



# The simple economics of bank fragility

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## Abstract

Banks are linked through the interbank deposit market, participations like syndicated loans and deposit interest rate risk. The similarity in exposures carries the potential for systemic breakdowns. This potential is either strong or weak, depending on whether the linkages remain or vanish asymptotically. It is shown that the linearity of the bank portfolios in the exposures, in combination with a condition on the tails of the marginal distributions of these exposures, determines whether the potential for systemic risk is weak or strong. We show that if the exposures have marginal normal distributions the potential for systemic risk is weak, while if e.g. the Student distributions apply the potential is strong.

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## 1. Introduction

The asset sides of the bank balance sheets contain mutual exposures in the interbank deposit market and participations in syndicated loans. Therefore large losses due to exogenous factors, such as an operational failure within a bank, lead to a chain reaction in the interbank market. Similarly, a failure by a large company renegeing on its (syndicated) loan immediately affects a sizable part of the banking sector. This makes that banks are directly connected. Since banks engage in similar activities like mortgages concentrated in specific areas, or loans to specific sectors of the economy, banks are also indirectly linked as they are exposed to the same macro-risk drivers. For example, the monetary policy determined short-term interest rate is in many countries the driving force behind mortgage defaults. Moreover, through their proprietary trades banks are exposed to the same market risks. This implies that the asset sides of different banks balance sheets hold the same risks (or risk factors), albeit in different proportions. The motive for this risk sharing rather than risk concentration by financial intermediaries is the very basic pressure for risk diversification. Somewhat ironically, while diversification reduces the frequency of individual bank failures, since smaller shocks can be easily borne by the system, at the same time diversification makes the bank sector prone to systemic breakdowns in case of very large (non-macro) shocks, which otherwise would only have isolated impact.

The liability sides of banks, balance sheets are perhaps even more alike than the asset sides. The liability side consists of an important part of deposits. Rewards on deposits are very closely aligned across banks due to competition and the ease by which the public can switch out of deposits into other asset categories. Movements in the macro interest rates induce the public to substitute between asset categories, and hence an interest rate move can rapidly change the size of deposit holdings by the public. Thus the liquidity needs of the banks may rapidly change.

Hence, either through contagion after an idiosyncratic shock, or due to a macro-shock such as an interest rate hike, the values of bank portfolios move in tandem. The similarity in bank portfolio exposures carries the potential for systemic breakdowns of the clearing and payment system, if many banks can fail simultaneously. Our aim is to exploit the properties of the banking sector to characterize the potential for systemic risk qualitatively. The fortunes of the banking sector as indicated by the balance sheet items, are sooner or later also reflected in the value of bank equity. This enables us to characterize systemic failure in terms of the joint bank equity price movements, which are driven by the interdependent bank portfolios. Two qualitatively different intensity levels of joint adverse movements in bank stock returns are recognized.

We will say that the potential for systemic breakdown is either *weak* or *strong*, depending on whether the linkages vanish or remain asymptotically as the loss levels are increased. Random variables are either asymptotically independent or dependent, regardless their correlation. If asymptotically independent, the dependency when present, eventually dies out completely as the losses become more and more extreme. It is shown that the linearity of portfolios in the exposures or risk factors, in combination with a simple condition on the tails of the marginal distributions of these

exposures, determines whether the potential for systemic risk is weak or strong. If the exposures have normal distributions, there is weak potential for systemic risk, while if e.g. the Student distributions apply the potential is strong. We show that given the apparent fat tail property of the marginal distributions of the risk drivers, the financial intermediaries, equity returns are necessarily asymptotically dependent.

Several researchers have started to measure bank loss interdependencies more elaborately. One line of research employs correlation analysis, see [Estrella \(2001\)](#), [Bikker and Van Lelyveld \(2002\)](#), [De Nicolo and Kwast \(2002\)](#).<sup>1</sup> This literature shows that the risk diversification motive is an important driving force behind financial intermediaries merger activities. At the same time this literature finds evidence for stronger loss dependencies for financial conglomerates than for more homogeneous banks. Both findings are consistent with our theory, saying that the dependency hinges on the balance sheets of different banks containing the same items. A conglomerate by nature has a wider (diversified) portfolio, containing more items that potentially are also part of other conglomerates balance sheets. So conglomerates are better at absorbing small shocks, but at the same time are more exposed to shocks from anywhere in the system.

Regression analysis has also been used to identify the risk drivers. An interesting recent contribution is by [Bae et al. \(2003\)](#), see also [Minderhout \(2003\)](#), in which joint extremes are partly explained by a number of macrovariables through a multinomial logistic regression. The correlation based literature suffers from the fact that the correlation measure is very much tied to the multivariate normal distribution which is focussed on the dependency in the center. In fact, there exists quite a bit of evidence that marginal distributions of (bank) stock returns are not normally distributed, especially in the tail area.

The line of research based on statistical extreme value theory to study the loss dependencies, is semi-parametric in nature and hence does not suffer from the normality supposition. Moreover, since only the tail part is modeled parametrically, estimation only uses data from the tail area and hence is not biased towards the center. Examples from this literature are [Starica \(1999\)](#), [Straetmans \(2000\)](#), [Longin and Solnik \(2001\)](#), [Hartmann et al. \(2004\)](#), and [Poon et al. \(2004\)](#). This research evaluates the dependency in the limit. Evaluating the dependency in the limit is certainly a strong interpretation of what constitutes a crash. Nevertheless, what happens in the limit is also informative about what happens at extreme but finite sample points. This literature found evidence for both cases, strong and weak dependency.

Today there does not exist a cogent economic explanation for the observed limiting dependency. We show that standard economic theory can explain the asymptotic dependence in the financial sector from the structure of the balance sheets of different financial intermediaries. We provide an economic rationale for the case of asymptotic dependence, combining the fat tail feature of the marginal distributions with the linearity of portfolios. Moreover, we do not stop short by providing

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<sup>1</sup> For a broad survey of the contagion literature based on correlation analysis see [De Bandt and Hartmann \(2001\)](#).

and explaining the classification in weak and strong potential for systemic risk, but we also suggest a measure for the amount of systemic risk. This then provides a scale along which supervisors may classify systemic risk and take decisions. The amount of asymptotic dependence provides a scale along which one can judge financial fragility or the amount of systemic risk. In the end such a scale may be useful to determine the level of supervision and regulation that one wants to impose on the banking sector.

Our reduced form approach has the virtue of simplicity, and therefore is able to connect the highly stylized theoretical models of the banking industry, which often ignore important statistical features, and purely statistical based models that only indirectly relate to the underlying economic explanations. A drawback of the approach is that it cannot without further identification distinguish between different sources of systemic failure such as contagion and macro risk drivers. The theory encompasses the different explanations offered for fragility, though, but the reduced form nature of the explanation does not directly distinguish between the different explanations when applied to real data. For policy this is may constitute a drawback. It is nevertheless important to be able to show and understand how the observed strong dependency between bank returns in stress situations derives simply from the elementary economic motive for diversification in combination with the marginal fat tail property of the underlying risk drivers. In this way we can bridge the gap that exists between more fully developed economic models, as in e.g. [Allen and Gale \(2000\)](#), and the summary statistical based evidence, which often does not go beyond the market speak of increased correlation during times of market stress.

### *1.1. Empirical motivation*

The paper is explicitly focussed on the bank interdependencies during times of turbulence. To motivate the issue of extreme dependencies graphically, consider the following crossplot of 3283 daily logarithmic equity returns for two Dutch banks ABN-AMRO and ING bank (from April 3, 1991 to February 10, 2003). The Dutch banking sector is dominated by three large banks and a cooperative. These banks, moreover, used to hold considerable stakes in each other, apart from having similar exposures in their loan portfolios and through the interbank market (the latter exposure is currently more EU oriented). The two bank returns are clearly not independent, the correlation measure is a sizable  $\rho = 0.73$ .<sup>2</sup>

What can we say about the dependency in the tails? This requires a benchmark against which the crossplot can be judged. To address this question, we take the estimated correlation coefficient together with the means and variances of the two marginal distributions and generate an equal amount of bivariate normally distributed pseudo random numbers. The normal resample is shown in the next figure, with the same scale as the previous figure. The plots differ markedly in the extreme

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<sup>2</sup> The Dutch banking sector exhibits higher correlations than many other countries' banking sectors, see [Bikker and Van Lelyveld \(2002\)](#).

North-East and South-West corners. The true data contain many more outliers than the normal distribution would suggest. These outliers, moreover, are located along the diagonal, and thus mostly occur jointly. The correlation cum normal assumption does not adequately capture the dependency in the tail areas (of the distribution) that is observed in the crossplot of Figs. 1 and 2.

The discrepancy between the two figures is not atypical for asset markets in general. The empirical literature has concluded the concept of normal based correlation does not adequately capture the dependency structure. See e.g. Longin and Solnik

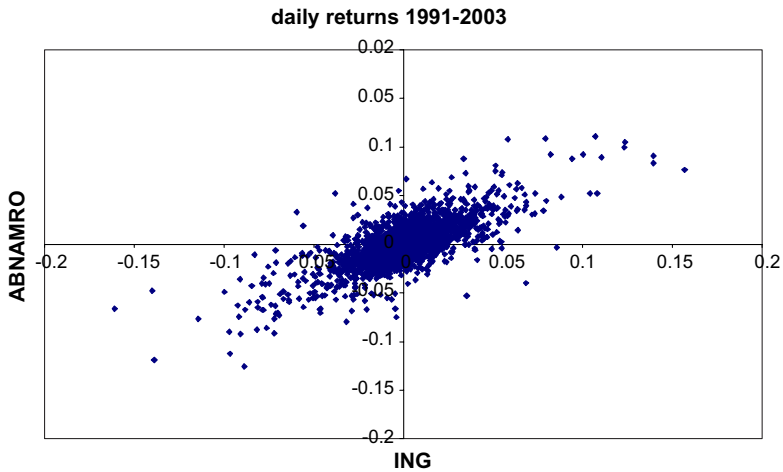


Fig. 1.

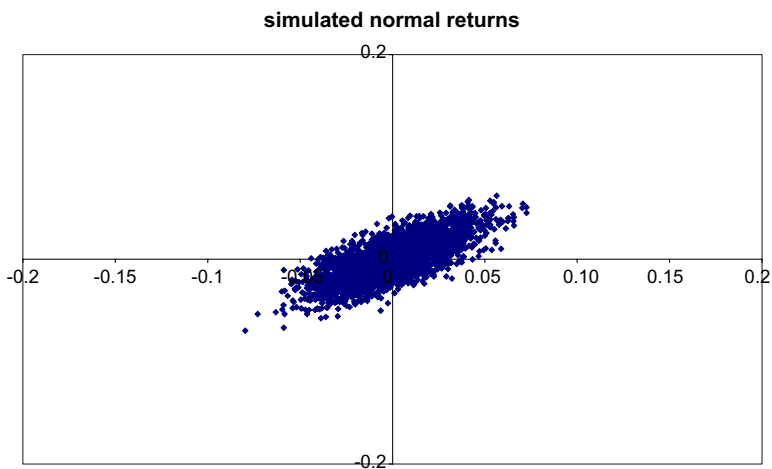


Fig. 2.

(2001) and King and Wadhvani (1990). In the literature this is mostly known as the changing correlation feature'. To investigate this data feature further, several researchers have calculated the so called conditional correlation measure. Boyer et al. (1997) show, however, that the conditional correlation measure for a bivariate normal distribution with given correlation can vary considerably depending on the conditioning sets, even when the true data generating mechanism is the bivariate normal distribution. Thus changes in correlation may be an artefact derived from the very act of conditioning; this was further illustrated in Forbes and Rigobon (2002). The correlation measure, moreover, as a measure of dependency is tightly connected with the multivariate normal law. Embrechts et al. (2002) contains an extensive and insightful discussion of the pitfalls of correlation analysis when the data are non-normal. We briefly return to this issue when we propose our preferred measure for extreme dependence.

The remainder of this paper proceeds as follows. In Section 2 we discuss the linear linkages and fat tails which are so prevalent in financial markets. A discussion and comparison of different measures to characterize linkages between banks during periods of market stress is provided in Section 3. We propose a scale for judging systemic risk. The central results of the paper regarding the relationship between the asset return distributions' marginal tail properties and the degree of tail dependence between portfolio return distributions under affine bank portfolios are derived in Section 4. The two cases of thin tailed and fat tailed marginals are treated in two separate sections. Finally, Section 5 contains a summary and some policy implications.

## 2. Affine portfolios and fat tails

The purpose of the paper is to provide an economic explanation for the observed limiting dependency in the data. To do this, we capitalize on the economic structure of the financial sector in combination with the heavy tail properties of asset return distributions.

Even though the crossplots are suggestive, we have little economic intuition as to why asset returns should be asymptotically dependent or independent. A simple economic rationale is developed in Section 4. Before we do so, we briefly review the economic theory of asset market crises, argue the affine structure of bank portfolios in the underlying risks and we provide evidence on the fat tail property of equity returns. Economic theory has not paid attention to the type of dependency that one might observe statistically, it has mostly focussed on the possible sources behind a systemic crisis.

Economic theory classifies financial crises broadly into two categories. The first category sees crises as caused by bad outcomes in underlying economic variables and is labelled for obvious reasons as fundamentals based. For example, Gorton (1988) makes forcefully the point that most episodes of banking instability in US history seem to have been related to business cycle downturns rather than occurring randomly. Krugman (1979) shows how unsustainable large budget deficits can lead to currency attacks. Detragiache (2001) discusses how lower barriers to capital

mobility may lead domestic depositors to drain domestic liquidity, thereby triggering a banking crisis. Even though these causes are fundamentals based, the first category also comprises bubble equilibria (in which the fundamentals can play the role of the martingale in the homogeneous solution). The other category holds that such crises are the expression of an occasional inherent malfunctioning of financial institutions or markets which causes an extrinsic to the economic fundamentals, and it is generally known as sunspots based equilibria. For example, [Diamond and Dybvig \(1983\)](#) show that bank depositor runs can occur as a self-fulfilling prophecy, which would imply that they happen more or less randomly. More recently, greater attention has been paid to the breadth of financial crises, relating to the notion of systemic risk. Similar to a single bank failure, the joint occurrence of bank failures can be related to common macroeconomic shocks and direct economic exposures, to multiple equilibria and to contagion. [Acharya and Yorulmazer \(2002\)](#) for example show that as long as different investment banks hold stakes in the same companies (to diversify and reduce risk), bank stocks are necessarily interdependent. [Allen and Gale \(2000\)](#) model theoretically the idea of contagion by the spreading of bank failures through interbank exposures. The verdict on the relevance of the contagion based crisis versus macrofactors affecting the entire bank sector directly, is still out though.

In the present note we do not take a position regarding the two views of self-fulfilling and fundamentals-based financial crises. Neither do we distinguish between macrorisk drivers and contagion. We rather concentrate on the systemic significance of a crisis directly. This enables us to use a reduced form approach, which is relatively simple and robust, and hence can act as a go between the highly stylized theoretical models, which often ignore important statistical features, and purely statistical based models that only vaguely relate to the underlying economic explanations. The theory can encompass the different explanations offered for fragility, though, but the reduced form nature of the explanation cannot directly distinguish between the different explanations when applied to bank portfolio data or bank equity data. This would require further identification of the shock sources. Take the market model for example. One might wonder whether it is the shocks in the market factor, the idiosyncratic shocks, or both, that are behind the systemic dependency. Estimation of the betas would enable identification of the two contributing sources. To distinguish between the different causes may be important for certain policy actions which aim to reduce the fragility. If the source of systemic risk is contagion, this calls for a reduction in the interbank exposures. If the threat comes from macrorisks, this calls for a stabilization of macropolicies. Regardless the sources, we believe it is important to be able to show how the observed strong dependency between bank returns in stress situations derives simply from the elementary economic motive for diversification in combination with the marginal fat tail property. We discuss the linearity of bank portfolios in further detail.

### *2.1. Affine portfolios*

It is widely observed that financial institutions are linked. There are several factors that contribute to the interdependency. One important linkage stems from the

sizable interbank deposit market, see [Allen and Gale \(2000\)](#), or [Elsinger et al. \(2002\)](#). Other contributing factors are the syndicated loans and more indirectly, similar exposures stemming from proprietary investments in equities, mortgages, etc. On the liability side deposit contracts all move in similar directions since they all respond to the same interest rate movements. This linkage is thus often just a linear relation between the returns on the projects financed by the different financial institutions. The linearity is due to the structure of e.g. the interbank deposit market and the syndicated loans market. Assume that each project has an independently distributed return. We impose independence between projects, so as not to build in the dependency in advance. Nevertheless we recognize that there are macrorisk drivers such as the business cycle, which affect all projects to a greater or lesser extent simultaneously. We can allow for such effects, but it is shown afterwards that such macroshocks only reinforce our arguments. Since, if project returns are driven by an affine combination of independent risk factors, such as in Roll's APT (with orthogonalized factors), the same result applies.

To further motivate the linear linkage structure due to loan syndication, consider the following excerpts from an article by Carrick Mollenkamp on syndicated loans in the *Wall Street Journal*, Friday September 20–22, 2002, European edition.

#### Deals & Deal Makers

For banking investors stung by this week's bad loan warning from J.P. Morgan Chase & Co., the question now is: Who might be next? The unsettling answer could be: almost anyone.

J.P. Morgan's announcement late Tuesday that it will need \$1.4 billion (1.44 billion) in credit costs for soured loans highlights what could become a much broader industry problem, as mega-banks like J.P. Morgan, Citi-group Inc. and Bank of America Corp. have moved to syndicate, or sell, their loans to other banks and investors to spread the risk.

On the one hand, that practice has helped the big banks avoid the kind of blowups that could bring them down. Indeed, bad loans as a percentage of the total of corporate loans are now well below where they were in past recessions, totaling 1.7% at J.P. Morgan and 1.4% at Citigroup. While syndication has helped shield individual banks from trouble, it also means that when problems arise, they quickly cascade across the balance sheets of other banks. What frustrates investors now is that no one knows for sure all of the loans that tripped up J.P. Morgan – and which other banks could be left holding the bag.

The article nicely points out there is a trade-off from being connected through a network. On the one hand, being connected smears out the risks over multiple institutions. Adverse movements which might have toppled a single bank, therefore have no effect since the multiple banks are now carrying this risk together. On the other hand, a network is more conducive to systemic risk than a number of banks operating in isolation. A very large shock may topple the entire system, since no bank is able to bear its share in the adverse movement. The article later continues with:



J.P. Morgan's problems come at a time when the US's biggest banks already are under scrutiny. Federal bank regulators recently wrapped up an annual review of the loans that the biggest banks syndicate. During the review, which is called a Shared National Credit review, regulators identify problem loans and then can tell banks to classify the loans as nonperforming.

It is a massive review that looks at about \$2 trillion of loan commitments. Analysts and money managers now believe that regulators have told some banks that syndicated loans to downgrade those loans. When that happens, all of the banks in the syndicate – not just the one that arranged the deal – are forced to write down those loans.

Syndicated loans are a clear example of linear dependency. Banks hold participations (shares) in large loans extended to firms, and hence the performance of these loans directly affects the performance of the bank's equity.

The motive for holding (linear) portfolios rather than specialized portfolios is to diversify risk. This motive is very basic and we take it as given, so that we do not have to model this drive for diversification any further (we do not enter into a utility based supporting derivation). Apart from the risk averse behavior induced by concave utility functions, there is also a societal drive for risk aversion since the regulatory framework in place also induces risk diversification and risk transfer. For example, the Basle I accord is widely regarded as the stimulus for banks to transfer credit portfolios to insurance companies. This in part is the reason why in the recent recession e.g. insurance companies' equities were doing worse than bank equity.

To prevent any misunderstandings, note that the portfolios may contain financial instruments which are non-linear functions of some underlying asset, as in the case of options. For the theory below, we only need to know whether the return distribution of the financial instrument is heavy tailed or not. Thus it is sufficient to know that the underlying asset induces the heavy tails on the return distribution of the instrument, but it is not necessary to know how this happens. Matters are different when both the instrument and the underlying asset are part of the portfolio. In that case the assumption of independence between the portfolio components clearly fails. For instruments that are linear in the underlying assets, such as a forward contract, this can be handled by applying Feller's convolution theorem presented below. In the same spirit, CAPM type dependence among the assets can be easily handled within our setup (and is therefore omitted for consideration of space). But the non-linear dependence between an option's return and the return distribution of the underlying asset is more complicated and is outside the scope of the present paper.<sup>3</sup>

In the rest of the paper we will consider two bank portfolios which are different linear combinations of two projects. Consider two syndicated loans with i.i.d. returns  $X$  and  $Y$ . To keep the presentation straightforward we limit ourselves to two dimensional loan portfolios; larger portfolios can be handled, but do not add new qualitative structure. We can allow the projects to be time dependent, but we need the cross

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<sup>3</sup> A possible approach is via the option beta and higher order approximations.

sectional independence. Below we comment on this, see Remark 1. The loans for the projects are underwritten by two investment banks or sold on to two financial intermediaries, like commercial banks or insurance companies. Let bank one hold the portfolio

$$Q = (1 - \gamma)X + \gamma Y, \quad (2.1)$$

while the loan portfolio of bank two is

$$W = \gamma X + (1 - \gamma)Y. \quad (2.2)$$

Here  $\gamma$  is restricted to be in the interval between zero and one. Note that the correlation between the two portfolios is <sup>4</sup>

$$\rho = \frac{2\gamma(1 - \gamma)}{\gamma^2 + (1 - \gamma)^2} = 1 - \frac{1 - 4\gamma(1 - \gamma)}{1 - 2\gamma(1 - \gamma)}.$$

Hence for  $\gamma \in (1/2, 1)$  the correlation is nonzero.

The use of linear models is by no means limited to this portfolio example. One can reinterpret the  $X$  and  $Y$  as orthogonal risk factors as in the popular Arbitrage Pricing Theory for explaining equilibrium equity returns (Ross, 1976; Roll and Ross, 1980). Other examples are e.g. the monetary model from the exchange rate literature.

## 2.2. Fat tails

The other feature that we will exploit, is the stylized fact that individual asset return distributions exhibit tails which are fatter than the normal distribution. Individual empirical asset return distributions yield more frequent crashes than would be predicted by the normal distribution. Since the seminal work by Mandelbrot (1963), numerous studies have found evidence for this non-normality. The relative occurrence of stock market extremes has by far received most of the attention; see e.g. Blattberg and Gonedes (1974), Jansen and De Vries (1991), Longin (1996) or Jondeau and Rockinger (2003). Bond market extremes have been considered in Hartmann et al. (2004). In applications to risk management this data feature plays an important role when devising stress tests. Thus fat tails and tail dependence of asset returns have by now been extensively documented in the empirical literature.

Embrechts et al. (2002) and Bradley and Taqqu (2003) discuss non-normal distribution settings in which correlation can be used and cannot be used as a measure of dependence. Moreover, this latter literature contains extensive discussion of alternative dependency measures such as copulas. This literature however, does not address which types of copulas are natural in economics and finance (also see Remark 1). In this paper we start to address this issue. We show how the marginal tail thickness relates theoretically to the bivariate tail dependence of returns in standard asset pricing models. In other words, by combining elementary financial models with marginal

<sup>4</sup> To see this, note that since  $X$  and  $Y$  are i.i.d.,  $\sigma_x^2 = \sigma_y^2$  and hence  $\sigma_q^2 = \sigma_w^2 = \gamma^2\sigma_x^2 + (1 - \gamma)^2\sigma_x^2$ . The portfolios covariance is  $\text{Cov}(Q, W) = 2\gamma(1 - \gamma)\sigma_x^2$ .

return distributions that are heavy tailed, we derive an economically relevant dependency structure.

### 3. Measures of dependency

The often used correlation coefficient is quite natural within the setup of linear models, but since it is a global measure it does not reflect all the relevant information in the tail area when the marginal distributions are non-normal. Moreover, it is a rather indirect measure. We discuss an alternative measure which directly links losses with the associated probabilities and which is explicitly focussed on the tail area.

#### 3.1. The correlation measure

The standard measure of dependency is the coefficient of correlation  $\rho$ . As is well known the means, variances and the correlation coefficient of a pair of random variables completely characterize the bivariate normal distribution under linear dependency. In our setup the linearity condition is satisfied due to the linearity of the bank portfolios. One must ask, however, how well  $\rho$  captures the dependency if the data are not normally distributed. Specifically, one wonders whether  $\rho$  adequately captures the interdependency at crisis levels. Boyer et al. (1997) noticed that even if the normal model applies, verifying the market speak of increased correlation during times of crisis by calculating conditional correlation coefficients can be illusory. Forbes and Rigobon (2002) show that, indeed, if one corrects  $\rho$  not much of a correlation change can be identified around crisis times. Moreover, the empirical literature finds little support for normality in stress situations, see De Nicolo and Kwast (2002).

One of the problems associated with the concept of correlation is that the data may be dependent, while the correlation coefficient is zero. Consider e.g. the discrete uniform distribution on the 8 points  $(\pm 1, \pm 1)$ ,  $(\pm 2, \pm 2)$  as in Fig. 3. Due to the symmetry it is immediate that  $\rho = 0$ , though the data are not independent. If  $x = -1$ ,  $y$  cannot be equal to 2, and  $P\{Y > 1 | X > 1\} = 1/2$ , while unconditionally  $P\{Y > 1\} = 1/4$  only. Thus  $\rho$  does not capture the dependency that is in the data.<sup>5</sup>

These examples, though, are less relevant within the linear framework (2.1) and (2.2), where the correlation is necessarily non-zero as long as  $\gamma \in (0, 1)$ . Another drawback is that the first two moments need to be bounded, but again for the asset risks faced by banks this is also not so relevant. Very relevant, though, is the criticism that the correlation measure is a measure for dependency in the center, which gives little weight to tail events when evaluated empirically. This becomes important if the marginal distributions are non-normal and the correlation measure sends the wrong signal. For example, consider the distribution concentrated on the four points

<sup>5</sup> The bivariate Student- $t$  distribution with density  $f(x, y) = \frac{1}{2\pi} [1 + \frac{x^2 + y^2}{v}]^{-(v+2)/2}$  constitutes another popular example, see Embrechts et al. (2002). Even if  $\rho = 0$  the model still exhibits dependence. This follows because the joint density cannot be factorized into the marginal dfs.

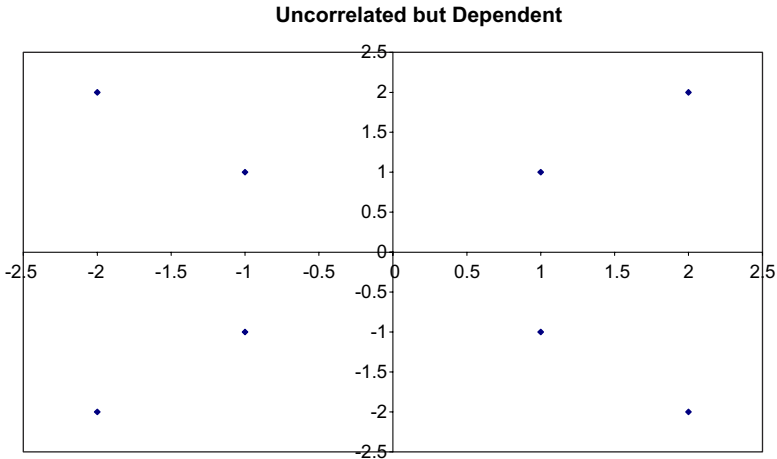


Fig. 3.

$(-\sqrt{2}, -\sqrt{2})$ ,  $(\sqrt{2}, \sqrt{2})$ ,  $(-1, 1)$  and  $(1, -1)$ ; where the first two points each have pointmass  $1/6$ , and the last two points have mass  $1/3$ , see Fig. 4. Again, one verifies that the correlation is zero, but now the dependency in the tails is opposite to and compensated by the dependency in the center. To study systemic risk features, one needs a measure of dependency that is exclusively focussed on the tail area.

Another important concern is that the economist who evaluates investments within expected utility theory frameworks, is not so much interested in the correlation measure itself; he rather has an interest in the trade-offs between risk measured as a probability and the gains or losses, which are the quantiles of the return distribution. As such the correlation is only an intermediate step in the calculation of this trade-off between quantile and probability. Therefore we like to turn to a measure

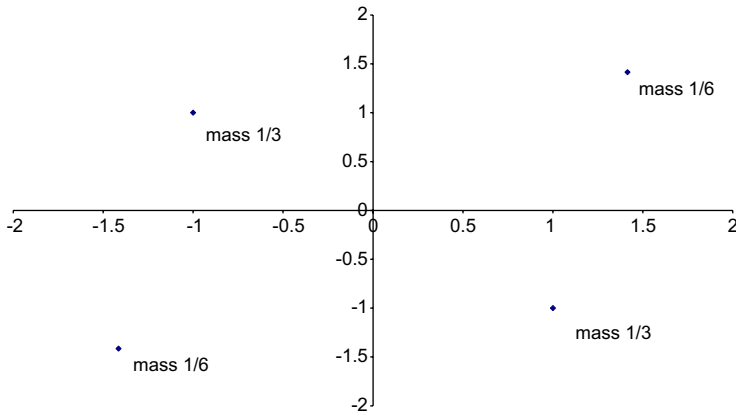


Fig. 4.

which is not conditioned on a particular multivariate distribution and which directly reflects the probabilities and associated crash levels.

### 3.2. Systemic risk measure

What is worrying for supervisors and industry representatives is that a heavy loss of one bank goes hand in hand with a heavy loss of another bank, creating systemic risk. More specifically, one asks given that  $W > s$ , what is the probability that  $Q > s$ , where  $Q$  and  $W$  are the bank equity loss returns and  $s$  is the common high loss level.<sup>6</sup> Since we are interested in the extreme linkage probabilities, we will try to directly evaluate these probabilities, bypassing the correlation concept.

If two random variables  $Q$  and  $W$  are not independent, having some information about one variable, say  $W$ , implies that one has also information about the other variable,  $Q$ . This can be readily expressed as a conditional probability  $P\{Q > s | W > s\}$ . We will, however, adopt the related probability measure that conditions on any bank failure, without indicating the specific bank. This is the linkage measure

$$\frac{P\{Q > s\}}{1 - P\{Q \leq s, W \leq s\}} + \frac{P\{W > s\}}{1 - P\{Q \leq s, W \leq s\}} \quad (3.1)$$

proposed in Huang (1992) and employed by Hartmann et al. (2004). The linkage measure, even though it is the sum of two conditional probabilities, reflects the expected number of bank failures given that at least one bank has collapsed. To see this, let  $\kappa$  denote the number of simultaneously crashing banks, i.e., returns exceeding  $s$ , and write the conditionally expected number of bank crashes given a collapse of at least one bank as  $E\{\kappa | \kappa \geq 1\}$ .

From elementary probability theory we have that

$$\begin{aligned} E\{\kappa | \kappa \geq 1\} &= 1 \frac{P\{Q > s, W \leq s\} + P\{Q \leq s, W > s\}}{1 - P\{Q \leq s, W \leq s\}} + 2 \frac{P\{Q > s, W > s\}}{1 - P\{Q \leq s, W \leq s\}} \\ &= \frac{P\{Q > s\} + P\{W > s\}}{1 - P\{Q \leq s, W \leq s\}}. \end{aligned} \quad (3.2)$$

The conditional expectation measure  $E\{\kappa | \kappa \geq 1\}$  also has the advantages that it can be easily extended beyond the bivariate setting. Moreover, one does not need to condition on a specific bank failure, whereby one would look only into one direction in the plane of failures.

To develop some intuition for this measure as a device for measuring dependence during times of market stress, consider two polar cases.

*Case 1.* If  $Q$  and  $W$  are independent and identically distributed (i.i.d.) and writing  $p = P\{Q > s\}$ , e.g. when  $\gamma = 1$ , then

<sup>6</sup> For simplicity we take the two quantiles on which we condition equal to  $s$ .

$$E\{\kappa \mid \kappa \geq 1\} = \frac{2p}{1 - (1-p)^2} = \frac{2}{2-p}.$$

In the limit  $p \rightarrow 0$  as  $s \rightarrow \infty$ , and hence  $E\{\kappa \mid \kappa \geq 1\} \rightarrow 1$ .

Case 2. If  $Q = W$ , i.e. if  $\gamma = 1/2$ , and writing  $p = P\{Q > s\}$ , then

$$E\{\kappa \mid \kappa \geq 1\} = \frac{2p}{1 - (1-p)} = 2.$$

Clearly, even as  $p \rightarrow 0$ , still  $E\{\kappa \mid \kappa \geq 1\} = 2$ .

These two cases show that  $1 \leq E\{\kappa \mid \kappa \geq 1\} \leq 2$ . In case the return pair is completely independent (Case 1),  $E\{\kappa \mid \kappa \geq 1\}$  reaches its lower bound for very large quantiles  $s$ , which implies that the returns are also asymptotically independent. On the other hand, if the data are completely dependent, then in the limit ( $s \rightarrow \infty$ ),  $E\{\kappa \mid \kappa \geq 1\}$  is equal 2 (complete asymptotic dependence). Also notice that even though in the first case the bank stock returns are independent, the dependency measure  $E\{\kappa \mid \kappa \geq 1\}$  is higher than 1 at all finite levels of  $p$  since even with independent returns, there is a nonzero probability that ‘two banks will crash, given that at least one bank fails’.

As for the intermediate case of imperfectly correlated returns ( $\rho \neq 0, |\rho| < 1$ ), either  $E\{\kappa \mid \kappa \geq 1\} = 1$  (asymptotic independence) or  $1 < E\{\kappa \mid \kappa \geq 1\} \leq 2$  (asymptotic dependence), if the quantile  $s$  gets large. In particular, one cannot rule out that bank returns are asymptotically independent in the presence of a nonzero correlation. Thus bank stock returns can be: (1) independent, (2) dependent but asymptotically independent, or (3) dependent and asymptotically dependent. In the first case the joint distribution factors into the product of the two marginal distributions. The interesting possibilities are the cases two and three. In these latter cases there is a distinction between the strength of the dependency in the tail area. For example, the correlated bivariate normal with  $|\rho| \neq 1$ , though dependent, it is asymptotically independent. The data in the cross plots suggest that the normal model is not appropriate due to the fact that the data exhibit stronger dependency in the tail area than is suggested by the bivariate normal plot.

### 3.3. Fragility scale

If one confronts supervisors, regulators and financial industry officials with the question of how much systemic risk there actually is, this either provokes a heated debate or deep silence. The reason is that we do not have a good definition of what we mean by systemic risk and hence we do not have a unique measure for financial fragility. We did discuss problems with using standard measures like the correlation concept. It would already be a step forward to possess a relative measure for comparing the fragility of different systems. This would enable regulators or supervisors to be more or less stringent, depending on whether the particular banking system at hand is more or less fragile than another system. The measure (3.2) can at least be used as a relative measure of market fragility. It provides a scale along which we can classify the fragility of the banking sector. When this scale equals one, i.e. when-

ever  $E\{\kappa|\kappa \geq 1\} = 1$  in the limit, we dub the fragility or linkage as weak (asymptotic independence). But if  $E\{\kappa|\kappa \geq 1\} > 1$ , the fragility is strong (asymptotic dependence). If the former case applies, the banking system is stable, whereas in the latter case it is subject to systemic risk and hence more fragile. But in the case of strong fragility, the amount varies between 1 and 2.<sup>7</sup> Thus supervisors should adopt a supervisory and perhaps regulatory stance which varies in strength with the level of the proposed fragility scale.

#### 4. Weak and strong financial fragility

Within the affine model of bank portfolios, we are now ready to prove that the limiting value of (3.2) critically depends on the tail properties of the marginal distributions of the portfolio components or risk factors.

##### 4.1. Asset returns with light tails

In this section we consider two specific light tailed distributions: the exponential and the normal distribution. Recall that bank one has portfolio returns  $Q = (1 - \gamma)X + \gamma Y$ , while the loan portfolio return of bank two is  $W = \gamma X + (1 - \gamma)Y$ . For simplicity shortselling is not considered, and  $\gamma$  is restricted to be in the interval between one half and one (the other case is completely analogous). We start with the exponential distribution and adopt the convention that  $Q$  and  $W$  denote the loss returns, so that the bank loss returns are modeled as positive numbers.

**Proposition 1.** *Suppose the project loss returns  $X$  and  $Y$  in the portfolios (2.1) and (2.2) are independently exponentially distributed, with density  $f(x) = \exp(-x)$ , then for  $\gamma \in (1/2, 1)$*

$$\lim_{s \rightarrow \infty} E\{\kappa | \kappa \geq 1\} = \lim_{s \rightarrow \infty} \frac{P\{Q > s\} + P\{W > s\}}{1 - P\{Q \leq s, W \leq s\}} = 1.$$

**Proof.** Consider  $P\{Q \leq s, W \leq s\}$  and exploit the symmetry

$$P\{Q \leq s, W \leq s\} = P\{X \leq s, Y \leq s\} + 2P\left\{0 \leq X \leq s, s \leq Y \leq \frac{s}{\gamma} - \frac{1-\gamma}{\gamma}X\right\}.$$

By integration

$$\begin{aligned} 1 - P\{Q \leq s, W \leq s\} &= 1 - (1 - e^{-s})^2 - 2 \int_0^s e^{-x} \int_s^{\frac{s}{\gamma} - \frac{1-\gamma}{\gamma}x} e^{-y} dy dx \\ &= \frac{2\gamma}{2\gamma - 1} e^{-\frac{s}{\gamma}} - \frac{1}{2\gamma - 1} e^{-2s}. \end{aligned}$$

<sup>7</sup> If desired, one can also distinguish between different levels of weak fragility, see Coles et al. (1999).

For  $P\{Q>s\}$  we find

$$\begin{aligned}
 P\{Q > s\} &= 1 - P\left\{0 \leq X \leq \frac{s}{1-\gamma}, 0 \leq Y \leq \frac{s}{\gamma} - \frac{1-\gamma}{\gamma}X\right\} \\
 &= 1 - \int_0^{\frac{s}{1-\gamma}} e^{-x} \int_0^{\frac{s}{\gamma} - \frac{1-\gamma}{\gamma}x} e^{-y} dy dx = \frac{\gamma}{2\gamma-1} e^{-\frac{s}{\gamma}} - \frac{1-\gamma}{2\gamma-1} e^{-\frac{s}{1-\gamma}}
 \end{aligned}$$

Thus

$$E\{\kappa \mid \kappa \geq 1\} = \frac{2\gamma e^{-\frac{s}{\gamma}} - 2(1-\gamma)e^{-\frac{s}{1-\gamma}}}{2\gamma e^{-\frac{s}{\gamma}} - e^{-2s}} = \frac{2\gamma - 2(1-\gamma)e^{-\frac{s(2\gamma-1)}{\gamma(1-\gamma)}}}{2\gamma - e^{-s\frac{2\gamma-1}{\gamma}}}$$

Hence, since  $\gamma \in (1/2, 1)$  taking limits yields

$$\lim_{s \rightarrow \infty} E\{\kappa \mid \kappa \geq 1\} = \lim_{s \rightarrow \infty} \frac{2\gamma - 2(1-\gamma)e^{-\frac{s(2\gamma-1)}{\gamma(1-\gamma)}}}{2\gamma - e^{-s\frac{2\gamma-1}{\gamma}}} = 1. \quad \square$$

Thus the multivariate distribution induced by the two portfolios  $(1-\gamma)X + \gamma Y$  and  $\gamma X + (1-\gamma)Y$  is asymptotically independent, even though the correlation coefficient is

$$\rho = 1 - \frac{1 - 4\gamma(1-\gamma)}{1 - 2\gamma(1-\gamma)} > 0.$$

Now suppose that  $X$  and  $Y$  are i.i.d. standard normally distributed. It is immediate that the portfolios  $Q$  and  $W$  are multivariate normally distributed with the same correlation coefficient as above.

**Proposition 2.** *If  $X$  and  $Y$  follow independent standard normal distributions and  $\gamma \in (1/2, 1)$ , then  $\lim_{s \rightarrow \infty} E\{\kappa \mid \kappa \geq 1\} = 1$ , so that the fragility is weak.*

In order to prove this claim we use Sibuya’s (1960) approach and the following asymptotic expansion for the tail probability of a normally distributed random variable:

$$P\{\theta X > s\} \sim \frac{1}{\sqrt{2\pi}} \frac{\theta}{s} \exp\left(-\frac{1}{2}\left(\frac{s}{\theta}\right)^2\right), \quad \text{for } s \text{ large} \tag{4.1}$$

(see e.g. Abramowitz and Stegun, 1972, p. 932).

**Proof.** First note that

$$E\{\kappa \mid \kappa \geq 1\} = \frac{P\{Q > s\} + P\{W > s\}}{1 - P\{Q \leq s, W \leq s\}} = \frac{1}{1 - \frac{P\{Q > s, W > s\}}{P\{Q > s\} + P\{W > s\}}}.$$



The following bound for the factor in the denominator will be exploited:

$$\begin{aligned} \frac{P\{Q > s, W > s\}}{P\{Q > s\} + P\{W > s\}} &= \frac{P\{Q > s, W > s\}}{2P\{Q > s\}} \leq \frac{P\{Q + W > 2s\}}{2P\{Q > s\}} \\ &= \frac{P\{X + Y > 2s\}}{2P\{(1 - \gamma)X + \gamma Y > s\}}. \end{aligned}$$

By the properties of the normal distribution,

$$P\{X + Y > 2s\} = P\{\sqrt{2}X > 2s\} \sim \frac{1}{\sqrt{2\pi}} \frac{\sqrt{2}}{2s} \exp\left(-\frac{1}{2} \left(\frac{2s}{\sqrt{2}}\right)^2\right)$$

and

$$\begin{aligned} P\{(1 - \gamma)X + \gamma Y > s\} &= P\left\{\sqrt{(1 - \gamma)^2 + \gamma^2} X > s\right\} \\ &\sim \frac{1}{\sqrt{2\pi}} \frac{\sqrt{(1 - \gamma)^2 + \gamma^2}}{s} \exp\left(-\frac{1}{2} \frac{s^2}{(1 - \gamma)^2 + \gamma^2}\right). \end{aligned}$$

Putting the numerator and denominator together and recalling  $\gamma \in (\frac{1}{2}, 1)$ , gives

$$\frac{P\{Q > s, W > s\}}{P\{Q > s\} + P\{W > s\}} \leq \frac{\sqrt{2}}{4\sqrt{(1 - \gamma)^2 + \gamma^2}} \exp -s^2 + \frac{1}{2} \frac{1}{(1 - \gamma)^2 + \gamma^2} s^2 \rightarrow 0$$

as  $s \rightarrow \infty$ .

It follows that

$$\lim_{s \rightarrow \infty} E\{\kappa \mid \kappa \geq 1\} = 1. \quad \square$$

These asymptotic independence results are by no means limited to the exponential or normal distributions. A similar procedure can be used to verify the asymptotic independence for many other types of joint distributions. The normal distribution appears most interesting, though, since it is so often assumed in theoretical and empirical work on equity returns and in other asset pricing applications. Note that we have just shown that the multivariate normal implies that systemic breakdowns essentially cannot occur. This optimistic view will be tempered by the results when we assume that the tails of the (marginal) loss distributions are fat.

#### 4.2. Asset returns with heavy tails

Prior to relating the tail fatness of bank portfolio returns to their degree of asymptotic dependence, we need a formal definition of what the term ‘fat tails’ exactly means. We adopt the following convention. A random variable exhibits heavy tails if its distribution function  $F(s)$  far into the tails has a first order term identical to the Pareto distribution, i.e.

$$F(s) = 1 - s^{-\alpha}L(s) \quad \text{as } s \rightarrow \infty, \tag{4.2}$$

where  $L(s)$  is a slowly varying function such that

$$\lim_{t \rightarrow \infty} \frac{L(ts)}{L(t)} = 1, \quad s > 0. \tag{4.3}$$

It can be easily shown that conditions (4.2) and (4.3) are equivalent to

$$\lim_{t \rightarrow \infty} \frac{1 - F(ts)}{1 - F(t)} = s^{-\alpha}, \quad \alpha > 0, \quad s > 0. \tag{4.4}$$

If (4.4) holds, the distribution is said to vary regularly at infinity. The tail index  $\alpha$  can be interpreted as the number of bounded moments. Since not all moments are bounded, we speak of heavy tails. Distributions like the Student- $t$ ,  $F$ -distribution, Burr distribution, sum-stable distributions with unbounded variance all fall into this class. It can be shown that the unconditional distributions of the ARCH and GARCH processes also belong to this class, see De Haan et al. (1989) for a proof. Student- $t$  distributions are often used in the empirical modelling of the unconditional return of equity returns, see e.g. Blattberg and Gonedes (1974), while GARCH processes are extremely popular conditional models, see King and Wadhvani (1990).

To derive our result, we need Feller’s convolution theorem (Feller, 1971, VIII. 8).

**Theorem 1.** *Let  $X$  and  $Y$  be i.i.d. random variables with regularly varying symmetric tails, i.e. as  $s \rightarrow \infty$*

$$P\{X \leq -s\} = P\{Y \leq -s\} = P\{X > s\} = P\{Y > s\} = s^{-\alpha}L(s).$$

*Then for the tail of the distribution of the sum of  $X + Y$  (twofold convolution) as  $s \rightarrow \infty$*

$$P\{X + Y \leq s\} = 1 - 2s^{-\alpha}L(s). \tag{4.5}$$

Thus Theorem 1 says that

$$P\{X + Y > s\} \approx 2P\{X > s\}.$$

Some intuition for this result can be obtained as follows. By the independence of  $X$  and  $Y$ , the probability of being in the lower South-West square equals

$$P\{X \leq s, Y \leq s\} = P\{X \leq s\}P\{Y \leq s\} = [1 - s^{-\alpha}L(s)]^2 = 1 - 2s^{-\alpha}L(s) + O(s^{-2\alpha}).$$

The theorem therefore maintains that the latter probability, ignoring the order term, is equal to the probability of being below the line  $X + Y = s$ ; that is, for large  $s$ ,

$$P\{X \leq s, Y \leq s\} \approx P\{X + Y \leq s\}.$$

So these quite distinct areas, although they partially overlap, carry the same probability mass. How can this be? Note that the two areas intersect the two axes at the same points  $(0, s)$  and  $(s, 0)$ , and that the marginal probabilities on being beyond these points are  $\Pr\{Y > s\}$  and  $\Pr\{X > s\}$  respectively. What transpires is that for a large threshold  $s$  the probability mass on any area far away from the origin is determined solely by where such an area cuts the axes, and the marginal probability mass that is loaded along these axes above such points. Thus the interior area is (relatively) empty. For example, consider the area in the upper North-East corner:

$$\Pr\{X > s, Y > s\} = \Pr\{X > s\} \Pr\{Y > s\} = s^{-2\alpha} L(s)^2.$$

This mass is clearly of smaller order than  $2s^{-\alpha}L(s)$ , so that it can be ignored for large  $s$ . In the proof of Theorem 1, one shows that this is true for any area excluding the axes. To summarize, for large quantiles  $s$ , all mass concentrates along the axes, so that lines and planes that cut both axes at the same points separate the same probability mass. This eventual concentration of probability mass along the axes implies

**Corollary 1.** *Let  $X$  and  $Y$  be i.i.d. random variables with regularly varying symmetric tails, i.e. as  $s \rightarrow \infty$ ,*

$$P\{X \leq -s\} = P\{Y \leq -s\} = P\{X > s\} = P\{Y > s\} = s^{-\alpha}L(s).$$

*Let  $\gamma \in [1/2, 1]$ . Then for the joint probability  $s \rightarrow \infty$ ,*

$$P\{(1 - \gamma)X + \gamma Y \leq s, \gamma X + (1 - \gamma)Y \leq s\} = 1 - 2\gamma^\alpha s^{-\alpha}L(s)$$

*as  $s \rightarrow \infty$ .*

**Proof.** Note that the two portfolio lines of the portfolios  $Q$  and  $W$ , cut the axes at  $(s/(1 - \gamma), 0)$  and  $(s/\gamma, 0)$  along the  $x$ -axis, while along the  $y$ -axis these points are, respectively  $(0, s/\gamma)$  and  $(0, s/(1 - \gamma))$ . For  $\gamma \in [1/2, 1]$ , one has that  $s/(1 - \gamma) \geq s/\gamma$ . Using the intuition behind the Feller Theorem 1 that only the mass along the axes counts, it follows that along both axes we have to deduct the mass which is above the points that are nearest to the origin, i.e.  $(s/\gamma, 0)$  and  $(0, s/\gamma)$ . The mass above the point  $(s/\gamma, 0)$  is  $\gamma^\alpha s^{-\alpha}L(s)$ , and similarly along the other axis. Adding up the two probabilities and subtraction from one then gives the claim.  $\square$

We can now give the main result.

**Proposition 3.** *Let  $X$  and  $Y$  be i.i.d. random variables with regularly varying tails, i.e. as  $s \rightarrow \infty$ ,*

$$P\{X \leq -s\} = P\{Y \leq -s\} = P\{X > s\} = P\{Y > s\} = s^{-\alpha}L(s).$$

*Then for  $\gamma \in [1/2, 1]$ ,*

$$\lim_{s \rightarrow \infty} E\{\kappa \mid \kappa \geq 1\} = 1 + \left(\frac{1}{\gamma} - 1\right)^\alpha.$$

**Proof.** By definition,

$$\lim_{s \rightarrow \infty} E\{\kappa \mid \kappa \geq 1\} = \lim_{s \rightarrow \infty} \frac{P\{(1 - \gamma)X + \gamma Y > s\} + P\{\gamma X + (1 - \gamma)Y > s\}}{1 - P\{(1 - \gamma)X + \gamma Y \leq s, \gamma X + (1 - \gamma)Y \leq s\}}.$$

Use Corollary 1 for the denominator. For the numerator adapt Feller's (1971, VIII.8) convolution theorem 1, to show that

$$P\{(1 - \gamma)X + \gamma Y > s\} = P\{\gamma X + (1 - \gamma)Y > s\} = [\gamma^\alpha + (1 - \gamma)^\alpha] s^{-\alpha}L(s).$$

Thus

$$\lim_{s \rightarrow \infty} E\{\kappa \mid \kappa \geq 1\} = \lim_{s \rightarrow \infty} \frac{2[\gamma^\alpha + (1 - \gamma)^\alpha]s^{-\alpha}L(s)}{2\gamma^\alpha s^{-\alpha}L(s)} = 1 + \left(\frac{1}{\gamma} - 1\right)^\alpha. \quad \square$$

The two portfolios returns  $Q$  and  $W$  are asymptotically dependent, since  $\lim_{s \rightarrow \infty} E\{\kappa \mid \kappa \geq 1\} > 1$ . Thus the crisis linkage for this class of distributions is strong and the financial system appears relatively fragile, exhibiting systemic risk. Note that since  $dE\{\kappa \mid \kappa \geq 1\}/d\gamma < 0$ , the asymptotic dependency is increasing in the amount of diversification. Thus systemic risk increases due to diversification. The asymptotic dependence is declining as the tail thickness declines, since  $dE\{\kappa \mid \kappa \geq 1\}/d\alpha < 0$ . This corroborates the results for the normal and exponential distributions.

**Remark 1.** We like to point out that Proposition 3 does not imply that there are no joint distributions that have heavy tailed marginals, positive correlation and asymptotic independence. In fact one can easily verify that for e.g. the bivariate Gumbel–Pareto distribution constructed from the Farlie–Gumbel–Morgenstern copula,

$$F(x, y) = (1 - x^{-\alpha})(1 - y^{-\alpha})(1 + \beta x^{-\alpha}y^{-\alpha}), \quad \alpha > 0, \quad 0 < \beta < 1,$$

the marginals exhibit Pareto shapes, i.e.,  $F_x(s) = F_y(s) = 1 - s^{-\alpha}$  and that the two variates are not independent. Nevertheless, the distribution exhibit asymptotic independence. In this sense the assumption about the linearity of asset returns in the fundamentals in Proposition 3 is crucial. One can also construct joint distributions, where the marginals have exponential type thin tails, but which nevertheless exhibit asymptotic dependence. But neither of these two examples has a firm footing in economics, whereas the linearity cum heavy tails based asymptotic dependence is implied by elementary financial structure.

A systematic analysis of bank crisis linkages implied by non-linearly dependent asset returns (or more general asset pricing) models is beyond the scope of this paper and left to future research. The above result, however, implies that if the dependency arises from the linear properties of the problem (portfolio), the marginals cannot have normal or exponential type tails to obtain asymptotic dependency. Currently, several researchers model the dependency by choosing a specific copula. The proposition suggests that if the economics imply that the dependency arises from the linearity of the problem, then one should restrict oneself to the much smaller subclass of copulas which are consistent with linear dependence.

### 5. Conclusion

We gave a simple statistical condition and a linearity property of bank portfolios which, when taken together, are sufficient for explaining the fragility of the banking system. The condition is technical and reflects the stylized fact that many asset re-

turns have marginal distributions which exhibit fatter tails than the normal distribution. The portfolio property views bank balance sheets as linear combinations of underlying risks, both on the asset and the liability sides. Banks are linked through the interbank deposit market, participations like syndicated loans and deposit interest rate risk. This implies that different bank stock returns become dependent because they have common risk drivers, albeit with different weights. It was shown that the tail condition in combination with the linearity property of the bank portfolios in the exposures determines whether the potential for systemic risk is weak or strong.

The potential for systemic breakdown is strong, in the sense that the linkages remain asymptotically, if the marginal distributions have heavy tails. Per contrast, if the exposures have marginal normal distributions, the potential for systemic risk is weak, even though the portfolios and hence bank stock returns are correlated. It is by now well known that financial returns exhibit heavy tails and are thus non-normally distributed. This implies that extreme market conditions tend to occur more frequently than could be expected on the basis of the normal distribution, which is used so often in standard asset pricing approaches. Thus if instead of the normal distribution, the exposures are e.g. Student-*t* distributed, the potential for systemic breakdown is strong.

We therefore showed that the similarity in exposures, while understandable from the point of view of risk diversification by an individual bank, carries the potential for systemic breakdowns. Thus in affine models of the financial system, the fragility of the system or its systemic stability hinges critically on the fact that different banks nevertheless hold exposures to the same risk drivers.

The fat tail data feature and the linearity property have policy implications. By pursuing stable monetary and fiscal policies, instead of drastic changes in variables like interest rates, public authorities can diminish fat tails in the macro risk drivers. In specific circumstances of large market-driven fluctuations, strong counteracting measures to provide liquidity may nevertheless be advisable (credit lines by the central bank to the commercial banks). Similarly, regulation and changes in financial oversight such as proposed by the Basle committee should be gradual. Supervision and regulation, including anti-trust measures, can reduce the systemic risk. Paradoxically, this may involve allowing for more rather than less risk concentration within a single bank. Risk concentration leads to more frequent individual bank failures, but segregation reduces the potential for systemic breakdown. From the narrow perspective of a bank regulator, it may appear preferable to shift risk to other sectors, such as the insurance sector. This is indeed that happened as a result of Basle I regulation (and why the insurance industry is currently in worse shape than the banking industry). But for the society at large there are limits to the desirability of conglomeration as this endangers the systemic stability.

Directions for future research emerge from the paper. If the linearity is indeed seen as the main cause for the asymptotic dependence, then parametric specifications using specific copulas can focus on the subclass which is consistent with this linearity. But such an investigation is beyond the ambition of the present paper. It would also be of interest to investigate the consequences of non-linear dependence if in a

portfolio both the underlying asset and an option written on this asset are present. An important policy question generated by this research regards the standard VaR measure. Since a result of our analysis is that joint crashes are more likely to occur than under the normal distribution, this implies that a VaR excess is more likely to occur jointly. Hence, if a bank acts on a VaR excess by trying to restructure its portfolio, then more likely than not other banks also have to liquidate parts of their portfolios. This leads to fire sales and further VaR violations. Since the VaR measure is univariate in nature, it suggests more room to maneuver than will actually be available in times of severe stress.

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