Online Appendix to "Riding Bubbles"

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Overview of Online Appendix

This online appendix is available at the homepage of one of the authors. It contains additional information on the paper "Riding Bubbles". Section A discusses the robustness checks and Section B shows our results for all 48 industries. Section C provides a detailed explanation and results of the simulations discussed in the paper in Section 4.1. In Section D we show the mathematical details for the derivations discussed in Section 5.3 and 5.4 of the paper.

The tables belonging to each section are directly included after each section. Plain references refer to equations, figures and tables in the original paper. References to equations and tables in the appendix are preceded by a capital letter.

A Robustness Checks

This section contains the robustness checks. Sections A.1 to A.3 examine the robustness of our method to obtain bubble signals in Section 1.2. Section A.4 provides robustness checks on our portfolio results. Sections A.5 and A.6 discuss in detail the robustness checks on the investor's asset allocation decision in Section 3.

In the robustness checks we focus on the risk-return trade-off and on the optimal portfolio choice that we report in Tables 4 and 6 for the standard parameter settings. Results of robustness checks for the other analyses in the original paper are available on request.

A.1 Significance Level of Bubble Signal

We examine the robustness of our results to changing the significance level of the structural break test in Equation (2) and the subsequent t-test on $\alpha^{\mathrm{p}}_{i,t}$. Because confidence levels of 99% and 95% are often used in the empirical literature, we consider these confidence levels as obvious alternative choices. Therefore, we replicate our analysis, that assumed a confidence level of 97.5%, for these two common confidence levels.

Table A.1 shows the summary statistics of the abnormal returns following positive and negative bubble signals for different confidence levels. As to be expected, the higher the confidence level, the smaller the number of positive bubble signals. It seems that there is a slightly positive relation between the returns following positive signals and the confidence levels. For example, for the CAPM, the standardized abnormal returns increase from 0.15 if we use a confidence level of 95%, to 0.19 for a confidence level of 97.5%, and 0.20 for a confidence level of 99%. Given an average idiosyncratic return volatility of 4.17% for the CAPM-based estimates, the monthly abnormal returns rise from 63 basis points to 79 basis points and 83 basis points as we increase the confidence level. For results based on the Fama-French and Carhart models the pattern is similar. Theoretically, it is not obvious what one should expect. A higher confidence level could be associated with a better quality of the bubble signal, that is, we detect more 'true' bubbles and make less mistakes. An alternative proposition is that we detect stronger bubbles as the confidence level increases and miss positive signals for weaker bubbles. In that case, the returns should be higher if the bubble continues. However, stronger bubbles might crash sooner.

Most importantly, for all different settings, we find that the standardized abnormal returns are significantly larger when the bubble signal is positive than if it is negative. As in our main specification in Table 4, we also find that volatility is economically and statistically larger when the bubble signal is positive. Consistent with our main results in Table 4, Table A.1 shows that differences in downside risk measures are not or only marginally significant when we use the CAPM. However, downside risk is generally significantly higher after a positive bubble signal than after a negative signal from signals derived from the Fama-French or Carhart models.

[Table 1 about here.]

Since the distribution of abnormal returns following positive and negative bubble signals is similar for the different confidence levels, we expect to observe the same for the asset allocation shown in Table A.2. For all confidence levels and asset pricing models, the increase in the optimal weight following a positive bubble signal is economically and statistically significant. Consequently, the expected utility is also consistently larger after a positive signal than after a negative signal, and the certainty equivalent is always significantly positive. Overall, we conclude that changing the confidence levels of our method to obtain bubble signals does not alter our findings.

[Table 2 about here.]

A.2 Break Interval and Estimation Period

In the derivation of the bubble signal, we set the maximum possible length of the break interval ζ_U to five years and choose an estimation period T of ten years. Here we examine whether our results are sensitive to changing these parameters. First, we analyze how changing the maximum length of the break interval affects our results. We reduce ζ_U to three years and increase it to seven years. Formally, we allow ζ in Equation (2) to vary from 12 to 36 and 12 to 84 while keeping the estimation period T at 120 months. In the standard setting, ζ could vary from 12 to 60. Second, we halve the estimation period (i.e., we set $T = 60$) for a maximum break interval of three years.

Table A.3 shows that the mean abnormal returns after a positive bubble signal is always significantly higher than if the signal is negative. The magnitude of the returns is similar across the different estimation settings. For example, for the Carhart model, the mean standardized abnormal return ranges from 0.097 after a positive signal for break intervals up to seven years to 0.118 when the maximum interval is three years. Based on an average idiosyncratic return volatility of 3.81% for the Carhart model, these figures translate into monthly abnormal returns ranging from 0.37% to 0.45%. If we restrict the estimation period to five years (i.e., $T = 60$), then the standardized abnormal return equals 0.114; that is an abnormal return of 0.43% per month. As with our previous results, the volatility estimates are consistently higher after positive bubble signals than after negative signals. The magnitude of the estimates is very similar across the different specifications. Similarly, the results for downside risk measures are robust. We conclude that the abnormal returns after bubbles signals are not sensitive to length of the break interval or the estimation period.

[Table 3 about here.]

The same holds for the asset allocation presented in Table A.4. The increase in optimal weight in response to a positive bubble signal is significant at the 5% level in all specifications. The p-values for the difference in utility and the certainty equivalent are well below the 5% level. As with our previous findings, the increase in optimal weight following a positive bubble signal is somewhat extreme for the CAPM. For the Fama-French and Carhart models the changes in weight are more realistic. For example, for break intervals with a maximum length of seven years, the optimal weight increases from zero after a negative signal to 1.03 after positive signals for abnormal returns based on the Fama-French model. It rises from 0.12 to 1.00 for the Carhart model returns. The changes in utility and the certainty equivalent return confirm our findings for the weights and previous results.

[Table 4 about here.]

A.3 Changes in Crash Definition

The end of a bubble can be associated with one or more crashes. Therefore, the bubble signal is negative if we observe a crash that is at least twice the standard deviation of abnormal returns during the previous six months. Both the horizon and size of the crash seem like reasonable choices, but, ultimately, they are arbitrary. We analyze whether our results are robust to modifying these parameters. We start by modifying the crash window and then analyze the effect of modifying the crash size.

In Table A.5 we replicate the risk and return trade-off following positive and negative bubble signals for two different crash windows. In the first specification, we set the crash window to zero. That is a strong test because we effectively remove the crash criterion. Even if there was a crash, the bubble signal could be positive. In the second specification, we double the length of the crash window and set it equal to 12 months. For all specifications, the mean abnormal return following a positive bubble signal is significantly higher than if the signal is negative. If we do not take crashes into account at all, the monthly standardized abnormal return increases from 0.003 after a negative bubble signal to 0.20 after a positive bubble signal for the CAPM-based results. Given an idiosyncratic return volatility of 4.17% for the CAPM, monthly abnormal returns increase from 1 basis point if the signal is negative to 83 basis points after a positive signal. For the Fama-French model, the standardized abnormal return rises is about zero after a negative signal and 0.10 after a positive signal. Given an average idiosyncratic return volatility of 3.86%, the difference in returns is 39 basis points per month. For the Carhart model, the standardized abnormal return is 0.011 after a negative signal and 0.089 after a positive signal. Because the idiosyncratic return volatility is 3.81% for the Carhart model, the difference in abnormal returns is 30 basis points per month. The magnitude of the differences in returns is very similar to our main results in the paper.

The volatility estimates are also significantly larger following positive bubble signals than after negative bubble signals. The downside risk measures, value-at-risk and expected shortfall are generally larger following a positive bubble signal than after a negative signal, but the difference is not always statistically significant.

[Table 5 about here.]

Table A.6 shows that the weight in the risky asset is significantly higher after a positive than after a negative signal. For abnormal returns based on the CAPM, the optimal weight increases from 0.04 after a negative signal to 1.66 following a positive bubble signal. The changes in optimal weight are more realistic but still sizable for the Fama-French and Carhart models. For the Fama-French model, the optimal weight rises from zero to 0.80 and for the Carhart model, it changes from 0.13 to 0.66 when the bubble signal becomes positive. Overall, we conclude that riding bubbles is the optimal strategy regardless of whether the investor considers crashes or at which horizon. Our results for the expected utility and the certainty equivalent support this conclusion.

[Table 6 about here.]

Table A.7 presents the risk-return trade-off after positive and negative bubble signals for different crash sizes. We apply a stricter boundary of 2.25 times the standard deviation of abnormal returns. We also loosen the restriction and include crashes up 1.75 times the standard deviation of abnormal returns. The results for the two different crash definitions confirm our original findings. The returns following positive bubble signals are consistently larger than after negative signals. The magnitude of the difference is similar for different crash sizes. As with our previous results, the volatility estimates are always larger after a positive bubble than after a negative signal. For the Fama-French and Carhart models, downside risk measures, such as VaR and ES, are higher following positive rather than negative signals.

[Table 7 about here.]

Table A.8 confirms that it is optimal for an investor to ride bubbles, regardless of crash size. Across all specifications, we find that the optimal weight allocated to the risky asset is significantly larger after a positive bubble signal than after a negative signal. The investor's utility is higher if she receives a positive bubble signal rather than a negative one. The certainty equivalent is positive and statistically significantly.

[Table 8 about here.]

A.4 Portfolio Returns: Tests and Value-weighted Results

A.4.1 WLS and GLS Tests

We construct tests based on weighted least squares (WLS) and generalized least squares (GLS) to correct for heteroskedasticity, contemporaneous correlation between abnormal returns, and to address the large difference between the number of positive bubble signals and negative bubble signals. To explain how we construct the tests, we use η^{B}_{t+1} to denote the abnormal return of the bubble portfolio with industries having $B_{i,t} = 1$, and η^{NB}_{t+1} for the no-bubble portfolio. Testing for a difference in means can result from a simple linear model

$$\eta^{b}_{t+1} = \alpha_b + u_{b,t+1},\ \mathrm{E}[u_{b,t+1}] = 0,\ b = \mathrm{B},\ \mathrm{NB}. \tag{A.1}$$

The variation in the number of industries and the selection of industries, as well as the general time-variation in volatility lead to heteroskedasticity in the portfolio returns,

$$\mathrm{E}[u^2_{b,t+1}] = \mathrm{Var}[\eta^{b}_{t+1}] = \omega^{b}_{t+1},\ b = \mathrm{B},\ \mathrm{NB}. \tag{A.2}$$

For a GLS-test, we allow for contemporaneous correlation between $u_{\mathrm{B},t+1}$ and $u_{\mathrm{NB}t+1}$,

$$\mathrm{E}[u_{\mathrm{B},t+1}u_{\mathrm{NB},t+1}] = \mathrm{Cov}[\eta^{\mathrm{B}}_{t+1}, \eta^{\mathrm{NB}}_{t+1}] = \omega^{\mathrm{B,NB}}_{t+1}, \tag{A.3}$$

whereas a WLS-test assumes $\omega^{\mathrm{B,NB}}_{t+1} = 0$. In both settings, we assume that lead-lag correlation between the abnormal returns is absent, that is,

$$E[u_{\mathrm{B},t+1}u_{\mathrm{NB},s+1}] = \mathrm{Cov}[\eta^{\mathrm{B}}_{t+1}, \eta^{\mathrm{NB}}_{s+1}] = 0,\ \forall s \neq t \tag{A.4}$$

We model the portfolio variance as

$$\omega^{b}_{t+1} = \left(\sum_{i=1}^{n} w^2(i, \mathcal{B}_t, b)\sigma^2_{i,t} + \sum_{i=1}^{n} \sum_{j=i+1}^{n} 2w(i, \mathcal{B}_t, b)w(j, \mathcal{B}_t, b)\sigma_{i,j,t} \right) \omega^{b}, \tag{A.5}$$

where $w(i, \mathcal{B}_t, b)$ gives the portfolio weight for industry i at time t in portfolio b depending on the set of bubble signals $\mathcal{B}_t$ at time t, and $\sigma^2_{i,t}$ and $\sigma_{i,j,t}$ are the variance and covariance of the residuals from Equation (1). The volatility of the bubble and no-bubble portfolios likely differs because one can expect volatility to be higher if there is a bubble. Therefore we include a separate multiplicative effect in the form of a coefficient ω^b.[1] For GLS, we model the covariance between the portfolios as:

$$\omega_{t+1}^{\mathrm{B,NB}} = \sum_{i=1}^{n} \sum_{j=i}^{n} w(i, \mathcal{B}_t, \mathrm{B}) w(j, \mathcal{B}_t, \mathrm{NB}) \sigma_{i,j,t}. \tag{A.6}$$

When WLS is applied, the linear model in (A.1) can be estimated with separate OLS-regressions for the bubble and the no-bubble portfolio, in which the dependent and explanatory variables are scaled by the inverse of the portfolio volatilities. For GLS, we first estimate the WLS-regressions and determine the coefficients ω^{B} and ω^{NB}, which we use in a second stage to determine the GLS-estimates. In both cases, we use the estimation results to test $\alpha_{\mathrm{B}} = \alpha_{\mathrm{NB}}$.

A.4.2 Value-weighted Results

To confirm the robustness of our findings in Section 2, we construct value-weighted portfolios. Based on the signals at the beginning of month t, we construct a bubble and a no-bubble portfolio. We use the total market capitalization of each industry at the beginning of month t to determine its weight in the portfolio. We report the results in Table A.9. Consistent with the small-firm effect, returns, volatilities and risk measures are slightly lower for the value-weighted portfolios than for the equally-weighted portfolios in Table 3. The bubble and no-bubble portfolio are equally affected by the different

[1] For WLS this coefficient does not matter, since WLS requires that heteroskedasticity is modeled up to multiplication with a constant.

weighing scheme. As a consequence, the differences between the bubble and no-bubble portfolios do not change much. The difference between the average abnormal returns of the bubble and the no-bubble portfolios for the CAPM is 0.70% per month, when the industries in the portfolios are equally weighted, and 0.67% when they are value weighted. For the Fama-French model (Carhart model) we observe a return difference of 0.25 (0.30) for equally-weighted portfolios and 0.18 (0.23) for value-weighted portfolios.

[Table 9 about here.]

The test results in Table 9(b) indicate that in all settings, the average abnormal return for the bubble portfolio is significantly larger than that for the no-bubble portfolio. The results for WLS-based tests show significant differences beyond the 1% level. However, WLS-based tests ignore contemporaneous correlation. The GLS-test allow for correlation, and we see a decrease in the statistics. Still, the differences between the abnormal returns remain significant at the 5% level.

A.5 Alternative Utility Functions

To examine the robustness of our results for the power-utility function in Section 3, we replicate our results for investors who have different utility functions. Because returns after positive bubble signals might not be normally distributed and have a higher downside risk, we focus on utility functions that explicitly take into account aversion to higher moments and downside-risk. In Section A.5.1, we analyze the allocation decisions of investors who are averse to higher moments such as skewness and kurtosis. In Section A.5.2, we focus on mean-semivariance investors.

A.5.1 Skewness, Kurtosis, and Mean-Variance Preferences

We investigate whether and how our findings change as investors have different preferences for skewness and kurtosis. We start with a very general utility function based on the different moments of the return distribution, and then substitute the parameters implied by the power-utility function for the variance, skewness, and kurtosis. To examine the sensitivity of our findings to these moments, we vary the skewness and kurtosis parameters. Finally, we set the parameters equal to the ones of a mean-variance utility function.

We approximate the utility function of an investor by a Taylor expansion around his reference point of wealth $\bar{W}_{t+1}$ (see for instance Harvey and Siddique, 2000; Jondeau and Rockinger, 2006; Guidolin and Timmermann, 2008):

$$U(W_{t+1}) = \sum_{k=0}^{\infty} \frac{1}{k!} \frac{\partial^k U(\bar{W}_{t+1})}{\partial W_{t+1}^k} (W_{t+1} - \bar{W}_{t+1})^k. \tag{A.7}$$

where we assume that $\bar{W}_{t+1} = W_t$ as in Harvey and Siddique (2000) and consequently $W_{t+1} = W_t(1 + r_{\mathrm{f},t+1} + w\eta_{t+1})$. We apply a Taylor expansion up to the fourth order:

$$\begin{aligned} U(W_{t+1}) =& U(W_t) + U^{(1)}(W_t)W_t(r_{\mathrm{f},t+1} + w\eta_{t+1}) + \frac{1}{2}U^{(2)}(W_t)W_t^2(r_{\mathrm{f},t+1} + w\eta_{t+1})^2 + \\ & \frac{1}{6}U^{(3)}(W_t)W_t^3(r_{\mathrm{f},t+1} + w\eta_{t+1})^3 + \frac{1}{24}U^{(4)}(W_t)W_t^4(r_{\mathrm{f},t+1} + w\eta_{t+1})^4 + \\ & O(W_{t+1}^4), \end{aligned} \tag{A.8}$$

where $O(W_{t+1}^4)$ contains the higher-order terms. When we take expectations, the resulting equation shows how higher-order moments of the portfolio return enter the utility function:

$$\begin{aligned}\mathrm{E}[U(W)] \approx&\kappa_0 + \kappa_1 \mathrm{E}[r_{\mathrm{f},t+1} + w\eta_{t+1}] + \kappa_2 \mathrm{E}[(r_{\mathrm{f},t+1} + w\eta_{t+1})^2]+\\ &\kappa_3 \mathrm{E}[(r_{\mathrm{f},t+1} + w\eta_{t+1})^3] + \kappa_4 \mathrm{E}[(r_{\mathrm{f},t+1} + w\eta_{t+1})^4],\end{aligned} \tag{A.9}$$

with $\kappa_k = U^{(k)}(W_t)W_t^k/k!$. The optimal weight allocated to the risky asset w^{H} to maximize this expression should satisfy:

$$\begin{aligned}\mathrm{E}[\eta_{t+1}] + 2\frac{\kappa_2}{\kappa_1}\mathrm{E}\left[(r_{\mathrm{f},t+1} + w^{\mathrm{H}}\eta_{t+1})\eta_{t+1}\right] + 3\frac{\kappa_3}{\kappa_1}\mathrm{E}\left[(r_{\mathrm{f},t+1} + w^{\mathrm{H}}\eta_{t+1})^2\eta_{t+1}\right] +\\ 4\frac{\kappa_4}{\kappa_1}\mathrm{E}\left[(r_{\mathrm{f},t+1} + w^{\mathrm{H}}\eta_{t+1})^3\eta_{t+1}\right] = 0.\end{aligned} \tag{A.10}$$

This equation implies that κ_0 does not affect w^{H}, so we can ignore it. Dividing by κ_1 allows us to remove this coefficient as well.

If the utility function is not known, then the values of the parameters κ_k are not determined. Necessary and desirable properties of utility functions, such as risk aversion and decreasing absolute risk aversion, restrict only the signs of the parameters: the uneven parameters are positive and the even parameters are negative (see Scott and Horvath, 1980). Therefore, we define $\kappa_2' = -\kappa_2'/\kappa_1$, $\kappa_3' = \kappa_3'/\kappa_1$ and $\kappa_4' = -\kappa_4'/\kappa_1$, which yields

$$\begin{aligned}\mathrm{E}\left[U'(W)\right] =&\mathrm{E}[r_{\mathrm{f},t+1} + w\eta_{t+1}] - \kappa_2' \mathrm{E}\left[(r_{\mathrm{f},t+1} + w\eta_{t+1})^2\right] +\\ &\kappa_3' \mathrm{E}\left[(r_{\mathrm{f},t+1} + w\eta_{t+1})^3\right] - \kappa_4' \mathrm{E}\left[(r_{\mathrm{f},t+1} + w\eta_{t+1})^4\right],\end{aligned} \tag{A.11}$$

where all coefficients should be positive.

We then substitute the parameters implied by the power-utility function with $\gamma = 2$ in Section 3: $\kappa_1 = W^{1-\gamma} = W^{-1}$, $\kappa_2' = \gamma/2 = 1$, $\kappa_3' = (1+\gamma)\gamma/6 = 1$, $\kappa_4' = (2+\gamma)(1+\gamma)\gamma/24 = 1$ and solve for the optimal weight using numerical techniques. The first rows of Table A.10, in Panels A, B, and C show the optimal weights, the expected utilities, and the certainty equivalent. We find that the results are consistently similar to the comparable scenario for the exact specification of the power-utility function in Table 6, indicating that a fourth-order approximation is reasonably precise.

We vary the investor's preferences for skewness and kurtosis as we move down each panel. As κ_3' increases, the investor becomes more averse to negative skewness and has a stronger preference for positive skewness. Her allocation to the risky asset after a positive signal increases, because the abnormal returns after positive bubbles signals are slightly positively skewed. For the CAPM the increases are quite pronounced, but for the Fama-French and Carhart models, they take on more moderate values and the effects seem more realistic. Accordingly, there is also an increase in the investor's utility and the certainty equivalent. If the investor becomes more concerned about kurtosis and κ_4' rises, then the weight after a positive bubble signal decreases slightly. For example, if the weight on kurtosis doubles from one to two, then the optimal weight after a positive signal decreases from 1.67 to 1.56 for the CAPM. For the Fama-French and Carhart models, the optimal weights after positive signals decrease from 0.92 to 0.91 and from 0.91 to 0.90, respectively. In all cases, the decrease is only small and the optimal weight after a positive signal is significantly larger than after a negative signal. Similarly, utility is significantly higher after a positive signal than after a negative one and the certainty equivalent is positive.

[Table 10 about here.]

We investigate the optimal allocation of a mean-variance investor by setting κ_3' and κ_4' equal to zero. We set κ_2' equal to one and two, which implies risk-aversion levels of two and four. For the mean-variance investor, the optimal weight declines as risk-aversion increases. However, even with a risk aversion of four, the optimal weight allocated to the risky asset is much larger if a positive bubble signal is received than if the signal is negative. For the CAPM, the optimal weight is 0.79 if the signal is positive compared to 0.04 for a negative signal. For the Fama-French and Carhart models, the optimal weight increases from about zero to 0.45 and 0.06 to 0.44 when the signal changes from negative to positive. These increases are not only economically large but also statistically significant. The same holds for the increases in utility and the positive certainty equivalent.

Overall, these results confirm our findings for the power-utility investor. Investors who care particularly about skewness and kurtosis and mean-variance investors would also ride bubbles.

A.5.2 Mean - Semivariance Utility

Value-at-risk (VaR) and expected shortfall (ES) in Table 4 in Section 2 show that abnormal returns after positive signals have a larger downside risk than returns after negative signals. Therefore, we investigate whether our conclusions hold for investors who are particularly averse to losses.

We choose an investor with a mean-semivariance utility function U^{SV}. In contrast to utility functions featuring VaR or ES, it has the desirable property that it is concave for losses. Following Harlow and Rao (1989), we link the utility function to the portfolio return

r_{p}:

$$U^{\mathrm{SV}}(r_{\mathrm{p}};k)=\begin{cases} r_{\mathrm{p}}-\gamma(k-r_{\mathrm{p}})^2 & \text{for } r_{\mathrm{p}}\leq k \\ \\ r_{\mathrm{p}} & \text{for } r_{\mathrm{p}}>k, \end{cases} \tag{A.12}$$

where k is the target return. Realizations of a portfolio return below k lead to a discount of utility. We follow Bawa and Lindenberg (1977) and assume that the target return equals the risk-free rate.

The second term of Equation (A.12) leads to the semi-variance with respect to k:

$$\mathrm{SV}_k[r_{\mathrm{p}}]=\int_{-\infty}^{k}(k-r_{\mathrm{p}})^2\,\mathrm{d}F(r_{\mathrm{p}}), \tag{A.13}$$

where $F(r_{\mathrm{p}})$ is the cumulative distribution function of r_{p}. The investor combines this utility function with the expression for the portfolio return in Equation 4 and solves:

$$\max_{w}\mathrm{E}\big[U^{\mathrm{SV}}(r_{\mathrm{f},t+1}+w\eta_{t+1};r_{\mathrm{f},t+1})|B_t\big]=\max_{w}\{r_{\mathrm{f},t+1}+w\,\mathrm{E}[\eta_{t+1}|B_t]-\gamma\mathrm{SV}_0[w\eta_{t+1}|B_t]\}. \tag{A.14}$$

Assuming that the threshold k equals the risk-free rate implies considering the semi-variance of the risky part of the portfolio with respect to zero, $\mathrm{SV}_0[w\eta_{t+1}|B_t]$. Due to this result, zero becomes the investor's variance threshold, and we can write $\mathrm{SV}_0[w\eta_{t+1}|B_t]=w^2\mathrm{SV}_0[\mathrm{sgn}(w)\cdot\eta_{t+1}|B_t]$. Solving for the optimal weight w^{SV} leads to:

$$w^{\mathrm{SV}}=\frac{\mathrm{E}[\eta_{t+1}|B_t]}{2\gamma\mathrm{SV}_0[\mathrm{sgn}(\mathrm{E}[\eta_{t+1}|B_t])\cdot\eta_{t+1}|B_t]}, \tag{A.15}$$

where we replace $\mathrm{SV}_0[\mathrm{sgn}(w)\cdot\eta_{t+1}|B_t]$ by $\mathrm{SV}_0[\mathrm{sgn}(\mathrm{E}[\eta_{t+1}|B_t])\cdot\eta_{t+1}|B_t]$, because the sign of the weight is solely determined by the sign of the expected abnormal return.

Table A.11 shows the optimal weight, the expected utility, and the certainty equivalent for an investor with a risk-aversion coefficient of two. The optimal weight allocated to the risky asset is substantially higher after a positive signal than after a negative signal. For the CAPM, it rises from 0.08 after a negative bubble signal to an amazing 2.38 when the signal is positive. Thus, the investor would leverage his wealth by a factor larger than one. The allocations for the Fama-French and Carhart models are more reasonable. For abnormal returns based on the Fama-French model, the weight in the risky asset increases from 0.02 after a negative signal to 1.10 when the signal is positive. For the Carhart model, the optimal weight rises from 0.13 to 1.08 when the signal changes from negative to positive. In all cases, the optimal weight is statistically significantly higher after positive signals rather than negative signals. Similarly, we find that expected utility is higher following a positive bubble signal than after a negative signal. The certainty equivalent demanded by the investor for not updating his portfolio is always positive, sizable, and statistically significantly different from zero. Overall, we conclude that even for an investor who is particularly concerned with downside risk, riding bubbles is the optimal strategy.

[Table 11 about here.]

A.6 Changes in the Risk-free Rate

In the analysis of the optimal portfolio, we assume that the risk-free rate equals its long-run average. In this section, we investigate how this assumption affects our results. Again, we use the approximation for the optimality condition:

$$0 = \mathrm{E}[(1 + r_{\mathrm{f},t+1} + w^*\eta_{t+1})^{-\gamma}\eta_{t+1}|B_t] \approx \mathrm{E}\left[\mathrm{e}^{-\gamma(r_{\mathrm{f},t+1}+w^*\eta_{t+1})}\eta_{t+1}|B_t\right] = \mathrm{e}^{-\gamma r_{\mathrm{f},t+1}}\,\mathrm{E}\left[\mathrm{e}^{-\gamma w^*\eta_{t+1}}\eta_{t+1}|B_t\right].$$

If the approximation was exact, then the risk-free rate would be irrelevant. The optimality condition would be completely determined by the second part, $\mathrm{E}\left[\mathrm{e}^{-\gamma w^* \eta_{t+1}} \eta_{t+1} | B_t\right] = 0$. Because we use monthly abnormal returns the approximation is almost exact. Therefore, the choice of the risk-free rate has only a negligible influence.

As alternative specifications, we use a risk-free rate of zero and a rate that is twice the long-run average. For both settings, we use our base case risk-aversion level of two. The impact of the changes of the risk-free rate on our results is limited to the third decimal of the estimates. Because the results are virtually identical, they are available on request.

Table A.1: Standardized Abnormal Returns After Bubble Signals With Different Confidence Levels

(a) CAPM, 95%

Signal	Negative		Positive		p-value
# Obs	36852		2844		
Mean	0.006	(0.011)	0.15	(0.032)	< 0.001
Median	−0.013	(0.009)	0.11	(0.034)	< 0.001
Volatility	1.04	(0.016)	1.16	(0.028)	< 0.001
VaR(0.95)	1.61	(0.028)	1.69	(0.055)	0.067
ES(0.95)	2.25	(0.053)	2.21	(0.071)	0.70

(b) CAPM, 99%

Signal	Negative		Positive		p-value
# Obs	38559		1137		
Mean	0.011	(0.011)	0.20	(0.048)	< 0.001
Median	−0.01	(0.009)	0.20	(0.048)	< 0.001
Volatility	1.04	(0.016)	1.19	(0.033)	< 0.001
VaR(0.95)	1.61	(0.027)	1.73	(0.096)	0.063
ES(0.95)	2.25	(0.051)	2.28	(0.11)	0.34

(c) Fama-French model, 95%

Signal	Negative		Positive		p-value
# Obs	37437		2259		
Mean	−0.0006	(0.009)	0.11	(0.032)	< 0.001
Median	−0.024	(0.007)	0.060	(0.026)	< 0.001
Volatility	1.06	(0.015)	1.20	(0.027)	< 0.001
VaR(0.95)	1.66	(0.027)	1.78	(0.062)	0.017
ES(0.95)	2.29	(0.049)	2.41	(0.090)	0.098

(d) Fama-French model, 99%

Signal	Negative		Positive		p-value
# Obs	39024		672		
Mean	0.003	(0.008)	0.15	(0.060)	< 0.001
Median	−0.020	(0.007)	0.076	(0.066)	0.008
Volatility	1.07	(0.015)	1.30	(0.043)	< 0.001
VaR(0.95)	1.66	(0.027)	1.95	(0.11)	0.004
ES(0.95)	2.29	(0.048)	2.55	(0.16)	0.052

(e) Carhart model, 95%

Signal	Negative		Positive		p-value
# Obs	37242		2214		
Mean	0.01	(0.008)	0.095	(0.037)	< 0.001
Median	−0.016	(0.008)	0.072	(0.033)	< 0.001
Volatility	1.08	(0.015)	1.24	(0.029)	< 0.001
VaR(0.95)	1.67	(0.027)	1.87	(0.082)	0.002
ES(0.95)	2.30	(0.044)	2.54	(0.13)	0.003

(f) Carhart model, 99%

Signal	Negative		Positive		p-value
# Obs	38785		671		
Mean	0.012	(0.008)	0.14	(0.062)	0.002
Median	−0.013	(0.008)	0.095	(0.061)	0.010
Volatility	1.08	(0.015)	1.34	(0.043)	< 0.001
VaR(0.95)	1.67	(0.028)	1.97	(0.11)	0.003
ES(0.95)	2.31	(0.044)	2.67	(0.20)	0.011

This table reports summary statistics and downside risk measures for the pooled set of standardized abnormal returns resulting from different confidence levels of 95% and 99% in the bubble signal detection. The abnormal returns are based on rolling regressions of the CAPM (Panels A–B), the Fama-French model (Panels C–D) and the Carhart model (Panels E–F) in Equation (1). For each regression, we construct an abnormal return for the period after the estimation window as in Equation (3). To correct for time-varying volatility, we standardize the abnormal return by dividing by the residual volatility of the regression model. We split the abnormal returns according to the detection of a bubble. In the column titled Negative (Positive) we report the results negative (positive) bubble signals. For each statistic, we construct standard errors, reported in parentheses, and p-values based on 1,000 temporal bootstraps. The column titled p-value reports the results of tests for equality of the statistics for negative and positive bubble signals under the null hypothesis of no distributional difference.

Table A.2: Optimal Portfolio Choice and Expected Utility for Different Confidence Levels

Model	Level	w_{NB}		w_{B}		p-value	$V_{\text{B}}/V_{\text{NB}}$	p-value	λ	p-value
CAPM	95%	0.06	(0.12)	1.43	(0.30)	< 0.001	2.48	< 0.001	5.02	< 0.001
	97.5%	0.08	(0.12)	1.70	(0.35)	< 0.001	3.18	< 0.001	7.38	< 0.001
	99%	0.12	(0.12)	1.71	(0.43)	< 0.001	3.31	< 0.001	7.48	< 0.001
Fama-French	95%	0.00	(0.10)	0.97	(0.29)	0.001	1.65	< 0.001	2.38	< 0.001
	97.5%	0.00	(0.10)	0.93	(0.35)	0.003	1.66	0.001	2.43	< 0.001
	99%	0.03	(0.10)	1.12	(0.47)	0.013	2.03	0.008	3.56	0.008
Carhart	95%	0.11	(0.09)	0.81	(0.32)	0.004	1.47	0.002	1.33	0.002
	97.5%	0.12	(0.09)	0.91	(0.35)	0.010	1.63	0.004	1.76	0.006
	99%	0.14	(0.09)	1.05	(0.46)	0.028	1.91	0.006	2.59	0.012

This table reports the optimal portfolio w_{NB} (w_{B}) of an investor who receives a negative (positive) bubble signal. We assume that the investor has a power-utility function with coefficient of relative risk aversion $\gamma = 2$. We report the optimal portfolios as fractions of wealth. We consider different confidence levels for the bubble signal detection. The investment opportunity is the typical industry with idiosyncratic volatilities equal to their pooled averages (CAPM, 4.17%; Fama-French model, 3.86%; and Carhart model, 3.81%). The risk-free rate equals its long-term average of 0.305% per month. Abnormal returns are based on the CAPM and the Fama-French and Carhart models. Based on the optimal portfolios, we calculate the ratio of expected utilities $V_{\text{B}}/V_{\text{NB}}$. We calculate the certainty equivalent return λ that an investor requires for not changing his portfolio from w_{NB} to w_{B} in % per year. We use 1,000 temporal bootstraps to calculate standard errors (reported in parentheses) and p-values. We test $w_{\text{NB}} = w_{\text{B}}$, $V_{\text{B}} = V_{\text{NB}}$ and $\lambda = 0$ under the null hypothesis of no difference in distribution between the cases with positive and negative bubble signals.

Table A.3: Standardized Abnormal Returns After Bubble Signals for Different Break Intervals and Estimation Windows

(a) CAPM, Bubble max. 7 Years

Signal	Negative		Positive		p-value
# Obs	38124		1572		
Mean	0.009	(0.011)	0.18	(0.047)	< 0.001
Median	−0.01	(0.009)	0.16	(0.053)	< 0.001
Volatility	1.04	(0.015)	1.28	(0.039)	< 0.001
VaR(0.95)	1.61	(0.027)	1.82	(0.10)	< 0.001
ES(0.95)	2.24	(0.052)	2.37	(0.094)	0.11

(b) CAPM, Bubble max. 3 Years

Signal	Negative		Positive		p-value
# Obs	37564		2132		
Mean	0.008	(0.011)	0.17	(0.036)	< 0.001
Median	−0.012	(0.009)	0.14	(0.037)	< 0.001
Volatility	1.04	(0.016)	1.13	(0.026)	0.002
VaR(0.95)	1.62	(0.028)	1.64	(0.065)	0.32
ES(0.95)	2.25	(0.052)	2.17	(0.085)	0.82

(c) CAPM, Estimation 5 Years

Signal	Negative		Positive		p-value
# Obs	38709		1419		
Mean	0.016	(0.01)	0.19	(0.047)	< 0.001
Median	−0.002	(0.008)	0.14	(0.050)	< 0.001
Volatility	1.08	(0.015)	1.20	(0.031)	< 0.001
VaR(0.95)	1.67	(0.028)	1.80	(0.10)	0.030
ES(0.95)	2.30	(0.047)	2.31	(0.11)	0.40

(d) Fama-French model, Bubble max. 7 Years

Signal	Negative		Positive		p-value
# Obs	38405		1291		
Mean	0.002	(0.008)	0.10	(0.045)	< 0.001
Median	−0.022	(0.007)	0.077	(0.043)	< 0.001
Volatility	1.06	(0.015)	1.31	(0.034)	< 0.001
VaR(0.95)	1.66	(0.027)	1.95	(0.11)	< 0.001
ES(0.95)	2.28	(0.048)	2.69	(0.13)	0.005

(e) Fama-French model, Bubble max. 3 Years

Signal	Negative		Positive		p-value
# Obs	38323		1373		
Mean	0.001	(0.008)	0.12	(0.038)	< 0.001
Median	−0.022	(0.007)	0.081	(0.038)	< 0.001
Volatility	1.07	(0.015)	1.23	(0.029)	< 0.001
VaR(0.95)	1.66	(0.027)	1.83	(0.089)	0.008
ES(0.95)	2.29	(0.048)	2.46	(0.11)	0.074

(f) Fama-French model, Estimation 5 Years

Signal	Negative		Positive		p-value
# Obs	39001		1127		
Mean	0.004	(0.008)	0.15	(0.044)	< 0.001
Median	−0.019	(0.008)	0.12	(0.038)	< 0.001
Volatility	1.12	(0.015)	1.25	(0.033)	0.001
VaR(0.95)	1.72	(0.026)	1.83	(0.079)	0.078
ES(0.95)	2.38	(0.042)	2.51	(0.13)	0.12

(g) Carhart model, Bubble max. 7 Years

Signal	Negative		Positive		p-value
# Obs	38183		1273		
Mean	0.012	(0.008)	0.097	(0.050)	< 0.001
Median	−0.014	(0.008)	0.081	(0.048)	< 0.001
Volatility	1.08	(0.014)	1.33	(0.034)	< 0.001
VaR(0.95)	1.67	(0.027)	2.01	(0.11)	< 0.001
ES(0.95)	2.30	(0.044)	2.75	(0.14)	< 0.001

(h) Carhart model, Bubble max. 3 Years

Signal	Negative		Positive		p-value
# Obs	38082		1374		
Mean	0.011	(0.008)	0.12	(0.043)	< 0.001
Median	−0.016	(0.008)	0.10	(0.042)	< 0.001
Volatility	1.08	(0.015)	1.24	(0.029)	< 0.001
VaR(0.95)	1.67	(0.028)	1.84	(0.083)	0.009
ES(0.95)	2.31	(0.045)	2.48	(0.13)	0.065

(i) Carhart model, Estimation 5 Years

Signal	Negative		Positive		p-value
# Obs	38744		1138		
Mean	0.016	(0.007)	0.11	(0.043)	0.006
Median	−0.011	(0.008)	0.11	(0.041)	< 0.001
Volatility	1.14	(0.015)	1.29	(0.033)	< 0.001
VaR(0.95)	1.76	(0.023)	1.85	(0.079)	0.12
ES(0.95)	2.43	(0.043)	2.69	(0.16)	0.019

This table reports summary statistics and downside risk measures for the pooled set of standardized abnormal returns resulting from different break intervals and estimation windows in the bubble detection method. We consider a maximum length of the break interval (ζ_U) of seven years (Panels A, D, G), three years (Panels B, E, H) and an estimation window of five years with a maximum break interval of three years (Panels C, F, I). For each regression, we construct an abnormal return for the period after the estimation window as in Equation (3). To correct for time-varying volatility, we standardize the abnormal return by dividing by the residual volatility of the regression model. We split the abnormal returns according to the detection of a bubble. In the column titled Negative (Positive) we report the results for negative (positive) bubble signals. For each statistic, we construct standard errors, reported in parentheses, and p-values based on 1,000 temporal bootstraps. The column titled p-value reports the results of tests for equality of the statistics for negative and positive signals under the null hypothesis of no distributional difference.

Table A.4: Optimal Portfolio Choice and Expected Utility for Different Break Intervals and Estimation Periods

Model	Setting	w_{NB}		w_{B}		p-value	$V_{\text{B}}/V_{\text{NB}}$	p-value	λ	p-value
CAPM	Base	0.08	(0.12)	1.70	(0.35)	< 0.001	3.18	< 0.001	7.38	< 0.001
	Long	0.09	(0.12)	1.59	(0.35)	< 0.001	2.78	< 0.001	5.92	< 0.001
	Short1	0.11	(0.12)	1.39	(0.37)	< 0.001	2.70	< 0.001	5.42	< 0.001
	Short2	0.17	(0.10)	1.53	(0.41)	< 0.001	2.80	< 0.001	5.42	< 0.001
Fama-French	Base	0.00	(0.10)	0.93	(0.35)	0.003	1.66	0.001	2.43	< 0.001
	Long	0.00	(0.10)	1.03	(0.34)	< 0.001	1.78	0.001	2.86	0.003
	Short1	0.00	(0.10)	0.83	(0.34)	0.007	1.53	0.005	1.94	0.003
	Short2	0.04	(0.08)	1.20	(0.35)	< 0.001	2.07	< 0.001	3.67	< 0.001
Carhart	Base	0.12	(0.09)	0.91	(0.35)	0.010	1.63	0.004	1.76	0.006
	Long	0.12	(0.09)	1.00	(0.36)	0.001	1.73	< 0.001	2.12	< 0.001
	Short1	0.13	(0.09)	0.72	(0.38)	0.035	1.42	0.021	1.07	0.032
	Short2	0.15	(0.08)	0.83	(0.35)	0.030	1.52	0.012	1.34	0.026

This table reports the optimal portfolio w_{NB} (w_{B}) of a rational investor if she derives a negative (positive) bubble signal. We assume that the investor has a power-utility function with coefficient of relative risk aversion $\gamma = 2$. We report the optimal portfolios as fractions of wealth. We consider a maximum length of the break interval (i.e., ζ_U) of seven years (setting "Long") and three years (setting "Short1"), and a setting where we also decrease the estimation window to five years with a maximum break interval of three years (setting "Short2"). The investment opportunity is the typical industry with idiosyncratic volatilities equal to their pooled averages (CAPM, 4.17%; Fama-French model, 3.86%; and Carhart model, 3.81%). The risk-free rate equals its long-term average of 0.305% per month. Abnormal returns are based on the CAPM and the Fama-French and Carhart models. Based on the optimal portfolios, we calculate the ratio of expected utilities $V_{\text{B}}/V_{\text{NB}}$. We also calculate the certainty equivalent return λ that an investor requires for not changing his portfolio from w_{NB} to w_{B}, in % per year. We use 1,000 temporal bootstraps to calculate standard errors (reported in parentheses) and p-values. We test $w_{\text{NB}} = w_{\text{B}}$, $V_{\text{B}} = V_{\text{NB}}$ and $\lambda = 0$ under the null hypothesis of no difference in the distribution based on positive and negative bubble signals.

Table A.5: Standardized Abnormal Returns After Bubble Signals With Different Crash Windows

(a) CAPM, No Window

Signal	Negative		Positive		p-value
# Obs	37095		2601		
Mean	0.003	(0.011)	0.20	(0.037)	< 0.001
Median	−0.013	(0.008)	0.16	(0.041)	< 0.001
Volatility	1.03	(0.015)	1.23	(0.029)	< 0.001
VaR(0.95)	1.61	(0.028)	1.71	(0.057)	0.041
ES(0.95)	2.25	(0.053)	2.24	(0.075)	0.55

(b) CAPM, 12-Month Window

Signal	Negative		Positive		p-value
# Obs	38168		1528		
Mean	0.01	(0.011)	0.18	(0.040)	< 0.001
Median	−0.011	(0.009)	0.18	(0.044)	< 0.001
Volatility	1.04	(0.015)	1.15	(0.034)	< 0.001
VaR(0.95)	1.62	(0.028)	1.69	(0.068)	0.16
ES(0.95)	2.25	(0.052)	2.16	(0.087)	0.78

(c) Fama-French model, No Window

Signal	Negative		Positive		p-value
# Obs	37945		1751		
Mean	0.0009	(0.008)	0.10	(0.040)	< 0.001
Median	−0.022	(0.007)	0.049	(0.032)	0.005
Volatility	1.06	(0.015)	1.31	(0.030)	< 0.001
VaR(0.95)	1.65	(0.027)	1.94	(0.091)	< 0.001
ES(0.95)	2.28	(0.049)	2.61	(0.11)	0.005

(d) Fama-French model, 12-Month Window

Signal	Negative		Positive		p-value
# Obs	38686		1010		
Mean	0.002	(0.008)	0.14	(0.046)	< 0.001
Median	−0.021	(0.007)	0.078	(0.047)	0.002
Volatility	1.07	(0.015)	1.26	(0.035)	< 0.001
VaR(0.95)	1.66	(0.027)	1.83	(0.10)	0.020
ES(0.95)	2.29	(0.048)	2.48	(0.13)	0.085

(e) Carhart model, No Window

Signal	Negative		Positive		p-value
# Obs	37690		1766		
Mean	0.011	(0.008)	0.089	(0.045)	< 0.001
Median	−0.014	(0.008)	0.043	(0.040)	0.010
Volatility	1.07	(0.015)	1.34	(0.030)	< 0.001
VaR(0.95)	1.66	(0.028)	1.99	(0.078)	< 0.001
ES(0.95)	2.29	(0.045)	2.67	(0.12)	< 0.001

(f) Carhart model, 12-Month Window

Signal	Negative		Positive		p-value
# Obs	38425		1031		
Mean	0.011	(0.008)	0.13	(0.047)	< 0.001
Median	−0.014	(0.008)	0.12	(0.048)	< 0.001
Volatility	1.08	(0.015)	1.28	(0.034)	< 0.001
VaR(0.95)	1.67	(0.028)	1.88	(0.094)	0.011
ES(0.95)	2.31	(0.045)	2.49	(0.12)	0.063

This table reports summary statistics and downside risk measures for the pooled set of standardized abnormal returns resulting from different crash windows in the bubble signal extraction. The abnormal returns are based on rolling regressions of the CAPM (Panels A and B), the Fama-French model (Panels C and D) and the Carhart model (Panels E and F) in Equation (1). For each regression, we construct an abnormal return for the period after the estimation window as in Equation (3). To correct for time-varying volatility, we standardize the abnormal return by dividing by the residual volatility of the regression model. We split the abnormal returns according to the detection of a bubble. In the column titled Negative (Positive) we report the results for negative (positive) bubble signals. For each statistic, we construct standard errors, reported in parentheses, and p-values based on 1,000 temporal bootstraps. The column titled p-value reports the results of tests for equality of the statistics for negative and positive bubble signal under the null hypothesis of no distributional difference.

Table A.6: Optimal Portfolio Choice and Expected Utility for Different Crash Windows

Model	Months	w_{NB}		w_{B}		p-value	$V_{\text{B}}/V_{\text{NB}}$	p-value	λ	p-value
CAPM	0	0.04	(0.12)	1.66	(0.31)	< 0.001	3.23	< 0.001	7.88	< 0.001
	6	0.08	(0.12)	1.70	(0.35)	< 0.001	3.18	< 0.001	7.38	< 0.001
	12	0.11	(0.12)	1.66	(0.38)	< 0.001	2.99	< 0.001	6.49	< 0.001
Fama-French	0	0.00	(0.10)	0.80	(0.31)	0.003	1.53	0.001	1.94	0.001
	6	0.00	(0.10)	0.93	(0.35)	0.003	1.66	0.001	2.43	< 0.001
	12	0.00	(0.10)	1.19	(0.38)	0.003	2.08	0.002	3.99	0.003
Carhart	0	0.13	(0.09)	0.66	(0.33)	0.037	1.35	0.015	0.89	0.020
	6	0.12	(0.09)	0.91	(0.35)	0.010	1.63	0.004	1.76	0.006
	12	0.13	(0.09)	1.08	(0.38)	0.003	1.88	0.002	2.57	0.002

This table reports the optimal portfolio w_{NB} (w_{B}) of a rational investor if she derives a negative (positive) bubble signal. We assume that the investor has a power-utility function with coefficient of relative risk aversion $\gamma = 2$. We report the optimal portfolios as fractions of wealth. We consider different crash windows (zero, six and twelve months, six being the base case) over which the investor looks for crashes when constructing the bubble signal. The investment opportunity is the typical industry with idiosyncratic volatilities equal to their pooled averages (CAPM, 4.17%; Fama-French model, 3.86%; and Carhart model, 3.81%). The risk-free rate equals its long-term average of 0.305% per month. Abnormal returns are based on the CAPM and the Fama-French and Carhart models. Based on the optimal portfolios, we calculate the ratio of expected utilities $V_{\text{B}}/V_{\text{NB}}$. We calculate the certainty equivalent return λ that an investor requires for not changing his portfolio from w_{NB} to w_{B} in % per year. We use 1,000 temporal bootstraps to calculate standard errors (reported in parentheses) and p-values. We test $w_{\text{NB}} = w_{\text{B}}$, $V_{\text{B}} = V_{\text{NB}}$ and $\lambda = 0$ under the null hypothesis of no difference in distribution between the cases with positive and negative bubble signals.

Table A.7: Standardized Abnormal Returns After Bubble Signals With Different Crash Size

(a) CAPM, Crash Below -1.75σ

Signal	Negative		Positive		p-value
# Obs	38028		1668		
Mean	0.009	(0.011)	0.19	(0.041)	< 0.001
Median	−0.011	(0.009)	0.16	(0.043)	< 0.001
Volatility	1.04	(0.015)	1.18	(0.036)	< 0.001
VaR(0.95)	1.62	(0.028)	1.64	(0.073)	0.37
ES(0.95)	2.25	(0.052)	2.19	(0.097)	0.70

(b) CAPM, Crash Below -2.25σ

Signal	Negative		Positive		p-value
# Obs	37511		2185		
Mean	0.006	(0.011)	0.19	(0.037)	< 0.001
Median	−0.013	(0.008)	0.16	(0.042)	< 0.001
Volatility	1.04	(0.016)	1.19	(0.029)	< 0.001
VaR(0.95)	1.61	(0.028)	1.70	(0.057)	0.058
ES(0.95)	2.25	(0.052)	2.19	(0.078)	0.73

(c) Fama-French model, Crash Below -1.75σ

Signal	Negative		Positive		p-value
# Obs	38550		1146		
Mean	0.002	(0.008)	0.11	(0.045)	< 0.001
Median	−0.021	(0.007)	0.072	(0.039)	0.002
Volatility	1.07	(0.015)	1.26	(0.034)	< 0.001
VaR(0.95)	1.66	(0.026)	1.88	(0.093)	0.004
ES(0.95)	2.29	(0.048)	2.61	(0.14)	0.011

(d) Fama-French model, Crash Below -2.25σ

Signal	Negative		Positive		p-value
# Obs	38230		1466		
Mean	0.002	(0.009)	0.10	(0.041)	< 0.001
Median	−0.022	(0.007)	0.059	(0.033)	0.004
Volatility	1.06	(0.015)	1.26	(0.029)	< 0.001
VaR(0.95)	1.66	(0.027)	1.86	(0.080)	0.003
ES(0.95)	2.29	(0.048)	2.56	(0.12)	0.009

(e) Carhart model, Crash Below -1.75σ

Signal	Negative		Positive		p-value
# Obs	38319		1137		
Mean	0.011	(0.008)	0.12	(0.048)	< 0.001
Median	−0.014	(0.008)	0.087	(0.044)	0.001
Volatility	1.08	(0.015)	1.28	(0.033)	< 0.001
VaR(0.95)	1.67	(0.028)	1.94	(0.091)	< 0.001
ES(0.95)	2.30	(0.045)	2.58	(0.13)	0.014

(f) Carhart model, Crash Below -2.25σ

Signal	Negative		Positive		p-value
# Obs	37974		1482		
Mean	0.011	(0.008)	0.093	(0.045)	0.002
Median	−0.014	(0.008)	0.057	(0.040)	0.004
Volatility	1.08	(0.015)	1.30	(0.031)	< 0.001
VaR(0.95)	1.67	(0.028)	1.96	(0.084)	< 0.001
ES(0.95)	2.30	(0.044)	2.65	(0.14)	0.001

This table reports summary statistics and downside risk measures for the pooled set of standardized abnormal returns resulting from different crash sizes in the bubble detection method. A crash is defined as a residual below -1.75 (Panels A, C, and E) or -2.25 standard deviations (Panels B, D, and F). The abnormal returns are based on rolling regressions of the CAPM (Panels A and B), the Fama-French model (Panels C and D) and the Carhart model (Panels E and F) in Equation (1). For each regression, we construct an abnormal return for the period after the estimation window as in Equation (3). To correct for time-varying volatility, we standardize the abnormal return by dividing it by the residual volatility of the regression model. We split the abnormal returns according to the detection of a bubble. In the column titled Negative (Positive) we report the results for negative (positive) bubble signals. For each statistic, we construct standard errors, reported in parentheses, and p-values based on 1,000 temporal bootstraps. The column titled p-value reports the results of tests for equality of the statistics for positive and negative bubble signals under the null hypothesis of no distributional difference.

Table A.8: Optimal Portfolio Choice and Expected Utility for Different Crash Sizes

Model	Size	w_{NB}		w_{B}		p-value	$V_{\text{B}}/V_{\text{NB}}$	p-value	λ	p-value
CAPM	Small	0.10	(0.12)	1.70	(0.38)	< 0.001	3.16	< 0.001	7.15	< 0.001
	Normal	0.08	(0.12)	1.70	(0.35)	< 0.001	3.18	< 0.001	7.38	< 0.001
	Large	0.07	(0.12)	1.67	(0.33)	< 0.001	3.13	< 0.001	7.27	< 0.001
Fama-French	Small	0.00	(0.10)	0.91	(0.38)	0.009	1.64	0.007	2.35	0.005
	Normal	0.00	(0.10)	0.93	(0.35)	0.003	1.66	0.001	2.43	< 0.001
	Large	0.00	(0.10)	0.83	(0.34)	0.007	1.53	0.005	1.94	0.003
Carhart	Small	0.13	(0.09)	1.00	(0.38)	0.006	1.75	0.001	2.15	0.001
	Normal	0.12	(0.09)	0.91	(0.35)	0.010	1.63	0.004	1.76	0.006
	Large	0.13	(0.09)	0.72	(0.35)	0.031	1.41	0.012	1.04	0.026

This table reports the optimal portfolio w_{NB} (w_{B}) of a rational investor if she derives a negative (positive) bubble signal. We assume that the investor has a power-utility function with coefficient of relative risk aversion $\gamma = 2$. We report the optimal portfolios as fractions of wealth. We vary the size of the threshold for crashes between -1.75, -2 (the base case) and -2.25 of the standard deviation of the residual returns. The investment opportunity is the typical industry with idiosyncratic volatilities equal to their pooled averages (CAPM, 4.17%; Fama-French model, 3.86%; and Carhart model, 3.81%). The risk-free rate equals its long-term average of 0.305% per month. Abnormal returns are based on the CAPM and the Fama-French and Carhart models. Based on the optimal portfolios, we calculate the ratio of expected utilities $V_{\text{B}}/V_{\text{NB}}$. We calculate the certainty equivalent return λ that an investor requires for not changing his portfolio from w_{NB} to w_{B} in % per year. We use 1,000 temporal bootstraps to calculate standard errors (reported in parentheses) and p-values. We test $w_{\text{NB}} = w_{\text{B}}$, $V_{\text{B}} = V_{\text{NB}}$ and $\lambda = 0$ under the null hypothesis of no difference in distribution between the cases with positive and negative bubble signals.

Table A.9: Value-weighted Portfolios based on Bubble Signals

(a) Portfolio characteristics

	CAPM		Fama-French model		Carhart model	
	Bubble	No-Bubble	Bubble	No-Bubble	Bubble	No-Bubble
Months with signals	69%	100%	64%	100%	64%	100%
Average No. of Industries	3.25	42.78	2.40	43.48	2.40	43.52
Std. Dev. of No. of Industries	2.64	3.69	1.49	3.26	1.51	3.25
Average Portfolio Turnover	1.76	0.95	1.34	0.95	1.32	0.95
Average return	1.63%	0.92%	1.37%	0.92%	1.52%	0.90%
Volatility	6.33%	4.54%	6.25%	4.53%	6.27%	4.53%
Average abnormal return	0.67%	0.00%	0.17%	-0.01%	0.25%	0.02%
Abnormal return volatility	4.11%	0.43%	3.87%	0.48%	3.68%	0.44%
Information Ratio	0.163	0.000	0.043	-0.011	0.067	0.034
VaR(0.95)	-4.92%	-0.47%	-5.97%	-0.56%	-5.57%	-0.51%
ES(0.95)	-6.94%	-0.93%	-8.21%	-1.05%	-7.94%	-1.06%

(b) Tests on equality of average abnormal returns

	CAPM		Fama-French model		Carhart model	
	Bubble	No-Bubble	Bubble	No-Bubble	Bubble	No-Bubble
Average, OLS	0.75%	0.03%	0.19%	0.00%	0.28%	0.04%
Average, WLS	0.60%	-0.01%	0.32%	-0.01%	0.35%	0.02%
	(0.11)	(0.02)	(0.12)	(0.01)	(0.12)	(0.01)
	[< 0.001]		[0.005]		[0.005]	
Average, GLS	0.40%	-0.01%	0.21%	-0.01%	0.23%	0.03%
	(0.10)	(0.02)	(0.10)	(0.02)	(0.10)	(0.01)
Test, GLS	4.21		2.17		2.08	
	[< 0.001]		[0.030]		[0.038]	

This table reports descriptive statistics on bubble portfolio ("Bubble") and no-bubble portfolio ("No-Bubble") in Panel A. For every month $t + 1$, we form two value-weighted portfolios based on the signal we receive at the end of month t. The no-bubble portfolio consists of the industries for which we received a negative bubble signal. The bubble portfolio contains the industries for which we received a positive signal. If there is no positive signal, the bubble portfolio is not invested. The table shows the average number of industries in each portfolio per month and its standard deviation, and the percentage of months for which each portfolio is invested. We calculate the average turnover of the portfolio as the average absolute change in portfolio weights. For each portfolio, we report the average return and volatility, and the average abnormal return and its standard deviation in % per month, where the abnormal returns are corrected for the risk factor exposures of the industries as in Equation (3). The information ratio is calculated as the ratio of average excess return and its standard deviation. We also report the realized Value-at-Risk and Expected Shortfall for a confidence level of 95%, based on the abnormal returns. Panel B report the results of testing for equality of the average abnormal return of the bubble and no-bubble portfolios. We address heteroscedasticity in the portfolio returns by estimating the averages by WLS and (feasible) GLS-regressions. We use the variance and covariance of the residuals of Equation (1) to estimate the variance of the portfolio abnormal returns. Standard errors are in parenthesis. The rows title Test report the t-statistic for a equality in abnormal returns with p-values below in brackets.

Table A.10: Optimal Portfolio Choice With Utility Defined Over Higher-Order Moments

(a) Abnormal Returns Based on the CAPM

κ_2'	κ_3'	κ_4'	$w_{\mathrm{NB}}^{\mathrm{H}}$		$w_{\mathrm{B}}^{\mathrm{H}}$		p-value	$V_{\mathrm{B}}^{\mathrm{H}}/V_{\mathrm{NB}}^{\mathrm{H}}$	p-value	λ	p-value
1	1	1	0.08	(0.12)	1.67	(0.33)	< 0.001	3.17	< 0.001	7.31	< 0.001
1	2	1	0.08	(0.12)	1.96	(0.49)	< 0.001	3.06	< 0.001	8.24	< 0.001
1	4	1	0.08	(0.12)	5.98	(2.55)	< 0.001	2.02	< 0.001	20.93	< 0.001
1	1	2	0.08	(0.12)	1.56	(0.28)	< 0.001	3.42	< 0.001	7.01	< 0.001
1	1	4	0.08	(0.12)	1.42	(0.23)	< 0.001	6.79	< 0.001	6.59	< 0.001
1	2	2	0.08	(0.12)	1.75	(0.35)	< 0.001	3.09	< 0.001	7.73	< 0.001
1	4	4	0.08	(0.12)	1.91	(0.38)	< 0.001	2.97	< 0.001	8.70	< 0.001
1	0	0	0.08	(0.12)	1.59	(0.31)	< 0.001	3.28	< 0.001	6.93	< 0.001
2	0	0	0.04	(0.06)	0.79	(0.15)	< 0.001	3.54	< 0.001	3.44	< 0.001

(b) Abnormal Returns Based on the Fama-French model

κ_2'	κ_3'	κ_4'	$w_{\mathrm{NB}}^{\mathrm{H}}$		$w_{\mathrm{B}}^{\mathrm{H}}$		p-value	$V_{\mathrm{B}}^{\mathrm{H}}/V_{\mathrm{NB}}^{\mathrm{H}}$	p-value	λ	p-value
1	1	1	0.00	(0.10)	0.92	(0.35)	0.003	1.66	0.002	2.43	0.003
1	2	1	0.00	(0.10)	0.96	(0.38)	0.005	1.65	0.002	2.49	0.003
1	4	1	0.00	(0.10)	1.05	(0.68)	0.010	1.32	0.003	2.65	0.003
1	1	2	0.00	(0.10)	0.91	(0.33)	0.002	1.68	0.002	2.41	0.003
1	1	4	0.00	(0.10)	0.89	(0.30)	0.002	1.72	0.002	2.37	0.003
1	2	2	0.00	(0.10)	0.94	(0.36)	0.005	1.65	0.002	2.47	0.003
1	4	4	0.00	(0.10)	0.98	(0.39)	0.006	1.64	0.003	2.57	0.003
1	0	0	0.00	(0.10)	0.91	(0.33)	0.002	1.67	0.002	2.38	0.003
2	0	0	0.00	(0.05)	0.45	(0.17)	0.002	1.70	0.002	1.19	0.003

(c) Abnormal Returns Based on the Carhart model

κ_2'	κ_3'	κ_4'	$w_{\mathrm{NB}}^{\mathrm{H}}$		$w_{\mathrm{B}}^{\mathrm{H}}$		p-value	$V_{\mathrm{B}}^{\mathrm{H}}/V_{\mathrm{NB}}^{\mathrm{H}}$	p-value	λ	p-value
1	1	1	0.12	(0.09)	0.91	(0.36)	0.008	1.63	0.001	1.76	0.001
1	2	1	0.13	(0.09)	0.94	(0.40)	0.009	1.62	0.001	1.82	0.001
1	4	1	0.13	(0.09)	1.02	(0.68)	0.012	1.31	0.001	1.95	0.001
1	1	2	0.12	(0.09)	0.90	(0.35)	0.008	1.64	0.001	1.74	0.001
1	1	4	0.12	(0.09)	0.87	(0.32)	0.007	1.68	0.001	1.71	0.003
1	2	2	0.13	(0.09)	0.93	(0.38)	0.009	1.62	0.001	1.80	0.001
1	4	4	0.13	(0.09)	0.96	(0.41)	0.010	1.61	0.001	1.87	0.001
1	0	0	0.12	(0.09)	0.89	(0.35)	0.007	1.64	0.001	1.73	0.001
2	0	0	0.06	(0.04)	0.44	(0.17)	0.007	1.66	0.001	0.86	0.003

This table reports the optimal portfolio weight after a negative (positive) bubble signal $w_{\mathrm{NB}}^{\mathrm{H}}$ ($w_{\mathrm{B}}^{\mathrm{H}}$). The investor's utility function is defined over moments up to order four of the portfolio return distribution, $\mathrm{E}\left[U'(W_{t+1})\right] = \mathrm{E}[r_{\mathrm{f},t+1} + w\eta_{t+1}] - \kappa_2' \,\mathrm{E}\left[(r_{\mathrm{f},t+1} + w\eta_{t+1})^2\right] + \kappa_3' \,\mathrm{E}\left[(r_{\mathrm{f},t+1} + w\eta_{t+1})^3\right] - \kappa_4' \,\mathrm{E}\left[(r_{\mathrm{f},t+1} + w\eta_{t+1})^4\right]$. We consider different values for the moment weights κ_2', κ_3' and κ_4'. The base case of $\kappa_2' = \kappa_3' = \kappa_4' = 1$ is the approximation of a power-utility function with $\gamma = 2$. We consider abnormal returns η_{t+1} constructed from the CAPM, the Fama-French and the Carhart models. The investment opportunity is the typical industry with idiosyncratic volatilities equal to their pooled averages (CAPM, 4.17%; Fama-French model, 3.86%; and Carhart model, 3.81%). The risk-free rate equals its long-term average of 0.305% per month. Based on the optimal portfolios, we calculate the ratio of expected utilities, $V_{\mathrm{B}}^{\mathrm{H}}/V_{\mathrm{NB}}^{\mathrm{H}}$. We also calculate the certainty equivalent return λ that an investor requires for not changing his portfolio from $w_{\mathrm{NB}}^{\mathrm{H}}$ to $w_{\mathrm{B}}^{\mathrm{H}}$. We use 1,000 temporal bootstraps to calculate standard errors (reported in parentheses). We test $w_{\mathrm{NB}}^{\mathrm{H}} = w_{\mathrm{B}}^{\mathrm{H}}$, $V_{\mathrm{B}}^{\mathrm{H}} = V_{\mathrm{NB}}^{\mathrm{H}}$ and $\lambda = 0$ under the null hypothesis of no difference in distribution between the cases with positive and negative bubble signals.

Table A.11: Optimal Portfolio Choice of a Mean-Semivariance Investor

	$w_{\text{NB}}^{\text{SV}}$		w_{B}^{SV}		p-value	$V_{\text{B}}^{\text{SV}}/V_{\text{NB}}^{\text{SV}}$	p-value	λ	p-value
CAPM	0.08	(0.13)	2.38	(0.65)	< 0.001	4.11	< 0.001	10.61	< 0.001
Fama-French	0.02	(0.10)	1.10	(0.49)	0.006	1.79	0.003	2.79	0.003
Carhart	0.13	(0.10)	1.08	(0.51)	0.008	1.77	0.001	2.18	0.001

This table reports the optimal weight after a negative (positive) bubble signal, that is w_{NB}^{H} (w_{B}^{H}). The investor is equipped with a mean-semivariance utility function. The optimal portfolio is given by $w_B^{\text{SV}} = \text{E}[\eta_{t+1}|B_t]/(2\gamma \text{SV}_0[\text{sgn}(\text{E}[\eta_{t+1}|B_t]) \cdot \eta_{t+1}|B_t])$. We set the parameter γ of aversion to semivariance equal to two. We consider abnormal returns η_{t+1} constructed based on the CAPM, the Fama-French model and Carhart model. The investment opportunity is the typical industry with idiosyncratic volatilities equal to their pooled averages (CAPM, 4.17%; Fama-French model, 3.86%; and Carhart model, 3.81%). The risk-free rate equals its long-term average of 0.305% per month. Based on the optimal portfolios, we calculate the ratio of expected utilities, $V_{\text{B}}^{\text{SV}}/V_{\text{NB}}^{\text{SV}}$. We also calculate the certainty equivalent return λ that an investor requires for not changing his portfolio from $w_{\text{NB}}^{\text{SV}}$ to w_{B}^{SV}. We use 1,000 temporal bootstraps to calculate standard errors (reported in parentheses). We test $w_{\text{NB}}^{\text{SV}} = w_{\text{B}}^{\text{SV}}$, $V_{\text{B}}^{\text{SV}} = V_{\text{NB}}^{\text{SV}}$ and $\lambda = 0$ under the null hypothesis of no difference in distribution after positive and negative bubble signals.

B Industry Specific Information

To ensure that our findings are not driven by a few industries we replicate our main results for each industry. Table B.1 provides descriptive statistics on the industry returns. The average return on the industries is 12.4% per year and its standard deviation is 26.2%. The minimum and maximum values indicate that several industries experienced extreme returns. We investigate this issue further and find that most of these extreme values occur during the Great Depression at the beginning of our sample period.

[Table 1 about here.]

Tables B.2 to B.4 present the abnormal return estimates for each industry. For all three asset pricing models, we find that the returns of a few industries are significantly larger or smaller than zero. The deviations are however generally economically small and do not show a clear pattern. The average of the mean abnormal returns across industries is indistinguishable from zero for the Fama-French model. For the Carhart model and the CAPM it is significantly positive. However, it is economically small because it is only 0.02 for the CAPM and 0.01 for the Carhart model. The standardized volatility estimates of the industries often differ significantly from one for all three asset pricing models. The deviations are always positive, indicating that the asset pricing models commonly predict too low residual volatilities. The results also show that the abnormal industry returns are sometimes positively or negatively skewed, but pooled skewness is positive for all asset pricing models. Kurtosis is always larger than three, indicating fat tails. The market beta for the pooled set of industry returns is close to one and the coefficients of HML and MOM are close to zero. However, some of the industries have a positive or negative exposure to these factors. The pooled set of abnormal returns has a slightly positive exposure to SMB, perhaps because we equal weight the industries in our analysis.

[Table 2 about here.]

[Table 3 about here.]

[Table 4 about here.]

Tables B.5 to B.7 correspond to Table 7 in the paper. They present descriptive statistics of the bubble signals and bubbles. We first report descriptive statistics on the signals. Subsequently, we define one uninterrupted sequence of bubble signals, starting from the earliest breakpoint, as "one bubble" and present information on these bubbles. The bubble signals we receive are widely distributed across the 48 industries. For most industries the fraction of observations identified as a possible bubble signal is below ten percent. The only exception to that is the computer industry ("Comps") for the Fama-French model and the real estate industry ("RlEst") for the CAPM-based results. That is not a surprise because these two industries experienced a major bubble during our sample period. The first intercept, $\alpha^{\mathrm{a}}_{i,t}$, tends to be slightly negative for most industries, but it is economically small. The intercept following the breakpoint, $\alpha^{\mathrm{p}}_{i,t}$, is in all cases positive and in most cases large in magnitude. Consistent with our method to detect the bubble signal, the measure "Strength" then also shows that $\alpha^{\mathrm{p}}_{i,t}$ easily passes the test for being significantly positive for most industries. While for some of the industries average number of months from the breakpoint to the signal is shorter or longer, we do not see here either that any of the industries could be dominating our findings.

The last three columns of each table show the number of bubbles for each industry. Most industries experience at least one bubble period. An exception to this is the utility industry which does not experience any bubble for any of the three asset pricing models. For the CAPM, we also observe that the industry “FabPr” did not experience a bubble and for the Fama-French and Carhart models the “Guns” industry did not experience a bubble. Part of the explanation could be that we have a shorter time-series for some of these industries. However, there is also no evidence that any of these industries experienced a famous bubble (see Baker and Wurgler (2006)) over the last century.

Across all industries, we find large positive abnormal returns and raw returns during bubbles. The annual raw returns during bubbles for the different asset pricing models range from a minimum of about 10% to a maximum of about 126%. The maximum of 126% is for the gold industry. It actually represents the 1979-1980 boom in gold prices. The standardized abnormal returns are centered around their pooled average, ranging from about 0.25 to one.

[Table 5 about here.]

[Table 6 about here.]

[Table 7 about here.]

Table B.8 presents the abnormal returns after positive bubble signals. For most industries the average abnormal returns after bubbles are positive. However, for none of these industries is the return extraordinarily large in the sense that it would disproportionately affect our results. The positive abnormal returns of many industries contribute to the overall positive average abnormal returns. For few a industries, we find negative abnormal returns following bubbles. This finding indicates that the investor might also only experience the deflation or crash following a bubble. For the gold industry, for example, the return is very negative. This finding stresses the fact that riding bubbles can be risky, as they can deflate or crash very quickly.

[Table 8 about here.]

Table B.1: Descriptive Statistics of Industry Returns

Industry	Start Date	# Obs.	Mean	Volatility	Skewness	Kurtosis	Min.	Max.
Agric	1926:07	1002	11.9	26.2	1.65	22.1	−36.5	91.3
Food	1926:07	1002	11.8	16.9	0.02	5.7	−27.7	32.8
Soda	1963:07	558	13.6	23.1	0.15	4.0	−26.3	38.9
Beer	1926:07	1002	14.7	25.8	1.81	22.1	−29.2	89.2
Smoke	1926:07	1002	13.7	20.4	0.06	3.4	−25.0	33.3
Toys	1926:07	1002	12.1	35.5	2.78	36.2	−43.3	140.4
Fun	1926:07	1002	14.3	32.8	0.63	9.2	−44.5	69.8
Books	1926:07	1002	11.8	27.0	0.85	7.2	−34.9	56.0
Hshld	1926:07	1002	11.1	21.0	0.35	12.6	−35.3	59.2
Clths	1926:07	1002	10.4	21.2	0.31	4.8	−30.9	41.2
Hlth	1969:07	486	12.3	30.1	−0.08	2.7	−41.1	36.4
MedEq	1926:07	1002	13.3	22.1	−0.13	1.8	−26.6	30.5
Drugs	1926:07	1002	13.2	20.5	0.26	7.4	−35.6	40.3
Chems	1926:07	1002	12.5	22.1	0.34	6.7	−33.3	47.0
Rubbr	1944:07	786	12.6	20.2	−0.13	2.7	−30.5	32.1
Txtls	1926:07	1002	11.3	28.0	1.01	9.6	−32.6	59.3
BldMt	1926:07	1002	11.4	24.2	0.37	6.3	−32.3	41.8
Cnstr	1926:07	1002	12.3	33.5	0.90	6.8	−38.0	67.8
Steel	1926:07	1002	11.6	29.6	1.34	13.6	−32.5	80.8
FabPr	1963:07	558	7.1	25.6	−0.15	2.8	−28.9	39.5
Mach	1926:07	1002	12.5	25.4	0.45	7.4	−33.4	51.9
ElcEq	1926:07	1002	14.3	26.8	0.58	8.6	−34.5	59.6
Autos	1926:07	1002	13.0	28.1	1.20	14.4	−36.4	81.9
Aero	1926:07	1002	16.9	33.0	0.91	7.7	−40.4	72.0
Ships	1926:07	1002	11.2	27.8	0.76	7.5	−34.4	63.4
Guns	1963:07	558	13.3	23.8	−0.11	1.8	−30.1	32.9
Gold	1963:07	558	13.2	36.1	0.77	5.6	−33.7	78.5
Mines	1926:07	1002	12.6	24.5	0.03	4.1	−34.5	46.1
Coal	1926:07	1002	15.5	31.8	0.87	6.8	−38.0	77.5
Oil	1926:07	1002	12.9	21.2	0.28	4.1	−29.7	39.2
Util	1926:07	1002	10.5	19.7	0.07	7.5	−33.0	43.2
Telcm	1926:07	1002	9.9	16.1	−0.01	3.2	−21.6	28.2
PerSv	1927:07	990	11.6	33.0	1.52	13.4	−39.3	84.8
BusSv	1926:07	1002	12.5	26.2	0.45	7.9	−40.4	56.7
Comps	1926:07	1002	14.8	25.7	0.07	4.5	−34.6	53.4
Chips	1926:07	1002	13.4	30.8	0.45	6.0	−42.1	64.7
LabEq	1926:07	1002	13.1	24.3	−0.27	1.9	−33.2	25.4
Paper	1936:07	882	12.9	23.4	0.29	5.0	−35.8	47.9
Boxes	1926:07	1002	12.8	21.7	0.15	5.6	−29.3	43.4
Trans	1926:07	1002	10.7	25.3	1.07	13.0	−34.5	65.4
Whlsl	1926:07	1002	9.6	26.1	0.58	11.1	−44.5	59.2
Rtail	1926:07	1002	11.8	20.9	0.01	5.0	−30.3	37.8
Meals	1926:07	1002	12.4	23.3	−0.34	2.6	−31.3	31.5
Banks	1926:07	1002	14.3	25.1	0.12	4.9	−33.7	42.3
Insur	1926:07	1002	12.3	26.6	1.03	15.6	−45.4	73.7
RlEst	1926:07	1002	9.1	34.2	0.77	6.9	−52.6	59.3
Fin	1926:07	1002	13.3	27.3	0.49	9.1	−39.2	67.2
Other	1926:07	1002	8.9	26.0	−0.01	3.5	−33.3	41.9
Pooled	-	45456	12.4	26.2	0.78	11.8	−52.6	140.4

This table reports summary statistics on the 48 US industries as defined in Fama and French (1997). For each industry we report the start date, the number of available return observations, their mean (in % per year), volatility (in % per year), skewness, kurtosis, minimum (in %) and maximum (in %).

Table B.2: Summary Statistics of Abnormal Returns (CAPM)

Industry	$\bar{\beta}$	Mean	Volatility	Skewness	Kurtosis	Min.	Max.
Agric	0.89	0.02	1.01	0.32*	4.56	−3.39	5.54
Food	0.74	0.08*	1.09*	0.36*	5.70	−4.90	5.66
Soda†	0.86	0.00	1.22*	0.20	5.71	−5.17	5.91
Beer	0.90	0.06	1.02	0.36*	7.04	−4.22	7.12
Smoke	0.66	0.09*	1.09*	−0.05	4.01	−4.97	4.08
Toys	1.21	−0.02	1.00	0.21*	5.91	−5.49	5.02
Fun	1.28	0.04	1.02	0.12	5.24	−5.15	5.67
Books	1.14	−0.01	1.02	0.05	5.79	−5.92	5.72
Hshld	0.89	0.03	1.04	−0.26*	6.55	−6.27	5.03
Clths	0.95	−0.01	1.06*	0.08	4.80	−4.96	4.51
Health†	1.08	−0.04	1.02	−0.49*	5.35	−4.76	2.89
MedEq	0.94	0.05	1.00	−0.07	4.66	−4.44	5.24
Drugs	0.88	0.08*	1.05*	0.05	6.33	−5.64	6.32
Chems	1.01	−0.02	1.05*	0.24*	5.15	−4.07	6.21
Rubbr†	1.04	0.02	1.11*	0.29*	5.70	−4.26	5.32
Txtls	1.09	−0.01	1.11*	0.26*	6.24	−3.90	6.98
BldMt	1.13	−0.04	1.08*	0.08	5.60	−4.69	5.67
Cnstr	1.38	−0.02	1.01	0.55*	5.32	−4.44	5.60
Steel	1.27	−0.04	1.06*	0.74*	6.56	−2.98	7.59
FabPr†	1.06	−0.07	1.09*	0.23*	4.67	−3.80	4.84
Mach	1.18	−0.04	1.04	0.19*	3.82	−3.84	3.68
ElcEq	1.21	0.03	1.01	0.12	3.66	−4.09	4.29
Autos	1.11	−0.01	1.06*	0.29*	7.63	−6.52	6.17
Aero	1.14	0.04	1.00	0.08	5.07	−4.55	3.77
Guns†	0.83	0.05	1.12*	−0.80*	8.53	−7.18	4.51
Gold†	0.69	0.03	1.13*	1.14*	9.47	−3.19	8.35
Ships	1.09	−0.02	1.05*	0.69*	6.73	−3.92	6.63
Mines	0.97	0.03	1.06*	0.21*	3.82	−3.56	4.81
Coal	1.00	0.06	1.12*	0.93*	7.97	−4.05	8.15
Oil	0.87	0.06	1.04	0.27*	4.05	−3.60	4.21
Util	0.66	0.04	1.04	0.07	3.75	−3.30	3.69
Telcm	0.64	0.08*	1.08*	1.03*	9.74	−3.96	9.18
PerSv†	1.08	−0.01	1.02	−0.02	4.22	−4.29	4.13
BusSv	1.09	0.04	1.01	0.42*	4.49	−3.14	4.20
Comps	1.15	0.06	1.06*	0.44*	5.04	−4.14	5.65
Chips	1.37	0.00	1.02	0.11	4.50	−4.99	4.97
LabEq	1.14	0.04	1.04	0.32*	4.94	−4.03	5.63
Paper†	1.12	−0.02	1.04	0.48*	6.09	−4.36	6.62
Boxes	0.98	0.04	1.04	−0.21*	3.85	−4.07	3.33
Trans	1.13	−0.03	1.02	0.48*	4.95	−3.40	5.44
Whshl	1.08	0.00	1.03	0.17*	6.78	−6.27	5.56
Rtail	0.96	0.04	1.04	−0.15	4.33	−4.08	4.56
Meals	1.01	0.05	1.02	0.14	4.69	−4.73	3.89
Banks	0.96	0.02	1.02	−0.27*	5.39	−5.02	4.21
Insur	0.97	0.01	1.03	0.07	6.18	−5.73	5.58
RlEst	1.18	−0.06	1.11*	0.64*	11.63	−6.19	9.15
Fin	1.23	0.04	1.04	−0.17*	5.89	−6.61	4.05
Other	1.11	−0.04	1.07*	−0.09	4.58	−4.36	4.28
Pooled	1.03	0.02*	1.05*	0.22*	5.75	−7.18	9.18

This table reports the results of the rolling regressions of the market model in Equation (1) with a 120-month estimation window. For each regression, we construct an abnormal return for the period after the estimation window as in Equation (3). To correct for time-varying volatility, we standardize the abnormal return by dividing it by the residual volatility of the regression model. A dagger after an industry name indicates that fewer observations are available. An asterisk denotes a significant difference from zero in case of means and skewness coefficients, and a significant difference from one in case of volatility, all at a 5% confidence level. We calculate standard errors of the skewness coefficient as $\sqrt{6/T}$ (see Tabachnick and Fidell, 2001).

Table B.3: Summary Statistics of Abnormal Returns (Fama-French model)

industry	$\bar{\beta}_{\mathrm{RM}}$	$\bar{\beta}_{\mathrm{SMB}}$	$\bar{\beta}_{\mathrm{HML}}$	Mean	Vol.	Skew.	Kurt.	Min.	Max.
Agric	0.85	0.47	−0.05	0.01	1.02	0.52*	5.12	−3.25	6.20
Food	0.78	0.00	−0.02	0.09*	1.11*	0.40*	5.44	−4.83	5.56
Soda†	0.99	−0.10	0.22	0.00	1.23*	0.57*	6.62	−4.46	6.36
Beer	0.89	0.26	0.00	0.05	1.05*	0.25*	6.54	−3.94	7.08
Smoke	0.72	−0.05	0.07	0.08*	1.11*	−0.12	4.18	−5.58	4.11
Toys	1.08	0.83	−0.16	−0.02	1.01	0.19*	5.62	−5.52	5.27
Fun	1.20	0.56	−0.04	0.03	1.03	0.00	6.33	−6.78	5.10
Books	1.06	0.49	−0.04	−0.01	1.03	−0.04	5.60	−6.40	5.23
Hshld	0.92	0.04	−0.21	0.07	1.08*	−0.68*	13.16	−9.91	5.38
Clths	0.89	0.53	0.17	−0.06	1.09*	0.01	6.48	−5.60	5.80
Health†	1.03	0.61	−0.02	−0.04	1.09*	−1.10*	9.27	−7.11	2.96
MedEq	0.85	0.19	−0.26	0.09*	1.04	−0.03	4.75	−4.83	5.23
Drugs	0.89	−0.14	−0.42	0.14*	1.09*	0.04	4.98	−5.16	5.02
Chems	1.12	−0.12	0.07	−0.05	1.07*	0.21*	4.90	−4.04	5.93
Rubbr†	0.96	0.66	0.16	−0.03	1.11*	0.29*	4.97	−4.18	4.88
Txtls	1.04	0.67	0.42	−0.07	1.13*	0.45*	6.12	−4.21	6.65
BldMt	1.16	0.23	0.04	−0.08*	1.09*	0.24*	4.65	−4.56	5.23
Cnstr	1.27	0.69	0.25	−0.06	1.02	0.27*	5.19	−4.44	5.70
Steel	1.23	0.24	0.49	−0.10*	1.10*	0.57*	5.93	−3.72	7.31
FabPr†	1.03	0.64	0.13	−0.09	1.12*	0.19	4.57	−3.60	4.93
Mach	1.16	0.27	0.17	−0.08*	1.06*	0.19*	3.76	−3.86	3.54
ElcEq	1.19	0.04	−0.19	0.05	1.04	0.12	3.39	−3.64	4.41
Autos	1.20	0.10	0.33	−0.07	1.08*	0.36*	6.37	−5.52	6.30
Aero	1.06	0.53	0.20	0.02	1.02	0.15	4.71	−4.86	4.17
Guns†	0.91	0.24	0.41	0.00	1.13*	−0.48*	8.62	−7.24	5.72
Gold†	0.69	0.57	0.27	0.01	1.15*	1.33*	10.58	−3.63	8.71
Ships	1.03	0.33	0.41	−0.04	1.09*	0.64*	5.90	−4.52	6.18
Mines	0.94	0.41	0.33	0.00	1.08*	0.26*	4.15	−3.84	5.34
Coal	0.89	0.35	0.41	0.06	1.14*	0.87*	7.21	−4.15	7.51
Oil	0.96	−0.37	0.31	0.03	1.07*	0.11	3.96	−3.52	4.03
Util	0.77	−0.10	0.23	0.01	1.07*	0.05	3.90	−3.61	3.92
Telcm	0.70	−0.13	0.04	0.07	1.10*	1.07*	10.45	−3.69	9.61
PerSv†	0.96	0.69	−0.11	−0.01	1.05	−0.18*	5.15	−5.33	3.97
BusSv	0.89	0.49	−0.23	0.09*	1.02	0.34*	4.68	−3.65	5.06
Comps	1.07	0.08	−0.52	0.12*	1.05*	0.44*	4.89	−4.00	5.69
Chips	1.24	0.37	−0.34	0.03	1.04	0.01	4.07	−4.88	4.19
LabEq	1.07	0.30	−0.46	0.08*	1.05	0.23*	4.15	−3.76	4.30
Paper†	1.20	0.08	0.27	−0.07	1.04	0.39*	5.62	−4.32	6.25
Boxes	1.01	0.00	−0.05	0.05	1.07*	−0.25*	3.95	−4.22	3.61
Trans	1.05	0.24	0.54	−0.10*	1.06*	0.20*	4.45	−4.01	5.58
Whshl	0.98	0.60	−0.06	−0.01	1.04	−0.10	7.15	−5.52	5.55
Rtail	0.97	0.16	−0.14	0.06	1.08*	−0.12	4.55	−4.59	4.82
Meals	0.94	0.57	−0.04	0.05	1.05*	0.06	4.49	−4.96	4.11
Banks	1.08	0.09	0.09	0.00	1.03	0.20*	4.34	−4.03	4.14
Insur	1.07	−0.13	0.01	0.00	1.02	0.33*	4.45	−3.78	4.46
RlEst	1.06	1.04	0.26	−0.13*	1.12*	0.92*	11.33	−5.20	9.31
Fin	1.21	0.17	0.26	0.01	1.07*	−0.11	5.80	−6.90	4.10
Other	1.06	0.41	−0.21	−0.03	1.12*	0.17*	5.17	−4.61	5.48
Pooled	1.01	0.29	0.05	0.01	1.07*	0.21*	5.77	−9.91	9.61

This table reports the results of the rolling regressions of the Fama-French model in Equation (1) with a 120-month estimation window. For each regression, we construct an abnormal return for the period after the estimation window as in Equation (3). To correct for time-varying volatility, we standardize the abnormal return by dividing by the residual volatility of the regression model. A dagger after an industry name indicates that fewer observations are available. An asterisk denotes a significant difference from zero in case of means and skewness coefficients, and a significant difference from one in case of volatility, all at a 5% confidence level. We calculate standard errors of the skewness coefficient as $\sqrt{6/T}$ (see Tabachnick and Fidell, 2001).

Table B.4: Summary Statistics of Abnormal Returns (Carhart model)

Industry	$\bar{\beta}_{\text{RM}}$	$\bar{\beta}_{\text{SMB}}$	$\bar{\beta}_{\text{HML}}$	$\bar{\beta}_{\text{MOM}}$	Mean	Vol.	Skew.	Kurt.	Min.	Max.
Agric	0.85	0.48	−0.04	0.06	−0.01	1.03	0.53*	5.18	−3.21	6.26
Food	0.78	0.00	−0.01	0.00	0.10*	1.13*	0.48*	4.97	−3.54	5.50
Soda†	0.98	−0.09	0.19	−0.07	0.03	1.25*	0.55*	6.45	−4.51	6.37
Beer	0.89	0.26	0.03	0.08	0.05	1.07*	0.26*	6.47	−3.89	7.26
Smoke	0.72	−0.05	0.05	−0.06	0.10*	1.12*	−0.09	4.00	−5.47	4.08
Toys	1.08	0.86	−0.13	0.06	−0.03	1.02	0.23*	5.16	−4.98	5.13
Fun	1.18	0.58	−0.04	0.04	0.01	1.04	−0.18*	6.53	−7.37	4.84
Books	1.06	0.49	−0.06	−0.03	0.01	1.07*	0.10	5.79	−6.38	6.12
Hshld	0.93	0.06	−0.20	0.06	0.05	1.09*	−0.65*	13.95	−10.27	5.51
Clths	0.89	0.51	0.16	−0.14	−0.01	1.11*	0.16	5.66	−5.07	6.07
Health†	1.04	0.60	−0.01	0.06	−0.04	1.12*	−0.85*	7.46	−6.11	3.13
MedEq	0.86	0.21	−0.24	0.07	0.08*	1.05*	−0.02	4.86	−4.67	5.28
Drugs	0.90	−0.12	−0.43	0.05	0.12*	1.11*	−0.01	5.00	−5.18	5.19
Chems	1.11	−0.12	0.05	−0.06	−0.02	1.10*	0.06	4.79	−4.46	5.25
Rubbr†	0.96	0.68	0.17	0.02	−0.03	1.12*	0.22*	4.90	−4.23	4.76
Txtls	1.03	0.65	0.40	−0.13	−0.04	1.13*	0.31*	5.03	−3.69	6.39
BldMt	1.16	0.23	0.04	−0.04	−0.06	1.11*	0.27*	4.72	−4.46	5.22
Cnstr	1.28	0.67	0.24	−0.04	−0.04	1.03	0.31*	4.98	−4.19	5.68
Steel	1.23	0.24	0.47	−0.10	−0.08*	1.11*	0.52*	5.56	−3.61	6.87
FabPr†	1.01	0.66	0.10	−0.10	−0.08	1.13*	0.28*	4.14	−3.71	4.32
Mach	1.15	0.27	0.15	−0.09	−0.05	1.07*	0.21*	3.65	−3.82	4.00
ElcEq	1.20	0.05	−0.18	0.00	0.05	1.05*	0.10	3.67	−4.54	4.68
Autos	1.19	0.07	0.30	−0.19	−0.01	1.08*	0.14	5.89	−5.89	6.25
Aero	1.05	0.56	0.23	0.07	0.01	1.04	0.13	4.65	−4.45	4.27
Guns†	0.90	0.24	0.40	−0.04	0.01	1.14*	−0.38*	8.37	−7.20	5.87
Gold†	0.72	0.61	0.29	0.14	0.00	1.16*	1.24*	10.21	−3.91	8.66
Ships	1.03	0.33	0.40	−0.03	−0.04	1.11*	0.60*	6.40	−4.74	6.33
Mines	0.94	0.41	0.31	−0.06	0.01	1.09*	0.23*	3.95	−3.52	5.26
Coal	0.91	0.33	0.41	0.01	0.07	1.15*	0.84*	6.99	−4.22	7.47
Oil	0.96	−0.37	0.31	0.04	0.03	1.10*	0.03	3.77	−3.55	4.01
Util	0.76	−0.11	0.22	−0.03	0.02	1.08*	0.09	3.93	−3.87	3.80
Telcm	0.69	−0.12	0.02	−0.06	0.09*	1.12*	1.00*	9.80	−3.67	9.52
PerSv†	0.96	0.69	−0.10	−0.01	0.01	1.05*	−0.17*	4.93	−5.07	3.95
BusSv	0.89	0.51	−0.22	0.04	0.07*	1.04	0.30*	5.22	−4.99	5.26
Comps	1.07	0.10	−0.52	−0.01	0.11*	1.06*	0.49*	4.71	−3.61	5.63
Chips	1.23	0.37	−0.35	−0.05	0.04	1.05*	0.02	4.12	−4.92	4.33
LabEq	1.07	0.33	−0.45	0.05	0.07	1.06*	0.31*	4.49	−3.75	4.44
Paper†	1.20	0.07	0.23	−0.10	−0.03	1.06*	0.29*	5.38	−5.46	5.53
Boxes	1.01	0.02	−0.07	−0.03	0.05	1.09*	−0.19*	4.03	−4.51	4.58
Trans	1.04	0.23	0.52	−0.08	−0.08*	1.07*	0.18*	4.30	−4.08	5.44
Whshl	0.99	0.61	−0.04	0.08	−0.02	1.06*	−0.01	6.72	−5.45	5.66
Rtail	0.97	0.15	−0.16	−0.10	0.09*	1.09*	−0.10	4.36	−4.96	4.61
Meals	0.95	0.57	−0.04	−0.03	0.06	1.06*	0.12	4.23	−4.88	4.06
Banks	1.08	0.08	0.06	−0.13	0.04	1.04	0.15	4.03	−4.24	3.80
Insur	1.06	−0.13	0.01	−0.05	0.02	1.04	0.35*	4.37	−3.63	4.43
RlEst	1.06	1.07	0.26	0.03	−0.14*	1.11*	0.79*	10.14	−5.24	8.69
Fin	1.20	0.17	0.24	−0.05	0.03	1.09*	−0.11	5.74	−6.85	4.38
Other	1.06	0.40	−0.22	−0.02	−0.02	1.14*	0.17*	4.99	−4.59	5.43
Pooled	1.01	0.29	0.05	−0.02	0.01*	1.09*	0.20*	5.59	−10.27	9.52

This table reports the results of the rolling regressions of the Carhart (1997) Four-Factor Model (Carhart model) in Equation (1) with a 120-month estimation window. For each regression, we construct an abnormal return for the period after the estimation window as in Equation (3). To correct for time-varying volatility, we standardize the abnormal return by dividing it by the residual volatility of the regression model. A dagger after an industry name indicates that fewer observations are available. An asterisk denotes a significant difference from zero in case of means and skewness coefficients, and a significant difference from one in case of volatility, all at a 5% confidence level. We calculate standard errors of the skewness coefficient as $\sqrt{6/T}$ (see Tabachnick and Fidell, 2001).

Table B.5: Statistics of Positive Bubble Signals and Bubbles per Industry (CAPM)

Industry	% of sample	$\alpha^{a}_{i,t}$	$\alpha^{p}_{i,t}$	Strength	Length	# Bubbles	Return	StAR
Agric	1.59	−0.00057	0.042	3.40	21.0	2	40.7	0.55
Food	8.39	−0.00118	0.016	4.46	28.9	3	23.5	0.58
Soda†	4.11	−0.00505	0.028	2.93	23.0	2	37.7	0.44
Beer	1.93	−0.00541	0.025	2.87	26.1	2	41.3	0.41
Smoke	6.12	−0.00188	0.035	3.91	28.5	5	41.3	0.76
Toys	7.03	−0.00623	0.032	3.28	27.2	3	39.1	0.55
Fun	4.42	−0.00542	0.023	3.59	36.5	2	26.9	0.50
Books	8.16	−0.01479	0.028	3.08	37.7	4	34.7	0.39
Hshld	3.85	−0.00319	0.016	3.49	37.9	3	26.6	0.52
Clths	8.96	−0.00910	0.023	2.99	34.8	4	21.9	0.36
Hlth†	9.02	−0.01027	0.036	3.15	38.2	3	41.0	0.43
MedEq	5.44	−0.00516	0.023	2.96	36.4	5	35.0	0.45
Drugs	7.03	−0.00275	0.021	3.67	38.9	4	30.1	0.57
Chems	3.85	−0.00500	0.009	2.59	38.9	3	19.2	0.36
Rubber†	2.70	−0.00477	0.019	3.07	38.3	3	27.1	0.46
Txtls	6.35	−0.01235	0.019	2.53	37.0	3	25.6	0.37
BldMt	6.24	−0.00710	0.014	2.64	43.3	3	24.9	0.36
Cnstr	9.75	−0.00711	0.028	3.31	35.0	4	40.5	0.44
Steel	2.49	−0.00673	0.025	3.10	28.0	2	26.0	0.54
FabPr†	0.00	–	–	–	–	-	–	–
Mach	2.49	−0.00461	0.015	2.90	18.9	5	14.5	0.45
ElcEq	6.12	−0.00624	0.014	2.87	37.1	2	19.6	0.41
Autos	1.47	−0.00492	0.022	2.77	28.2	2	24.7	0.36
Aero	8.96	−0.00879	0.021	2.83	36.8	4	38.8	0.47
Guns†	4.34	−0.00085	0.048	3.45	21.8	2	33.8	0.76
Gold†	0.68	0.00419	0.076	3.56	14.7	1	95.1	0.78
Ships	1.70	−0.00211	0.041	3.73	19.7	3	39.2	0.83
Mines	7.71	−0.00449	0.024	3.16	37.2	4	31.5	0.49
Coal	4.65	−0.00370	0.049	3.48	21.9	4	51.7	0.66
Oil	1.25	−0.00180	0.020	2.98	19.8	2	37.7	0.59
Util	0.00	–	–	–	–	-	–	–
Telcm	2.95	0.00109	0.019	4.52	26.8	3	36.2	0.99
PerSv†	5.17	−0.01076	0.025	3.12	42.1	2	31.2	0.46
BusSv	7.60	−0.00458	0.015	2.83	35.6	5	19.2	0.48
Comps	5.67	−0.00410	0.024	3.13	27.4	3	42.5	0.54
Chips	4.20	−0.00525	0.025	3.61	27.4	2	44.7	0.64
LabEq	1.59	0.00114	0.028	3.52	26.7	3	31.0	0.56
Paper†	2.49	−0.00173	0.025	3.43	21.0	2	21.2	0.69
Boxes	7.82	−0.01158	0.021	2.62	38.9	3	19.3	0.27
Trans	6.58	−0.00569	0.015	2.68	33.6	3	12.0	0.45
Whlsl	2.61	−0.00339	0.022	3.94	27.0	3	13.3	0.61
Rtail	9.30	−0.00515	0.015	3.31	37.7	4	22.9	0.47
Meals	3.74	−0.00451	0.026	3.84	38.2	4	39.2	0.49
Banks	7.03	−0.00381	0.018	3.01	33.0	4	27.6	0.45
Insur	1.02	−0.00214	0.037	3.23	15.8	2	31.6	0.71
RlEst	10.66	−0.01067	0.025	2.90	38.5	3	28.1	0.31
Fin	8.62	−0.00420	0.014	3.72	37.1	3	27.1	0.55
Other	0.00	–	–	–	–	-	–	–
Pooled	4.95	−0.00609	0.023	3.21	33.9	3	30.2	0.49

This table reports for each industry the fraction of the complete sample that is classified as a positive bubble signal in percent (column 2), the average values of several properties of bubble signals (columns 3–6), and the number of bubbles (column 7), the average raw returns during bubbles (column 8, in % per year), and the average standardized abnormal return during bubbles (column 9, "StAR"). To construct the signals we regress the industry returns on a constant and the market return. If a ten-year series of industry returns shows evidence of an upward structural break in the constant and the constant is significantly positive after the break, an investor detects a bubble. A bubble has ended if a crash has occurred in the last six months, where a crash is defined as a residual below -2 times its standard deviation. Critical values for the structural break test correspond with a 97.5% confidence level, and are obtained from Andrews (1993). We denote the constant before the structural break by $\alpha^{a}_{i,t}$ and the one after it by $\alpha^{p}_{i,t}$. The t-statistic of $\alpha^{p}_{i,t}$ gives the "strength" of the bubble. "Length" is the number of months passed since the structural break.

Table B.6: Statistics of Positive Bubble Signals and Bubbles per Industry (Fama-French model)

Industry	% of sample	$\alpha^{\mathrm{a}}_{i,t}$	$\alpha^{\mathrm{p}}_{i,t}$	Strength	Length	# Bubbles	Return	StAR
Agric	1.25	−0.00268	0.051	3.93	17.4	2	35.8	0.61
Food	7.26	−0.00141	0.015	4.23	27.9	4	23.8	0.57
Soda†	3.88	−0.00479	0.030	2.97	21.4	2	37.5	0.43
Beer	1.93	−0.00777	0.023	2.74	39.4	2	40.8	0.46
Smoke	5.10	−0.00279	0.031	3.66	30.4	3	34.9	0.61
Toys	3.51	−0.00007	0.038	4.04	19.3	2	50.7	0.84
Fun	0.57	−0.00537	0.030	2.75	19.4	3	35.2	0.58
Books	5.10	−0.01434	0.027	3.01	28.0	5	39.6	0.47
Hshld	4.76	−0.00408	0.014	3.21	38.0	2	22.6	0.49
Clths	2.27	−0.00670	0.020	2.92	20.2	4	34.9	0.59
Hlth†	3.83	−0.01039	0.037	2.71	16.7	2	38.8	0.37
MedEq	5.33	−0.00575	0.023	3.00	35.9	4	34.2	0.55
Drugs	5.90	−0.00122	0.022	3.67	33.7	4	24.9	0.61
Chems	3.06	−0.00598	0.011	2.45	36.2	3	18.8	0.39
Rubber†	0.60	−0.00610	0.022	2.66	19.5	2	31.5	0.69
Txtls	6.01	−0.00419	0.015	2.95	39.6	5	29.5	0.36
BldMt	2.49	−0.00663	0.015	2.33	26.0	4	19.8	0.43
Cnstr	2.83	−0.01172	0.034	2.60	19.0	5	47.5	0.56
Steel	3.06	−0.00733	0.020	2.84	18.9	4	28.4	0.57
FabPr†	0.91	−0.01665	0.025	2.32	32.5	2	23.8	0.39
Mach	4.08	−0.00639	0.012	2.79	24.1	4	17.9	0.41
ElcEq	1.81	−0.00287	0.019	3.06	20.7	3	23.5	0.54
Autos	0.57	−0.00760	0.025	2.56	14.2	2	41.3	0.61
Aero	5.33	−0.00771	0.021	2.75	37.5	4	29.4	0.37
Guns†	0.00	–	–	–	–	-	–	–
Gold†	0.68	0.00231	0.079	3.52	15.0	1	107.9	0.84
Ships	0.79	−0.00369	0.031	3.10	18.1	4	39.4	0.68
Mines	8.28	−0.00500	0.024	2.91	28.9	6	32.4	0.44
Coal	2.83	−0.00348	0.057	3.70	20.2	4	60.2	0.75
Oil	0.68	0.00232	0.031	3.98	20.2	2	48.2	0.83
Util	0.00	–	–	–	–	-	–	–
Telcm	3.29	0.00068	0.018	4.07	25.1	3	32.4	0.79
PerSv†	2.53	−0.01122	0.023	2.40	21.0	1	25.4	0.38
BusSv	0.57	−0.00169	0.012	2.98	21.0	2	32.3	0.68
Comps	10.32	−0.00440	0.019	3.02	39.4	4	35.4	0.43
Chips	4.54	−0.00400	0.024	4.03	31.8	2	41.7	0.65
LabEq	1.59	0.00229	0.030	3.74	17.1	2	11.5	0.82
Paper†	2.36	−0.00398	0.021	2.94	19.7	2	21.2	0.53
Boxes	4.42	−0.01069	0.022	2.78	33.4	3	19.2	0.45
Trans	0.79	−0.00497	0.019	2.52	15.0	2	37.6	0.52
Whlsl	1.59	−0.00433	0.015	2.71	29.3	4	19.5	0.43
Rtail	9.86	−0.00577	0.014	3.00	37.8	5	22.9	0.43
Meals	7.26	−0.00219	0.022	3.43	35.7	3	36.8	0.40
Banks	4.65	−0.00431	0.018	2.88	26.6	3	26.3	0.40
Insur	0.45	−0.00403	0.043	3.05	13.5	1	36.6	0.62
RlEst	1.93	−0.00645	0.027	2.80	27.0	1	26.1	0.30
Fin	6.92	−0.00442	0.010	2.97	33.2	3	31.1	0.46
Other	0.11	−0.00075	0.014	3.86	15.0	1	50.3	1.04
Pooled	3.39	−0.00500	0.022	3.15	30.2	3	31.4	0.51

This table reports for each industry the fraction of the complete sample that is classified as a positive bubble signal in percent (column 2), the average values of several properties of bubble signals (columns 3–6), the number of bubbles (column 7), the average raw returns during bubbles (column 8, in % per year), and the average standardized abnormal return during bubbles (column 9, "StAR"). To construct the signals we regress the industry returns on a constant and Fama and French (1993)'s three factors. If a ten-year series of industry returns shows evidence of an upward structural break in the constant and the constant is significantly positive after the break, an investor detects a bubble. A bubble has ended if a crash has occurred in the last six months, where a crash is defined as a residual below -2 times its standard deviation. Critical values for the structural break test correspond with a 97.5% confidence level, and are obtained from Andrews (1993). We denote the constant before the structural break by $\alpha^{\mathrm{a}}_{i,t}$ and the one after it by $\alpha^{\mathrm{p}}_{i,t}$. The t-statistic of $\alpha^{\mathrm{p}}_{i,t}$ gives the "strength" of the bubble. "Length" is the number of months passed since the structural break.

Table B.7: Statistics of Positive Bubble Signals and Bubbles per Industry (Carhart model)

Industry	% of sample	$\alpha^{\mathrm{a}}_{i,t}$	$\alpha^{\mathrm{p}}_{i,t}$	Strength	Length	# Bubbles	Return	StAR
Agric	1.26	−0.00276	0.050	3.88	18.7	2	35.5	0.49
Food	6.39	−0.00124	0.015	4.12	26.3	4	24.3	0.62
Soda†	3.42	−0.00164	0.031	3.19	20.8	2	37.5	0.50
Beer	1.26	−0.00930	0.020	2.51	36.6	2	38.6	0.40
Smoke	5.48	−0.00259	0.032	3.50	29.5	4	33.0	0.65
Toys	3.65	−0.00165	0.034	3.73	20.9	2	46.8	0.73
Fun	1.03	−0.00634	0.023	2.56	24.7	3	33.1	0.45
Books	5.82	−0.01490	0.028	3.07	29.1	4	40.6	0.47
Hshld	4.79	−0.00460	0.014	2.86	37.2	2	23.1	0.46
Clths	3.08	−0.00623	0.020	2.80	21.3	4	34.8	0.62
Hlth†	3.83	−0.00963	0.040	2.87	16.7	2	35.6	0.38
MedEq	5.37	−0.00593	0.023	2.91	36.0	4	33.9	0.54
Drugs	5.59	−0.00232	0.021	3.44	33.5	4	24.9	0.59
Chems	4.00	−0.00393	0.013	2.79	32.0	4	22.2	0.48
Rubber†	0.75	−0.00491	0.020	2.77	17.2	2	30.2	0.71
Txtls	6.74	−0.00394	0.014	2.90	39.0	5	31.0	0.42
BldMt	2.97	−0.00625	0.014	2.35	27.2	4	20.4	0.47
Cnstr	1.71	−0.01013	0.029	2.58	21.9	5	42.9	0.49
Steel	3.08	−0.00636	0.021	2.86	18.3	4	30.1	0.61
FabPr†	2.51	−0.01106	0.015	2.43	47.2	2	22.3	0.40
Mach	4.45	−0.00567	0.011	2.69	30.0	4	16.0	0.44
ElcEq	2.51	−0.00409	0.016	2.68	24.7	3	23.9	0.53
Autos	1.03	−0.00489	0.027	2.81	14.4	2	39.8	0.62
Aero	6.16	−0.00791	0.020	2.74	40.1	4	28.0	0.34
Guns†	0.00	–	–	–	–	-	–	–
Gold†	0.23	−0.00345	0.069	3.06	14.0	1	126.5	0.93
Ships	0.91	−0.00277	0.032	3.17	17.9	4	38.0	0.71
Mines	8.45	−0.00590	0.024	2.82	30.9	7	31.5	0.43
Coal	2.63	−0.00424	0.057	3.67	20.2	4	60.1	0.75
Oil	0.68	0.00026	0.025	3.37	21.2	2	48.2	0.76
Util	0.00	–	–	–	–	-	–	–
Telcm	3.42	0.00033	0.018	3.94	24.0	3	33.7	0.81
PerSv†	2.53	−0.01139	0.023	2.39	21.5	1	25.4	0.43
BusSv	0.68	−0.00099	0.013	3.15	23.0	2	30.3	0.70
Comps	7.99	−0.00335	0.019	3.15	41.8	3	31.5	0.40
Chips	4.57	−0.00384	0.024	4.08	31.8	2	41.5	0.68
LabEq	1.26	0.00174	0.029	3.53	16.5	2	15.1	1.05
Paper†	2.62	−0.00366	0.021	2.91	19.4	2	21.2	0.55
Boxes	4.22	−0.00958	0.024	2.93	34.2	3	21.4	0.42
Trans	1.03	−0.00506	0.017	2.43	17.2	3	33.1	0.50
Whlsl	1.37	−0.00428	0.014	2.55	22.3	3	20.4	0.52
Rtail	8.45	−0.00377	0.014	3.24	39.1	5	23.2	0.50
Meals	6.16	−0.00314	0.022	3.38	33.8	2	35.6	0.42
Banks	5.02	−0.00273	0.019	3.00	26.9	3	24.8	0.36
Insur	0.57	−0.00285	0.044	3.12	13.6	1	36.6	0.62
RlEst	1.83	−0.00512	0.029	2.87	26.6	1	26.1	0.33
Fin	6.74	−0.00426	0.011	2.99	33.2	3	31.1	0.48
Other	0.23	−0.00039	0.016	3.51	17.5	2	33.2	1.05
Pooled	3.39	−0.00487	0.021	3.11	30.4	3	31.0	0.52

This table reports for each industry the fraction of the complete sample that is classified as a positive bubble signal in percent (column 2), the average values of several properties of the positive bubble signals (columns 3–6), the number of bubbles (column 7), the average raw returns during bubbles (column 8, in % per year), and the average standardized abnormal return during bubbles (column 9, "StAR"). To construct the signals we regress the industry returns on a constant and Carhart (1997)'s four factors. If a ten-year series of industry returns shows evidence of an upward structural break in the constant and the constant is significantly positive after the break, an investor detects a bubble. A bubble has ended if a crash has occurred in the last six months, where a crash is defined as a residual below -2 times its standard deviation. Critical values for the structural break test correspond with a 97.5% confidence level, and are obtained from Andrews (1993). We denote the constant before the structural break by $\alpha^{\mathrm{a}}_{i,t}$ and the one after it by $\alpha^{\mathrm{p}}_{i,t}$. The t-statistic of $\alpha^{\mathrm{p}}_{i,t}$ gives the "strength" of the bubble. "Length" is the number of months passed since the structural break.

Table B.8: Standardized Abnormal Returns per Industry After Positive Bubble Signals

	CAPM	Fama-French model	Carhart model
Agric	0.03	0.16	0.10
Food	0.50	0.53	0.52
Soda†	0.29	−0.21	0.15
Beer	0.29	0.12	−0.19
Smoke	0.44	0.45	0.41
Toys	0.22	0.51	0.44
Fun	0.30	−0.73	−0.90
Books	0.18	0.13	0.20
Hshld	−0.01	0.26	0.11
Clths	0.07	−0.19	−0.17
Health†	−0.03	−0.01	−0.14
MedEq	0.19	0.20	0.15
Drugs	0.28	0.21	0.20
Chems	−0.04	−0.32	−0.24
Rubbr†	0.12	−0.93	−1.29
Txtls	0.15	0.15	0.28
BldMt	0.03	−0.10	−0.14
Cnstr	0.22	−0.19	−0.72
Steel	0.23	0.15	0.27
FabPr†	–	−0.16	−0.19
Mach	−0.27	−0.36	−0.28
ElcEq	0.19	−0.12	−0.14
Autos	−0.21	−0.57	−0.20
Aero	0.22	0.07	0.14
Guns†	0.20	–	–
Gold†	−1.29	−1.12	−1.49
Ships	0.23	−1.74	−1.71
Mines	0.09	0.11	−0.03
Coal	0.54	0.22	0.33
Oil	−0.45	0.65	0.60
Util	–	–	–
Telcm	0.72	−0.05	0.29
PerSv†	0.12	0.02	0.04
BusSv	0.23	0.05	0.10
Comps	0.09	0.29	0.32
Chips	0.37	0.61	0.66
LabEq	−0.16	−0.33	0.07
Paper†	0.38	0.06	0.19
Boxes	0.15	0.22	0.28
Trans	0.09	−0.20	−0.42
Whshl	0.27	−0.84	−1.02
Rtail	0.34	0.16	0.24
Meals	−0.03	0.24	0.26
Banks	0.07	−0.05	−0.01
Insur	−0.40	−0.77	−0.75
RlEst	0.15	−0.13	−0.09
Fin	0.38	0.20	0.24
Other	–	−4.20	−2.86
Pooled	0.19	0.11	0.11

This table report the standardized average abnormal returns per industry after a positive bubble signal. In the derivation of the signals and the construction of the abnormal returns, we use the CAPM, the Fama-French model or the Carhart model.

C Simulations

We investigate how many bubble signals are likely to be just noise or a general misspecifications of the asset pricing models using simulations[2]. A statistical interpretation of our simulations would be that we analyze the size of a type I error, that is, how often do we falsely reject the null hypothesis of a negative bubble signal. Because we use a sequence of statistical tests to obtain the bubble signals, it is a priori not obvious how frequently we obtain a positive signal by mere chance.

We design the simulation as a bootstrap as in Kosowski et al. (2006). We construct pseudo returns $\tilde{r}_{i,t}$ as

$$\tilde{r}_{i,t} = \tilde{\boldsymbol{\beta}}'_{i,t}\boldsymbol{f}_t + \tilde{\varepsilon}_{i,t}, \quad t = 1, 2, \ldots, m \tag{C.1}$$

where $\tilde{\boldsymbol{\beta}}_{i,t}$ is the pseudo factor exposure, $\boldsymbol{f}_t$ is the actual factor realization at time t, $\tilde{\varepsilon}_{i,t}$ is a pseudo error term, and m gives the total number of observations. We first address how obtain the pseudo factor exposure, and then how we create the pseudo error term term.

We resample the pseudo factor exposure $\tilde{\boldsymbol{\beta}}_{i,t}$ from the set of estimates for $\boldsymbol{\beta}_{i,t}$ that result from the estimation of Equation (1). Because we estimate this equation in a moving window framework, we have $m - T$ different estimations from which we can draw. These estimations are numbered from T to m. Resampling from these estimations means that we draw a sequence j_t, $t = 1, 2, \ldots, m$ of random numbers in the range T to m. We take

[2]We analyze the effect of more closely define misspecifications such as an omitted risk factor or an omitted structural break in a risk factor in Section D of this appendix and Sections 5.3 and 5.4 of the main paper.

the estimates from estimation j_t for $\tilde{\boldsymbol{\beta}}_{i,t}$, that is, $\tilde{\boldsymbol{\beta}}_{i,t} = \hat{\boldsymbol{\beta}}_{i,j_t}$.

To construct a pseudo-sample that resembles the original sample as close as possible, we put restrictions on j_t. The index j_t for pseudo-observation t has to come from the subset of index numbers that include the original observation t in their estimation window,

$$\max\{t, T\} \leq j_t \leq \min\{t + T - 1, m\}. \tag{C.2}$$

This ensures that the drawing $\hat{\boldsymbol{\beta}}_{i,j_t}$ has actually been estimated for a set of observations that included the original time t observation. The min and max operators make sure that the start and end of the sample are properly taken into account.

We create the error term $\tilde{\varepsilon}_{i,t}$ in two different ways. In the first approach, we draw the error terms independently over time and they are equal to zero in expectation. By drawing independently over time we remove any bubble effect. This approach introduces pure noise in the sample of pseudo returns, and shows how pure noise affects our results. In the second approach, the error terms are drawn independently over time, but do not necessarily have expectation equal to zero. In this second approach, the asset pricing model is not correctly specified. It shows how misspecification can affect our results.

In both cases the estimations of Equation (1) form the basis. Each estimation yields a set of T residuals $\varepsilon_{i,t,\tau}$. The volatility of these sets $\sigma_{i,t}$ varies over time. For the first approach, we draw from the set of standardized residuals for each estimation,

$$\hat{u}_{i,t,\tau} = \hat{\varepsilon}_{i,t,\tau}/\hat{\sigma}_{i,t} = \left(r_{i,t-\tau} - \hat{\alpha}_{i,t,\tau} - \hat{\boldsymbol{\beta}}'_{i,t}\boldsymbol{f}_{t-\tau}\right)/\hat{\sigma}_{i,t}. \tag{C.3}$$

If we rejected a structural break in the estimation for time t, we substitute $\hat{\alpha}_{i,t}$; if not, we substitute $\hat{\alpha}^{\mathrm{a}}_{i,t}$ when τ lies before the structural break or $\hat{\alpha}^{\mathrm{p}}_{i,t}$ when τ lies after it. In the second approach, we draw from the set of the standardized abnormal returns (i.e. the error terms, $\hat{\varepsilon}_{i,t,\tau}$, plus intercept $\hat{\alpha}_{i,t,\tau}$):

$$\hat{v}_{i,t,\tau} = \big(r_{i,t-\tau} - \hat{\boldsymbol{\beta}}'_{i,t}\boldsymbol{f}_{t-\tau}\big)/\hat{\sigma}_{i,t} = \big(\hat{\varepsilon}_{i,t,\tau} + \hat{\alpha}_{i,t,\tau}\big)/\hat{\sigma}_{i,t}. \tag{C.4}$$

To draw from either of these sets for pseudo return t we need two random index numbers, the first $k_{1,t}$ to determine from which estimation we draw a residual, and the second $k_{2,t}$ to determine which residual in an estimation we draw. The index $k_{1,t}$ consists of the complete set of estimations, that is T to m. The index $k_{2,t}$ ranges from 1 to T within each estimation. We multiply the drawing with the volatility that corresponds with the estimation that we drew for $\tilde{\beta}_{i,t}$, that is, σ_{i,j_t}. To summarize, for the analysis of the effect of noise we construct

$$\tilde{r}_{i,t} = \hat{\boldsymbol{\beta}}'_{i,j_t}\boldsymbol{f}_t + \hat{\sigma}_{i,j_t}\hat{u}_{i,k_{1,t},k_{2,t}}, \tag{C.5}$$

and for the analysis of the effect of misspecification

$$\tilde{r}_{i,t} = \hat{\boldsymbol{\beta}}'_{i,j_t}\boldsymbol{f}_t + \hat{\sigma}_{i,j_t}\hat{v}_{i,k_{1,t},k_{2,t}}. \tag{C.6}$$

We use different index number j_t and $k_{1,t}$ because we do not put any restrictions on $k_{1,t}$. For the selection of β, we use the more narrow window to keep a possible pattern intact. We use a larger set to draw the error, because that leads to a better quality pseudo return. We construct a temporal bootstrap to account for correlations among industries. It means that we use the same numbers j_t, $k_{1,t}$ and $k_{2,t}$ for each industry to construct one pseudo sample. We repeat this procedure to create many pseudo samples.

In Table C.1, we compare the bubble signals and bubbles derived from 1,000 simulated data sets to our original data set. Because the temporal bootstrap assumes a fixed size of the cross section, we construct simulated data sets for the 40 industries for which returns from 1926 are available. Since all our results are stated per industry, this does not influence our results. For both, the noise simulations (Equation (C.3)) and the misspecification simulation (Equation (C.4)) we find around 15-16 bubble signals per industry. In the "real" data, we find 43 signals per industry for the CAPM and about 29 for the Fama-French and Carhart models. Comparing the number of bubble signals from the simulated data to the number of signals from the real data shows that our signal derivation method is rather noisy. Up to half of the bubble signals we obtain could be attributed to noise or a misspecification. The signals we obtain from the simulated data look very similar to the signals we derive from the original data set. In both cases, the intercept before the break, $\alpha^{\mathrm{a}}_{i,t}$, is slightly negative, but statistically indistinguishable from zero. The intercept following the breakpoint, $\alpha^{\mathrm{p}}_{i,t}$, is large and positive. The strength of the bubble signals from the simulated as well as real data is close to three and the length of the signal since the breakpoint centers around 30 months. Given the large similarity between the bubble signals from the real data and the simulated data, it should come as no surprise that the resulting bubbles look very similar. Again, we find a relatively large number of bubbles in the simulated. Just like the "real" bubbles, these bubbles are characterized by large positive raw and abnormal returns.

[Table 1 about here.]

As one should expect, Table C.2 shows no difference in mean abnormal returns following positive and negative bubble signals for the simulated data. Similarly, there is also no evidence of significant differences in volatility or downside risk following positive and negative signals in the simulated data.

Overall, we conclude that a sizable number of positive bubble signals can potentially be attributed to noise or a misspecification of the asset pricing model. These “false” signals however cannot contribute to explaining our findings of positive abnormal returns and a higher risk after positive bubble signals based on the real data. It thus seems that the “true” signals we extract have such a strong power to predict subsequent returns that our results are economically and statistically meaningful despite the noisiness of the signals.

[Table 2 about here.]

Table C.1: Observed versus Simulated Bubble Signals and Bubbles

	CAPM					Fama-French model					Carhart model				
	obs.	noise		misspec.		obs.	noise		misspec.		obs.	noise		misspec.	
(a) *Bubble signals*															
Positive Signals (in %)	4.9	1.7	(0.31)	1.8	(0.30)	3.3	1.8	(0.30)	1.8	(0.29)	3.3	1.7	(0.28)	1.7	(0.29)
α^{a} ($\times 10^{-2}$)	-0.61	-0.54	(0.09)	-0.47	(0.08)	-0.50	-0.51	(0.08)	-0.44	(0.08)	-0.49	-0.49	(0.08)	-0.42	(0.08)
α^{p} ($\times 10^{-2}$)	2.28	2.26	(0.18)	2.31	(0.17)	2.17	2.06	(0.16)	2.11	(0.16)	2.13	2.04	(0.16)	2.11	(0.16)
Strength	3.21	2.94	(0.09)	3.05	(0.10)	3.15	2.93	(0.09)	3.03	(0.10)	3.11	2.92	(0.08)	3.04	(0.10)
Length	33.9	29.8	(1.92)	30.4	(1.89)	30.2	30.3	(1.75)	30.5	(1.88)	30.4	30.1	(1.89)	30.6	(1.84)
(b) *Bubbles*															
Bubbles per Ind.	3.05	1.83	(0.20)	1.89	(0.19)	2.98	1.88	(0.19)	1.86	(0.18)	3.00	1.86	(0.18)	1.86	(0.18)
Raw Return (p.a.)	30.2	32.9	(1.91)	33.7	(1.83)	31.4	30.6	(1.68)	31.0	(1.73)	31.0	29.9	(1.63)	30.8	(1.67)
St. Abn. Ret.	0.49	0.52	(0.03)	0.54	(0.03)	0.51	0.52	(0.02)	0.55	(0.03)	0.52	0.53	(0.03)	0.55	(0.03)

This tables shows descriptive statistics of the bubble signals and bubbles as we derive them from the actually observed data and from two simulation settings. The statistics that result from the actual data are in the columns listed "obs." and correspond with Tables 2 and 7. The simulations settings are based on Kosowski et al. (2006). In the first setting, called "noise", the idiosyncratic uncertainty in the pseudo returns comes from a zero-mean standardized error distribution (see Equation (C.3)). In the second setting, called "misspecification", it comes from the standardized abnormal return distribution (see Equation (C.4)). We construct 1,000 sets of pseudo-returns for the 40 industries for which returns over the full sample period (July 1926 – December 2009) are available, and apply our bubble identification procedure to it. For each statistic, we report its average and standard deviation over all simulated samples. The standard deviations are in parentheses.

Table C.2: Standardized Abnormal Returns after Simulated Negative and Positive Bubble Signals

	(a) CAPM, noise simulation					(b) CAPM, misspecification simulation				
Signal	Negative		Positive		p-value	Negative		Positive		p-value
mean	−0.0041	(0.0093)	−0.0013	(0.045)	0.459	0.019	(0.0089)	0.026	(0.045)	0.413
median	−0.028	(0.0086)	−0.026	(0.049)	0.478	−0.0077	(0.0083)	−0.0022	(0.048)	0.422
volatility	1.03	(0.0041)	1.08	(0.049)	0.149	1.03	(0.0041)	1.08	(0.048)	0.126
VaR(0.95)	1.61	(0.017)	1.68	(0.112)	0.272	1.58	(0.018)	1.65	(0.105)	0.238
ES(0.95)	2.20	(0.025)	2.29	(0.165)	0.307	2.17	(0.027)	2.26	(0.156)	0.302

	(c) Fama-French model, noise simulation					(d) Fama-French model, misspecification simulation				
Signal	Negative		Positive		p-value	Negative		Positive		p-value
Mean	−0.0004	(0.0071)	0.0022	(0.047)	0.465	0.014	(0.0084)	0.026	(0.046)	0.380
Median	−0.023	(0.0073)	−0.024	(0.051)	0.496	−0.011	(0.0084)	0.0009	(0.048)	0.401
Volatility	1.03	(0.0033)	1.08	(0.047)	0.126	1.03	(0.0043)	1.09	(0.046)	0.111
VaR(0.95)	1.62	(0.014)	1.69	(0.110)	0.273	1.62	(0.015)	1.68	(0.103)	0.272
ES(0.95)	2.20	(0.020)	2.28	(0.157)	0.322	2.18	(0.020)	2.27	(0.149)	0.306

	(e) Carhart model, noise simulation					(f) Carhart model, misspecification simulation				
Signal	Negative		Positive		p-value	Negative		Positive		p-value
Mean	−0.0005	(0.0072)	0.0031	(0.046)	0.468	0.018	(0.0075)	0.033	(0.046)	0.358
Median	−0.022	(0.0074)	−0.020	(0.050)	0.469	−0.0067	(0.0077)	0.0077	(0.050)	0.388
Volatility	1.03	(0.0032)	1.09	(0.044)	0.103	1.04	(0.0033)	1.09	(0.047)	0.114
VaR(0.95)	1.64	(0.014)	1.71	(0.102)	0.241	1.62	(0.015)	1.69	(0.106)	0.278
ES(0.95)	2.21	(0.019)	2.29	(0.149)	0.292	2.18	(0.020)	2.27	(0.152)	0.315

This tables shows summary statistics of the standardized abnormal returns after bubble signals based on two simulation settings (cf. Table 4). The simulations settings are based on Kosowski et al. (2006). In the first setting, called "noise", the idiosyncratic uncertainty in the pseudo returns comes from a zero-mean standardized error distribution (see Equation (C.3)). In the second setting, called "misspecification", it comes from the standardized abnormal return distribution (see Equation (C.4)). We construct 1,000 sets of pseudo-returns for the 40 industries for which returns over the full sample period (July 1926 – December 2009) are available, and apply our bubble identification procedure to it. For each statistic, we report its average and standard deviation over all simulated samples. The standard deviations are in parentheses. The columns labelled p-values report the results of the test for equality of the statistics for negative vs. positive bubble signals.

D Misspecification and bubble detection

We investigate how our method to derive bubble signals is affected by possible misspecification of the asset pricing model in Section 5.3 and 5.4 of the paper. Here, we provide the underlying derivations.

We would like to determine the effect of a structural break in a factor exposure, the effect of an omitted risk factor, and a combination of both. In these three cases, the true model reads:[3]

$$r_t = \alpha + \beta_t x_t + \gamma w_t + \upsilon_t,\ \mathrm{E}[\upsilon_t] = 0,\ \mathrm{E}[\upsilon_t^2] = \sigma_\upsilon^2,\ \mathrm{E}[x_t \upsilon_t] = 0,\ \mathrm{E}[w_t \upsilon_t] = 0. \tag{D.1}$$

Our interest is in x_t. In case of a structural break, β_t changes at one point, but this change is ignored. In case of an omitted risk factor the true exposure β_t is constant, and unequal to zero, but the factor is omitted in the derivation of the bubble signal. In the last case, these two effects show up combined. In showing the effect of these misspecifications we follow the asymptotic setup of Andrews (1993). This means that we derive asymptotic results under the assumption that the number of observations both before and after a structural break approach infinity, while the fractions of observations before the structural break and after the structural break remain constant.

The setup in Equation (D.1) applies to models with more factors as well. We can write any multi-factor model as:

$$\begin{aligned} r_t &= \alpha + \beta_t x_t + \gamma w_t + \boldsymbol{\delta}' \boldsymbol{f}_t + \upsilon_t = \alpha + \beta_t x_t + \gamma (w_t + \boldsymbol{\delta}' \boldsymbol{f}_t / \gamma) + \upsilon_t \\ &= \alpha + \beta_t x_t + \gamma \tilde{w}_t + \upsilon_t, \end{aligned}$$

[3]We use a slightly different notation here to simplify the derivations.

with $\tilde{w}_t \equiv w_t + \boldsymbol{\delta}' \boldsymbol{f}_t / \gamma$.

D.1 Structural break in factor exposure

When a structural break is present in the factor exposure towards the risk factor x_t, the true model reads as in Equation (D.1), with the further specification:

$$\beta_t = \begin{cases} \beta^{\mathrm{a}} & \text{for } t \leq \xi T \\ \beta^{\mathrm{p}} & \text{for } t > \xi T, \end{cases} \tag{D.2}$$

where $\xi \in (0, 1)$ gives the fraction of observations before the structural break.[4] So, the true model exhibits a structural break in the exposure to x_t and there is no structural break in the intercept.

The first step of our method to obtain a bubble signal only allows for a structural break in the intercept. It estimates the model:

$$\begin{aligned} r_t &= a_t + b x_t + c w_t + e_t, \ \mathrm{E}[e_t] = 0 \\ a_t &= \begin{cases} a^{\mathrm{a}} & \text{for } t \leq \xi T \\ a^{\mathrm{p}} & \text{for } t > \xi T \end{cases} \end{aligned} \tag{D.3}$$

with OLS. To estimate this model, a sample of size T is available, with ξT observations before the structural breakpoint, and $(1 - \xi) T$ observations thereafter. We use $\boldsymbol{r}$, $\boldsymbol{x}$ and $\boldsymbol{w}$ to denote the vector of observations and $\boldsymbol{u}$ for the vector of error terms. A superscript a (p) denotes the subvectors before (after) the structural break. First we derive the OLS

[4] Based on the model in Equations (1) and (2), we have $\xi = (T - \zeta - 1)/T$.

estimates. We define the auxiliary matrix:

$$\boldsymbol{Z}_T = \begin{pmatrix} \boldsymbol{\imath}_{\xi T} & 0 & \boldsymbol{x}^{\mathrm{a}} & \boldsymbol{w}^{\mathrm{a}} \\ 0 & \boldsymbol{\imath}_{(1-\xi)T} & \boldsymbol{x}^{\mathrm{p}} & \boldsymbol{w}^{\mathrm{p}} \end{pmatrix},$$

where $\boldsymbol{\iota}_m$ denotes a vector of length m filled with ones. Standard regression theory gives the estimates for the coefficients:

$$\begin{pmatrix} \hat{a}^{\mathrm{a}} \\ \hat{a}^{\mathrm{p}} \\ \hat{b} \\ \hat{c} \end{pmatrix} = \left(\boldsymbol{Z}_T'\boldsymbol{Z}_T\right)^{-1} \boldsymbol{Z}_T' \begin{pmatrix} \boldsymbol{r}^{\mathrm{a}} \\ \boldsymbol{r}^{\mathrm{p}} \end{pmatrix}. \tag{D.4}$$

Next, we use asymptotic theory to derive the properties of these estimates. We use m_x^n to denote the n^{th} moment of the variable x_t, and similar for the other variables; and m_{xw} for the comoment of x and w. We assume that the moments of the explanatory variables are constant over time, and do not change with the structural break. We calculate:

$$\boldsymbol{Z}_T'\boldsymbol{Z}_T = \begin{pmatrix} \xi T & 0 & \boldsymbol{\imath}_{\xi T}'\boldsymbol{x}^{\mathrm{a}} & \boldsymbol{\imath}_{\xi T}'\boldsymbol{w}^{\mathrm{a}} \\ 0 & (1-\xi)T & \boldsymbol{\imath}_{(1-\xi)T}'\boldsymbol{x}^{\mathrm{p}} & \boldsymbol{\imath}_{(1-\xi)T}'\boldsymbol{w}^{\mathrm{p}} \\ \boldsymbol{\imath}_{\xi T}'\boldsymbol{x}^{\mathrm{a}} & \boldsymbol{\imath}_{(1-\xi)T}'\boldsymbol{x}^{\mathrm{p}} & \boldsymbol{x}'\boldsymbol{x} & \boldsymbol{x}'\boldsymbol{w} \\ \boldsymbol{\imath}_{\xi T}'\boldsymbol{w}^{\mathrm{a}} & \boldsymbol{\imath}_{(1-\xi)T}'\boldsymbol{x}^{\mathrm{p}} & \boldsymbol{x}'\boldsymbol{w} & \boldsymbol{w}'\boldsymbol{w} \end{pmatrix},$$

and use this to define:

$$\boldsymbol{\Sigma}_{zz} \equiv \lim_{T\to\infty} \frac{1}{T}\boldsymbol{Z}_T'\boldsymbol{Z}_T = \begin{pmatrix} \xi & 0 & \xi m_x^1 & \xi m_w^1 \\ 0 & (1-\xi) & (1-\xi)m_x^1 & (1-\xi)m_w^1 \\ \xi m_x^1 & (1-\xi)m_x^1 & m_x^2 & m_{xw} \\ \xi m_w^1 & (1-\xi)m_w^1 & m_{xw} & m_w^2 \end{pmatrix}. \tag{D.5}$$

In a similar fashion we calculate:

$$\boldsymbol{Z}_T' \begin{pmatrix} \boldsymbol{r}^{\mathrm{a}} \\ \boldsymbol{r}^{\mathrm{p}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\imath}_{\xi T}' \boldsymbol{r}^{\mathrm{a}} \\ \boldsymbol{\imath}_{(1-\xi)T}' \boldsymbol{r}^{\mathrm{p}} \\ \boldsymbol{x}^{\mathrm{a}\prime} \boldsymbol{r}^{\mathrm{a}} + \boldsymbol{x}^{\mathrm{p}\prime} \boldsymbol{r}^{\mathrm{p}} \\ \boldsymbol{w}^{\mathrm{a}\prime} \boldsymbol{r}^{\mathrm{a}} + \boldsymbol{w}^{\mathrm{p}\prime} \boldsymbol{r}^{\mathrm{p}} \end{pmatrix} = \begin{pmatrix} \xi T \alpha + \beta^{\mathrm{a}} \boldsymbol{\imath}_{\xi T}' \boldsymbol{x}^{\mathrm{a}} + \gamma \boldsymbol{\imath}_{\xi T}' \boldsymbol{w}^{\mathrm{a}} + \boldsymbol{\imath}_{\xi T}' \boldsymbol{u}^{\mathrm{a}} \\ (1-\xi) T \alpha + \beta^{\mathrm{p}} \boldsymbol{\imath}_{(1-\xi)T}' \boldsymbol{x}^{\mathrm{p}} + \gamma \boldsymbol{\imath}_{(1-\xi)T}' \boldsymbol{w}^{\mathrm{p}} + \boldsymbol{\imath}_{(1-\xi)T}' \boldsymbol{u}^{\mathrm{p}} \\ \boldsymbol{\imath}_T' \boldsymbol{x} \alpha + \beta^{\mathrm{a}} \boldsymbol{x}^{\mathrm{a}\prime} \boldsymbol{x}^{\mathrm{a}} + \beta^{\mathrm{p}} \boldsymbol{x}^{\mathrm{p}\prime} \boldsymbol{x}^{\mathrm{p}} + \gamma \boldsymbol{x}' \boldsymbol{w} + \boldsymbol{x}' \boldsymbol{u} \\ \boldsymbol{\imath}_T' W \alpha + \beta^{\mathrm{a}} \boldsymbol{x}^{\mathrm{a}\prime} \boldsymbol{w}^{\mathrm{a}} + \beta^{\mathrm{p}} \boldsymbol{x}^{\mathrm{p}\prime} \boldsymbol{w}^{\mathrm{p}} + \gamma \boldsymbol{w}' \boldsymbol{w} + \boldsymbol{w}' \boldsymbol{u} \end{pmatrix},$$

where we have substituted the true model for r_t. We use this result to define:

$$\boldsymbol{\Sigma}_{zy} \equiv \lim_{T\to\infty} \frac{1}{T} \boldsymbol{Z}_T' \boldsymbol{Y}_T = \begin{pmatrix} \xi\big(\alpha + \beta^{\mathrm{a}} m_x^1 + \gamma m_w^1 + m_u^1\big) \\ (1-\xi)\big(\alpha + \beta^{\mathrm{p}} m_x^1 + \gamma m_w^1 + m_u^1\big) \\ \alpha m_x^1 + \big(\xi\beta^{\mathrm{a}} + (1-\xi)\beta^{\mathrm{p}}\big) m_x^2 + \gamma m_{xw} + m_{xu} \\ \alpha m_w^2 + \big(\xi\beta^{\mathrm{a}} + (1-\xi)\beta^{\mathrm{p}}\big) m_{xw} + \gamma m_w^2 + m_{wu} \end{pmatrix}. \tag{D.6}$$

Consequently, we find:

$$\operatorname{plim}_{T\to\infty} \begin{pmatrix} \hat{a}^{\mathrm{a}} \\ \hat{a}^{\mathrm{p}} \\ \hat{b} \\ \hat{c} \end{pmatrix} = \begin{pmatrix} \alpha + (1-\xi)(\beta^{\mathrm{a}} - \beta^{\mathrm{p}}) m_x^1 \\ \alpha - \xi(\beta^{\mathrm{a}} - \beta^{\mathrm{p}}) m_x^1 \\ \xi\beta^{\mathrm{a}} + (1-\xi)\beta^{\mathrm{p}} \\ \gamma \end{pmatrix}. \tag{D.7}$$

As expected, $\hat{b}$ converges to a weighted average of β^{a} and β^{p}, where the weight depends on the proportions of the sample before and after the structural break. The deviations of a^{a} and a^{p} from the true intercept α reflect the size of the structural break $\beta^{\mathrm{a}} - \beta^{\mathrm{p}}$, the proportion ξ and the average value of x_t.

The structural break test in our method to obtain the bubble signal also uses the variance of the estimator. Therefore, we derive the variation of the residuals e_t. We have:

$$e_t = \begin{cases} (1-\xi)(\beta^{\mathrm{a}} - \beta^{\mathrm{p}})(x_t - m_x^1) + \upsilon_t & \text{for } t \leq \xi T \\ -\xi(\beta^{\mathrm{a}} - \beta^{\mathrm{p}})(x_t - m_x^1) + \upsilon_t & \text{for } t > \xi T \end{cases}$$

so σ_e^2 becomes

$$\sigma_e^2 = \xi(1-\xi)(\beta^{\mathrm{a}} - \beta^{\mathrm{p}})^2\sigma_x^2 + \sigma_\upsilon^2, \tag{D.8}$$

where σ_x^2 is the (population) variance of x_t. This expression shows that the misspecification leads to an increase in the residual variance. Applying standard regression theory gives the desired result:

$$\sqrt{T}\left(\begin{pmatrix} \hat{a}^{\mathrm{a}} \\ \hat{a}^{\mathrm{p}} \\ \hat{b} \\ \hat{c} \end{pmatrix} - \begin{pmatrix} \alpha + (1-\xi)(\beta^{\mathrm{a}} - \beta^{\mathrm{p}})m_x^1 \\ \alpha - \xi(\beta^{\mathrm{a}} - \beta^{\mathrm{p}})m_x^1 \\ \xi\beta^{\mathrm{a}} + (1-\xi)\beta^{\mathrm{p}} \\ \gamma \end{pmatrix}\right) \rightarrow N\left(\mathbf{0}, \boldsymbol{\Sigma}_{zz}^{-1}\sigma_e^2\right). \tag{D.9}$$

The test statistic for a structural break is based on the difference $\hat{a}^{\mathrm{p}} - \hat{a}^{\mathrm{a}}$, for which we have:

$$\mathrm{plim}_{T\to\infty}\, \hat{a}^{\mathrm{p}} - \hat{a}^{\mathrm{a}} = (\beta^{\mathrm{p}} - \beta^{\mathrm{a}})m_x^1$$
$$\sqrt{T}\left(\hat{a}^{\mathrm{p}} - \hat{a}^{\mathrm{a}} - (\beta^{\mathrm{p}} - \beta^{\mathrm{a}})m_x^1\right) \rightarrow N\left(0, \frac{1}{\xi(1-\xi)}\sigma_e^2\right).$$

This means that the expected value of the statistic for the structural break test on the intercept, when there actually is a structural break in the factor is given by:

$$\chi_{\mathrm{SBF}} = \frac{\sqrt{T\xi(1-\xi)}(\beta^{\mathrm{p}} - \beta^{\mathrm{a}})m_x^1}{\sigma_e^2} = \frac{\sqrt{T}(\beta^{\mathrm{p}} - \beta^{\mathrm{a}})m_x^1}{\sqrt{(\beta^{\mathrm{p}} - \beta^{\mathrm{a}})^2\sigma_x^2 + \frac{1}{\xi(1-\xi)}\sigma_\upsilon^2}}, \tag{D.10}$$

and we conclude that the statistic depends on the average value of the factor m_x^1, its variance σ_x^2, the residual variance of the returns σ_u^2, the size of the true structural break $\Delta \equiv \beta^{\mathrm{p}} - \beta^{\mathrm{a}}$, and the location of the structural break $\boldsymbol{\xi}$.

We also analyze the sensitivities of the statistic for different inputs. As the size and location of the structural break show up in both the numerator and the denominator, we rewrite the statistic as:

$$\chi_{\text{SBF}} = m_x^1 \sqrt{T} \left(\sigma_x^2 + \sigma_v^2 \left(\xi - \xi^2 \right)^{-1} \Delta^{-2} \right)^{-1/2}.$$

It is straightforward to see that the statistic is increasing in the factor average m_x^1 and in the absolute size of the structural break Δ, and decreasing in the factor and residual variances σ_x^2 and σ_v^2. To find the effect of the location of the structural break, we differentiate χ with respect to ξ:

$$\frac{\mathrm{d}\chi_{\text{SBF}}}{\mathrm{d}\xi} = \frac{1}{2} m_x^1 \sqrt{T} \left(\sigma_x^2 + \sigma_v^2 \left(\xi - \xi^2 \right)^{-1} \Delta^{-2} \right)^{-3/2} \sigma_v^2 \left(\xi - \xi^2 \right)^{-2} \Delta^{-2} (1 - 2\xi).$$

The statistic is convex with respect to $\xi \in [0, 1]$ so it is maximized for $\xi = 1/2$, i.e., when the structural break is located in the middle of the sample.

D.2 Omitted risk factor

When the risk factor x is omitted, it means that the true model reads as in (D.1), where the factor exposure may be constant, i.e. $\beta_t = \beta$ or show a structural break as in Equation (D.2). As the presence of a structural break in β is the more general case (with no break implying $\beta^{\mathrm{a}} = \beta^{\mathrm{p}}$), we make the derivations under that assumption and discuss subsequently what a constant exposure to an omitted risk factor implies.

The first step in identifying the bubble signal allows for a break in the intercept, but it would in this case ignore the factor x_t. Consequently, we estimate a reduced version of the model in Equation (D.3):

$$r_t = a_t + c_t w_t + e_t,\ \mathrm{E}[e_t] = 0$$
$$a_t = \begin{cases} a^{\mathrm{a}} & \text{for } t \leq \xi T \\ a^{\mathrm{p}} & \text{for } t > \xi T, \end{cases} \tag{D.11}$$

with OLS. The assumptions on the sample and notation are the same as in the previous subsection. For deriving the estimators, we also follow the same approach as in the previous subsection. First we define an auxiliary matrix:

$$\boldsymbol{Z}_T^* = \begin{pmatrix} \boldsymbol{\imath}_{\xi T} & 0 & \boldsymbol{w}^{\mathrm{a}} \\ 0 & \boldsymbol{\imath}_{(1-\xi)T} & \boldsymbol{w}^{\mathrm{p}} \end{pmatrix},$$

which is simply $\boldsymbol{Z}_T$ without the column $(\boldsymbol{x}^{\mathrm{a}}, \boldsymbol{x}^{\mathrm{p}})$. We use this matrix to construct the coefficient estimates:

$$\begin{pmatrix} \hat{a}^{\mathrm{a}} \\ \hat{a}^{\mathrm{p}} \\ \hat{c} \end{pmatrix} = \left(\boldsymbol{Z}_T^{*\prime} \boldsymbol{Z}_T^*\right)^{-1} \boldsymbol{Z}_T^{*\prime} \begin{pmatrix} \boldsymbol{r}^{\mathrm{a}} \\ \boldsymbol{r}^{\mathrm{p}} \end{pmatrix}. \tag{D.12}$$

To derive the asymptotic properties of the estimators, we define two limiting matrices:

$$\boldsymbol{\Sigma}_{zz^*} \equiv \lim_{T\to\infty} \frac{1}{T} \boldsymbol{Z}_T^{*\prime} \boldsymbol{Z}_T^* = \begin{pmatrix} \xi & 0 & \xi m_w^1 \\ 0 & (1-\xi) & (1-\xi) m_w^1 \\ \xi m_w^1 & (1-\xi) m_w^1 & m_w^2 \end{pmatrix} \tag{D.13}$$

and

$$\boldsymbol{\Sigma}_{zy^*} \equiv \lim_{T\to\infty} \frac{1}{T} \boldsymbol{Z}_T^{*\prime} \boldsymbol{Y}_T = \begin{pmatrix} \xi\big(\alpha + \beta^{\mathrm{a}} m_x^1 + \gamma m_w^1 + m_u^1\big) \\ (1-\xi)\big(\alpha + \beta^{\mathrm{p}} m_x^1 + \gamma m_w^1 + m_u^1\big) \\ \alpha m_w^2 + \big(\xi\beta^{\mathrm{a}} + (1-\xi)\beta^{\mathrm{p}}\big) m_{xw} + \gamma m_w^2 + m_{wu} \end{pmatrix}. \tag{D.14}$$

We use these two matrices to derive the asymptotic values of the estimators:

$$\mathrm{plim}_{T\to\infty} \begin{pmatrix} \hat{a}^{\mathrm{a}} \\ \hat{a}^{\mathrm{p}} \\ \hat{c} \end{pmatrix} = \begin{pmatrix} \alpha + \beta^{\mathrm{a}} m_x^1 - \dfrac{\sigma_{xw}}{\sigma_w^2} \bar{\beta} m_w^1 \\ \alpha + \beta^{\mathrm{p}} m_x^1 - \dfrac{\sigma_{xw}}{\sigma_w^2} \bar{\beta} m_w^1 \\ \gamma + \dfrac{\sigma_{xw}}{\sigma_w^2} \bar{\beta} \end{pmatrix}, \tag{D.15}$$

where σ_{xw} denotes the (population) covariance of x_t and w_t, σ_w^2 denotes the (population) variance of w_t and we use the shorthand notation $\bar{\beta} = \xi\beta^{\mathrm{a}} + (1-\xi)\beta^{\mathrm{p}}$.

The estimate for $\hat{c}$ consists of two terms, the true exposure to the factor w_t, γ, and a term that is related to the omitted factor. As could be expected, part of the exposure to the omitted factor comes in via the correlation between x_t and w_t. The fraction σ_{xw}/σ_w^2 would simply be the regression coefficient of x_t on w_t. The factor $\bar{\beta} = \xi\beta^{\mathrm{a}} + (1-\xi)\beta^{\mathrm{p}}$ reflects the structural break, and is the weighted average of the factor exposure before and after the structural break. If there is no structural break in the omitted factor, this factor would reduce to β.

The estimates for $\hat{a}^{\mathrm{a}}$ and $\hat{a}^{\mathrm{p}}$ consist of three terms. The first is the true intercept α. The second term shows up because the average of the omitted factor multiplied by its exposure is captured by the intercept. The third term is a correction related to the entrance of the omitted risk factor via w_t in $\hat{c}$. Therefore, it is simply the second term of $\hat{c}$ multiplied with the mean of w_t.

Before we derive the variance of the estimators, we first consider the difference between $\hat{a}^{\mathrm{a}}$ and $\hat{a}^{\mathrm{p}}$:

$$\operatorname{plim}_{T\to\infty} \hat{a}^{\mathrm{p}} - \hat{a}^{\mathrm{a}} = (\beta^{\mathrm{p}} - \beta^{\mathrm{a}})m_x^1.$$

This expression is equal to our result in the previous section, and shows that the limiting bias in the difference is the product of the size of the structural break and the average of the omitted factor. This expression shows that an omitted factor only affects the bubble signal, if the return exhibits a structural break in its exposure towards this factor. So, an omitted exposure to a factor that does not exhibit a structural break has no influence on whether we obtain a positive or negative bubble signal.

As a first step towards the variance of the estimators we consider the residuals:

$$e_t^* = \begin{cases} \beta^{\mathrm{a}}(x_t - m_x^1) - \dfrac{\sigma_{xw}}{\sigma_w^2}\bar{\beta}(w_t - m_w^1) + \upsilon_t & \text{for } t \le \xi T \\[2ex] \beta^{\mathrm{p}}(x_t - m_x^1) - \dfrac{\sigma_{xw}}{\sigma_w^2}\bar{\beta}(w_t - m_w^1) + \upsilon_t & \text{for } t > \xi T \end{cases}$$

The residual variance is again constructed in the usual fashion, yielding:

$$\begin{aligned} \sigma_{e^*}^2 &= \big(\xi\beta_1^2 + (1-\xi)\beta_2^2\big)\sigma_x^2 - 2\big(\xi\beta^{\mathrm{a}} + (1-\xi)\beta^{\mathrm{p}}\big)\frac{\sigma_{xw}}{\sigma_w^2}\bar{\beta}\sigma_{xw} + \frac{\sigma_{xw}^2}{\sigma_w^4}\bar{\beta}^2\sigma_w^2 + \sigma_\upsilon^2 \\ &= \big(\xi\beta_1^2 + (1-\xi)\beta_2^2\big)\sigma_x^2 - 2\frac{\sigma_{xw}^2}{\sigma_w^2}\bar{\beta}^2 + \frac{\sigma_{xw}^2}{\sigma_w^2}\bar{\beta}^2 + \sigma_\upsilon^2 \\ &= \big(\xi\beta_1^2 + (1-\xi)\beta_2^2\big)\sigma_x^2 - \rho_{xw}^2\bar{\beta}^2\sigma_x^2 + \sigma_\upsilon^2 \\ &= \big(\xi\beta_1^2 + (1-\xi)\beta_2^2 - \rho_{xw}^2\bar{\beta}^2\big)\sigma_x^2 + \sigma_\upsilon^2, \end{aligned} \tag{D.16}$$

where ρ_{xw} is the correlation between x_t and w_t. Also in this case, we see that the variance of the residuals consists of the original variance of the errors σ_υ^2 and an extra term related to the misspecified model with regard to x_t. The increase with respect to the error variance is largest when the omitted factor is unrelated to other factors in the model, i.e., $\rho_{xw} = 0$. When there is some correlation, the factor w_t can to some extend provide information on the omitted factor, and consequently the variance is reduced. When the correlation is perfectly positive or negative, the reduction is maximal, and Equation (D.16) reduces to Equation (D.8). Of course, in this particular situation, knowing w_t implies knowing x_t (up to a linear transformation) and the factor is not really omitted.

Finally we consider the test statistic. The asymptotic distribution of the estimators is quite similar to that in Equation (D.9):

$$\sqrt{T}\left(\begin{pmatrix}\hat{a}^{\mathrm{a}}\\ \hat{a}^{\mathrm{p}}\\ \hat{c}\end{pmatrix}-\begin{pmatrix}\alpha+\beta^{\mathrm{a}}m_x^1-\dfrac{\sigma_{xw}}{\sigma_w^2}\bar{\beta}m_w^1\\ \alpha+\beta^{\mathrm{p}}m_x^1-\dfrac{\sigma_{xw}}{\sigma_w^2}\bar{\beta}m_w^1\\ \gamma+\dfrac{\sigma_{xw}}{\sigma_w^2}\bar{\beta}\end{pmatrix}\right)\to N\left(\mathbf{0},\boldsymbol{\Sigma}_{zz*}^{-1}\sigma_{e^*}^2\right). \tag{D.17}$$

From this result, we derive the test statistic for the structural break test on the intercept, when there actually is a structural break in the omitted factor:

$$\chi_{\mathrm{OFB}}=\frac{\sqrt{T\xi(1-\xi)}(\beta^{\mathrm{p}}-\beta^{\mathrm{a}})m_x^1}{\sigma_{e^*}^2}. \tag{D.18}$$

As we have established that $\sigma_{e^*}^2 \geq \sigma_e^2$, we find $\chi_{\mathrm{OFB}} \leq \chi_{\mathrm{SFB}}$. It implies that omitting a factor to which the exposure exhibits a structural break actually reduces the bias in the test statistic. Consequently, investigating the effect of a structural break in the factor exposure towards already included factors gives an upper bound to the effect that omitted factors with a structural break can have.

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