



## Capabilities as menus: A non-welfarist basis for QALY evaluation

Han Bleichrodt<sup>a,\*</sup>, John Quiggin<sup>b</sup>

<sup>a</sup> Erasmus School of Economics & iMTA/iBMG, Erasmus University Rotterdam, The Netherlands

<sup>b</sup> School of Economics, The University of Queensland, Brisbane, Australia

### ARTICLE INFO

#### Article history:

Received 19 October 2011

Received in revised form 21 May 2012

Accepted 10 October 2012

Available online 30 October 2012

#### JEL classification:

D63

D70

I14

#### Keywords:

Capability approach

QALYs

Menu-dependent choice

### ABSTRACT

Quality-Adjusted Life Years (QALYs) are the most widely used measure of health in economic evaluations of health care. Within a welfarist framework QALYs are consistent with people's preferences under stringent assumptions. Several authors have argued that QALYs are a valid measure of health within an extra-welfarist framework. This paper studies the applicability of QALYs within the best-known extra-welfarist framework, Sen's capability approach. We propose a procedure to value capability sets and provide a foundation for QALYs within Sen's capability approach. We show that, under appropriate conditions, the ranking of capabilities can be represented locally by a QALY measure and that a willingness to pay for QALYs can be defined. The validity of QALYs as a general measure of health requires the same stringent conditions as in a welfarist framework.

© 2012 Elsevier B.V. All rights reserved.

### 1. Introduction

The provision and funding of health care are among the most important responsibilities of governments in modern societies. In the absence of effective markets for most health care services, it is necessary to make decisions regarding the amount that should be spent on health care services and the allocation of those services between individuals and across different kinds of services.

Over recent decades, attempts have been made in many countries to improve the allocation of health care resources using economic evaluations of health care. The central tool in this approach has been the concept of Quality Adjusted Life Years (QALYs) (Gold et al., 1996). QALYs have intuitive appeal and provide practical guidance in making the difficult trade-offs that are inevitable in any system of health care provision where a government or insurer must decide which services to fund. However, attempts to provide a secure foundation for the QALY approach in terms of economic theories of welfare, which we review in the next subsection, have proved problematic. The conditions under which QALYs represent preferences are stringent and unlikely to hold in full generality (Bleichrodt and Pinto, 2012).

A different justification for using QALYs in health policy is based on a questioning of the notion of 'welfarism' (Culyer, 1989; Hurley, 2000; Brouwer et al., 2008). In this context, 'welfarism' may be summed up as the idea that policy should aim at maximizing a social welfare function, the only arguments of which are the lifetime utility profiles of the individual members of society. According to the extra-welfarist approach advocated by Culyer and others, the evaluation of policy decisions should not be exclusively based on individual utilities, but should also take account of other factors, such as concepts of equity.

The best developed alternative to welfarism has been based on the concept of capabilities, put forward by Sen (1985) and further developed by Nussbaum (2000) and Robeyns (2006) among many others. Although the details of the formulation vary, the crucial idea is to distinguish between capabilities, represented as a person's opportunities to achieve well-being, and achieved functionings, the actual outcomes realized by individuals given their capabilities, preferences, and social situation.

While the capability approach has been much discussed, there has been less progress in its formal theoretical development and empirical application (Schokkaert, 2009). Kuklys and Robeyns (2005) and Fleurbaey (2005) presented formal models, but subsequent development of these models has been limited. Anand (2005) and Cookson (2005) argue that the capability approach should be applied in health economics and could be integrated with a QALY-based analysis. Coast et al. (2008) give a qualified endorsement

\* Corresponding author.

E-mail address: [bleichrodt@ese.eur.nl](mailto:bleichrodt@ese.eur.nl) (H. Bleichrodt).

to this view, but note that the capability approach will require adaptation to the health context.

The aim of this paper is to explore the extent to which a QALY-based evaluation of health services can be used within the capability approach. That is, we study the question of whether QALYs can be justified under a specific extra-welfarist framework. To do so, we must address several open questions in the capability approach, the most important one being how capabilities can be valued numerically (Schokkaert, 2009; Fleurbaey, 2009).

The central idea in our formalization of the capability approach, following suggestions of Sen (1991) and Sugden (1993) is to model capabilities as menus, i.e. as sets of functionings, in the analytical framework developed by Kreps (1979).<sup>1</sup>

In Kreps' framework, an agent takes decisions in two stages. In the first stage, he chooses a menu, a capability set of possible functionings. In the second stage, he chooses a functioning from this menu. This modeling strategy permits us to capture an essential part of Sen's capability theory, namely, that people have the freedom to choose the kind of lives they want to lead. Freedom of choice is intrinsically valuable in Sen's theory and our model allows it to be valued as such.

The key contribution of Kreps was to show that, under very weak conditions, any ranking over menus may be derived from the maximization of a state-contingent utility function over final outcomes. The state-contingency may reflect an agent's uncertainty about his future preferences or about the future availability of outcomes. Using this idea, we derive an evaluation rule for capabilities in terms of a state-contingent utility function over functionings. This rule can be interpreted as the evaluation rule for a society in which people's preferences are not known in advance and where freedom of choice is valued.

We next consider the relationship between capabilities and QALYs. We derive conditions under which QALYs can be used as a local approximation to a ranking over capabilities. That is, we derive when the use of QALYs is justified in the capability approach. Our central result shows that, under appropriate conditions, any ranking of capabilities gives rise to a 'shadow price' for QALYs, which is locally consistent with the given ranking and which can be interpreted as a local approximation of the willingness to pay for a QALY. However, the global validity of QALYs requires stringent conditions, similar to those discussed in the welfarist framework by Bleichrodt and Quiggin (1999).

## 2. The limitations of QALYs under welfarism

Before considering the interpretation of health in terms of capabilities, we briefly review the standard QALY approach. The central idea of the QALY approach is to value improvements in health in terms of the additional years of life in full health that would be regarded as being equally beneficial and to add these improvements using unweighted summation. Cost-effectiveness then requires that, given two equally costly services, the one that gives the larger improvement in QALYs should be provided first. Given a consistent application of cost-effectiveness analysis to health services with a given budget, there will exist a shadow QALY price  $p^*$  (typical values are of the order of €30,000) such that all and only those services for which the cost per QALY gained is less than  $p^*$  will be approved.

The QALY approach is popular and widely used in practical cost-effectiveness studies. However, attempts to found QALYs in

economic welfare theory have met with only limited success. Early contributions that sought to provide an economic foundation for QALYs involved reasoning based on the expected utility (EU) theory of choice under uncertainty. Axioms on preferences over lotteries, with health profiles as payoffs, were used to derive QALY-maximization as a social objective (Pliskin et al., 1980; Bleichrodt et al., 1997; Miyamoto et al., 1998). This approach encountered a number of difficulties.

First, it became evident that individuals do not, in general, satisfy the assumptions of EU in making choices under uncertainty (Starmer, 2000). Evidence of violations of EU for health can be found, among others in Llewellyn-Thomas et al. (1982), Oliver (2003), and Bleichrodt et al. (2007). The observed violations of EU entailed a reformulation of the axiomatic basis of the QALY model to avoid reliance on the EU assumptions (Bleichrodt and Quiggin, 1997; Miyamoto, 1999; Bleichrodt and Miyamoto, 2003). Empirical tests of these reformulations have led to mixed results, but the general finding is that the basic QALY model, in which life-years are multiplied by a quality weight, is too restrictive (Bleichrodt and Pinto, 2005; Bleichrodt and Filko, 2008).

Second, there was no obvious justification for extending individual preferences for QALY-maximization to a social objective of maximizing the expected QALY benefit derived from health services. Several authors have argued and presented empirical evidence in favor of equity-weighting QALYs (for an overview see Tsuchiya, 2012) and various alternatives for 'QALY-utilitarianism' were put forward, notably including Williams' (1997) fair innings principle. Axiomatic foundations for these alternative models have been provided in Bleichrodt et al. (2004).

Third, the standard welfare-theoretic justifications for QALY-maximization considered individual welfare solely in terms of health status, without taking account of other aspects of welfare such as the consumption of market goods and services. This approach appeared adequate in terms of cost-effectiveness analysis, but proved problematic when broader considerations were taken into account. Bleichrodt and Quiggin (1999) provided necessary and sufficient conditions for QALY maximization to be consistent with the maximization of an individual lifetime welfare objective. As subsequent discussion made clear, these conditions are stringent (Klose, 2002; Bleichrodt and Quiggin, 2002).

## 3. The capability approach

The limitations of QALYs under welfarism have led to justifications of QALYs under an extra-welfarist framework. In this paper we focus on the most well known of these extra-welfarist frameworks, Sen's capability approach. As emphasized by Robeyns (2006) and Schokkaert (2009), the capability approach (Sen, 1985, 1992) is essentially a mode of thinking about normative issues, which specifies an evaluation space. The core characteristic of the capability approach is its focus on what people are able to do and be. Together these doings and beings, which Sen calls *functionings*, constitute a life and make a life valuable. Examples of functionings are being adequately nourished, avoiding premature mortality, and being happy.

A key distinction in the capability approach is between means and ends. Only ends are important and means are instrumental in reaching the ends. This illustrates that functionings are not equal to commodities. Commodities are objects which a person might use to achieve a valuable life, whereas functionings are an aspect of living itself. The distinction between means and ends is not always obvious, however. Good health, for instance, is a means to be able to work or be happy, but it is also an end in itself. Functionings are not equal to utility in the classical economic sense of metrics

<sup>1</sup> A similar approach may be found in the literature on valuing freedom deriving largely from the work of Pattanaik and Xu (1990, 2000) and further developed by Puppe (1996), Nehring and Puppe (1999), and Xu (2003).

of happiness. For Sen, being happy is just one of many elements of being that may be relevant to an overall evaluation of well-being.

The other central concept in the capability approach is the distinction between functionings and capabilities, that is, between the realized and the effectively possible. In choosing what kind of life to live an individual chooses between functionings. The set of available functionings is the person's *capability set*. The capability set represents the person's opportunities to achieve well-being or, alternatively stated, his freedom of choice. This freedom is central in Sen's theory and of intrinsic value. The focus on the importance of freedom illustrates that the capability approach belongs to the liberal school of thought in political philosophy.

#### 4. Two open questions

##### 4.1. Capabilities or functionings

Two important questions pervade the application of the capability approach (Fleurbaey, 2009). The first concerns the normative issue of whether the evaluation of an individual's situation should be based on capabilities alone or on both capabilities and functionings. This concern is related to the question whether the appropriate metric for interpersonal comparisons corresponds to achievements or to opportunities. Normative economics is increasingly interested in opportunity-based theories in which value is attached to the size and richness of an individual's opportunity set (Sen, 1992; Arrow, 1995; Roemer, 1998; Sugden, 2004). According to these theories, public policy and theories of justice should be concerned with maximizing individuals' opportunity sets and should not be concerned with preference satisfaction. A normative reason to focus on opportunity sets instead of preference satisfaction is that individuals must take responsibility for how they use their opportunities. A more pragmatic reason lies in the wealth of behavioral data showing that preferences are unstable and inconsistent and, hence, an unreliable guide for public policy.

There are several counterarguments against an exclusive focus on capabilities. First, functionings are "constitutive of a person's being" (Sen, 1992, p. 39) and, consequently, they should be valued. Second, theories of responsibility are harsh on the losers. Should we punish individuals for the sins of their youth? And if so, how long are individuals deemed to be responsible for past actions? And are there limits to this responsibility (Schokkaert, 2009)? The evaluation model we will propose in the current paper takes account both of the richness of the capability set, and of the quality of the functionings in the set. Hence, it values both the individual's freedom and the achieved functionings and belongs to the category of theories about what Sen terms "refined functionings".

##### 4.2. Valuing capability sets

The second open question is how to value capability sets? If we want to use the capability approach to derive overall conclusions about the welfare impact of specific (health care) programs, it is necessary to value capability sets. Unfortunately, the capabilities literature has very little to say on how this can be done. Sen (1985) suggests declaring one capability set better than another if all individuals involved agree on this, but this suggestion is not very helpful in public policy. To carry out a meaningful economic evaluation requires a complete ranking of capability sets. However, Sen's proposal only leads to a partial ranking of capability sets. Moreover, it has paradoxical consequences in the sense that it precludes that several plausible conditions on social choice jointly hold (Brun and Tungodden, 2004). Gaertner and Xu (2006, 2008) suggest ranking capability sets in terms of a standard of living, the development of

which over time is uncertain. Their ranking is in terms of a class of distance functions, implying that the resulting ranking of capability sets is, once again, only partial.

A subtle issue in the valuation of capability sets is what the role of individual preferences should be. The capability approach was proposed as an alternative to welfarism with its exclusive focus on individual preferences. According to Sen (1985), some functionings are intrinsically valuable and should not depend on people's preferences. In Sen (1985) he declared his belief that a purely subjectivist view of well-being is "ultimately rejectable" and that "the limits of objectivity extend well into the assessment of well-being". Later contributions somewhat qualified Sen's position and there is now wide agreement that individual preferences have a role to play in valuing capabilities and that it is possible to respect individual preferences while avoiding a return to welfarism (Fleurbaey, 2009; Schokkaert, 2009).

Once we allow individual preferences to play a role in valuing capability sets, other problems emerge. Most importantly, do we introduce current preferences or take account of future preferences about which the individual is possibly uncertain? Several authors have argued that that to makes sense of opportunities requires the consideration of potential future preferences (Jones and Sugden, 1982; Arrow, 1995). On the other hand, Sen (1991) and Puppe (1995) argue that preference for freedom and uncertainty about future preferences are not necessarily linked.

As Schokkaert (2009, p. 549) concludes: "The problem of the evaluation of opportunity sets remains open". The purpose of this paper is to suggest a solution to this problem that takes account of the above issues. Our proposed solution allows a role for individual preferences while avoiding welfarism, values freedom, and takes account of the uncertainty about future preferences.

#### 5. The model

##### 5.1. Functionings

Let us now formalize the above discussion. In Sen's (1985, 1992) capability approach there are two key elements: functionings and capability sets. A *functioning*  $f$  is a vector  $(f_1, \dots, f_n)$  summarizing the activities,  $j = 1, \dots, n$ , undertaken by an individual.<sup>2</sup> Activities  $f_j$  include consumption levels for market goods and services, work undertaken in the labor market, and measures of non-market activities such as going for walks, participation in family and social life, and so on. Aspects of health quality may be part of the functioning vector if they contribute directly to welfare, but this need not be the case.

Let  $\mathcal{F}_j$  denote the set of possible values of functioning activity  $j$ , and let  $\mathcal{F} = \prod_{j=1}^n \mathcal{F}_j$  denote the set of functionings, which is the Cartesian product of the  $n$  different sets  $\mathcal{F}_j$ . We will assume that each of the  $\mathcal{F}_j$  is compact. An example is the case where each  $\mathcal{F}_j$  is an interval which can be represented without loss of generality in the form  $[0, M_j]$ . Functionings are denoted as  $f, g, h$ .

##### 5.2. Capabilities

The capability set is the set of *achievable* functionings. This set can be written as

$$C = \mathcal{X}(c(e)). \quad (1)$$

In Eq. (1),  $e$  is an individual's *initial endowment*. For simplicity and to clarify the main ideas, we will only focus on the case where

<sup>2</sup> What we call 'functioning' is also sometimes called 'state of being'.

the initial endowment consists of health and wealth. Extensions to richer initial endowments are straightforward. The initial wealth endowment can be spent on goods and services, one of which is health care. These goods and services are converted through the function  $c(\cdot)$  into a vector of objective characteristics in the Gorman (1959) and Lancaster (1966) tradition. Finally, the objective characteristics are converted through the *technology relation*  $\mathcal{X}$  into the set of achievable functionings. Eq. (1) captures the notion that goods and services do not carry value by themselves but are a means to the end of producing functionings through the technology relation  $\mathcal{X}$ . We will denote generic capability sets by  $x, y, z$ .

### 5.3. The decision problem

We analyze the position of an agent choosing an allocation of resources between health care and general resources, and of health care resources between different services. The agent can either be a health policy maker or a representative individual. The agent is taken to have an endowment of initial health status and resources, but not to have specified preferences over achievable functionings (see Section 5.4 for more on this). Thus, the individual is not a ‘representative agent’ in the sense used in economic modeling. Rather her position is closer to that of a decision maker seeking ‘reflective equilibrium’ behind a Rawlsian veil of ignorance.

Eq. (1) assumes that the initial endowment and the technology relation  $\mathcal{X}$  are the same for all individuals. In particular, initial health status is given and can be improved by the use that is made of health care and other resources. This improvement is the same for all individuals because the technology relation is the same for all individuals. One way to extend the above model would be to allow for heterogeneity in initial health status and to make the initial endowment and the technology relation  $\mathcal{X}$  individual-specific. This extension is beyond the scope of the present paper, but we will consider some implications in the subsequent discussion.

We will assume that the initial wealth endowment  $w_e$  can be allocated to health care expenditures and other expenditures to generate functionings  $f = f(q, w)$ , where  $q$  denotes health,  $w$  denotes wealth minus health expenditures, and we replace the composite function  $\mathcal{X} \circ c$  by  $f$  for notational convenience. We will define the health cost function  $h(q)$  as the expenditure  $h$  required to produce health status  $q$ . It is immaterial for our analysis whether health expenditure stands for the expenditure on one composite good health care or for a vector of expenditures on different treatments.

A functioning  $f = f(q, w)$  is *feasible* if and only if

$$h(q) + w \leq w_e. \quad (2)$$

The primary question for health policy is to determine the socially optimal choice of  $q$  given the initial endowments and the technology as described. This choice may be broken into questions of cost-effectiveness (for given health expenditures  $h$ , choose  $q$  such that  $q$  is maximized), and budget allocation (assuming cost-effective choices of  $q$ , determine  $h$ ).

The central claim of the capability approach is that the best way of approaching these questions is to consider the capability set  $\mathcal{C}$  arising from particular choices of  $h$  and  $q$ . The elements of the capability sets are feasible functionings and a typical capability set is  $\{f^1, f^2, \dots, f^m\}$ . The capability sets represent the individual's opportunities for achieving well-being. The larger the capability set the larger is the number of feasible functionings available to the individual and the larger are the individual's possibilities for achieving well-being. In other words, the capability set represents the individual's freedom of choice.

### 5.4. Evaluating capability sets

To make the capability approach operational we should find a way to evaluate capability sets. The approach we take is based on Kreps' (1979) model of preference for flexibility, which was later extended by Dekel et al. (2001) and Kopylov (2009). We consider an ordering  $\succsim$  defined over capability sets  $x, y, z$ . The ordering can either belong to a social policy maker or to a representative agent. The ordering  $\succsim$  is assumed to be a *weak order*: *transitive* (for all  $x, y, z$ , if  $x \succsim y$  and  $y \succsim z$  then  $x \succsim z$ ) and *complete* (for all  $x, y$ , either  $x \succsim y$  or  $y \succsim x$ ). Strict order  $\succ$  and indifference  $\sim$  are defined as usual. We assume that the ordering over capability sets is *nontrivial*, i.e. there exist capability sets  $x$  and  $y$  such that  $x \succ y$ . In other words, not all capability sets are considered equivalent. A function  $v$  represents the ordering  $\succsim$  if for all  $x, y \in \mathcal{C}$ ,  $x \succsim y \Leftrightarrow v(x) \geq v(y)$ .

Under the standard economic model, the ordering  $\succsim$  over capability sets  $x, y$  is induced from a preference relation  $\succsim$  over functionings. More precisely, define  $x \succsim y$  if and only if for all  $g \in y$  there exists  $f \in x$  such that  $f \succsim g$ . That is, capability set  $x$  is ranked above capability set  $y$  if for all functionings  $g$  in  $y$  there is a functioning  $f$  in  $x$  that is at least as good as  $g$ . The standard economic model cannot, however, incorporate the intrinsic value of freedom, which is central to the capability approach. This follows, because the above definition implies that if  $x \succsim y$  then  $x \sim x \cup y$ . If for all functionings  $g$  in  $y$  there is a functioning  $f$  in  $x$  such that  $f$  is at least as good as  $g$ , then expanding the capability set  $x$  by adding the functionings in  $y$  to it does not lead to an improvement in welfare. Hence, the possibility that the capability set  $x \cup y$  offers more freedom of choice is not valued and it can never be the case that  $x \cup y$  is strictly preferred to both  $x$  and  $y$ .<sup>3</sup>

To incorporate a preference for larger and richer opportunity sets, and thus for freedom of choice, we define the ordering  $\succsim$  over capability sets instead of over functionings. Preferences over functionings are not given and the decision maker may contemplate an array of different preference relations. It is for this reason that we mentioned in Section 5.3 that the position of the representative agent is closer to a Rawlsian decision maker seeking reflective equilibrium behind a veil of ignorance than that of a standard economic agent.

To capture Sen's (1992) idea that freedom of choice is valuable, we assume that  $\succsim$  satisfies *monotonicity*: for all capability sets  $x, y \in \mathcal{C}$ , if  $y \subseteq x$  then  $x \succsim y$ . In words, if capability set  $y$  is contained in  $x$  and, hence all functionings available in  $y$  are also available in  $x$ , then  $y$  cannot be strictly better than  $x$ . In contrast with the standard economic model,  $x \cup y$  may be strictly preferred to both  $x$  and  $y$ . Strictly ordering  $x \cup y$  above both  $x$  and  $y$  implies that more choice is preferred and, hence, that freedom of choice is positively valued.

Apart from the idea that freedom is valuable we will impose one more condition, which is again adopted from Kreps (1979). Suppose that adding capability set  $y$  to capability set  $x$  has no value to a decision maker, for example because the functionings included in  $y$  are all of low value to the decision maker compared to what is available in  $x$ . The extra opportunities offered by  $y$  have therefore no value to him. We can express this as  $x \sim x \cup y$ .<sup>4</sup> Given this preference, it seems plausible that if we enlarge the individual's capabilities to

<sup>3</sup> By completeness either  $x \succsim y$  or  $y \succ x$  or both. If  $x \succ y$  then  $x \sim x \cup y$ . If  $y \succ x$  then  $y \sim x \cup y$ . If both  $x \succ y$  and  $y \succ x$  then  $x \sim x \cup y \sim y$ . Hence,  $x \cup y \succ x$  and  $x \cup y \succ y$  are impossible.

<sup>4</sup> The fact that we allow for the possibility that  $x \sim x \cup y$  shows that larger capability sets are not necessarily better and illustrates that our approach differs from Pattanaik and Xu (1990) and other non-preference-based measurements of freedom.

$x \cup z$  by adding any set  $z$  to  $x$ , then adjoining  $y$  to  $x \cup z$  should still have no value to the individual. If the functionings included in  $y$  are all of low value to the individual if he has  $x$  available, then they should also be of low value when he has the larger set  $x \cup z$  available. We will refer to this condition as *irrelevance*: for all  $x, y \in C$ , if  $x \sim x \cup y$  then for all  $z \in C$ ,  $x \cup z \sim x \cup y \cup z$ .

### 5.5. The state-dependent utility representation

Kreps (1979) showed that if monotonicity and irrelevance jointly hold then  $\succsim$  can be represented by

$$V(x) = \sum_{s=1}^S \max_{f \in x} U_s(f). \quad (3)$$

The different  $s$  can be interpreted as states that could reflect the individual's uncertainty about his future preferences or about the availability of future functionings, but they could also refer to a set of reasonable or potential preferences.  $U_s(f)$  is a real-valued state-dependent utility function defined over functionings.<sup>5</sup> We can write Eq. (3) as

$$V(x) = \sum_{s=1}^S \max_{f \in x} \lambda_s v_s(f). \quad (4)$$

where the  $\lambda_s$  are decision weights. In some choice problems these weights can be interpreted as subjective probabilities of the subjective states. However, when utility is state-dependent these subjective probabilities cannot be uniquely determined and, hence, they do not have a clear behavioral interpretation. Moreover, a social ordering may place weight on the availability of options, even if the probability that these options will actually be selected is zero. As an example, I may be entirely confident that I would not wish to visit Antarctica. Nevertheless, I may object to a state of affairs in which I am prevented from doing so, either by legal restrictions or by a lack of resources.

To understand Eq. (4), imagine that the representative agent chooses a capability set  $x \in C$  knowing that at some ex post stage, he will learn what his preferences are. He then chooses the optimal functionings from capability set  $x$  according to these ex post preferences once he knows what they are. Ex ante, these preferences are aggregated by summing the maximum utilities across states.

A drawback of Kreps' axiomatization is that the states  $s$  are not uniquely defined and that Eqs. (3) and (4) are essentially ordinal representations. This "problem" was solved by Dekel et al. (2001) by letting menus, capability sets in our framework, consist of probability distributions over functionings and was later generalized by Kopylov (2009) to menus as abstract convex compact spaces. Because the non-uniqueness of Kreps' representation is no problem for our subsequent analysis and probability distributions over functionings are intuitively less plausible when considering capabilities, we do not further pursue these alternative approaches in this paper. The important thing for our analysis is that by adopting monotonicity and irrelevance we can value capability set through the sum of a collection of state-dependent utility functions over functionings. If we need cardinal utility, as in the construction of QALY league tables, then we can obtain this by imposing the axioms of Dekel et al. (2001) or Kopylov (2009).

### 5.6. The two-stage decision problem

Given Eq. (4), we are faced with a two-stage decision problem. In the first stage an optimal vector  $(q, w)$  is chosen subject to the restriction that  $h(q) + w \leq w_e$ . This choice ensures that the resulting capability set is optimal. The second stage then entails choosing the most preferred functioning from the optimal capability set given the realization of  $s$ , encompassing preferences, relative prices and other contingent factors.

In Appendix A we show that both the first- and the second-stage problems are well-defined and that solutions to these decision problems exist. This analysis requires the introduction of several technical conditions, stated in the Appendix, and can be summarized by the following two results:

**Result 1.** Under the assumptions stated in Appendix A,  $U_s(x) = \max_{f \in x} U_s(f)$  is well defined for each  $s$ .

**Result 2.** Under the assumptions stated in Appendix A, the first-stage problem is well-defined.

### 5.7. A local QALY evaluation

Let us now analyze the role of QALYs within this framework. Using Result 2 and Eq. (3), we can implicitly define a *capability valuation function*  $V(q, w)$ . The Proof of Result 2 shows that  $V$  is continuous, and increases in health and wealth. The health attribute in  $V(q, w)$  can consist of various dimensions, such as longevity and various quality of life dimensions such as mobility, self-care and pain. We assume that the set of health states is a subset of  $\mathbb{R}^n$ , i.e. the different health attributes can be expressed numerically. This assumption may be too strong, e.g. sometimes it may not be possible to express quality of life numerically. We present the more general case in Appendix B where we drop the assumption that the set of health states is a subset of  $\mathbb{R}^n$ .

If the functions  $U_s$  in Eq. (3) are twice differentiable then  $V(q, w)$  is also twice differentiable.<sup>6</sup> Let  $z = (q, w)$ . Differentiability of  $V$  implies that there exists a linear function  $dV: \mathbb{R}^{n+1} \rightarrow \mathbb{R}$  such that

$$\lim_{k \rightarrow 0} \frac{V(z+k) - V(z) - dV(k)}{\|k\|} = 0, \quad (5)$$

where  $\|k\|$  denotes the Euclidean norm of  $k$ . The linear function  $dV$  can be represented by the gradient vector, the vector of partial derivatives,  $\nabla V = (\partial V / \partial q_1, \dots, \partial V / \partial q_n)$  of  $V$ , in the sense that for all  $k \in \mathbb{R}^{n+1}$   $dV(k) = \nabla V \cdot k$ .

The existence of a linear function  $dV$  in Eq. (5) implies that both cost-effectiveness analysis and cost-benefit analysis are consistent with the use of QALY measures in a neighborhood of the optimal  $(q^*, w^*)$  as we will show next. Without loss of generality, suppose that  $q_1$  denotes longevity. If we normalize  $V$  such that  $(\partial V / \partial q_1) = V_1(q^*, w^*) = 1$ , then the partial derivative  $(\partial V / \partial q_k) = V_k(q^*, w^*)$  is the increase in health attribute  $k$  that would be ranked equally with a unit increase in longevity (with health characteristics  $q^*$ ). That is, the partial derivatives express the rates of trade-off between health attributes and longevity and we can compare health gains in terms of a QALY measure. Thus, we obtain a QALY measure for any local change in health

$$\nabla V_q(q^*, w^*) \cdot dq, \quad (6)$$

where  $\nabla V_q(q^*, w^*)$  is the health gradient vector evaluated at the optimum  $(q^*, w^*)$ . Note that the optimum  $q^*$  does not necessarily mean full health, and indeed in general it will not mean full health,

<sup>5</sup> To be precise, Kreps only showed that (3) holds if the set of functionings is finite. Kopylov (2009) derives (3) for infinite  $\mathcal{F}$  from a different set of conditions.

<sup>6</sup> This follows from the implicit function theorem (Rudin, 1976, Theorem 9.28).

but the level of health that generates the optimal capability set for the decision maker. That is, it is the solution to the first-stage decision problem, the determination of the optimal capability set.

Because wealth is one of the arguments of the capability valuation function  $V$  we can also compute the willingness to pay for a unit increase in longevity (with health characteristics  $q^*$ ). This is defined as  $1/V_w(q^*, w^*)$ .<sup>7</sup> In other words,  $1/V_w$  is the willingness to pay for a QALY when the decision maker's position is  $(q^*, w^*)$ . More general, we can determine the decision maker's willingness to pay for an increase in health by  $dq$  as

$$\left( \nabla V_q \cdot \frac{dq}{V_w} \right) (q^*, w^*). \quad (7)$$

Eqs. (6) and (7) are local approximations and are valid only in a neighborhood of a given  $(q^*, w^*)$ . They are applicable to valuing alternative health improvements for a decision maker who is at an optimal position, given the constraints of the first-stage problem. Eqs. (6) and (7) also imply that QALY-based measures can be used to compare individuals with comparable health and wealth  $(q^*, w^*)$ . Because Eqs. (6) and (7) are local approximations, they are not applicable to comparisons between individuals in widely separated positions. In those cases, the restrictions on preferences consistent with QALY maximization, derived by Bleichrodt and Quiggin (1999) still apply and alternative measures may have to be used instead. An example is Fleurbaey's (2005) full-health equivalent income measure.

The QALY measure derived in Eq. (6) depends on health and wealth and differs from that usually considered in the literature. The standard QALY measure compares some given health status  $q$  with the health vector associated with some given number of years in full health. By contrast, the evaluation here is undertaken at the health status  $q^*$ , the optimal health of the decision maker given the initial endowments and the technology as described. In our view, the latter approach may be regarded as an improvement, especially in the light of the capability approach. Even with the best of health care, nutrition and so on, the capabilities of a 70 year-old are not the same as those of a 25-year old. So, if we are comparing interventions that increase longevity and alternatives that increase other aspects of health capability, we would not want to evaluate them against the hypothetical yardstick of an individual living 45 years after age 25, while enjoying the capabilities of a healthy 25-year old.<sup>8</sup>

Similarly, the QALY measure in Eq. (6) is dependent on the wealth level  $w^*$ . It does not provide a basis for comparison between individuals with significantly different endowments. The analytical framework presented here therefore leaves open the question of whether a common QALY measure should be applied to evaluate the health-related capabilities of individuals with different levels of wealth. Some extra-welfarist views of the ethics of health care take as axiomatic the principle that all health care benefits should be evaluated equally, regardless of who benefits and, in particular, regardless of ability to pay. They might interpret Eq. (6) as proof that the current use of QALYs is incompatible not only with welfarism but also with a preference-based extra-welfarist valuation method.

## 6. Welfarism, freedom of choice, and consequentialism

We now consider whether, as we have claimed, the analytical framework developed in the previous section provides an

extra-welfarist basis for the allocation of health care resources and, subject to appropriate constraints, for the use of QALYs in the evaluation of health outcomes.

The analysis developed in the previous section shows that, under fairly weak conditions, any ranking of capabilities by a decision-maker may be represented by a state-contingent utility function defined over the elements of the capability set. Further, given any set of feasible resource allocations, the optimal choice may be derived as the maximum of a function  $V(q, w)$  where  $q$  is a QALY value and  $w$  is wealth.

Before discussing the difference between welfarism and extra-welfarism, it is useful to consider conditions under which the utility function  $U_s$  in Eq. (3) is state-independent. The following condition ensures this

*Condition (Stability)* for all  $x, y \in C$ , either  $x \succcurlyeq x \cup y$  or  $y \succcurlyeq x \cup y$ .

Stability together with monotonicity and irrelevance implies that each capability set is equivalent to its most preferred element. In other words, we have the following result.

**Result 3.** Under monotonicity, irrelevance, and stability, the ordering  $\succcurlyeq$  over  $C$  can be represented by

$$V(x) = \max\{u(f) : f \in x\}$$

That is, given the set  $x$ , the individual can do no better than choose the most preferred element  $f^*(x)$  and this is achievable with the single-element choice set  $\{f^*(x)\}$ . It is clear from this observation that Result 3 excludes any relevance of freedom of choice.

### 6.1. Welfarism and extra-welfarism

The term 'welfarist' along with alternatives such as 'non-welfarist' and 'extra-welfarist' has been the subject of vigorous debate. As is common in such cases, there is no generally accepted definition of this term. We will define 'welfarism' to mean the evaluation of policies in terms of a preference-based evaluation of the functionings they produce (or, in the case of uncertainty, of the probability distribution over functionings). Under this definition welfarism focuses on the maximization of individual satisfaction and is characterized by Result 3. Freedom of choice is irrelevant. The key objection from the liberal approach against welfarism is that it fails to distinguish obtaining what one wants from being satisfied. People may value freedom of choice for the reasons outlined below even though they may end up with an outcome from which they derive the same hedonic experience.

As shown by Result 3, the approach developed here includes welfarism as a special case. The model as presented here allows for both objective (or paternalist) versions of welfarism in which the ranking of functionings is determined by an objective criterion, such as a QALY model with weights, derived from evidence on the actual quality of life under particular conditions, and subjective (or preference-based) versions in which QALY weights are determined by individual preferences.

### 6.2. Freedom of choice

The main difference between Eq. (3) and Result 3 is that people may strictly prefer larger capability sets or, in other words, they may prefer freedom of choice. The literature on the value of freedom distinguishes two approaches to measure freedom: a non-preference-based approach and a preference-based approach (Dowding and Van Hees, 2009). The first approach is closer to extra-welfarism and Sen's argument that freedom of choice is intrinsically valuable irrespective people's preferences. It originates from Pattanaik and Xu (1990) who derived rules that imply

<sup>7</sup>  $V_w = \frac{\partial V}{\partial w}$ .

<sup>8</sup> For empirical evidence that people's notion of acceptable health changes with age see Brouwer et al. (2005)

that the value of freedom is determined by the size of the opportunity set. This approach has encountered at least three difficulties (Dowding and Van Hees, 2009).

First, simple measures of size, such as the cardinality of the choice set, take no account of the dissimilarity between alternatives. Adding an alternative that is substantially different from those already available provides more freedom than adding an alternative that is barely distinguishable from an alternative that is already available. Later attempts to take the diversity of the alternatives in the opportunity set into account have only proved partly successful. Second, it ignores the opportunity aspect of freedom: the set of available alternatives may be very large but this is of little use if they are highly unattractive. Finally, it ignores the psychological costs of choosing and the possibility of negative freedom. Adding alternatives may lead to a decrease in freedom, because they preclude certain types of activities (Van Hees, 1998). Kuklys and Robeyns (2005) and Fleurbaey (2005) proposed to model freedom of choice in the capability approach by including a variable that captures the intrinsic value of choice in the utility function over functionings. Their proposal also belongs to this non-preference-based approach of valuing freedom.

The second preference-based approach to value freedom takes account of individual preferences over alternatives in the evaluation of opportunity sets. This approach was adopted by Puppe (1996), see also Nehring and Puppe (1999) and Xu (2003).

There are several reasons why people value freedom of choice. The first reason, suggested by Kreps (1979), is uncertainty about future preferences. For instance, a 20-year old may not be fully sure how important he will consider health to be, relative to income, when he is 50. To account for this preference uncertainty, larger and richer capability sets can be preferred. Similarly, a social planner may be unsure about the preferences of people in society on whose behalf he acts and may therefore prefer larger capability sets.

A different source of uncertainty for the decision maker could be the future availability of functionings. In this case the decision maker knows his future preferences (or the preferences of people in society), but he is unsure whether all functionings will still be available when he has to choose among the available functionings (Barberà and Grodal, 2011). For example, if health is a functioning then some health states may no longer be available due to unexpected illness.

On the other hand, freedom of choice may be valuable precisely because preferences are potentially unstable and therefore not a satisfactory basis for ranking outcomes. The literature on behavioral economics has uncovered many inconsistencies in people's preferences, and individuals may act on different preferences at different times and in different situations. In the presence of such preference instability, the flexibility offered by larger and richer opportunity sets is valuable (Sugden, 2004).

A possible compromise between non-preference-based and preference-based approaches arises if the decision maker gives positive weight to any preferences he considers 'reasonable' (Jones and Sugden, 1982; Sugden, 1998). Even if the decision maker believes that it is unlikely that he will act on some preferences he may wish to include these preferences in his evaluation because he believes they are reasonable. For example, I may be sure that to treat kidney failure I will prefer transplant to dialysis, but I believe that in some case a preference for dialysis is reasonable and I, therefore, would not want to exclude such preferences from consideration. Similarly, a policy maker may wish to base his resource allocation decisions on all preferences that he considers reasonable.

This discussion shows that, while our approach places weight on preferences, it is more flexible than traditional welfarism. Under welfarism adding an outcome that will never be chosen cannot carry value. Under our framework it can. In our model preferences

are defined over capability sets and a decision maker can prefer capability set  $x$  over capability set  $y$  even though the object that he ultimately chooses belongs to both capability sets.

### 6.3. Consequentialism

Finally, it may be useful to consider whether the model proposed here is consistent with consequentialism, which we will define as the principle that a policy should be evaluated according to its actual or (in the case of uncertainty) expected outcome. Clearly, an evaluation rule based on achieved functionings is consequentialist. We claim, consistently with Sen (1979), that an evaluation rule based on the capabilities generated by a given policy is also consistent with consequentialism. At least in the health context, the typical components of a capability (mobility, life expectancy, absence of pain) are the direct consequences of medical interventions, while the realized functionings (walking to the shops, working in a given occupation) are indirect consequences.

By contrast, a good deal of health policy debate is concerned with non-consequentialist principles. For example, the most contentious component of the US health reforms introduced under the Affordable Care Act is the 'individual mandate', requiring everyone to purchase health insurance. There is widespread, though not universal, agreement that, by overcoming adverse selection problems, the mandate reduces social costs and (with appropriate side payments) yields at least a potential Pareto-improvement. Most criticism of the mandate takes no account of these consequences but is based on the claim that a mandate to purchase a particular product is unconstitutional, a violation of human rights or both.

The distinction between consequentialist and non-consequentialist principles is not sharp. A consequentialist opponent of the mandate might argue that the loss of liberty it entails is itself a consequence, even if the cost savings it generates are enough to expand the individual's budget set. Nevertheless, it seems useful to draw the distinction in such a way that both capability-based and functioning-based policy evaluation rules are classified as consequentialist, while rights-based approaches to policy debates are not.

## 7. Concluding comments

The idea that public policy should aim at expanding the capabilities available to people rather than at specific improvements in welfare has considerable appeal. However, except in very limited contexts, it has not been developed in an operational form. Conversely, while the use of the QALY measure to evaluate health interventions is well-developed in an operational sense, attempts to provide a welfare-theoretic basis for QALY measures and hence to establish consistency between cost-effectiveness analysis and cost-benefit analysis, have proved unsatisfactory, except in special cases.

In this paper, we have shown that, using concepts from the literature on choice and flexibility, and in particular the model of Kreps (1979), it is possible to develop a formal representation of capabilities. Further, this representation allows for an evaluation of health interventions in which QALYs may be interpreted as local measures of changes in capabilities. To derive a full representation, implying that QALYs can be used for individuals with different wealth and health positions would require similarly stringent conditions as those used by Bleichrodt and Quiggin (1999) to establish the link between cost-effectiveness analysis and cost-benefit analysis. Hence, one message that emerges from this paper is that it is hard to justify the general use of QALYs even within a capabilities

framework. That said, it should be kept in mind that our operationalization of the capabilities framework rested on some specific assumptions, e.g. on the role of preferences in valuing capability sets and the freedom of choice. We believe that these assumptions are reasonable but other proposals have been made. However, it is as yet unclear how these other proposals can be operationalized and thus whether a justification for the use of QALYs can be provided under them.

There is considerable scope for expansion of the work presented here. Most importantly, the analysis presented here applies to the case of a single individual who may be considered as representative of a class of potential patients with similar initial health status and financial wealth. Extension of the analysis to the case of a heterogeneous population represents a considerable challenge. For example, a difference in initial endowments would mean that priority should be given to the least advantaged if society aims to provide people with equal capability sets (Sen, 1992, p. 12). Allowing for individual differences in “producing” functionings from health and wealth could lead to priority being given to those with the least capacity to benefit from health care. Such a priority would not only be inconsistent with QALY maximization, but also with the aim of equalizing the distribution of health care resources across individuals with similar health conditions, which underlies the extensive literature on the measurement of inequalities in health.

Another direction for extension would be to relax the assumption that a greater range of choice is always desirable. The obvious counter-example is that of addictive or unhealthy behaviors, where patients may seek interventions that reduce the effective availability of these behaviors. This point is related to the literature on self-control (Gul and Pesendorfer, 2001) and raises questions about paternalism and limits to freedom.

This research is in its early stages. Nevertheless, we believe an analysis based on capabilities may help to better understand the principles on which health care resources should be allocated and, in particular, on the strengths and limitations of the QALY approach, than do the standard frameworks of cost-effectiveness analysis and cost-benefit analysis.

### Acknowledgements

We are grateful to Marc Fleurbaey and two anonymous reviewers for helpful comments. Han Bleichrodt’s research was made possible through a grant from the Netherlands Organization for Scientific Research (NWO). John Quiggin’s research was made possible by a Federation Fellowship from the Australian Research Council.

### Appendix A. Proofs of Results

#### A.1. Preliminaries

The first and the second-stage are linked by the relation

$$\mathcal{X}(q, w) = \{f \in \mathcal{F} : f \text{ is feasible given } (q, w)\}.$$

We start by analyzing the second-stage solution. To derive this we have to introduce some additional assumptions of a technical nature. First, we assume that a topology exists on the capability set  $\mathcal{C}$ . The ordering  $\succ$  is continuous with respect to this topology. That is, for any capability set  $y \in \mathcal{C}$  the sets  $\{x : x \succ y\}$  and  $\{x : y \succ x\}$  are both open in the topology on  $\mathcal{C}$ .

The capability set  $\mathcal{C}$  is compact and convex. The convex combination  $\alpha x + (1 - \alpha)y$ ,  $\alpha \in [0, 1]$  of two capability sets  $x, y \in \mathcal{C}$  is defined as: for all  $f \in x$ ,  $g \in y$ ,  $\alpha f + (1 - \alpha)g$  belongs to  $\alpha x + (1 - \alpha)y$ . That is, the linear combination  $\alpha f + (1 - \alpha)g$  is feasible. We further assume

that if  $(q, w') < (q, w)$  then  $\mathcal{X}(q', w') \subset \mathcal{X}(q, w)$ . That is, the capability set expands when health and wealth increase. Finally, we assume that  $\mathcal{X}$  is an upper semi-continuous correspondence. That is, at all  $(q, w)$  if  $\lim_{n \rightarrow \infty} (q^n, w^n) = (q, w)$ ,  $f^n \in \mathcal{X}(q^n, w^n)$ ,  $\lim_{n \rightarrow \infty} f^n = f$  implies that  $f \in \mathcal{X}(q, w)$ . In words, if  $(q^n, w^n)$  is a sequence that converges to  $(q, w)$  and  $f^n \in \mathcal{X}(q^n, w^n)$  is a sequence of feasible functionings that converges to  $f$  then  $f$  is also feasible given health and wealth  $(q, w)$ . We can now state the proof of our first result, which says that the second-stage solution exists.

**Proof of Result 1.**  $\mathcal{X}$  is a correspondence from the set of  $(q, w)$  into the set of functionings.  $\mathcal{X}$  is upper semi-continuous. The set  $\mathcal{F}$  is compact by Tychonoff’s theorem (Dugundji, 1966, p. 224). Continuity implies that the  $U_s$  are continuous. It follows by the upper semi-continuous maximum theorem (Berge, 1963, p. 116) that the function  $g(q, w) = \max_{f \in \mathcal{X}} \{U_s(f(q, w))\}$  is well-defined and is continuous from above over the set of  $(q, w)$ .  $\square$

For the Proof of Result 2 we further assume that  $\mathcal{C}$  is separable, i.e. it contains a countable order dense subset.

**Proof of Result 2.** Because we assume convexity of  $\mathcal{C}$ , we can no longer use Eq. (3), because convexity implies that  $\mathcal{C}$  is infinite. Instead we use Theorem 4 in Kreps (1979) which shows that  $\succ$  can be represented by a function  $V(u_1, \dots, u_s)$ , which is strictly increasing in each of the  $u_s$  and  $u_s = \sup_{f \in \mathcal{X}} U_s(f)$ . Because  $\mathcal{F}$  is compact,  $u_s = \sup_{f \in \mathcal{X}} U_s(f)$  is well-defined. By Tychonoff’s theorem,  $\mathcal{F}^S$  is compact and the maximization of  $V$  is well-defined by the upper semi-continuous maximum theorem. Strict increasingness of  $V$  in  $(q, w)$  follows from increasingness of the capability set. Continuity of  $V$  follows from continuity.  $\square$

**Proof of Result 3.** We can define a subspace topology on  $\mathcal{F}$  derived from the topology on  $\mathcal{C}$ . The subspace topology is compact and separable. Define a preference relation  $\succ^*$  over functionings from the preference relation over capability sets as: for all functionings  $f, g \in \mathcal{F}$ ,  $f \succ^* g$  iff  $f \succ f \cup g$ . It is easily verified that this preference relation over functionings is complete. For transitivity, suppose that  $f \succ^* g$  and  $g \succ^* h$ . Then  $f \succ f \cup g$  and  $g \succ g \cup h$ . By irrelevance  $f \cup g \succ f \cup g \cup h$ . By monotonicity  $f \cup g \cup h \succ f \cup h$ . Thus, we have  $f \succ f \cup g \succ f \cup g \cup h \succ f \cup h$ . By transitivity of  $\succ$  we obtain  $f \succ f \cup h$  and thus  $f \succ^* h$ .

It is obvious that  $f \succ g$  iff  $f \succ^* g$ . Hence,  $\succ^*$  is continuous with respect to the subspace topology on  $\mathcal{F}$  and we can define a continuous utility function  $U$  over the set of functionings  $\mathcal{F}$ . Because  $\succ^*$  is continuous and  $\mathcal{F}$  is compact, there is a maximal element in  $\mathcal{F}$ . Similarly any subset of elements of  $\mathcal{F}$  has a maximal element. It follows that each capability set  $x$  has a maximal element. Let the maximal element of  $x \in \mathcal{C}$  be  $f^*$ . Hence, for all  $f \in \mathcal{C}$ ,  $f^* \succ^* f$ . It follows from repeated application of irrelevance that  $f^* \succ \bigcup_{f \in x} f = x$ . By monotonicity,  $x \succ f^*$ . Hence,  $x \sim f^*$  and  $V(x) = U(f^*)$ . Result 3 follows.  $\square$

Because  $\succ^*$  is continuous and  $\mathcal{F}$  is compact, there is a maximal element in  $\mathcal{F}$ . Similarly any subset of elements of  $\mathcal{F}$  has a maximal element. It follows that each capability set  $x$  has a maximal element. Let the maximal element of  $x \in \mathcal{C}$  be  $f^*$ . Hence, for all  $f \in \mathcal{C}$ ,  $f^* \succ^* f$ . It follows from repeated application of irrelevance that  $f^* \succ \bigcup_{f \in x} f = x$ . By monotonicity,  $x \succ f^*$ . Hence,  $x \sim f^*$  and  $V(x) = U(f^*)$ . Result 3 follows.  $\square$

**Appendix B. Extension to more general health spaces**

In the analysis of the main paper, we assumed that health was a subset of  $\mathbb{R}^n$ . This assumption may be too restrictive, because health states do not correspond directly to subsets of the real numbers. In this appendix we show that Eqs. (6) and (7) can also be derived in a more general framework. Let the set of health states be any linear space with a norm  $\|\cdot\|$  defined on it. We assume that the capability valuation function  $V$  is Fréchet differentiable as in Machina (1982). Fréchet differentiability is the natural notion of differentiability on normed spaces. A real-valued function  $V$  on an open subset  $A$  of



a normed linear space  $\mathcal{Z}$  is said to be *Fréchet differentiable* at  $z \in A$  if there exists a continuous linear functional  $T_z$  on  $\mathcal{Z}$  where, given  $\varepsilon > 0$ , there exists a  $\delta(\varepsilon, z) > 0$  such that  $|V(z+k) - V(z) - T_z(k)| < \varepsilon \|k\|$  for all  $k \in \mathcal{Z}$ ,  $\|k\| < \delta$ . Or, alternatively stated:

$$\lim_{k \rightarrow 0} \frac{|V(z+k) - V(z) - T_z(k)|}{\|k\|} = 0.$$

An equivalent way of stating this is by writing

$$V(z+k) - V(z) = T_z(k) + o(\|k\|)$$

where  $o$  denotes a function which is zero at zero and of a higher order than its argument. By the Hahn–Banach theorem we can extend  $T_z$  to  $\mathcal{Z}$ . The subscript  $z$  in  $T_z$  serves as a reminder that  $T$  will generally be different at different  $z$ .

Hence, it follows that

$$\frac{dV}{dz_i} = \frac{dT_z}{dz_i} + \frac{do \|dz_i\|}{dz_i}.$$

Because the derivative of the higher order term  $o$  is zero at zero it follows that

$$\frac{dV}{dz_i} = \frac{dT_z}{dz_i} = \nabla dz_i.$$

Hence, the change in  $V$  due to a change in each of the  $x_i$  can be written as a linear function of these changes:

$$dV = \sum_{i=1}^n \beta_i dx_i,$$

and locally we obtain a QALY-type representation.

## References

- Anand, P., 2005. Capabilities and health. *Journal of Medical Ethics* 31, 299–303.
- Arrow, K.J., 1995. A note on freedom and flexibility. In: Basu, K., Pattanaik, P.K., Suzumura, K. (Eds.), *Choice, Welfare and Development: A Festschrift in Honor of Amartya K. Sen*. Oxford University Press, Oxford, pp. 7–16.
- Barberà, S., Grodal, B., 2011. Preference for flexibility and the opportunities of choice. *Journal of Mathematical Economics* 47, 272–278.
- Berge, C., 1963. *Topological Spaces*. Dover, New York.
- Bleichrodt, H., Pinto, J.L., 2012. Conceptual foundations for health utility measurement. In: Jones, A.M. (Ed.), *The Elgar Companion to Health Economics*. Edward Elgar, Cheltenham, UK, pp. 371–382.
- Bleichrodt, H., Diecidue, E., Quiggin, J., 2004. Equity weights in the allocation of health care: the rank-dependent QALY model. *Journal of Health Economics* 23, 157–171.
- Bleichrodt, H., Miyamoto, J., 2003. A characterization of quality-adjusted life-years under cumulative prospect theory. *Mathematics of Operations Research* 28, 181–193.
- Bleichrodt, H., Quiggin, J., 1999. Life-cycle preferences over consumption and health: when is cost-effectiveness analysis equivalent to cost-benefit analysis? *Journal of Health Economics* 18, 681–708.
- Bleichrodt, H., Quiggin, J., 1997. Characterizing QALYs under a general rank dependent utility model. *Journal of Risk and Uncertainty* 15, 151–165.
- Bleichrodt, H., Wakker, P.P., Johannesson, M., 1997. Characterizing QALYs by risk neutrality. *Journal of Risk and Uncertainty* 15, 107–114.
- Bleichrodt, H., Abellan-Perpiñan, J.M., Pinto-Prades, J.L., Mendez-Martinez, I., 2007. Resolving inconsistencies in utility measurement under risk: tests of generalizations of expected utility. *Management Science* 53, 469–482.
- Bleichrodt, H., Filko, M., 2008. New tests of QALYs when health varies over time. *Journal of Health Economics* 27, 1237–1249.
- Bleichrodt, H., Pinto, J.L., 2005. The validity of QALYs under non-expected utility. *Economic Journal* 115, 533–550.
- Bleichrodt, H., Quiggin, J., 2002. Life-cycle preferences over consumption and health: a reply to Klose. *Journal of Health Economics* 21, 167–168.
- Brouwer, W.B.F., Culyer, A.J., van Exel, N.J.A., Rutten, F.F.H., 2008. Welfarism vs. extra-welfarism. *Journal of Health Economics* 27, 325–338.
- Brouwer, W.B.F., van Exel, N.J.A., Stolk, E.A., 2005. Acceptability of less than perfect health states. *Social Science & Medicine* 60, 237–246.
- Brun, B.C., Tungodden, B., 2004. Non-welfaristic theories of justice: Is 'the intersection approach' a solution to the indexing impasse? *Social Choice and Welfare* 22, 49–60.
- Coast, J., Smith, R., Lorgelly, P., 2008. Should the capability approach be applied in health economics? *Health Economics* 17, 667–670.
- Cookson, R., 2005. QALYs and the capability approach. *Health Economics* 14, 817–829.
- Culyer, A.J., 1989. The normative economics of health care finance and provision. *Oxford Review of Economic Policy* 5, 34–58.
- Dekel, E., Lipman, B.L., Rustichini, A., 2001. Representing preferences with a unique subjective state space. *Econometrica* 69, 891–934.
- Dowding, K., Van Hees, M., 2009. Freedom of choice. In: Anand, P., Pattanaik, P.K., Puppe, C. (Eds.), *The Handbook of Rational & Social Choice*. Oxford University Press, Oxford, pp. 374–392.
- Dugundji, J., 1966. *Topology*. Allyn and Bacon, Boston.
- Fleurbaey, M., 2005. Health, wealth and fairness. *Journal of Public Economic Theory* 7, 253–284.
- Fleurbaey, M., 2009. Beyond GDP: the quest for a measure of social welfare. *Journal of Economic Literature* 47, 1029–1075.
- Gaertner, W., Xu, Y., 2008. A new class of measures of the standard of living based on functionings. *Economic Theory* 35, 201–215.
- Gaertner, W., Xu, Y., 2006. Capability sets as the basis for a new measure of human development. *Journal of Human Development* 7, 311–321.
- Gold, M.R., Siegel, J.E., Russell, L.B., Weinstein, M.C., 1996. *Cost-Effectiveness Analysis in Health and Medicine*. Oxford University Press, New York.
- Gorman, W.M., 1959. Separable utility and aggregation. *Econometrica* 27, 469–481.
- Gul, F., Pesendorfer, W., 2001. Temptation and self-control. *Econometrica* 69, 1403–1435.
- Hurley, J., 2000. An overview of the normative economics of the health care sector. In: Culyer, A.J., Newhouse, J.P. (Eds.), *Handbook of Health Economics*. Elsevier Science, Amsterdam.
- Jones, P., Sugden, R., 1982. Evaluating choice. *International Review of Law and Economics* 2, 47–65.
- Klose, T., 2002. Life-cycle preferences over consumption and health: a comment. *Journal of Health Economics* 21, 161–166.
- Kopylov, I., 2009. Finite additive utility representations for preferences over menus. *Journal of Economic Theory* 144, 354–374.
- Kreps, D.M., 1979. A preference for flexibility. *Econometrica* 47, 565–576.
- Kuklys, W., Robeyns, I., 2005. Sen's capability approach to welfare economics. In: Kuklys, W. (Ed.), *Amartya Sen's Capability Approach: Theoretical Insights and Empirical Applications*. Springer Verlag, Berlin, pp. 9–30.
- Lancaster, K., 1966. A new approach to consumer theory. *Journal of Political Economy* 74, 132–157.
- Llewellyn-Thomas, H., Sutherland, H.J., Tibshirani, R., Ciampi, A., Till, J.E., Boyd, N.F., 1982. The measurement of patients' values in medicine. *Medical Decision Making* 2, 449–462.
- Machina, M., 1982. 'Expected utility' analysis without the independence axiom. *Econometrica* 50, 277–323.
- Miyamoto, J.M., 1999. Quality-adjusted life-years (QALY) utility models under expected utility and rank dependent utility assumptions. *Journal of Mathematical Psychology* 43, 201–237.
- Miyamoto, J.M., Wakker, P.P., Bleichrodt, H., Peters, H.J.M., 1998. The zero-condition: a simplifying assumption in QALY measurement and multiattribute utility. *Management Science* 44, 839–849.
- Nehring, K., Puppe, C., 1999. On the multi-preference approach to evaluating opportunities. *Social Choice and Welfare* 16, 41–63.
- Nussbaum, M.C., 2000. *Women and Human Development: The Capabilities Approach*. Cambridge University Press, Cambridge, New York, and Melbourne.
- Oliver, A.J., 2003. A quantitative and qualitative test of the Allais paradox using health outcomes. *Journal of Economic Psychology* 24, 35–48.
- Pattanaik, P.K., Xu, Y., 2000. On ranking opportunity sets in economic environments. *Journal of Economic Theory* 93, 48–71.
- Pattanaik, P.K., Xu, Y., 1990. On ranking opportunity sets in terms of freedom of choice. *Récherches Économiques De Louvain* 56, 383–390.
- Pliskin, J.S., Shepard, D.S., Weinstein, M.C., 1980. Utility functions for life years and health status. *Operations Research* 28, 206–223.
- Puppe, C., 1995. Freedom of choice and rational decisions. *Social Choice and Welfare* 12, 137–153.
- Puppe, C., 1996. An axiomatic approach to "preference for freedom of choice". *Journal of Economic Theory* 68, 174–199.
- Robeyns, I., 2006. The capability approach in practice. *Journal of Political Philosophy* 14, 351–376.
- Roemer, J.E., 1998. *Equality of Opportunity*. Harvard University Press, Cambridge, MA.
- Rudin, W., 1996. *Principles of Mathematical Analysis*, 3rd Ed. McGraw-Hill, Singapore.
- Schokkaert, E., 2009. The capabilities approach. In: Anand, P., Pattanaik, P.K., Puppe, C. (Eds.), *The Handbook of Rational & Social Choice*. Oxford University Press, Oxford, pp. 542–566.
- Sen, A.K., 1992. *Inequality Re-Examined*. Clarendon Press, Oxford.
- Sen, A.K., 1991. Welfare, preference and freedom. *Journal of Econometrics* 50, 15–29.
- Sen, A.K., 1985. *Commodities and Capabilities*. North Holland, Amsterdam.
- Sen, A.K., 1979. Personal utilities and public judgements: or what's wrong with welfare economics. *The Economic Journal* 89, 537–558.
- Starmer, C., 2000. Developments in non-expected utility theory: the hunt for a descriptive theory of choice under risk. *Journal of Economic Literature* 28, 332–382.

- Sugden, R., 2004. The opportunity criterion: consumer sovereignty without the assumption of coherent preferences. *American Economic Review* 94, 1014–1033.
- Sugden, R., 1998. The metric of opportunity. *Economics and Philosophy* 14, 307–337.
- Sugden, R., 1993. Welfare, resources, and capabilities: a review of inequality reexamined by Amartya Sen. *Journal of Economic Literature* 31, 1947–1962.
- Tsuchiya, A., 2012. Distributional judgements in the context of economic evaluation. In: Jones, A.M. (Ed.), *The Elgar Companion to Health Economics*, Cheltenham, UK, pp. 406–414.
- Van Hees, M., 1998. On the analysis of negative freedom. *Theory and Decision* 45, 1175–1197.
- Williams, A., 1997. Intergenerational equity: an exploration of the 'fair innings' argument. *Health Economics* 6, 117–132.
- Xu, Y., 2003. On ranking compact and comprehensive opportunity sets. *Mathematical Social Sciences* 45, 109–119.