Solutions to Exercises Behavioral Economics

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Chapter 1

EXERCISE 1.4.2.

a) EU(0.80:200; 0.20:39) = 2.83; CE = 150.32
b) EU(0.80:400; 0.20:78) = 3.27; CE = 300.64
c) EU(0.80:2000; 0.20:390) = 4.56; CE = 1503.2.

It may be noticed that when multiplying all outcomes in the prospect by 2 or 10 (or any other positive constant), the CE is multiplied by the same number. The corresponding theoretical property is called constant relative risk aversion. If you multiply the outcomes of the prospect in part a) by another positive constant λ of your own choice, then you will see that the CE is also multiplied by λ. If you know this condition, it could save some calculation work in parts b and c. □

EXERCISE 1.4.4

a) If the client insures and pays −CE, the wealth level is CE for sure, both if the bike is stolen and if not, and this is equivalent to no insurance, so this is the maximum the client pays.
b) By the law of large numbers, the insurance company pays EV per client to reimburse the bikes stolen. But it receives −CE in return. So the profit is EV−CE > 0.

c) The Rotterdam client has CE = −10.16, and the risk premium is −10 − (−10.16) = 0.16. The Amsterdam client has CE = −10.26, and the risk premium is −10 − (−10.26) = 0.26. The insurance company can make some more money on average in Amsterdam, but not much money is to be made from these rational clients.

This exercise illustrates that risk premiums can be important. Can serve to make more money! □

EXERCISE 1.5.2. The preferences cannot be modeled by EU because they reveal a violation of transitivity. Ignoring the upper of the three available acts in all choice situations, so, focusing on the lower two, we get cyclic inequalities

$EU(s_1{:}100, s_2{:}100, s_3{:}200, s_4{:}200, s_5{:}300, s_6{:}300) <$

$EU(s_1{:}200, s_2{:}200, s_3{:}300, s_4{:}300, s_5{:}100, s_6{:}100) <$

$EU(s_1{:}300, s_2{:}300, s_3{:}100, s_4{:}100, s_5{:}200, s_6{:}200) <$

$EU(s_1{:}100, s_2{:}100, s_3{:}200, s_4{:}200, s_5{:}300, s_6{:}300)$.

These cannot be because the first and last act are the same.

This exercise illustrates once more that majority rules often violate transitivity, something underlying the Condorcet paradox and Arrow's voting paradox.

Although a violation of WARP (weak axiom of revealed preference) cannot be directly inferred from the data here, it can be seen that there is no extension to a choice function on the domain of all finite choice sets that can satisfy WARP. □

EXERCISE 1.6.1. Additive separability:

Consider

$$(x_0,\ldots, x_{j-1},\mathbf{c_j},x_{j+1},\ldots) \succcurlyeq (y_0,\ldots, y_{j-1},\mathbf{c_j},y_{j+1},\ldots)$$

Write the corresponding inequality of the discounted utilities. The left- and right-hand side have a common term $\delta^j U(c_j)$. The inequality does not change if we replace it by a common term $\delta^j U(d_j)$. We get an inequality of discounted utilities implying

$$(x_0,\ldots, x_{j-1},\mathbf{d_j},x_{j+1},\ldots) \succcurlyeq (y_0,\ldots, y_{j-1},\mathbf{d_j},y_{j+1},\ldots).$$

This is what additive separability requires.

Stationarity:

$(x_0,x_1,\ldots,x_n) \succcurlyeq (y_0,y_1,\ldots,y_m) \Rightarrow \sum_{j=0}^{n}\delta^j U(x_j) \geq \sum_{i=0}^{m}\delta^i U(y_i) \Rightarrow U(c_0) + \delta\sum_{j=0}^{n}\delta^j U(x_j) \geq$

$U(c_0) + \delta\sum_{i=0}^{m}\delta^i U(y_i) \Rightarrow (c_0,x_0,x_1, ...,x_n) \succcurlyeq (c_0,y_0,y_1, ...,y_m)$. □

EXERCISE 1.6.2. The preferences cannot be modeled by DU because they violate transitivity. Thus,

DU(100, 100, 200, 200, 300, 300, 0, 0, …) <

DU(200, 200, 300, 300, 100, 100, 0, 0, …) <

DU(300, 300, 100, 100, 200, 200, 0, 0, …) <

DU(100, 100, 200, 200, 300, 300, 0, 0, …)

cannot be because the first and last act are the same.

This exercise is similar to Exercise 1.5.2. □

EXERCISE 1.7.1.

a) $1 = U(b) = \sum_{j=1}^{3} a_j u_j(b) = \sum_{j=1}^{3} a_j 1 = \sum_{j=1}^{3} a_j$.

b) $x^1 \sim x^2 \sim x^3$ implies

$U(x^1) = U(x^2) = U(x^3)$, or

$\sum_{j=1}^{3} a_j u_j(x^1) = \sum_{j=1}^{3} a_j u_j(x^2) = \sum_{j=1}^{3} a_j u_j(x^3)$, or

$a_1 \times 1 + a_2 \times 0 + a_3 \times 0 = a_1 \times 0 + a_2 \times 1 + a_3 \times 0 = a_1 \times 0 + a_2 \times 0 + a_3 \times 1$, or

$a_1 = a_2 = a_3$. Because the a_j's add up to 1, each must be ⅓.

c) $U(x^j) = a_j \times 1 + 0 = ⅓$ for each j. Hence, $U(y) = ⅓$. For each agent j, $u_j(y) = p$. $U(y) = \sum_{j=1}^{3} a_j u_j(y) = \sum_{j=1}^{3} ⅓ p = p$. Hence, $p = ⅓ = u_j(y) = U(y)$ for all j. □

Chapter 2

EXERCISE 2.3.1. Assume probabilistic sophistication. If $P(R_A) > P(R_K)$, then the upper preference would be $\preccurlyeq$ instead of $\succ$,[1] so this cannot be. Similarly, if $P(R_A) = P(R_K)$, then the upper preference would be ~ instead of $\succ$ because the two prospects would generate the same probability distribution (p:15, 1−p:0) and, hence, the same V value. Hence $P(R_K) > P(R_A)$ must still hold. Similarly, $P(B_K) > P(B_A)$ must still hold. We obtain the same contradiction as in the powerpoint slides. □

EXERCISE 2.4.1.

(a) For contradiction, we assume stationarity, which implies:

$(0,0,0,0,0,0,c,r,0,\ldots) \succ_0$

$(0,0,0,0,0,c,r,0,0,\ldots) \succ_0$

$(0,0,0,0,c,r,0,0,0,\ldots) \succ_0$

$(0,0,0,c,r,0,0,0,0,\ldots) \succ_0$

$(0,0,c,r,0,0,0,0,0,\ldots) \succ_0$

$(0,c,r,0,0,0,0,0,0,\ldots) \succ_0$

$(c,r,0,0,0,0,0,0,0,\ldots)$

where each preference follows from the one below by inserting a common 0 upfront, and the lowest preference is assumed in the question.

By transitivity,

$(0,0,0,0,0,0,c,r,0,\ldots) \succ_0$

$(c,r,0,0,0,0,0,0,0,\ldots)$

contradicting the last preference assumed in the question.

In words, *with all preferences referring to $\succcurlyeq_0$*, if this year the agent prefers postponing by at least one year, then by stationarity she should prefer every postponement by at least one more year according to this year's preferences. It means that she prefers postponing as long as possible, which is until year $t = 6$. But we assumed the opposite, being that she rather does the job this year than postponing till year $t = 6$: a contraction has resulted. (For specialists: this solution did not use time

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[1] Note that this reasoning does not need the SEU formula, but only probabilistic sophistication.

invariance, a term not defined during the course.)

(b) The naïve agent will postpone this year, erroneously thinking that she will do the job next year or soon after and surely before the last year. But it is plausible to speculate that history repeats next year, she will again prefer to postpone, and each year again, and she ends up doing the job the last year, which is what she dispreferred this year.

(c) The sophisticated agent knows that if she postpones this year, then (as a plausible speculation) she will postpone again next year and, similarly, every year, ending up the last year which she dislikes. So, it boils down to now or at the end. Hence, she will do the job this year, preferring this now to doing it the last year.

FURTHER COMMENTS ON COMMON MISTAKES

Students may erroneously think that the sophisticated agent could just make a plan in year 0 and could then stick with it, as if she had a commitment device and could commit to his plan in year 0. But this is not the case. The big problem of the sophisticated agent, and in fact the only thing that precludes full rationality for her, is that she cannot commit. She *knows* what she will do in the future, but she cannot control it. Such students will then often go on claiming that the agent would do the job in year 5, apparently assuming that her prior preference in year 0 was for doing the job in year 5. The latter need not be. The exercise never specifies what the preferences of the agent in year 0 are over doing the job in year 2, 3, 4, or 5. Many students, similarly, think that the agent in year 0 would simply solve an optimization problem and choose the then most preferred future year and do it then. Again, the agent does not have the commitment device to implement such a policy. Many things from traditional rationality theories work differently in behavioral models!

This whole exercise is subtle, as is the whole material on behavioral intertemporal choice, because classical natural consistency conditions need not hold. Many students make general claims about agent's preferences taking these as consistent in one way or the other and forgetting that the agent's preferences can be different in different times. In a way, the agent is not one agent, but several mutually disagreeing agents. Preferences should always be related to the relevant decision timepoint. Students often use vague expressions to describe preference conditions and implications such as "preferences remain the same" without specifying whether "the same" compares preferences at different decision times or not, and whether in this comparison stopwatch time or calendar time is to be kept constant, for instance. □

EXERCISE 2.5.1. Homo economicus will answer $Z = 10$ because the two questions then are identical.

By scale compatibility, in Question 1 salary is overweighted and the person will be less willing to sacrifice salary, leading to an overestimation of Y. In Question 2, vacation days are overweighted, and the person will be less willing to sacrifice vacation days, inducing an overestimation of Z. The overestimation in Y will add to the overestimation of Z. Hence we can expect $Z > 10$. This was confirmed in empirical studies (Delquié 1993). □