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# Equity weights in the allocation of health care: the rank-dependent QALY model

Han Bleichrodt<sup>a,\*</sup>, Enrico Diecidue<sup>b</sup>, John Quiggin<sup>c</sup>

<sup>a</sup> *iMTA/iBMG, Erasmus University, P.O. Box 1738, 3000 DR Rotterdam, The Netherlands*

<sup>b</sup> *INSEAD, Singapore, Singapore*

<sup>c</sup> *University of Queensland, Brisbane, Australia*

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## Abstract

This paper introduces the rank-dependent quality-adjusted life-years (QALY) model, a new method to aggregate QALYs in economic evaluations of health care. The rank-dependent QALY model permits the formalization of influential concepts of equity in the allocation of health care, such as the fair innings approach, and it includes as special cases many of the social welfare functions that have been proposed in the literature. An important advantage of the rank-dependent QALY model is that it offers a straightforward procedure to estimate equity weights for QALYs. We characterize the rank-dependent QALY model and argue that its central condition has normative appeal.

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## 1. Introduction

The use of quality-adjusted life-years (QALY)<sup>1</sup> has become standard practice in the analysis of the cost-effectiveness of medical interventions. The use of the QALY measure is commonly associated with the assumption that health care resources should be allocated so as to achieve the maximal health gain as measured by additional QALYs. Many authors (Broome, 1988; Harris, 1988; Lockwood, 1988; Culyer, 1989; Wagstaff, 1991; Dolan, 1998) have raised concerns about the equity implications of this allocation rule.

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\* Corresponding author. Fax: +31-10-408-9092.

*E-mail address:* [bleichrodt@bmg.eur.nl](mailto:bleichrodt@bmg.eur.nl) (H. Bleichrodt).

<sup>1</sup> Although we speak of QALYs throughout the paper, our analysis also applies to other measures of health, such as healthy-years equivalents (HYEs).

The basic problem is presented by Williams (1997) in terms of the ‘fair innings’ argument. Is it equitable that an increase of, say, one QALY should be valued equally whether it accrues to someone who is already ‘rich’ in years or to someone whose life in the absence of treatment will be short and miserable? Williams suggests that an appropriate response may be to use ‘equity weights’, but expresses the concern (p. 28) that:

there is a danger such weights become arbitrary and capricious and come to be used to fudge outcomes in ways that would not be acceptable if their basis were exposed. One safeguard against this is to have some underlying (or ‘over-arching’) general principle enunciated, which can be confronted with evidence so that its various implications can be explored in a quantitative way.

Williams argues that the fair-innings principle may provide the basis for such an approach, but observes that much work needs to be done in developing the conceptual basis of a ‘fair innings’. In particular, it is important to determine the implications of any general principle, considered in isolation or in combination with other principles which may seem desirable. A set of principles may appear unexceptionable when considered separately, but may produce unpalatable implications when considered jointly, or may be mutually inconsistent. The best way to avoid such undesirable outcomes is through the derivation of equity weights from clearly stated conditions. The acceptability or otherwise of these conditions may then be assessed both in isolation and with respect to their joint implications.

The object of this paper is to present a general allocation rule which incorporates equity weights and to derive the conditions on which this rule depends. Under this rule, the equity weight assigned to an individual depends on his rank when individuals are ranked in terms of their expected lifetime QALYs. That is, the equity weights depend on the relative positions of the individuals. We refer to this allocation rule as the rank-dependent QALY model. Models of rank dependence are widely used in decision under uncertainty (Quiggin, 1981; Yaari, 1987; Schmeidler, 1989; Tversky and Kahneman, 1992) and the measurement of inequality (Weymark, 1981; Ebert, 1988; Yaari, 1988). Rank dependence was used to give a preference foundation for the QALY model in Bleichrodt and Quiggin (1997, 1999) and in Miyamoto (1999). Here we introduce the concept of rank dependence for the social evaluation of QALY profiles.

We give a preference foundation for the rank-dependent QALY model and we will argue that the model depends on reasonable conditions. An important advantage of the rank-dependent QALY model for empirical research and practical applications is that it provides a simple way to estimate equity weights from individual choices, as we show in Section 2.5. Williams and Cookson (2000, p. 1905) argue that the “great challenge [for health economists] is to bridge the gap between the economic requirement to estimate precisely targeted equity–efficiency trade-offs, and the psychological capabilities of respondents to think about equity and efficiency in such a tightly defined manner.” The rank-dependent QALY model can bridge the gap to which Williams and Cookson refer: the model can incorporate both efficiency concerns and a wide array of equity concerns and its empirical elicitation is straightforward and not cognitively demanding.

The paper is organized as follows. Section 2 is the central section of the paper. It describes the rank-dependent QALY model and shows that the model can incorporate equity concerns into QALY-based decision making and that it has several important social welfare functions

as special cases, including unweighted aggregation, generally referred to as QALY utilitarianism, and Rawls (1971) maximin rule. We also show how the equity weights can be elicited in the rank-dependent QALY model. An interesting feature of the rank-dependent QALY model is that it can be decomposed into an efficiency term and an equity term. This decomposition may be used to provide a welfare economic foundation for the measurement of equity in health and health care (Van Doorslaer et al., 1993).

Sections 3 and 4 are more technical and contain a derivation of the rank-dependent QALY model. Our analysis relies to a large extent on the observation, based on the work of Harsanyi (1953, 1955), Atkinson (1970), and Rawls (1971), that there is a close connection between models of choice under uncertainty and models of social choice. Section 3 presents a translation of de Finetti's (1931) famous book-making principle to the context of social choices over QALY profiles. We use the book-making principle to derive conditions for the optimality of QALY utilitarianism. Section 4 argues that the full form of the book-making principle is too restrictive in social choice. We introduce a weaker version, the comonotonic book-making principle. The comonotonic book-making principle is the central condition in our derivation of the rank-dependent QALY model. We argue that the comonotonic book-making principle has normative appeal, and thereby give a defense for the use of the rank-dependent QALY model in the economic evaluation of health care. Section 5 concludes. Proofs of the results presented throughout the paper are given in Appendix A.

## 2. The model

### 2.1. Background

We consider a policy maker who has to choose between health care programs leading to different allocations of expected lifetime QALYs among the individuals in society. Let  $n$  be the number of individuals in society. Let  $Q_i$  denote the expected number of QALYs received by individual  $i$  throughout his lifetime. A QALY profile  $(Q_1, \dots, Q_n)$  specifies the expected number of lifetime QALYs received by each individual. We denote the set of all QALY profiles by  $Q$ . Occasionally we use the notation  $(p_1, Q_1; \dots; p_m, Q_m)$ . This notation means that proportion  $p_i$  of the population receives  $Q_i$  QALYs,  $i = 1, \dots, m$ . A social preference relation  $\succsim$  is defined over  $Q$ , where  $\succsim$  denotes "at least as preferred as." The relation  $\succsim$  reflects a policy maker's preferences over QALY profiles. More specific assumptions with respect to  $\succsim$  will be made in Sections 3 and 4. A social welfare function  $W$  is a function that assigns a real-valued index to every conceivable QALY profile  $Q$  and that represents  $\succsim$ . That is, if for all  $q^1, q^2 \in Q$ ,  $W(q^1) \geq W(q^2)$  if and only if  $q^1 \succsim q^2$ .

The best known social welfare function is the utilitarian form  $W(q) = \sum_{i=1}^n (1/n) Q_i$ . This form is generally referred to as QALY utilitarianism and is widely used in economic evaluations of health care. Harsanyi (1953) gave a defense of utilitarianism in terms of expected utility (EU). Harsanyi assumed that every person could occupy any position in society with equal (known) probability. He showed, under these assumptions, that people who are expected utility maximizers will choose to maximize the sum of utilities across society.<sup>2</sup>

<sup>2</sup> To be precise, Harsanyi derived the function  $W(q) = \sum_{i=1}^n (1/n) U_i(Q_i)$ .

Critics of QALY utilitarianism have frequently attacked the view that a given increment of QALYs should be valued equally, no matter to whom it accrues. These critics have argued that we should rank individuals and apply different weights depending on the relative position of an individual. Rawls (1971), for instance, proposed a principle of justice focusing on the worst-off individual.<sup>3</sup> Like Harsanyi, Rawls derived a social choice rule from preferences under uncertainty. However, Rawls attempted a more detailed consideration of how individual preferences under uncertainty might translate into social decisions. Rawls introduced the idea of a ‘veil of ignorance’, behind which individuals seek to agree on appropriate social choice rules without knowing what position they will hold in society or what particular preferences they will have. In particular, they do not know what their risk attitudes will be.

A key point of disagreement between Rawls and Harsanyi is that Rawls rejects EU as a basis for individual decision-making under uncertainty. Rawls’ criticisms of utilitarianism as a social choice criterion are closely related to his criticisms of EU as a theory of choice under uncertainty. In particular, Rawls’ claim that the aggregation process involved in utilitarianism ‘does not take individuals seriously’ follows directly from a rejection of the sure thing principle for choice under uncertainty. On the basis of his criticisms of EU, Rawls suggests the use of a maximin rule under uncertainty. That is, individuals should choose the action under which the harm suffered in the worst state of the world is as small as possible.

Rawls’ approach seems to raise particular difficulties in the context of medical resource allocation. In this context, the worst off at any point in time are those at or near the point of death. The worst off in a lifetime sense are infants who die shortly after birth. There does not seem to be any limit to the resources that can be poured into the objective of achieving marginal extensions of life in circumstances of this kind. Hence, any allocation of medical resources to anyone other than the terminally ill would fail Rawls’ criterion of justice. As noted, Rawls would presumably not advocate such an allocation in practice, but would seek to allocate resources to the worst-off group. But, it is difficult to see how to formalize this idea unless the maximin rule is abandoned.

## 2.2. The rank-dependent QALY model

Rawls’ critique of utilitarianism has been subject to some reassessment in the light of the literature on generalized expected utility theory (Schmidt, 1998; Starmer, 2000). The general tendency of this literature is to endorse much of the critique of EU theory put forward both by Rawls and by earlier critics such as Allais (1953), but not to accept Rawls’ maximin alternative. Let us therefore formulate an alternative allocation rule that is sensitive to whom health benefits accrue but that is more flexible than Rawls’ maximin principle.

Suppose that the QALY profile  $q \in Q$  is rank-ordered so that  $Q_1 \geq \dots \geq Q_n$ . Every QALY profile can be written in this manner by reordering the individual expected lifetime QALYs in decreasing order. The rank-dependent QALY model evaluates the rank-ordered

<sup>3</sup> In his informal discussion, Rawls (1971) suggested that the appropriate focus should be on the worst-off group in the community, representing, say, those in the bottom quintile of the income distribution, rather than on the worst-off person. This idea is not formalized, and seems easier to apply in relation to monetary income than in the case of health status.

QALY profile  $q$  as

$$W(q) = \sum_{i=1}^n \pi_i Q_i \quad (1)$$

where the  $\pi_i$  are *equity weights* that are defined as  $\pi_1 = w(1/n)$ ,  $\pi_j = w(j/n) - w((j-1)/n)$ ,  $1 < j \leq n$ . The equity weights indicate the attention given to individual  $i$  in the evaluation of QALY profile  $q$ . The function  $w$  is defined from  $[0, 1]$  into  $[0, 1]$  and satisfies  $w(0) = 0$ ,  $w(1) = 1$ , and  $w(p) \geq w(q)$  if  $p > q$ . It follows from the definition of  $w$  that the equity weights are nonnegative and sum to one.

The intuition behind the rank-dependent QALY model is that the policy maker's concern for an individual (or group of individuals) depends on how badly-off the individual is in comparison with the other individuals in society. To illustrate, consider a QALY profile (1/3, 60; 1/3, 50; 1/3, 40) where one-third of the individuals receive 60 QALYs, one-third receive 50 QALYs, and one-third receive 40 QALYs. Suppose that the policy maker is averse to inequalities in health. Then the policy maker will show more concern for the group getting 40 QALYs, than for either of the other two equally sized groups, because the group getting 40 QALYs is the worst-off. Say that the equity weight assigned to the group getting 40 QALYs,  $\pi_3$ , is equal to 1/2. Because the policy maker is inequality-averse, he will pay most of his remaining attention to the group receiving 50 QALYs (say  $\pi_2 = 1/3$ ). Because the equity weights sum to one it follows that  $\pi_1 = 1/6$  and the attention paid to the group with the largest QALY gain will be relatively small. With the given weights, in fact, the policy maker places the same value on an increment of one QALY accruing to group 3 as on an increment of one QALY accruing to each of the other groups ( $\pi_1 + \pi_2 = \pi_3 = 1/2$ ). Similarly, since  $\pi_2 = 2\pi_1$ , the policymaker places twice the value on an increment of one QALY to group 2 (the group receiving 50 QALYs) as on an increment of one QALY to group 1 (the group receiving 60 QALYs). Thus, the following interventions would be equally valued:

- (a) an increment of 3 QALYs, from 60 to 63, for group 1;
- (b) an increment of 1.5 QALYs, from 50 to 51.5, for group 2;
- (c) an increment of one QALY, from 60 to 61 and 50 to 51, for groups 1 and 2;
- (d) an increment of one QALY, from 40 to 41, for group 3; and
- (e) an increment of 0.5 QALYs for the entire population.

Suppose next that members of group 2, who initially received 50 QALYs, now only get 30 QALYs, so that the QALY profile becomes (1/3, 60; 1/3, 30; 1/3, 40). Under the rank-dependent QALY model, the weights accruing to the three groups depend on the ranking of the groups in terms of the number of QALYs they obtain. Since the ranking has changed, the equity weights also change, to (1/6, 1/2, 1/3). Thus, even though nothing has changed for the group receiving 40 QALYs (the amount of QALYs and the size of the group have not changed), their equity weight decreases because they are no longer the worst-off group.

Finally, suppose that members of group 1 get 65 QALYs instead of 60 QALYs. Since this change does not affect the ranking, it does not affect the equity weights and, therefore, does not affect the evaluation of marginal interventions. That is, the model is sensitive only to

the ranking of groups and not to the magnitude of the differences between them. Obviously this feature has both costs and benefits. On the one hand, the range of social welfare judgements the model can accommodate is limited. Moreover, small changes in expected lifetime QALYs can lead to abrupt changes in the equity weights if the rank-ordering of the individuals is affected. On the other hand, the model is more parsimonious and more easily tested and estimated. The continuing popularity of rank-dependent models in the analysis of choice under uncertainty suggests that, in this context, the model has achieved an appropriate trade-off between parsimony and explanatory power. The fact that the rank-dependent QALY model encompasses such popular models as utilitarianism, Rawlsian maximin and the Gini model (see Sections 2.4 and 2.6) suggests that the same may be true in the context of social choice.

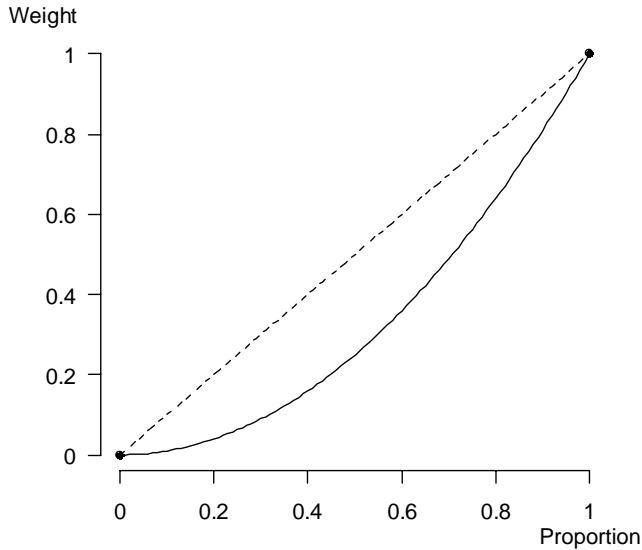
### 2.3. Attitudes towards inequality

The policy maker's attitudes towards inequality in health outcomes can be characterized through the function  $w$ . We say that a policy maker is *inequality-averse* if his preferences satisfy the Pigou–Dalton *Principle of Transfers*: a transfer of QALYs from people who receive relatively many QALYs to those who receive relatively few QALYs is always desirable, as long as the transfer does not affect anybody's position in the QALY ranking. The policy maker is *inequality seeking* if the above transfer is considered undesirable; he is *inequality neutral* if he is indifferent with respect to the above transfer.

Consider again the QALY profile  $(1/3, 60; 1/3, 50; 1/3, 40)$  of the above example. We noted that if the policy maker is inequality-averse then  $\pi_1 = w(1/3) < 1/3$  and  $\pi_3 = 1 - w(2/3) > 1/3$ , so that  $w(2/3) < 2/3$ . Hence, to get the (inequality-averse) preferences of the above example we must have  $w(1/3) < 1/3$  and  $w(2/3) < 2/3$ . This suggests as a general rule that to have inequality aversion we must have  $w(p) < p$  for all  $p \in (0, 1)$ . The results of Quiggin (1993) show that the condition  $w(p) < p$  is sufficient to ensure that policy makers will always prefer an equal allocation to an unequal one with the same number of QALYs. The stronger requirement of aversion to Pigou–Dalton transfers requires convexity of  $w$ . Fig. 1 displays a function  $w$  that corresponds to inequality aversion. It can be shown in a similar way that a concave  $w$  corresponds to inequality seeking. If  $w$  is linear, so that  $w(p) = p$  for all  $p \in (0, 1)$ , then the policy maker is inequality neutral. This latter case corresponds to QALY utilitarianism.

### 2.4. Special cases

The rank-dependent QALY model is both consistent with QALY utilitarianism (let  $w(j) = j$  for all  $j \in [0, 1]$ ) and with Rawls' (1971) maximin rule (let  $w((n-1)/n) = 0$ ). It can also accommodate all social welfare functions that lie between QALY utilitarianism and maximin, that is, social welfare functions that give different weight to individuals that are ranked differently but do not give all weight to the worst-off individual. For example, the rank-dependent QALY model is consistent with the social welfare functions depicted in Figs. 1, 3, 5, 8, 11, and 12 in Williams and Cookson (2000). The social welfare functions in Williams and Cookson (2000) that the rank-dependent QALY model cannot accommodate are those in which the equity weights are (partially) determined by factors other than the health of a person and those that have no maximand.



Note: people are ranked from highest to lowest in terms of QAL Ys gained

Fig. 1. A function  $w$  that corresponds to inequality aversion.

The rank-dependent QALY model is consistent with several of the proposals for equity weighting QALYs that have been put forward in the health economics literature. For example, the rank-dependent QALY approach can formalize Williams' (1997) fair innings approach. The Cobb-Douglas social welfare function proposed by Dolan (1998) is a special case of the rank-dependent QALY model if QALYs are not evaluated linearly but by the logarithmic function.

### 2.5. Empirical elicitation

An important advantage of the rank-dependent QALY model is that the empirical elicitation of the equity weights is straightforward. The first step in the empirical elicitation consists of fixing two gauge outcomes  $M$  and  $m$  with  $M > m$ . To illustrate, let  $M = 60$  QALYs and let  $m = 30$  QALYs. The next step consists of selecting several proportions  $p_1, \dots, p_k$ . For each of these proportions  $p_i$  we determine  $w(p_i)$  by asking subjects for the number  $X_i$  that makes them indifferent between the profile giving  $X_i$  expected lifetime QALYs to all individuals in the population under consideration and the profile giving  $M = 60$  QALYs to proportion  $p_i$  of the population and  $m = 30$  QALYs to proportion  $1 - p_i$  of the population. It then follows from the rank-dependent QALY model that

$$X_i = 60w(p_i) + 30(1 - w(p_i))$$

or

$$w(p_i) = \frac{X_i - 30}{30}.$$

In general, five or six of such indifferences suffice to elicit the function  $w$ . The function  $w$  can then be used to obtain the equity weights, as described in Section 2.2.

2.6. An abbreviated social welfare function

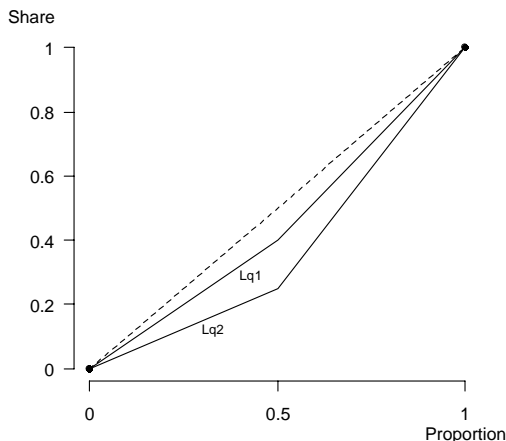
The rank-dependent QALY model permits us to express the social welfare associated with a QALY profile as the product of the mean number of QALYs and a number that reflects the equality rating of the QALY profile. This decomposition makes explicit the point that there is an efficiency–equity trade-off in the rank-dependent QALY model.

The Lorenz curve for health is defined as

$$L\left(\frac{j}{n}\right) = \sum_{i=j}^n \frac{1}{n} \frac{Q_i}{\mu_q}$$

where  $\mu_q$  denotes the mean number of QALYs of profile  $q$ . The Lorenz curve for health is comparable with the well-known Lorenz curve for income and indicates the shares of the total number of QALYs received by fractions  $1/n, 2/n, \dots$ , cumulated upwards from the individual (or group) who receives the lowest number of lifetime QALYs. Define  $\gamma = \sum_{i=1}^n (Q_i/\mu_q)\pi_i$ . The term  $Q_i/\mu_q$  indicates the slope of the Lorenz curve for health at the point  $\sum_{i=j}^n (i/n)$ . The terms  $(Q_i/\mu_q)\pi_i$  indicate how much weight the policy maker gives to changes in QALY shares at different points of the Lorenz curve for health.

If the policy maker is inequality-averse, he will pay more attention to the QALY shares of those individuals who are worst-off. It follows that for two QALY profiles  $q^1, q^2 \in Q$ , if the policy maker is inequality-averse, then  $\gamma_{q^1} > \gamma_{q^2}$  indicates that  $q^1$  is closer to complete equality than  $q^2$ . Hence, the term  $\gamma_q$  can be interpreted as the equality rating of QALY profile  $q$ . Fig. 2 gives a simple illustration that  $\gamma$  can be interpreted as an equality rating.



Note: people are ranked from lowest to highest in terms of QAL Ys gained

Fig. 2. Two Lorenz curves for health.



The figure shows the Lorenz curves for health corresponding to two QALY profiles  $q^1$  and  $q^2$ . Profile  $q^1$  is more equal than profile  $q^2$  because its Lorenz curve for health ( $L_{q^1}$ ) lies within the Lorenz curve for  $q^2$  ( $L_{q^2}$ ). The slope of  $L_{q^1}$  exceeds that of  $L_{q^2}$  for  $p < 0.5$  and falls short of the slope of  $L_{q^2}$  for  $p > 0.5$ . If the policy maker is inequality-averse, he gives more weight to the lower part of the Lorenz curve for health (the part for which  $p < 0.5$ ) and therefore  $\gamma_{q^1} > \gamma_{q^2}$ . Hence, if  $q^1$  is more equal than  $q^2$ , then  $\gamma$  reflects this and  $\gamma$  can, therefore, be interpreted as an equality rating.

The rank-dependent QALY model implies that the social welfare of QALY profile  $q$ ,  $W(q)$ , is equal to  $\mu_q \gamma_q$ . That is, the social welfare of profile  $q$  can be expressed as the product of an efficiency term (the mean number of QALYs of the profile) and an equity term (the equality rating of the profile). The most common index for assessing equality is the Gini coefficient. The *Gini coefficient of equality* is defined as twice the area under the Lorenz curve. It can be shown that if  $\pi_i = (2i - 1)/n^2$ , the equality rating  $\gamma$  is equal to the Gini coefficient of equality (Blackorby and Donaldson, 1978). Hence, the rank-dependent QALY model contains the social welfare function underlying the Gini coefficient as a special case.<sup>4</sup> The Gini coefficient and related indices, such as the Kakwani index, are widely used in the measurement of equity in health and health care (Van Doorslaer et al., 1993). This work has been criticized for having no foundation in welfare economics. The analysis of this section shows that the rank-dependent model can serve to provide such a welfare economic foundation. This topic, however, is beyond the scope of the present paper.

### 3. The book-making principle

Let us now turn to the characterization of the rank-dependent QALY model. Crucial in our characterization is a consistency condition, which is a reformulation of de Finetti's book-making principle (de Finetti, 1931) for social evaluations of QALY profiles. The book-making principle is based on the idea that a number of good decisions, when taken together, should still be good. In terms of the economic evaluation of health care, it means the following. Suppose that the policy maker has performed a given number, say  $m$ , comparisons between QALY profiles. Abstracting from differences in costs, we may say that the policy maker has performed  $m$  different cost utility analyses. Suppose that in the first cost utility analysis profile  $q^1$  is preferred to profile  $r^1$  ( $q^1 \succ r^1$ ); in the second cost utility analysis  $q^2$  is preferred to  $r^2$ ; and so on for all the  $m$  comparisons. Hence, for all  $m$  comparisons, implementing the health care program that leads to  $q^j$  instead of the program that leads to  $r^j$ ,  $j = 1, \dots, m$ , is a good thing. It then seems natural to expect that the joint implementation of the programs that lead to  $q^1, \dots, q^m$  will result in a more favorable QALY profile than the joint implementation of the programs that lead to  $r^1, \dots, r^m$ . If on the other hand the joint implementation of the programs that lead to  $q^1, \dots, q^m$  results in a lower number of QALYs for each individual than the joint implementation of the programs that lead to  $r^1, \dots, r^m$ , it appears that some inconsistency has arisen. The book-making principle excludes this apparent inconsistency. That is, the book-making principle excludes the possibility that when the recommendations of a number of cost utility analyses are carried out jointly, social

<sup>4</sup> For this reason, Weymark (1981) refers to the rank-dependent model as the generalized Gini model.

welfare actually decreases. The book-making principle can be considered as a consistency condition for cost-utility analysis. The book-making principle is implicitly assumed in the economic evaluation of health care. All we do here is to make this assumption explicit.

### 3.1. QALY utilitarianism

We assume that the social preference relation  $\succsim$  is a *weak order*, that is, it is *complete* (for all  $\mathbf{q}, \mathbf{r} \in \mathcal{Q}$ , either  $\mathbf{q} \succsim \mathbf{r}$  or  $\mathbf{r} \succsim \mathbf{q}$ ) and *transitive* (for all  $\mathbf{q}, \mathbf{r}, \mathbf{s} \in \mathcal{Q}$ , if  $\mathbf{q} \succsim \mathbf{r}$  and  $\mathbf{r} \succsim \mathbf{s}$ , then  $\mathbf{q} \succsim \mathbf{s}$ ). We further assume that there exists a constant equivalent for each QALY profile. The *constant equivalent* of QALY profile  $\mathbf{q} = (Q_1, \dots, Q_n)$  is the number of QALYs  $c$  which, if distributed equally, would give the same level of social welfare as  $\mathbf{q}$ . That is,  $(c, \dots, c) \sim (Q_1, \dots, Q_n)$ . The concept of a constant equivalent is commonly used in the welfare economics literature. For example, Kolm (1969) uses the notion of representative income, which, if equally distributed among the individuals, results in the same social welfare as the existing income distribution. Atkinson (1970) called this income level the *equally distributed equivalent income*.

Let  $\mathbf{s} = (S_1, \dots, S_n)$  be an initial profile of expected lifetime QALYs. In cost-utility analysis changes with respect to the initial profile are evaluated. Let  $\varepsilon_{i_q}$  denote the number of QALYs individual  $i$  gains by comparison with the initial profile after the implementation of a health care program that leads to profile  $\mathbf{q}$ . Abstracting from differences in costs, cost utility analysis concludes that the health care program that leads to QALY profile  $\mathbf{q}$  is preferred to the program that leads to QALY profile  $\mathbf{r}$  if  $(S_1 + \varepsilon_{1_q}, \dots, S_n + \varepsilon_{n_q}) \succsim (S_1 + \varepsilon_{1_r}, \dots, S_n + \varepsilon_{n_r})$ . For notational convenience, we will denote profiles  $(S_1 + \varepsilon_{1_q}, \dots, S_n + \varepsilon_{n_q})$  as  $(\varepsilon_{1_q}, \dots, \varepsilon_{n_q})$ . That is, we only denote the QALY gains. It should be kept in mind, however, that social preferences are defined over profiles of expected lifetime QALYs, as we explained in Section 2.

We assume that QALY gains can be added over programs. Let  $\sum_{j=1}^m Q_i^j$  be the gain in QALYs for individual  $i$  when profiles  $\mathbf{q}^1, \dots, \mathbf{q}^m$  are jointly implemented. Similarly,  $\sum_{j=1}^m R_i^j$  denotes the gain in QALYs for individual  $i$  when profiles  $\mathbf{r}^1, \dots, \mathbf{r}^m$  are jointly implemented.

**Definition 1.** A book consists of the preferences  $\mathbf{q}^j \succsim \mathbf{r}^j$ ,  $j = 1, \dots, m$ , but  $\sum_{j=1}^m Q_i^j < \sum_{j=1}^m R_i^j$  for all individuals  $i$ .

The aim of the book-making principle is to exclude the possibility of books. Hence, we say that the *book-making principle* holds if no book can be made.

Let  $\rho$  be a *permutation*, that is, a function which specifies a reordering of the individuals in society. Let  $\mathbf{q}_\rho$  denote the QALY profile that is obtained after application of permutation  $\rho$ . For example, if  $\mathbf{q} = (2, 3)$  and  $\rho(1) = 2, \rho(2) = 1$  then  $\mathbf{q}_\rho = (3, 2)$ . The preference relation satisfies *anonymity* if for all permutations  $\rho$  and for all QALY profiles  $\mathbf{q} \in \mathcal{Q}$ ,  $\mathbf{q} \sim \mathbf{q}_\rho$ . In words, anonymity says that permuting the QALYs among individuals does not affect social preference. It implies that social preference does not depend on the identity of a QALY recipient. Anonymity is widely used in social choice theory and is generally considered appealing (Sen, 1970).

We can now state:

**Theorem 1.** *The following two statements are equivalent.*

1. *QALY utilitarianism holds.*
2. *The preference relation  $\succsim$  is a weak order that satisfies anonymity, for each QALY profile there exists a constant equivalent, and the book-making principle holds.*

A proof of [Theorem 1](#) is given in [Appendix A](#).

#### 4. The comonotonic book-making principle

[Theorem 1](#) is perhaps a surprising result. The assumptions of weak order, anonymity, and the existence of a constant equivalent are widely accepted, while the book-making principle seems a necessary consistency requirement for economic evaluations of health care. However, [Theorem 1](#) shows that these four, seemingly innocuous, conditions imply that the social welfare function must take the QALY-utilitarian form. Hence, [Theorem 1](#) provides a rationale for the use of QALY utilitarianism in the economic evaluation of health care. To justify equity weighting QALYs, at least one of the four conditions must be modified. We will consider modifying the book-making principle, because the other three conditions are reasonable. We will argue that the book-making principle, although plausible at first sight, is too strong because it does not pay attention to the relative position of individuals and therefore ignores equity considerations. Instead we will propose a weaker version of the book-making principle, which takes account of the ranking of individuals in terms of QALYs and, thereby, incorporates equity concerns. Let us first give an example, which illustrates why the (unrestricted) book-making principle is too strong.

**Example 1.** Consider two individuals who are identical in every respect. Let  $q^1 = (20, 0)$ . That is, in profile  $q^1$  the first individual gains 20 QALYs and the second nothing. Let  $q^2 = (0, 20)$  be the “mirror-image” of  $q^1$ , that is, the profile in which the second individual gains 20 QALYs and the first individual nothing. If the policy maker is inequality-averse then the following preferences are plausible:

$$(9, 9) \succsim (20, 0)$$

and

$$(9, 9) \succsim (0, 20).$$

But  $(9, 9) + (9, 9) = (18, 18) < (20, 20) = (20, 0) + (0, 20)$ , in violation of the book-making principle.

The reason for the violation of the book-making principle in [Example 1](#) is that the inequality in  $q^1$  is neutralized by the inequality in  $q^2$  and vice versa. In other words, the two profiles are complementary. To exclude violations of the book-making principle, we must exclude the possibility of complementarity between the profiles. That is, we must restrict the

set of QALY profiles for which we require the book-making principle to hold to those QALY profiles for which complementarity cannot arise. Complementarity cannot arise if the rank position of each individual in terms of QALYs gained is the same across all programs and is equal to the rank in the initial allocation of QALYs. QALY profiles for which this holds are *comonotonic*. Formally, two QALY profiles  $\mathbf{q} = (\varepsilon_{1q}, \dots, \varepsilon_{nq})$  and  $\mathbf{r} = (\varepsilon_{1r}, \dots, \varepsilon_{nr})$  are said to be *comonotonic* if for all  $i, j \in \{1, \dots, n\}$ , if  $S_i \leq S_j$  (where  $S_i$  and  $S_j$  denote, as before, the individuals' endowments in the initial allocation of QALYs), then also  $\varepsilon_{iq} \leq \varepsilon_{jq}$  and  $\varepsilon_{ir} \leq \varepsilon_{jr}$ . Thus, we cannot have, for example, that the gains of individual  $i$  are larger than the gains of individual  $j$  in profile  $\mathbf{q}$  but smaller in profile  $\mathbf{r}$ . Comonotonicity was introduced by [Schmeidler \(1989\)](#) for decision under uncertainty. [Ben-Porath and Gilboa \(1994\)](#) used comonotonicity to characterize preferences over income distributions.

It is easily verified that the profiles  $\mathbf{q}^1$  and  $\mathbf{q}^2$  in [Example 1](#) are not comonotonic: in profile  $\mathbf{q}^1$  the first individual gains more QALYs than the second, 20 versus 0, but in profile  $\mathbf{q}^2$  the first individual gains less QALYs than the second, 0 versus 20. [Example 1](#) suggests that we might generalize the book-making principle to capture equity concerns by imposing it only when all QALY profiles are comonotonic. A *comonotonic book* is a book as in [Definition 1](#) with the extra restriction that the QALY profiles  $\{\mathbf{q}^1, \dots, \mathbf{q}^m, \mathbf{r}^1, \dots, \mathbf{r}^m\}$  are all comonotonic. That is, for all QALY profiles the ranking of the individuals in terms of QALY gains must be the same and must coincide with the ranking in the initial allocation.

The *comonotonic book-making principle* says that no comonotonic books are permitted. Because the profiles  $\mathbf{q}^1$  and  $\mathbf{q}^2$  in [Example 1](#) are not comonotonic, [Example 1](#) does not violate the comonotonic book-making principle. The comonotonic book-making principle only excludes the existence of books for profiles that are comonotonic, that is, for profiles for which it is not possible to neutralize the inequality in one profile by the inequality in the other. The comonotonic book-making principle is a less restrictive condition than the book-making principle, because we no longer require the principle to hold for all QALY profiles, but only for those QALY profiles that are comonotonic.

We are now in a position to characterize the rank-dependent QALY model.

**Theorem 2.** *The following two statements are equivalent.*

1. *The rank-dependent QALY model holds.*
2. *The preference relation  $\succsim$  is a weak order, for each QALY profile there exists a constant equivalent, and the comonotonic book-making principle holds.*

A proof of [Theorem 2](#) is given in [Appendix A](#). Note that we no longer impose anonymity. Anonymity is implied by the other three conditions. In the rank-dependent QALY model the weight an individual receives in social welfare evaluations depends on his rank. Individuals that occupy the same rank will get the same weight regardless of who they are. Hence, the identity of individuals plays no role in social welfare judgments, which is what anonymity asserts.

[Theorem 2](#) offers a case for the use of the rank-dependent QALY model in economic evaluations of health care. As noted before, weak order and the existence of a constant equivalent are generally accepted conditions. The crucial condition in the representation is, therefore, the comonotonic book-making principle. The comonotonic book-making principle implies

that if we follow the recommendations of  $m$  cost utility analyses that do not affect the relative positions of individuals in society, society cannot be made worse off (in the sense that every individual is worse off). That is, profiles are compared only if they yield the same ranking of individuals. We may say, somewhat loosely, that the comonotonic book-making principle is equivalent to the restriction of the book-making principle to cases where no equity concerns apply. We believe that this condition is reasonable and has normative appeal.

## 5. Conclusion

This paper has introduced the rank-dependent QALY model, a new model to evaluate QALY profiles. It provides a way to incorporate equity concerns into cost-utility analysis and is consistent with many types of social welfare function that have been proposed in the literature, including QALY utilitarianism, Rawls' maximin, and the Gini social welfare function.

The central preference condition in the characterization of the rank-dependent QALY model is the comonotonic book-making principle. We have argued that the comonotonic book-making principle has normative appeal. This is not to say that the comonotonic book-making principle is beyond dispute. It may be argued that the equity weights should depend not only on individuals' ranks but also on the number of QALYs they receive. This allows, for example, to have equity weights that vary with the severity of illness. A disadvantage of doing so is that the equity weights come to depend more on the outcomes, measured in QALYs. This dependence makes the estimation of the equity weights more complicated. In the rank-dependent QALY model the equity weights have to be elicited once and can then be applied to all social evaluations. This is no longer possible if the equity weights depend also on the number of QALYs. There is a trade-off between greater generality and practical applicability.

An important advantage of the rank-dependent QALY model is that it offers a straightforward procedure for estimating equity weights from preferences over comonotonic QALY profiles. Several papers have argued that equity weights should be applied to QALYs, but relatively little empirical work has been done (Williams and Cookson, 2000). The work that has been done is somewhat *ad hoc* in nature. Typically, a specific social welfare function is selected *a priori* and the equity weights are estimated under the assumption that this function represents social preferences. Scant attention is paid to the equity implications of the choice of the social welfare function. We have argued that the rank-dependent QALY model depends on reasonable conditions and is consistent with important concepts of equity in health. We hope that the rank-dependent QALY model will prove useful in future empirical work on the elicitation of equity weights for QALY-based decision making.

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## Appendix A.

### A.1. Proof of Theorem 1

That (1) implies (2) is straightforward. Hence, we assume (2) and derive (1). By the proof of Theorem 5 in Diecidue and Wakker (2002), weak order, the existence of a constant equivalent, and the book-making principle imply that  $W(\mathbf{q}) = \sum_{i=1}^n p_i Q_i$  for nonnegative and uniquely determined  $p_i$  that sum to one. Let  $i, j \in \{1, \dots, n\}$  and let  $\mathbf{q}$  be a QALY profile that has  $Q_i \neq Q_j$ . Consider the profile  $\mathbf{q}^1$  which is obtained from  $\mathbf{q}$  by giving  $Q_i$  to individual  $j$  and  $Q_j$  to individual  $i$  and leaving the QALY gain of all other individuals unchanged. By anonymity,  $\mathbf{q} \sim \mathbf{q}^1$ , and, because  $Q_i \neq Q_j$ , we must have  $p_i = p_j$ . Because  $i$  and  $j$  were chosen arbitrarily,  $p_i = p_j$  for all  $i, j \in \{1, \dots, n\}$ . This establishes QALY utilitarianism.

### A.2. Proof of Theorem 2

That (1) implies (2) is straightforward. Hence, we assume (2) and derive (1). Let  $\rho$  be a permutation of  $\{1, \dots, n\}$ . Define  $Q^\rho = \{\mathbf{q} \in Q : Q_{\rho(1)} \succ \dots \succ Q_{\rho(n)}\}$ . Because all  $\mathbf{q} \in Q^\rho$  are comonotonic, the book-making principle holds on  $Q^\rho$ , and, by Theorem 5 in Diecidue and Wakker (2002), there exists an additive representation  $W^\rho(\mathbf{q}) = \sum_{i=1}^n p_i^\rho Q_i$  for  $\succ$  on  $Q^\rho$  with the  $p_i^\rho$  uniquely determined, nonnegative, and summing to one. Define  $w^\rho(\sum_{i=1}^j (i/n)) = \sum_{i=1}^j p_i^\rho$ . This gives the rank-dependent QALY model on  $Q^\rho$ .

For any two permutations  $\rho$  and  $\rho'$ , the common domain  $Q^\rho \cap Q^{\rho'}$  is nonempty. (It contains for example the constant QALY profiles.) The additive representations  $W^\rho(\mathbf{q})$  and  $W^{\rho'}(\mathbf{q})$  coincide on this common domain. This implies that  $w^\rho = w^{\rho'} = w$  for all permutations  $\rho, \rho'$ . Hence, the rank-dependent QALY model holds on  $Q$ .

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