DEPARTMENT OF ECONOMICS   CAMBRIDGE, MASSACHUSETTS 02139

30 July 1991

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Dear Mr. Fishburn:

Here are my recollections.

1.   von Neumann-Morgenstern (c. 1942?) presented nothing persuasive to the generation of scholars who found

$$\text{Max } E(U()) \qquad U() \text{ determinate up to } |a|U()+b:$$

gratuitous. When I read your anniversary article I meant to write to you and expand on why N-M enjoyed so low a press rating in this regard.

In particular my pre-Marschak Japanese-English paper gave the Machina-like Ysidro example:

$$\text{Max } \left(\sum p_j x_j\right)^{\frac{1}{2}} / \left(\sum p_j x_j^{-1}\right)^{\frac{1}{2}}$$

that seemed rationally admissible and which N-M simply forbade without explication. (Johnny v. N. never conceded anything: Either he could not perceive the point [hard to believe] or was too vain to admit there was a problem [hard to believe]. Oscar never understood the nuances at all.)

2.   Marschak [1950] was the first to show me the "independence axiom" or some equivalent. [I gave it that name.] For some months I found it arbitrary. I corresponded with Jimmie Savage--to no resolution. Then he wrote in effect: "If you were Ysidro you could make book against yourself." I capitulated, at first not quite for the right reason. As I noted a few years back, the terror of being made book against--which was already in Ramsey [1930, 1926]--is a bit of a scarecrow. What got me was reflection on what "mutually-exclusive" means to an ex ante contemplator of stochastic outcomes.

I would not pity a scholar who violated the I.A., but I would consider it unaesthetic in myself to want to do so (and to pay for the privilege of being allowed to do so!)

Jimmie Savage's palaver about "sure things", I decided, came to much the same as my "mutually exclusive" interpretations.

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3. Friedman and Savage [1948] did <u>not</u> have the independence axiom. Jimmie tried on me their axiom, and then admitted that my claim of a non-sequitur was correct. That is why, on the basis of my rebuttal, he composed the 1952 "correction." (I can't remember whether it called itself that, or called itself an extension.)

People like Charnes-Cooper and Wold were dealing with gasoline blends that violated $E(U())$ and the N-M axioms. I voiced widely the suspicion that N-M had slipped in a mickey in the form of a zeroth axiom that escaped noticing because it was built into the definitions of the entities of their system. The very young Malinvaud confirmed this conjecture of mine at the 1952 Paris Colloquium.

about what? → *Marschak)*

4. I vaguely remember that Herman Rubin may have helped Jascha in the 1947-50 period. I remember arguing with Dalkey at RAND c. 1949 or 1949-50. Where Nash came into the picture I cannot now remember. A glance at his early papers might help. Marschak was my hero in the matter. Later he (and Ian Little? and Savage? and de Finetti?) discovered Ramsey had it all in a few cryptic throw-away lines.

5. In 1950 at the Cambridge World Math Congress, my pal Bob Bishop stated an axiom for $E(U())$ systems that <u>I'll</u> write as follows:

$f(x,y) = z$ is a symmetric monotone-increasing smooth mean
$f(x,x) = x$, $f(x,y) = f(y,x)$, $\partial f/\partial x$, $\partial^2 f/\partial x \partial y$, $\partial^2 f/\partial x^2$ exist, etc.;

Theorem:

$f(x,y)$ is an <u>associative</u> <u>mean</u>

with $f(x,y) = F^{-1}[\frac{1}{2}F(x)+\frac{1}{2}F(y)]$, $|a|F()+b$ admissible,

iff

$$f(\ f(x,f(x,y)),\ f(f(x,y),y)) = f(x,y)$$

Andy Gleason has an example showing that continuous $f(x,y)$ is not enough. Try your hand!

Sincerely,

Paul [Samuelson]

Paul A. Samuelson

PAS/jmm