

Behavioral Economics

Peter P. Wakker

Behavioral economics:

makes economics more empirically realistic,
by

using more realistic models of economic agents.

Borrows much from psychology.

Allows emotions/irrationalities.



In short:

Replaces homo economicus by homo sapiens.

That is, replace
Mr. Spock



by Homer
Simpson



Foundations of classical economics

(now challenged by behavioral economics),
underlie all classical econ courses.

Is often still taken for granted.

Today I:

- specify those foundations &
- show the problems that led to the behavioral approach.

So you can understand why behavioral approach came about.

Extra pro: you learn about history and foundations of classical economics.

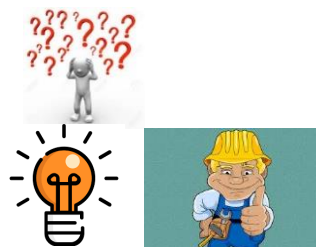
Course will thus be a bit on history of economics.



Historical line often is best didactical line!

Starting today a bit, at the end, and mostly next meetings: the answers of the behavioral approach.

Today host of problems ...
Next meetings solutions.



Practical point: all references can be found in the annotated bibliography on my homepage, at <http://personal.eur.nl/wakker/refs/webrfrncs.docx>

Outline of the 3 Parts

PART I. (Ordinal) homo economi- cus and his empirical problems

Ch. 1. (Ordinal) homo economicus



Ch. 2. (Empirical) problems for homo
economicus



Ch. 3. Beginning of behavioral
economics, 1970-1980:
intermezzo



PART II. Behavioral theories

Ch. 4. Behavioral theories for decision under risk



Ch. 5. Behavioral theories for intertemporal choice



Ch. 6. Behavioral theories for welfare



Ch. 7. Breakaways from revealed preference



PART III. Behavioral applications

Ch. 08. Behavioral applications in game theory



Ch. 09. Behavioral applications in risky choice



Ch. 10. Behavioral applications in intertemporal choice and self-control



Ch. 11. Preference reversals and framing



Part I:
(Ordinal) homo
economicus and
his empirical
problems

Outline of Part 1

Chapter 1. (Ordinal) homo economicus



- 1.1. Birth of ordinal homo economicus
- 1.2. From choice to preference
- 1.3. From preference to utility
- 1.4. Decision under risk: expected utility
- 1.5. Decision under uncertainty: expected utility
- 1.6. Intertemporal decisions: discounted
utility
- 1.7. Welfare theory: utilitarianism
- 1.8. Further results

Chapter 2. (Empirical) problems for homo economicus



- 2.1. Problem for social choice (welfare)
(Arrow'51)
- 2.2. Problem for risk (Allais'53)
- 2.3. Problem for uncertainty (Ellsberg'61)
- 2.4. Problem for intertemporal choice (Strotz'56)
- 2.5. Most serious problem: preference reversals
(Lindman'71; Slovic & Lichtenstein'71)
- 2.6. Emotions in game theory (ultimatum game)
- 2.7. Summarizing the two main ordinal principles

Chapter 3. Beginning of behavioral economics, 1970-1980: Intermezzo

3.1. Biases and heuristics



Chapter 1

(Ordinal) homo
economicus



Chapter 1. (Ordinal) homo economicus

- 1.1. Birth of ordinal homo economicus
- 1.2. From choice to preference
- 1.3. From preference to utility
- 1.4. Decision under risk: expected utility
- 1.5. Decision under uncertainty: expected utility
- 1.6. Intertemporal decisions: discounted utility
- 1.7. Welfare theory: utilitarianism
- 1.8. Further results



Up to 20th century: no strict rules on empirical status of economics (or other social sciences). Psych. inputs & introspection were freely used.



Bentham (1789) and others (Bernoulli 1738),

on utility: is something concrete, just existing.

Say, a happiness in your heart; how good things are for you



Samuelson (ordinalist):

“For Edgeworth [pre-ordinal] utility was as real as his morning jam.”



Many (confused) debates:

- Smith (1776):

If I pay more for diamond than for water,
then is

$U(\text{diamond}) > U(\text{water})?$

But I need water more than diamond??



Big changes in all social sciences in beginning of 20th century:

± 1930 philosophers (Vienna circle, logical positivism, verificationism):

everything should be empirically testable.

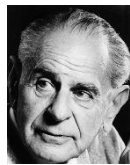
No metaphysics

(unobservables such as “the world is deterministic”,

nor

“utility measures happiness in your heart.”)

(Carnap 1923; Popper 1935 added a refinement—falsifiability).



Empirical status of all social sciences, including economics, came under scrutiny.



Economics 1900-1930: ordinal revolution!

Empirical primitive: choice behavior.

Revealed preference paradigm.

Everything should be verifiable/falsifiable through observable choices.



Introspection = psychology \neq economics.

Choice maximizes utility: is calculated self-interest.

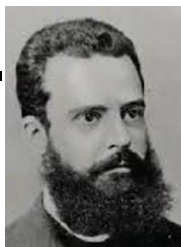
This formalized homo economicus.

(Also, \pm 1930, in psychology, similarly: “behaviorism.”)



Influential references:

Pareto (1906), Robbins (1932), Hicks & Allen (1934).



Most (all?) your courses were on homo economicus:



At first, many positive results.
(So now: foundations of classical economics.)



Chapter 1. (Ordinal) homo economicus

- 1.1. Birth of ordinal homo economicus
- 1.2. From choice to preference
- 1.3. From preference to utility
- 1.4. Decision under risk: expected utility
- 1.5. Decision under uncertainty: expected utility
- 1.6. Intertemporal decisions: discounted utility
- 1.7. Welfare theory: utilitarianism
- 1.8. Further results



From (multi-)choice to binary choice (= preference): revealed preference theory

Assume you choose a from $\{a,b,c,d\}$.

Reveals preferences $a \succ b$, $a \succ c$, $a \succ d$.

Conversely, if we know these preferences, then we know that you will choose a from $\{a,b,c,d\}$.

Revealed preference theory studies when and how choices are related to binary preferences and vice versa.

Famous idea: Samuelson's weak axiom of revealed preference (WARP).

Explained next:



Notation:

X : set of **prospects** (e.g., commodity bundles)

C : **choice function**: to every nonempty finite subset A of X it assigns a nonempty subset $C(A) \subset A$ (the “best” prospects in A , i.e., which the decision maker is willing to choose)

WARP:

there are no $x, y \in X$, finite $A, B \subset X$, such that both:

$x \in C(A), y \in A$,

(i.e, x revealed weakly preferred to y)

and

$y \in C(B), x \in B \setminus C(B)$

(i.e., y revealed strictly preferred to x).



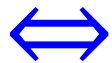
For a binary (“preference”) relation \succsim on X ,

C maximizes \succsim if

$$C(A) = \{x \in A : x \succsim y \text{ for all } y \in A\}.$$

Theorem (\pm Samuelson).

C maximizes a weak order \succsim



C satisfies WARP.



Weak order: transitive and complete.



Chapter 1. (Ordinal) homo economicus

- 1.1. Birth of ordinal homo economicus
- 1.2. From choice to preference
- 1.3. From preference to utility
- 1.4. Decision under risk: expected utility
- 1.5. Decision under uncertainty: expected utility
- 1.6. Intertemporal decisions: discounted utility
- 1.7. Welfare theory: utilitarianism
- 1.8. Further results



From preference (binary choice) to utility (unitary evaluation)

U represents \succsim if
 $x \succsim y \Leftrightarrow U(x) \geq U(y)$.
 U is called utility.

THEOREM. \succsim is a continuous weak order
(transitive & complete)



\succsim can be represented by a continuous utility U .



This was a behavioral foundation of utility.
Mas-Colell et al. (1995) used the, naïve, term
“rational” to refer to this model.



Utility, preference, and everything, can be measured, verified/falsified, from choice.

No more and no less.

If you interpret utility as an index of happiness, then you do so at your own risk.

Ordinal economists wish not get involved in such metaphysics (by their standards).



Utility is ordinal in

- **mathematical sense**: only its ordering of prospects matters; its differences do not matter.
- **conceptual sense**: no interpretations of happiness/introspection.

Many results were derived from this ordinal basis.

Main breakthrough: Hicks & Allen (1934):
market & consumer demand only need ordinal utility
(Pareto, 1906, had observed this partially before).

Now come some ordinal achievements, which played a big role in the subsequent behavioral approach.



Chapter 1. (Ordinal) homo economicus

1.1. Birth of ordinal homo economicus

1.2. From choice to preference

1.3. From preference to utility

1.4. Decision under risk: expected utility

1.5. Decision under uncertainty: expected utility

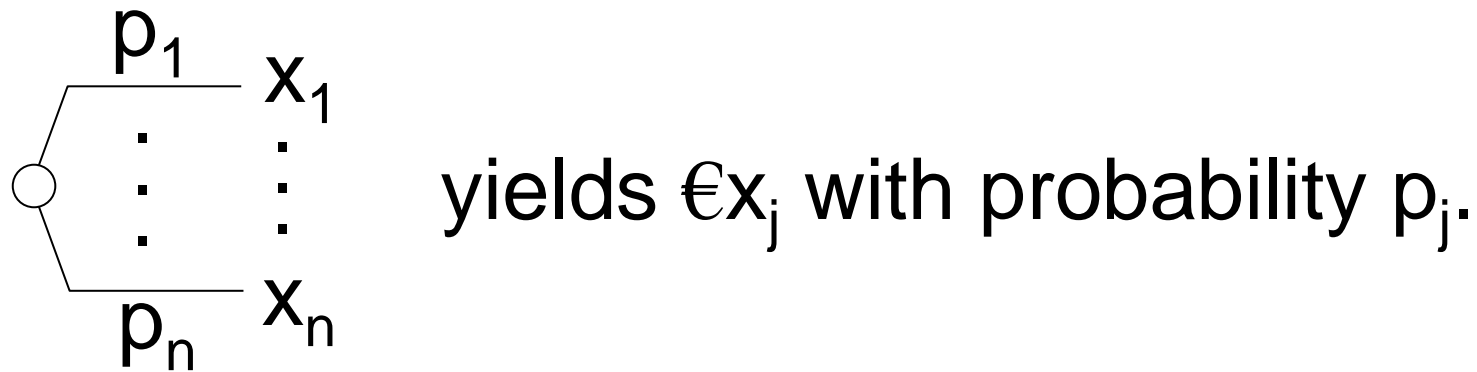
1.6. Intertemporal decisions: discounted
utility

1.7. Welfare theory: utilitarianism

1.8. Further results



Decision under risk: prospects are probability distributions over money.



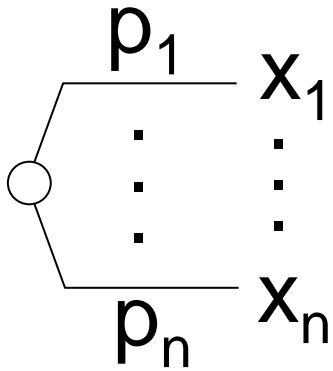
Also denoted $(p_1:x_1, \dots, p_n:x_n)$ or, short, x .



Bernoulli (D.):



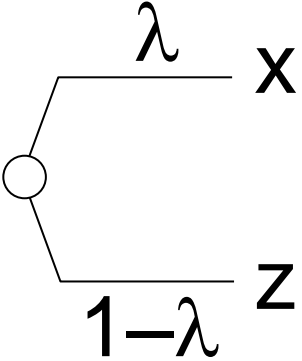
Expected utility (EU)



$$\rightarrow p_1 U(x_1) + \dots + p_n U(x_n)$$



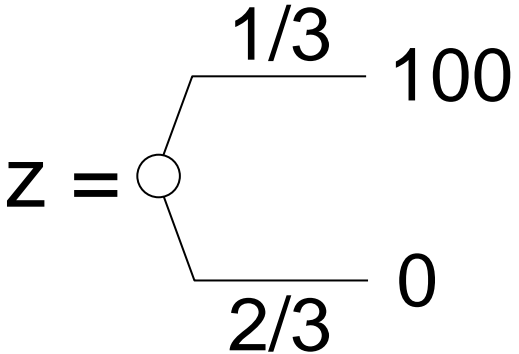
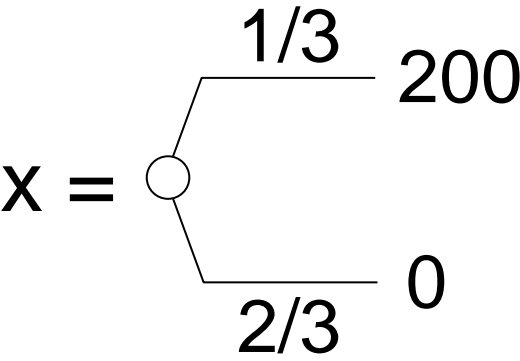
Notation:



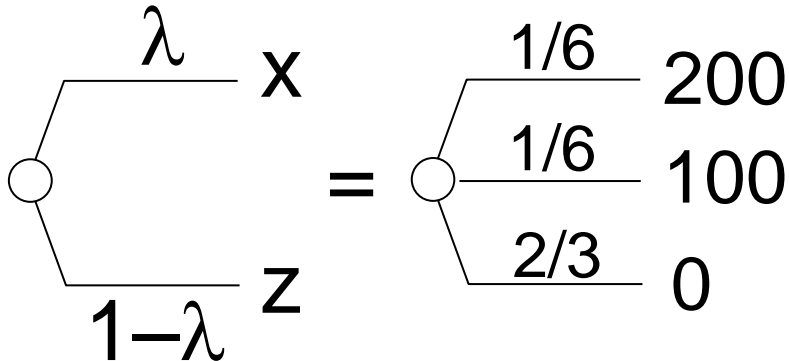
$= \lambda x + (1-\lambda)z$ is probabilistic mixture.

It gives prospect x with probability λ and prospect z with probability $1-\lambda$.

Example:

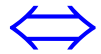


$\lambda = 0.5$



THEOREM.

Expected utility

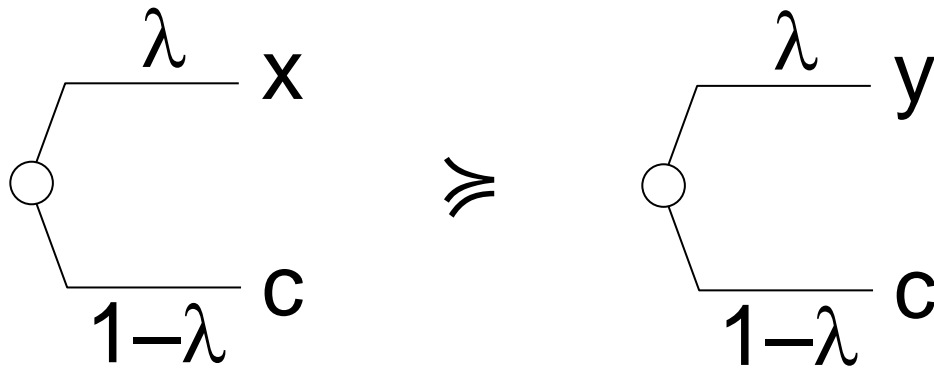


- (1) \succsim is a weak order
- (2) Continuity in probability
- (3) Independence (defined next).

This is a behavioral foundation (also called axiomatization or representation theorem or preference foundation) of EU.



Independence: if $x \succcurlyeq y$, then for all prospects c and $\lambda > 0$:



“Improving the upper branch (x for y) improves the whole thing.”

Roughly, **EU** \Leftrightarrow **independence!**

The condition looks convincing in the abstract.

I think it IS convincing, normatively.



Expected utility is the basis of most in economics.



Chapter 1. (Ordinal) homo economicus

1.1. Birth of ordinal homo economicus

1.2. From choice to preference

1.3. From preference to utility

1.4. Decision under risk: expected utility

1.5. Decision under uncertainty: expected utility

1.6. Intertemporal decisions: discounted
utility

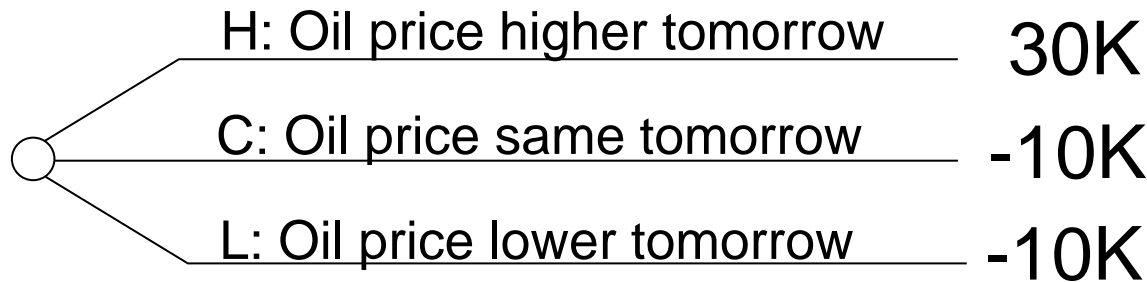
1.7. Welfare theory: utilitarianism

1.8. Further results



K: €1000

Speculate on financial asset, and pay 10K to receive 40K if oil price increases tomorrow?

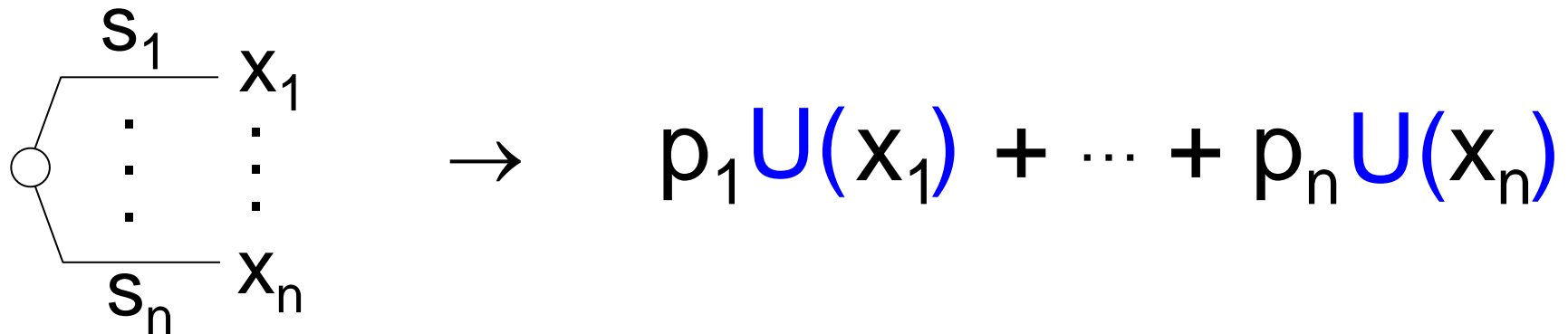


Decide how? No (“objective”) probabilities available. Savage (1954) gave a brilliant behavioral foundation (we have no time to look into it) Of:

choose subjective probabilities $P(H)$, $P(C)$, and $P(L)$, and utility U . Then, with $U(0)=0$, accept asset if $P(H)U(30K) + P(C)U(-10) + P(L)U(-10) > 0$.



Savage (1954) in general: (subjective) expected utility



Here s_1, \dots, s_n are uncertain events. Exactly one will happen, the others won't, but you are uncertain which will happen, and don't know any ("objective") probabilities of them.

Then choose subjective probabilities p_j and maximize expected utility.



Chapter 1. (Ordinal) homo economicus

1.1. Birth of ordinal homo economicus

1.2. From choice to preference

1.3. From preference to utility

1.4. Decision under risk: expected utility

1.5. Decision under uncertainty: expected utility

1.6. Intertemporal decisions: discounted
utility

1.7. Welfare theory: utilitarianism

1.8. Further results



Intertemporal decisions: prospects are income streams.



(x_0, x_1, \dots) yields x_0 this month, x_1 next month, and so on; all $x_j \geq 0$. We assume that only finitely many x_j s are nonzero: you are mortal.

Samuelson (1937) proposed (constant-)discounted utility:



$$\sum \delta^j U(x_j)$$

with:

- $\delta > 0$ the discount factor &
- U utility, continuous, strictly increasing, $U(0)=0$.

Usually $\delta < 1$: impatience.



THEOREM (Koopmans 1960, 1972).

Constant discounted utility holds



(1) weak ordering

(2) continuity

(3) monotonicity (prefer increases in income)

(4) additive separability:

$$(x_0, \dots, x_{j-1}, \mathbf{c}_j, x_{j+1}, \dots) \succcurlyeq (y_0, \dots, y_{j-1}, \mathbf{c}_j, y_{j+1}, \dots)$$



$$(x_0, \dots, x_{j-1}, \mathbf{d}_j, x_{j+1}, \dots) \succcurlyeq (y_0, \dots, y_{j-1}, \mathbf{d}_j, y_{j+1}, \dots)$$

(preference does not depend on common outcomes)

(5) stationarity: see next page.



Stationarity:

$$(x_0, x_1, \dots) \succcurlyeq (y_0, y_1, \dots)$$



$$(c_0, x_0, x_1, \dots) \succcurlyeq (c_0, y_0, y_1, \dots)$$



(delaying income does not affect current preference).

By repeated application it implies:

$$(x_0, x_1, \dots) \succcurlyeq (y_0, y_1, \dots)$$



$$(c_0, \dots, c_t, x_0, x_1, \dots) \succcurlyeq (c_0, \dots, c_t, y_0, y_1, \dots).$$

Example.

x: 1 year-long €100/month salary increase.

y: ½ year-long €190/month salary increase.

You say: x now \succcurlyeq y now (you are patient now).

Then also x in 4 years \succcurlyeq y in 4 years.

You are as patient for future options as for present. \square



Chapter 1. (Ordinal) homo economicus

1.1. Birth of ordinal homo economicus

1.2. From choice to preference

1.3. From preference to utility

1.4. Decision under risk: expected utility

1.5. Decision under uncertainty: expected utility

1.6. Intertemporal decisions: discounted
utility

1.7. Welfare theory: utilitarianism

1.8. Further results





Welfare theory (Harsanyi 1955): prospects x are probability distributions over social states s .

Assume n agents with preferences \succsim_j , $j = 1, \dots, n$, over prospects.

\succsim denotes the preference relation for society of a benevolent social planner with no self interests.

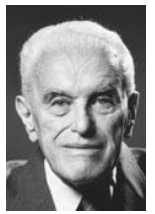
Assume that all preferences \succsim_j , and also \succsim , maximize expected utility, with utility functions u_j and U , respectively. Assume **Pareto optimality**:

$$x \succsim_j y \text{ for all } j \Rightarrow x \succsim y.$$

Then we get utilitarianism, as the following theorem shows.



THEOREM (Harsanyi 1955). Under some richness of structure:



Utilitarianism holds: $U(x) = \sum a_j u_j(x)$ where each agent j maximizes EU w.r.t. u_j , society maximizes EU with respect to U , and $a_j \geq 0$ is weight of agent j .



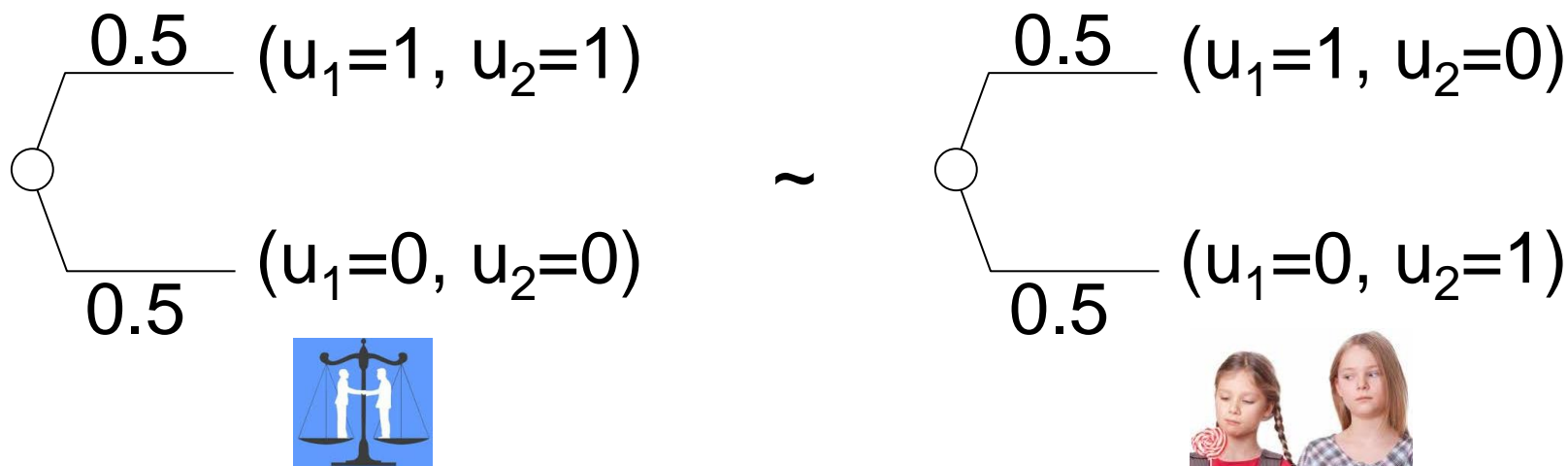
All \succsim_j , and also \succsim , satisfy the EU axioms. Further, Pareto optimality holds. \square

Each agent only cares about self. Planner averages without any equity consideration!



Harsanyi's theorem caused a **sensation**.
None of the axioms seems objectionable.
Yet they rule out equity/fairness considerations,
whereby a person is weighted more if poor than
if rich.

I immediately give an **objection**.



Pareto optimality implies the indifference. Yet
many prefer the left arrangement for always
bringing equity. Many debates ...

Chapter 1. (Ordinal) homo economicus

- 1.1. Birth of ordinal homo economicus
- 1.2. From choice to preference
- 1.3. From preference to utility
- 1.4. Decision under risk: expected utility
- 1.5. Decision under uncertainty: expected utility
- 1.6. Intertemporal decisions: discounted utility
- 1.7. Welfare theory: utilitarianism
- 1.8. Further results



Further successes for ordinal approach:



- Nash (1950) proved equilibrium in games.
- Debreu (1959) proved equilibrium in markets.



Both results were awarded a Nobel prize.



Chapter 2

(Empirical) problems
for homo
economicus

Outline of Ch. 2

- 2.1. Problem for social choice (Arrow'51)
- 2.2. Problem for risk (Allais'53)
- 2.3. Problem for uncertainty (Ellsberg'61)
- 2.4. Problem for intertemporal choice (Strotz'56)
- 2.5. Most serious problem: preference reversals
(Lindman'71; Slovic & Lichtenstein'71)
- 2.6. Emotions in game theory (ultimatum game)
- 2.7. Summarizing the two main ordinal principles

First problem, and signal of more serious ones to come, for ordinal approach:



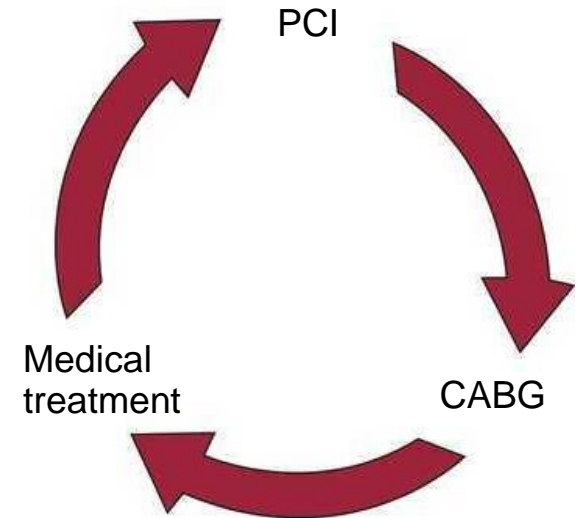
Arrow's (1951) voting paradox: for ordinal preferences, a few natural conditions lead to a paradox.

Basic problem can be seen in the Condorcet paradox:

	First preference	Second preference	Third preference
Heart team member 1	CABG	PCI	Medical treatment
Heart team member 2	PCI	Medical treatment	CABG
Heart team member 3	Medical treatment	CABG	PCI

(They were going by majority preference:)

They got stuck on what to do!



Majority preference violates transitivity: **undesirable!**

Explanation: For CABG \succ PCI, we count PCI \succ_2 CABG as only one preference but it should be more because it is extremem and stronger.

Ordinal approach seeks to ignore such “cardinal” information, formally through Arrow’s “independence of irrelevant alternatives” (not explained here).

Lesson to be learnt: for good welfare decisions, we have to incorporate cardinal info about how much people prefer things.

Something like utility differences ...

This lesson does not sit well with the ordinal spirit ...

Reactions came,
hoping to save ordinal homo economicus.

...

We do not elaborate.

Arrow's paradox is a problem, but not
enough so for people to give up ordinalism.

Chapter 2. (Empirical) problems for homo economicus

2.1. Problem for social choice (Arrow'51)

→ 2.2. Problem for risk (Allais'53)

2.3. Problem for uncertainty (Ellsberg'61)

2.4. Problem for intertemporal choice (Strotz'56)

2.5. Most serious problem: preference reversals
(Lindman'71; Slovic & Lichtenstein'71)

2.6. Emotions in game theory (ultimatum game)

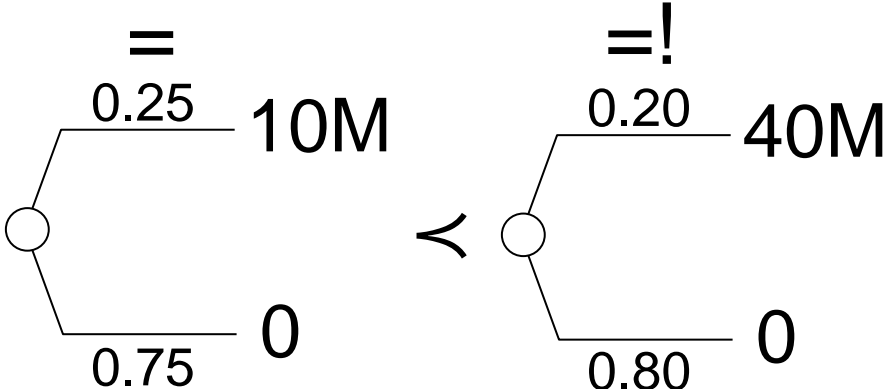
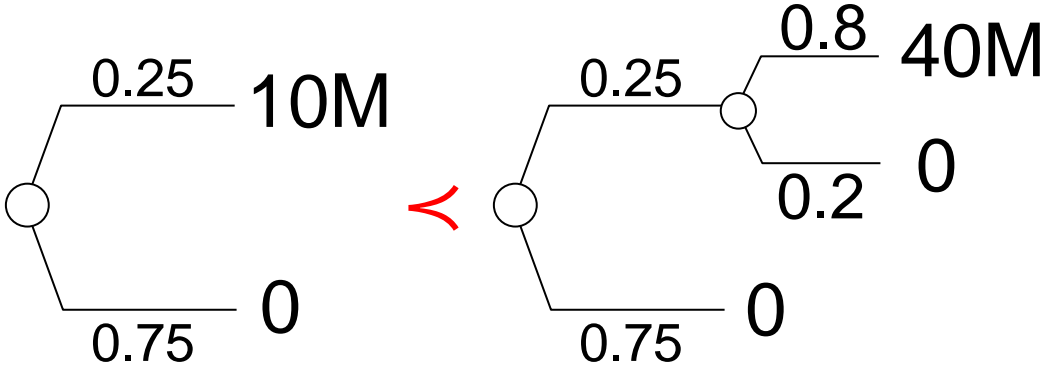
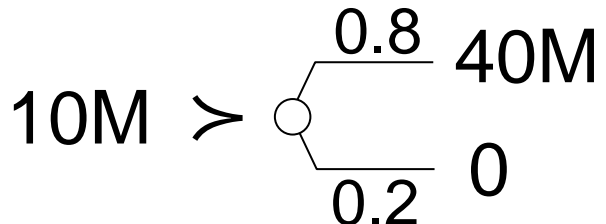
2.7. Summarizing the two main ordinal principles

2nd problem for ordinal approach: Allais' (1953) paradox.

Later confirmed empirically. Violates expected utility.

Preferences below are majority prefs, and violate independence, so EU.

M: €10⁶



< iso ? violates independence. 



First reactions:

- just weird anomaly in lab with extreme amounts and probabilities.
- subjects are confused and will learn better in the market.

Allais' paradox is a problem, but not enough so, for people to give up ordinalism.

Other classical problem for expected utility:
coexistence of gambling & insurance
(risk seeking \neq risk aversion).

Still not enough to give up ordinalism.

Ordinal champion Arrow said it this way
(1971, p. 90):

“I will not dwell on this point extensively,
emulating rather the preacher, who,
expounding a subtle theological point to his
congregation, frankly stated:

Brethren, here there is a great
difficulty; let us face it firmly
and pass on.”

Ellsberg (1961) paradox.

Presented next.



Is for the practically prevailing case that probabilities are unknown; concerns Savage (1954).

Is more fundamental problem for EU.

Long time not well understood.

Led to field of

“Decision under ambiguity.”

Chapter 2. (Empirical) problems for homo economicus

2.1. Problem for social choice (Arrow'51)

2.2. Problem for risk (Allais'53)

2.3. Problem for uncertainty (Ellsberg'61)

2.4. Problem for intertemporal choice (Strotz'56)

2.5. Most serious problem: preference reversals
(Lindman'71; Slovic & Lichtenstein'71)

2.6. Emotions in game theory (ultimatum game)

2.7. Summarizing the two main ordinal principles

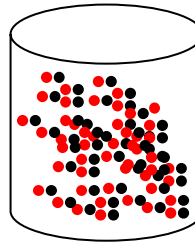
Ellsberg paradox

Notation:

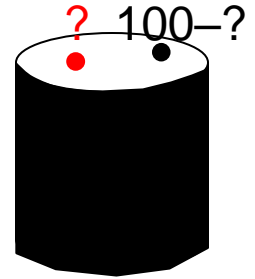
$(R_K: \text{€}15)$: get €15 if the ball drawn from K is red; get nothing otherwise.

Known urn K Ambiguous urn A

50 R
50 B



100 R&B
in unknown
proportion



$(R_K: \text{€}15) \succ (R_A: \text{€}15)$

$(B_K: \text{€}15) \succ (B_A: \text{€}15)$

$P(R_K) \succ P(R_A)$

$P(B_K) \succ P(B_A)$

$\frac{1}{1}^+ \succ \frac{1}{1}^+$



Common

interpretation:

Violates subjective probabilities:

Violates Savage's (1954) subjective expected utility. (More fundamentally, **violates subjective probabilities.**)

Chapter 2. (Empirical) problems for homo economicus

- 2.1. Problem for social choice (Arrow'51)
- 2.2. Problem for risk (Allais'53)
- 2.3. Problem for uncertainty (Ellsberg'61)
- 2.4. Problem for intertemporal choice (Strotz'56)
- 2.5. Most serious problem: preference reversals (Lindman'71; Slovic & Lichtenstein'71)
- 2.6. Emotions in game theory (ultimatum game)
- 2.7. Summarizing the two main ordinal principles

Fourth problem for ordinal approach (Strotz 1956 & Thaler 1981): consider the following decision situations.

now
€100

or

in 4 months
€105

Choose, in 4 years, between, at that moment:

now
€100

or

in 4 months
€105

in 4 years
€100

or

in 4 years + 4 months
€105

Common prefs
irrational!?



Strotz ... reactions are discussed later.

Nonconstant discount functions, to accommodate the common preferences, were introduced later. This is part of the behavioral approach also discussed later.

Chapter 2. (Empirical) problems for homo economicus

- 2.1. Problem for social choice (Arrow'51)
- 2.2. Problem for risk (Allais'53)
- 2.3. Problem for uncertainty (Ellsberg'61)
- 2.4. Problem for intertemporal choice (Strotz'56)
- 2.5. Most serious problem: preference reversals (Lindman'71; Slovic & Lichtenstein'71)
- 2.6. Emotions in game theory (ultimatum game)
- 2.7. Summarizing the two main ordinal principles

If problems so far were signals of things to come, then now the most serious blow to ordinal approach & homo economicus. The ultimate plague sent to the classical model.



Most serious blow for ordinalism:
preference reversals (Lichtenstein & Slovic '71;
Lindman '71).



\$-prospect:

Prob. 0.31: \$16
nil otherwise

P-prospect:

Prob. 0.97: \$4
nil otherwise

Choice-question:

Which of these two prospects would you choose?

\$-prospect:

Prob. 0.31:

\$16

nil otherwise

P-prospect:

Prob. 0.97:

\$4

nil otherwise

Money-value question:

Determine for each prospect its **subjective** monetary value **for you**.

\$-prospect:

Prob. 0.31: \$16
nil otherwise

P-prospect:

Prob. 0.97: \$4
nil otherwise

Common finding

- Choice:
majority prefers P-prospect.
- Monetary evaluation:
majority assigns higher monetary value to ...
\$-prospect!

~~X~~ transitivity:

\$-prospect \sim its monetary value $>$ monetary
value of P-prospect \sim P-prospect $>$ \$-prospect

First reaction:

economists could not believe.

The experimental economists Grether & Plott'79:

“This paper reports the results of a series of experiments designed to discredit the psychologists’ works as applied to economics.”

Did very careful experiment, with real incentives and all that.

Could only confirm preference reversal. The psychologists were fully right.

This blow was right at the heart of economics; at what Mas-Colell called rational.

It turned the table.

After, ordinal economics was never again what it had been before.

Possible explanation of preference reversal:



\$-prospect:

Prob. 0.31: \$16
nil otherwise

P-prospect:

Prob. 0.97: \$4
nil otherwise

“Scale compatibility” explains (part of)
preference reversal:



When determining monetary values, subjects focus too much on the better payment \$16. So, \$-bet fares better in valuation than in choice.

Preference reversals are now well established. Serve as **smoking gun** for inconsistent evaluations, and for detecting biases. Many more biases have been found.

Chapter 2. (Empirical) problems for homo economicus

2.1. Problem for social choice (Arrow'51)

2.2. Problem for risk (Allais'53)

2.3. Problem for uncertainty (Ellsberg'61)

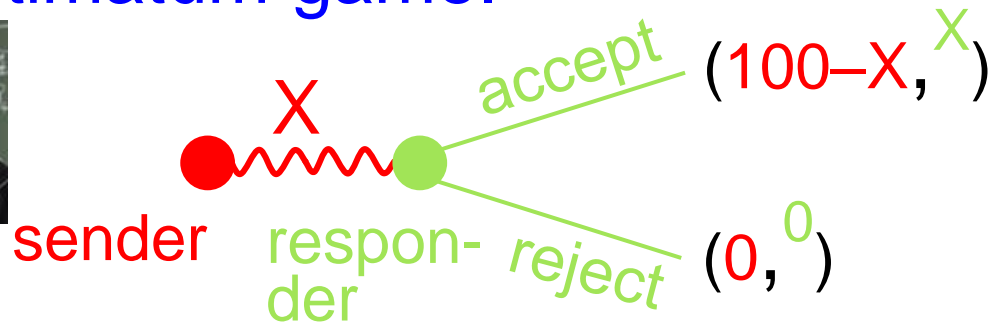
2.4. Problem for intertemporal choice (Strotz'56)

2.5. Most serious problem: preference reversals
(Lindman'71; Slovic & Lichtenstein'71)

→ 2.6. Emotions in game theory (ultimatum game)

2.7. Summarizing the two main ordinal principles

Güth, Schmittberger, & Schwarze (1982): the ultimatum game.



$$0 \leq X \leq 100$$

What will happen for homo economicus?

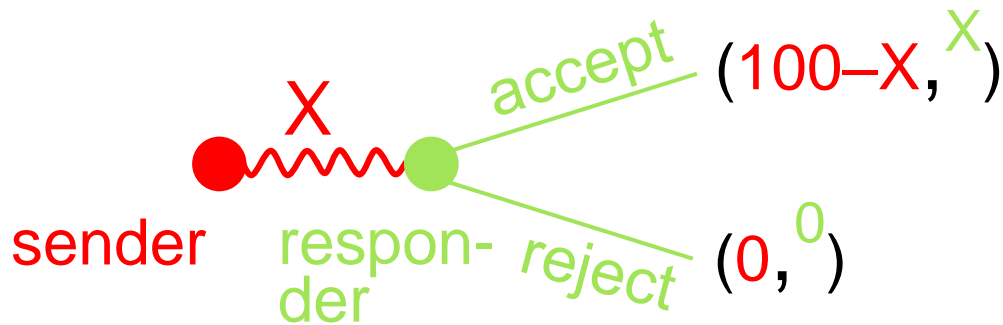
Backward induction:



$X=0$, (accept all $X > 0$, accept $X=0$)

Outcome is $(100, 0)$.

Güth, Schmittberger, & Schwarze (1982): the ultimatum game.



$$0 \leq X \leq 100$$

What happens for homo sapiens (behavioral)?
Offers below $X=10$ are almost always **rejected**.

Modal offer: $X=50$!

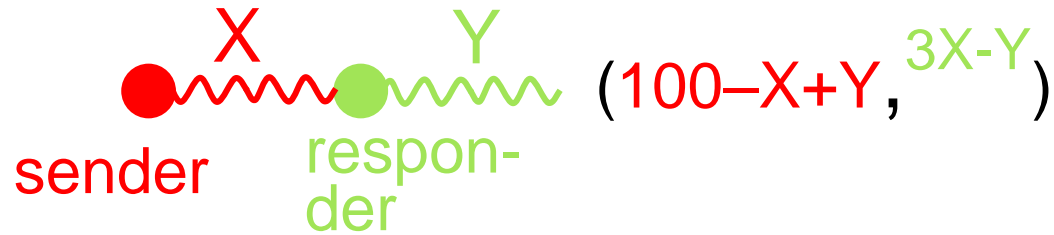
Modal outcome is $(50, 50)$.

People are driven by fairness, equity, altruism ...

Other-regarding preferences. Very different from classical economics!



Berg, Dickhaut, McGabe (1995): the trust game.



$$0 \leq X \leq 100$$

$$0 \leq Y \leq 3X$$

Explanation:

sender receives 100.

Sends X to **responder**, keeps rest.

Different from ultimatum game: X is multiplied by 3.

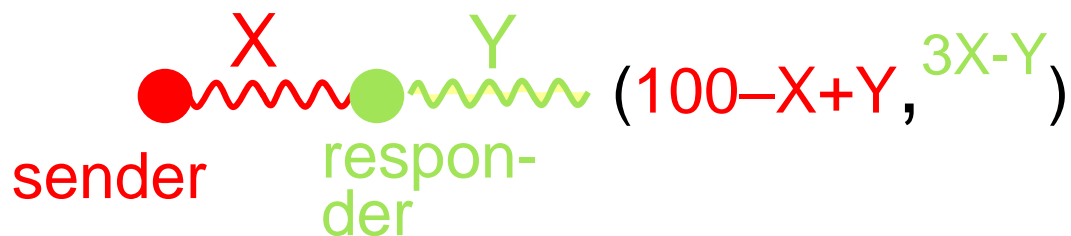
Then from $3X$, **responder** gives Y back to **sender**, and keeps rest.

What will happen for homo economicus?



Backward induction: ($X=0$, $Y=0$). Outcome is $(100, 0)$.

Berg, Dickhaut, McGabe (1995): the trust game.



$$0 \leq X \leq 100$$

$$0 \leq Y \leq 3X$$

What will happen for **homo sapiens** (behavioral)? 

Modal sending: $X=50$.

Modal response: $Y=X$.

Modal outcome is $(100, 100)$.

Aside question: what would happen if players could perfectly well cooperate?

$X = 100$.

Goeree & Holt (2001): test of Nash equilibrium (NE)



	L	C	NN	R
T	200 ⁵⁰	0 ⁴⁵	10 ³⁰	20 ⁻²⁵⁰
B	0 ⁻²⁵⁰	10 ⁻¹⁰⁰	30 ³⁰	50 ⁴⁰

What will happen for homo economicus?

You can think ...

NE: (T,L) and (B,R).



What would you play in this game if column?

What will happen for homo sapiens (behavioral)?



	L (26%)	C (8%)	NN (68%)	R (0%)
T (68%)	200 ⁵⁰	0 ⁴⁵	10 ³⁰	20 ⁻²⁵⁰
B (32%)	0 ⁻²⁵⁰	10 ⁻¹⁰⁰	30 ³⁰	50 ⁴⁰

This game was tested in an experiment by Goeree & Holt (2001, *American Economic Review*), with real incentives. The percentages are the percentages of subjects choosing the corresponding strategies.

Chapter 2. (Empirical) problems for homo economicus

2.1. Problem for social choice (Arrow'51)

2.2. Problem for risk (Allais'53)

2.3. Problem for uncertainty (Ellsberg'61)

2.4. Problem for intertemporal choice (Strotz'56)

2.5. Most serious problem: preference reversals
(Lindman'71; Slovic & Lichtenstein'71)

2.6. Emotions in game theory (ultimatum game)

2.7. Summarizing the two main ordinal principles

Two views of ordinal economics & homo economicus:

- (1) Revealed preference paradigm: use only choice-based inputs
- (2) The best empirical model we will ever get is the normative model. Arrow (1951

Econometrica p. 406): “In view of the general tradition of economics, which tends to regard rational behavior as a first approximation to actual, I feel justified in lumping the two classes of theory together.”

Relatedly, the earlier Newton (1687): “I can calculate the motion of heavenly bodies, but not the madness of people.”



This course focuses on generalizing (2). On the more fundamental (1), only brief remarks at the end.

Chapter 3

Beginning of
behavioral
economics; 1970-
1980: intermezzo



Outline of Ch. 3

→ 3.1. Biases and heuristics





Kahneman & Tversky developed biases & heuristics in the mid-1970s (e.g., Science 1974). Already tried to model irrationalities.

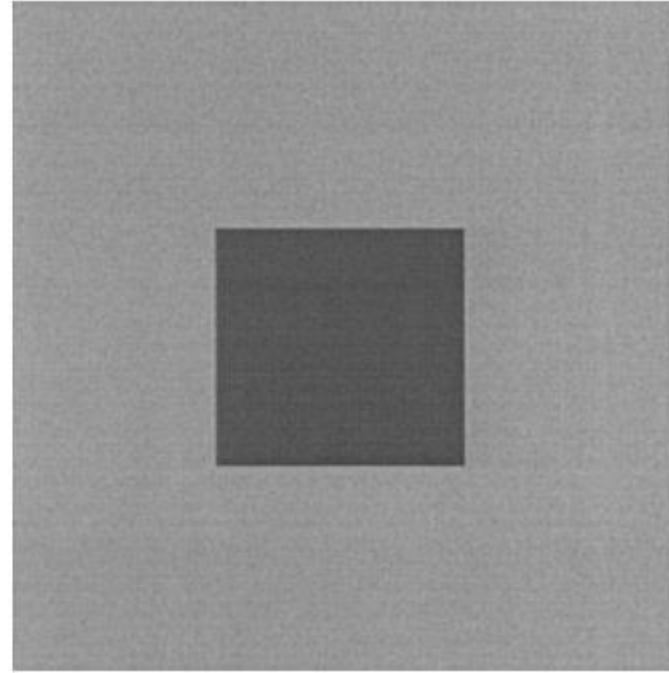
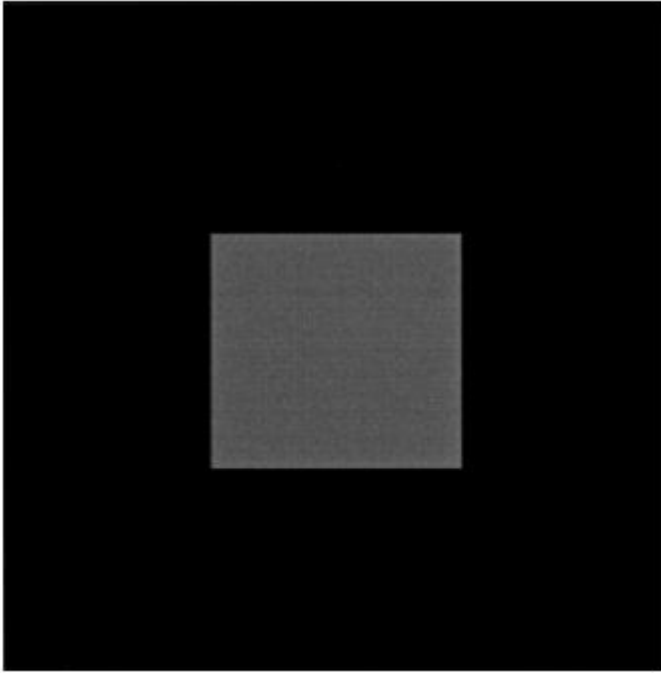
But, pre-behavioral, psychology-style; nonquantitative; warming up for the real thing ... Were inspired by the aforementioned problems (and Simon's 1955 bounded rationality, not presented here). **Heuristics:** simple, useful rules, sometimes generating biases.

Like our vision. Our eyes use context to interpret.

Example:



Which middle square is darker?



Neither! Equally dark! The good heuristic of relating to context sometimes biases us.

Our decision-brain works like vision:
uses simplifying heuristics that are mostly
useful.

But sometimes go horribly wrong, generating
illusions/biases.

Psychologists Kahneman & Tversky
discovered several such heuristics & biases.

One:

representativeness heuristic:

when judging how probable it is that object A
belongs to class B, people go by similarity.

Example:



Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations. Which is more probable?

(a) Linda works in a bank.

(b) Linda works in a bank and is active in the feminist movement.

Obviously, it is (a). But most people say (b). Linda represents (b) more than (a).



Many other biases were discovered, especially in the 1970s:
availability, anchoring and adjustment, ...
and so on.

Useful discoveries. But no great predictability yet.

Not quantitative.

No hard predictions.

More useful for psychology than for economics.

We focus on the latter.

The first good quantitative theory:
1979 prospect theory.



Other development (following up on Hicks & Allen (1934))
in 1970s economists searched much for
micro-foundations of **macro-phenomena**.
E.g. “rational expectations.”
They took micro no longer for granted.
More interest in getting micro right.

Time was getting ready for behavioral!



*Part II:
Behavioral
theories*

Outline of Part 2

Chapter 4. Behavioral theories for decision under risk



- 4.1. Reaction to the 2nd problem (§2.2); a behavioral theory for risk: prospect theory
- 4.2. Chew's weighted utility
- 4.3. Gul's disappointment aversion
- 4.4. Quiggin's (1982) RDU and Tversky & Kahneman's (1992) PT
- 4.5. Biseparable utility as a version of PT
- 4.6. Neo-additive utility as a version of PT
- 4.7. Afterword & moderating views

Chapter 5. Behavioral theories for intertemporal choice



- 5.1. Decreasing impatience
- 5.2. Hyperbolic discounting
- 5.3. Quasi-hyperbolic discounting
- 5.4. Unit invariance discounting
- 5.5. Discussion

Chapter 6. Behavioral theories for welfare

6.1. Fehr-Schmidt inequality aversion



Chapter 7. Breakaways from revealed preference



7.1. Kahneman's experienced utility

7.2. Happiness studies

Chapter 4



Behavioral theories for decision under risk



Ch. 4



- 4.1. Reaction to the 2nd problem (§2.2); a behavioral theory for risk: prospect theory
- 4.2. Chew's weighted utility
- 4.3. Gul's disappointment aversion
- 4.4. Quiggin's (1982) RDU and Tversky & Kahneman's (1992) PT
- 4.5. Biseparable utility as a version of PT
- 4.6. Neo-additive utility as a version of PT
- 4.7. Afterword & moderating views

Kahneman & Tversky's (1979) prospect theory: a new risk theory.

But implications are much broader.

It provided the first explicit & **tractable** deviation from rationality. Before, thought to be impossible.

Repeating two citations:

Arrow (1951 *Econometrica* p. 406):

“In view of the general tradition of economics, which tends to regard rational behavior as a first approximation to actual, I feel justified in lumping the two classes [normative and descriptive] of theory together.”

Relatedly, the earlier Newton (1687): “I can calculate the motion of heavenly bodies, but not the madness of people.”



Kahneman & Tversky could have said: “We **can** calculate the madness of people.”

Prospect theory was first to:

- capture emotions beyond rationality, yet
- allow quantitative measurements/predictions.



Thus, it initiated the behavioral approach.

Is most-cited paper ever in an economic journal (Merigó, Rocafort, & Aznar-Alarcón 2016).

Original 1979 prospect theory (OPT) used, roughly, the following formula, deviating from expected utility. It, modestly, restricted itself to no more than two nonzero outcomes. We only consider gains (≥ 0).

$$\begin{array}{l}
 p_1 \quad x_1 \\
 \hline
 p_2 \quad x_2 \\
 \hline
 1-p_1-p_2 \quad 0
 \end{array}
 \rightarrow w(p_1)U(x_1) + w(p_2)U(x_2) = \text{OPT}$$

U is utility; $U(0) = 0$ has to be under PT.

$w: [0,1] \rightarrow [0,1]$ is probability weighting.

$w(0) = 0, w(1) = 1$, and w is strictly increasing.

$w(p) = p$ gives EU.

The formula is for $x_1 \neq x_2$.

For only one nonzero x_1 , drop x_2 term (say $p_2=0$).



Formula on preceding slide is
often used in the literature for OPT.

Kahneman&Tversky'79 used a slightly different—better—formula when $p_1 + p_2 = 1$.

Biggest novelty: reference dependence.

0 is reference point.

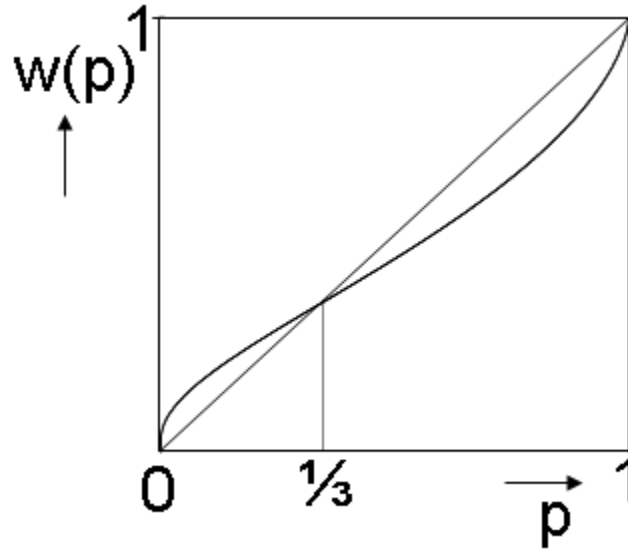
It depends on psychological framing.

U behaves differently for gains than for losses.

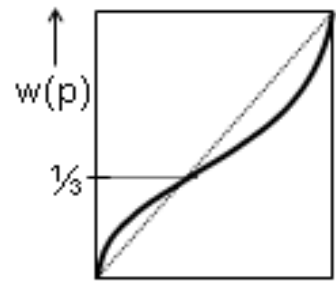


We focus on gains (≥ 0) and on w ,
the other novelty.

Common empirical finding: w has inverse-S shape:



Captures psychological aspects of risk attitudes beyond EU.

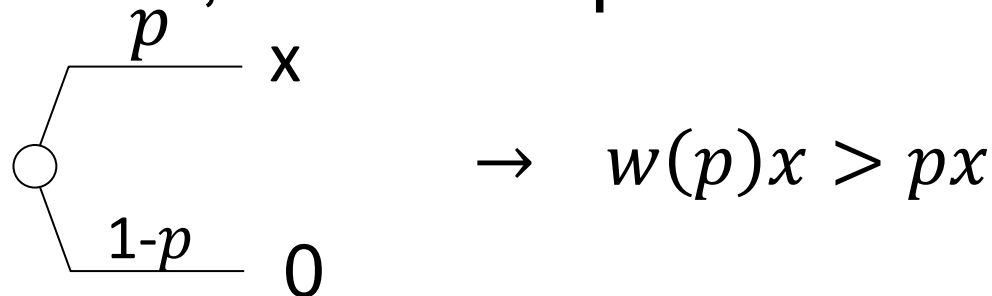


Overweighting of small probabilities p deviates from the widespread economic belief in universal risk aversion.

Can generate risk seeking for gains: preferring a risky prospect to its expected value. Assume $U(x) = x$ (holds approximately for small x).



Then, for small p :



Explains gambling: people overweight a $1/10^6$ probability of gaining a lottery.



Also **helps to explain insurance**: people overweight a $1/10^6$ probability of their house catching fire.

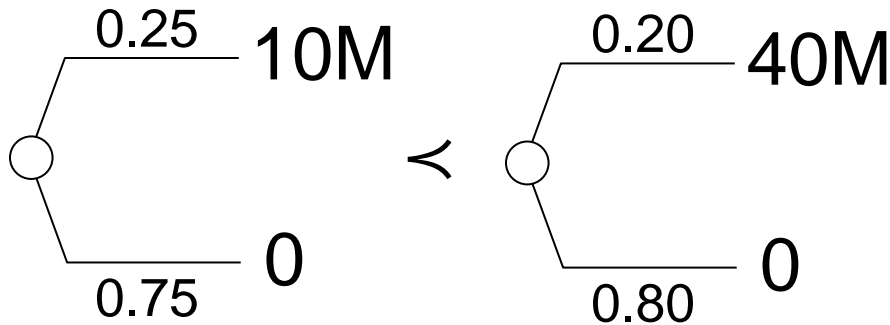
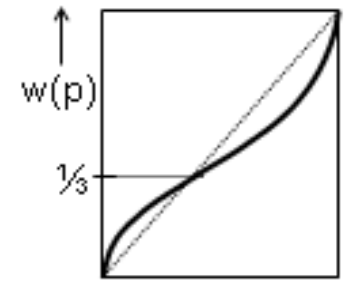
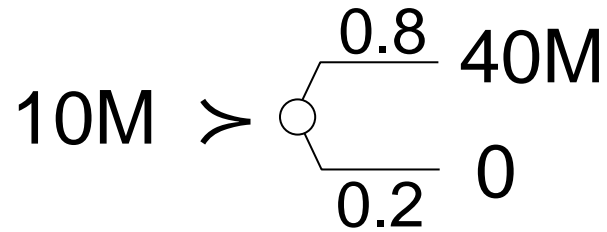
Co-existence of gambling and insurance—a classical paradox—is thus resolved. They even have the same cause: overestimation of small probabilities.



Prospect theory can also accommodate Allais' paradox:

Allais' (1953) paradox:

M: €10⁶



Assume the reasonable $U(10M) = 0.75 \times U(40M)$.
Then $w(0.8) < 0.75$, a common finding, explains the first preference.
 $\frac{w(0.20)}{w(0.25)} > 0.8 > 0.75$, again, commonly found, explains the second preference.

Some calculations to illustrate:

Assume

$$U(\alpha) = \alpha^\theta \text{ with } \theta = 0.2075 \text{ (then } \frac{U(10M)}{U(40M)} = 0.75).$$

(If you know about relative risk aversion:
relative risk aversion index is $1 - 0.2075 = 0.7925$.)

Assume (as in preceding figure)

$$w(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}} \text{ with } \gamma = 0.61.$$

Preparation: It is often useful to calculate **certainty equivalent** (CE) of any prospect: sure outcome equally preferred. If the OPT value of a prospect is OPT, then

$$U(CE) = OPT.$$

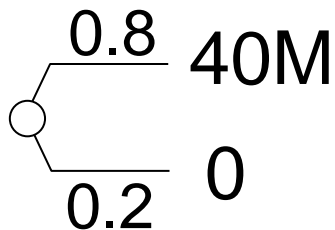
So:

$$CE = U^{-1}(OPT).$$



10M

>



$$\text{OPT} = U(10M) = 28.35;$$

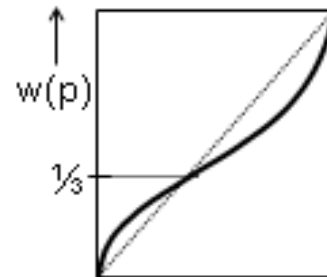
$$\text{CE} = U^{-1}(28.35) = 10M$$

>

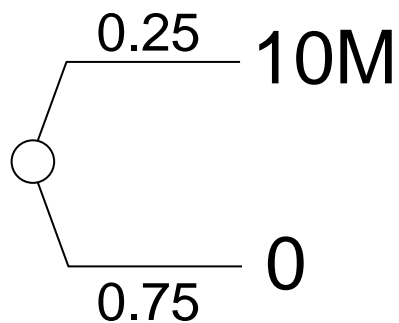
$$\text{OPT} = w(0.8)U(40M) =$$

$$0.61 * 37.79 = 22.96;$$

$$\text{CE} = U^{-1}(22.96) = 3.62M$$



Strong
pessimism



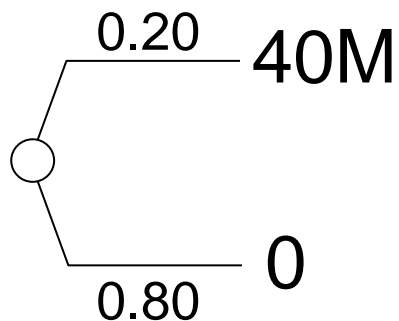
<

$$\text{OPT} = w(0.25)U(10M) =$$

$$0.29 * 28.35 = 8.24;$$

$$\text{CE} = U^{-1}(8.24) = 0.026M$$

<



$$\text{OPT} = w(0.20)U(40M) =$$

$$0.26 * 37.79 = 9.86;$$

$$\text{CE} = U^{-1}(9.86) = 0.061M$$



OPT had some **theoretical problems**.

The absence of any behavioral foundation could have signaled it.

OPT cannot handle many outcomes well—too much overweighting there.

Main problem: OPT was discovered to violate **stochastic dominance** (increasing an outcome improves the prospect). No wonder that no behavioral foundation could be found! Problems were fixed later; we will see later.

Now first comes the first well-known (albeit not the first; see later) nonEU risk theory with a behavioral foundation: weighted utility (Chew 1983).

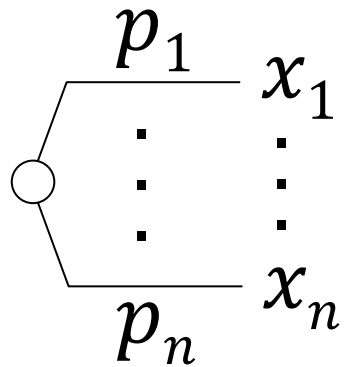
Chapter 4. Behavioral theories for decision under risk



- 4.1. Reaction to the 2nd problem (§2.2); a behavioral theory for risk: prospect theory
- 4.2. Chew's weighted utility
- 4.3. Gul's disappointment aversion
- 4.4. Quiggin's (1982) RDU and Tversky & Kahneman's (1992) PT
- 4.5. Biseparable utility as a version of PT
- 4.6. Neo-additive utility as a version of PT
- 4.7. Afterword & moderating views



Weighted utility:



A diagram of a lottery wheel on the left, with a circle on the left side and lines extending to outcomes x_1 through x_n on the right. Probabilities p_1 through p_n are written above and below the outcomes respectively. An arrow points from the wheel to the following equation:

$$\frac{\sum p_j f(x_j) U(x_j)}{\sum p_j f(x_j)}$$

Without any f (or, f constant) it is just EU.

But now $f(x_j) > 0$ comes in:

x_j with high f values get extra weight.

One should normalize weights: hence the denominator.

Most common case:

pessimism: low outcomes are overweighted; i.e., f is decreasing.



Weighted utility/pessimism is intuitive.
Can accommodate Allais' paradox:
e.g., if much attention for the bad outcome 0.
So: $f(0)$ is big.

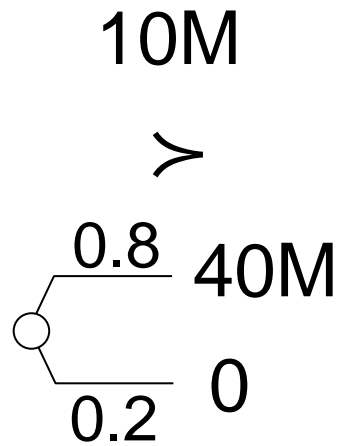
An extreme case to illustrate:

$f(0) = 100, f = 1$ elsewhere.

So, $f(10M) = f(40M) = 1$.

Say U is the identity ($U(\alpha) = \alpha$).

Write WU (weighted utility) for the theory.

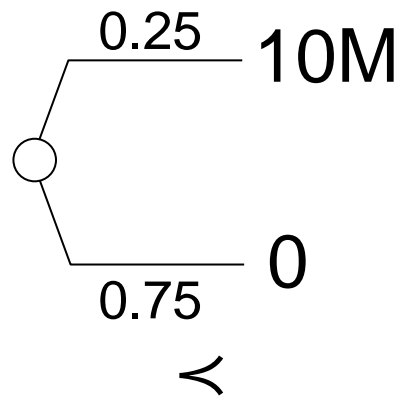


$$WU = U(10M) = 10M;$$

$$CE = U^{-1}(10M) = 10M$$

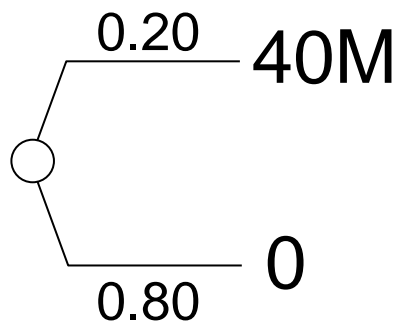
$$WU = \frac{0.8 \times 1 \times 40M + 0.2 \times 100 \times 0}{0.8 \times 1 + 0.2 \times 100} = 1.54M;$$

$$CE = U^{-1}(1.54M) = 1.54M;$$



$$WU = \frac{0.25 \times 1 \times 10M + 0.75 \times 100 \times 0}{0.25 \times 1 + 0.75 \times 100} = 0.033M;$$

$$CE = U^{-1}(0.033M) = 0.033M;$$



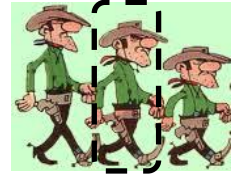
$$WU = \frac{0.20 \times 1 \times 40M + 0.80 \times 100 \times 0}{0.20 \times 1 + 0.80 \times 100} = 0.1M;$$

$$CE = U^{-1}(0.1M) = 0.1M;$$

Strong 0-aversion



Main preference condition in behavioral foundation of WU is betweenness:



for all prospects x, y , $0 \leq \lambda \leq 1$:

$$x \succcurlyeq y \implies x \succcurlyeq \lambda x + (1 - \lambda)y \succcurlyeq y.$$

Is intuitive!

Weakens independence.

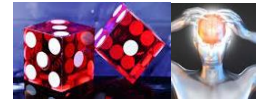
Further, another condition (“weak substitution”) was used, not defined here.

Behavioral foundation: the two conditions hold if and only if WU holds.

This start of WU was better than of OPT.
WU could satisfy stochastic dominance.
But, only in limited domains; not if U unbounded.

However:

- WU did not fit data as well as OPT.
- WU cannot explain co-existence of gambling and insurance.
- $f(x)$ is less psychological than $w(p)$.
(Risk attitude should refer to (feelings about) probabilities!)



Hence, WU did not become very popular.

WU models pessimism in an appealing manner.

WU deserves further attention.

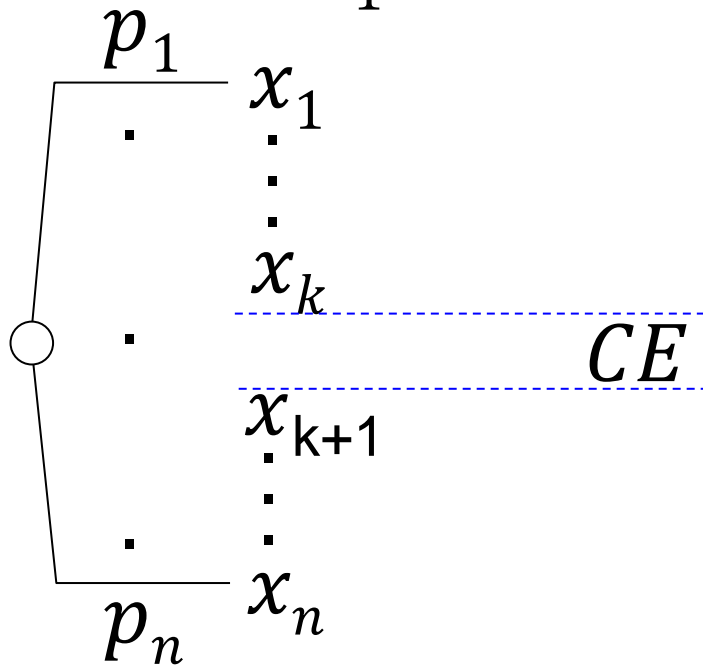
Chapter 4. Behavioral theories for decision under risk



- 4.1. Reaction to the 2nd problem (§2.2); a behavioral theory for risk: prospect theory
- 4.2. Chew's weighted utility
- 4.3. Gul's disappointment aversion
- 4.4. Quiggin's (1982) RDU and Tversky & Kahneman's (1992) PT
- 4.5. Biseparable utility as a version of PT
- 4.6. Neo-additive utility as a version of PT
- 4.7. Afterword & moderating views



Assume $x_1 \geq \dots \geq x_k \geq CE \geq x_{k+1} \geq \dots \geq x_n$



$x_1 \dots x_k$: **elation.** $x_{k+1} \dots x_n$: **disappointment**

Disappointment aversion (DA) theory =

$$\sum_{i=1}^k p_i U(x_i) + \sum_{j=k+1}^n (1 + \beta) p_j U(x_j)$$

$$\sum_{i=1}^k p_i + \sum_{j=k+1}^n (1 + \beta) p_j$$

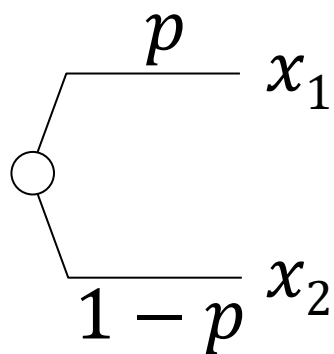
With no β it is EU.
 Have to normalize weights.
 But now, with $\beta > 0$,
 disappointment is
 overweighted.



DA theory satisfies betweenness, as did Chew's weighted utility (WU). Gul used one more complex condition to provide a **behavioral foundation**, as did Chew-not given here.

- Unlike WU, DA satisfies stochastic dominance throughout.
- **Big pro:** DA has only one parameter (β) more than EU, so is very tractable.
- Disappointment aversion is intuitive.
- **Big drawback:** definition is implicit.
To know CE, should know DA functional.
But to know DA functional, should know CE.
Circularity; hard to calculate!

If a prospect has only two outcomes $x_1 \geq x_2$, then we have **explicit expression of DA**. Then only the worst outcome is overweighted, and we need not know CE to do that:

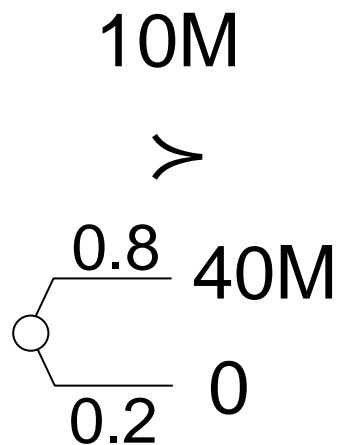


$$\rightarrow \frac{pU(x_1) + (1-p)(1+\beta)U(x_2)}{p + (1-p)(1+\beta)}$$

Some calculations to illustrate, and show that DA can accommodate the Allais paradox. Assume

$$U(\alpha) = \alpha^\theta \text{ with } \theta = 0.5$$

and $\beta = 4$.

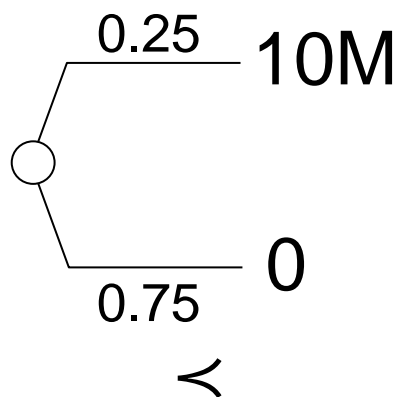


$$DA = U(10M) = 3162;$$

$$CE = U^{-1}(3162) = 10M$$

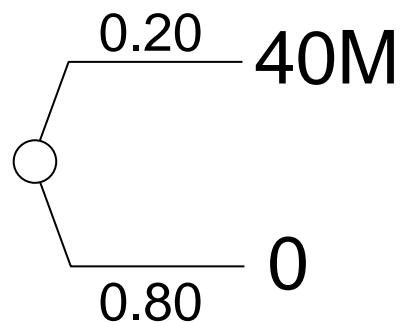
$$DA = \frac{0.8 \times U(40M) + 5 \times 0.2 \times 0}{0.8 + 5 \times 0.2} = 2810;$$

$$CE = U^{-1}(2810) = 7.90M;$$



$$DA = \frac{0.25 \times U(10M) + 5 \times 0.75 \times 0}{0.25 + 5 \times 0.75} = 198;$$

$$CE = U^{-1}(198) = 0.039M;$$



$$DA = \frac{0.20 \times U(40M) + 5 \times 0.80 \times 0}{0.20 + 5 \times 0.8} = 301;$$

$$CE = U^{-1}(301) = 0.090M;$$

Strong 0-aversion



Chapter 4. Behavioral theories for decision under risk



- 4.1. Reaction to the 2nd problem (§2.2); a behavioral theory for risk: prospect theory
- 4.2. Chew's weighted utility
- 4.3. Gul's disappointment aversion
- 4.4. Quiggin's (1982) RDU and Tversky & Kahneman's (1992) PT
- 4.5. Biseparable utility as a version of PT
- 4.6. Neo-additive utility as a version of PT
- 4.7. Afterword & moderating views



Quiggin (1982) solved the problem of OPT of violating stochastic dominance. He invented the right formula to transform probabilities, satisfying stochastic dominance. His theory is called **rank-dependent utility (RDU)**.

He also gave a behavioral foundation (preceding Chew 1983!). But his paper remained unnoticed for several years, until Chew discovered it and propagated its idea.



Tversky & Kahneman (1992) gladly incorporated Quiggin's idea into their new prospect theory, called cumulative prospect theory or, preferably, just prospect theory (PT).

They added views on utility and loss aversion. We will not discuss those.

PT is nowadays the most popular risk theory, almost exclusively used besides (its special case of) EU.

Betweenness theories were mostly discarded because of a poor empirical performance in the so-called probability triangle. But this domain is unfavorable to betweenness (Wakker, Erev, & Weber 1994, p. 196). Betweenness is an appealing property.

Even if PT works well for many subjects, betweenness will work well for others. Further, RDU and PT have their own problems. Better theories remain to be developed, probably combining the preceding theories.

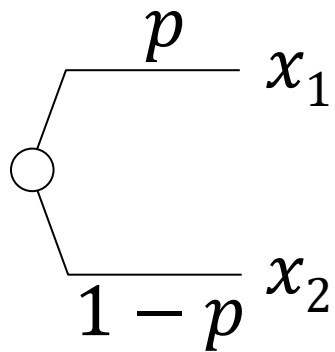
We will not consider RDU and PT in full generality (done in my field course Risk & Rationality). We only consider two simple and important special cases.

Chapter 4. Behavioral theories for decision under risk



- 4.1. Reaction to the 2nd problem (§2.2); a behavioral theory for risk: prospect theory
- 4.2. Chew's weighted utility
- 4.3. Gul's disappointment aversion
- 4.4. Quiggin's (1982) RDU and Tversky & Kahneman's (1992) PT
- 4.5. Biseparable utility as a version of PT
- 4.6. Neo-additive utility as a version of PT
- 4.7. Afterword & moderating views

Biseparable utility (Miyamoto 1988; Ghirardato & Marinacci 2001) makes assumptions only for two-outcome prospects. Assume $x_1 \geq x_2 \geq 0$.



$$\rightarrow w(p)U(x_1) + (1 - w(p))U(x_2)$$

Again, U is utility and $w: [0,1] \rightarrow [0,1]$ is probability weighting. Now the ranking of outcomes matters, with the worst outcome weighted differently than the best outcome. **Importantly, this theory satisfies stochastic dominance.**

Biseparable utility imposes no restrictions on prospects with three or more outcomes (except the obvious weak ordering and stochastic dominance).



Biseparable utility still agrees with OPT for prospects with only one nonzero outcome. Hence, the explanation of Allais' paradox and calculations given before for OPT hold equally well here. Biseparable utility can accommodate these phenomena just like OPT does. So, the calculations are not repeated here.



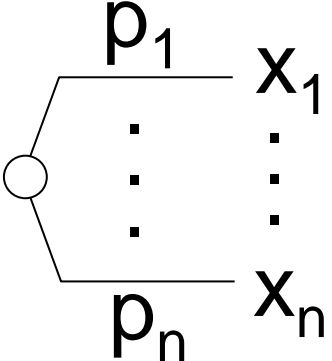
Chapter 4. Behavioral theories for decision under risk



- 4.1. Reaction to the 2nd problem (§2.2); a behavioral theory for risk: prospect theory
- 4.2. Chew's weighted utility
- 4.3. Gul's disappointment aversion
- 4.4. Quiggin's (1982) RDU and Tversky & Kahneman's (1992) PT
- 4.5. Biseparable utility as a version of PT
- 4.6. Neo-additive utility as a version of PT
- 4.7. Afterword & moderating views

Now another useful special case of PT: neo-additive utility. Captures main ideas of prospect theory (except loss aversion). And is simple!

Expected utility EU (with $p_j > 0$):



A diagram of a lottery is shown on the left. It consists of a small circle on the left, with two lines extending from it to the right. The top line is labeled p_1 above it and x_1 to its right. Between the top and bottom lines, there are three vertical dots. The bottom line is labeled p_n below it and x_n to its right. An arrow points from this diagram to the right, where the expected utility formula is written: $p_1 U(x_1) + \dots + p_n U(x_n)$. The terms $U(x_1)$ and $U(x_n)$ are highlighted in blue.

$$p_1 x_1 \begin{matrix} \vdots \\ \vdots \\ \vdots \end{matrix} p_n x_n \rightarrow p_1 U(x_1) + \dots + p_n U(x_n)$$

neo-additive utility (NAU):

give extra weight to best and worst outcomes.

Convex combination of
EU & maximal & minimal utility.

neo-additive utility (NAU) is

$$\sigma \times U(\min) + \tau \times U(\max) + (1 - \sigma - \tau) EU$$

where:

$\sigma \geq 0$ = pessimism index;

$\tau \geq 0$ = optimism index.

min = worst outcome “possible.”

max = best outcome “possible.”

$$\sigma + \tau \leq 1.$$

The meaning of “possible” depends on the context.

Here: with positive probability.

(Later, in behavioral game theory, modified to fit game theory.)





Is psychologically convincing. People do pay special attention to extreme outcomes, with hope and fear.

Gambling is driven by hope, and insurance by fear.

Also, NAU is often used under “ambiguity” (uncertainty about probabilities). Especially σ is then used, as safeguard against probabilities worse than thought.

Common values:

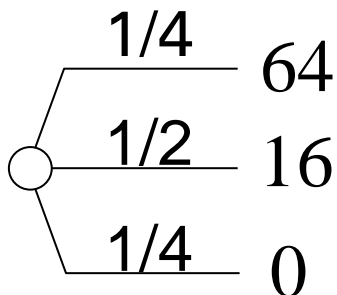
$$\sigma = 0.2.$$

$$\tau = 0.1.$$



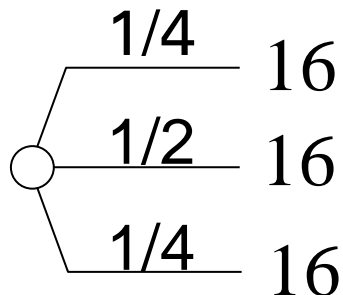
EXAMPLE

Risky



vs.

Safe



Which is preferred? Always assume that

$$U(\alpha) = \sqrt{\alpha}.$$

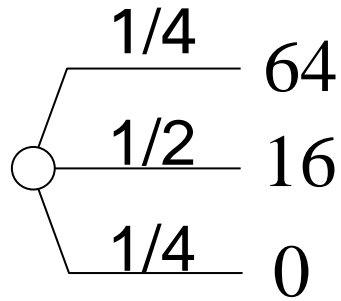
(1) Under expected utility

(2) Under NAU with $\sigma = 0.2$, $\tau = 0.1$.

You can now calculate.

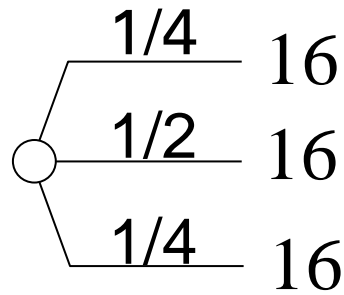


Risky



vs.

Safe



$$U(\alpha) = \sqrt{\alpha}.$$

Answers:

$$EU(\text{Risky}) = \frac{1}{4} * 8 + \frac{1}{2} * 4 + \frac{1}{4} * 0 = 4.$$

$EU(\text{Safe}) = 4$. Under EU they are indifferent.

NAU of Risky =

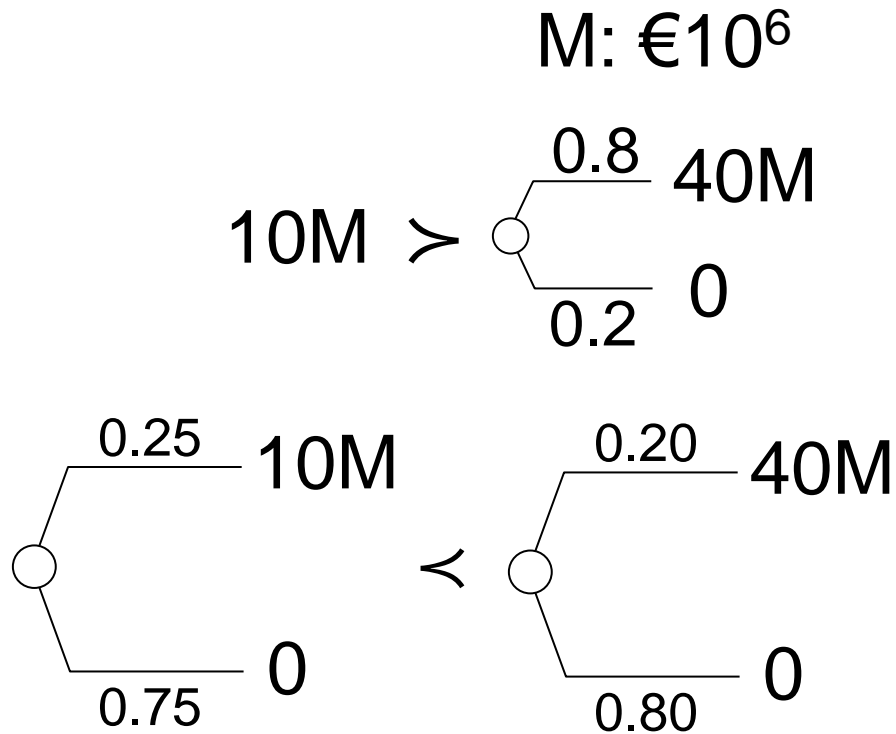
$$0.2 * 0 + 0.1 * 8 + 0.7 * 4 = 3.6.$$

NAU of Safe =

$$0.2 * 4 + 0.1 * 4 + 0.7 * 4 = 4. \text{ Now Safe is preferred.}$$



NAU can explain Allais paradox



Say $U(0) = 0$, $U(40M) = 1$, $U(10M) = 0.8$.

Under EU we'd have indifferences.

NAU with $\sigma = 0.2$, $\tau = 0.1$ works.

Calculations can show:

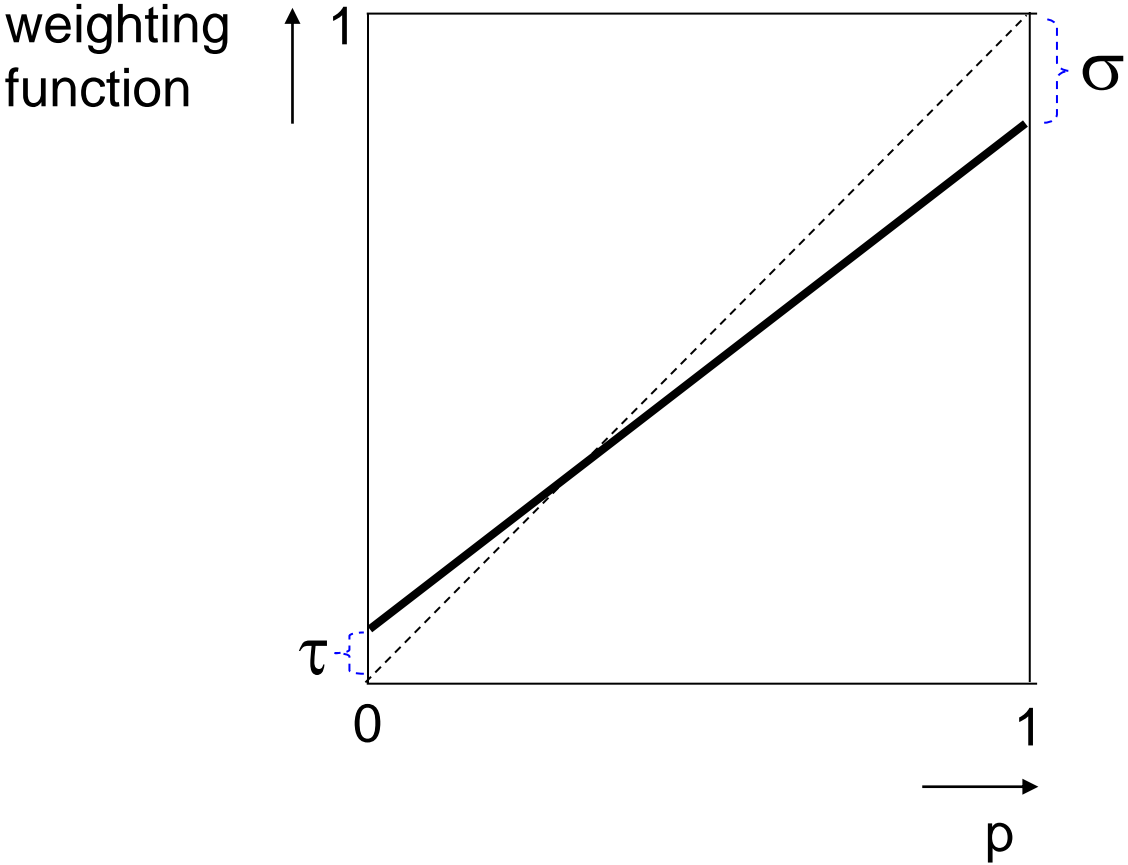
in lower choice max 40M tips the balance to the right;

in upper choice min 0 tips the balance against right.

Allais cleverly understood and used this psychology!



For two-outcome prospects, NAU is biseparable utility with weighting function:



Chapter 4. Behavioral theories for decision under risk



- 4.1. Reaction to the 2nd problem (§2.2); a behavioral theory for risk: prospect theory
- 4.2. Chew's weighted utility
- 4.3. Gul's disappointment aversion
- 4.4. Quiggin's (1982) RDU and Tversky & Kahneman's (1992) PT
- 4.5. Biseparable utility as a version of PT
- 4.6. Neo-additive utility as a version of PT
- 4.7. Afterword & moderating views

All models can accommodate the Allais paradox and the main violations of expected utility.

Prospect theory is currently most popular, but still has its problems. It clearly is not the ultimate theory.

Developing a better theory, solving the problems of PT, will be a Nobel-prize advance!



A final word on reception/dissemination in the field:

Counterreactions to initial claims by Kahneman & Tversky & behavioral stream came in 1980s from:

- experimental economists
- psychologists, primarily Gigerenzer.



Kahneman & Tversky overemphasized irrationality of choice.

Experimental economists: subjects in experiments often were just misunderstanding. Good incentives & learning reduce irrationalities.

Gigerenzer: heuristics often work well.

Truth is in the middle: irrationalities are not as pronounced as originally suggested, but they still are important.

Chapter 5



Behavioral theories for intertemporal choice

Outline of Ch. 5



- 5.1. Decreasing impatience
- 5.2. Hyperbolic discounting
- 5.3. Quasi-hyperbolic discounting
- 5.4. Unit invariance discounting
- 5.5. Discussion

We only considers single nonzero outcomes,
dated outcomes.



Notation. $(t: \alpha)$: receive outcome α at timepoint t ;
nothing (status quo) elsewhere.

The, problematic, separability play no role here.
We focus on (non)constant discounting &
(non)stationarity.

Assumed evaluation: $(t: \alpha) \rightarrow D(t)U(\alpha)$.

D is discount function; U is utility function.

$U(0) = 0$; U is continuous and strictly increasing.

D is continuous and nonincreasing (impatience);
usually, $D(0) = 1$.

(Classical) constant discounting: $D(t) = \delta^t$;

i.e., $D(t) = e^{-rt}$ (for $e^{-r} = \delta$).

Implies stationarity (homework).

By impatience, $\delta \leq 1$.



Stationarity is also called **constant impatience**:

$$(0: \sigma) \sim (\ell, \lambda) \Rightarrow (d: \sigma) \sim (d + \ell: \lambda).$$



In words:

if

for a small outcome σ -now

you are now willing to wait a time ℓ for a
larger outcome λ ,

then

you are now just as (im)patient if all outcomes are
delayed by d .

We will generalize a bit. But first notation:

Notation to help memory:

s : soon time (was time 0 above);

ℓ : late time;

$s < \ell$;

σ : small outcome (coming “soon”);

λ : large outcome (coming “late”).

$\lambda > \sigma$;

$d > 0$ (delay);

Constant **impatience/stationarity** (now s general)

$$(s: \sigma) \sim (\ell, \lambda) \Rightarrow (d + s: \sigma) \sim (d + \ell: \lambda).$$

Empirically prevailing (and violating it):

decreasing impatience:

$$(s: \sigma) \sim (\ell, \lambda) \Rightarrow (d + s: \sigma) < (d + \ell: \lambda).$$

Later, after delay d , people are willing to wait more.

They are especially impatient at present ($s = 0$).

We saw before:

$$(Now: 100) > (4 \text{ months}: 105) \ \&$$

$$(4 \text{ years}: 100) < (4 \text{ years} + 4 \text{ months}: 105).$$

That is, with month as time unit:

$$(0: 100) > (4: 105) \ \& \ (48: 100) < (52: 105).$$

To formally get decreasing impatience: take $\varepsilon > 0$ with

$$(0: 100 - \varepsilon) \sim (4: 105) \ \& \ (48: 100 - \varepsilon) < (52: 105):$$

decreasing impatience, found “indirectly”!

Another indirect way to observe decreasing impatience:

$$(0: \sigma) \sim (\ell, \lambda) \ \& \ (d: \sigma) \sim (d + \ell + \varepsilon: \lambda)$$

for some $\varepsilon > 0$, under strict impatience.



Chapter 5. Behavioral theories for intertemporal choice



- 5.1. Decreasing impatience
- 5.2. Hyperbolic discounting
- 5.3. Quasi-hyperbolic discounting
- 5.4. Unit invariance discounting
- 5.5. Discussion

Assume (k to be specified)

$$(0: \sigma) \sim (\ell, \lambda) \ \& \ (d: \sigma) \sim (kd + \ell: \lambda).$$

Decreasing impatience: $k > 1$.

Assume special pattern of decreasing impatience:

(with ℓ fixed) not only for d , but for **all** d'

$$(0: \sigma) \sim (\ell, \lambda) \ \& \ (d': \sigma) \sim (kd' + \ell: \lambda)$$

with always that same $k > 1$.

i.e., extra time kd' from ℓ on

is exchanged against extra present time d'

at a constant exchange rate $k > 1$.

Suggests special **misperception** of time,

linear:

clock for delays from ℓ on

runs at different speed, by k , than

for delays at present.



Prelec & Loewenstein (1992) showed:
preceding form of decreasing impatience holds iff:



$$D(t) = \frac{1}{(1 + at)^{b/a}}$$

for some $a \geq 0, b > 0$.

For $a = 0$: $D(t)$ is to be taken as e^{-bt} (constant discounting).

In general,

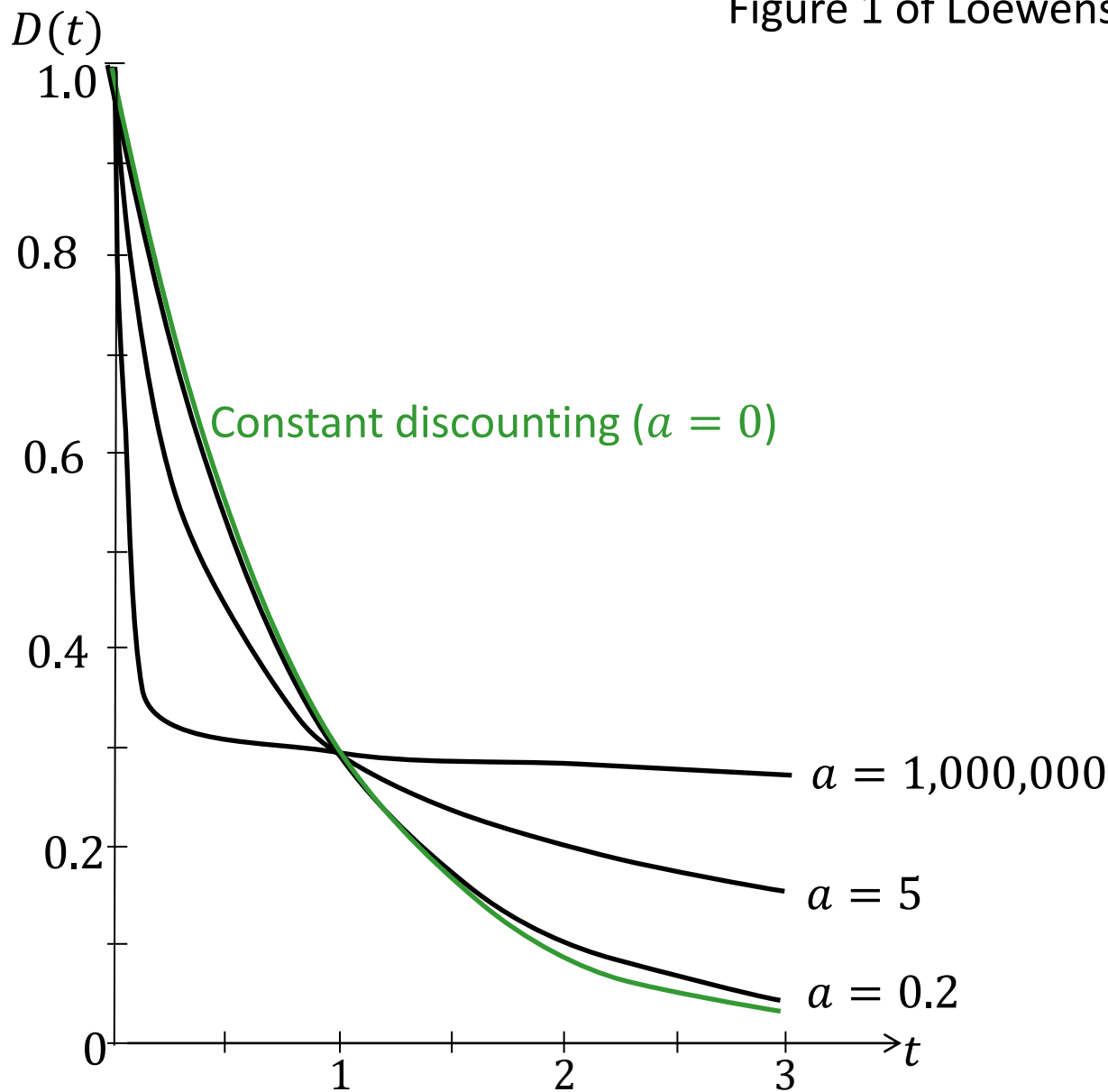
- the higher b the more impatience (more discounting), &
- the higher a the more **decrease in** impatience (more nonstationarity).

$D(t)$: **generalized hyperbolic discounting.**

(Special case of $a = b$ is **hyperbolic discounting**, by Herrnstein (1981).)

In the literature, “hyperbolic discounting” is often used informally for any kind of nonconstant discounting.

Figure 1 of Loewenstein & Prelec (1992)



b was always chosen so as to have $D(1) = 0.3$.

Now come calculations to illustrate
generalized hyperbolic discounting;
accommodating decreasing impatience.

DU: discounted utility; PV: **present value** ($= U^{-1}(DU)$).

We take, besides $D(t) = \frac{1}{(1+at)^{b/a}}$:

$U(\alpha) = \sqrt{\alpha}$, $a = 0.05$, and $b = 0.01$.

(0:100) $DU = U(100) = 10;$
 $PV = U^{-1}(10) = 100.$

>

(4:105) $DU = D(4)U(105) = 0.96 * 10.25 = 9.88;$
 $PV = U^{-1}(9.88) = 97.62.$

(48:100) $DU = D(48)U(100) = 0.78 * 10 = 7.83;$
 $PV = U^{-1}(7.83) = 61.29.$

<

(52:105) $DU = D(52)U(105) = 0.77 * 10.25 = 7.93;$
 $PV = U^{-1}(7.93) = 62.90.$

Strong impatience due to presence effect

Chapter 5. Behavioral theories for intertemporal choice



- 5.1. Decreasing impatience
- 5.2. Hyperbolic discounting
- 5.3. Quasi-hyperbolic discounting
- 5.4. Unit invariance discounting
- 5.5. Discussion

Violations of constant discounting are by far the strongest if the present (time $t = 0$; in preceding examples, $s = 0$) is involved. This is called the **immediacy bias** or **present bias**.



Hence, pragmatic/simple to assume only such violations, with constant discounting at all times $t > 0$:

quasi-hyperbolic discounting,
also known as the **β - δ model**.

Introduced by Phelps & Pollak (1968);
popularized in economics by Laibson (1997) .



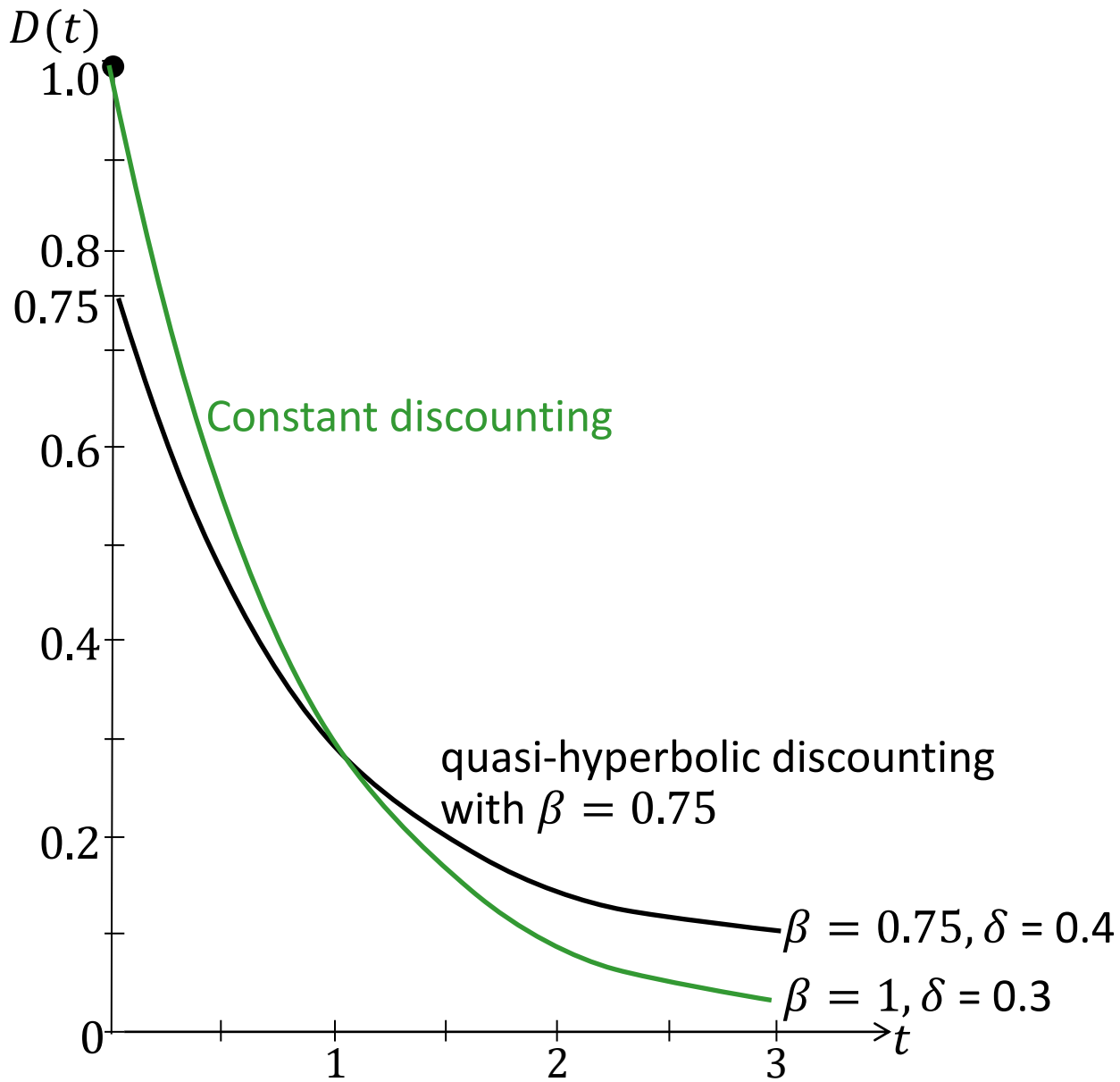
$D(t) = 1 (= \delta^t)$ at $t = 0$;

$D(t) = \beta\delta^t$ at $t > 0$.

Here $0 \leq \beta \leq 1$ and $0 \leq \delta \leq 1$.

Everything except the present gets punished by an extra weight β .

Decisions not involving the present all have the same constant β , which can be dropped then, still giving constant impatience there.



To show that quasi-hyperbolic discounting can accommodate decreasing impatience, we present calculations.

DU : discounted utility; PV : present value = $U^{-1}(DU)$.

We take $U(\alpha) = \sqrt{\alpha}$; $\delta = 0.995$ (per month; = 0.94 annually); $\beta = 0.75$.

$$(0:100) \quad \begin{aligned} DU &= U(100) = 10; \\ PV &= U^{-1}(10) = 100. \end{aligned}$$

$>$

$$(4:105) \quad \begin{aligned} DU &= D(4)U(105) = 0.74 * 10.25 = 7.53; \\ PV &= U^{-1}(7.53) = 56.74. \end{aligned}$$

$$(48:100) \quad \begin{aligned} DU &= D(48)U(100) = 0.59 * 10 = 5.90; \\ PV &= U^{-1}(5.90) = 34.76. \end{aligned}$$

$<$

$$(52:105) \quad \begin{aligned} DU &= D(52)U(105) = 0.58 * 10.25 = 5.92; \\ PV &= U^{-1}(5.92) = 35.07. \end{aligned}$$

Strong impatience due to presence effect

Quasi-hyperbolic is pragmatic and tractable. Is the analog of neo-additive utility for risk.

Chapter 5. Behavioral theories for intertemporal choice



- 5.1. Decreasing impatience
- 5.2. Hyperbolic discounting
- 5.3. Quasi-hyperbolic discounting
- 5.4. Unit invariance discounting
- 5.5. Discussion

Assume

$$(0: \sigma) \sim (\ell, \lambda) \text{ \& } (d: \sigma) \sim (d': \lambda).$$

Constant impatience: $d' = d + \ell$;

decreasing impatience: $d' > d + \ell$.

We do not commit to decreasing impatience in this section; allowing increasing: $d' < d + \ell$.

Unit invariance:

if we change time unit by factor $\mu > 0$, and then modify outcome λ (to λ') to get back the first indifference (so that we can again inspect from d' to what extent impatience is decreasing or increasing),

then should get back same result in sense that:

$$(\mu 0: \sigma) \sim (\mu \ell, \lambda') \Rightarrow (\mu d: \sigma) \sim (\mu d': \lambda').$$

Say we took years iso months. Then $\mu = 12$.

Here people perceive time proportionally.

Attitude towards months is as towards years.

Ebert & Prelec (2007): unit invariance holds iff



$$D(t) = e^{-(rt)^\delta}$$

for some $r \geq 0, \delta > 0$.

Nowadays called **unit invariance family**.

The bigger r , the more impatience;
the smaller δ , the more *decrease* in impatience.

Is like constant discounting but time is perceived nonlinearly, through power function.

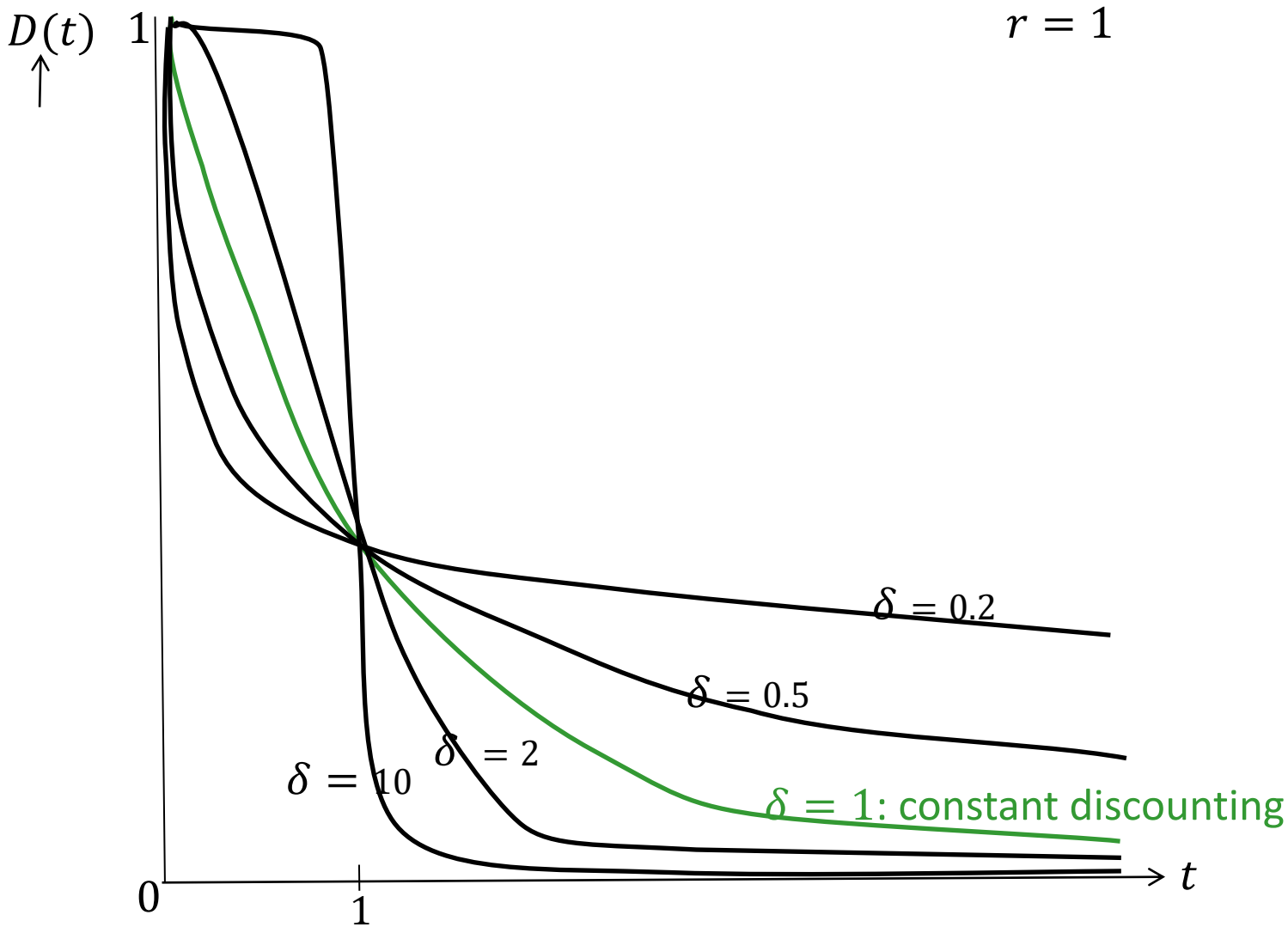


$\delta < 1$: time is transformed concavely \Rightarrow decreasing impatience.

$\delta > 1$: time is transformed convexly \Rightarrow increasing impatience!



Bleichrodt, Rohde, & Wakker (2009) argued for adding $\delta \leq 0$. We will not analyze this case.



Note that $\delta > 1$ is allowed, giving increasing impatience.

We give calculations.

DU : discounted utility; PV : present value = $U^{-1}(DU)$.

We accommodate decreasing impatience, taking

$U(\alpha) = \sqrt{\alpha}$, $r = 0.003$, and $\delta = 0.5$.

$$(0:100) \quad \begin{aligned} DU &= U(100) = 10; \\ PV &= U^{-1}(10) = 100. \end{aligned}$$

>

$$(4:105) \quad \begin{aligned} DU &= D(4)U(105) = 0.90 * 10.25 = 9.18; \\ PV &= U^{-1}(9.18) = 84.34. \end{aligned}$$

$$(48:100) \quad \begin{aligned} DU &= D(48)U(100) = 0.68 * 10 = 6.84; \\ PV &= U^{-1}(6.84) = 46.82. \end{aligned}$$

<

$$(52:105) \quad \begin{aligned} DU &= D(52)U(105) = 0.67 * 10.25 = 6.90; \\ PV &= U^{-1}(6.90) = 47.66. \end{aligned}$$

Strong impatience due
to presence effect

Chapter 5. Behavioral theories for intertemporal choice



- 5.1. Decreasing impatience
- 5.2. Hyperbolic discounting
- 5.3. Quasi-hyperbolic discounting
- 5.4. Unit invariance discounting
- 5.5. Discussion

All models can accommodate the empirically prevailing decreasing impatience, strongest at $t = 0$ (immediacy effect).

Quasi-hyperbolic is most popular today. But too crude for refined analyses.

Generalized hyperbolic family is more refined. But does not fit data very well. Has some analytical drawbacks.

Drawback of both quasi- and generalized hyperbolic: **can only accommodate decreasing impatience**. Even if this is the prevailing empirical finding, there is interest in increasing impatience:

- (1) In every study a considerable subset of individuals will exhibit increasing impatience, so it is needed for fitting at the individual level.
- (2) Several studies even find prevailing increasing impatience.
- (3) Increasing impatience is not counterintuitive: at first waiting may not be so bad, but after a wait the extra wait may become more annoying.

Hence the interest in unit invariance. But, is new and needs more study. **In fact, this field is “underdeveloped.”** Not many good families yet.

Better families to be invented by young researchers.

Chapter 6

Behavioral theories for welfare



Outline of Ch. 6

6.1. Fehr-Schmidt inequality aversion



Fehr-Schmidt (1999) welfare model with inequality aversion



Assume a welfare allocation (x_1, \dots, x_n) over n agents.

Welfare V_1 of agent 1 is:

$$x_1 - \sum_{j: x_j > x_1} b_1 (x_j - x_1) - \sum_{j: x_j < x_1} a_1 (x_1 - x_j)$$

with $b_1 \geq 0$, $a_1 \geq 0$.

Every $x_i \neq x_j$ decreases welfare, through inequality aversion.

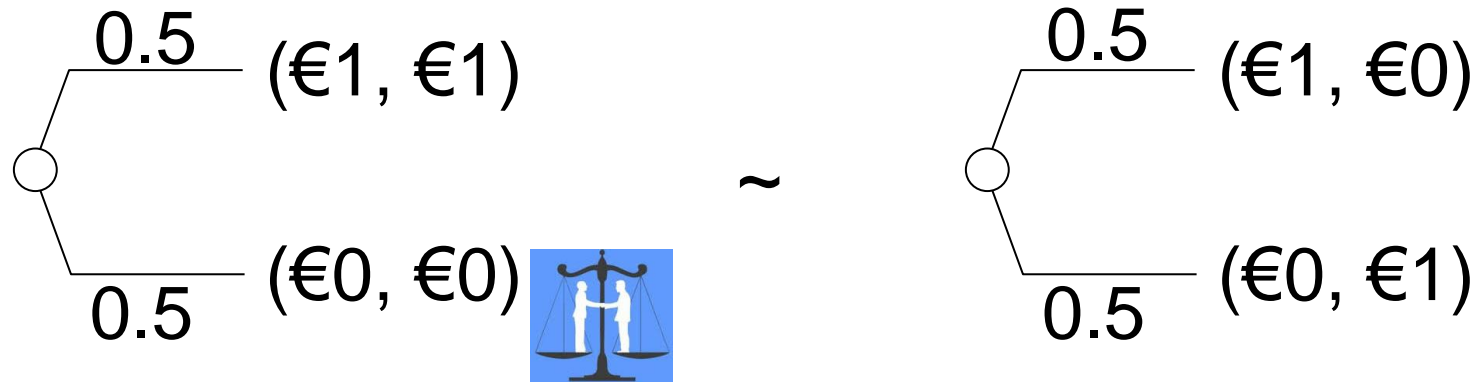
b_1 : aversion to being Behind (concerns j richer than 1)

a_1 : aversion to being Ahead (concerns j poorer than 1).

a - and b -parameters are subjective.

Mostly, $\frac{1}{n-1} \geq b_1 \geq a_1$ (impact of ineq. av. not too big)

Harsanyi-utilitarianism's problematic prediction, taking utility linear:



Fehr-Schmidt sheds new light on it.

Predicts preference for left:

(1, 1) has value 1 for agent 1.

(0, 0) has value 0 for agent 1.

Expected “utility” is 0.5 in left situation.

(1, 0) has value $1 - a_1$ for agent 1.

(0, 1) has value $-b_1$ for agent 1.

Expected utility is $0.5 - b_1/2 - a_1/2$ in right situation.

Agent 1, and agent 2 likewise, prefer left situation. So does social planner.

Debatable: different interpretations of utility. We let it be

There are many models on other-regarding preferences. Extend and deviate from Fehr-Schmidt.

More applications of Fehr-Schmidt will come later in this meeting, in game theory.

Chapter 7

Breakaways from revealed preference



Outline of Ch. 7



7.1. Kahneman's experienced utility

7.2. Happiness studies



Kahneman's experienced utility:

Many anomalies of revealed preference, especially regarding perception of time duration.

⇒ use **introspective** utility then.



Only ask for experienced utility exactly when it is experienced, to avoid memory transformations.

Fundamental breakaway from ordinalism.

Quasi-behavioral foundation:

Kahneman, Wakker, & Sarin (1997).

Chapter 7. Breakaways from revealed preference






7.1. Kahneman's experienced utility

7.2. Happiness studies



Many **economic** studies today measure introspective happiness. Derive much from it.

TI has a reputation here:

- van Praag, Bernard M.S. (1968) “*Individual Welfare Functions and Consumer Behavior.*” North-Holland, Amsterdam. 
- van Praag, Bernard M.S. & Ada Ferrer-i-Carbonell (2004) “*Happiness Quantified.*” Oxford University Press, Oxford. 
- Ruut Veenhoven maintains the *world data base on happiness.* 
- EHERO: Erasmus Happiness Economics Research Organization:
<http://www.eur.nl/english/ehero/>

Part III: Behavioral Applications

Outline of Part 3

Chapter 8. Behavioral applications in game theory



- 8.1. Game theory with neo-additive players
- 8.2. Not choosing best, but choosing better: quantal response equilibrium (a more psychological approach to game theory)
- 8.3. Fairness emotions in game theory (Fehr-Schmidt model)

Chapter 9. Behavioral applications in risky choice



- 9.1. Applying prospect theory to health: improving quality of life measurements
- 9.2. Applying prospect theory to insurance

Chapter 10. Behavioral applications in intertemporal choice and self- control



10.1. Naive and sophisticated nonstationarity

10.2. Elaborated example: procrastination



Chapter 11. Preference reversals and framing

11.1. The evaluability hypothesis

11.2. Context effects

11.3. Endowment effects

11.4. Epilogue of Ch. 11: applications of preceding versions of preference reversals

Chapter 8

Behavioral applications in game theory



Ch. 8



- 8.1. Game theory with neo-additive players
- 8.2. Not choosing best, but choosing better: quantal response equilibrium (a more psychological approach to game theory)
- 8.3. Fairness emotions in game theory (Fehr-Schmidt model)

Neo-additive players explain data better than classical theory. Consider the following game:

	L	R
T	6, 4.01	-9, 4
B	5, 9.01	9, 9

There will only be pure equilibria in what follows.
 What is classical Nash eq. (NE) if players do EU?

You think ...

(T,L). L strictly dominates R.

P.s.: the column player dearly hopes for row B iso T,
 but row player decides.

What would you/a-wise-person do as row player?

You think ...

Column player may be:



What would neo-additive players do?

	L	R
T	6 4.01	-9 4
B	5 9.01	9 9

Both players are neo-additive

(NAU with pessimism $\sigma = 0.2$ & optimism $\tau = 0.1$):

column player 2 still plays strictly dominating L.

Row player? Thinks $P(L) = 1$ (let us assume).

Rule: every strategy is "possible," also if probability 0.

So, R is "possible."

You think ...

Value T = $0.2 * (-9) + 0.1 * 6 + 0.7 * 6 = 3$.

Value B = $0.2 * 5 + 0.1 * 9 + 0.7 * 5 = 5.4$.

Player 1 chooses B!

Eq. is (B,L).

Is empirically plausible.



	L	R
T	6, 4.01	-9, 4
B	5, 9.01	9, 9

The preceding game illustrates a known saying in behavioral game theory:
 a classical game theorist looks only one side when crossing the street.
 A behavioral game theorist looks both sides.
 With car drivers like Homer Simpson (homo sapiens),
 better look both sides.



Now for another game, seen before.

	L (26%)	C (8%)	NN (68%)	R (0%)
T (68%)	200 ⁵⁰	0 ⁴⁵	10 ³⁰	20 ⁻²⁵⁰
B (32%)	0 ⁻²⁵⁰	10 ⁻¹⁰⁰	30 ³⁰	50 ⁴⁰

Was tested by Goeree & Holt (2001, *American Economic Review*), with real incentives.

Percentages indicate % subjects choosing strategies.

There are two pure eq. under EU:

(T,L) and (B,R).

	L (26%)	C (8%)	NN (68%)	R (0%)
T (68%)	200 ⁵⁰	0 ⁴⁵	10 ³⁰	20 ⁻²⁵⁰
B (32%)	0 ⁻²⁵⁰	10 ⁻¹⁰⁰	30 ³⁰	50 ⁴⁰

What are eq. (only pure) under neo-additive utility (NAU) with pessimism $\sigma = 0.2$ and optimism $\tau = 0.1$ for both players?

You think ...

Easy to see: now **NN** strictly better than all others. Value of **NN** = 30.

To wit, its main competitor, **C**, in its most favorable case ($P(\mathbf{T}) = 1$) has lower NAU:

$0.2 \cdot (-100) + 0.1 \cdot 45 + 0.7 \cdot 45 = 16 < 30$. In all other cases, values of competitors of **NN** are much worse. So, $P(\mathbf{NN}) = 1$.

Then value **T** = $0.2 \cdot 0 + 0.1 \cdot 200 + 0.7 \cdot 10 = 27$.

Value **B** = $0.2 \cdot 0 + 0.1 \cdot 50 + 0.7 \cdot 30 = 26$.

Eq. is (**T**, **NN**).

For a pure eq., this is best result given the data.

A success for behavioral game theory!

Weak point in previous analysis, as with all of game theory so far:

Predicts deterministically that with certainty (probability 1) people choose what the theory gets as best (and if randomize then with certainty only among the best).

The best **L** in the first game, and **NN** in second, were chosen with $p = 1$ according to the solution. Not psychologically realistic. With Homer Simpson (homo sapiens) you never know! Data are nondeterministic.

To be fixed next.

Chapter 8. Behavioral applications in game theory



8.1. Game theory with neo-additive players

~~8.2. Not choosing best, but choosing better: quantal response equilibrium (a more psychological approach to game theory)~~

8.3. Fairness emotions in game theory (Fehr-Schmidt model)

In reality, homo sapiens makes mistakes/is insecure. Does choose better strategies more often. But does not choose the best with probability 1. There is randomness. (“Choosing various betters iso best.”)

NE is different. Players only choose best. Randomized strategies exist in NE, but always only over best, and never involving nonbest. $P(\text{best}) = 1$ always. Here we are going to deviate fundamentally. But first a preparation on probabilistic choice.

Preparation: random individual choice (Luce 1959). Choice from objects x_1, \dots, x_n with utilities $U(x_j)$ is proportional to utility.

$$P(x_j) = \frac{U(x_j)}{U(x_1) + \dots + U(x_n)}$$

Called linear choice model. Nice initiating idea, but not really very good. Has problems (e.g. $U < 0$ cannot be). More popular and better is logistic choice model:

$$P(x_j) = \frac{e^{\lambda U(x_j)}}{e^{\lambda U(x_1)} + \dots + e^{\lambda U(x_n)}} ; \lambda \geq 0: \text{parameter.}$$

In both models: people choose “better iso best:” better things are more likely to be chosen.

Further properties of logistic choice model:

$$P(x_j) = \frac{e^{\lambda U(x_j)}}{e^{\lambda U(x_1)} + \dots + e^{\lambda U(x_n)}} \quad . \quad \lambda \geq 0: \text{parameter.}$$

$$\lambda = 0: P(x_j) = \frac{1}{1 + \dots + 1} = 1/n: \text{pure randomness!}$$

Utility does not matter.
Decision maker understands nothing.

We will see: as λ grows, choices tend to the rationality of always choosing the best.

THEOREM. If $\lambda \rightarrow \infty$ and $U(x_j)$ is the unique maximum, then $e^{\lambda U(x_i)} / e^{\lambda U(x_j)} \rightarrow 0$ for each $i \neq j$ and, hence, $P(x_j) \rightarrow 1$ (so, surely choosing best).

Proof: consider $\lambda \rightarrow \infty$. Assume

$U(x_i) - U(x_j) < 0$.

$$\frac{e^{\lambda U(x_i)}}{e^{\lambda U(x_j)}} = e^{\lambda(U(x_i) - U(x_j))} \text{ goes to } 0 \text{ as } \lambda \rightarrow \infty$$

because $U(x_i) - U(x_j) < 0$.

□

Thus, the bigger λ , the more rationality.

$\lambda=0$: total randomness;

$\lambda=\infty$: perfect rationality.

(One application: λ is well suited to study learning!)

Probabilistic individual choice is plausible. **Empirically:** if you ask subjects something again 5 minutes later, then they often change choice: inconsistency.

Some researchers think that it is intrinsic to preference. You can see by **introspection:** if you ask yourself what you prefer, then you often feel uncertain. Probabilistic preference may be intrinsic within yourself.

We will now reconsider **game theory using probabilistic choice.** More psychologically realistic, but, as a price to pay, more difficult mathematically.

Quantal response equilibrium (QRE)

(only for static games)

1. Each player plays each strategy with some probability (\approx randomized)
2. For each player, each strategy has an (expected) utility given the probabilities of opponents' strategy choices
3. For each player, the probability distribution over his own strategies is determined by their expected utilities according to logistic choice or to some other probabilistic choice rule.

Note circularity:

Utilities depend on choice probabilities, and choice probabilities depend on utilities. More precisely, we get circularity in the following principles:

Principle 1: your choice probabilities depend on expected utilities of your strategies

Principle 2: those expected utilities depend on your opponent's choice probabilities

Principle 3: your opponent's choice probabilities depend on expected utilities of his strategies

Principle 4: those expected utilities depend on your choice probabilities.

We have a cycle. Implicit equalities & definitions.

Determining QRE involves solving implicit equations.

Question: do QREs exist?

Answer: always!

Can be hard to compute or analyze ...

QREs were introduced by McKelvey & Palfrey (1995, Games and Economic Behavior).

Example: hide-and-seek

We first use Luce's linear choice model (not empirically plausible, but we do for simplicity).

All payoffs are increased by 3 to avoid negative utility. You are row player:

	Seek-High	Seek-Low
Hide-High	0 ⁶	3 ³
Hide-Low	3 ³	2 ⁴

Can find QRE algebraically:

	q Seek-High	1-q Seek-Low
p Hide-High	0 ⁶	3 ³
1-p Hide-Low	3 ³	2 ⁴

$$EU(\text{HH}) = 3 - 3q \text{ (principle 2);}$$

$$EU(\text{HL}) = 3q + 2(1-q) = 2 + q \text{ (principle 2);}$$

$$p = \frac{EU(\text{HH})}{(EU(\text{HH}) + EU(\text{HL}))} = \frac{(3-3q)}{((3-3q) + (2+q))} = \frac{(3-3q)}{(5-2q)} \text{ (principle 1);}$$

$$EU(\text{SH}) = 6p + 3(1-p) = 3 + 3p \text{ (principle 4);}$$

$$EU(\text{SL}) = 3p + 4(1-p) = 4-p \text{ (principle 4);}$$

$$q = \frac{(3+3p)}{((3+3p) + (4-p))} = \frac{(3+3p)}{(7+2p)} \text{ (principle 3).}$$

THEOREM. Because QRE always exists, the equations above have at least one solution. It is unique here and is:

$$p = 0.36$$

$$q = 0.53.$$

PROOF.

$$p = (3-3q)/(5-2q); q = (3+3p)/(7+2p).$$

$$p = \frac{3 - 3\left(\frac{3+3p}{7+2p}\right)}{5 - 2\left(\frac{3+3p}{7+2p}\right)} = \frac{3(7+2p) - 3(3+3p)}{5(7+2p) - 2(3+3p)} =$$

$$\frac{21 + 6p - 9 - 9p}{35+10p - 6 - 6p} = \frac{12 - 3p}{29 + 4p} ;$$

$p(29+4p) = 12-3p$; $4p^2+29p = 12-3p$;
 $4p^2 + 32p = 12$. Can use abc formula: $p = 0.36$. (I always use different derivation, as follows:)

$$4p^2 + 32p = 12; p^2 + 8p = 3;$$
$$(p+4)^2 - 4^2 = 3; (p+4)^2 = 3 + 16;$$
$$p+4 = \pm\sqrt{3 + 16};$$
$$p+4 = \sqrt{19};$$
$$p = \sqrt{19} - 4 = 0.36;$$

$$p = 0.36!$$

$$\text{Then } q = (3+3p)/(7+2p) = 0.53.$$

This gives QRE. \square

As a double-check, we next verify that the (circular) conditions for QRE are indeed satisfied.

We calculate EUs of strategies using the choice probabilities ($p = 0.36$, $q = 0.53$) found, and then check that the choice probabilities agree with the EUs obtained.

	(q =) 0.53 Seek-High	0.47 Seek-Low
(p =) 0.36 Hide-High	0 ⁶	3 ³
0.64 Hide-Low	3 ³	2 ⁴

We check QRE for $p = 0.36$ & $q = 0.53$:

$$EU(\text{HH}) = 3 - 3q = 1.41;$$

$$EU(\text{HL}) = 2 + q = 2.53;$$

$$p = 1.41 / (1.41 + 2.53) = 0.36; \text{ correct!}$$

$$EU(\text{SH}) = 3 + 3p = 4.08;$$

$$EU(\text{SL}) = 4 - p = 3.64;$$

$$q = 4.08 / (4.08 + 3.64) = 0.53; \text{ correct again!}$$

All utilities and probabilities are mutually consistent, as the circular QRE equations require.

	(q =) 0.53 Seek-High	0.47 Seek-Low
(p =) 0.36 Hide-High	0 ⁶	3 ³
0.64 Hide-Low	3 ³	2 ⁴

Afterthoughts about hide-and-seek game using Luce's linear choice model:

QRE deviates from NE ($p = q = \frac{1}{4}$).

QRE's p and q are higher. Empirically plausible (shift towards the 50-50 randomness of not understanding).

Usually p and q are too high. Still q of QRE quite high.

Luce's linear choice model is good but not very good.

Hide and seek game continued. Now analyzed with the better logistic choice theory. (Then adding 3 to all outcomes does not matter.)

	q Seek-High	1-q Seek-Low
p Hide-High	0 ⁶	3 ³
1-p Hide-Low	3 ³	2 ⁴

Now:

$$EU(\text{HH}) = 3 - 3q \text{ (as we saw before);}$$

$$EU(\text{HL}) = 2 + q \text{ (as we saw before);}$$

$$p = \exp(\lambda(3-3q)) / (\exp(\lambda(3-3q)) + \exp(\lambda(2+q))).$$

$$EU(\text{SH}) = 3 + 3p \text{ (as we saw before);}$$

$$EU(\text{SL}) = 4 - p \text{ (as we saw before);}$$

$$q = \exp(\lambda(3+3p)) / (\exp(\lambda(3+3p)) + \exp(\lambda(4-p))).$$

I did not try algebra. I let the computer calculate numerically.

Gave the following results, for various λ .

Increasing λ : they learn more and more.

λ	p	q
0	0.50	0.50
0.2	0.44	0.54
0.5	0.35	0.55
1	0.26	0.51
2	0.21	0.42
4	0.21	0.33
6.1	0.22	0.30

Total randomness;
here p should go down, and q should go up.

Still p should go down, and q should go up.

Still p should go down, and q should go up.

Still p should go down; q less clear.

Still p should go down; q now also.

Still p should go down; q also.

Still p should go down; q also.

Close to rational NE. My computer couldn't calculate higher exponentials. Fascinating empirical studies into human learning can be done here.



Now for empirical studies into QRE. Some studies found that QRE better fits than “level-k thinking” (not explained here).

We next consider [data from Ochs](#) (1995 Games and Economic Behavior), reanalyzed by McKelvey & Palfrey (1995, Games and Economic Behavior) using QRE.

Ochs carried out three (asymmetric) matching penny games.

		q	1-q
		L	R
p	T	1 ⁰	0 ¹
1-p	B	0 ¹	1 ⁰

Game 1

NE:
 $p = 1/2;$
 $q = 1/2;$

		q	1-q
		L	R
p	T	9 ⁰	0 ¹
1-p	B	0 ¹	1 ⁰

Game 2

NE:
 $p = 1/2;$
 $q = 1/10;$

		q	1-q
		L	R
p	T	5 ⁰	0 ¹
1-p	B	0 ¹	1 ⁰

Game 3

NE:
 $p = 1/2;$
 $q = 1/6;$

Ochs let each subject play one of these games 52 times, to see how subjects learned. We split up into 4 periods, 1-16, 17-32, 33-48, 49-52.



For game 1: everyone and all theories predict $p = q = \frac{1}{2}$. This was found. Not discussed further.

Game 2 (NE: $p=0.5$, $q = 0.1$).

The third column gives actually observed average p and q .

Fifth column gives λ that best fits data.

Fourth column gives QRE's p and q .

TABLE IX
DATA AND ESTIMATES FOR OCHS. GAME 2

Period	n	Actual		Predicted		λ
		p	q	p	q	
1-16	128	0.541	0.326	0.645	0.347	1.951
17-32	128	0.649	0.228	0.645	0.228	3.763
33-48	128	0.578	0.250	0.648	0.241	3.475
48-52	64	0.626	0.200	0.636	0.197	4.638
All	448	0.595	0.258	0.649	0.254	3.241

	q	$1-q$
p	9^0	0^1
$1-p$	0^1	1^0

Discussion: QRE fits data well. $p > 0.5$ (considering own payoff 9) is well-known empirical fact. λ indeed grows (roughly). p close to 0.5 at first because subjects then do not understand the whole game well. q converges to NE (p to come later).

Game 3 (NE: $p=0.5$, $q = 1/6$).

The third column gives actually observed average p and q .

Fifth column gives λ that best fits data.

Fourth column gives QRE's p and q .

TABLE X
DATA AND ESTIMATES FOR OCHS, GAME 3

Period	n	Actual		Predicted		λ
		p	q	p	q	
1-16	128	0.527	0.366	0.615	0.383	1.856
17-32	128	0.573	0.393	0.610	0.405	1.568
33-48	128	0.610	0.302	0.614	0.301	3.306
48-52	128	0.455	0.285	0.500		∞
All	512	0.542	0.336	0.619	0.331	2.656

	q	$1-q$
p	5 ⁰	0 ¹
$1-p$	0 ¹	1 ⁰

As before, QRE fits data well. Considering own payoff 5 gives weaker effect than in Game 2 (where own payoff was 10). λ again grows (roughly), and again convergence to NE.

Conclusion: QRE closer to homo sapiens than NE. Neo-additive too. Psychological improvements. But more difficult maths ... These findings confirm the value of behavioral game theory.

Chapter 8. Behavioral applications in game theory

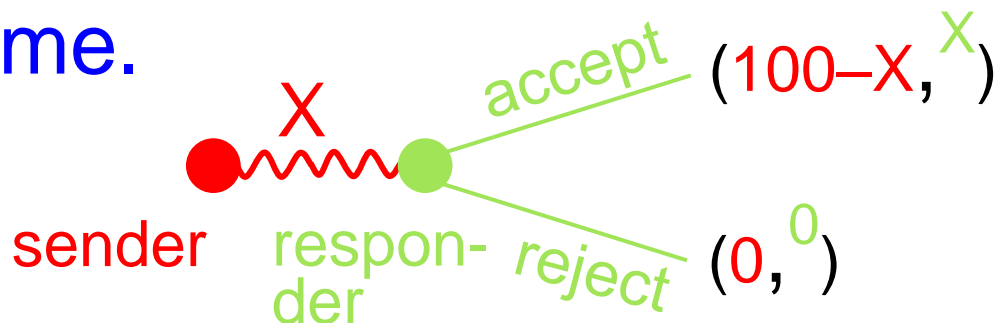


8.1. Game theory with neo-additive players

8.2. Not choosing best, but choosing better: quantal response equilibrium (a more psychological approach to game theory)

8.3. Fairness emotions in game theory (Fehr-Schmidt model)

Fehr-Schmidt (1999) analysis of ultimatum game.



$$0 \leq X \leq 100$$

Clearly $X \leq 50$. Analysis for responder:

accept if $X - b_r * ((100 - X) - X) \geq 0$.

$$X(1+2b_r) \geq b_r 100.$$

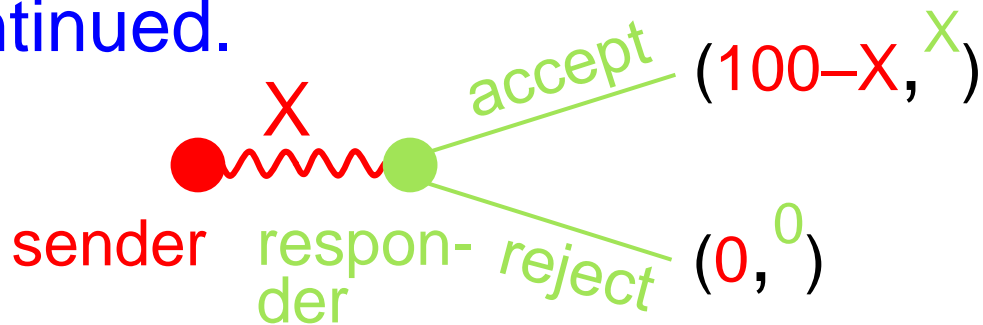
$$X \geq b_r 100 / (1+2b_r).$$

$X=50$ is always accepted.

For $b_r=1$, $X > 33$ is accepted.

$X=10$ is accepted iff $b_r \leq 1/8$. Data: this almost never happens.

Fehr-Schmidt (1999) analysis of ultimatum game;
continued.



$$0 \leq X \leq 100$$

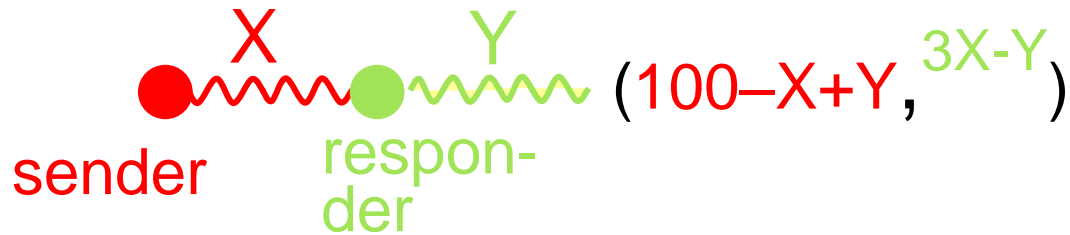
Assume $X \leq 50$. Analysis for sender:

if accept, then welfare is $100 - X - a_p(100 - X) - X = X(2a_p - 1) + 100(1 - a_p)$.

If $a_p > 0.5$, then increasing in X , so the maximal $X = 50$ is optimal (it is surely accepted).

If $a_p < 0.5$, then decreasing in X . So, would do $X = 0$ if X were surely accepted. In reality, X must be bigger to have good chance of acceptance. So, some $0 < X < 50$..

Trust game (no Fehr-Schmidt here)



$$0 \leq X \leq 100$$

$$0 \leq Y \leq 3X$$

What will happen for homo sapiens?

Modal sending: $X=50$.

Modal respons: $Y=X$.

Modal outcome is $(100, 100)$.

Fehr-Schmidt cannot explain responder. Other motives (reciprocity) play a role.

(Given $Y = X$, FS does perfectly well explain $X = 50$.)

There is much beyond the ordinal economic model in game situations ...

Applications:

- Name your price. In some restaurants you can pay what you want. People typically pay fairly.
- In the US, can tip waiters as you like. People tip fairly.

Many other examples.

People care for fairness, equality, can be altruistic, and so on ...

We need behavioral economics for homo sapiens ...

Chapter 9



Behavioral applications in risky choice

Outline of Ch. 9

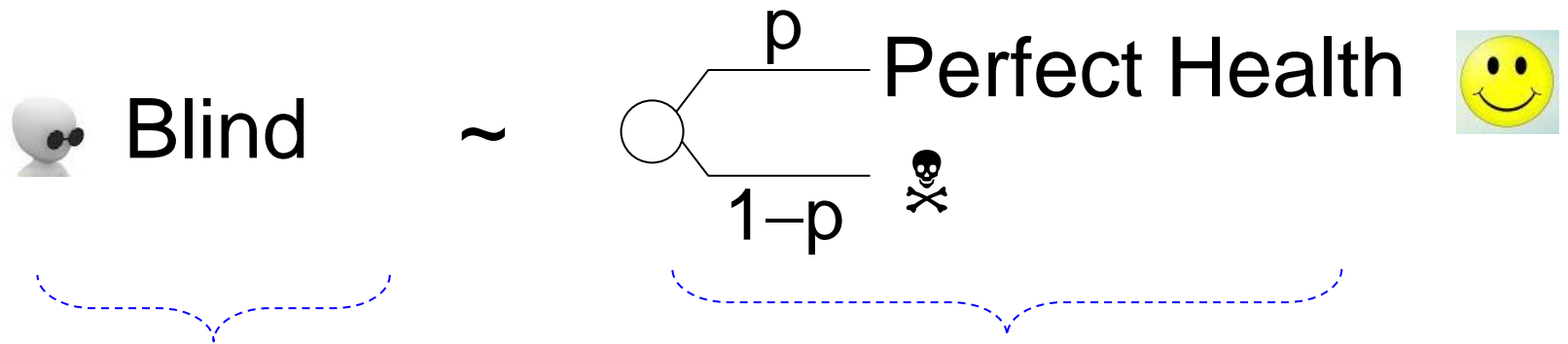


- 9.1. Applying prospect theory to health: improving quality of life measurements
- 9.2. Applying prospect theory to insurance

Standard gamble question to measure
(quality of life =) utility of being blind.

Uses conventional scaling in health:

$U(\text{perfect health}) = 1$; $U(\text{skull}) = 0$. Assume:



Under EU:

$$U(\text{Blind}) = p$$

expected utility =

$$p \times 1 + (1-p) \times 0$$

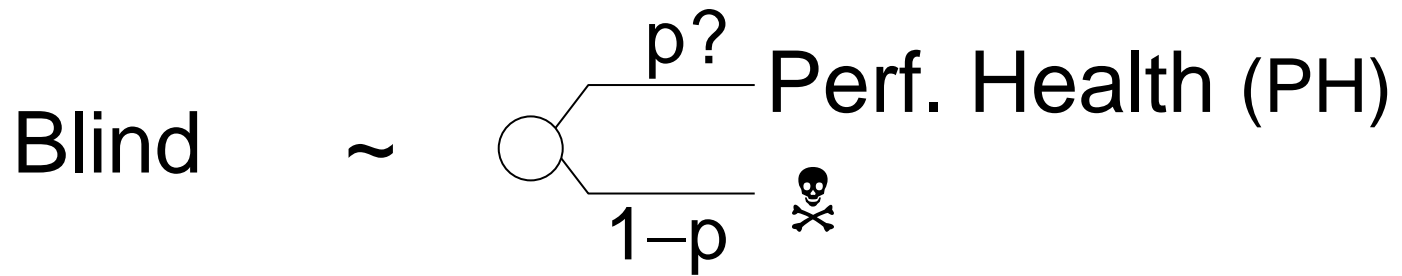
$$= p$$

Common conclusion: $U(\text{blind}) = p$.

Is common measurement method in health.

Based on expected utility; i.e., on homo economicus!?

Analysis using prospect theory



THEOREM:

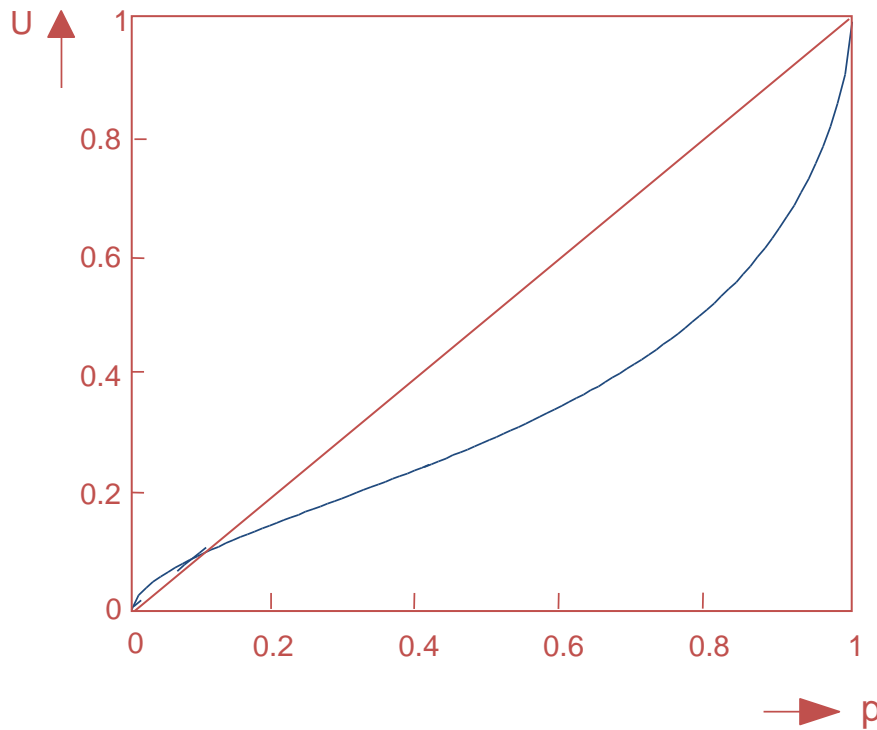
Prospect theory: $U(\text{blind}) = \frac{w(p)}{w(p) + \lambda w(1-p)}$

□

w: probability weighting & λ loss aversion as in Kahneman & Tversky (1979).

Proof: see Bleichrodt, Pinto, & Wakker (2001).

They used common empirical findings about w and λ to obtain the following graph:



p is probability
chosen in standard
gamble question

Standard Gamble Utility Curve corrected for prospect theory

Those authors showed that inconsistencies in utility measurements, complicating classical methods, disappear under prospect theory.

Prospect theory restores consistent utility!

Improves quality of life analyses/policy recommendations.

Chapter 9. Behavioral applications in risky choice



- 9.1. Applying prospect theory to health: improving quality of life measurements
- 9.2. Applying prospect theory to insurance

Many many other applications, and different insights, under prospect theory. One more:



Insurance: first example of risk aversion taught in basic micro classes. Used to illustrate concave utility. Right!? Well, under EU that is ... Now behavioral. Insurance is about losses.

Prospect theory: utility for losses is more **convex** than concave! Yes!

Is entire opposite of what “they” taught you.

So, how about insurance then?

Prospect theory: nothing to do with utility.

Is due to probability weighting: overweighting of small probabilities.

The primary example of concave utility that the ordinal economists taught you, was completely off.

For insurance, prospect theory turns everything upside down!



Chapter 10



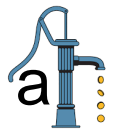
Behavioral applications in
intertemporal choice and
self-control

Outline of Ch. 10



→ 10.1. Naive and sophisticated nonstationarity

10.2. Elaborated example: procrastination



Example [arbitrage out of nonstationarity]. Imagine you own a “soon small” payment. Compare, at $t=0$, to a late large payment. You pay €0.10 to exchange.



You violate stationary: pref. changes if moving consumptions forward by 4 years:



Arbitrage! I can pump €0.10 out of you:

Here ($t=0$) I offer to exchange small for large, charging €0.10.

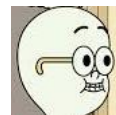
Here I offer to change back large for small.



Arbitrage: I gain €0.10. You are in original situation less €0.10.

In general, two ways to live with nonstationarity:

1. **Naïve**: plans now on future actions according to present preferences. Does not realize that future self will deviate.



2. **Sophisticated**: plans now on future actions, but reckons with future selves doing what they want. Given that makes the best of a bad situation.



Imagine, in previous example, you are informed beforehand that I plan to offer exchange in 4 years. If sophisticated, you will not fall victim; only if naive.

Getting-up in morning example:

best to get up on time with alarm within reach.

Naïve will not realize that tomorrow she will turn off alarm and oversleep.



Sophisticated knows, and puts alarm out of reach.



Neither gets best, but sophisticated is better off.

Chapter 10. Behavioral applications in intertemporal choice and self- control



10.1. Naive and sophisticated nonstationarity

→ 10.2. Elaborated example: procrastination

Another illustration of irrationalities:

procrastination

(O'Donoghue & Rabin, many papers).



Analyzed using quasi-hyperbolic discounting:

$$D(t) = 1 (= \delta^t) \text{ at } t = 0;$$

$$D(t) = \beta \delta^t \text{ at } t > 0.$$

$$0 \leq \beta \leq 1 \text{ and } 0 \leq \delta \leq 1.$$

So, everything except the present gets punished by an extra weight β .

Example of report to be written

- Report to be written within 4 weeks.
- Assume $\delta=1$ (no regular discounting/impatience). Only β , the overweighting of the present, matters.
- There will be immediate costs but delayed rewards.



You usually go to the cinema on Saturday.

Schedule:

- mediocre movie this week
- good movie next week
- great movie in 2 weeks
- best movie in 3 weeks



You must write a report within 4 weeks.
Must skip one movie for it.

Which movie to be skipped?

Value of going to the movie:

This week	Next week	In 2 weeks	In 3 weeks
3	5	8	13

Report **must** be written.
(Value comes after 4 weeks.
Can be ignored in the analysis.)



Decisions of stationary person (so, $\beta = 1$) at various times: (trivial ...)

This week	Next week	In 2 weeks	In 3 weeks
3	5	8	13

Decision today: this week!

If not done this week:

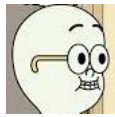
Decision next week: next week!

If not done next week:

Decision in 2 weeks: in 2 weeks!

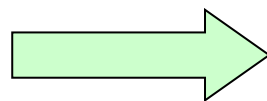
If not done in 2 weeks:

Decision in 3 weeks: in 3 weeks!



Decisions of naïve person with $\beta = 1/2$

	This week	Next week	In 2 weeks	In 3 weeks
Undiscounted costs	3	5	8	13
perspective of this week	3	2.5	4	6.5
perspective of next week		5	4	6.5
perspective in 2 weeks			8	6.5



Does it last moment, in 3 weeks.



Decisions of sophisticated person with $\beta = 1/2$

	This week	Next week	In 2 weeks	In 3 weeks
Costs	3	5	8	13
perspective of this week	3	2.5		
perspective of next week		5		6.5
perspective in 2 weeks			8	6.5



Does it next week.

Conclusion:

- Stationary person does it this week (is best)
- Sophisticated person does it next week
- Naive person does it in 3 weeks (is worst)

Time inconsistencies lead to many problems of self-control and so on. Much literature and trade based on this.



Clocky®

The patented alarm clock that runs away and hides to get you out of bed. Clocky gives you one chance to get up. But if you snooze, Clocky will jump off your nightstand and wheel around your room looking for a place to hide, beeping all the while. You'll have to get out of bed to silence his alarm. Clocky is kind of like a misbehaving pet, only he will get up at the right time. Patent 7355928. Batteries not included. Add batteries to your order below.

Size: 5.25" x 3.5" x 3.5"

\$50

color

aqua

quantity

1

Add To Bag

Checkout

Download Instructions & Warranty



Pictures of Clocky in Bedroom



Chapter 11



Preference reversals and framing

Outline of Ch. 11



- 11.1. The evaluability hypothesis
- 11.2. Context effects
- 11.3. Endowment effects
- 11.4. Epilogue of Ch. 11: applications of preceding versions of preference reversals

Preference reversals initiated many follow-ups, challenging the basic axioms of revealed preference.

Not very quantitative models.

Hsee (1993; [evaluability hypothesis](#))



You want a music dictionary, planning to spend between €10 and €50. You are in a used-book store.

Torn's EVALUATION:

they have one, with 20,000 entries, a torn cover, and as new otherwise.

What is the maximum price you want to pay?



Sparse's EVALUATION:

they have one, with 10,000 entries (sparse) and as new otherwise.

What is the maximum price you want to pay?



CHOICE: choose between those two dictionaries.

Hsee found:

Sparse best in evaluations.

Torn best in choice.

Again, preference reversal.

Explanation: evaluability hypothesis.

In evaluation, people don't know how to assess nr. of entries, ignore it, and overweigh cover.

In choice, people can compare nr. of entries.

Other example of evaluability hypothesis:

In grocery stores, we can easily evaluate prices, but not quality. So we choose cheapest.

Consequence: more and more we get cheap low-quality food.

Yet other example of evaluability hypothesis:

When comparing two cakes, we prefer the sweeter one, not aware of loss of other tastes.

Consequence: cakes in Holland are too sweet.



Chapter 11. Preference reversals and framing

11.1. The evaluability hypothesis

→ 11.2. Context effects

11.3. Endowment effects

11.4. Epilogue of Ch. 11: applications of preceding versions of preference reversals



Simonson & Tversky (1992, context effect).



n=106 subjects chose between (choice percentages added)
elegant pen (**36%**) vs. \$6 (64%).



n=115 subjects got same choice, but with a less elegant, regular, pen added as 3rd prospect.



(homo economicus & WARP):

both percentages will decrease some in 2nd case, by people moving to the regular pen.



However:

Regular pen: 2%; elegant pen: **46%!;** \$6: (52%).



Systematic violations of WARP & transitivity.
Bad pen makes elegant pen stand out more clearly.

Known as **attraction effect** or **asymmetric dominance effect** or **decoy effect**.

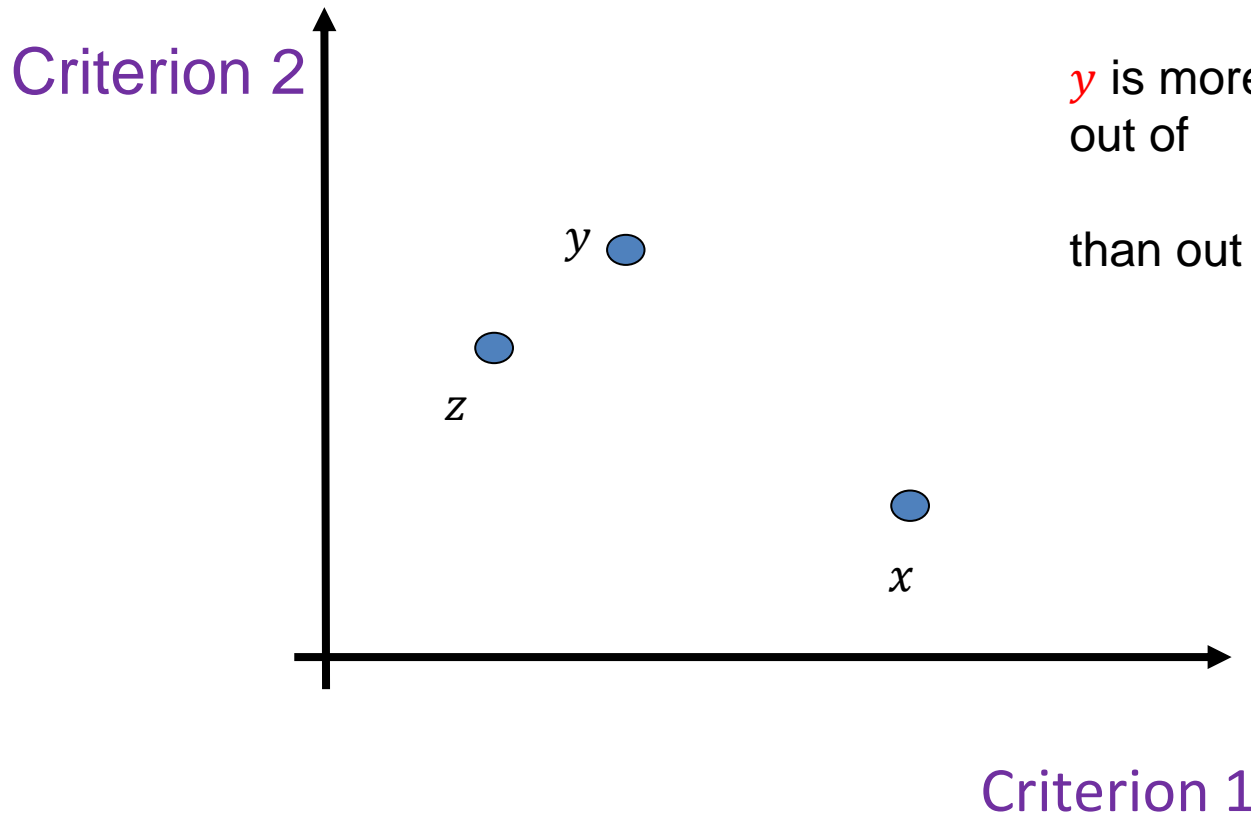
Widely used in marketing.

The Decoy Effect

y is chosen out of $\{x, y\}$ with some probability.

y is more likely to be chosen out of

than out of $\{x, y, z\}$
 $\{x, y\}$!



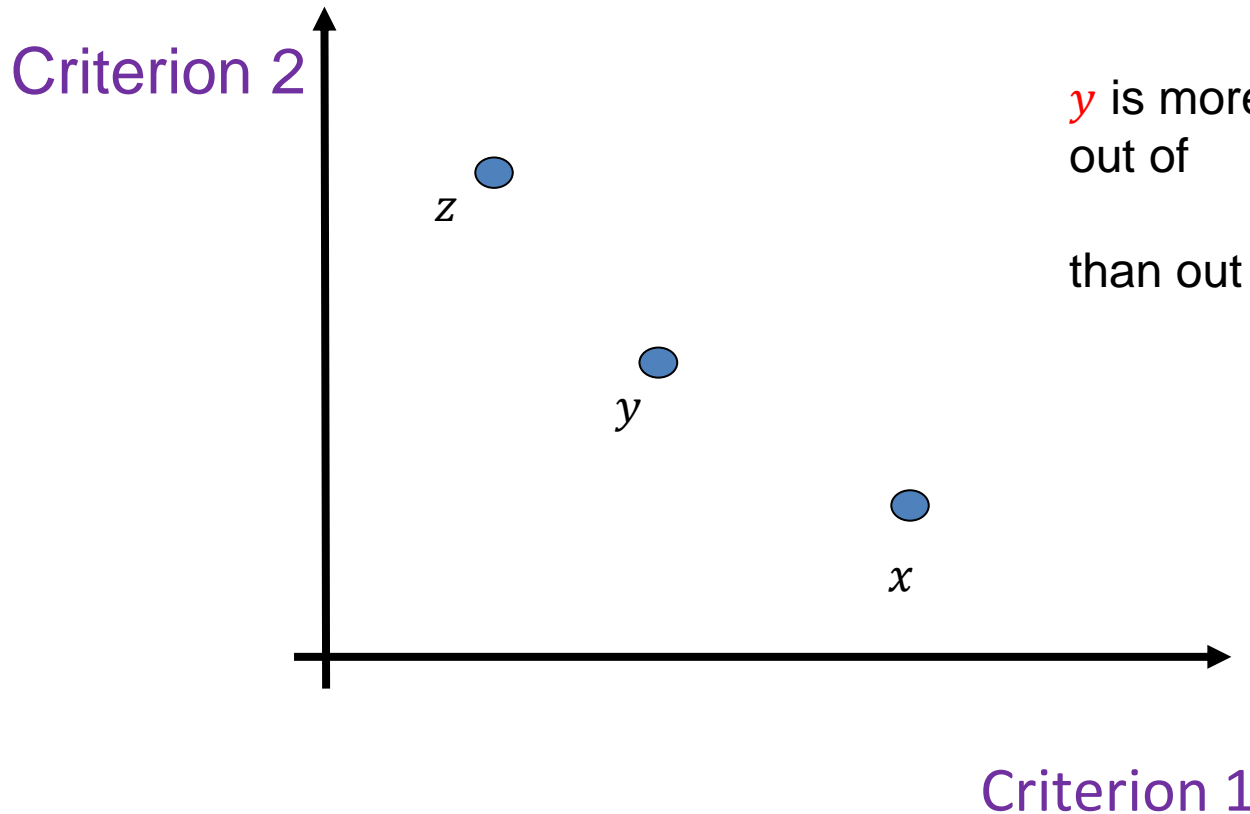
Other such violation of WARP:
compromise effect.



Other such violation of WARP:

The Compromise Effect

y is chosen out of $\{x, y\}$ with some probability.



y is more likely to be chosen out of

than out of

$\{x, y, z\}$

$\{x, y\}$!

Ex-President Trump

From 4 Jan 2020 NY Times:

“WASHINGTON — In the chaotic days leading to the death of [Maj. Gen. Qassim Suleimani, Iran’s most powerful commander](#), top American military officials put the option of killing him — which they viewed as the most extreme response to recent Iranian-led violence in Iraq — on the menu they presented to President Trump.

They didn’t think he would take it. In the wars waged since the Sept. 11, 2001, attacks, Pentagon officials have often offered improbable options to presidents to make other possibilities appear more palatable.”

I.e., they often tried to use the compromise effect. With unexpected consequence in case of Trump-Suleimani.



Many other such violations of WARP, e.g. regret theory (Loomes & Sugden 1982).

And so on ...

Even the most basic axioms-of-revealed-preference/transitivity are often violated. Life is really more complex.

Widely applied in marketing.

Chapter 11. Preference reversals and framing

11.1. The evaluability hypothesis

11.2. Context effects

→ 11.3. Endowment effects

11.4. Epilogue of Ch. 11: applications of preceding versions of preference reversals



Endowment effect: Bateman et al. (1997),
and many others.

Combines framing & **loss aversion*** to
generate violations of revealed preference
axioms.

* “losses loom larger than gains”

Here comes a characteristic finding:





1. Subjects are endowed with coffee mug.
2. Are asked willingness to accept (WTA):
minimum $\text{€}X$ for which they sell mug.
homo economicus: $\text{€}X \sim \text{mug}$.
3. Later, subjects are asked willingness to pay (WTP):
maximum amount $\text{€}Y$ they pay to buy the mug. Homo economicus: $\text{€}Y \sim \text{mug}$.
4. Homo economicus: $X = Y$ (modulo a small income effect)
5. Homo sapiens: $WTP = Y \approx X/2 = WTA/2$.

WTP-WTA discrepancy.



Explanation: the endowment effect.

What you own and have to give up,
you value more than
what you do not own but can get.

Is about the same as loss aversion.



Application (text from Thaler 1980; read it):



Until recently, credit card companies banned their affiliated stores from charging higher prices to credit card users. A bill to outlaw such agreements was presented to Congress. When it appeared likely that some kind of bill would pass, the credit card lobby turned its attention to form rather than substance. Specifically, it preferred that any difference between cash and credit card customers take the form of a cash discount rather than a credit card surcharge. This preference makes sense if consumers would view the cash discount as an opportunity cost of using the credit card but the surcharge as an out-of-pocket cost.

Explanation:

surcharge perceived as: giving up money you own;
cash discount perceived as: getting money not owned.

Endowment effect: former is felt more than latter.



Remarkable classic reference:

Bentham, Jeremy (1828-43) [1782-7], "The Rationale of Reward." John Bowring [ed.], *The Works of Jeremy Bentham*, Part VII, 297-364.



Argued:

payment with high salaries & fines for bad performance works better than low salaries and rewards for good performance, even if the same in final wealth.

Endowment effects, as most behavioral principles, have been known to mankind all through history.

But were not imposed on homo economicus.

No-one knew how to do quantitative economic analysis with it.



Chapter 11. Preference reversals and framing

11.1. The evaluability hypothesis

11.2. Context effects

11.3. Endowment effects

→ 11.4. Epilogue of Ch. 11: applications of preceding versions of preference reversals



Applications of preceding versions of preference reversals:

Are extensively used to manipulate people, for good or bad. Central to marketing.

But applications not yet very quantitative. Maybe more psychology/marketing than economics.



Before final conclusion:

Big impact (or cause?) of behavioral:
economics became **experimental** at micro level.



Final conclusion



Behavioral economics provides empirical models more realistic than the classical models that were based on the ordinal homo economicus. It provides new insights and models in virtually all fields in economics.

Also in your field ...

And also for your own decisions ...

The End