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AN EMS TARGET ZONE MODEL IN DISCRETE TIME

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SUMMARY

The discrete time analogue of the continuous time Krugman target zone model is developed in order to capture the typical volatility clusters and fat-tailed distributed innovations of exchange rates. It is shown that under these more general stochastic conditions the S-shaped relation between exchange rate and fundamentals is preserved, but is less pronounced. The model is tested for its S-shape and stochastic properties. Two clearly distinct sets of EMS currencies are detected on the basis of the curvature features. One-step-ahead realignment probabilities are used as an alternative evaluation method. © 1998 John Wiley & Sons, Ltd.

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1. INTRODUCTION

The major innovation in the analysis of target zones was Krugman's (1991) contribution, which provides an explicit rational expectations solution for the behaviour of the exchange rate within a fully credible band. By assuming that the fundamentals follow a regulated Brownian motion, Krugman shows that the log exchange rate becomes an S-shaped function of the fundamentals which approaches the boundaries smoothly. This paper develops the analogue target zone analysis in discrete time.[1] It is shown by simple calculus that the solution in discrete time is also characterized by the S-shape if the distribution of the innovation is symmetric and unimodal. We note that this set of conditions is much weaker than the assumption of normality which underlies continuous time analysis. In contrast to continuous time analysis there is no smooth pasting because in discrete time one can jump to the margin of the band from strictly within the band. Nevertheless, the margins still provide the necessary boundary conditions which tie down the solution.

The motives to develop discrete time analysis are threefold: simplicity, non-normality, and the empirical specification. The simplicity of the approach derives from the usage of standard calculus. Discrete time analysis does not rely on stochastic calculus, and the main result is a straightforward application of Leibniz's rule. The importance of allowance for non-normality is empirical. In spite of the strong economic intuition behind the S-shape, the empirical evidence for

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[1] In Pesaran and Samiei (1992), Donald and Maddala (1992) and Lee (1994) the discrete time solution with current expectations is discussed. Pesaran and Samiei (1995) and De Jong (1993, ch. 4) discuss the existence of a solution with future expectations through a fixed-point argument. This paper characterizes the solution.

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the S-effect is weak. See, for example, on the one hand, Flood, Rose, and Mathieson (1991) and Smith and Spencer (1992), while, on the other hand, there are the results of Rose and Svensson (1991, 1994) and Svensson (1993). The first two articles heavily rely on the presumption of normality (by testing for the hyperbolic sine–cosine solution through the use of the method of simulated moments). The latter three articles test robustly, by using OLS, for mean reversion. Because mean reversion appears to be significant under the latter approach, it can be important to take care of the non-normality of the data. It is well known that even under a free float forex returns are fat-tailed distributed and exhibit clusters of high and low volatility (see e.g. Koedijk *et al*., 1990; Hols and De Vries, 1991; Baillie and McMahon, 1989). It is shown below that these features actually flatten the S-shape *vis-à-vis* the case of normal innovations. Thus the mean reversion (rotation effect) is increased, but the amount of curvature is decreased. In contrast, certain types of realignments (see Bertola and Caballero, 1992) lower the amount of mean reversion. The empirical specification below directly follows from the discrete time model and is designed to capture both the possible mean reversion and curvature effects. Interestingly, we identify four EMS countries for which no rotation and curvature effects are found and two countries for which all effects are significant. After estimating the model and testing for the S-effect, we also infer the degree of credibility of the EMS zone through a prediction test. This is done by calculating one-step-ahead realignment probabilities.

2. THEORY

To set up the discrete time target zone model we first consider the two limiting cases of a free float and a perfectly credible target zone, and then consider the intermediate case in which the two are nested. Subsequently we discuss the implications of alternative assumptions concerning the stochastic process of the fundamentals. Our empirical specification and a realignment forecasting formula are derived towards the end of the section.

The reduced-form free float monetary model in discrete time has the following canonical structure:

$$s_t = \frac{1}{1+\eta} z_t + \frac{\eta}{1+\eta} E_t[s_{t+1}]$$

where η is the semi-interest elasticity of money demand, $\eta > 0$, s_t denotes the logarithm of the spot exchange rate at time t, and z_t is the set of fundamentals. Defining $f_t = z_t/(1+\eta)$ and $\lambda = \eta/(1+\eta)$, so that $0 < \lambda < 1$, we can simplify the notation and write

$$s_t = f_t + \lambda E_t[s_{t+1}] \tag{1}$$

Assume that f_t follows an AR(1) process:

$$f_{t+1} = \tau f_t + \varepsilon_{t+1} \tag{2}$$

and where $E_t[\varepsilon_{t+1}] = 0$. It is easy to check that if $\tau < 1/\lambda$

$$s_t = \frac{1}{1-\tau\lambda} f_t \tag{3}$$

is a particular solution. The assumptions so far comprise a number of different cases. The standard finance martingale solution arises when $\tau = 1$, so that $s_t = z_t$. With $|\tau| < 1$ the AR(1) process of the fundamentals is stationary, and hence the fundamentals and exchange rate are mean reverting.[2] Even though the data do not point towards explosive processes, we can allow for $|\tau| > 1$.

Apart from the expected part in equation (2), it is useful to discuss the structure of the innovations ε. The unconditional d.f. of forex returns is known to be heavy tailed (see Koedijk *et al.*, 1990; Hols and De Vries, 1991), and the conditional distribution is strongly heteroskedastic (see Baillie and McMahon, 1989). These features can be built in by letting ε follow a stochastic volatility process with Student-t distributed innovations, say. In the continuous time approach the fundamentals are mostly assumed to follow Brownian motion, i.e. a random walk with normal innovations. The Ito stochastic calculus then yields a neat explicit solution of s in terms of f for the case of a fully credible target zone. Within the continuous time paradigm it is nevertheless difficult to do justice to the discrete time sample path properties of fat-tailed innovations cum volatility clustering.[3] In the discrete time approach, though, it is straightforward to incorporate these typical features of the innovations.

Consider then a fully credible target zone in discrete time. The global nature of the forex market only permits indirect regulation of s through f. To keep s from exiting the target zone, assume that f is regulated at the symmetric boundaries $c + \bar{f}$ and $c - \bar{f}$, $\bar{f} > 0$, about the centre fundamental c as follows:

$$f_{t+1} = \begin{cases} c + \bar{f} & \text{if } \tau f_t + \varepsilon_{t+1} \geqslant c + \bar{f} \\ \tau f_t + \varepsilon_{t+1} & \text{if } c - \bar{f} \leqslant \tau f_t + \varepsilon_{t+1} < c + \bar{f} \\ c - \bar{f} & \text{if } c - \bar{f} > \tau f_t + \varepsilon_{t+1} \end{cases} \tag{4}$$

Note that the regulation takes place only at the boundaries as in Krugman (1991). However, the parameter τ can be used to model intramarginal interventions by setting $\tau < 1$. Intramarginal interventions were for some time common practice in the EMS (see Dominguez and Kenen, 1992). However, in contrast to the continuous time model, we show below that the intervention is a.s. not incipient. Technically speaking, potential regulation of f must also be available to counter any speculative bubble in s, i.e. the homogenous solutions to equation (1). By the backward induction argument these bubbles then never arise, and therefore this type of regulation is never needed in practice.

Given this regulation, it is immediate that for all i the expectation $E_t[f_{t+\mathrm{i}}]$ is bounded. Therefore the unique solution follows from the method of recursive forward substitution:

$$s_t = \sum_{i=0}^{\infty} \lambda^i E_t[f_{t+\mathrm{i}}] \tag{5}$$

[2] Froot and Obstfeld (1991) and Delgado and Dumas (1992) treat this case within continuous time, i.e. when the fundamentals follow an Ornstein–Uhlenbeck process. The target zone analysis then involves solving the more complicated Kummer's equation. In the discrete time analysis mean reversion does not present extra difficulties. Pesaran and Samiei (1995) also consider the case of AR(1) as well as higher-order autoregressive processes within the discrete time framework.

[3] The Brownian motion approximation massages away the fat-tail property. Also, given the typical parameter values for the volatility equation, a non-normal stable Ornstein–Uhlenbeck process would be more appropriate than the normal version.

In order to be able to further characterize the solution in terms of the predetermined variables we need to say a little more about the d.f. $G(\varepsilon)$ of the innovations. Specifically, we assume the following:

Assumption 1

(i) $G(\varepsilon)$ is antisymmetric around the point (0, 1/2), i.e. $1 - G(x) = G(-x)$
(ii) $G(\varepsilon)$ is unimodal around 0
(iii) $G(\varepsilon_t)$ does not depend on f_{t-i}, $i = 1, 2, \ldots$

The symmetry is needed in view of the symmetric band. A d.f. $G(\varepsilon)$ is unimodal around 0 if $G(\varepsilon)$ is convex in $\varepsilon < 0$ and concave in $\varepsilon > 0$. In other words, under (i) and (ii) the density is increasing up to 0 and decreasing thereafter. This seems a mild condition on forex returns. There is a wide class of d.f.'s that satisfy (i) and (ii). Examples are the Student-t d.f.'s, the logistic d.f., the symmetric stable d.f.'s, and the uniform d.f. Note that the special case of the normal distribution, which is used in the continuous time framework, also fits (i) and (ii). But the gain of Assumption 1 instead of normality is that one can easily capture the heavy tail property of forex returns.[4] If the ε_t are i.i.d., then (iii) is clearly satisfied. In view of the well-known clusters of volatility, however, it is desirable to allow for some dependency in the second moment. But, as will become clear, this dependency needs to be restricted in the sense of (iii) (c.f. Merton, 1973, Th. 10). The characterization of the solution will be in terms of the following definition:

Definition 1. We will say that the function $k(x)$ is convexoconcave and antisymmetric about c (abbreviated as CCA) if $k(x)$ is twice differentiable, convex for $x < c$, concave for $x > c$, and antisymmetric: $k(c) - k(c - x) = k(c + x) - k(c)$ for $x > 0$.

Note that any d.f. $G(\varepsilon)$ which satisfies conditions (i) and (ii) from Assumption 1 is CCA about 0.

We can now state and prove the main lemma that paves the road for the discrete time target zone analysis.

Lemma 1
Let the function $k(f)$ be CCA about c and let the d.f. $G(\varepsilon)$ be differentiable and satisfy the conditions (i)–(iii) of Assumption 1. Then for the regulated fundamental f_{t+1} as defined in equation (4), the following function of f_t

$$e_{t+1}(f_t) = E[k(f_{t+1}) \mid f_t] \tag{6}$$

is CCA about c. □

A proof of the lemma is given in Appendix A. To provide some intuition for this result consider the linear case $k(f) = f$ with $c = 0$ and $\tau = 1$ in equation (4). The linear case constitutes the second element of the infinite sum (5). Splitting the integral in equation (6) yields (omitting time subscripts)

$$e(f) = -\bar{f}G(-\bar{f} - f) + \int_{-\bar{f}-f}^{\bar{f}-f} (f + \varepsilon)g(\varepsilon)\, d\varepsilon + \bar{f}[1 - G(\bar{f} - f)] \tag{7}$$

[4] The discrete time with current expectations approach that is followed by Lee (1994) and Pesaran and Ruge-Murcia (1996) allows for non-normal d.f.'s as well.

The use of Leibniz's rule then yields $\mathrm{d}e(f)/\mathrm{d}f = G(\bar{f} - f) - G(-\bar{f} - f)$, while $\mathrm{d}^2e(f)/\mathrm{d}f^2 = -g(\bar{f} - f) + g(-\bar{f} - f)$. Clearly, $\mathrm{d}e(f)/\mathrm{d}f > 0$, while the single peakedness and symmetry of the density $g(\varepsilon)$ implies that $\mathrm{d}^2e(f)/\mathrm{d}f^2 \gtreqless 0$ as $f \lesseqgtr 0$. Thus, $e(f)$ is convexoconcave when $k(f)$ is linear. Observe, moreover, that $e(f)$ is strictly CCA as long as $G(\varepsilon)$ is not the uniform distribution (linearity of $k(f)$ implies that $k(f)$ is not strictly CCA).

The lemma implies that the class of functions which are CCA about the same argument c is closed under the stated functional operation. This permits repeated use of the lemma. Consider the problem of characterizing the third element of the infinite sum (5). By the law of iterated expectations

$$e_{t+2}(f_t) = E_t[E_{t+1}[f_{t+2}]]$$

By the lemma the inner expectation $E_{t+1}[f_{t+2}]$ is CCA in f_{t+1}. Applying the lemma once more then shows that $e_{t+2}(f_t)$ is CCA in f_t. A straightforward induction argument shows that $e_{t+i}(f_t)$ is CCA in f_t about c for all $i = 1, 2, \ldots$ Moreover, we note that the CCA property about c is preserved under multiplication by positive scalars and addition of two functions which are CCA about the same c.[5] We have obtained the following result.

Proposition 1 The target zone solution $s_t(f_t)$ to the reduced-form exchange rate model (1), given the regulated AR(1) process (4) with CCA distributed innovations, is CCA in f_t. □

The proposition shows that the mean feature of the continuous time target zone model, i.e. the S-shaped bending of the function $s(f)$, is also exhibited by the discrete time model. The intuition is the same: while depreciating, the currency depreciates by less than by what is signalled through the fundamentals in anticipation of the future expected appreciation once the upper edge of the band will be reached. Hence s exhibits mean reversion throughout its band. This conclusion follows regardless the value of the autoregressive parameter τ. The interpretation of τ is as follows. The standard case is $\tau = 1$ when the fundamentals are a martingale and interventions are conducted only at the margins. It is well known, however, that the EMS has experienced periods of considerable intramarginal interventions (see Dominguez and Kenen, 1992). This feature can be captured by considering values $|\tau| < 1$; in fact any process such that $f_{t+1} = k(f_t + \varepsilon_{t+1})$, where $k(.)$ is CCA would be appropriate for this purpose.[6] Thus Proposition 1 applies both to the case of marginal and to the case of intramarginal interventions.

It is useful to discuss the relation of Proposition 1 with the existing literature on discrete time. In Pesaran and Samiei (1992) and Pesaran and Ruge-Murcia (1996) the model with current instead of future expectations is analysed along the lines of Donald and Maddala (1992) and Lee (1994). The analysis is potentially simpler than with future expectations, because only a single expectation has to be evaluated. The method allows for different distributional assumptions, but the normal d.f. is used most frequently. In de Jong (1994) and Pesaran and Samiei (1995) the discrete time model with future expectations is studied. A fixed-point argument as in Deaton and Laroque (1992) is used to establish the existence of a solution; in practice a finite number of backward recursions is used to obtain a numerical solution. In this respect the value added of our

[5] Let $e(f)$ and $k(f)$ be CCA about c, i.e. e'', $k'' \gtreqless 0$ as $f \lesseqgtr c$. Define $h(f) = ae(f) + k(f)$, $a \geqslant 0$. Then $h'' = ae'' + k'' \gtreqless 0$ as $f \lesseqgtr c$. Similarly $h(c) - h(c - x) = h(c + x) - h(c)$.

[6] The exact CCA property is generally not preserved if the fundamentals follow a higher-order AR process. See Pesaran and Samiei (1995) for the case of an AR(2) process.

Proposition 1 is that we are able to *characterize* the solution in terms of S-shaped behaviour under mild distributional restrictions without reliance on an approximate numerical procedure.

The S-shaped nature of the solution corresponds with the continuous time cum normal innovations analysis. But we can push the characterization a little further. The shape of S-curved function $s(f)$ in discrete time differs from the continuous time model in that it does not reach the edges smoothly, but at a positive angle. This follows immediately from the fact that all the $e_{t+i}(f_t)$ in sum (5) are CCA so that $\mathrm{d}e_{t+i}(f_t = \bar{f})/\mathrm{d}f_t \geqslant 0$, and the fact $e_t(f_t) = f_t$ so that $\mathrm{d}e_t(f_t)/\mathrm{d}f_t = 1 > 0$.[7] It follows that:

Proposition 2 In the discrete time model there is no smooth pasting. □

The intuitive reason for this result is that in the continuous time model the expected change in the fundamental jumps and becomes non-zero only at the edges, because only then is the fundamental restricted to moving one way (placing its expected future value inside the band). But in the discrete time model the expected change in the fundamental is already non-zero inside the band, because at any epoch the innovation can be so large that it crosses the band. Hence the expected changes in f and s are continuous functions of f. Thus in the discrete time model there is no need to make s independent from f at the edges to circumvent arbitrage opportunities. It also follows that interventions are almost always sizeable (non-incipient), because the unregulated process crosses the band by a discrete amount.

It was already noted that Assumption 1 admits other d.f.'s than the normal d.f. (see also Pesaran and Ruge-Murcia, 1996). The issue of what the effects are of other distributional assumptions for the target zone solution $s(f)$ has, to the best of our knowledge, not been addressed in the literature. If anything, forex return innovations tend to be non-normal heavy tailed, and we analyse the implications. The feature of heavy tails can be formalized through the concept of regular variation. A d.f. $G(x)$ is said to vary regularly at infinity if

$$\lim_{t\to\infty} \frac{1 - G(tx)}{1 - G(t)} = ax^{-\alpha} \tag{8}$$

for $a > 0$ and $\alpha > 0$. In essence, equation (8) says that $G(x)$ has a Pareto-type tail (i.e. the right tail of the density declines by a power). For example, the Student-t, when α equals the degrees of freedom, and the non-normal stable distributions satisfy equation (8); but the normal d.f. does not (because the tail of the density declines exponentially fast).

A sensible test function for investigating the effect of tail fatness is the 'Symmetric Pareto' d.f.

$$G(x) = \begin{cases} \frac{1}{2}[1 - x]^{-\alpha} & \text{if } x < 0 \\ 1 - \frac{1}{2}[1 + x]^{-\alpha} & \text{if } x \geqslant 0 \end{cases} \tag{9}$$

and where $\alpha > 2$, so that the variance is bounded. Evidently this $G(x)$ varies regularly at infinity. From the results below equation (7) it immediately follows that (assuming $\tau = 1$, $c = 0$):

$$\mathrm{d}e(f)/\mathrm{d}f = 1 - \tfrac{1}{2}[1 + \bar{f} + \mathrm{f}]^{-\alpha} - \tfrac{1}{2}[1 + \bar{f} - f]^{-\alpha} \tag{10}$$

[7] Moreover, it can be shown that in terms of the original fundamentals, $z_t = f_t/(1 + \eta)$, the derivative of s_t with respect to z_t at the edges lies strictly between 0 and 1.

Now consider a change in the tail fatness through a change in the tail index α:

$$\begin{aligned} \mathrm{d}^2 e(f)/\mathrm{d}f\mathrm{d}\alpha = {} & \tfrac{1}{2}[1+\bar{f}+f]^{-\alpha} \log(1+\bar{f}+f) \\ & + \tfrac{1}{2}[1+\bar{f}-\mathrm{f}]^{-\alpha} \log(1+\bar{f}-f) > 0 \end{aligned} \tag{11}$$

By induction, assume $\mathrm{d}^2 e_m(f)/\mathrm{d}f\mathrm{d}\alpha > 0$. Then

$$\frac{\mathrm{d}^2 e_{m+1}(f)}{\mathrm{d}f\mathrm{d}\alpha} = \int_{-\bar{f}-f}^{\bar{f}-f} \frac{\mathrm{d}^2 e_m(f)}{\mathrm{d}f\mathrm{d}\alpha} g(\varepsilon)\,\mathrm{d}\varepsilon + \int_{-\bar{f}-f}^{\bar{f}-f} \frac{\mathrm{d}e_m(f)}{\mathrm{d}f} \frac{\mathrm{d}g(\varepsilon)}{\mathrm{d}\alpha} \mathrm{d}\varepsilon \tag{12}$$

It is immediate that the first integral is unambiguously positive. The factor $\mathrm{d}e_m(f)/\mathrm{d}f$ in the integrand of the second integral is positive by the fact $e_m(f)$ is CCA. It is straightforward to check that a sufficient condition for $\mathrm{d}g(\varepsilon)/\mathrm{d}\alpha$ to be positive as well is: $\bar{f} < \exp(1/\alpha) - 1$. This leads to:

Proposition 3 If the innovations have a symmetric Pareto d.f. and if the margins are not too large, i.e. $\bar{f} < \exp(1/\alpha) - 1$, then an *increase in the tail fatness* (lower α) *decreases* the slope of the S-curved solution $s(f)$ for all f. □

Empirically one finds α to be in the neighbourhood of 3 (see e.g. Koedijk *et al.*, 1990). While the normal d.f. can be viewed as having $\alpha = \infty$ (recall that the Student-t approximates the normal d.f. as the degrees of freedom increase without bound). The proposition then may at least provide a partial explanation for why the empirical literature, like Flood *et al.* (1991), does not find a strong S-effect. Because the estimation method like the method of simulated moments heavily relies on the normality assumption. A heavy-tailed d.f. implies that $s(f)$ will be less responsive to changes in f in comparison with the normal d.f. But by the same device the rotation effect is stronger.

Thus empirical work such as Rose and Svensson (1991) and Svensson (1993) which is focused on testing this latter aspect through regression analysis, i.e. the mean reversion effect due to the band, should fare better. This observation also provides an important motivation for our empirical specification below.

The other dominant distributional characteristic of forex return data are the clusters of high and low volatility. In line with Assumption 1 (iii), we consider a variant of the stochastic volatility models. Take again the two-sided Pareto law (9). But now suppose that the tail index α is random and driven by a stationary process such that $\alpha > 2$ always. The latter condition ensures that the conditional variance of x, i.e. $\mathrm{Var}(x \mid \alpha) = 2/(\alpha-1)(\alpha-2)$, is a meaningful concept. For example, one might think of α as being generated by a Markov switching model, and where the switching probabilities are independent from the history of the Pareto innovations. The implications for the typical target zone S-shaped curvature can then be readily inferred from Proposition 3.

Corollary 1 In times of high volatility, i.e. low conditional α, the solution $s(f)$ is less curved and shows a stronger mean reversion effect *vis-à-vis* times of low volatility. □

Thus far we have taken the target zone to be fully credible, but for an application to EMS data this rosy picture does not fit the facts. A band is only partially credible if it admits the possibility of realignment. Svensson (1991) and Bertola and Caballero (1992) elegantly develop specific realignment scenarios within the continuous time framework. Bertola and Caballero (1992) consider realignments which occur at the margins, while Svensson (1991) considers intramarginal realignments. In the latter set-up a realignment occurs with some constant probability p without regard for the place in the band (note: in discrete time the Bernoulli distribution replaces the continuous time Poisson distribution on the number of realignments per unit of time). And with probability $1 - p$ there is no realignment, for f_{t+1} is as in equation (4). Suppose, as in Svensson (1991), that a realignment shifts both the fundamental f and the centre fundamental c by some i.i.d. innovation θ with mean jump size $E[\theta]$, i.e. the realignments are put into effect through an (additional) change in the fundamentals which causes c to be relocated as well. Note that we allow for $\theta = \varepsilon$. Because θ cancels out from all terms that involve the difference $c - f$, it follows that we only have to add $pE[\theta]$ to equation (7). It is straightforward to show that $E[\theta]$ enters linearly in all higher order terms as well.

Proposition 4 Given a fixed probability p on a fundamentals-driven intramarginal realignment with mean jump size $E[\theta]$ of f and c, the solution $s(f)$ is again CCA in f. It involves, in addition to the fully credible solution, the drift term $E[\theta]/(1 - \lambda p)$. □

The intuition behind the preservation of the CCA property is that the realignment is driven by the fundamentals, most particularly so if we take $\theta = \varepsilon$.[8] Instead, a realignment can be envisioned as being triggered by a large change in the fundamentals. Moreover, during a realignment the place of the fundamental in the band relative to the centre often changes. This type of realignment scheme is investigated by Bertola and Caballero (1992). Specifically, the realignment resets the centre and fundamental at a certain threshold, say $\bar{f}$ or $-\bar{f}$, once the fundamentals exceed this threshold (the fundamentals are not fully regulated inside the band). It can be shown (see e.g. Koedijk *et al.*, 1993) that the change in the relative position of f *vis-à-vis* c implies that the CCA property is lost. Nevertheless, it is still the case that $s(f)$ lies in between the free float and fully credible solution.

Propositions 1–4 provide a number of testable hypotheses concerning the behaviour of $s(f)$ in a discrete time (partially) credible target zone. Most notably, if the zone is credible and the distribution of the innovation is CCA, then we expect to find an S-shape but without the smooth pasting property. Due to the heavy tail property we also expect the C-curve to be quite flat, and that the flatness varies with the amount of conditional volatility. A number of alternative scenarios were identified under which the S-shape would not materialize. For example, if the relative position of the fundamental *vis-à-vis* the centre can be altered in the event of a realignment, or if the fundamentals follow a higher-order ARMA process, then the CCA property is lost. A specification which is amenable to testing these results is developed below.

In going from the theory to empirics the first difficulty one encounters is the mere absence of a reliable fundamentals model (cf. Meese and Rogoff, 1983). Thus to uncover the nonlinear relationship between s and f, as in Flood *et al.* (1991) and Pesaran and Samiei (1992), one runs into the difficulty of having to specify f. Not knowing f may therefore strongly impair the results. Here we want to circumvent this difficulty as follows.

Rewrite equation (1) and use Proposition 1 to obtain

$$E_t[s_{t+1}] = \frac{1}{\lambda}(s_t(f_t) - f_t) \tag{13}$$

[8] Pesaran and Ruge-Marcia (1995) recently considered a similar realignment scheme within the framework of current expectations.

Now recall from the arguments preceding Proposition 2 that $\mathrm{d}s_t/\mathrm{d}f_t \neq 0$ on $[c-\bar{f}, c+\bar{f}]$. Hence by the implicit function theorem we may express the expectation as a function in s_t:

$$E_t[s_{t+1}] = \frac{1}{\lambda}(s_t - f_t(s_t)) = a(s_t) \tag{14}$$

say. Also,

$$\mathrm{d}a/\mathrm{d}s = (1 - \frac{1}{(\mathrm{d}s/\mathrm{d}f)}/\lambda >)0$$

as $\mathrm{d}s/\mathrm{d}f > 1$. Furthermore $a(s)$ is CCA on $(s(c-\bar{f}), s(c+\bar{f}))$, because $\mathrm{d}s/\mathrm{d}f$ is decreasing to the right of c and vice versa to the left of c. Note that the CCA property obtains under the same conditions as the CCA-ness of $s(f)$, such as is stated in Propositions 1 and 4.

Given rational expectations, we may write

$$s_{t+1} = E_t[s_{t+1}] + \phi_{t+1} \tag{15}$$

where the innovation ϕ_{t+1} is orthogonal to the information set: $E[\phi_{t+1} \mid s_t] = 0$. Upon combining equation (14) and (15) we obtain

$$s_{t+1} = a(s_t) + \phi_{t+1} \tag{16}$$

Thus s_{t+1} is expressed as the sum of a CCA function in s_t and an orthogonal innovation. The benefit of this transformation is twofold. The arguments on the left-hand side and in the expected part on the right-hand side are both observable and the function $a(s)$ exhibits all the properties of $f(s)$ as discussed in Propositions 1–4 and Corollary 1. Also note that $a(s)$ is expected to be linear under a free float.

To test for Propositions 1–4 and Corollary 1 the function $a(s)$ is developed into a third-order Taylor approximation around the central parity q. The tests on the specific non-linearities implied by the target zone theory are then reduced to tests about the regression coefficients (see Cramer, 1975, ch. 5). Rose and Svensson (1991) and Svensson (1993) adopt similar specifications, but these are not directly derived from the shape of $a(s)$ in discrete time. Moreover, these articles only use a linear approximation. The Taylor approximation reads

$$s_{t+1} = q_t + \beta_1(s_t - q_t) + \beta_2(s_t - q_t)^2 + \beta_3(s_t - q_t)^3 + \psi_{t+1} \tag{17}$$

The error term ψ comprises the innovation ϕ and the rest term from the Taylor expansion. The coefficient β_i stands for the ith derivative of $a(s)$ at $s = q$. In the case of a credible target zone as in Proposition 1, the hypothesis is that $0 < \beta_1 < 0$. At q the second derivative of $a(s)$ is zero by the CCA property, but is positive to the left of q_t and negative to the right of the central parity. To test for this we dummy up $(s_t - q_t)^2$ by a dummy for the sign of $s_t - q_t$. To our knowledge, this distinction has not been made previously in the literature. In this way we can test for $\beta_2^+ < 0$, $\beta_2^- > 0$ and $-\beta_2^+ = \beta_2^-$. Evidently, $\beta_1 < 1$, so that s_t is mean reverting under a target zone regime. This has to be contrasted with the alternative hypothesis of a non-credible target zone or free-float regime when $\beta_1 = 1$ and $\beta_2 = \beta_3 = 0$. In this case s_t contains a unit root. Therefore the specification to be estimated is written as

$$\begin{aligned} s_{t+1} - s_t = \beta_0 - \delta(s_t - q_t) + \beta_2^-(s_t - q_t)^2 + \beta_2^+(s_t - q_t)^2 \\ + \beta_3(s_t - q_t)^3 + \psi_{t+1} \end{aligned} \tag{18}$$

where $\delta = 1 - \beta_1$.

Before we can estimate the model, the properties of the error term ψ have to be discussed. Recall that ψ consists of a rest term from the Taylor expansion of $a(s)$ and the innovation ϕ which is orthogonal to $a(s)$. While we have been able to characterize the convexity properties of the conditional expectation $a(s)$, we cannot be so clear about the distributional aspects of ϕ. Given the weak assumption of $G(\varepsilon)$, not much can be said about the unconditional distribution of ϕ in the case of the fully credible target zone (Cox and Miller, 1965, p. 63). The conditional distribution of the innovation ϕ_{t+1}, given s_t, under the conditions of Proposition 1 has a bounded support and is skewed except at $s_t = q_t$. But nevertheless $E[\phi_{t+1} \mid s_t] = 0$. In case of the Svensson partially credible band (recall Proposition 4) the conditional distribution of ϕ_{t+1} becomes a mixture of two distributions, one of which has bounded support. Other realignment schemes and the case of a free float may also imply that the support of ϕ becomes unbounded. What is known, though, is that ϕ cannot be homoscedastic due to any of the above regulations. Moreover, heteroscedasticity is also a well-established empirical regularity of freely floating exchange rates. Another issue is that ψ_{t+1} and s_t may be correlated because ψ_{t+1} comprises the rest term from the Taylor expansion of $a(s_t)$. Given the above difficulties, our approach will be to adopt two different estimation procedures. First, we neglect the rest term problem and rely on the othogonality of the innovation ϕ, and use straightforward OLS. Note that this approach does not commit on whether the support of ψ is bounded or not. We report both conventional and Newey–West standard errors to take care of the heteroskedasticity of the innovation. Second, we also look into instrumenting the estimators to circumvent the possible correlation between the regressors and ψ.

3. EMPIRICAL RESULTS

In this section the target zone equation (18) is estimated on the basis of weekly spot rate quotations for the Belgian, Danish, French, Dutch, Irish and Italian currencies *vis-à-vis* the Deutschemark. Data were taken from Datastream. The period covered runs from Wednesday 4 April 1979 until 23 September 1992, giving 704 observations. The log differences of the exchange rates display the usual features like fat tails, non-constant variance and some skewness due to the strength of the deutschemark.

Table I contains the OLS parameter estimates which directly follow from equation (18). Under the hypotheses of Propositions 1 and 4, the S-effect materializes if $\beta_0 = 0$, $0 < \beta_1 < 1$ or equivalently if $\delta > 0$ in equation (18), $\beta_2^- = -\beta_2^+ > 0$, and $\beta_3 < 0$. Alternatively, if the target zone is non-credible in the sense of a *de facto* free float or under a Bertola and Caballero realignment scheme, these restrictions should not be met.

As is apparent from Table I, Holland and Ireland are the only two countries which meet all conditions of a fully credible target zone for the full sample period. For these two countries the mean-reversion effect is quite strong, as δ is in the order of 0·25, and significant. Noteworthy are also the significant non-linear effects as captured by the β_2 and β_3 coefficients. Moreover, we cannot reject equality of β_2^- and $-\beta_2^+$ according to the standard F-test. The results for Holland and Ireland clearly show that the S-effect is present for these currencies. This is in contrast to the findings of Svensson (1993) and Rose and Svensson (1994), where it is reported that non-linear effects do not play any role whatsoever.[9] One reason for this failure to detect non-linearities is due

[9] In Appendix B we report results from the instrumental variables method for Holland to address the issue of correlation between ψ and $(s_t - q_t)$. While the IV estimates come with higher standard errors, the point estimates are hardly affected and concern for biased estimates need not to be great.

Table I. Parameter estimates of discrete time target zone model[a]

	BEL	DEN	FRA	HOL	IRE	ITA
β_0	−4·3 E-5	−6·7 E-4	−5·25 E-4	−5·4 E-5	−1·9 E-4	−6·4 E-4
	(1·0 E-3)	(2·0 E-4)	(2·36 E-4)	(9·6 E-5)	(2·2 E-4)	(2·8 E-4)
	[0·7 E-3]	[2·0 E-4]	[2·0 E-4]	[0·0]	[3·0 E-4]	[3·0 E-4]
δ	−8·7 E-2	1·1 E-1	5·2 E-2	2·79 E-1	1·9 E-1	1·8 E-2
	(0·20)	(1·1 E-1)	(1·2 E-1)	(7·3 E-2)	(7·6 E-2)	(5·0 E-2)
	[0·24]	[1·1 E-1]	[1·0 E-1]	[8·5 E-2]	[8·8 E-2]	[6·0 E-2]
β_2^-	−3·09	10·83	16·37	33·64	15·90	−4·0 E-1
	(4·57)	(14·49)	(17·50)	(13·69)	(7·99)	(2·39)
	[8·08]	[14·23]	[17·19]	[16·90]	[10·85]	[2·68]
β_2^+	19·07	−9·83	−14·91	−30·13	−23·77	−1·10
	(12·42)	(14·22)	(16·56)	(13·04)	(7·39)	(2·98)
	[16·06]	[13·92]	[16·93]	[16·98]	[9·37]	[3·64]
β_3	−19·11	−393·18	−741·33	−1103·56	−493·74	10·65
	(25·73)	(455·00)	(578·06)	(558·82)	(182·38)	(27·99)
	[55·60]	[422·19]	[620·99]	[663·68]	[278·22]	[31·52]
R^2	0·264	0·024	0·008	0·044	0·039	0·015
SE	0·020	0·004	0·005	0·002	0·005	0·006

[a] This table reports coefficient estimates for equation (18), with standard errors in parentheses and heteroscedastic consistent standard errors in square brackets. The goodness of fit R^2 and standard error (SE) of the regression are also reported.

to the sign switch in the second-order derivative when f moves from the convex to the concave part. In a previous version of this paper we showed that if one collapses β_2^- and β_2^+ into a single coefficient, then its estimate becomes insignificantly different from 0.

For the other EMS countries, Belgium, Denmark, France and Italy, we do not find significant mean-reversion or non-linear effects as a result of the target zone. But the coefficients for France have the same sign and magnitudes as for Holland and Ireland. It should be stressed that these findings again differ substantially from Svensson (1993) and Rose and Svensson (1994) where significant mean-reversion effects are reported for each country. The explanation for these results may be the omitted variable bias. If the rotation effect, i.e. β_1, and the bending effect, i.e. β_2^-, β_2^+ and β_3, are collapsed into a single mean reversion coefficient, then this coefficient may be significant, whereas this is not the case when the effects are separated out. When we constrain the approximation in equation (17) only to the first-order effect, we also find the mean reversion coefficient to be significant for each of the six countries (see Koedijk *et al.*, 1993).

On the basis of the results reported in Table I we conclude that Holland and Ireland were the only two countries in which the EMS system was credibly adhered to, because significant S-effects were found for their currencies. Our results also indicate that there was no credible target zone for Belgium, Denmark and Italy over the sample period as both mean-reversion and non-linear effects were found to be absent. For France the coefficients have the right sign and sizes, but the standard errors are too large for the S-effect to be significant.

To validate this conclusion regarding the two more or less distinct groups of EMS countries we also judge the model through a prediction test on the realignment forecasts. Such a prediction test is a direct measure of the economic significance of the EMS, while the statistical significance of the coefficients is only an indirect measure of the economic significance. For the four countries which did not pursue a credible target zone policy the curvature in $s(f)$ is absent. This implies that as s approaches the margins of the band, a devaluation (revaluation) becomes more likely, and hence the model should signal upcoming realignments. To construct devaluation probabilities we make some specific assumptions regarding the d.f. $G(\psi)$. It is well known that freely floating rates are approximately a martingale with clusters of volatility and fat-tailed conditional innovations. Therefore we adopt the GARCH(1.1) cum Student-t with v degrees of freedom model:

$$\psi_{t+1} = x_{t+1}[h_{t+1}(v-2)/v]^{1/2} \tag{19}$$

$$h_{t+1} = \omega + \gamma\psi_t^2 + \zeta h_t \tag{20}$$

and where x_t are i.d.d. Student-t innovations. We employ the following shorthand notation for the exchange rate model:

$$s_{t+1} = \beta y_t + \psi_{t+1} \tag{21}$$

where y_t is a vector of explanatory variables such as in equation (18). Let u denote the upper bound on the currency band. The one-step-ahead devaluation probability can then be constructed as follows:

$$P_t\{s_{t+1} > u\} = P_t\left\{x_{t+1} > \frac{u - \beta y_t}{[h_{t+1}(v-2)/v]^{1/2}}\right\} \tag{22}$$

For the GARCH model the conditional volatilities h are found by iterating equation (20) back in time. We employed two versions of the exchange rate model (21): the empirical counterpart to equation (18) and the simple martingale model (because the latter assumption could not be rejected for France, Denmark, Italy and Belgium). Both models generated almost identical results.

Figures 1 and 2 record the one-week-ahead devaluation probabilities for France and Denmark. In contrast, in Figures 3 and 4 we also report the 'devaluation probabilities' for the two credible EMS countries using the same methodology. The dashed bars indicate an actual devaluation of the currency against the deutschemark. Holland is clearly the country that stands out. Apart from a weak signal of a devaluation probability in 1983, this country appears to have been fully credible against the deutschemark over the sample period. For Ireland there is a clear distinction between the period before and after 1987. Before 1987, the Irish punt was not fully credible as can be seen from the probabilities which signalled the devaluations in this period. After 1987 the picture for the Irish currency changed; except for a short period in 1990, there was no sign that a devaluation was imminent and hence the punt was apparently perceived as being a credible EMS member. The results for France and Denmark stand in sharp contrast to the results for Holland and Ireland. A remarkable feature of the figure for France is that even though the sequence of realignments decreased during the latter half of the 1980s, the bouts of upheaval measured by devaluation probabilities did not lessen in frequency or in size. If anything, they appear to have increased

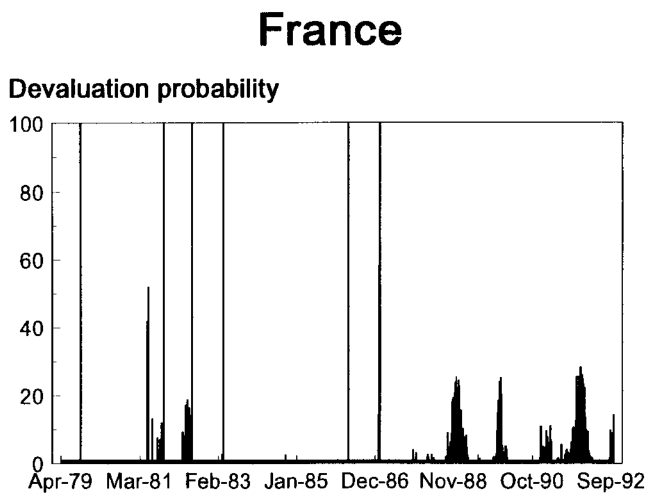

Figure 1. France: devaluation probability

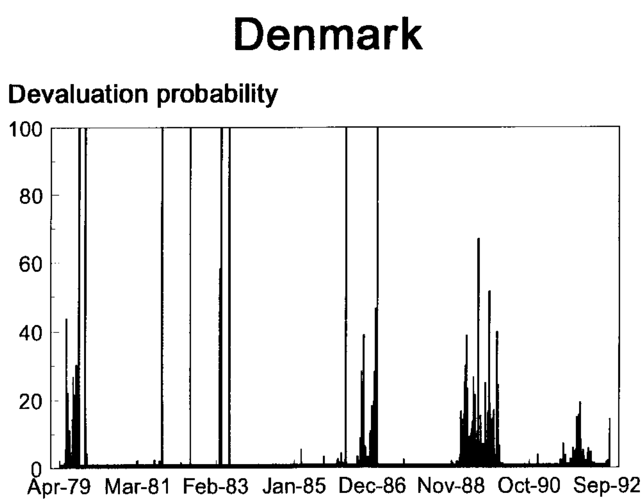

Figure 2. Denmark: devaluation probability

rather than decreased. This clearly suggests that the French franc was not perceived as being fully credible in the second half of the 1980s. A similar conclusion holds for the Danish krona.

4. CONCLUSION

The target zone literature started by adopting the continuous time framework, assuming normal innovations and a band from which the fundamental cannot escape. Krugman (1991) showed that under these conditions exchange rates stay within a band as well, and follow a regulated Brownian motion. This implies the elegant and intuitively appealing S-shaped behaviour of exchange rates. Empirical evidence in favour of these models has, however, been weak.

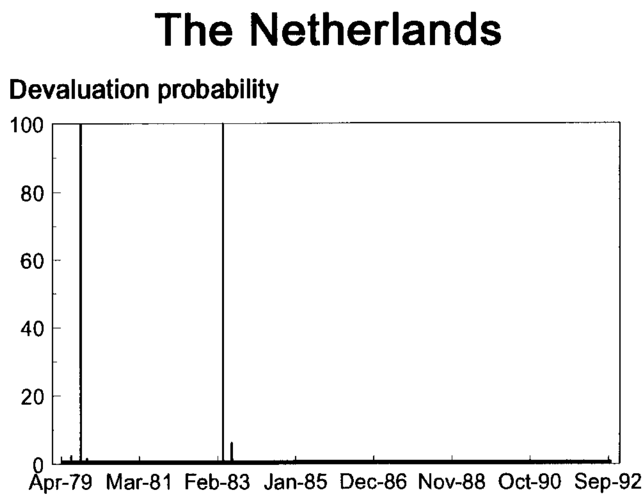

Figure 3. Netherlands: devaluation probability

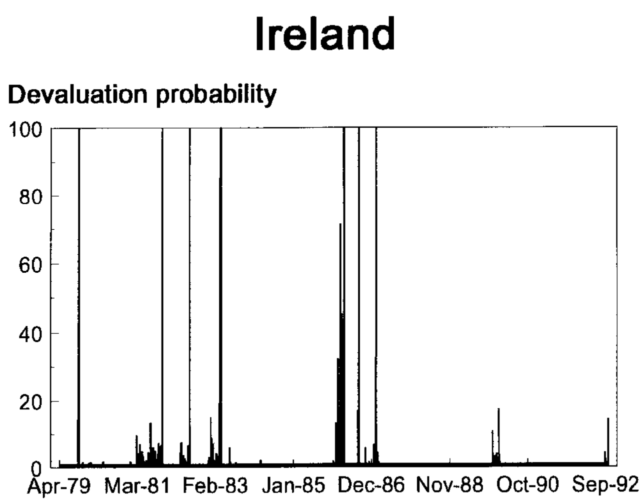

Figure 4. Ireland: devaluation probability

We derived the discrete time analogue to the standard continuous time model. The discrete time model displays the main feature of the continuous time model, both with marginal and with intramarginal interventions and also under the Svensson realignment scheme. But in contrast to the continuous time analysis the convexoconcavity does not hinge on the assumption of normality. In particular, we can allow for fat-tailed innovations. Also, we showed that the more pronounced the tail fatness is, the flatter the S-curve will be. Given that forex return innovations are typically fat-tailed this provides an explanation for the observed difficulty in empirical work which presupposes normality to recover the S-curvature.

Apart from the tail-shape and martingale nature, a third major property of forex returns is their conditional heteroskedasticity. Within the framework of a stochastic volatility model, which, *inter alia*, preserves the curvature properties, the flatness of the S-curve was shown to vary with the amount of conditional volatility. A difference between the continuous time and discrete time model is that the latter model does not exhibit a smooth pasting property. This is due to the fact that the unregulated fundamentals can jump outside the band from within the band.

The empirical work was facilitated by the fact that we were able to show that under a target zone regime the expected future spot rate $E_t[s_{t+1}]$ as a function of the current spot rate s_t exhibits the same curvature properties as $s_t(f_t)$, i.e. when s_t is written as a function of the fundamental. A third-order Taylor approximation to the S-shape was used to falsify the null hypothesis on EMS data. For Ireland and Holland, the null could not be rejected. For these two countries we found both significant and sizeable mean reversion (rotation effect), as well as the bending effect (curvature effect). But for France, Italy, Belgium and Denmark neither of these effects were present, suggesting that the latter four countries followed a *de facto* free float.

By making extra assumptions concerning the nature of the innovations, the same conclusions concerning exchange credibility are obtained by calculating one-step-ahead realignment probabilities. In line with the significant S-effect for Ireland and Holland, the risk of an upcoming realignment is essentially zero. But for the other countries, as in the case of France, realignment probabilities jumped up repeatedly, even in the period of 1987–92, when the EMS was regarded as a stable arrangement. An interesting exercise for future work is to investigate specification (13) directly by means of a well-specified fundamentals model. The two approaches (13) and (18) could then be compared.

APPENDIX A

Here we prove lemma 1. For ease of presentation we delete time subscripts and set $c = 0$, so that $k(0) = 0$.

Proof Splitting the integral we can write

$$\begin{aligned} e(f) &= E[k(f + \varepsilon) \,|\, f] \\ &= \int_{-\infty}^{-\bar{f}-\tau f} k(-\bar{f}) g(\varepsilon)\, \mathrm{d}\varepsilon + \int_{-\bar{f}-\tau f}^{\bar{f}-\tau f} k(\tau f + \varepsilon) g(\varepsilon)\, \mathrm{d}\varepsilon + \int_{\bar{f}-\tau f}^{\infty} k(\bar{f}) g(\varepsilon)\, \mathrm{d}\varepsilon \end{aligned}$$

We first prove that $e(f)$ is antisymmetric by showing that $e(-f) = -e(f)$. To this end use the fact that $k(-\tau f + \varepsilon) = -k(\tau f - \varepsilon)$. Define $\phi = -\varepsilon$, and apply the change of variables technique by exploiting the symmetry of the density $g(-\phi) = g(\phi)$. Taking, for example, the middle term in the above expression gives for $f = -x$:

$$\begin{aligned} \int_{-\bar{f}+\tau x}^{\bar{f}+\tau x} k(-\tau x + \varepsilon) g(\varepsilon)\, \mathrm{d}\varepsilon &= -\int_{-\bar{f}+\tau x}^{\bar{f}+\tau x} k(\tau x - \varepsilon) g(\varepsilon)\, \mathrm{d}\varepsilon \\ &= \int_{-\bar{f}-\tau x}^{\bar{f}-\tau x} k(\tau x + \phi) g(-\phi) \,|-1|\, \mathrm{d}\phi = -\int_{-\bar{f}-\tau x}^{\bar{f}-\tau x} k(\tau x + \phi) g(\phi)\, \mathrm{d}\phi \end{aligned}$$

Together with similar manipulations for the other two terms yields the antisymmetry of $e(f)$.

The convexoconcavity follows from the application of Leibniz's rule. Differentiating once gives

$$\frac{\mathrm{d}e(f)}{\mathrm{d}f} = \int_{-\bar{f}-\tau f}^{\bar{f}-\tau f} \frac{\mathrm{d}k(\tau f + \varepsilon)}{\mathrm{d}f} g(\varepsilon)\,\mathrm{d}\varepsilon$$

and differentiating again yields

$$\frac{\mathrm{d}^2 e(f)}{\mathrm{d}f^2} = -\frac{\tau \mathrm{d}k(\bar{f})}{\mathrm{d}f} g(\bar{f} - \tau f) + \int_{-\bar{f}-\tau f}^{\bar{f}-\tau f} \frac{\mathrm{d}k^2(\tau f + \varepsilon)}{\mathrm{d}f^2} g(\varepsilon)\,\mathrm{d}\varepsilon + \frac{\tau \mathrm{d}k(-\bar{f})}{\mathrm{d}f} g(-\bar{f} - \tau f)$$

The first derivative $\mathrm{d}e(f)/\mathrm{d}f$ is positive because $k(x)$ is CCA by assumption so that $\mathrm{d}k(x)/\mathrm{d}x > 0$.

By using the antisymmetry of $k(x)$ and the change of variable technique as above, we can manipulate the second derivative as follows:

$$\begin{aligned}\frac{\mathrm{d}^2 e(f)}{\mathrm{d}f^2} &= \tau \frac{\mathrm{d}k(\bar{f})}{\mathrm{d}f}[g(-\bar{f} - \tau f) - g(\bar{f} - \tau f)] + \int_{-\bar{f}}^{\bar{f}} \frac{\mathrm{d}^2 k(\phi)}{\mathrm{d}f^2} g(\phi - \tau f)\,\mathrm{d}\phi \\ &= -\tau \frac{\mathrm{d}k(\bar{f})}{\mathrm{d}f}[g(\bar{f} - \tau f) - g(-\bar{f} - \tau f)] + \int_{0}^{\bar{f}} \frac{\mathrm{d}^2 k(\phi)}{\mathrm{d}f^2}[g(\phi - \tau f) - g(-\phi - \tau f)]\,\mathrm{d}\phi\end{aligned}$$

where $\phi = \varepsilon + \tau f$. Because $k(x)$ is CCA, evidently $\mathrm{d}^2 k(x)/\mathrm{d}x^2 < 0$ when $x > 0$. The antisymmetry and unimodality assumptions about $G(\varepsilon)$ imply that for $x > 0$:

$$g(x - \tau f) \gtrless g(-x - \tau f) \quad \text{as} \quad f \gtrless 0$$

It follows that $e(f)$ is convexoconcave about 0:

$$\frac{\mathrm{d}^2 e(f)}{\mathrm{d}f^2} \gtrless 0 \quad \text{as} \quad f \lessgtr 0$$

APPENDIX B

We present the IV estimates for Holland in Table AI. Standard errors are in parentheses. As instruments we used the explanatory variables lagged by one period and ten periods. In addition we used $(s_{t-1} - s_{t-2})$, $(s_{t-2} - s_{t-3})$, $(s_{t-1} - s_{t-2})^3$, as well as the ten-period lagged version of these latter three instruments.

Table AI.

α	δ	β_2^-	β_2^+	β_3
−4·7 E-6	1·97 E-1	29·661	−26·550	−1249·35
(1·03 E-4)	(0·146)	(29·213)	(27·655)	(1178·361)
R^2 − 0·036				
SE − 0·002				

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