Are Counterfactual Decisions Relevant for Dynamically Consistent Updating under Nonexpected Utility?*

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ABSTRACT. This paper proposes a new updating method that preserves dynamic consistency in nonexpected utility. Given nonseparability of disjoint events, preferences conditional on an observed event also depend on counterfactual outcomes, i.e., outcomes that would have resulted outside of the conditioning event; this point has been well-understood in the literature. This paper argues that, as a consequence, also counterfactual decisions are relevant. A new "strategic" method for updating then follows.

KEYWORDS: dynamic consistency, resolute choice, updating.

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1. INTRODUCTION

This paper proposes a new updating method for nonexpected utility, called strategic updating. Strategic updating follows the dynamic decision principles of resolute choice advocated by McClennen (1988, 1990) and Machina (1989, 1991), in particular it preserves dynamic consistency. It deviates, however, from the method of resolute updating generally adopted in the literature and called committed updating in this paper. Under committed updating, a fixed choice is assumed at counterfactual decision nodes, so that there is no more counterfactual decision to be taken. Under strategic updating, not only counterfactual outcomes, but also counterfactual decisions remain relevant.

Let me emphasize that strategic updating does not induce new preference behavior, different from resolute choice. The main result of this paper, Theorem 5.1, shows that resolute choice maximizes the strategically updated functional at every decision node of every tree. That is, strategic updating is fully in line with resolute choice. The claim of this paper is therefore that the natural way for updating resolute choice is strategic, and not committed.

If strategic updating is considered undesirable, I hope the reader will not hold that against the analysis of this paper, but will instead question the premise of the analysis, being resolute choice. In that case, this paper can be interpreted as a negative result for resolute choice. I hope that this paper, even if interpreted as a criticism of resolute choice, nevertheless contributes to our understanding of dynamic consistency in nonexpected utility.

The outline of the paper is as follows. Section 2 summarizes the difficulties in applying nonexpected utility to dynamic choice and updating. Section 3 defines resolute choice, committed updating, and strategic updating. In the next section, Example 4.1 repeates principles of revealed preference, in particular "menu independence" (every choice option has an intrinsic value, independent of competing options). That principle underlies the concept of utility, and has been generally accepted in preference theories such as consumer demand theory. Example 4.2 is a special case of Example 4.1. It applies the revealed preference principles to updating under resolute choice. Menu independence then leads to strategic updating, and not committed updating. Theorem 5.1 states the result formally, for general decision trees. Section 6 presents some examples where committed updating does result. In each case the result is due to confounding factors, not representative of risk attitude, such as extraneous commitments or hidden nodes. Section 7 presents applications of strategic updating. Section 8, finally, rephrases the argument of this paper in terms of inseparability of events, and concludes.

2. NONEXPECTED UTILITY IN DYNAMIC DECISIONS

Today's preference is the update of yesterday's preference (Machina, 1989, p. 1652). Hence, decision models should be able to model updated preference. Indeed, updating is a central topic of debate in the modern nonexpected utility theories. New impulses have come from game theory, where the consistency requirement for equilibrium hinges crucially on the method of updating after an opponent's move (Dow & Werlang, 1994; Eichberger & Kelsey, 1994; Haller, 1995; Hendon, Jacobsen, Sloth, & Tranaes, 1995; Klibanoff, 1995; Lo, 1995; Ghirardato & Le Breton, 1996; Mukerji & Shin, 1997).

Updating is relatively simple in expected utility where, because of separability of disjoint events, the technique of dynamic optimization can be used (Bellman, 1954; Streufert, 1990). Tractability is guaranteed because one can forget about counterfactual events from the past (consequentialism) and implementability is guaranteed because one adheres to prior plans (dynamic consistency). One of the most serious challenges to nonexpected utility was put forward by Hammond (1988) (see also Karni & Safra, 1989, and Sarin, 1992). Hammond made the surprising discovery that, essentially, the technique of dynamic optimization implies expected utility in static decisions. Hence the technique cannot be used and must be abandoned by nonexpected utility, and new methods of updating must be developed.

A range of responses to Hammond's discovery has been provided. Karni & Safra (1990) advocate sophisticated choice of Strotz (1956) and Pollak (1968), abandoning dynamic consistency. Grant, Kajii, & Polak (1997a,b) and the references therein abandon reduction of compound lotteries. The most common response is to preserve dynamic consistency and reduction of compound lotteries, and abandon consequentialism (Machina, 1989, 1991; McClennen, 1988, 1990). The resulting approach is called *resolute choice* and is the subject of this paper. Strotz (1956) suggested, in a context without uncertainty, that such an approach cannot be implemented unless a precommitment device would be available. Machina and McClennen give arguments for implementability without such a device. The critical implication of resolute choice is that the value of a real strategy depends on counterfactual events. Such dependency, while obvious in game theory (Harsanyi & Selten, 1988; Shin, 1991; Asheim, 1997), is debated in individual decision making.

A drawback of resolute choice is its intractability. One's current decision depends on all the events that might have happened in the past but didn't. Machina (1989) suggests that in a complete analysis that is indeed the case, but counterfactual events from long ago may be ignored if their impact on a distant future has faded away. We will follow Machina and others by not entering lifetime decision trees, but using simple decision trees in the examples and illustrations. Let me emphasize that this paper assumes resolute choice throughout, and argues for normative strategic updating *under that assumption*. A detailed discussion of the pros and cons of resolute choice is outside the scope of this paper.

If one uses Choquet-expected utility (Schmeidler, 1989; Gilboa, 1987) to derive decisions from nonadditive measures, then methods for updating nonadditive beliefs can generate methods for updating preferences. Various update methods for nonadditive beliefs have been proposed (Dempster, 1967; Denneberg, 1994; Gilboa & Schmeidler, 1993; Haller, 1995; Jaffray, 1992, 1994; Lehrer, 1996; Mukerji, 1996; Shafer, 1976). For some of these, unfortunately, Choquet expected utility would not be closed under updating (Ghirardato, 1997). As pointed out by Gilboa & Schmeidler (1993), Dempster-Shafer updating can be related to Choquet expected utility by assuming superior counterfactual outcomes, and Bayesian updating by assuming inferior counterfactual outcomes. For the purpose of this paper, it suffices to note that both updating methods can be related to fixed counterfactual decisions.

3. COMMITTED AND STRATEGIC UPDATING

Let V be the nonexpected utility functional that represents prior preference over probability distributions over outcomes. *Resolute choice* adopts the following prior optimization method for solving decision trees.

(1) List all the strategies available in the decision tree.
(2) For each strategy, calculate the generated probability distribution over outcomes.
(3) Choose the best available probability distribution.
(4) Follow the belonging strategy throughout the decision tree.

Thus, the decision trees can be solved by prior "normal form" optimization over strategies. Step 2 is based on reduction of compound lotteries, step 3 on the basic rationality principles of revealed preference, and step 4 on dynamic consistency.

We assume henceforth that an event E has been observed, leading to a decision node that we call node 1. Paths from node 1 onwards are denoted by X, Y, Z, R, and are called *real*. The complementary event E^c is now known not to be true. Paths from the resulting decision node 2 onwards are denoted by A, B, C and are called *counterfactual*[1]. Thus, paths through the tree can be denoted by XA, RC, etc.; they are identified with probability distributions over outcomes.

[1]Counterfactual decisions are sometimes called forgone decisions in the literature.

V_E denotes the derived functional (still to be explained) that represents the decision maker's updated preferences given E. Under resolute choice, V_E should be in complete agreement with prior preference. The common method of resolute updating, routinely followed in all papers that I am aware of, is committed updating. It assumes a fixed counterfactual strategy A, and evaluates real options R by V(RA) (Eichberger & Grant, 1997; Eichberger & Kelsey, 1996; Gilboa & Schmeidler, 1993; Jaffray, 1994, Figure 1; Lo, 1995, 1996; Machina & Schmeidler, 1992).

In general, however, several counterfactual strategies will be available. A crucial question for committed updating then is, of course, which counterfactual strategy A should be chosen in the committed updating functional V(RA). The only paper that, to the best of my knowledge, considers cases with several such counterfactual decisions, is Machina (1989; see also its twin Machina, 1991). In all examples in that paper, however, the choice between counterfactual decisions is trivially governed by stochastic dominance and hence there is no real issue of counterfactual decisions (in Machina's Figure 12, which does have nontrivial counterfactual decisions, the decision maker is misinformed and does not know about those decisions). Parts of the text in Machina (1989), in particular Footnote 29, suggest the following procedure, and personal communications have confirmed that it is the generally accepted procedure in the field. First, the optimal prior strategy is determined, denoted XA throughout the rest of this paper.[2] Next, the counterfactual part A of XA is taken as the fixed counterfactual strategy to be used in updating.

Strategic updating does not assume a fixed choice outside of E. Now any real option R is evaluated by $\max_C V(RC)$, i.e., the counterfactual strategy is optimized given R. $V_E(\cdot)$ thus is the pointwise optimum over all functionals $V(\cdot R)$.

In summary:

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[2]To avoid formulas with suprema, I restrict attention to decision trees where the relevant maximization problems have solutions. Sometimes I use formulations for the case of unique solutions, leaving the general formulations to the reader.

• *Committed updating*: $V_E(\bullet) = V(\bullet A)$. (3.1)

• *Strategic updating*: $V_E(\bullet) = \max_C V(\bullet C)$ over all counterfactual C. (3.2)

4. A CHOICE-BASED INTERPRETATION OF UPDATED PREFERENCE AND AN EXAMPLE

This section discusses the pros and cons of the updating methods in the context of an example. The next section gives formal statements of the claims made here. In the first example, which does not yet consider uncertainty or dynamic choice, general foundations of revealed preference are repeated. The second example is a special case of the first, with uncertainty and resolute choice involved. The principles of the first example then naturally lead to strategic updating. Other studies of uncertainty that invoke principles of revealed preference are Hammond (1976, 1988) and Green & Oswald (1991).

EXAMPLE 4.1 [Revealed Preference]. Consider a choice set {X,Y,Z} containing three options, where options can be anything such as commodity bundles, houses, welfare allocations, lotteries, income profiles, etc. An economist observes the choice of a rational consumer. If the consumer is only willing to choose X, then we infer $X \succ Y$ and $X \succ Z$. The preference between Y and Z then cannot be inferred from this choice situation (Kreps, 1988). To find out about the preference between Y and Z, the economist discards option X and presents[3] the choice set {Y,Z} to the consumer. If the consumer now is only willing to choose Z, then we conclude that $Z \succ Y$. In this manner, choices correspond with preferences and vice versa. □

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[3]possibly only as a hypothetical thought experiment (Kreps, 1988)

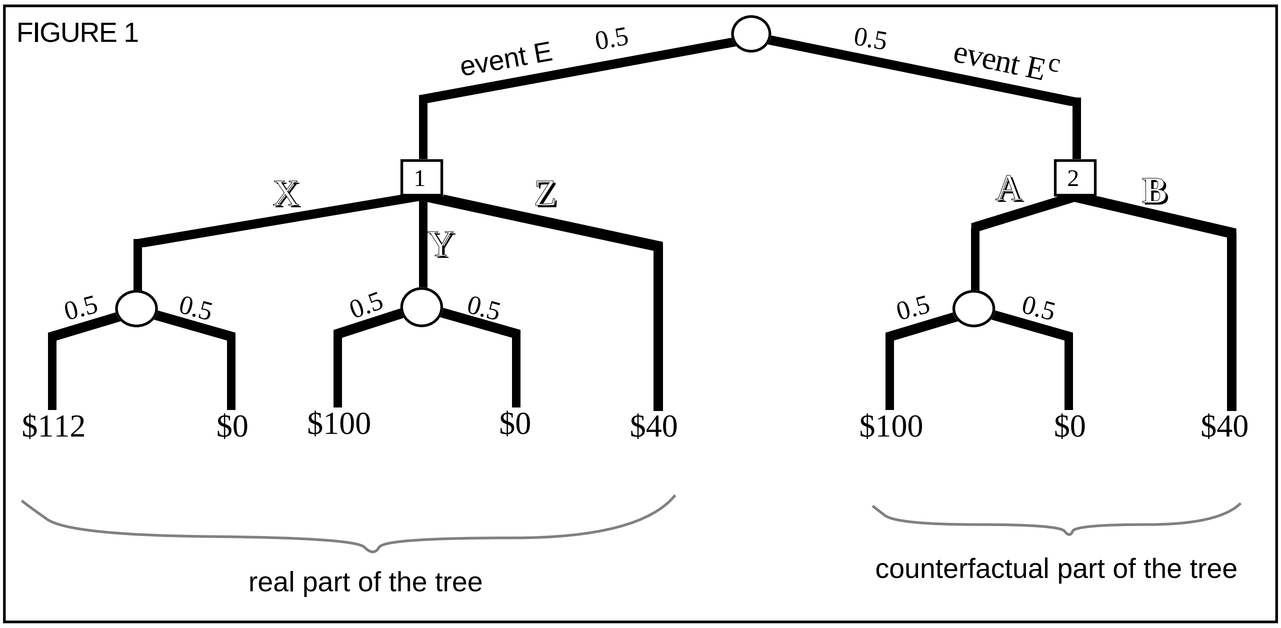
In the observations of the consumer's choices in the example, "ceteris paribus" assumptions must be imposed. The removal of X from the choice set should not affect other "relevant" aspects in the choice situation. Thus, we formulate the principle of *menu independence*. Choice should be based on an intrinsic value of objects. That intrinsic value should not be affected by the alternative choice options that are available, neither by their presence or absence nor by their nature. In later discussions a crucial point will indeed be that:

$$\text{The value } V(Z) \text{ should be independent of } X. \tag{4.1}$$

The term menu-independence was proposed by Sen (1997). Other terms are monadic value (Burks, 1977, p. 277), absence of attraction effect (Huber, Payne, & Puto, 1982), #principle of individuation by justifiers (Broome, 1991),# and context-independence (Tversky & Simonson, 1993; see also Tversky, 1969). The condition is a special case of framing-invariance (Kreps, 1990, p. 28) and underlies revealed preference axioms such as independence of irrelevant alternatives (Nash, 1950; Arrow, 1959) and other conditions (Samuelson, 1938; Ville, 1946; Houthakker, 1950; Sen, 1971; Tian, 1993). Classical examples have been advanced in which menu-independence is not reasonable (Luce & Raiffa, 1957, Section 13.3; Kreps, 1990, p. 28; Sen, 1997). In such cases, however, there is no clear meaning for preference (or nonexpected utility functionals), and standard optimization is not possible. Hence, we follow the traditions of normative analyses and assume menu independence.

EXAMPLE 4.2 [Resolute Updating]. Consider node 1 in Figure 1, where a rational consumer has three choice options X,Y, and Z, and an economist observes the preference relation of the consumer over the choice options. The choice options are now lotteries over money. There has been a prehistory, where in the past it was possible that the decision maker would have ended up at decision node 2 instead of 1. At this time, decision node 2 and its choice options are counterfactual, i.e., they constitute a risk born in the past but not actualized.

FIGURE 1



We assume that the prior preference relation over strategies is represented by the functional V described in (4.2), where XA designates the probability distribution over outcomes generated by the strategy choice X at node 1 and A at node 2; etc.

$$V(XA) > V(ZB) > V(XB) > V(YA) > V(YB) = V(ZA). \qquad (4.2)$$

(4.2) constitutes a violation of independence, hence expected utility, because V(XA) > V(XB) but V(ZA) < V(ZB). ZB has the lowest expected value, but is ranked high because of the certainty effect. It is the only riskless prior strategy, and the only one that guarantees a positive gain. The preferences result from rank-dependent utility, as well as cumulative prospect theory, when the commonly found value and probability transformation functions are substituted.[4]

[4]Tversky & Kahneman (1992) estimate the value function as $x^{0.88}$ and the probability transformation function as $\frac{p^\gamma}{(p^\gamma+p^{1-\gamma})^{1/\gamma}}$ for gains, with $\gamma = 0.61$. Then, with V the rank-dependent utility functional, the V-values in (2.1) are 26.0, 25.7, 25.6, 24.2, 23.9, and 23.9, respectively, in agreement with the ordering in

Under resolute choice, the consumer chooses X at node 1, and this implies $V_E(X) > V_E(Y)$ and $V_E(X) > V_E(Z)$. To find out about the preference ordering of Y and Z, the economist follows the revealed preference approach and discards option X from node 1. Figure 2a results. (Figure 2b will be discussed later.)

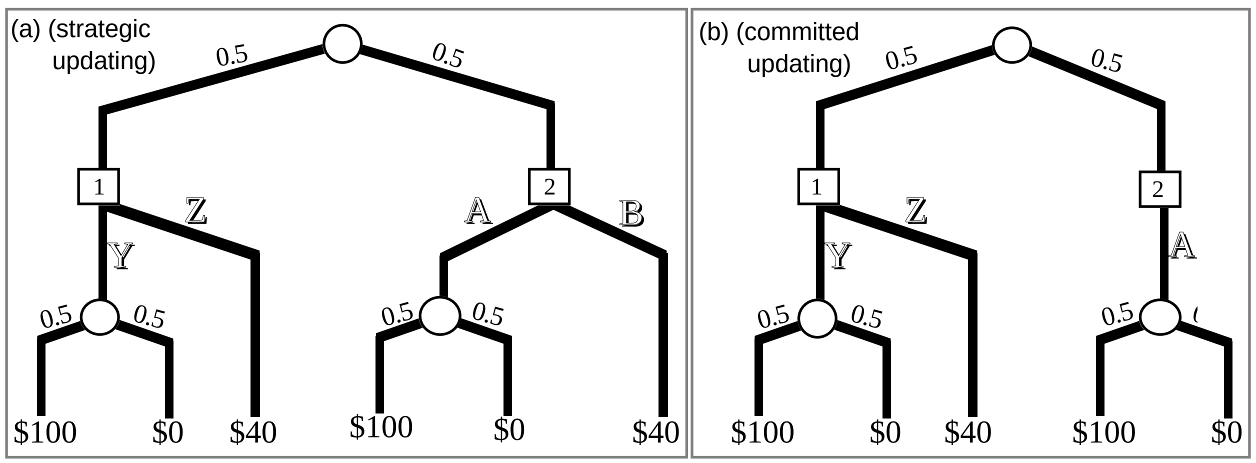


FIGURE 2

The consumer, in the spirit of resolute choice, adheres to his prior preference relation. Now ZB is the optimal strategy from the prior perspective in Figure 2a and therefore the consumer prefers Z at node 1. The economist thus observes the choice Z and $V_E(Z) > V_E(Y)$ follows. Note that the choice of Z is based on the value V(ZB), not on the value V(ZA).

It is not relevant here whether the economist knows about the prehistory, i.e., about node 2, or not. It may well be that node 2 is "hidden" to the economist, and that he only observes X, Y, and Z. □

In general, for every pair of real options R, R', a choice from {R, R'} corresponds with the inequality $\max_C V(RC) > \max_C V(R'C)$ where C ranges over the counterfactual options.

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the example; cumulative prospect theory coincides because only gains are involved. Let me emphasize that rational optimality, not empirical realism, is the primary interest of this example.

Thus the strategic updating functional $V_E(\bullet)$, the pointwise maximum of the functionals $V(\bullet C)$ over the counterfactual C, represents resolute choice. It satisfies menu independence.

Readers who advocate committed updating may dispute the validity of the ceteris paribus assumption when moving from Figure 1 to Figure 2a. They may prefer Figure 2b, where not only the real option X has been discarded, but also the only counterfactual option left is A, derived from the optimal prior strategy XA. In Figure 2b, the consumer chooses Y because V(YA) > V(ZA), which indeed agrees with committed updating $V_E(R) = V(RA)$.

Let me explain why Figure 2b leads to a violation of menu independence. Had we chosen an option X' instead of X such that X'B was optimal a priori (e.g., an X' similar to Z), then B would have been kept at node 2 under committed updating. The updated value of Z would have been V(ZB) instead of V(ZA) and Z would have been preferred to Y at node 1. Thus, the value of Z at node 1 depends on the nature of X, through the counterfactual A generated by X. Figure 2b seems to be based on contradictory assumptions: At node 1 it is assumed that X is not available, yet the restriction to A at node 2 is based on the availability of X at node 1.

5. GENERAL DECISION TREES

The next theorem extends the results of the preceding section to general decision trees. It considers various choice sets B of available strategies at a real decision node 1. The proof readily follows from substitution of definitions.

THEOREM 5.1. Let V_E be the strategically updated preference functional. For every choice set B at node 1, X = argmax(V_E) if and only if X is the resolute choice, i.e., for some counterfactual A, XA is the a priori optimal strategy. □

It is therefore a *mathematical fact* that resolute choice maximizes the strategically updated preference functional at any decision node. Menu independence is satisfied. In the spirit of resolute choice, the strategically updated functional does depend on the set of counterfactual strategies that are available in the tree, just like the committed updating functional does.

A comparison to consumer demand theory may be clarifying. There, the following procedure is adopted.

(1) For every commodity bundle x, the value V(x) ("utility") is determined intrinsically, independently of competing commodity bundles (menu independence).

(2) From any budget set B, argmax(V) is chosen.

Resolute choice follows the same procedure at the real decision node, with the following substitutions:

consumer demand	commodity bundels x	budget sets B	"utility" V
resolute choice	real strategies X	choice sets B	strategically updated V_E

Committed updating cannot play the same role as strategic updating because it violates menu independence. To determine the value of a real strategy at node 1 under committed updating, one first has to inspect the whole set B of available strategies at node 1. One then has to determine the optimal prior strategy (thus already determining the optimal real strategy), to lay down the counterfactual strategy. Only after that can one determine the value of any real strategy. That value changes when competing real strategies at node 1 are added or deleted.

Let me briefly describe the procedure at other decision nodes than node 1 in general decision trees. *Past decisions* are decisions that are logically necessary to reach the current decision node. All past decisions leading to node 1 are obviously assumed fixed for updating. One therefore drops all other decisions at past decision nodes, and their belonging strategies, that would move away from node 1. The dropping of those belonging strategies becomes important when, for an "unfavorable" choice set B at node 1, prior strategies would become optimal that would not pass through node 1. In agreement with the principles of revealed preference theory, such superior alternatives must be removed before a choice from the real choice set B can be revealed. All decisions following previous *chance* moves away from node 1 are now counterfactual, and a counterfactual strategy describes choices at every counterfactual decision node.

6. EXAMPLES LEADING TO COMMITTED UPDATING-PREFERENCES

In this section, examples of committed updating are considered. Commitment to a fixed choice is obvious, and even logically necessary, for past decision nodes. The case is, however, different for counterfactual decisions. Those were never materialized and the assumed decision there is up to one's imagination and plans. The term "counterfactual decision" instead of the more common "forgone decision" was chosen in this paper to maximally avoid any confusion beween past and counterfactual decisions. Next some examples are discussed.

EXAMPLE 6.1 [Extraneous Precommitment].
(a) Consider again Example 4.2. Imagine that, in deviation from the preceding analysis, the decision maker would choose the counterfactual A because of an extraneous precommitment. Then under resolute choice the preference at node 1 is $X \succ Y \succ Z$, in agreement with committed updating.
(b) [Parental Example of Machina (1989, p. 1643)]. This example, sometimes called "Machina's mom," is illustrated in Figure 3.

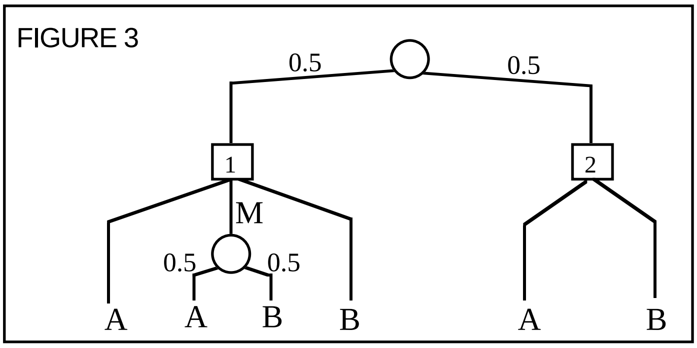


FIGURE 3

A mother should give an indivisible treat to either her daughter Abigail (A) or her son Benjamin (B). She would equally well want A as B. For equity reasons, she rather lets a coin toss decide than choose A or B herself. Heads leads to node 1 in Figure 3 and tails to node 2. Prior to the toss, the mother announces that she will choose A at node 1 and B at node 2. Heads shows up, leading to node 1. Benjamin then recommends an updated preference of M (toss again) over A. The mother's updated preference is, however, A $\succ$ M $\succ$ B, that is, the committed updated preference. The fixed counterfactual choice B at node 2 is essential for the mother's updated preference.

DISCUSSION OF (a) AND (b). The updated preference in (a) is caused by the extraneous precommitment and does not reflect risk attitude. Many kinds of extraneous commitments can be thought of, leading to any kind of updated preference. (b) is a special case of (a). In (b), equity considerations and social interaction, and not risk attitude, have generated the extraneous commitment. It has often been suggested that extraneous precommitments, if available, should be explicitly incorporated in the model (Strotz, 1956, p. 173; Hammond, 1976, p. 162; Kohlberg & Mertens, 1986, footnote 3; Asheim, 1997, p. 428). For an alternative analysis of (b), see Grant (1995). □

EXAMPLE 6.2 [Misinformed Decision Maker]. Consider again Example 4.2. Imagine that the decision maker originally thought that X would be available, but upon arrival at node 1 discovers it is not. Assume that he knows that he would have chosen A at node 2, thinking that X would be available at node 1. In that case it is reasonable to assume a fixed choice A at node 2, leading to the committed updating preference.

DISCUSSION. The updated preference is caused by wrong information and does not reflect risk attitude. Any kind of wrong information can be thought of, leading to any kind of updated preference relation. Cases with misinformed decision makers are discussed by Machina (1989, "hidden nodes"). □

EXAMPLE 6.3 [Reconsidered Choice]. At the beginning of the decision tree the decision maker plans on a strategy, say XA in Figure 1. After receipt of the information E, he reconsiders his choice X conditional on E, and therefore re-evaluates the available options in an updated manner. In doing so he assumes he would not have reconsidered his choice at node 2, i.e., would have chosen A there. Hence the committed preference results at node 1.

DISCUSSION. Although a foundation of normative individual updating on plans and then changes of mind may be questioned (Shin, 1991), reconsidered choice is often invoked and is worth discussing. When the choice X at node 1 is reconsidered and hence is no more sure, the reason for the choice A at node 2 also disappears. There is no rational reason for the decision maker to feel committed to a counterfactual decision as he did in the example. If reconsidered choice can at all be meaningful at node 1, it should be accompanied by reconsideration at the counterfactual node 2. Then ZB, suggested by strategic updating, deserves consideration more than YA, suggested by committed updating. □

7. EXAMPLES OF STRATEGIC UPDATING

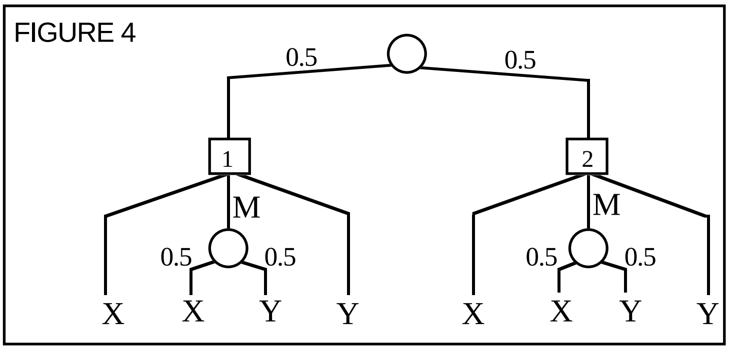
This section presents three examples. The first shows that strategically updated preferences can be considered a special case of induced preferences.

EXAMPLE 7.1 [Optimization with Hidden Variables]. Suppose a consumer chooses from two-dimensional commodity bundles (R,C) and maximizes a utility function V(R,C). We can, however, only observe the first commodity R; the second commodity is *hidden*. Assume that the maximum is attained at (X,A). It seems more natural to define the value of R as U(R) = [$\max_C$V(R,C) over all C] than as U(R) = V(R,A). The former functional is sometimes called the induced preference functional (Kelsey & Milne, 1995; Kreps & Porteus, 1979; Machina, 1984).

DISCUSSION. Updated preference can be considered a special case of optimization with hidden variables. The choice at node 1 in Figure 1 is, seemingly, between X, Y, and Z, but in reality is between each of these when combined with A or B at node 2. Strategic updating agrees with induced preference. □

In the next example, the prescripts of strategic updating may not be appealing to some readers. The reason is, I think, not that strategic updating would be incorrect under resolute choice or inferior to committed updating. Rather, the very assumption of resolute choice, underlying all of the analysis, may be unconvincing to such readers. Their intuition may be guided by consequentialism, where preferences depend neither on counterfactual outcomes nor on counterfactual decisions.

EXAMPLE 7.2 [Consequentialistic Intuitions]; see Figure 4.



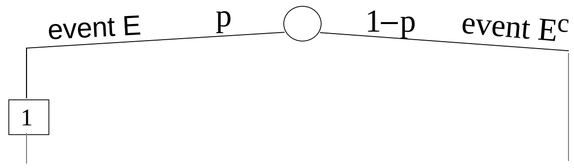
Assume that prior preferences order the available strategies as follows: $\frac{1}{2}X + \frac{1}{2}Y \succ \frac{1}{4}X + \frac{3}{4}Y \sim \frac{3}{4}X + \frac{1}{4}Y \succ X \sim Y$, reflecting quasiconvexity (love for probabilistic mixing). A priori there are three optimal strategies, XY, YX, and MM, where M abbreviates $\frac{1}{2}X + \frac{1}{2}Y$ ("Mix"). Under committed updating, the updated preference relation at node 1 can be $X \succ M \succ Y$ (prior strategy was XY), or $Y \succ M \succ X$ (prior strategy was YX), or $M \succ X \sim Y$ (prior strategy was MM); all these three updated preferences are equally plausible under committed updating. Thus, under committed updating, the decision maker may strictly prefer X because he *would have* chosen Y at node 2, and thus evaluates X as $\frac{1}{2}X + \frac{1}{2}Y$. He will then disprefer the other options because he adheres to his choice of Y at node 2.

Under strategic updating, the updated preference relation is $X \sim Y \sim M$, i.e., all options at node 1 are indifferent. Then the decision maker can again defend the choice X because he *would have* chosen Y at node 2, and he can evaluate X as $\frac{1}{2}X + \frac{1}{2}Y$. Here, however, the decision maker would as well want to choose any of the other options at node 1 and evaluate all of these as $\frac{1}{2}X + \frac{1}{2}Y$, under the argument that he *would* make the corresponding optimal choice at node 2.

DISCUSSION. The reader may find the conclusion of either updating, that one may choose X at node 1 but still evaluate it as $\frac{1}{2}X + \frac{1}{2}Y$ for no other reason than "promising" that one planned to choose Y in the counterfactual event, unsatisfactory. The argument may be that such "vacuous promises" of what was planned to be done in events that are known not to occur, must be irrelevant for the evaluation of the real situation. This argument is, however, based on consequentialist intuitions. In resolute choice one must accept the relevance of counterfactual situations for evaluations of real situations. A consequentialist updating at node 1 in Figure 4 would yield the preferences $M \succ X \sim Y$ independently of what might have happened at node 2.

I hope that intuitive discussions of the prescriptions of strategic updating will lead to further insights into resolute choice for dynamic decisions. □

FIGURE 5



Next we discuss a paradox for committed updating, put forward by Border & Segal (1994). We assume that the functional V satisfies proper differentiability conditions.[5] Border and Segal consider lotteries of the form (p,R; 1−p,C) (see Figure 4, with R at node 1 and C for event E^c), and let p tend to 0. Then these lotteries will be compressed more and more within the direct neighborhood of C. In that neighborhood V is approximately linear, i.e., it tends to expected utility. Hence the functional $R \mapsto V(RC)$ approaches expected utility. Next note that, from a prior perspective, our current state becomes more and more unlikely as time proceeds. Hence, if one interprets event E in Figure 5 as our current state, and C as the (fixed) counterfactual strategy, then this result implies that preferences converge to expected utility in the long run.

Things are more complicated if there are more than one counterfactual strategy, which is the common practical case. The next example shows that convergence to expected utility then need not occur under strategic updating.

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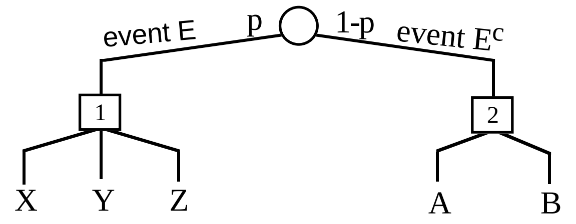
[5]Following Border & Segal (1994), we assume the Hausdorff metric on preference relations.

FIGURE 6



EXAMPLE 7.3 [No Convergence to Expected Utility under Strategic Updating]; see Figure 6. The Appendix demonstrates that the assumptions made hereafter can be satisfied by nonexpected utility functionals. We assume that p converges to 0, and analyze the case under strategic updating.

Assume that lottery Y results from the half-half probability mixture of X and Z, with an additional \$1; i.e., $Y = (\frac{1}{2}X + \frac{1}{2}Z) \oplus \$1$.[6] Further, assume that for all p close to 0, the decision maker is indifferent between (p,X; 1–p,A) and (p,Z; 1–p,B), but prefers these two lotteries to any of the other lotteries that can be obtained from the decision tree. We assume that the decision maker observes E, and first study his updated preferences according to strategic updating. At the end, we discuss committed updating. A priori, two plans were optimal, (p,X; 1–p,A) and (p,Z; 1–p,B). Therefore, conditional on E, the two options X and Z are optimal and are indifferent.[7] They are strictly preferred to $Y = (\frac{1}{2}X + \frac{1}{2}Z) \oplus \$1$. Under strict stochastic dominance, the latter lottery is strictly preferred to $\frac{1}{2}X + \frac{1}{2}Z$. Hence the updated preferences violate the betweenness axiom, thus also expected utility.

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[6]We use the + sign in the mixing of probabilities and the ⊕ sign to designate that all amounts of money in the lottery are increased by an amount, \$1 in this case.

[7]This can be further clarified by replacing X by X ⊕ x for a small positive amount x, tending to 0. For each small positive x, there is a strict prior preference for (X⊕x,p; A,1–p) over all other options, so the dynamically consistent decision maker strictly prefers, conditional on E, each X⊕x to Y and Z. Similarly, there is a strict posterior preference for each Z⊕x over Y and X for small positive x. This reasoning provides an alternative, continuity-based, argument for the preferences of strategic updating.

The updated preference relation not only violates expected utility, but also is not very close to any expected utility preference, no matter how small p is. A formal proof is provided in the Appendix.

Under committed updating, the decision maker can, arbitrarily, choose an optimal prior decision, say (p,X; 1–p,A). For his updated preference given E he then assumes the counterfactual A. As p tends to 0, the preference relation then tends to expected utility. By betweenness, the preference $X \succ Z \succcurlyeq Y$ is excluded and the updated preference $X \succ Y \succ Z$ results, in agreement with expected utility. So under committed updating, the paradoxical convergence to expected utility can be maintained. □

Example 7.3 has followed the format of Figure 5 that is assumed throughout the analysis of Border and Segal. The p and 1–p branches do not change as p tends to 0. In particular, the number of chance nodes and decision nodes does not change for p tending to 0 both for the model of Border and Segal and for Example 7.3. In practice, the probability of reaching a current decision node will, as time proceeds, tend to 0 in different ways than in Figure 5. If a decision tree is changed by inserting additional resolution of uncertainty prior to a decision node 1, then it is natural to assume that the further resolution of uncertainty will also create new chance nodes (and decision nodes) elsewhere in the tree. Of course, these can alter the decision situation in many ways. Also, it then becomes less realistic to assume consumption only at terminal nodes. Then there will be intermediate consumptions which further complicate the analysis. In general, it seems that little can be said about the limiting behavior of updated preferences when consequentialism (thus expected utility) is abandoned.

8. CONCLUSION

This paper has discussed the updating of preferences conditional on an event E, assuming resolute choice where preferences are not separable over disjoint events. It is well-known that the optimal decision conditional on E then depends on counterfactual outcomes, conditional on the complementary event E^c. Therefore, maximization conditional on E cannot be discussed independently of what happens in E^c. The crucial point that I want to convey to the reader is that, similarly, maximization conditional on E^c then cannot be discussed prior to the decision under E either! The two maximization problems simply cannot be disentangled. This is a price one has to pay for giving up expected utility. Because the decision on E^c cannot be assumed fixed when updating preferences on E, "strategic" updating is called for. Counterfactual decisions depend as much on real ones as vice versa.

The role of counterfactual decisions raises new research questions, for instance about the properties that V_E, the strategically updated functional conditional on E, inherits from V. Aversion to mean-preserving spreads will be kept, but comonotonic independence, betweenness, and differentiability may be lost. Another question concerns what additional restrictions can serve to increase tractability (Jaffray, 1997).

The main message of this paper has been that choosing a fixed counterfactual strategy for updating, as routinely done in the literature, is not self-evident. It is trivially appropriate if there is only one counterfactual strategy available, and for that case the existing results remain useful. For general trees, however, it needs further discussion.

APPENDIX. ELABORATION OF EXAMPLE 7.3

Throughout, EU abbreviates expected utility. Assumptions for the nonEU preference functional V are described that imply the behavior of Example 7.3 in the decision tree. First, we assume the indifferences, explained later (A.3):

$$A \sim B \tag{A.1}$$

and

$$pX + (1-p)A \sim pZ + (1-p)B \tag{A.2}$$

for all $p > 0$ close to 0 (i.e., $p_f > p > 0$ for some fixed p_f). These preferences could be satisfied under EU and then would imply that not only A and B are indifferent, but also X and Z. Under EU (with strict stochastic dominance), however, Y would then be strictly preferred to X and Z and therefore, conditional on event E, Y would be chosen, contrary to our assumptions. Therefore V has to be a nonEU functional. We further assume the following preferences:

For all $p > 0$ close to 0, (A.3)
$pX + (1-p)A$ and $pZ + (1-p)B$
are strictly preferred to all of the following four lotteries:
$pX + (1-p)B$, $pY + (1-p)A$, $pY + (1-p)B$, $pZ + (1-p)A$.

Before discussing (A.3), first some more comments are given on (A.2). The easiest way to obtain (A.2) for nonEU preferences is to take, in addition to (A.1), $X = A$ and $Z = B$, and this may be the best way to study the example at first reading. For a discussion of the "outcomewise dynamic consistency" condition in Border & Segal (1994), however, it is necessary that all lotteries are distinct. Hence, to cover all versions of dynamic consistency discussed by Border and Segal, we consider the general case where all lotteries can be distinct.

We describe a way for constructing the preferences where in neighborhoods of A and B expected utility holds "locally," with local utility functions U_A and U_B.[8] Other constructions are possible, of course. The indifferences in (A.2) follow for p close to 0 if X and A are indifferent under EU with U_A, and similarly Z and B are indifferent under EU with U_B: Then $pX + (1-p)A$ is indifferent to A and $pZ + (1-p)B$ is indifferent to B and, by (A.1), (A.2) follows. To obtain the strict preferences in (A.3), U_A must be different from U_B. Under EU with U_A, A and X are indifferent and are both preferred to B and Z and even to Y, but under EU with U_B, B and Z are indifferent and are both preferred to A and X and, again, to Y.

We write $\succcurlyeq'$ for the updated preference relation, and prove:

OBSERVATION A.1. The updated preference relation is not a limit of EU preference relations.

PROOF. Assume $\succcurlyeq$ were a limit of EU preferences $\succcurlyeq_n$. We have $X \oplus \$1 \succ' Z \succ' Y$. Hence, for large enough n, $X \oplus \$1 \succ_n Z \succ_n Y \succ_n \frac{1}{2}(X \oplus \$1) + \frac{1}{2}Z \succ_n Z$; the first two preferences follow for n large enough (compare Border & Segal, 1994, p. 176), the third preference follows from strict stochastic dominance, and the last preference follows from betweenness for $\succcurlyeq_n$ and the first preference. We have obtained a preference cycle, hence a contradiction. □

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[8] Then V is linear in those neighborhoods and the "local utility functions" described in Machina (1989) are equal to U_A and U_B, respectively.

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