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INTERTEMPORAL TRADEOFFS FOR GAINS AND LOSSES: AN EXPERIMENTAL MEASUREMENT OF DISCOUNTED UTILITY*

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This article provides a parameter-free measurement of utility in intertemporal choice and presents new and more robust evidence on the discounting of money outcomes. Intertemporal utility was concave for gains and convex for losses, consistent with a hypothesis put forward by Loewenstein and Prelec (1992). Discount rates declined over time but less so than previously observed under the assumption of linear utility. For approximately 40% of our subjects constant discounting provided the best fit. The remaining 60% were most consistent with Harvey's (1986) power discounting. Our data provide little support for the popular quasi-hyperbolic model, which is widely used in economics today. We observed an asymmetry in the discounting of gains and losses that, unlike earlier findings, cannot be explained by a framing effect.

Many economic decisions involve outcomes that occur at different points in time. Employees decide how much money to put aside for retirement, people adopt healthy lifestyles to improve their future health and governments take actions to reduce the future effects of global warming. To analyse such decisions, economists have typically used discounted utility models. These models combine an instantaneous utility function that reflects attitudes towards outcomes with a discount function that captures the effect of the passage of time. The most widely used discounted utility model in economics is constant discounting in which the discount function is determined by a constant rate of discount. Empirical studies on time preference have observed, however, that discount rates are not constant but decrease over time, a phenomenon referred to as decreasing impatience (Frederick *et al.*, 2002; Read, 2004). These findings have led to the development of alternative discounted utility models, commonly referred to as hyperbolic discounting. The hyperbolic discounting models are consistent with decreasing impatience and have become quickly popular in economics. Today many economic applications are based on hyperbolic discounting, in particular on quasi-hyperbolic discounting, a model first proposed by Phelps and Pollak (1968) and made popular by Laibson (1997).[1]

Empirical measurement of discounted utility models is complex, because it requires the simultaneous elicitation of the instantaneous utility function and the discount function. Previous studies have side-stepped this problem and have assumed specific functional forms for utility and discounting. In particular, most studies have assumed

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[1] Examples of applications based on quasi-hyperbolic discounting include Laibson (1997), Bernheim *et al.* (2001), Harris and Laibson (2001), Krusell and Smith (2003) and Salanié and Treich (2006) for saving, O'Donoghue and Rabin (1999) and Brocas and Carrillo (2001) for procrastination, Brocas and Carrillo (2000) for the value of information, Gruber and Köszegi (2001) for addiction, Bénabou and Tirole (2002) for self-confidence, Diamond and Köszegi (2003) for retirement, Karp (2005) for global warming and Paserman (2008) for job search.

linear utility. A drawback of making parametric assumptions is that the quality of the estimation comes to depend on the choice of functional forms. If utility is concave instead of linear then falsely assuming linear utility will lead to an overestimation of discount rates and, indeed, most empirical studies have observed high discount rates. Another limitation of assuming functional forms for utility is that no or only limited information is obtained on the utility function. Consequently, in spite of the importance of intertemporal preferences and discounted utility models in economics, there is to date no study that has actually measured the instantaneous utility function in intertemporal choice.

This article presents a new method for measuring both the instantaneous utility function and the discount function without making any assumptions about functional forms. It is in this sense that we refer to our method as parameter-free. Our method consists of two stages. In the first stage the instantaneous utility function is measured in such a way that no knowledge of the discount function is required. The second stage then uses these utilities to measure the discount function. The first stage, the measurement of utility, resembles the tradeoff method of Wakker and Deneffe (1996) for decision under uncertainty. The difficulty in translating their method to intertemporal choice is that the instantaneous utility function in intertemporal choice has different uniqueness properties from the utility function in decision under uncertainty. The combination of the adjusted tradeoff method with the second stage to provide a complete parameter-free measurement of discounted utility is the main novelty of our method.

An additional advantage of our method is that it allows measuring utility and discounting at the individual level and, therefore, takes account of heterogeneity in individual intertemporal preferences. We applied our method in an experimental study and, hence, this article is the first to provide a parameter-free measurement of the entire discounted utility model and both at the aggregate and at the individual level.

Our data allowed us to address several open empirical questions. First, we obtained evidence on the shape of the instantaneous utility function for gains and losses. Classical economics assumes that utility is everywhere concave. Loewenstein and Prelec (1992), by contrast, posit that people treat gains and losses differently and have concave utility for gains and convex utility for losses. They showed that this S-shape is able to explain intertemporal economic phenomena like the tendency of businesses to cut back on investment during periods of lower than anticipated profits and the tendency of consumers to save more from bonuses than from normal income. Our experiment involved both gains and losses, which made it possible to compare the predictions of classical economics with Loewenstein and Prelec's (1992) hypotheses.

Second, we could test whether the commonly observed pattern of declining discount rates persisted when the assumption of linear utility was relaxed and we could test the prevalence of decreasing impatience at the individual level.

Third, our data made it possible to compare the fit of constant discounting with that of its main hyperbolic alternative, quasi-hyperbolic discounting and other discounting models. Many studies have provided support for hyperbolic discounting (Ainslie, 1975; Thaler, 1981; Benzion *et al.*, 1989; Kirby and Marakovic, 1995)[2] but little insight exists

[2] For findings challenging hyperbolic discounting see Read (2001) and Read, Frederick, Orsel and Rahman (2005).

into which hyperbolic model most accurately describes intertemporal preferences. The popularity in economics of quasi-hyperbolic discounting rests on its theoretical tractability and not on its displayed descriptive superiority over other discounting models. The importance of testing the goodness of fit of hyperbolic discounting is highlighted by recent empirical studies that observed increasing instead of decreasing impatience (Chesson and Viscusi, 2003; Gigliotti and Sopher, 2004; Read, Airoldi and Loewenstein, 2005; Sayman and Öncüler, 2009). Hyperbolic discounting models cannot explain increasing impatience (Bleichrodt *et al.*, 2009). Therefore, we also compared the fit of the hyperbolic discounting models with a discounting model recently proposed by Ebert and Prelec (2007). Ebert and Prelec's (2007) model is as tractable as hyperbolic discounting but can accommodate both moderately decreasing and increasing impatience.

Finally, we could perform a more robust test for an asymmetry between the discounting of gains and losses observed in some earlier studies (Thaler, 1981, Benzion *et al.*, 1989). One explanation for the gain–loss asymmetry may be that it is an artifact of the assumption of linear utility. When utility is concave for gains, leading to an overestimation of discount rates for gains, and closer to linear for losses, leading to less distortion of discount rates for losses, then the gain–loss asymmetry will follow from the assumption of linear utility even when people have the same discount function for gains and for losses.

In what follows, the next Section reviews previous theoretical and empirical research on intertemporal choice. Section 2 presents our method for measuring discounted utility. The design and results of our experiment are described in Sections 3 and 4 and are discussed in Section 5. Section 6 concludes the article.

1. Background

We consider *temporal profiles* $(x_0,\ldots,x_T)$, where x_t denotes outcome x at time point t and time point 0 is the present. Outcomes can be money amounts but also binary prospects (M, p) denoting money amount M with probability p and nothing otherwise.

We examine preferences $\succcurlyeq$ over temporal profiles. As usual, $\sim$ denotes indifference. Preferences over outcomes are derived from preferences over *constant temporal profiles*, where $x_1 = \ldots = x_T = \alpha$. We define $\alpha \succcurlyeq \beta$ when $(\alpha,\ldots,\alpha) \succcurlyeq (\beta,\ldots,\beta)$, i.e. receiving α at all points in time is preferred to receiving β at all points in time.

The decision maker perceives outcomes relative to 0, receiving nothing. *Gains* are outcomes preferred to 0 and *losses* are outcomes less preferred than 0. We will only consider temporal prospects where all outcomes have the same sign, i.e. either all outcomes are gains or all outcomes are losses. A function V *represents* $\succcurlyeq$ when for all x, y, $x \succcurlyeq y$ if and only if $V(x) \geq V(y)$. Throughout the article, we assume that preferences over temporal profiles can be represented by *discounted utility*

$$V(x_0,\ldots,x_T) = \sum_{t=0}^{T} \lambda_t^i u(x_t),\ i = +,- \tag{1}$$

with the *time weights* λ_t^i positive and $\lambda_0^i = 1$ and u a real-valued *instantaneous utility function* that represents preferences over outcomes. The time weights can be different for gains and for losses. To keep the notation tractable, we will suppress the sign-dependence of the λ_t^i and simply write λ_t in what follows. Whether the time weights for gains or the time weights for losses apply will be clear from the decision context.

The time weights λ_t are unique and the utility function is unique up to unit. Equation (1) is general in the sense that it presumes nothing about the ordering or the relative magnitude of the λ_t. The main models of discounting are all special cases of (1). A preference foundation for discounted utility has been given by Krantz *et al.* (1971, Theorem 6.15).

The best-known special case of (1) is constant discounting, introduced by Samuelson (1937) and still the most widely used discounted utility model in economics. *Constant discounting* entails that the time weights λ_t in (1) are equal to $(1 + \delta)^{-t}$, where δ is the constant discount rate. Experimental evidence has challenged the descriptive validity of constant discounting. In this article we focus on two violations of constant discounting: *decreasing impatience*, the finding that discount rates are not constant but decrease over time, and the *gain–loss asymmetry*, the finding that people discount gains more than losses. One other deviation from constant discounting that we will briefly discuss is the *magnitude effect*, the finding that larger amounts are discounted less than smaller amounts.

Many studies have observed decreasing impatience. See for example Thaler (1981), Benzion *et al.* (1989), Shelley (1993) and Kirby and Marakovic (1995) for money amounts, and Chapman (1996), Lazaro *et al.* (2001) and van der Pol and Cairns (2002) for health outcomes. The common assumption in all these studies was linear intertemporal utility. Chapman (1996) also considered power utility. She elicited utility in an atemporal setting using introspective strength of preference judgments and then assumed that this function could also be applied to intertemporal choice. Andersen *et al.* (2008) used a similar strategy: they estimated power utility from decision under risk (assuming expected utility) and then applied this function to intertemporal choice. Whether utility is transferable across decision domains is highly controversial in economics. Arrow (1951), Savage (1954), Luce and Raiffa (1957) and Fishburn (1989) amongst others have argued against such transferability.

There is some controversy in the literature as to whether decreasing impatience holds in general or whether violations of constant discounting occur only in the first time interval. The latter hypothesis is referred to as the *immediacy effect* and underlies quasi-hyperbolic discounting, which will be discussed below. Some studies found support for the immediacy effect (Bleichrodt and Johannesson, 2001; Frederick *et al.*, 2002); others also found violations of constant discounting for later time intervals (Kirby and Herrnstein, 1995; Kirby, 1997; Lazaro *et al.*, 2001).

The gain–loss asymmetry is empirically less well-established than decreasing impatience. Thaler (1981) and Benzion *et al.* (1989) found evidence of the gain–loss asymmetry but Shelley (1993) showed that their findings could be explained by a framing effect. In a neutral frame, she found no evidence of a gain–loss asymmetry, a finding later confirmed by Ahlbrecht and Weber (1997).

1.1. *Alternative Discounting Models*

Several alternative discounting models have been proposed in response to the observed violations of constant discounting. These models were primarily designed to explain decreasing impatience. Table 1 displays the main models.

Table 1

Alternative Discount Functions for Constant Discounting

Model	Suggested by	Expression
Hyperbolic discounting	Loewenstein and Prelec (1992)	$\lambda_t = (1+\gamma t)^{-\alpha/\gamma}$, $\alpha,\gamma > 0$
Proportional discounting	Herrnstein (1981)	$\lambda_t = (1+\gamma t)^{-1}$, $\gamma > 0$
Power discounting	Harvey (1986)	$\lambda_t = (1+t)^{-\alpha}$, $\alpha > 0$
Quasi-hyperbolic discounting	Phelps and Pollak (1968)	$\lambda_t = \beta(1+\delta)^{-t}$, $0 < \beta \leq 1$, $\delta > 0$
Constant sensitivity	Ebert and Prelec (2007)	$\lambda_t = \exp[-(at)^b]$, $a,b > 0$

The parameter γ in hyperbolic discounting determines the departure from constant discounting. The limiting case of γ tending to zero yields constant discounting. Because α and γ are positive, the discount rates implied by hyperbolic discounting decrease over time, corresponding to decreasing impatience. By implication, hyperbolic discounting cannot account for increasing impatience. Loewenstein and Prelec (1992) assumed that the time weights were the same for gains and for losses. To explain the gain–loss asymmetry, they suggested that the utility function u in (1) is concave for gains and convex for losses and is more elastic for losses than for gains. Proportional discounting and power discounting are the special cases of hyperbolic discounting in which $\alpha = \gamma$ and $\gamma = 1$, respectively.

The parameter β in quasi-hyperbolic discounting reflects the special status of the first period. If $\beta = 1$ quasi-hyperbolic discounting reduces to constant discounting. The case $\beta < 1$ models the immediacy effect.

None of the above models is able to accommodate increasing impatience. This is a drawback, in particular for individual data analysis. Even though decreasing impatience is the common pattern (although as we mentioned before some studies observed increasing impatience even at the aggregate level) there will always be individuals who are increasingly impatient. Constant sensitivity (Ebert and Prelec, 2007) can model both increasing and decreasing impatience. The parameter a in constant sensitivity reflects impatience and the parameter b sensitivity to time. The parameter b reflects the degree of decreasing impatience. For $b < 1$ a decision maker is decreasingly impatient, for $b > 1$ he is increasingly impatient. For $b = 1$ constant sensitivity reduces to constant discounting.

2. Measurement Method

Our measurement method consisted of two stages. In the first stage, choices between temporal profiles were constructed such that the time weights λ_t cancelled, allowing us to measure utility without the need to know the time weights. In the second stage, we used the elicited utilities to measure the time weights. Hence, we could measure the time weights from the elicited utilities and no assumptions about the shape of utility had to be made.

Table 2 summarises our measurement method. By x_0y_t we denote the temporal profile that gives x now, y at time point t and nothing in all other periods. The second column of Table 2 describes the variable that was measured, the third column the indifference that we elicited, the fourth column the implication of the elicited

Table 2

Measurement Method

	Assessed Quantity	Indifference	Under General Discounting	Stimuli
Step 1	Losses $\ell^1,\ldots,\ell^k$	$L_0\ell_t^j \sim L_0^*\ell_t^{j-1}$	$u(\ell_t^j) - u(\ell_t^{j-1}) = u(\ell_t^1) - u(\ell_t^0)$	$T = 1$ year $L^* =$ (€2000, ½) $L =$ (€500, ½) $\ell^0 =$ €0
	Gains $g^1,\ldots,g^k$	$G_0 g_t^j \sim G_0^* g_t^{j-1}$	$u(g_t^j) - u(g_t^{j-1}) = u(g_t^1) - u(g_t^0)$	$T = 1$ year $G^* =$ (€2000, ½) $G =$ (€500, ½) $g^0 =$ €0
Step 2	Time weights losses λ_t^-	$z_0 \sim \ell_t^k$	$\lambda_t^- = -u(z)$	$t = 3$ mths, 6 mths, 1y , 2y, 3y, 4y
	Time weights gains λ_t^+	$z_0 \sim g_t^k$	$\lambda_t^+ = u(z)$	$t = 3$ mths, 6 mths, 1y, 2y, 3y, 4y

indifference in terms of discounted utility, and the final column the stimuli that we used in the experiment described in Section 3. All money amounts are in euros.

2.1. *First Stage: Measurement of Utility*

In the first stage we constructed sequences of gains $g^0,\ldots,g^k$ and losses $\ell^0,\ldots,\ell^k$ for which $u(g^j) - u(g^{j-1})$ and $u(\ell^j) - u(\ell^{j-1})$, $j = 1,\ldots, k$, are constant. These sequences allow us to measure utility. We will illustrate the procedure for gains. The first step in the measurement of utility was to select two *gauge outcomes* G^* and G and a *starting outcome* g^0. In our experiment, G^* was the prospect (€2000, 1/2), G the prospect (€500, 1/2) and g^0 was €0. We then elicited the gain g^1 such that a subject was indifferent between $G_0 g_t^1$ and $G_0^* g_t^0$. In terms of discounted utility, (1), this indifference implies that

$$u(g^1) - u(g^0) = \frac{u(G^*) - u(G)}{\lambda_t}. \tag{2}$$

The outcome g^1 was used as an input in the next question where we elicited g^2 such that indifference held between $G_0 g_t^2$ and $G_0^* g_t^1$. By (1) and a similar argument as above, this indifference implies that

$$u(g^2) - u(g^1) = \frac{u(G^*) - u(G)}{\lambda_t}. \tag{3}$$

Thus, $u(g^2) - u(g^1) = u(g^1) - u(g^0)$. We proceeded to elicit indifferences $G_0 g_t^j \sim G_0^* g_t^{j-1}$, $j = 3,\ldots,k$, and obtained a sequence of gains $g^0, \ldots, g^k$ for which successive elements were equally spaced in terms of utility: $u(g^j) - u(g^{j-1}) = u(g^1) - u(g^0)$ for $j = 2,\ldots,k-1$. Because G^* is better than G, the sequence of gains is increasing, i.e. $g^j > g^{j-1}$, $j = 1, \ldots, k$.

If all temporal profiles involve the same unit of time and have common final periods then we can freely choose the utility of two outcomes. Since we only used such temporal profiles, we set $u(g^k) = 1$ and $u(g^0) = 0$, yielding $u(g^j) = j/k$, $j = 0, \ldots, k$.

For losses, the elicitation was similar, except that the gauge outcomes L^* and L were such that L^* was worse than L. Hence, $\ell^j < \ell^{j-1}$ and the elicited sequence of losses was decreasing. We set $u(\ell^k) = -1$ and $u(\ell^0) = 0$, yielding $u(\ell^j) = -j/k$ for $j = 0,\ldots,k$.

2.2. *Second Stage: Measurement of the Time Weights*

To measure the time weights for gains, we elicited the gain z such that a subject was indifferent between z now and g^k at time point t. By (1) and the scaling $u(0) = 0$ and $u(g^k) = 1$ we obtain that $u(z) = \lambda_t$. By varying t, we could elicit different time weights. The elicited outcomes z typically did not belong to the sequence of gains elicited in the first stage and their utility was unknown. However, if subjects have positive time preference, which was the case for all subjects in our experiment reported below, z will lie between two elements of the elicited sequence and we could approximate the utility of z through the known utility of these elements of the sequence. This approximation will be good if successive elements of the sequence of gains are close. We return to the issue of approximation below. The measurement of the time weights for losses is similar except that now $-u(z) = \lambda_t$.

3. Experiment

3.1. *Subjects and Incentives*

Seventy students from different departments of the Erasmus University Rotterdam participated and were paid a fixed amount of €12.50. Before the actual experiment, we tested the design in several pilot sessions using other students and university staff as subjects.

Throughout the experiment we used hypothetical choices. There were several reasons for using hypothetical instead of real incentives. A first reason was the problem in organising payments in the future, some of which occurred in four years time. Second, because utility tends to be close to linear for small amounts (Wakker and Deneffe, 1996), we used large money amounts to capture the effect of utility curvature. Actually paying these amounts would have been prohibitively expensive. Third, there were ethical constraints on using real incentives for the losses part of the experiment. Finally, in hypothetical questions one can ask subjects to assume that there is no risk associated with future payments. With real stakes, subjects may consider the receipt of future money amounts uncertain, which could inflate the discounting of these amounts. We discuss the issue of using hypothetical rewards in more detail in Section 5.

3.2. *Procedure*

The experiment was run on a computer in individual sessions lasting between 30 and 45 minutes. Answers were entered into the computer by the interviewer, so that subjects could concentrate on the questions and the possibility of mistakes could be reduced. Throughout the experiment, subjects were encouraged to think aloud to obtain insight into the reasoning underlying their answers.

Indifferences were elicited through a series of choices. Previous evidence suggests that choice-indifferences lead to fewer inconsistencies than indifferences determined by matching, where subjects are directly asked to state their indifference value (Bostic *et al.*, 1990). Because we used choices, our study employs what Shelley (1993) refers to as a neutral frame and, hence, we could test whether the gain–loss asymmetry was caused by a framing effect only.

The interviewer used a scroll bar to vary the value of the outcome that we sought to elicit, starting with values for which preferences were clear and then 'zooming in' on the indifference value. Examples of the computer screens that subjects faced in the first and the second stage of the experiment are in Figures 1 and 2.

The discounted utility model was elicited first for gains and then for losses. We always started with the gains part because we learnt from the pilot sessions that this made it easier for subjects to understand the choice task. Both parts of the experiment were preceded by a practice question.

In the first part of the experiment we elicited a sequence of 6 elements both for gains and for losses. Risky prospects were selected for the gauge outcomes G^* and G and L^* and L to discourage heuristics like simply computing the difference in absolute values. Our results are robust to subjects' evaluation of prospects (e.g., according to expected utility or prospect theory) provided that the same theory is used throughout the

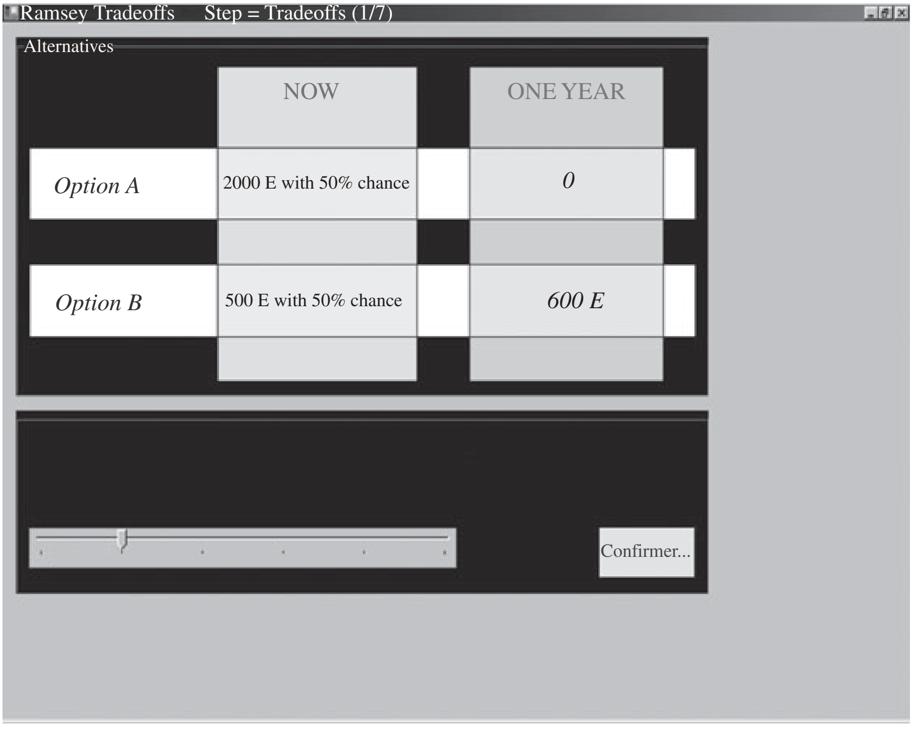

Fig. 1. *Example of the Display Subjects Faced in the Elicitation of Utility*

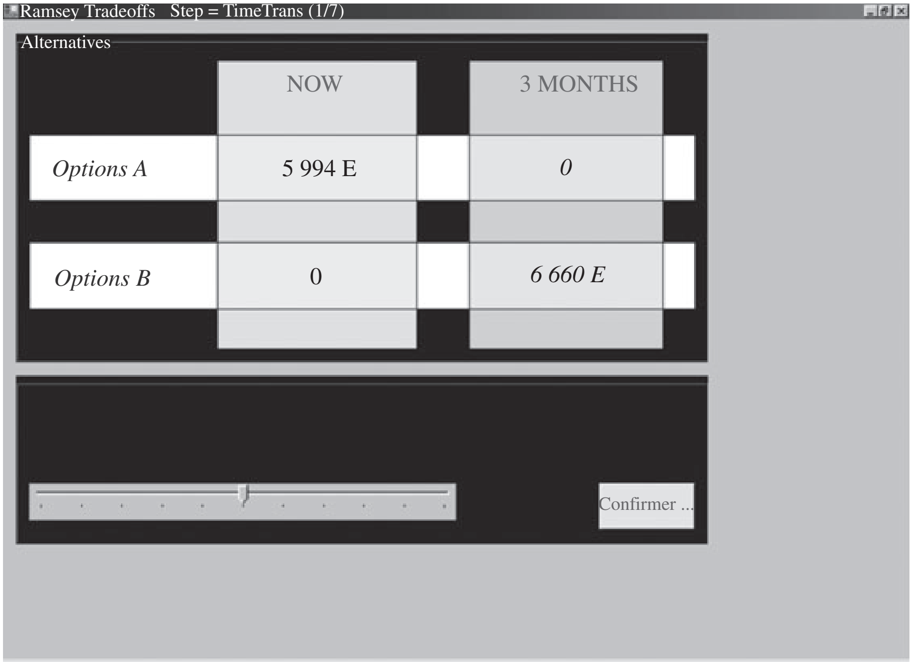

Fig. 2. *Example of the Display Subjects Faced in the Elicitation of the Time Weights*

experiment. A few subjects mentioned budgetary constraints in the losses questions. They were told to assume that an interest-free loan was available to pay off the losses.

In the second part of the experiment, we elicited the time weights for $t = 3$ months, 6 months, 1 year, 2 years, 3 years and 4 years, both for gains and for losses. The order in which the time weights were elicited was random. Both for gains and for losses, we tested for consistency by repeating the first elicitation at the end of each experimental task. That is, in the elicitation of the utility for gains [losses], we repeated the elicitation of $g^1\,[\ell^1]$ after $g^1,\ldots,g^6\,[\ell^1,\ldots,\ell^6]$ had been elicited and in the elicitation of the time weights we repeated the elicitation of the time weight that had been elicited first[3] after the time weights for 3 months,…, 4 years had been elicited.

3.3. *Analyses*

The results for means and medians were similar and, hence, we will only report the medians in the analysis of the aggregate data. Due to the presence of outliers, we focused on non-parametric tests to test for statistical significance. A significance level of 5% was used throughout.

To investigate the curvature of utility at the individual level, we computed

$$\partial_j^+ = (g^{j+1} - g^j) - (g^j - g^{j-1}), \quad j = 1,\ldots,5, \tag{4a}$$

and

[3] Recall that the order in which the time weights were elicited was random.

$$\partial_j^- = (\ell^{j+1} - \ell^j) - (\ell^j - \ell^{j-1}), \quad j = 1, \ldots, 5. \tag{4b}$$

That is, we explored the extent to which successive outcome intervals increase or decrease. We observed five values of ∂_j^+ and five values of ∂_j^- for each subject. A positive value of ∂_j^+ corresponds to a concave part of the utility function, because a larger increase in money is needed to obtain a given increase in utility (1/6) at higher amounts than at lower amounts. Likewise, a negative value of ∂_j^+ corresponds to a convex part of the utility function and a value of zero to linear utility. For losses, a positive value of ∂_j^- corresponds to a convex part of the utility function and a negative value of ∂_j^- to a concave part.

Both for gains and for losses, we classified a subject as having linear [concave, convex] utility if he had at least three linear [concave, convex] parts. We used a criterion of three instead of five similar parts, to account for response error.[4] If none of the three parts (linear, concave or convex) occurred more than twice, the subject was left unclassified.

To smoothen out irregularities in the data, we also analysed the data under specific parametric assumptions about utility. We examined two parametric families: the power family and the exponential family. Because the two functions yielded similar results we will only report the results for the power family. An advantage of the power family is that it is not sensitive to the selected unit of time (Baucells and Sarin, 2007*a*). Let $z = x/x^6$, $x \in [0,x^6]$. The *power family* is defined by $|z|^r$ if $r > 0$, by $\ln(z)$ if $r = 0$ and by $-|z|^r$ if $r < 0$. For gains [losses], $r < 1$ corresponds to concave [convex] utility and $r > 1$ to convex [concave] utility; the case $r = 1$ corresponds to linear utility both for gains and for losses. We estimated the parametric families both for the median data and for each individual separately. The estimation was performed by nonlinear least squares.

The estimates of the power coefficients were used to obtain another, parametric, classification of individual subjects. For gains [losses] we classified a subject as concave [convex] if his power coefficient was below 0.95, as linear if his power coefficient was between 0.95 and 1.05, and as convex [concave] if his power coefficient exceeded 1.05.

To compute the time weights we estimated the utility of the elicited outcome z by linear approximation. We also used approximation by the estimated power and exponential utility. This affected the results only marginally and we do not report these results separately.

From the elicited time weights we could estimate implied annual discount rates ρ_s as follows:

$$\lambda_s = \frac{1}{(1+\rho_s)^s}, \tag{5}$$

where s is time in years. To test whether the implied annual discount rates were constant, we computed $\rho_j - \rho_{j+1}$, $j = 3$ months,..., 3 years. To account for response error, if at least three of five of these differences were positive [negative, constant] then the subject was classified as decreasingly [increasingly, constantly] impatient, i.e. as having decreasing [increasing, constant] discount rates over time.

We also used the elicited time weights to estimate the parameter(s) in constant discounting, the different hyperbolic discounting models and the constant sensitivity

[4] Similar criteria were used by Fennema and van Assen (1999), Abdellaoui (2000), Etchart-Vincent (2004) and Abdellaoui *et al.* (2005).

model. Each model was estimated separately, so we did not assume that parameters that models had in common (e.g. ρ in constant discounting and ρ in quasi-hyperbolic discounting) were equal. The models were estimated by nonlinear least squares both for the median data and for each subject separately. To test whether the results were sensitive to the specification of the unit of time, we performed the estimations for different specifications of the unit of time (years, months and weeks).

Goodness of fit of the various discounting models was assessed by the sum of squared errors divided by the degrees of freedom. This takes into account that the discounting models differ in the number of parameters. To give an impression of overall goodness of fit we also computed R^2s adjusted for the degrees of freedom. Because the coefficients for the median data were very close to the medians of the estimated coefficients for the individual data, we will focus on the individual data.

4. Results

The data for two subjects were excluded from the analyses, because their answers did not correspond to their reasoning. The data for another subject were lost due to a computer crash. As a result, the data of 67 individuals (31 females) were included in the analysis.

The consistency of the data was good. Remember that we repeated four questions: in the first stage the elicitation of g^1 and ℓ^1 and in the second stage two randomly determined questions. None of the four differences between replication and original elicitation was significant ($p > 0.10$ in all cases). The replication of g^1 yielded a median value of 510 (interquartile range (IQR) = [390,750]) as opposed to a median value of 570 in the original elicitation (IQR = [270,750]). For losses the median replication value and the median original value were both equal to −750. The IQR was [−960,−450] in the replication and [−960,−540] in the original elicitation. Because in the second stage the question that was repeated varied, providing the medians is not informative. We computed the absolute differences between replication and original elicitation as a proportion of the original elicitation instead. Both for gains and for losses, these proportions were small, indicating good consistency. They were 4.3% for gains and 2.1% for losses.

4.1. *Utility*

For gains, the most common pattern was concave utility: 22 subjects had concave utility, 16 had linear utility and 7 had convex utility. The proportion of concave subjects was significantly higher than the proportion of convex subjects ($p < 0.01$). For losses, the individual data showed no clear pattern. Twenty-two subjects had convex utility, 20 had concave utility and 9 had linear utility. The proportion of subjects with convex utility did not differ significantly from the proportion with concave utility ($p > 0.10$).[5]

[5] Subjects classified as concave for gains 'erred' in 26.3% of their responses in the direction of convexity and in 8.2% of their responses in the direction of linearity. Subjects who were linear [convex] for gains erred in 13.8% [34.3%] of their responses in the direction of concavity [concavity] and in 10% [5.7%] in the direction of convexity [linearity]. For losses, subjects classified as convex [linear, concave] erred in 23.6% [2.2%, 28.0%] of their responses in the direction of concavity [concavity, convexity] and in 14.6% [6.7%, 6.0%] of their responses in the direction of linearity [convexity, linearity].

Table 3

Individual Parametric Classification of Utility Curvature for Gains and Losses

	Losses			
Gains	Concave	Linear	Convex	Total
Concave	5	13	20	38
Linear	2	6	4	12
Convex	7	4	6	17
Total	14	23	30	67

The parametric classification confirmed concavity for gains but showed more pronounced convexity for losses. Table 3 summarises the results of the individual analysis. There were significantly more subjects with concave utility than with convex utility for gains and significantly more subjects with convex utility than with concave utility for losses ($p < 0.01$ in both cases). The medians of the individual estimates of the power coefficients were 0.91 for gains and 0.96 for losses. They are consistent with slight concavity for gains and with slight convexity for losses. Both medians were not significantly different from 1, however ($p = 0.075$ and $p = 0.085$ respectively). Table 3 also shows that the most common pattern was concave utility for gains and convex utility for losses. Twenty subjects belonged to this category. There were only 5 subjects who behaved according to the common assumption in economics that utility is everywhere concave.

Let us now turn to the aggregate analysis. Figure 3 shows the shape of the utility for gains and the utility for losses based on the median data. The *x*-axis shows the medians of the elicited elements of the sequences,[6] the *y*-axis their utility. Both for gains and for losses we could reject the hypothesis that the difference between successive elements of the elicited sequence was constant, the case corresponding to linear utility ($p < 0.01$). For gains, the difference between successive elements of the

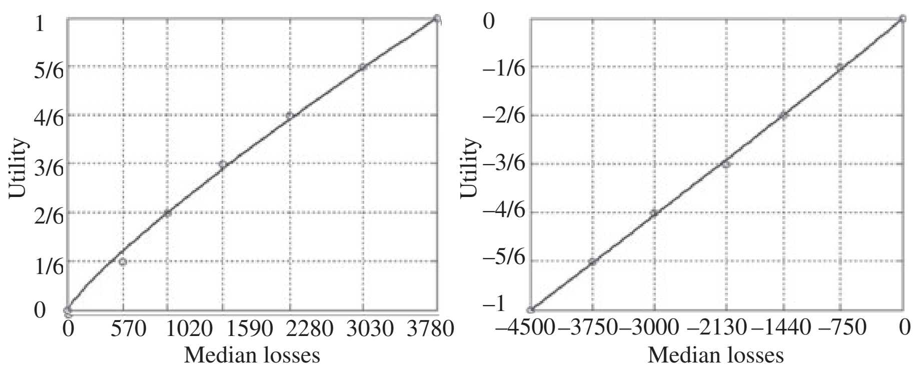

Fig. 3. *The Utility Function for Gains and for Losses Based on the Median Data*

[6] The interquartile ranges for the 6 elicited elements of the sequence for gains were [390, 750], [660, 1500], [900, 2205], [1260, 3000], [1500, 3750], and [1740, 4500] and for losses [−960, −510], [−1800, −960], [−2640, −1455], [−3600, −2085], [−4500, −2610], and [−5640, −3450].

sequence generally increased, consistent with concave utility; for losses, it slightly decreased consistent with slight convexity of utility. The estimates of the power coefficients based on the median data were 0.84 for gains and 0.97 for losses. Both differed significantly from 1 ($p < 0.01$ for gains and $p = 0.05$ for losses). Figure 3 also shows the estimated power functions. As the Figure shows, the fit of the estimations was very good.

4.2. *Time Weights*

The individual classification showed evidence of decreasing impatience. For gains, 55 subjects were decreasingly impatient and 12 were increasingly impatient. For losses, 47 subjects were decreasingly impatient, 18 were increasingly impatient and 2 were constantly impatient. The difference between the proportion of decreasingly impatient subjects and the proportion of increasingly impatient subjects was significant both for gains and for losses ($p < 0.01$ in both cases).

Table 4 shows the median time weights for each delay, as well as the median annual discount rates implied by these weights. The median implied discount rates were relatively low, both for gains and for losses. They are comparable to the discount rates obtained by Andersen *et al.* (2008) in spite of the differences in experimental design. The discount rates declined over time, consistent with decreasing impatience, but the decline was modest. The pattern of decreasing impatience was significant, however. The hypothesis that the implied annual discount rates were constant could be rejected both for gains and for losses ($p < 0.01$ in both cases).

Let us next turn to the estimation of the various discounting models. At the aggregate level the models did not differ much in terms of goodness of fit. For gains, constant sensitivity gave the best fit, for losses hyperbolic discounting. Table 5 shows that at the individual level the results were more conclusive. The data suggested a 40–60 dichotomy (see column 5 of Table 5): constant discounting fitted the data best for about 40% of the subjects, both for gains and for losses; the remaining subjects were nonconstant discounters. A majority of the non-constant discounters behaved most consistently with power discounting. For few subjects the popular quasi-hyperbolic and hyperbolic models provided the best fit. The results were not sensitive to the specification of the unit of time.

Two interesting observations can be made about the parameter estimates. First, the medians of the parameter β in quasi-hyperbolic discounting were close to 1, suggesting that the immediacy effect was limited in size. In fact, there were only 13 [7] subjects for whom β was significantly smaller than 1 for gains [losses].

The other notable parameter is the parameter b in the constant sensitivity model. Prelec (2004) developed an index of decreasing impatience, which reflects the extent to which people deviate from constant discounting. It is easy to show that one minus the parameter b in constant sensitivity is equal to Prelec's (2004) index. If b is less than [exceeds] 1 then the decision maker is decreasingly [increasingly] impatient. According to their b-values, 48 subjects were decreasingly impatient for gains and for losses with the remaining 19 subjects being increasingly impatient. For 18 [19] subjects b was significantly smaller than 1 for gains [losses]. Hardly any subjects were signifi-

Table 4

Time Weights and Implied Annual Discount Rates for Gains and Losses

Outcome		3 months	6 months	1 year	2 years	3 years	4 years
Gains	Median time weight	0.97 (0.94–0.99)	0.94 (0.90–0.98)	0.92 (0.79–0.95)	0.83 (0.67–0.92)	0.74 (0.49–0.89)	0.74 (0.35–0.87)
	Annual discount rate	12.8% (4.0–26.9)	12.3% (4.6–24.3)	8.3% (4.9–27.1)	9.7% (4.5–22.0)	10.8% (3.9–27.1)	8.0% (3.6–29.9)
Losses	Median time weight	0.98 (0.96–0.99)	0.97 (0.93–0.99)	0.95 (0.89–0.97)	0.90 (0.82–0.94)	0.87 (0.75–0.93)	0.83 (0.65–0.91)
	Annual discount rate	6.8% (3.8–14.6)	6.6% (2.3–16.1)	5.6% (3.3–12.0)	5.5% (3.1–10.7)	4.7% (2.4–10.0)	4.6% (2.4–11.3)

Note. Interquartile ranges (IQRs) are in parentheses.

Table 5

Medians of the Individual Parameter Estimates for the Discounting Models

Model	Parameter	Value	IQR	Best fit	Adj. R^2	IQR
Gains						
Constant	δ	0.10	0.02–0.23	23	0.78	0.55–0.91
Power	α	0.19	0.08–0.33	26	0.78	0.63–0.86
Proportional	γ	0.10	0.04–0.27	0	0.79	0.57–0.89
Quasi-hyperbolic	β	0.99	0.94–1.00	9	0.75	0.65–0.92
	δ	0.08	0.03–0.23			
Hyperbolic	γ	0.22	0.01–1.96	5	0.80	0.64–0.91
	α	0.14	0.08–0.37			
Constant sensitivity	a	0.07	0.02–0.23	4	0.82	0.65–0.93
	b	0.81	0.54–1.08			
Losses						
Constant	δ	0.05	0.02–0.10	26	0.78	0.47–0.90
Power	α	0.11	0.05–0.21	24	0.78	0.60–0.88
Proportional	γ	0.05	0.02–0.11	0	0.79	0.54–0.89
Quasi-hyperbolic	β	0.99	0.03–0.23	5	0.78	0.66–0.89
	δ	0.04	0.02–0.08			
Hyperbolic	γ	0.31	0.01–1.91	4	0.83	0.66–0.89
	α	0.10	0.05–0.21			
Constant sensitivity	a	0.03	0.00–0.08	8	0.81	0.46–0.91
	b	0.74	0.54–1.07			

cantly increasingly impatient: 3 for gains and 5 for losses. Overall, Table 5 shows that there was modestly decreasing impatience.

The final two columns of Table 5 show that the fit of the discounting models was good. The medians of the individual adjusted R^2s were all close to 0.80 and the interquartile ranges show that for the large majority of our subjects the fit was good to very good.

4.3. *Comparison of Gains and Losses*

Our data provide evidence of a gain–loss asymmetry in discounting. Table 4 shows that the time weights were higher for losses than for gains. The difference was significant for all delays. Most parameters in the various discounting models differed significantly between gains and losses. Exceptions are γ in hyperbolic discounting, β in quasi-hyperbolic discounting and b in constant sensitivity. These findings suggest that the deviations from constant discounting, the immediacy effect, captured through β in quasi-hyperbolic discounting, and the degree to which impatience decreases over time, captured through b in constant sensitivity, are similar for gains and for losses. Hence, our data suggest that the degree of impatience differs between gains and losses, but the change in impatience over time is similar. This conclusion only holds at the aggregate level. At the individual level, there was no significant relation between the change in impatience for gains and the change in impatience for losses: the correlation was only 0.05.

4.4. *The Effect on the Time Weights of Assuming Linear Utility*

Most previous studies that estimated discount rates assumed linear utility. Let us briefly explore the effect of imposing linear utility. The bias introduced by imposing linear utility was small at the aggregate level. The median time weights were lower under linear utility and, consequently, the annual discount rates were higher but the differences with the parameter-free time weights were generally not significant. The gain–loss asymmetry became more pronounced when linear utility was assumed. The above observations did not hold at the individual level: there we observed large differences between the parameter-free time weights and the time weights under linear utility.

The fit of the discounting models was significantly better when we used the utility-adjusted time weights than when we imposed linear utility. The conclusions about the relative fit of the discounting models were, however, hardly affected by assuming linear utility.

5. Discussion

Loewenstein and Prelec (1992) suggested that utility in intertemporal choice be concave for gains and convex for losses and showed that this shape could explain economic behaviour. Examples include the tendency of businesses to reduce investment during economic downturns and the disposition effect, the tendency of people to hold on to losing stocks and to real estate that has dropped in value, which depresses trading volume during market downturns. We found some evidence for their conjecture. Concave utility for gains and convex utility for losses was the most common pattern, but the degree of curvature was modest, suggesting that the shape of utility alone is unlikely to explain the aforementioned phenomena.

Interestingly, our findings on the degree of utility curvature were close to those obtained in decision under uncertainty and in decision under risk. Economists have traditionally argued that utility differs across domains and, hence, that the utility function that is relevant for decision under risk cannot be employed in other contexts, such as intertemporal decision making. In applied economics transferability of utility is, however, commonly assumed. For instance, in health economics measurements of intertemporal utility are routinely used in welfare comparisons. Our findings offer some tentative support for the transferability of utility.[7] Obviously, more robust evidence is required before any firm conclusions can be drawn.

The discount rates we observed are relatively modest compared to those observed in several earlier studies. One reason for this disparity may be the assumption of linear utility made in most previous studies: discounting utilities instead of money amounts tends to decrease discount rates. A second reason may be the size of the outcomes used. We used large money amounts. Previous evidence suggests that larger amounts are discounted less.

Our data suggest decreasing impatience, declining discount rates over time but only to a limited degree. For only 25% of our subjects was the parameter b in Ebert and Prelec's (2007) constant sensitivity model significantly less than 1, indicating

[7] For further evidence see Abdellaoui *et al.* (2007).

decreasing impatience. At the individual level, the case for decreasing impatience seems less convincing than at the aggregate level. One reason why we found less decreasing impatience than previous studies could be related to the elicitation procedure used. Ahlbrecht and Weber (1997) observed decreasing impatience in a matching task, where people are directly asked for their indifference value, but not in a choice task. Our elicitation procedure was also choice-based. On the other hand, most previous studies in the literature used matching tasks. Another reason could be the relative size of the outcomes used in our study. It is well known that larger outcomes are discounted less. Decreasing impatience may also be less pronounced for larger outcomes. We are not alone in finding limited support for decreasing impatience. Andersen *et al.* (2008) observed more support for constant impatience than for decreasing impatience. As mentioned in the Introduction, several recent studies even observed increasing instead of decreasing impatience (Chesson and Viscusi, 2003; Rubinstein, 2003; Gigliotti and Sopher, 2004; Read, Airoldi and Loewenstein, 2005; Sayman and Öncüler, 2009; Attema *et al.*, 2008). The case for decreasing impatience seems less convincing than is sometimes claimed.

Many economic studies have used the quasi-hyperbolic discounting model and, in particular, the existence of the immediacy effect, to explain phenomena like undersaving, underinvestment in retirement plans and addictive behaviour. Our data cast doubt on such explanations: the immediacy effect was very modest and many of our subjects did not display it at all. We do not pretend to provide the definitive verdict on the importance of the immediacy effect but our data at least caution against the use of quasi-hyperbolic discounting to explain pervasive economic phenomena.

Our study, like the great majority of studies on intertemporal choice, might be criticised for studying the discounting of money amounts. The constant discounting model was written over consumption and not over money amounts. This distinction is important, since a decision maker who faces no liquidity constraints should value money according to the market interest rate independently of his preferences (Cubitt and Read, 2007). Hence, choices over money do not necessarily measure the true discount function but some combination of the discount function, liquidity constraints, and boundedly rational thinking about money. We chose our setup for the following reasons. First, one of the aims of our study was to assess the impact of utility curvature on discounting. To enhance comparability with previous studies we chose not to deviate from the convention in the field of intertemporal choice to use money as the outcome domain.

A second reason is that if we do not consider preferences over money amounts then measuring the discounted utility model becomes problematic. It is hard to see how one could construct sequences of, say, candy bars like we did for money amounts and, to the best of our knowledge, no study has been able to measure the utility of consumption for intertemporal choice. Developing methods to measure the intertemporal utility for consumption is an important topic for future research but as yet this cannot be fulfilled and the above criticism seems to lead to the undesirable conclusion that economics is unable to quantify one of its principal models.

A third reason that helps to justify using money as outcomes is the following. There are good grounds to believe that the distinction between consumption and money amounts, while valid in theory, is less relevant for our study. Whereas a rational person should value money according to the market interest rate most of our subjects did not do so. Remember that we asked our subjects to think aloud while answering the questions. Hardly any of our subjects mentioned issues of borrowing and lending. The vast majority of our subjects referred to consumption opportunities when asked to explain their choices. In terms of Thaler's (1985) theory of mental accounting, subjects coded the outcomes not as investment possibilities but as additional or foregone consumption opportunities. The large individual differences in discount rates and the lower discounting of losses than gains (whereas borrowing rates tend to exceed saving rates) are inconsistent with a financial market explanation and suggest that this is not the main explanation for our findings. Benzion *et al.* (1989) tested whether a financial market explanation could explain their data but concluded that it was inconsistent with their findings.

Finally, there are advantages of using money amounts instead of consumption. Future consumption evokes emotions such as anticipation, dread, savouring and self-control that have nothing to do with the structure of the discount function. Using money reduces these problems.

We used hypothetical choices in our experiment. Studies that investigated the differences between real and hypothetical money amounts in intertemporal decision making provide no clear-cut evidence. Kirby and Marakovic (1995) found that discount rates were lower for hypothetical rewards. Coller and Williams (1999), on the other hand, found no effect after controlling for possible confounders. Summing up, Frederick *et al.* (2002, p. 389) conclude that 'there is, as yet, no clear evidence that hypothetical rewards are discounted differently than real rewards'. There has been considerable debate outside of the context of intertemporal choice whether hypothetical choices are representative of real decisions. Several authors have argued that there is no basis requiring the use of real incentives in experiments (Tversky and Kahneman, 1992; Loewenstein, 1999; Rubinstein, 2001). Indeed, empirical studies generally find that real and hypothetical incentives give qualitatively similar results, although real incentives tend to reduce data variability (Camerer and Hogarth, 1999; Hertwig and Ortmann, 2001). It should be mentioned though that some studies have observed systematic differences between real and hypothetical choices (Cummings *et al.*, 1995; Holt and Laury, 2002).

Our method used chained measurements, i.e. answers from previous questions were used as inputs in later stages. A possible danger of using chained measurements is error propagation: errors in earlier responses get transferred to later responses. Bleichrodt and Pinto (2000) and Abdellaoui *et al.* (2005) examined the effect of error propagation on chained measurements and concluded that it had little impact. Since we used a similar chaining process to their's, we are inclined to conclude that the effect of error chaining is limited in our study. Blavatskyy (2006) showed theoretically that the elicitation method we used is optimally efficient in the sense that it minimises the effect of error relative to other measurement procedures.

Both for gains and for losses, we measured six points of the instantaneous utility function and six points of the discount function. The reason for not collecting more data was that increasing the number of questions would risk subjects getting tired and less careful and, consequently, a decrease in the quality of the data. In decision analysis, a general rule is that five measurements suffice to determine functions with sufficient precision (von Winterfeldt and Edwards, 1986). There is a danger when the number of observations is small of outliers coming to drive the results. We do not believe that outliers have affected our conclusions. First, we spent considerable effort in obtaining good quality data by interviewing subjects individually and by asking them to explain their choices. Second, we included several consistency tests. None of these indicated substantial or significant deviations from the original measurements. Finally, the fit at the individual level was good. Median adjusted R^2s were all close to 0.80 and for the large majority of subjects the fit was good to very good.

Our measurement method amounts to measuring sequences of gains $g^1, \ldots, g^6$ and losses $\ell^1, \ldots, \ell^6$, which are subject-specific. Consequently the amounts g^6 and ℓ^6, used in the measurement of the discount function, will typically differ across subjects. Indeed, our data indicated substantial variation in these amounts: g^6 varied between €660 and €11,460 and ℓ^6 varied between −€8,700 and −€720. If there were a relationship between the sizes of g^6 and ℓ^6 and the discount rates then this may have affected our aggregate analyses. We tested whether such a relationship existed but found no evidence for it. The null hypothesis of no relationship could not be rejected ($p = 0.17$ both for gains and for losses). This is consistent with earlier findings that observed no magnitude effect for money amounts exceeding $200 (Read, 2004).

A crucial assumption in our method was that subjects behaved according to discounted utility. This model underlies all of the main discounting models used in the literature. A central property of discounted utility is intertemporal additivity. There is some evidence of violations of intertemporal additivity (Loewenstein and Sicherman, 1991, Frank and Hutchens, 1993, Baucells and Sarin, 2007*b*). It is not clear how important these violations are. As mentioned by Loewenstein and Prelec (1992), they seem particularly relevant when evaluating complete alternative sequences of outcomes like savings plans or multi-year salary contracts. In our experiment, we considered, however, elementary types of intertemporal choices. We tried to mitigate the possible effect of violations of intertemporal additivity by using prospects in the measurement of utility. We learned from pilot tests that using prospects made it more likely that people viewed things that happened at different points in time as separate and, hence, behaved more in line with intertemporal additivity.

6. Conclusion

This article has presented a parameter-free method to measure the discounted utility model in its entirety. Hence, we are the first to measure the utility function in intertemporal choice without imposing any assumptions and we provide more robust evidence on the discounting of monetary outcomes. We found concave utility for gains and slightly convex utility for losses, which supports a hypothesis put forward by Loewenstein and Prelec (1992). Our data confirmed decreasing impatience. The decrease was, however, modest and the fit of constant discounting was rather good. At the aggregate level, we observed no significant differences between the various discounted utility models that we considered. At the individual level, there was an approximate 40%–60% dischotomy between constant discounters and non-constant discounters. Of the non-constant discounted functions, Harvey's (1986) power discounting function fitted the data best. Our data were less supportive of the widely-used quasi-hyperbolic discounting model: it did not fit particularly well and the observed immediacy effect was small. Finally, we found evidence for a gain–loss asymmetry in the time weights, which contradicts earlier conclusions that the gain–loss asymmetry is due to a framing effect (Shelley, 1993; Ahlbrecht and Weber, 1997).

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