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Measuring Loss Aversion under Ambiguity: A Method to Make Prospect Theory Completely Observable

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Abstract We propose a simple, parameter-free method that, for the first time, makes it possible to completely observe Tversky and Kahneman's (1992) prospect theory. While methods exist to measure event weighting and the utility for gains and losses separately, there was no method to measure loss aversion under ambiguity. Our method allows this and thereby it can measure prospect theory's entire utility function. Consequently, we can properly identify properties of utility and perform new tests of prospect theory. We implemented our method in an experiment and obtained support for prospect theory. Utility was concave for gains and convex for losses and there was substantial loss aversion. Both utility and loss aversion were the same for risk and ambiguity, as assumed by prospect theory, and sign-comonotonic trade-off consistency, the central condition of prospect theory, held.

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1 Introduction

Loss aversion, the assumption that people are more sensitive to losses than to commensurate gains, is a central element of prospect theory (Kahneman and Tversky 1979; Tversky and Kahneman 1992) and key to explaining deviations from expected utility (Rabin 2000, pp. 1288–1289). There is abundant qualitative evidence for loss aversion, from both the lab and the field (Barberis 2013; Fox and Poldrack 2014; Wakker 2010). However, measuring loss aversion is difficult. It requires the simultaneous measurement of utility for gains and utility for losses, which is complicated by prospect theory's assumption that decision weighting for gains and losses may differ. As a result, existing measurements of loss aversion impose simplifying assumptions, typically linear utility for gains and losses and no probability weighting.

Abdellaoui et al. (2007) was the first to propose a method for measuring loss aversion that did not have to impose simplifying assumptions about utility or probability weighting. Their method is designed for decision under risk, where objective probabilities are known. In most real-world decisions (e.g., the success of new medicines, the dangers of climate change, returns on investments in R&D), objective probabilities do not exist or are unknown and such decisions under ambiguity are now widely studied in both the empirical and the theoretical literature. It is difficult to extend the method of Abdellaoui et al. (2007) to decisions under ambiguity.[1]

This paper introduces a method to measure loss aversion under ambiguity without making simplifying assumptions about prospect theory's parameters. It makes it possible, for the first time, to completely observe Tversky and Kahneman's (1992) prospect theory.[2] Parameter-free methods to measure prospect theory's other parameters have been introduced before. Wakker and Deneffe (1996) showed how utility for gains and losses can be measured separately. Abdellaoui (2000) and Bleichrodt and Pinto (2000) showed how probability weights can be measured in decision under risk. Abdellaoui et al. (2005) showed how event weights can be measured in decision under ambiguity. Abdellaoui et al. (2007) showed how loss aversion can be measured in decision under risk. There did not yet exist a method to measure loss aversion under ambiguity and this is what our paper achieves. Hence, this paper completes a program to make prospect theory observable.

Our method is simple and uses only one response mode, which reduces the cognitive burden on subjects. It can quantify loss aversion through three preference elicitations and it does not require the complete measurement of utility. Our method is based on the

[1] This extension requires finding events with decision weight ½, which can be complex.

[2] Throughout this paper we use the term prospect theory for the 1992 version of the theory and the term original prospect theory (OPT) for the 1979 version. Because we only consider two-outcome prospects, OPT is the special case of prospect theory for decision under risk in which probability weighting for gains and losses are the same.

trade-off method of Wakker and Deneffe (1996). In its original form the trade-off method can only measure the utility for gains and the utility for losses separately and, consequently, it cannot measure loss aversion. We extend the trade-off method so that it can measure the utility for gains and losses simultaneously, and thus loss aversion. This extension is not only useful from an empirical perspective, but also has theoretical merits. There is a close connection between measurements using the trade-off method and axiomatizations of decision theories (Köbberling and Wakker 2003). Our method may help to simplify existing preference characterizations and to develop new ones.

Because our method can completely measure prospect theory's utility function, it also permits new tests of prospect theory. We implement our method in an experiment and show that our measurements can easily be used to test the central condition of prospect theory, sign-comonotonic trade-off consistency. We also test whether both utility and loss aversion are the same under risk and ambiguity, as assumed by prospect theory. Our data is consistent with prospect theory. We could neither reject sign-comonotonic trade-off consistency nor in most cases the null hypotheses that utility and loss aversion were the same under risk and ambiguity. Utility had prospect theory's hypothesized shape, concave for gains and convex for losses, and there was substantial loss aversion.

2 Background

2.1 Binary prospect theory

Consider a decision maker who has to make a choice in the face of ambiguity. Ambiguity is modeled through a *state space* S. Exactly one of the states will obtain, but the decision maker does not know which one. Subsets E of S are *events* and E^c denotes the complement of E.

Prospects map states to outcomes. Outcomes are money amounts and more money is preferred to less. In our measurements, we will only use two-outcome prospects x_Ey, signifying that the decision maker obtains €x if event E occurs and €y otherwise. If probabilities are known, we will write x_py for the prospect that pays €x with probability p and €y with probability $1-p$. We will refer to x_Ey as an *ambiguous prospect* (meaning that probabilities are unknown) and to x_py as a *risky prospect* (meaning that probabilities are known).

The decision maker has preferences over prospects and we use the conventional notation $\succ$, $\succcurlyeq$, and $\sim$ to denote strict preference, weak preference, and indifference. Preferences are defined relative to a reference point x_0. *Gains* are payoffs higher than x_0 and *losses* are payoffs lower than x_0. A prospect is *mixed* if it involves both a gain and a loss. For mixed prospects, the notation x_Ey signifies that x is a gain and y is a loss. A *gain prospect* involves no losses (i.e., *both x and y are at least as great as* x_0) and a *loss prospect* involves no gains. For gain and loss prospects the notation x_Ey signifies that the absolute value of x exceeds the absolute value of y (i.e., for gains $x \geq y$ and for losses $x \leq y$).

Under *binary prospect theory* (*PT*) the decision maker's preferences over mixed prospects $x_E y$ are evaluated by:

$$W^+(E)U(x) + W^-(E^c)U(y), \quad (1a)$$

and preferences over gain or loss prospects by:

$$W^i(E)U(x) + \left(1 - W^i(E)\right)U(y), \quad (1b)$$

where $i=+$ for gains and $i=-$ for losses. U is a strictly increasing, real-valued *utility function* that satisfies $U(x_0)=0$. The utility function is a ratio scale and we are free to choose the utility of one outcome other than the reference point. U is an overall utility function that includes loss aversion. In empirical applications U is often decomposed in a basic utility function, which captures the decision maker's attitudes towards final outcomes and which can be interpreted as the rational part of utility, and a loss aversion coefficient λ capturing attitudes towards gains and losses (Köbberling and Wakker 2005; Köszegi and Rabin 2006; Sugden 2003). Our method does not require this decomposition. However, it does allow the decomposition of U into u, the basic utility function, and loss aversion λ if this is considered desirable.

The *event weighting functions* $W^i, i=+,-$, assign a number $W^i(E)$ to each event E such that

(i) $W^i(\varnothing) = 0$

(ii) $W^i(S) = 1$

(iii) W^i is *monotonic*: $E \supseteq F$ implies $W^i(E) \geq W^i(F)$.

The event weighting functions W^i depend on the sign of the outcomes and may be different for gains and losses. They need not be additive. For gains, binary PT contains most ambiguity models as special cases,[3] as was pointed out by Luce (1991) and Ghirardato and Marinacci (2001). These ambiguity models only differ when the number of outcomes is at least three. Equations (1a) and (1b) represent the extension of these models to include sign-dependence.

Binary PT evaluates mixed risky prospects $x_p y$ by

$$w^+(p)U(x) + w^-(1-p)U(y) \quad (2a)$$

and gain and loss risky prospects $x_p y$ by

$$w^i(p)U(x) + \left(1-w^i(p)\right)U(y), i = +,-. \quad (2b)$$

[3] For example, Choquet expected utility (Schmeidler 1989), maxmin expected utility (Gilboa and Schmeidler 1989), α-maxmin expected utility (Ghirardato et al. 2004), and contraction expected utility (Gajdos et al. 2008).

w^i is a strictly increasing *probability weighting function* that satisfies $w^i(0) = 0$ and $w^i(1) = 1$ and that, again, may differ between gains and losses. Hence, in the evaluation of risky prospects the event weighting functions W^i are replaced by probability weighting functions w^i. Equations (2a-b) include most theories of decision under risk as special cases.[4]

2.2 Previous evidence

Because we concentrate on utility and loss aversion in this paper, we will only discuss the empirical literature on these two elements of prospect theory. For an extensive review of the literature on probability weighting and event weighting see Wakker (2010) and Fox and Poldrack (2014).

Tversky and Kahneman (1992) assume that utility differs between gains and losses and is S-shaped: concave for gains and convex for losses. In addition, they assume that utility is steeper for losses than for gains, reflecting loss aversion. Nearly all the empirical evidence on utility comes from decision under risk. There is much evidence that utility for gains is indeed concave (Wakker 2010), but for losses the evidence is somewhat mixed. Although most studies found convex utility, some studies also found linear or concave utility (for example, Bruhin et al. 2010). For losses, utility usually was closer to linearity than for gains.

Empirical evidence on utility under ambiguity is scarce. Abdellaoui et al. (2005) confirmed that utility under ambiguity was concave for gains and slightly convex for losses. Their parametric estimates were close to those previously obtained under risk, but they did not directly measure utility under risk. Abdellaoui et al. (2011) and Vieider et al. (2013) measured utility under risk and under ambiguity for small stakes and under parametric assumptions about utility. Abdellaoui et al. (2011) found that utility was moderately concave for both risk and ambiguity, while Vieider et al. (2013) found linear utility.

Nearly all empirical measurements of loss aversion made simplifying assumptions, typically assuming linear utility and either ignoring probability weighting (Baltussen et al. forthcoming; Booij and van de Kuilen 2009; Pennings and Smidts 2003)[5] or assuming equal weighting for gains and losses (Gaechter et al. 2007). Of these studies, only Baltussen et al. (forthcoming) estimated loss aversion under both risk and ambiguity. They reported more loss aversion under ambiguity than under risk when subjects made their decisions in public, but not when they did so in private. Abdellaoui et al. (2007) measured loss aversion under risk without imposing simplifying assumptions on either utility or probability weighting. To the best of our knowledge, such "clean" estimates of loss aversion do not exist for decision under ambiguity.

Most studies found loss aversion coefficients around 2, meaning that losses weight approximately twice as much as absolutely commensurate gains (Booij et al. 2010; Fox and Poldrack 2014). A difficulty in comparing the results of these studies is that they

[4] For example, original prospect theory (Kahneman and Tversky 1979), rank-dependent expected utility (Quiggin 1981; Quiggin 1982), prospective reference theory (Viscusi 1989), and disappointment aversion theory (Gul 1991).

[5] Booij and van de Kuilen (2009) investigated the robustness of their findings by using probability weights estimated in other studies.

not only made different parametric assumptions, but also used different definitions of loss aversion.

Finally, even though binary PT is consistent with much of the empirical data on decision under risk and ambiguity and includes many models as special cases, there is some evidence challenging it. For example, Birnbaum and Bahra (2007) and Wu and Markle (2008) obtained violations of binary PT for mixed prospects. Because of this negative evidence, we included a test of sign-comonotonic trade-off consistency, the main condition underlying binary PT, in our experiment. This test is explained below.

3 Measurement method

Our method for measuring utility and loss aversion consists of three stages and is summarized in Table 1. In the first stage, a gain and a loss are elicited that connect utility for gains (measured in the second stage) with utility for losses (measured in the third stage). The measurements in the second and in the third stage employ the trade-off method of Wakker and Deneffe (1996) to determine a *standard sequence* of outcomes such that the utility difference between successive elements of the sequence is constant. The trade-off method is commonly used in decision theory (Wakker 2010), but thus far it could only be used to measure utility for gains and utility for losses separately. It could not be used to measure loss aversion, which requires that the utility for gains and the utility for losses can be compared. Our method measures utility for gains and utility for losses jointly by eliciting a standard sequence of outcomes that goes through the reference point, and, consequently, it can measure loss aversion. In all the derivations presented below we impose no parametric assumptions on utility and the weighting functions W^i and $w^i, i = +, -$. Hence, our method is parameter-free. Our method only asks subjects to respond in terms of money and uses no other response scale. This reduces the cognitive demands on subjects.

Table 1 Three-stage procedure to measure utility. The third column shows the quantity that is assessed in each of the three stages of the procedure. The fourth column shows the indifference that is elicited. The fifth column shows the stimuli used in our experiment. ℓ_{alt} and $k_{L_{alt}}$ were used to test binary PT (see Section 4 for explanation)

		Assessed quantity	Indifference	Choice variables
Stage 1		L	$G_E L \sim x_0$	G = €2000
		x_1^+	$x_1^+ \sim G_E x_0$	E = color of a ball drawn from an unknown Ellsberg urn (for the case of risk we replace E by p = ½)
		x_1^-	$x_1^- \sim L_{E^c} x_0$	$x_0 = 0$
Stage 2	Step 1	$\mathcal{L}$	$x_1^+{}_E \mathcal{L} \sim \ell_{E^c} x_0$	$\ell = -$ €300; $k_G = 6$
	Step 2 to k_G	x_j^+	$x_j^+{}_E \mathcal{L} \sim x_{j-1}^+{}_E \ell$	ℓ_{alt} = €0; $k_{Galt} = 3$
Stage 3	Step 1	$\mathcal{G}$	$\mathcal{G}_E x_1^- \sim g_E x_0$	g = €300; $k_L = 6$
	Step 2 to k_L	x_j^-	$\mathcal{G}_E x_1^- \sim g_E x_{j-1}^-$	

3.1 First stage: Connecting utility for gains and utility for losses

We start by selecting an event E that will be kept constant throughout the first stage and a gain G. Then we elicit the loss L for which $G_E L \sim x_0$. It follows from equation (1a) that

$$W^+(E)U(G) + W^-(E^c)U(L) = U(x_0) = 0. \quad (3)$$

We next elicit certainty equivalents x_1^+ and x_1^- such that $x_1^+ \sim G_E x_0$ and $x_1^- \sim L_{E^c} x_0$. The indifference $x_1^+ \sim G_E x_0$ implies that

$$U\left(x_1^+\right) = W^+(E)U(G). \quad (4)$$

The indifference $x_1^- \sim L_{E^c} x_0$ implies that

$$U\left(x_1^-\right) = W^-(E^c)U(L). \quad (5)$$

Combining Eqs. (3)- (5) gives

$$U\left(x_1^+\right) = -U\left(x_1^-\right). \quad (6)$$

Equation (6) defines the first elements x_1^+ and x_1^- of the standard sequences of gains and losses that we will elicit in the second and third stages.

For choice under risk, the elicitation of x_1^+ and x_1^- is similar except that the event E is replaced by a known probability p, and that the weights $W^+(E)$ and $W^-(E^c)$ are replaced by $w^+(p)$ and $w^-(1-p)$, respectively.

3.2 Second stage: Measurement of utility for gains

In the second stage, we elicit a standard sequence of gains. Let ℓ be a prespecified loss. We first elicit the loss $\mathcal{L} < \ell$ such that the decision maker is indifferent between the prospects $x_1^+{}_E\mathcal{L}$ and $\ell_{E^c} x_0$, where x_1^+ is the gain that was elicited in the first stage. We may select an event E' different from the event E used in the first stage, but, for notational convenience, we will continue using the symbol E for the selected event. In our experiment, we used the same event in all stages to simplify the tasks for the subjects. The indifference $x_1^+{}_E\mathcal{L} \sim \ell_{E^c} x_0$ implies that

$$W^+(E)U\left(x_1^+\right) + W^-(E^c)U(\mathcal{L}) = W^-(E^c)U(\ell). \quad (7)$$

Rearranging Eq. (7) and using $U(x_0)=0$ gives

$$U\left(x_1^+\right) - U(x_0) = \frac{W^-(E^c)}{W^+(E)}(U(\ell) - U(\mathcal{L})). \quad (8)$$

Next, we elicit the gain x_2^+ such that $x_2^+{}_E\mathcal{L}\sim x_1^+{}_E\ell$. From this indifference we obtain after rearranging

$$U\left(x_2^+\right) - U\left(x_1^+\right) = \frac{W^-(E^c)}{W^+(E)}(U(\ell) - U(\mathcal{L})). \tag{9}$$

Combining Eqs. (8) and (9) gives

$$U\left(x_2^+\right) - U\left(x_1^+\right) = U\left(x_1^+\right) - U(x_0). \tag{10}$$

We proceed by eliciting a series of indfferences $x_j^+{}_E\mathcal{L}\sim x_{j-1}^+{}_E\ell, j = 2, \ldots, k_G$, to obtain the sequence $\left\{x_0, x_1^+, x_2^+, \ldots, x_{k_G}^+\right\}$. It is easy to see that for all j, $U\left(x_j^+\right) - U\left(x_{j-1}^+\right) = U\left(x_1^+\right) - U(x_0)$. For decision under risk, we apply the above procedure with the event E replaced by a probability p (which can be different from the probability used in the first stage).

3.3 Third stage: Measurement of utility for losses

The standard sequence of losses is constructed similarly. We select a gain g and an event E and elicit the gain $\mathcal{G}$ such that $\mathcal{G}_E x_1^- \sim g_E x_0$.[6] We then proceed to elicit a standard sequence $\left\{x_0, x_1^-, x_2^-, \ldots, x_{k_L}^-\right\}$ by eliciting a series of indifferences $\mathcal{G}_E x_j^- \sim g_E x_{j-1}^-$, $j=2, \ldots, k_L$. For risk, we replace the event E by a probability p (which can be different from the probabilities used in the other two stages).

By combining the second and the third stages we have elicited a sequence $\left\{x_{k_L}^-, \ldots, x_1^-, x_0, x_1^+, \ldots, x_{k_G}^+\right\}$ that runs from the domain of losses through the reference point to the domain of gains and for which the utility difference between successive elements is constant. We can scale utility by selecting the utility of an arbitrary element. In the analyses reported below, we set $U\left(x_{k_G}^+\right) = 1$ from which it follows that $U\left(x_j^+\right) = j/k_G$ for $j=1, \ldots, k_G$, and $U\left(x_j^-\right) = -j/k_G$, for $j=1, \ldots, k_L$.

4 Experiment

We will next implement our method in an experiment. By exploring whether utility and loss aversion are the same for risk and ambiguity, we test prospect theory. As a second test of prospect theory, the experiment also contains a test of sign-comonotonic trade-off consistency, the central condition of prospect theory.

4.1 Experimental set-up

Subjects were 75 economics students of the Erasmus School of Economics, Rotterdam (29 female). Each subject was paid a flat fee of €10 for participation in the experiment.

[6] Again, we may select an event E'' different from the events employed in the other two stages.

Before conducting the actual experiment, the experimental protocol was tested in several pilot sessions.

The experiment was run on computers. Subjects answered the questions individually in sessions of at most two subjects. They first received instructions about the tasks and then completed five training questions. Subjects were told that there were no right or wrong answers and that they should go through the experiment at their own pace. They could approach the experimenter if they had any questions regarding the experiment. A session lasted 40 minutes on average.

The order in which utility under risk and ambiguity were measured was randomized between sessions. When a subject had completed the first part of the experiment, the experimenter would approach her to explain the next part. Within the risk and ambiguity elicitations, the order in which the gain sequence and the loss sequence were elicited was also randomized. The first stage, the elicitation of the amounts x_1^+ and x_1^-, always came first because it served as an input for the other stages.

We did not immediately ask subjects for their indifference values, but, instead, first used three binary choice questions to zoom in at them and only then asked subjects for their indifference value. Examples of this zooming-in procedure can be found in the Appendix. We applied a choice-based elicitation procedure as previous research suggests that it leads to more reliable results than directly asking for indifference values (Bostic et al. 1990).

4.2 Details

The method described in Section 3 requires the prior specification of some stimuli. The final column of Table 1 shows the stimuli we selected for the experiment. We made the common assumption that the reference point x_0 was equal to 0. In the risk condition, the outcome of a prospect was determined by drawing a ball from an urn containing five red balls and five black balls. Subjects could state which color they preferred to bet on with the chance of winning always equal to 50%. In the ambiguity condition, the outcome of a prospect was determined by drawing a ball from an urn containing ten red and black balls in unknown proportions. Again, subjects could select the color they preferred to bet on to avoid suspicion (Pulford 2009; Viscusi and Magat 1992).

For both gains and losses, we elicited six points of the utility function under risk and six points of the utility function under ambiguity. Next to these elicitations, we performed a second smaller sequence in the domain of gains where we used a different gauge amount ℓ. In the main elicitation we set $\ell = -€300$. In the second elicitation, where we only elicited x_2^+ and x_3^+, we set $\ell_{alt} = €\ 0$. Under binary PT the elicitations of x_2^+ and x_3^+ should not depend on the selected value of ℓ. This second elicitation tested sign-comonotonic trade-off consistency (Köbberling and Wakker 2003).[7]

Figures 1, 2, and 3 show the displays used under ambiguity. The screens under risk were similar, except that the two branches would simply say 50% rather than “Red” or “Black”. Figure 1 displays the typical decision that subjects had to make. Subjects faced a choice between two prospects denoted as alternatives A and B. They could not

[7] Köbberling and Wakker (2003) define sign-comonotonic trade-off consistency formally. In a nutshell, the condition holds because changing ℓ from -€300 into €0 does not change the rank-ordering and the sign (no loss is turned into a gain or vice versa) of each prospect’s payoffs. Then utility differences should not be affected according to prospect theory.

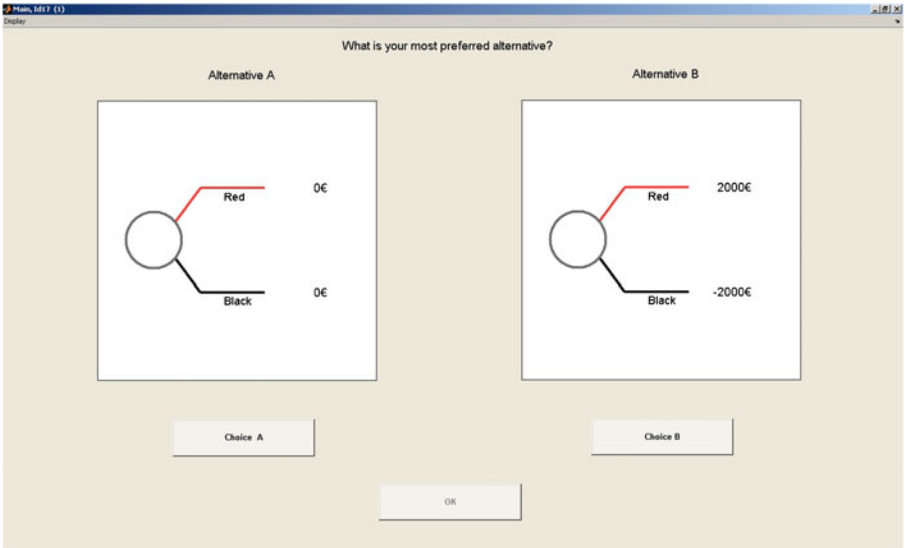

Fig. 1 Choice screen used in the ambiguity elicitations

state indifference. By choosing between the two prospects, the subject narrowed down the interval in which her indifference value should fall.

After narrowing down the interval thrice, we presented subjects with a scrollbar (Fig. 2). The scrollbar allowed subjects to specify their indifference value up to €1 precision. The range of the scrollbar was wider than the interval, so that subjects could correct any mistakes they might have made. The way in which subjects used the scrollbar also gives an indication of the quality of the data. If many subjects had provided answers that did not align with their previous choices, possibly even violating stochastic dominance, this would signal poor understanding of the task. After specifying a value with the scrollbar, subjects were asked to confirm their choice (Fig. 3). If they cancelled their choice, the process started over. If subjects confirmed their choice, they moved on to the next elicitation.

4.3 Analyses

4.3.1 Utility curvature

Two different methods were used to investigate utility curvature. In the first, nonparametric method, we calculated the area under the utility function. For both gains and losses, the domain of U was normalized to $[0, 1]$ by transforming every gain x_j^+ to the value x_j^+/x_6^+ and every loss x_j^- to x_j^-/x_6^-.[8] If utility is linear, the area under this normalized curve equals ½. For gains, we define utility to be convex [concave] if the

[8] Three subjects (two for risk and one for ambiguity) violated monotonicity so that x_6^- was not the largest loss. For these subjects we transformed losses x_j^- to $x_j^-/\{\min_{i=1,\ldots,6} x_i^-\}$.

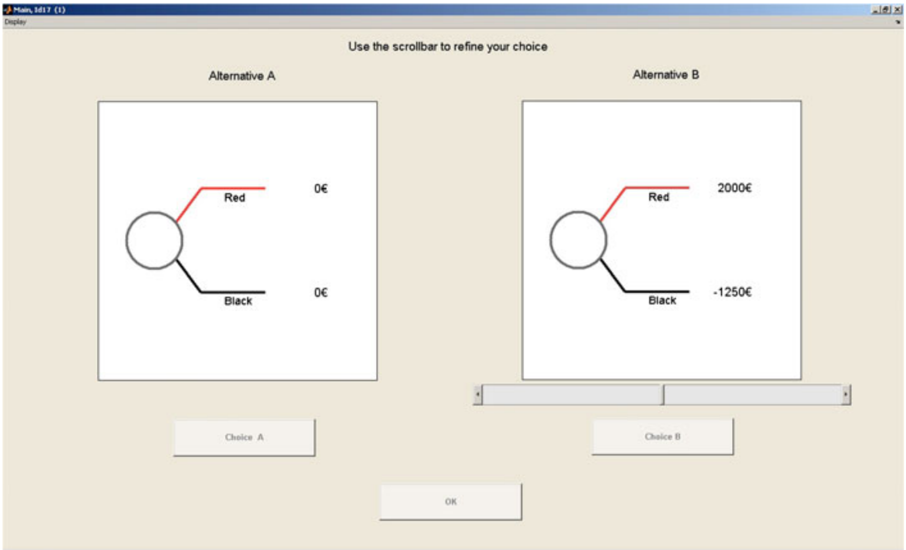

Fig. 2 Scrollbar screen used in the ambiguity elicitations

area under the curve is smaller [larger] than ½. For losses, utility is defined to be convex [concave] if the area under the curve is larger [smaller] than ½.

We also analyzed the utility function by parametric estimation. We employed the power/constant relative risk aversion (CRRA) family, x^{α}, the most commonly used parametric family. For gains [losses] $\alpha > 1$ corresponds to convex [concave] utility, $\alpha = 1$ corresponds to linear utility, and $\alpha < 1$ corresponds to concave [convex] utility. Estimation was by nonlinear least squares. We also performed a mixed-effects estimation in which each individual parameter was estimated as the sum of a fixed effect,

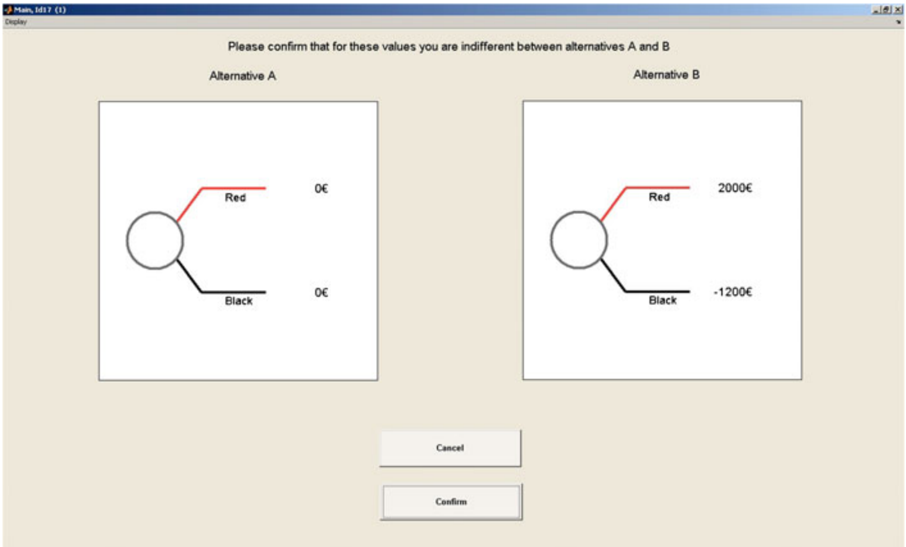

Fig. 3 Confirmation screen used in the ambiguity elicitations

common to all subjects, and an individual-specific random effect. The mixed-effects estimation led to the same conclusions and will therefore not be reported.

A potential problem in estimating a model like binary PT is collinearity between utility and the event weights. The trade-off method avoids this problem. By keeping event weighting fixed during the elicitation of utility, the event weights drop from the equations and utility can be measured independent of event weighting. Hence, collinearity is completely excluded. This is an additional advantage of our method.

4.3.2 Loss aversion

There exist several definitions of loss aversion. Abdellaoui et al. (2007) concluded that the definitions of Kahneman and Tversky (1979) and Köbberling and Wakker (2005) are empirically most useful and we will use these. Other definitions (Wakker and Tversky 1993; Bowman et al. 1999; Neilson 2002) turned out to be too strict for empirical purposes, leaving many subjects unclassified.

Kahneman and Tversky (1979) defined loss aversion as $-U(-x) > U(x)$ for all $x > 0$. To measure loss aversion coefficients, we computed $-U(-x_j^+)/U(x_j^+)$ and $-U(x_j^-)/U(-x_j^-)$ for $j = 1,...,6$, whenever possible.[9] Usually $U(-x_j^+)$ and $U(-x_j^-)$ could not be observed directly and had to be determined through linear interpolation. Some subjects occasionally violated stochastic dominance. In that case, it is impossible to estimate utility and we treated utility as missing for the amounts for which this happened. A subject was classified as loss averse if $-U(-x)/U(x) > 1$ for all observations, as loss neutral if $-U(-x)/U(x) = 1$ for all observations, and as gain seeking if $-U(-x)/U(x) < 1$ for all observations. To account for response error, we also used a more lenient rule, classifying subjects as loss averse, loss neutral, or gain seeking if the above inequalities held for more than half of the observations.

Köbberling and Wakker (2005) defined loss aversion as the kink of utility at the reference point. Formally, they defined loss aversion as $U'_\uparrow(0)/U'_\downarrow(0)$, where $U'_\uparrow(0)$ represents the left derivative and $U'_\downarrow(0)$ the right derivative of U at the reference point. To operationalize this definition, we computed each subject's coefficient of loss aversion as the ratio of $U(x_1^-)/x_1^-$ over $U(x_1^+)/x_1^+$, because x_1^- and x_1^+ are the loss and gain closest to the reference point. Given that $U(x_1^-) = -U(x_1^+)$, this ratio is equal to $x_1^+/-x_1^-$. Hence, the first stage of our method immediately gives an estimate of Köbberling and Wakker's (2005) loss aversion coefficient without the need to further measure utility. A subject was classified as loss averse if $x_1^+/-x_1^- > 1$, as loss neutral if $x_1^+/-x_1^- = 1$, and as gain seeking if $x_1^+/-x_1^- < 1$.

5 Results

For one subject the program crashed and we lost his data. Three subjects violated stochastic dominance in critical, early steps of the measurement procedure. Violations of stochastic dominance at these early measurements undermine subsequent answers

[9] These computations required that $-x_j^+$ was contained in $[x_6^-, 0)$ and $-x_j^-$ in $(0, x_6^+]$.

and subjects committing them were removed from the analyses. For the remaining 71 subjects, we could determine the entire utility function, for both risk and ambiguity.

5.1 Consistency checks

We included a number of repetitions to test for consistency. First, in each of the six standard sequences (the short and the long gain sequences and the loss sequence for both risk and ambiguity), we repeated the final iteration in the elicitation of $x_2^i, i = +, -$. Subjects made the same choice in 63.6% of the repeated choices. Reversal rates around one third are common in the literature (Stott 2006). Moreover, our consistency test was strict as we repeated the final choice of the iteration process and subjects were close to indifference in this choice. There were no differences in consistency between risk and ambiguity.

Furthermore, at the end of eliciting the long gain sequence, we elicited x_4^+ again, both for risk and for ambiguity. The correlation between the original measurement and the repeated measurement of x_4^+ was almost perfect.[10] For risk, Kendall's τ was 0.92, for ambiguity it was 0.94.

As a final indication of consistency, we compared whether the final answer provided by using the scrollbar fell within the interval as set up by the bisection procedure. Subjects provided answers that aligned with their original choices. Furthermore, when a subject's final answer was outside the bisection interval, it typically only violated the final choice, probably indicating that they were close to indifference at this point.

5.2 Sign-comonotonic trade-off consistency

As explained in Section 4, we elicited two sequences of gains, a longer one based on $\ell = -€300$, which we used in the main analysis, and a shorter one based on $\ell_{alt} = €0$. If our subjects behaved according to binary PT and satisfied sign-comonotonic trade-off consistency, then the values of x_2^+ and x_3^+ in the short sequence should be equal to those obtained in the long sequence.

We could not reject binary PT, for both risk and ambiguity. The correlation between the obtained values was substantial. For risk, Kendall's τ was 0.57 for x_2^+ and 0.51 for x_3^+. For ambiguity, these values were 0.70 for x_2^+ and 0.64 for x_3^+. All correlation coefficients differed from 0 ($p < 0.001$). Moreover, for ambiguity, we could not reject the null hypotheses that the values of x_2^+ and x_3^+ obtained in the short sequence were equal to those obtained in the long sequence (Wilcoxon test, both $p > 0.72$). For risk, the values of x_2^+ differed marginally ($p = 0.08$), but the values of x_3^+ did not differ ($p = 0.19$). Hence, even though x_3^+ was chained to x_2^+, the marginal difference for x_2^+ did not carry over to x_3^+.

5.3 The utility for gains and losses

Figure 4 shows the utility for gains and losses under risk (Panel A) and ambiguity (Panel B) based on the median data. At first sight, the utility functions were close. They

[10] We use the (standard) nomenclature of Landis and Koch (1977) to describe the strength of associations.

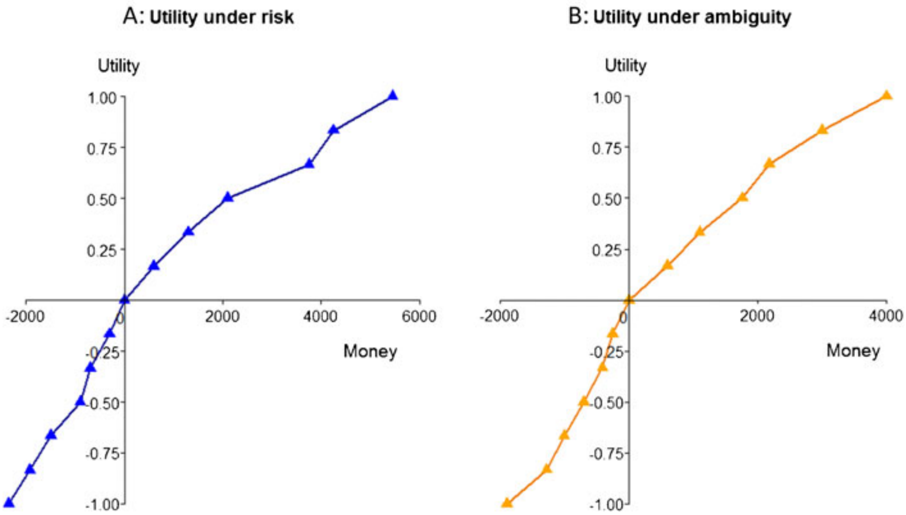

Fig. 4 The utility for gains and losses based on the median data

are consistent with the typical finding of convex utility for losses and concave utility for gains. Furthermore, the utility functions were steeper for losses than for gains, indicating loss aversion.

To investigate these patterns more thoroughly, we move to the individual level analysis. Table 2 shows that the classification of subjects according to the shape of their utility function was very similar for risk and ambiguity and we could not reject the null hypothesis that the overall distribution of classifications between the two conditions was the same (Fisher's exact test, $p = 0.97$). The common pattern was S-shaped utility: concave for gains and convex for losses. Less than 20% of the subjects behaved

Table 2 Classification of subjects according to the shape of their utility function. The table classifies the subjects according to the shape of their utility function based on the area under the normalized utility function. Panel A displays the results under risk. Panel B displays the results under ambiguity

	Losses			
Gains	Concave	Convex	Linear	Total
Panel A: Risk				
Concave	13	30	1	44
Convex	15	8	1	24
Linear	2	0	1	3
Total	30	38	3	71
Panel B: Ambiguity				
Concave	13	30	0	43
Convex	18	9	0	27
Linear	1	0	0	1
Total	32	39	0	71

according to the traditional assumption in decision theory that utility is concave throughout.

The parametric results confirmed the above conclusions. Table 3 shows the medians of the estimated individual CRRA functions. Utility was mostly concave for gains and convex for losses. Under both risk and ambiguity, 31 subjects (44%) had S-shaped utility.

For losses, we could not reject the null hypothesis that utility curvature was the same for risk and ambiguity, neither for the area measure (Wilcoxon test, $p = 0.31$), nor for the CRRA coefficients ($p = 0.94$). However, utility for gains was more concave under risk for both measures (both $p = 0.04$). The utilities under risk and under ambiguity were moderately correlated: Kendall's τ was 0.41 for gains and 0.46 for losses for the area measure, and 0.41 for gains and 0.42 for losses for the CRRA coefficients.

5.4 Loss aversion

Figure 5 displays the relations between the medians of x_j^+ and $-x_j^-$ under risk and under ambiguity. An advantage of our method is that it immediately reveals that there is loss aversion in the sense of Kahneman and Tversky (1979) when $x_j^+ > -x_j^-$.[11] As Fig. 5 clearly shows, this held true for all j, under both risk and ambiguity. We obtain an aggregate measure of loss aversion by regressing the x_j^+ on $\left(-x_j^-\right)$.The βs in Fig. 5 display the coefficients from this regression. Both βs (for risk and ambiguity) exceeded one (t-test, $p < 0.01$) and the values were close to those observed previously for risk (Fox and Poldrack 2014). We could not reject the hypothesis that the values of β were the same for risk and ambiguity (z-test, $p = 0.32$).

Moving to the individual level, we found that $x_j^+ > -x_j^-$ for all j (Wilcoxon test, all $p < 0.01$), which is consistent with the existence of loss aversion à la Kahneman and Tversky (1979). Furthermore, $x_j^+/-x_j^-$ did not differ between risk and ambiguity for any j (Wilcoxon test, all $p > 0.25$), which is consistent with the hypothesis of prospect theory that loss aversion is the same under risk and under ambiguity.

Table 4 shows the results of the individual analyses of loss aversion based on Kahneman and Tversky's (1979) and Köbberling and Wakker's (2005) definitions. The table clearly shows evidence of loss aversion, irrespective of the definition used and regardless of whether we took response errors into account. According to both definitions, the median loss aversion coefficients for risk and ambiguity did not differ (Wilcoxon test, both $p > 0.26$) and they were moderately correlated (both Kendall's $\tau > 0.37$, $p < 0.001$.

The two measures of loss aversion were substantially correlated. Kendall's τ was 0.78 for risk and 0.82 for ambiguity (all $p < 0.001$). It is comforting to observe that these two distinct measures—one of a local nature and relying on a single kink in the slope of the utility function, and the other global and relying on different absolute

[11] For a given j, x_j^+ and x_j^- have the same absolute value of utility by construction, $U\left(x_j^+\right) = -U\left(x_j^-\right)$, and, thus, $x_j^+ > -x_j^-$ implies that $U\left(x_j^+\right) < -U\left(-x_j^+\right)$, consistent with Kahneman and Tversky's definition of loss aversion ($U(x) < -U(-x)$ for all $x > 0$).

Table 3 Summary of individual parametric fittings of utility. The table depicts the results of fitting CRRA functions on each subject's choices individually. Shown are the median and interquartile range (IQR) for the resulting estimates

	Risk		Ambiguity	
	Gains	Losses	Gains	Losses
Median	0.87	0.93	0.94	0.91
IQR	[0.62–1.07]	[0.63–1.16]	[0.72–1.17]	[0.68–1.36]

utilities associated with the same absolute money amounts in the positive and negative domain—showed a high degree of consistency in classifying subjects.

6 Discussion

Our data is consistent with prospect theory. Both utility and loss aversion were close for risk and ambiguity, as assumed by prospect theory. The results also supported sign-comonotonic trade-off consistency, the central condition of prospect theory. Finally, utility was S-shaped, concave for gains and convex for losses and there was substantial loss aversion.

An easy response strategy in measurements using the trade-off method is to let the outcomes of the standard sequence increase by the difference between the gauge outcomes (L and ℓ in the sequence of gains G and g in the sequence of losses). This would bias the results in the direction of linear utility. We checked for this heuristic by counting the number of subjects for whom the outcomes of the standard sequence

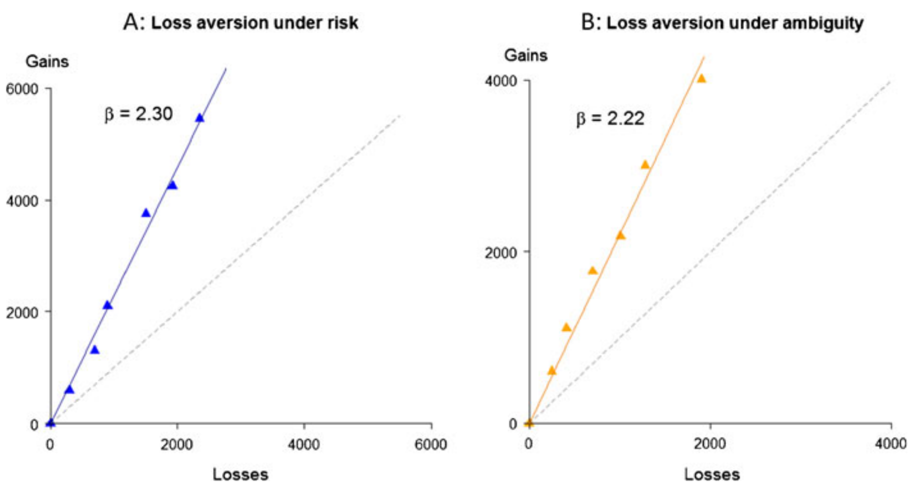

Fig. 5 The relation between median gains and median losses with the same absolute utility. Panel A displays the relation between median gains and losses under risk. Panel B displays this relation under ambiguity. The dashed line corresponds to the case where gains and losses of the same absolute utility would be equal. The straight line with slope β corresponds to the best fitting linear equation

Table 4 Results under the two definitions of loss aversion. The table depicts the results under the two definitions of loss aversion for both risk and ambiguity. The table displays how the coefficients are defined, their medians and interquartile ranges, and the number of loss averse, gain seeking, and loss neutral subjects. The numbers in parentheses for Kahneman and Tversky's definition correspond to the case where response errors are not taken into account

Definition	Coefficient	Condition	Median [IQR]	Loss averse	Gain seeking	Loss neutral
Kahneman and Tversky (1979)	$\frac{-U(-x)}{U(x)}$	Risk	2.21 [1.06–5.52]	58(46)	10(6)	1(1)
		Ambiguity	2.30 [1.12–7.29]	53(49)	16(10)	0(0)
Köbberling and Wakker (2005)	$\frac{x_1^+}{-x_1^-}$	Risk	1.88 [1.06–4.50]	56	12	3
		Ambiguity	2.00 [1.21–6.50]	56	14	1

(approximately) increased by the difference between the gauge outcomes but found little evidence to support it.

We used large payoffs because we were interested in studying both utility curvature and loss aversion. Utility curvature is typically modest over small intervals (Luce 2000; Wakker and Deneffe 1996) and we were concerned that it would be hard to detect differences between utility under risk and ambiguity for small stakes. Because we used large losses, all choices were hypothetical. It is impossible to find subjects willing to participate in an experiment where they can lose substantial amounts of money. Because all but one of the questions involved losses, we could not play out one of the gain questions for real either, as subjects would know immediately which question would be played out for real. The literature on the importance of real incentives is mixed. Most studies found that for small to modest stakes there is little or no effect of using real instead of hypothetical choices for the kind of tasks that we asked our subjects to perform, except that hypothetical responses tend to be noisier (Bardsley et al. 2010).

Our method is chained (adaptive) in the sense that previous responses are used in the elicitation of subsequent choices. Chaining may lead to error propagation, where errors made in one particular choice affect later choices. We checked for the impact of error propagation using the simulation methods developed by Bleichrodt and Pinto (2000) and Abdellaoui et al. (2005). In both simulations, we confirmed the conclusions from those studies that the impact of error propagation on measurements using the trade-off method was negligible.[12] We also repeated the parametric analysis of utility accounting for serial correlation in the error terms.[13] The estimates were similar to the ones reported in Section 5. Hence, we conclude that the chained nature of our measurements did not affect the results.

[12] Bleichrodt et al. (2010) also concluded that error propagation was negligible in their measurements using the trade-off method.

[13] We assumed that the error terms followed an AR(1) process $\epsilon_t + \rho\epsilon_{t-1} = u_t$ with u_t normally distributed with expectation 0 and variance σ^2 and estimated this using generalized least squares.

7 Conclusion

In many real-world problems probabilities are unknown. To apply prospect theory to such decision situations requires methods to measure its parameters. This paper shows how utility and loss aversion can be measured in decision under ambiguity. Our method, for the first time, makes prospect theory completely observable. By combining our measurements with the method of Abdellaoui et al. (2005), all prospect theory's parameters can be measured without imposing simplifying assumptions. Our paper completes a program to make prospect theory empirically observable. Our method allows new tests of prospect theory's assumptions and an experimental implementation showed support for two of these assumptions: that sign-comonotonic trade-off consistency holds and that both utility and loss aversion are the same for risk and ambiguity. We hope that by providing a simple way to measure prospect theory our method will foster applications.

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Appendix

Table 5 Three illustrations of the bisection method under risk

	Offered choices in elicitation L	Offered choices in elicitation x_1^+	Offered choices in elicitation x_2^-
1	**0** vs. (2000, 0.5; −2000)	(2000, 0.5;0) vs. **1000**	**(300, 0.5;−200)** vs. (800, 0.5;−700)
2	0 vs. **(2000, 0.5; −1000)**	**(2000, 0.5;0)** vs. 500	**(300, 0.5;−200)** vs. (800, 0.5;−450)
3	**0** vs. (2000, 0.5; −1500)	(2000, 0.5;0) vs. **750**	(300, 0.5;−200) vs. **(800, 0.5;−325)**
Slider	Start value: −1250 Interval: [−2000,−500]	Start value: 625 Interval: [250, 1000]	Start value: −388 Interval: [−576,−200]

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