SAMUELSON

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF ECONOMICS

CAMBRIDGE, MASSACHUSETTS 02139-4307

11 June 1992

PAS.
2 letters of June 11, 1992,
+ biblio. + CES footnote

Mr. Peter C. Fishburn: Letter #1
AT&T Bell Laboratories
600 Mountain Avenue
P.O. Box 636
Murray Hill, NJ 07974-0636

For Peter Wakker
articles attached

Dear Peter:

Travels and duties have kept me from reading your manuscript on independence axiom(s). My loss!

1. As I sit in a coffee shop, sans books and cum pen only, I recall that in your letter you asked something like:

> Was it only in the time of Sono, Leontief, et al., that economists dealt with additive utility functions such as

—no no. With indep. axiom

$$U(\text{hats}) + V(\text{gloves}) = \sum_1^n U_j(X_j)?$$

Put that way, the answer must be as follows.

2. From the beginning--Gossen (1854), Jevons (1871), Walras (1874), Edgeworth (1881), Marshall (1879, 1890), Launhardt (1885±), Auspitz und Lieben (1889), Fisher (1892), it was standard to assume

$$U(X_1,\ldots,X_n) = \sum_1^n U_j(X_j)$$

$$\partial^2/\partial X_i \partial X_j \quad = 0,\ i \neq j$$
$$\qquad\qquad < 0,\ i = j$$

We know

Accidentally, Edgeworth (1881) backed into $U(X_1,X_2)$ with $\partial^2 U/\partial X_1 \partial X_2 \neq 0$. (I seem to recall he began with independence in one set of variables; but when he re-expressed them in terms of certain natural linear combinations, substitution of the new variables into the original utility expressions lost the zero-ness of cross derivatives. Something like

$$U(X) + V(Y) \qquad\qquad x = X + Y$$
$$y = Y$$

$$\frac{\partial^2[U(x-y)+V(y)]}{\partial x \ \partial y} = -U''(x\text{-}y) > 0$$

this is not indep^ce

Fisher, c. 1892 as Gibbs' Yale student, discovered that "joint" functions could be used, just a few days before getting Edgeworth's 1881 book. But Auspitz and Lieben (1889) had already (in German) proposed $\partial^2U/\partial X_i\partial X_J$ as a measure of complementarity or rivalry. This is oddly called the Edgeworth-Pareto measure. Pareto in the early 1890s fought against independence. Like Fisher he glimpsed that $f[U(X) + V(Y)]$, $f'[\ ]>0$, would do as well as $U(X) + V(Y)$: only "indifference contours"--already in Edgeworth (1881)--mattered; and by altering $f''[\ ]$ we can diddle the algebraic sign of $\partial^2 f[\ ]/\partial X\ \partial Y$.

Paradoxically, Fisher and Frisch [1926, 1931,...], who knew all in my previous paragraph, for some reason thought that welfare economics needed preferred individual utilities that could be added into a Benthamite social utility total. So they needed for each person a cardinal measure of utility preferred above all the other infinity of cardinal indicators.

If U can be written as

$$U = F(X_1,\ldots,X_n), + g(X_{r+1},\ldots,X_n)$$

any non-linear $f[U]$ will lose the additive property. So they brutalized demand data to "identify" (up to scale and origin constants) unique F( ) and G( ) functions.

3. After 1925 we move into the era when one tests on (P_j,Q_j) demand data the hypothesis that one

$$U = F(X) + G(X)$$

function does exist. See 1927 Fisher in J.B. Clark Festschrift where he proves the following:

$$\text{Let } -\frac{\partial x_1}{\partial x_i} = {}^1R^i(x_1,\ldots,x_n)$$

$$= \frac{\partial u(\ \ )/\partial x_i}{\partial u(\ \ )/\partial x_1} \text{ an invariant MRS}$$

Then for ${}^1R^2(x_1,x_2)$, we must have

$$\frac{{}^1R^2(a_1,a_2)}{{}^1R^2(b_1,a_2)} = \frac{{}^1R^2(a_1,b_2)}{{}^1R^2(b_1,b_2)} \text{ for all } (a_j,b_j)$$

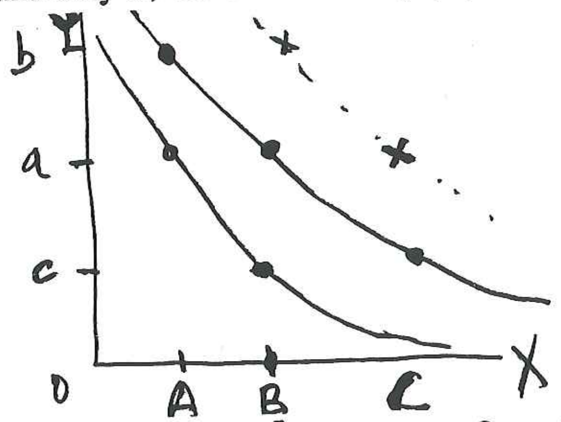
Leontief's

$$\frac{\partial \log {}^1R^2(x_1,x_2)}{\partial x_1\, \partial x_2} \equiv 0$$

follows easily.

The most beautiful result is that of Georgescu-Roegen in <u>Southern Economic Journal</u>.

If, and only if, there exists a U(X,Y) of the form F(X) + G(Y), will it be the case that:



"(A,a) indifferent to (B,c)"
and "(A,b) indifferent to (B,a) and indifferent to (C,c)"
must imply [for arbitrary A < B < C]
"(B,b) indifferent to (C,a)"

Similar to Thomsen-Blaschke.

This generalizes to $\Sigma_1^n U_j(X_j)$, $n \geq 2$; or to $f(x_1,\ldots,x_r) + g(x_{r+1},\ldots,x_n)$.

It is much deeper for n=2, than Debreu's well-known 1960 topological result for n>2.

(yes!)

4. All the above relates to deterministic demand. With Bernoulli-Laplace-Ramsey-Marschak-Savage stochastic decision-making, you are familiar with the topological story. (Note my mention of Marschak. Ken Arrow loved him, so I asked Ken why in his eulogy of J.M.'s accomplishments he left out the definitive 1950 independence axiom. Ken was genuinely puzzled. He had more or less forgotten that because (I am sure) of a feeling that already in the years just before 1950 all this was "understood." [Savage? Rubin? Chernoff? Arrow?] I can testify that, in mid-1950, Savage thought his 1948 Friedman-Savage axiom(s) did the business and was surprised when I showed him its defects. Ken, then and since, never seemed to focus on what von Neumann critics like me worried about. I suspect Rubin may have helped J.M. a lot. After you understand all, you perceive that Chernoff-Hurewicz axioms concerning subjective probability could illuminate utility axioms. But ex post ain't ex ante. (I may have told you all this before. I hope my accounts are consistent!) I believe so.

(see other stuff)

5. John Harsanyi (JPE, 1955) made a Bergson-grade contribution to welfare economics. He proved that a social welfare function W(Smith's beans, Smith's salt; Jones' beans, Jones' salt) must be capable of being written in the form:

U_1 (Smith's beans, Smith's salt)
$+ U_2$ (Jones' beans, Jones' salt)

Peter: I enclose my '84 T & D piece on this.

if

a) neither cares about the other's consumptions
b) the Bergson Ethical Observer turns happier when one of Smith or Jones turns happier and the other does not turn less happy
c) Smith and Jones each subscribe to Savage's sure-thing axiom(s) in stochastic situations
d) the Bergson Observer subscribes to Savage's axiom(s).

6. Houthakker and Samuelson, in Econometrica (1962, 1965), have worked out by duality theory demand-data singularities when there exists a $\Sigma U_j(X_j)$. (Example: What I buy of (X_1,X_2) can be predicted from $(P_1,P_2$, total spent on them), independently of $(P_3,P_4,\ldots;$ total income).

fter
52

7. Earlier, R.G.D. Allen (RES, c. 1934) and H.A.J. Green (monograph) have worked out properties of

$$q_i = Q^i(p_1,\ldots,p_n q,_i M) \quad , (i = b\ldots,n)$$

when

$$\frac{U_i'(q_i)}{p_i} = \lambda, \sum_1^n q_i p_i - M; \; U_i' > 0 > U_i''$$

Example: All goods have $\partial q_i/\partial M > 0$; $\partial q_i/\partial p_j \gtreqless 0$ depending on $-q_i U_i''(q_i)/U_i'(q_i) \gtrless 1$; etc. Also, one U_i'' could be somewhat positive. Then only that one good can have positive income elasticity. (N.B. I write at high speed and from unguarded memory; feel free to reverse a sign or inequality.)

In Houthakker [1960, Econometrica] and Samuelson [1965, Econometrica], duality theory is used to study additive preferences and additive preferences of the type $\Sigma_j a_j q_j^{\,b}$. This ties in with the vast CES literature [Bergson, 1937; Dickinson 1945; Champernowne (unknown date); Arrow-Chenery-Minhas-Solow 1961;...] Duality theory goes back to Hotelling, 1932; Samuelson 1937-; R. Roy, 1942; Shephard 1953; Legendre 1798;...

8. Duncan Luce (and I in my seminars) used to apply "independence" to ordinal choices among a lattice of discrete variables: (3 hats, 5 gloves) indifferent to (4 hats, 1 glove), etc.

9. Back around 1826, Abel worked out lots of quasi-independence theorems:

If

$$g[a,b] = g[b,a],\ h(c,d) = h(d,c);\ g[a+c,b] > g[a,b],\ h(a+c,b) > h(a,b), \quad \text{for } c > 0;$$

$$g[x,h(y,z)] = g[h(x,y),z], \text{ and } g[a,h(a,a)] = a;$$

then

$$g[x,h(y,x)] = \psi^{-1}[\frac{1}{3}\psi(x)+\frac{1}{3}\psi(y)+\frac{1}{3}\psi(z)],\ \psi'(\ ) > 0.$$

The Aczél modern lectures on functional equations are full of stuff like this. See enclosed Gale Festschrift manuscript for more such stuff.

10. My crude conclusion: After you understand independence, you lose some respect for it except in "mutually-exclusive stochastic outcomes."

What else is new?

Sincerely,

Paul

Paul A. Samuelson

PAS/jmm

P.S. If I can find a pre-copy of my Gale Festschrift piece, I'll send a copy. It is full of Jack Horner plums.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF ECONOMICS

CAMBRIDGE, MASSACHUSETTS 02139-4307

11 June 1992

Mr. Peter Fishburn: Letter #2
AT&T Bell Laboratories
600 Mountain Avenue
P.O. Box 636
Murray Hill, NJ 07974-0636

Dear Peter:

I am now at my office and can address the four Fishburn-Wakker Questions.

1. I now have no memory of 1950 Nash as being in the "independence" act. I knew its bargaining aspect (and my pal Bob Bishop knew it better), and regarded some of its "invariance" axiom(s) as contrived rather than natural. But since I referred to Nash in 1952, my present memory may have lost something.

Nash was a loner and I know of no "influences" on him. Alas, today's J.N. is not a reliable witness of those days.

2. Dalkey certainly spoke orally at Rand on these matters c.1949, and possibly in memos. (You strike me as too indulgent on J.v.N. His "mass" and other discussions of the time do not suggest to me that he sensed what was non-optimal about his expositions, which were ill-devised to alert me to 1949 Machina-like alternatives. Dalkey may have sniffed where the trouble lay but he never explained to me the relation between 1950 Marschak and J.v.N.--neither in 1950+ or before.)

3. I think by 1950 Savage did have the "sure-thing" principle in mind. After some complaining by me, he soon saw where his 1948 vision had been imperfect. (Probably in 1948 he thought he was writing up a valid version of it. Even Homer can nod.)

By the way, in your early pages and bibliography you say nought of Ramsey(*). Why? For an early hero, he stars. Also, Savage knew de Finetti well. Isn't he in the act?

Good question.

not really for utility.

I believe Savage had no particular familiarity with $\Sigma_1^n U_j(X_j)$ consumer demand theory. I wonder why you include it as a topic in your article, since your own discussion shows

would [a]gree.

(*) Not really direct stuff in his axioms which, much as in VNM, sort of does the equivalence thing. The most relevant matter seems to be in the page following his axioms. (Did I comment on this earlier?)

Peter,
This was my initial reaction, though for different reasons. You "convinced" me otherwise by the technical connections.

its lack of indecomposability. I advise excluding it in your revision. Write a separate piece if you must.

4. My earlier letter of today shows that, long prior to Sono, Leontief, and 1947 Samuelson, there was a vast literature on

$$\underset{qi}{\text{Max}} \sum_{1}^{n} U_j(q_j) \text{ s.t. } \Sigma p_j q_j = M$$

Only in the post 1930 period did the literature deal with <u>revealed</u> testings. I append references to several of my papers that are as germane as those you did cite.

Sincerely,

Paul

Paul A. Samuelson

PAS/jmm

P.S. The Chernoff axiomatizations are only tangentially related to the rest of your topics, I believe. (I would agree)

Some Additional References

Houthakker, H.S. (1960) "Additive Preferences", Econometrica 28(2), 244-57.

Samuelson, Paul A. (1965) "Using Full Duality to Show That Simultaneously Additive Direct and Indirect Utilities Implies Unitary Price Elasticity of Demand", Econometrica 33(4), 781-96.

Samuelson, Paul A. (1972) "Unification Theorem for the Two Basic Dualities of Homothetic Demand Theory", Proceedings of the National Academy of Sciences, U.S.A., 69(9):2673-74.

Samuelson, Paul A. (1974) "Complementarity: An Essay on the 40th Anniversary of the Hicks-Allen Revolution in Demand Theory", Journal of Economic Literature 12(4), 1255-89.