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COMPARATIVE ANALYSIS OF LITIGATION SYSTEMS: AN AUCTION-THEORETIC APPROACH

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A Appendix

A.1 *Proof of Proposition 2*

Suppose first that the equilibrium legal expenditures are a strictly increasing function of the amount a litigant stands to gain by winning: $e_i'(v_i) > 0$; this supposition will be verified below. Under this assumption, e_i^{-1} exists, and the expected payoff $\mathrm{E}U(e_i,v_i)$ of a party who expends e_i on legal services is as in (3) in the main text. Differentiating $\mathrm{E}U(e_i,v_i)$ in (3) with respect to e_i gives the first order condition for player i's optimal level of legal expenditures:

$$\frac{1}{e_j'\left[e_j^{-1}(e_i)\right]}\left\{v_i - \beta e_i - (1-\alpha)e_j\left[e_j^{-1}(e_i)\right]\right\}f\left[e_j^{-1}(e_i)\right] - \int_0^{e_j^{-1}(e_i)} \beta f(v_j)\mathrm{d}v_j$$

$$-\frac{1}{e_j'\left[e_j^{-1}(e_i)\right]}\left\{-\alpha e_i - (1-\beta)e_j\left[e_j^{-1}(e_i)\right]\right\}f\left[e_j^{-1}(e_i)\right] - \int_{e_j^{-1}(e_i)}^{\bar{v}} \alpha f(v_j)\mathrm{d}v_j = 0.$$

In a symmetric equilibrium, $e_i(v) = e_j(v) = e(v)$, so we may simplify the last expression to obtain the differential equation:

$$e'(v) = \frac{vf(v)}{\alpha - (\alpha-\beta)F(v)} + \frac{2(\alpha-\beta)f(v)}{\alpha-(\alpha-\beta)F(v)}e(v). \tag{11}$$

The solution to this differential equation is known as

$$e(v) = \int_0^v \frac{sf(s)}{\alpha-(\alpha-\beta)F(s)}\exp\left[\int_s^v \frac{2(\alpha-\beta)f(u)}{\alpha-(\alpha-\beta)F(u)}\mathrm{d}u\right]\mathrm{d}s. \tag{12}$$

Straightforward manipulation (12) yields (4) from the main text.
Note that the equilibrium expenditures $e(v)$ in (4) are strictly positive for all $v \in (0,\bar{v})$ since $\alpha[1-F(s)] + \beta F(s) > 0$ for $v \in (0,\bar{v})$, and $(\alpha, \beta) > 0$. We also verify that $e'(v) > 0$. Substituting (4) in the RHS of (11) yields

$$\frac{\mathrm{d}e(v)}{\mathrm{d}v} = \frac{f(v)}{\{\alpha[1-F(v)]+\beta F(v)\}^3}Q(v),$$

where

$$Q(v) = (2\alpha-\beta)\int_0^v sf(s)\{\alpha[1-F(s)]+\beta F(s)\}\mathrm{d}s + v\{\alpha[1-F(v)]+\beta F(v)\}^2.$$

Evidently, for $\alpha \geq \beta$, $Q(v) > 0$ and hence $e'(v) > 0$. When $\alpha = 0$, $Q(v)$ can be simplified to

$$Q(v) = \beta^2 \int_0^v F(s)^2 \mathrm{d}s > 0.$$

Thus if $\alpha = 0$ and $\beta > 0$ then $e'(v) > 0$. For the case $0 < \alpha < \beta$, we note that the factors multiplied by $2\alpha\beta$ can be rewritten as

$$\int_0^v sf(s)F(s)\mathrm{d}s - \int_0^v sf(s)[1 - F(s)]\ \mathrm{d}s + v[1 - F(v)]F(v)$$
$$= \int_0^v F(s)[1 - F(s)]\mathrm{d}s > 0.$$

Hence, $e'(v) > 0$ for $v \in (0, \bar{v})$.

Next, we verify that the second order condition holds. We follow Matthews' (1995) analysis for the first price auction and show that there is no incentive to misrepresent one's type, given that the opponent uses the equilibrium strategy. If a litigant bids as if his valuation is z while his true valuation is v, his expected payoff (3) becomes

$$\mathrm{E}U[e(z), v] = vF(z) - \beta e(z)F(z) - \mathrm{E}[e(v)] + \alpha \int_0^z e(v)f(v)\mathrm{d}v - \alpha e(z)[1 - F(z)]$$
$$+ \beta \int_z^{\bar{v}} e(v)f(v)\mathrm{d}v.$$

Compare this to the payoff when bidding according to his valuation $\mathrm{E}U[e(v),v]$. Define the difference in payoffs

$$\Delta(z, v) = \mathrm{E}U[e(z), v] - \mathrm{E}U[e(v), v].$$

Differentiating this difference Δ yields

$$\frac{\mathrm{d}\Delta(z, v)}{\mathrm{d}z} = vf(z) - [\alpha - (\alpha - \beta)F(z)]e'(z) + 2(\alpha - \beta)e(z)f(z).$$

Using (11) we can substitute out $e'(z)$

$$\frac{\mathrm{d}\Delta(z, v)}{\mathrm{d}z} = (v - z)f(z).$$

Integration then yields

$$\Delta(z, v) = (v - z)F(z) - \int_z^v F(x)\mathrm{d}x \leq 0$$

since $F(x)$ is non-decreasing.

Finally, we establish uniqueness. Since $\min\{\alpha[1 - F(v)] + \beta F(v)\} \geq \min(\alpha, \beta) > 0$, and the boundedness of the density imply that the linear differential equation (11) satisfies a Lipschitz condition. This implies existence and uniqueness, see Coddington and Levinson (1955, Theorems 2.3 and 2.2). Moreover, the solution can be continued to the boundaries $v = 0, \bar{v}$, see Coddington and Levinson (1955, section 1.4). Q.E.D

A.2 *Random Merit*

Here we relax assumption ($A2$). Suppose exogenous circumstances also affect the quality of the case. In particular, suppose that with probability π the quality of the case is determined as in the text, but with probability $1 - \pi$ the quality is determined by an independent random variable that awards the case to either party with probability 1/2. Payoffs are as follows, see (3), when $e_i > e_j$:

$$\pi\left[v_i - \beta e_i - (1-\alpha)e_j\right] + (1-\pi)\frac{1}{2}\left[v_i - \beta e_i - (1-\alpha)e_j\right]$$
$$+ (1-\pi)\frac{1}{2}\left[-\alpha e_i - (1-\beta)e_j\right]$$
$$= \frac{1+\pi}{2}v_i - \left(\beta\frac{1+\pi}{2} + \alpha\frac{1-\pi}{2}\right)e_i - \left[(1-\alpha)\frac{1+\pi}{2} + (1-\beta)\frac{1-\pi}{2}\right]e_j. \quad (13)$$

Similarly, if $e_i < e_j$ the payoffs are

$$\frac{1-\pi}{2}v_i - \left(\alpha\frac{1+\pi}{2} + \beta\frac{1-\pi}{2}\right)e_i - \left[(1-\beta)\frac{1+\pi}{2} + (1-\alpha)\frac{1-\pi}{2}\right]e_j. \quad (14)$$

Differentiating the implied expected payoff function yields the analogue of (3) relevant in the case of random merit:

$$e'(v) = \frac{\pi v f(v)}{\alpha((1+\pi)/2) + \beta((1-\pi)/2) + \pi(\beta-\alpha)F(v)} + \frac{2\pi(\alpha-\beta)f(v)}{\alpha((1+\pi)/2) + \beta((1-\pi)/2) + \pi(\beta-\alpha)F(v)}e(v).$$

Solving this differential equation gives the analogue to (4)

$$e(v) = \frac{\pi}{[\alpha\frac{1+\pi}{2} + \beta\frac{1-\pi}{2} + \pi(\beta-\alpha)F(v)]^2}H(v)$$

where

$$H(v) = \int_0^v sf(s)[\alpha\frac{1+\pi}{2} + \beta\frac{1-\pi}{2} + \pi(\beta-\alpha)F(s)]\mathrm{d}s.$$

The expected legal expenditures can therefore be written as:

$$\mathrm{E}[e(v)] = \int_0^{\bar{v}} \frac{\pi}{[\alpha((1+\pi)/2) + \beta((1-\pi)/2) + \pi(\beta-\alpha)F(v)]^2}H(v)f(v)\mathrm{d}v.$$

Integrating by parts and simplifying yields

$$\mathrm{E}[e(v)] = \frac{2\pi}{\alpha(1-\pi) + \beta(1+\pi)}\int_0^{\bar{v}} sf(s)[1-F(s)]\mathrm{d}s. \quad (15)$$

Comparing this expression to (8) (which obtains as a special case when $\pi = 1$), notice that α enters the expression for the expected equilibrium expenditures when $\pi \neq 0$. In this case, the American system and the Quayle proposal are no longer equivalent; under the latter the expected equilibrium expenditures are indeed lower (as hoped by the President's Council), since

$$TC^{Marshall} > TC^{American} > TC^{Quayle}.$$

Thus, assumption ($A2$) is not innocuous.

A.3 *Ex Ante Expected Expenditures in the Two-Stage Game: The Uniform Case*

We now provide the details underlying the example presented in Section 3 and Figure 2. Here, v is uniformly distributed on [0, 1] and $\alpha = \beta$. In this case, only players with valuations in excess of $\hat{v}$ will litigate, so conditional on being in the litigation stage, the opponent's valuation is uniformly distributed on the interval $[\hat{v}, 1]$. Thus, in instances where both parties litigate, the legal expenditure of a player with valuation $v \in [\hat{v}, 1]$ is

$$e(v) = \beta^{-2}\beta \int_{\hat{v}}^{v} s \frac{1}{1-\hat{v}}\, \mathrm{d}s$$
$$= \frac{1}{2\beta}\frac{\hat{v}^2 - v^2}{\hat{v} - 1}.$$

Notice that a player with valuation $\hat{v}$ loses with probability one but he nonetheless is willing to litigate because the rents earned if the rival had conceded exactly offsets any loss in the litigation stage. At trial, his own expenditures are zero by the above calculation but he pays the fraction $(1 - \beta)$ of the expenditures of his rival. The expected expenditures of his rival are

$$\mathrm{E}[e(v)|\hat{v}] = \int_{\hat{v}}^{1} \frac{1}{2\beta}\frac{\hat{v}^2 - v^2}{\hat{v} - 1}\frac{1}{1-\hat{v}}\mathrm{d}v$$
$$= \frac{1}{6\beta}(2\hat{v} + 1).$$

Thus, the expected utility of a player of type $\hat{v}$ in the litigation stage is (using (3)),

$$\mathrm{E}U^L(\hat{v}|\hat{v}) = -\beta e_i(\hat{v}) - (1-\beta)\mathrm{E}[e(v)|v > \hat{v}]$$
$$= -\frac{(1-\beta)}{6\beta}(2\hat{v} + 1).$$

Consequently, the critical value $\hat{v}$ in (10) satisfies

$$-\frac{(1-\beta)}{6\beta}(2\hat{v} + 1) = \frac{-\hat{v}^2}{2(1-\hat{v})}$$

so

$$\hat{v} = \hat{v}(\beta) = \frac{1}{\beta + 2}\left[\left(\frac{1-\beta}{2}\right) + \frac{1}{2}\sqrt{3}\sqrt{(\beta+3)(1-\beta)}\right].$$

Notice that $\hat{v}(\beta)$ is decreasing in β with $\hat{v}(0) = 1$ and $\hat{v}(1) = 0$. It follows that the *ex ante* fraction of players opting to litigate, $1 - F[\hat{v}(\beta)] = 1 - \hat{v}(\beta)$, tends to zero as β tends to zero. In contrast, all players opt for litigation when $\beta \geq 1$.

We may use these results to calculate the *ex ante* expected legal expenditures for a legal system with parameter $\alpha = \beta$ that takes into account both the incentives to litigate and the expected legal outlays conditional upon litigation. Since litigation expenditures arise only if both parties litigate, and this event occurs with probability $\{1 - F[\hat{v}(\beta)]\}^2$, we have for $\beta \in [0, 1]$ that $\{1 - F[\hat{v}(\beta)]\} \in [0, 1]$. For the uniform case ($F = v$), for instance,

$$\Pr(i \,\&\, j \text{ litigate})\mathrm{E}(\text{Legal Outlays}|i \,\&\, j \text{ litigate}) = [1 - F(\hat{v})]^2[2\mathrm{E}e(v|v > \hat{v})]$$
$$= [1 - \hat{v}(\beta)]^2\frac{2}{6\beta}[2\hat{v}(\beta) + 1].$$

When $\beta > 1$, all types will choose to litigate and thus $\hat{v} = 0$. In this case,

$$\Pr(i \,\&\, j \text{ litigate})\mathrm{E}(\text{Legal Outlays}|i \,\&\, j \text{ litigate}) = [1 - F(0)]^2 2\mathrm{E}e(v|v > 0)$$
$$= 2\mathrm{E}e(v)$$
$$= \frac{1}{3\beta}.$$

It follows from these equations that *ex ante* expected legal expenditures – taking into account both the incentives to litigate and expected expenditures per trial – are maximised when $\beta = 1$.