.29$ for outcome $x_1 = -1{,}600$, $\pi_2 = w^-(0.50) - w^-(0.25) = 0.16$ for outcome $x_2 = -200$, $\pi_3 = w^+(0.50) - w^+(0.25) = 0.13$ for outcome $x_3 = 1{,}200$, and $\pi_4 = w^+(0.25) = 0.29$ for outcome $x_4 = 1{,}600$. The value of F is $\pi_1 \lambda v(-1{,}600) + \pi_2 \lambda v(-200) + \pi_3 v(1{,}200) + \pi_4 v(1{,}600) = -215.70$. The other prospects are evaluated similarly. A program to calculate prospect-theory values, written by Veronika Köbberling, is available at http://www1.fee.uva.nl/creed/wakker/miscella/calculate.cpt.kobb/index.htm.

References

Kahneman, Daniel, Amos Tversky. 1979. Prospect theory: An analysis of decision under risk. *Econometrica* **47** 263–291.

Levy, Moshe, Haim Levy. 2002. Prospect theory: Much ado about nothing. *Management Sci.* **48** 1334–1349.

Luce, R. Duncan. 2000. *Utility of Gains and Losses: Measurement-Theoretical and Experimental Approaches*. Lawrence Erlbaum Publishers, London, UK.

Markowitz, Harry M. 1952. The utility of wealth. *J. Political Econom.* **60** 151–158.

Quiggin, John. 1982. A theory of anticipated utility. *J. Econom. Behaviour Organ.* **3** 323–343.

Starmer, Chris. 2000. Developments in non-expected utility theory: The hunt for a descriptive theory of choice under risk. *J. Econom. Literature* **38** 332–382.

Tversky, Amos, Daniel Kahneman. 1992. Advances in prospect theory: Cumulative representation of uncertainty. *J. Risk Uncertainty* **5** 297–323.

Note from the Editor-in-Chief. Levy and Levy, the authors of the paper "Prospect Theory: Much Ado About Nothing," also wrote the following paper: "Experimental Test of the Prospect Theory Value Function: A Stochastic Dominance Approach" (*Organization Behavior and Human Decision Processes* 89, 2002, pp. 1058–1081). The two papers present very similar experiments and results. The failure of Levy and Levy to cross-cite these papers is a violation of proper scholarly practice and may have contributed to the controversy surrounding their work.

Accepted by James E. Smith; received November 27, 2002. This paper was with the author 1 month for 2 revisions.