Value Function Elicitation:

A Comment on Craig R. Fox & Amos Tversky,

"A Belief-Based Account of Decision under Uncertainty"

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This note discusses the elicitation of value functions under prospect theory (Kahneman & Tversky 1979, Tversky & Kahneman 1992). Our purpose is to clarify some confusion concerning the method used by Fox & Tversky (1998) and Fox, Rogers, & Tversky (1996). Let $(x, p; y, p; 0, 1\text{-}2p)$ be a prospect that offers a prize of $\$x$ with probability p, $\$y$ with probability p, and nothing otherwise. These researchers obtain indifferences

$$(x, p; y, p; 0, 1-2p) \sim (a, p; b, p; 0, 1-2p) \tag{1}$$

with $x>y>0$ and $a>b>0$, respectively. Let $v(.)$ be the value function for monetary gains. The authors conclude from (1) that

$$v(x) - v(a) = v(b) - v(y). \tag{2}$$

This analysis is valid under expected utility theory (EU) and also under the original form of prospect theory (Kahneman & Tversky 1979; henceforth *OPT*). However, contrary to what was suggested by Fox & Tversky (1998, p. 883, first column, third paragraph, opening sentence) and Fox, Rogers, & Tversky (1996, p. 8 l. 15–18), this analysis is not consistent with the cumulative version of prospect theory (Tversky & Kahneman 1992; henceforth *CPT*). Under CPT, (1) implies that

$$w(p)v(x) + \big(w(2p)-w(p)\big)v(y) = w(p)v(a) + \big(w(2p)-w(p)\big)v(b), \tag{3}$$

where *w* denotes the weighting function. Hence

$$w(p)\big(v(x) - v(a)\big) \;=\; \big(w(2p) - w(p)\big)\big(v(b) - v(y)\big) \qquad (4)$$

and (2) does not hold unless $w(p) = w(2p) - w(p)$. Empirical studies suggest that $w(p) > w(2p) - w(p)$ ("lower subadditivity," Tversky & Fox 1995) so that (2) does not generally hold.

We wish to emphasize that the empirical results presented by Fox & Tversky (1998) rely only on the analysis of the value function applied to expected utility theory, and hence remain valid. In that paper, the two-stage model was fit by predicting cash equivalents for uncertain prospects from judged probabilities of target events and cash equivalents for chance prospects—and therefore made no assumptions concerning the shape of the value function. Cash equivalents for uncertain prospects violated the partition inequality that is implied by expected utility theory with risk aversion, but were entirely consistent with the pattern predicted by the two-stage model based on support theory and prospect theory. Hence, although the cited sentence may be misleading, all the empirical results and conclusions of Fox & Tversky (1998) remain valid and theoretically well-founded.

Fox, Rogers, & Tversky (1996) use indifferences as in (1) with $p = 1/6$. They obtain $x–a = y–b$ and interpret this as evidence for linearity of value in the context of cumulative prospect theory. However, if $w(1/6) > w(2/6) - w(1/6)$ (lower subadditivity) then *v* must be nonlinear, as can be seen from (4). Hence although the theoretically claimed implication of linear utility among professional options traders in Fox, Rogers, & Tversky (1996, p. 8, l. 15–18) holds under EU and OPT, it does not necessarily hold under CPT.

From a pragmatic perspective, linear utility remains compelling under CPT. First, the vast majority of options traders priced prospects according to expected value for each of the two matching tasks (75% and 80% of subjects, respectively). It is highly implausible that such a large number of respondents would have used nonlinear weighting and nonlinear value in precisely equal and opposite ways. Second, the same options traders also exhibited agreement with expected value when they were later asked to price chance prospects of the form ($150,*p*) where *p* varied from .1 to .9. Hence, although linearity of the value function cannot not theoretically proven, the evidence for this conclusion remains rather convincing as do the empirical implications derived thereof in the rest of their paper.[1]

To avoid misunderstandings, let us end by repeating that under the *cumulative* version of prospect theory, indifferences (1) and inferences (2) do not provide valid elicitations of value functions. Alternative, valid, methods have been recently advanced in the literature. Many papers have estimated the value function under cumulative prospect theory, jointly with the weighting function, by parametric fitting, usually assuming a power value function (Tversy & Kahneman 1992, Bernstein, Chapman, Christensen, Potts, & Elstein 1997, Bleichrodt & van Rijn 1999, Camerer & Ho 1994). Nonparametric methods for estimation have been used

[1] It may be further reassuring that the qualitative conclusions in their paper are not very sensitive to the shape of the value function. Assuming a power value function (cf. Tversky 1967),

$$v(x) = x^{\alpha}, \alpha > 0,$$

the primary conclusion (options traders' decision weights for prospects involving familiar stocks exhibit both lower and upper subadditivity, for both samples) remains valid for values of α from 0.01 to 1.72 (i.e., up to a considerable degree of risk-seeking), and the secondary conclusion (that judged probabilities exhibit bounded subadditivity) is unaffected by the estimation of the value function.

by Wakker & Deneffe (1996), Abdellaoui (1998), Bleichrodt & Pinto (1998), and Gonzalez & Wu (1999).

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