

The impact of bank size, capital structure and asset dependence on social welfare

Sander Muns^{a,b,1}, Chen Zhou^{b,c,2}

^a*CPB Netherlands Bureau for Economic Policy Analysis*

^b*Erasmus University Rotterdam and Tinbergen Institute, The Netherlands*

^c*De Nederlandsche Bank, The Netherlands*

Abstract

We investigate how social welfare is jointly affected by bank size, banks' capital structure, and asset dependence across banks. The model suggests that banks always prefer a capital ratio below the socially optimal level, while banks prefer an extra large size when cross-sectional dependencies are typically low. More stringent capital requirements result in larger banks under low bankruptcy costs, high financing costs and high taxes. To enhance social welfare, policies on capital are nevertheless more effective than policies on size. This is particularly true when dependencies are low, i.e., during economic booms. Our results are in support of the countercyclical capital buffers in the Basel III proposals and imply a negative association between the stringency of monetary policy and prudential policy.

Keywords:

banking regulation, bank size, capital requirements, asset dependence

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¹corresponding author, email: s.muns@cpb.nl.
postal address: Centraal Planbureau, P.O. Box 80510, 2508 GM The Hague, The Netherlands

²zhou@ese.eur.nl

1. Introduction

The financial crisis has forcefully shown that the transmission of adverse shocks on the financial system may have a severe effect on social welfare. Three fundamental characteristics of the financial system are crucial in this transmission mechanism. First, by their typically high leverage, the low capital ratio makes individual financial institutions highly sensitive to asset losses. Second, similar asset holdings across different institutions suggests that a single shock may hit multiple institutions simultaneously. Third, some financial institutions are extraordinarily large, and consequently their failure would impose a large shock to the system which is more severe than the failure of a group of small institutions. Using a stylized model, we investigate the impact of capital structure, asset interdependence and size on social welfare in a theoretical model. In addition, we evaluate policy interventions directly or indirectly designed towards these characteristics.

Thus, the stability of a financial system and the potential welfare loss once the system fails depend on three important bank characteristics: the capital structure, the asset portfolio composition, and the bank size. All three characteristics are a consequence of banks' management decisions over time. For each characteristic, banks tend to choose a level that is suboptimal for society. First, because equity financing is not tax deductible and banks' shareholders have limited liability in case of a default, bank owners prefer a lower level of capital than optimal for society. Second, after a major stress event, regulators tend to save banks ex-post to prevent a systemic banking crisis. In particular, large banks or a large number of smaller identical banks may trigger a major stress event. Banks anticipate such policies by growing large and holding similar asset portfolios ex-ante.

We build an equilibrium model in which banks make management decisions on capital, and size with the following considerations. First, banks determine their capital ratio by balancing the financing costs of debt and equity. Following the corporate finance literature, the tax shield is a key factor that favours debt financing, while a high level of debt corresponds to a high default probability. Second, large banks benefit from a more diversified portfolio, and thus a lower risk of assets. However, large banks may suffer from higher operational costs due to the complexity to manage such large banks. Third, similar to Ibragimov et al. [17] banks invest in correlated assets in a myopic way. They do not take into account the adverse effect on social welfare of correlated asset portfolios.

To evaluate the impact on social welfare, we consider two welfare measures. First, we define a welfare measure as the total surplus generated by bank's loans. Second, we define social welfare by subtracting from the former welfare measure the potential cost of a systemic crisis. This cost depends on the frequency of a systemic crisis defined as the probability that some fraction of debt in the financial system is not repaid.

A key feature of our theoretical model is that the three characteristics – capital ratio, asset interdependence and size – are endogenously interrelated. Importantly, this interrelation affects systemic risk and social welfare in an ambiguous way. For example, large banks invest in a more diversified portfolio of assets than small banks. Consequently the asset dependence between two large banks tends to be higher which raises systemic risk. On the other hand, systemic risk may also be lower, as large banks bear less individual risk due to their diversification benefits. Nonetheless, the lower individual risk of large banks gives more room to lever the balance sheet. This partly offsets the potential reduction in systemic risk due to diversification. This example demonstrates that a system consisting of a few large banks that are well diversified but also highly leveraged may or may not be more systemically risky. Our theoretical model considers the interaction between the three characteristics and provides a corresponding equilibrium analysis for policy evaluations.

Our main findings are as follows. First, without any regulation banks make a suboptimal decision on size and the capital ratio. Banks hold less capital than socially optimal while the size may be larger or smaller than socially optimal. Second, if the bankruptcy costs are low, while financing costs and taxes are high, imposing a higher capital requirement results in larger banks, and imposing higher capital requirements always increases social welfare. On the contrary, restricting bank size is not necessarily welfare improving in a crisis period where asset interdependence is high and investment opportunities are scarce.

The policy implication is that capital requirements are the preferred policy measure during economic booms as well as during crisis periods. In particular during economic booms where asset interdependence is low, capital requirements may have an additional mitigating effect on systemic risk through its indirect effect on a smaller preferred size by banks.

To the best of our knowledge, this paper is the first to address the joint impact of the three characteristics of banking on systemic risk and social welfare. The recent paper of De Nicolò et al. [11] is most related to our approach. They calibrate a dynamic model to assess the effect of capital

requirements on social welfare. They find that capital requirements affect both bank lending and social welfare with an inverted U-shape. Our model complements this paper by including size as an additional endogenous factor and by allowing for capital-size interaction effects. In addition, we calibrate our model by considering more severe shocks exhibiting heavy tails.

Our model on the choice of capital structure is related to the corporate finance literature on optimal capital structure. Starting with Modigliani and Miller [22], the theoretical corporate finance literature has studied at length the determinants of the optimal capital structure, e.g., Kraus and Litzenberger [18], Brennan and Schwartz [7], Bradley et al. [6], and Miao [21] among many others. The main message of this literature is that taxes and bankruptcy costs are the key determinants of the capital structure. Our model shows how these determinants affect social welfare through affecting banks' decision on the capital structure.

Although our model is a static equilibrium model, our study can be compared with the literature on the interrelation between the optimal capital ratio and the business cycle, see, e.g., Angeloni and Faia [4], and Repullo and Suarez [25]. Our static equilibrium model can be interpreted as the unconditional long-run effect. Alternatively, for a specific parameter choice we interpret our model as conditional on some state of the business cycle.

We model the asset dependence between banks to address the systemic risk in the financial system. This is in line with the literature relating systemic risk to banks' common exposure, see Acharya [1], Wagner [29], and Ibragimov et al. [17].³ For exogenous capital-size characteristics, Ibragimov et al. [17] find that the similarity of financial intermediaries has an adverse impact on systemic risk and hence social welfare. This externality depends crucially on the distribution of the shocks.

Lastly, this study also contributes to the emerging literature on the impact of bank size on banks' financing decisions and risk taking, and eventually systemic risk. On the one hand, Hughes and Mester [16] find that large banks have lower funding costs on uninsured deposits. On the other hand, Pais and Stork [23], Laeven et al. [19], and Boyd and Heitz [5] document a non-trivial adverse

³Systemic risk is usually attributed to the following channels: direct linkages from mutual exposures, indirect linkages from common exposures, and interdependence from information contagion. We focus on the indirect linkage channel. Broad surveys on systemic risk are in De Bandt and Hartmann [10], Santos [26], and Galati and Moessner [14].

effect of bank size on systemic risk. In addition, Davies and Tracey [9] report that large banks do not benefit from economies of scale after a correction for the potential bailout effect. Most of this literature are empirical studies to evaluate the size impact, whereas we provide a theoretical model.

The paper proceeds as follows. Section 2 presents the model and derives the banking equilibrium for a bank of a given size and a given capital ratio. In Section 3, we derive the optimal size and capital structure decision given the risk profile of the bank, and we consider the existence of a general equilibrium. Section 4 defines the social welfare function. A calibration of the model is in Section 5. Section 6 concludes. The proofs of the analytical results are postponed to the Appendix.

2. The model

2.1. Bank balance sheet and operations

The model is a one period model in a risk neutral world with a risk free rate r_f .⁴ We denote gross (net) interest rate with an upper(lower) case letter. In this general equilibrium model, banks choose total assets x (referred to as size) and the equity to total assets ratio y (referred to as capital ratio). Consequently, a bank's equity and debt is $E = xy$ and $D = x - E = x(1 - y)$, respectively.

To ensure the existence of banks, we adopt the standard assumption that investors in debt and equity cannot lend directly to entrepreneurs. This is consistent with the theory of financial intermediation in Diamond [12]. We assume that banks are perfectly competitive, as in Diamond and Dybvig [13], and De Nicolò et al. [11] among others.

Bank assets

The assets are loans to finance projects of investors such as entrepreneurs or mortgageholders. We simply refer to entrepreneurs. A bank earns a stochastic return on assets $R_A = \mu + \sigma X$ with mean $\mathbb{E}[R_A] = \mu$, standard deviation $\sigma(x, \mu)$, and X is a random variable with zero mean and unit variance. We denote the probability density function and the distribution function of X by f and F , respectively. The mean asset return μ is endogenously determined by the competition between banks (see section 3 for details). The standard deviation $\sigma(x, \mu)$ is a function of the size x and the mean asset return μ . It decreases with size due to the potential diversification benefits, i.e.,

⁴Risk neutral settings are also in Miao [21], Titman and Tsyplakov [28], Repullo and Suarez [25], and Allen et al. [2].

$\sigma_1(x, \mu) \leq 0$.⁵ Throughout the paper, we use f_i to indicate the partial derivative of a function f to its i -th argument, while f_x denotes the total derivative to a variable x .

Charging a higher loan rate to entrepreneurs increases the probability on a loan restructuring. In addition, a higher rate may particularly deter risk-averse entrepreneurs, and less risky projects are possibly matched to a cheaper source of financing. Hence, we assume that the standard deviation $\sigma(x, \mu)$ is non-decreasing with the mean asset return, i.e., $\sigma_2(x, \mu) \geq 0$.

Operational cost

Bank operations have a cost function $c(x)$ that depends on size x . We assume that the total operational cost $c(x)$ and the average operational cost $\bar{c}(x) = c(x)/x$ are both convex in x with $\bar{c}(x)$ minimized at some $x_0 \geq 0$. Intuitively, at some point there are diseconomies of scale in operational costs by inefficiencies such as bureaucracy costs. This assumption is supported by Davies and Tracey [9] who show that large banks do not benefit from economies of scale. Empirical evidence for diseconomies of scale in banking is in Allen and Rai [3].

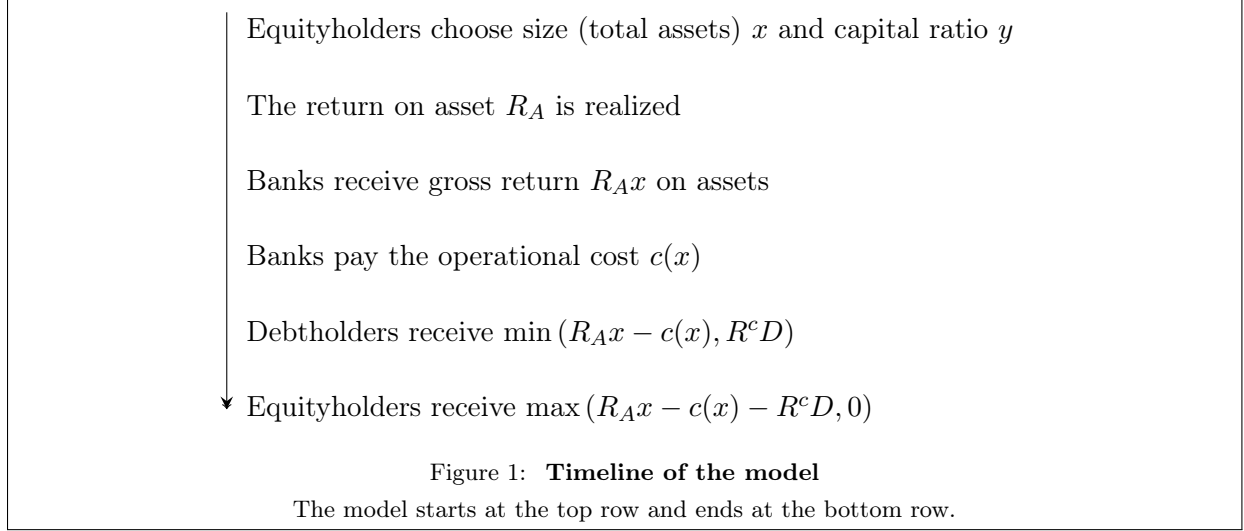
Debt financing

We assume that debt financing has a fixed maturity of one period. In addition, by assuming $y < 1$, we rule out the boundary case of no debt financing where $D = 0$. A bank promises to pay at the end of the period the gross contracted interest rate R_c on debt outstanding. It can only fulfil this promise if the return on assets R_A is sufficient. Namely, a low realization of R_A results into default if total cost exceeds total revenue: $c(x) + R_c D > R_A x$. In case of a default, debt holders incur an additional deadweight bankruptcy cost $\eta > 0$ on each unit of debt to recover their debt outstanding.⁶ In this risk neutral setting, risk aversion is another interpretation of this bankruptcy cost. Since holders of defaulting debt only receive $R_A x - c(x) - \eta D$ instead of the contracted $R_c D$, the expected gross return on debt is

$$\mathbb{E}[R_D] = \mathbb{E}\left[\min\left(\frac{R_A x - c(x)}{D}, R_c\right)\right] - \eta PD, \quad (1)$$

⁵Since σ refers to the standard deviation per unit of assets, we do not assume that the standard deviation of total assets decreases in size.

⁶A similar feature is in Strebulaev [27] and De Nicolò et al. [11]. Our bankruptcy cost differs from costs of illiquidity studied in Cifuentes et al. [8] and Wagner [30]. They model illiquidity costs by fire sale losses which is not a deadweight loss.



where the probability of default is $PD = \mathbb{P}(R_A x < c(x) + R_c D)$.

Equity financing

The equity holders are the residual claimants who receive the after-tax profit at the end of the period. The end-of-period equity value before taxes (E_{BT}) equals revenues minus the sum of operational costs and costs of debt financing. Accordingly, the initial expectation of the end-of-period equity is

$$\mathbb{E}[E_{BT}] = \mathbb{E}[[R_A x - c(x) - R_c D]^+] , \quad (2)$$

where $[x]^+ = \max(x, 0)$. The bank pays the corporate tax rate $\tau = 1 - \bar{\tau}$ on before-tax profits. Note that the cost of debt financing is tax deductible, while the cost of equity financing is not.

Figure 1 displays the sequence of events.

2.2. Banking equilibrium

In the banking equilibrium, the risk neutrality of investors implies that equity holders and debt holders earn on average the risk free rate on their investments. From these returns, we determine for given size x and given capital ratio y , the following three equilibrium values: (i) the mean return on assets (μ^*), (ii) the probability of default (PD^*), and (iii) the contracted interest rate on debt (R_c^*).

Proposition 1. *If an equilibrium exists for the size-capital ratio pair (x, y) , then a banking equilibrium is the triple (μ^*, PD^*, R_c^*) that satisfies the following three equations*

$$\mu^* = 1 + \bar{c}(x) + \frac{r_f}{\bar{\tau}}y + (r_f + \eta PD^*)(1 - y) \quad (3)$$

$$\mathbb{E}[\min(X, F^{-1}(PD^*))] = -\frac{(1 + r_f/\bar{\tau})y}{\sigma(x, \mu^*)} \quad (4)$$

$$\mathbb{E}\left[\min\left(\frac{R_A - \bar{c}(x)}{1 - y}, R_c^*\right)\right] = R_f + \eta PD^*. \quad (5)$$

Note that μ^* and PD^* can be solved simultaneously from (3) and (4) and then R_c^* follows from (5). Since F^{-1} is an increasing function, equation (4) can be rewritten as

$$PD^* = g\left(\frac{\sigma(x, \mu^*)}{(1 + r_f/\bar{\tau})y}\right). \quad (6)$$

where g is some increasing function that depends on the given cdf F and the parameters r_f and $\bar{\tau}$.

An implicit expression for PD^* follows by substituting (3) into the RHS in (6). The obtained equation has PD^* as the single unknown. Since both hand sides of the obtained equation are increasing in PD^* , there may exist multiple equilibriums, or a unique equilibrium, or even no equilibrium. The next proposition provides sufficient conditions for the existence and the uniqueness of the banking equilibrium.

Proposition 2. Existence: *For all (x, y) , a banking equilibrium (μ^*, PD^*, R_c^*) satisfying (3)–(5) exists if at least one of the following two conditions holds*

$$1 + \frac{r_f}{\bar{\tau}} \leq -\inf(X) \min_{\tilde{x}, \mu} \sigma(\tilde{x}, \mu) \quad (7)$$

$$\sigma_2(x, \mu) \equiv 0. \quad (8)$$

Uniqueness: *The equilibrium is unique if (8) holds.*

An equilibrium may not exist if none of the sufficient existence conditions hold. For example, debt holders may prefer a small PD to maximize the payment on their debt, while the equity holders in turn may prefer for any asset return μ a strictly higher PD to benefit from their limited liability. In such cases, (3) and (4) have no common solution (μ^*, PD^*) . As a consequence, the equityholders

and bondholders cannot agree upon an equilibrium (μ^*, PD^*, R_c^*) . Therefore, we will impose the condition (8) in the calibration in Section 5.

Suppose (8) holds, i.e., σ is invariant to the mean asset return μ . ~~We consider the impact of the financing cost r_f and the operational cost $\bar{c}(x)$ on the endogenous probability of default PD^* .~~ In this case (6) implies that PD^* does not depend on μ^* . Thus, increasing r_f has no feedback effect on PD^* through μ^* . From (6), PD^* decreases with respect to the financing cost r_f .

The average operational cost $\bar{c}(x)$ has no direct effect on PD^* (see (6)). Following the equilibrium equation (3), the asset return μ^* compensates any effect of the operational cost on PD^* . The random spread $R_A - \bar{c}(x)$ in the equilibrium equation (5) is then unaffected by changes in the operational cost. Therefore, PD^* (and R_c^* in (5)) is unaffected by the operational cost.

To study the impact of taxes and bankruptcy costs on the equilibrium, first assume that both are absent ($\tau = \eta = 0$). Then, equation (3) leads to $\mu^* = \bar{c}(x) + R_f$, which says that the required marginal revenue on assets equals its marginal cost. Having taxes and bankruptcy costs raises the marginal cost, and, consequently, the marginal revenue μ^* in equilibrium.

3. Competitive equilibrium

In this section we obtain equilibrium results when the size x and capital y are endogenously determined by the competitive setting in the banking market.

3.1. Model setup

As in the banking equilibrium in section 2.2, entrepreneurs are price takers since they stick to the offered loan rate of the banks. Following Diamond [12], we assume information asymmetry between entrepreneurs and banks. More specifically, banks know the aggregate return distribution of projects, but do not know the return distribution of a specific project. By contrast, entrepreneurs do have private information on their project. An entrepreneur initiates a project if the lowest offered rate is below the project's return in case the project is successful. Consequently, the entrepreneur extracts all rents beyond the lowest offered loan rate.

In the perfectly competitive banking industry, the entrepreneurs force banks to demand the lowest possible interest rate. Only banks that charge this minimal rate survive in this perfectly competitive setting. While the charged rate is ex-ante deterministic, the ex-post return on the

bank's assets, i.e., the loans to entrepreneurs, is random with mean μ . Thus, the competitive equilibrium (x^c, y^c) corresponds to the pair (x, y) with the minimal equilibrium mean asset return μ^c . In this competitive equilibrium, efficient banks choose ~~(i) the optimal size x^c where the marginal benefit of a lower volatility ($\sigma_1 < 0$) equals the marginal cost of a higher average operational cost ($\bar{c}' > 0$), and (ii) the optimal capital ratio y^c where the marginal benefit of a lower expected bankruptcy cost equals the marginal cost of a lower benefit from tax deductible debt financing.~~

The equilibrium values of the variables in the competitive equilibrium are denoted with a superscript c . We suppress asterisks for convenience. For instance, $(x^c, y^c) = \operatorname{argmin}_{\tilde{x}, \tilde{y}} \mu^*(\tilde{x}, \tilde{y})$ and

$$\begin{aligned} \mu^c &= \mu^*(x^c, y^c) & \mu_1^c &= \left. \frac{\partial}{\partial x} \mu^*(x, y) \right|_{(x=x^c, y=y^c)} = 0 \\ PD^c &= PD^*(x^c, y^c) & PD_1^c &= \left. \frac{\partial}{\partial x} PD^*(x, y) \right|_{(x=x^c, y=y^c)}. \end{aligned}$$

Denote $x^c(y) = \operatorname{argmin}_x \mu(x, y)$ and $y^c(x) = \operatorname{argmin}_y \mu(x, y)$ as the optimal (competitive) size x and the optimal (competitive) capital ratio y conditional on the other variable.

3.2. Good growth: theoretical analysis

We consider the impact of the size x and the capital ratio y on PD^c . To avoid clutter, we make the following mild technical assumptions on the competitive equilibrium (x^c, y^c) . The global minimum $\mu^c = \mu^*(x^c, y^c)$ is (i) unique, (ii) an interior point, and (iii) the determinant of the Hessian matrix of μ^* is strictly positive at (x^c, y^c) . The assumptions ensure that μ^c is a strict global minimum and the correspondences $x^c(y)$ and $y^c(x)$ are properly defined functions in a neighborhood of (x^c, y^c) . Note that we still allow for multiple local minimums.

We consider the following two conditions:

$$\frac{dx^c(y^c)}{dy} > 0 \qquad \frac{dy^c(x^c)}{dx} > 0 \qquad (C)$$

The conditions in (C) state that a higher capital ratio induces banks to choose a larger size, and vice versa. The conditions in (C) can be regarded as a good growth scenario since banks increase their capital ratio when growing larger.

In practice, it is not obvious if the conditions in (C) hold. Concerning the impact of capital on

size ($\frac{dx^c}{dy}$), a higher capital ratio may result in a larger desire to enjoy diversification benefits by increasing size. However, the more expensive equity financing may make project investments too expensive, and hence reduce the bank size. The conditions in (C) put a higher weight on the former effect. It will turn out that both conditions in (C) hold if either of the two holds.

The following proposition gives the impact of x and y on PD in the competitive equilibrium.

Proposition 3.

(i) If $\sigma_1^c = 0$ then $\frac{dy^c(x^c)}{dx} = \frac{dx^c(y^c)}{dy} = 0$ and all derivatives in Table 1 are zero at (x^c, y^c) except $\frac{\partial}{\partial y} PD^c = \frac{d}{dy} PD^c < 0$.

(ii) If $\sigma_1^c < 0$,

(1) the partial and total derivatives of PD and \bar{c} at an interior competitive equilibrium (x^c, y^c) are as given in Table 1.

(2) the conditions in (C) are equivalent to

$$(1 - y^c)PD_{12}^c < PD_1^c. \quad (9)$$

	$\frac{\partial}{\partial x}$	$\frac{d}{dx}$	$\frac{\partial}{\partial y}$	$\frac{d}{dy}$
PD^c	—	—*	—	—*
\bar{c}^c	+		0	+**

Table 1: **Derivatives of the default probability and of the average cost function.**

The table reports the sign of the partial derivatives and the total derivatives of the default probability $PD^*(x, y)$ and the average cost function $\bar{c}(x, y)$. The derivatives are with respect to size x and capital ratio y at an interior competitive equilibrium that satisfies $\sigma_1^c < 0$. The competitive equilibrium (x^c, y^c) corresponds to the banking equilibrium (μ^*, PD^*, R_c^*) in (3)–(5) with (x, y) such that the mean asset return μ^* is minimal. * if (C) holds. ** if and only if (C) holds.

3.3. Worse growth: numerical example

In practice, large banks tend to have a lower capital ratio. This indicates that the conditions in (C) and some of the results in Proposition 3 may not hold. We thus consider the case when the condition (C) fails to hold. Full analytical results such as in Table 1 cannot be derived for this case. Therefore, a numerical example demonstrates how the total derivatives depend on the parameters.

The average operational cost is convex and given by $\bar{c}(x) = x^\gamma$ with $\gamma > 1$. For analytical convenience, assume that $X \sim U[-\sqrt{3}, \sqrt{3}]$. This uniform distribution satisfies the assumptions $\mathbb{E}[X] = 0$ and $\text{Var}(X) = 1$. A similar assumption is in Acharya [1] and Angeloni and Faia [4]. The calibration study in Section 5 uses a t -distribution for the shocks.

The standard deviation of a unit of assets is $\sigma(x, \mu) = \frac{1}{\sqrt{3}} \left(\frac{\sigma_0}{x^\delta} \right)^2$. ~~Without loss of generality,~~ we assume $0 < \delta \leq \frac{1}{4}$ by the following reasoning. If assets are completely dependent, the standard deviation of total assets is linear in x . While the standard deviation per unit of assets is constant, which corresponds to $\delta = 0$. Since we assume that δ is strictly positive, we could choose a very small δ for the case with completely dependent assets. At the other extreme, assets are completely independent. This means that the standard deviation of total assets is proportional to \sqrt{x} , which corresponds to $\delta = \frac{1}{4}$. The assumption $0 < \delta \leq \frac{1}{4}$ implies that $\zeta := \gamma/\delta > 4$. Define the constants $\tilde{R}_f := \sqrt{1 + \frac{r_f}{\tau}}$, $r_\tau := r_f \left(\frac{1}{\tau} - 1 \right)$, $y_1 := \frac{1}{3} + \frac{2}{3} \sqrt{\frac{\zeta-1}{\zeta+2}}$, $\eta_a := \frac{r_\tau}{1 - \frac{1}{2} \left(\frac{1}{\zeta} \phi(y_1) \tilde{R}_f^\zeta \sigma_0^{-\zeta} r_\tau \right)^{\frac{1}{\zeta-1}}}$, $\eta_b := \frac{(2\sigma_0)^\zeta \zeta}{2\phi(y_1) \tilde{R}_f^\zeta}$, $\tilde{\sigma} := \left[\phi \left(\frac{\zeta}{\zeta+2} \right) \frac{\eta}{\zeta} \left| 2 - \frac{2r_\tau}{\eta} \right|^{1-\zeta} \right]^{1/\zeta} \tilde{R}_f$, and $x_{\max} := \left([\eta - r_\tau] \frac{\sigma_0}{\tilde{R}_f^2} \right)^{1/\delta}$.

The following proposition shows at the competitive equilibrium the relation between size and capital as well as the impact of size and capital on the probability of default. The condition $\eta \notin [\eta_a, \eta_b]$ is only needed to exclude a boundary equilibrium with $y^c = 1$.

Proposition 4. At the competitive equilibrium, the size x^c , the capital ratio y^c and the corresponding derivatives are as given in Table 2.

	$\eta < r_\tau$	$\eta \geq r_\tau \cap \eta \notin [\eta_a, \eta_b]$	
		$\sigma_0 < \tilde{\sigma}$	$\sigma_0 \geq \tilde{\sigma}$
x^c	$[0, x_{\max})$	$[x^c(y_1), x_{\max}]$	
x_y^c	+	-	
y^c	$[0, \frac{1}{3})$	$[\frac{1}{3}, y_1]$	
y_x^c	+	-	
PD_x^c	-		
PD_y^c	-	+	

Table 2: **Total derivatives and intervals at the competitive equilibrium.**

This table presents the intervals of the size x , and the capital ratio y . In addition, it presents the derivatives of x , y , and the default probability PD . The shocks follow a uniform distribution, and risk as in $\sigma(x, \mu) = \frac{1}{\sqrt{3}} \left(\frac{\sigma_0}{x^\delta} \right)^2$ which satisfies $\sigma_2(x, \mu) \equiv 0$. All statistics are at an interior competitive equilibrium (x^c, y^c) . This corresponds to (x, y) with the banking equilibrium (μ^*, PD^*, R_c^*) in (3)–(5) with the smallest μ^* . The parameters η and σ_0 represent the bankruptcy cost and the uncertainty in the economy, respectively. The definitions of r_τ , y_1 , η_a , η_b , $\tilde{\sigma}$, and x_{\max} are on p.12. A plus and a minus sign indicate a positive and a negative total derivative, respectively.

We start with interpreting the interrelation between size and capital. It follows from Table 2 that size and capital are positively related if the bankruptcy cost η is low. A low bankruptcy cost gives a low penalty on a higher default probability from debt financing. This results in low capital ratios and higher marginal diversification benefits when size increases. Consequently, a higher capital ratio results in larger banks in case capital ratios are below $\frac{1}{3}$.

The previous result indicates an important trade-off for implementing a capital requirement when the actual levels of capital are low. In that case, a more stringent capital requirement results into lower individual default probabilities, but larger banks. From the micro-prudential view of Table 2, since $PD_1^c \leq 0$ and $PD_2^c \leq 0$ (Table 1), the larger size reinforces the direct reduction in PD from the larger capital requirements. From a macro perspective, the upward effect on size counteracts the effect of the smaller individual default probabilities on systemic risk by a crowding effect on banks. Closed-form results are unavailable for this trade-off. Accordingly, we will elaborate on this trade-off in a calibration in Section 5.

Next, we consider the total effect on the probability of default. In this respect, we infer from Proposition 4 that large banks are preferred from a micro-prudential point of view. More specifically, an upward cap on size unambiguously increases the probability of default because $PD_x^c < 0$. Again, such a policy is only meaningful in a macro context which we discuss in Section 5.

The total effect of the capital ratio on the probability of default is ambiguous. If bankruptcy costs η and uncertainty σ_0 are both high, a higher capital requirement results in a higher default probability. The high bankruptcy cost and large uncertainty results in high capital and hence a smaller marginal diversification benefit that is dominated by the effect of the operational costs. This explains the negative relation between size and capital ($x_y^c < 0$). Together with a high uncertainty, this results in a dominant upward effect of the smaller size on the probability of default. Hence, this case demonstrates that a larger capital requirement may fail to achieve the micro-prudential goal of a smaller individual probability of default.

For this example, we can derive the impact of the policy parameters at the competitive equilibrium: the uncertainty σ_0 , the bankruptcy costs η , the financing costs r_f , and the tax rate τ .

Proposition 5. Provided an interior equilibrium exists (a sufficient condition is $\eta \notin [\eta_a, \eta_b]$), the sensitivities of the size x , the capital ratio y , and the probability of default PD to the parameters

σ_0 , η , r_f and τ are given in Table 3.

	$\eta < r_\tau$	$r_\tau \leq \eta < \zeta r_\tau$		$\eta \geq \zeta r_\tau$	
		$\sigma_0 < \tilde{\sigma}$	$\sigma_0 \geq \tilde{\sigma}$	$\sigma_0 < \tilde{\sigma}$	$\sigma_0 \geq \tilde{\sigma}$
$x_{\sigma_0}^c$	-				
x_η^c	+	?		+	
$x_{r_f}^c$?			+	
x_τ^c	?			+	
$y_{\sigma_0}^c$	-	+			
y_η^c	+			-	
$y_{r_f}^c$	-				
y_τ^c	-				
$PD_{\sigma_0}^c$	+	?	+	?	+
PD_η^c	-		?		-
$PD_{r_f}^c$?		-	?	-
PD_τ^c	?		-	?	-

Table 3: **Comparative statics at the competitive equilibrium.**

The table presents comparative statics in the competitive equilibrium for size x , capital ratio y , and default probability PD with respect to the uncertainty σ_0 , the bankruptcy cost η , the risk free rate r_f and the tax rate τ . The shocks have a uniform distribution and risk is constant in expected return μ : $\sigma_2(x, \mu) \equiv 0$. All statistics are at the competitive equilibrium (x^c, y^c) which is the pair (x, y) with the banking equilibrium (μ^*, PD^*, R_c^*) in (3)–(5) that has the smallest μ^* . The parameter $\zeta := \gamma/\delta$ follows from the average cost function $\bar{c}(x) = x^\gamma$ and the standard deviation of each unit of assets $\sigma(x, \mu) \propto \sigma_0 x^{-2\delta}$. The definitions of r_τ , η_a , η_b , and $\tilde{\sigma}$ are on p.12. A plus and a minus sign indicate a positive and a negative derivative, respectively.

We discuss the impact of the parameters on size and capital in more detail.

- (i) σ_0 : When the uncertainty σ_0 in the economy is high, the diversification benefit received by large banks is low. Therefore, the optimal bank size is lower and closer to the size that minimizes the operational cost. Because capital protects banks against unexpected shocks, banks tend to hold more capital when the economy is more uncertain. However, this relation is reversed when bankruptcy costs are low. In that case, banks choose an initial low capital ratio due to the low cost of debt. Such highly leveraged banks enjoy the potential upside shocks due to the uncertainty in the economy without suffering from the potential downside by equityholder's limited liability. Therefore, with uncertainty increasing, banks may leverage more to exploit the potential upward shocks.
- (ii) η : A higher bankruptcy cost η gives banks a stronger incentive to reduce their default probability by means of diversification. The positive relation between η and size reflects this incentive.

Therefore, banks are willing to hold more capital and prefer a larger size. In conjunction with the endogenous relation between size and capital, we observe that η is positively related to the capital ratio when η is low whereas this relation is negative when η is high.

- (iii) r_f : A higher risk free rate r_f corresponds to an equally higher expected return on debt and equity. This makes debt relatively more attractive due to its tax deductibility. Hence, the capital ratio decreases when the risk free rate increases, which results in a smaller size. The latter is only observed if the bankruptcy cost is high. Otherwise, the endogenous relation between size and capital makes the relation between the risk free rate and size ambiguous.
- (iv) The explanation for the effect of the tax rate is similar to the explanation with the risk free rate. Tax deductibility and costs of financing are again the relevant channels.

Finally, the impact of the policy parameters on the probability of default PD are mostly in line with the impact on size and the capital ratio. A high uncertainty in the economy, a low bankruptcy cost, a low financing cost, and a low tax rate correspond to a high PD .

4. Social welfare

For a financial system consisting of a number of banks, we study the effect of size, the capital ratio, and the policy parameters on social welfare. ~~Social welfare is defined as the total surplus generated by the banking system discounted by a penalty for the associated systemic risk.~~ We define measures for the two welfare components total surplus and systemic risk in section 4.1 and section 4.2, respectively. Section 4.3 introduces a model for the dependence between shocks. ~~The calibration is postponed to Section 5.~~

4.1. Total surplus

By risk neutrality for the net return on equity and debt ($\mathbb{E}[r_E] = \mathbb{E}[r_D] = r_f$),⁷ and the tax effect of equity ($\mathbb{E}[r_E] = \bar{\tau} (\mathbb{E}[E_{BT}] - E) / E$), the operations of each bank generate the surplus

$$(\mathbb{E}[E_{BT}] - E - r_f E) + (\mathbb{E}[D_{\text{end}}] - D - r_f D) = \left(\frac{1}{\bar{\tau}} - 1\right) r_f E + 0. \quad (10)$$

⁷The expectations of the return distributions are of course under the risk neutral measure.

where $\mathbb{E}[D_{\text{end}}]$ and $\mathbb{E}[E_{BT}]$ are the end-of-period values of debt and before-tax equity, respectively. The left hand side (LHS) contains two end-of-period values for the expected surplus: the before-tax surplus of the equity holders, and the surplus of the debt holders.

Debt does not generate any surplus, while the taxation of equity generates a surplus. The right hand side (RHS) of (10) represents this surplus. The surplus is zero if taxes are absent ($\bar{\tau} = 1 - \tau = 1$). Given the size of equity, an increase in the corporate tax rate τ (a decrease in $\bar{\tau}$) increases the surplus (10) of a bank since an increase in τ leads to a higher asset return μ^* on bank loans to entrepreneurs. Otherwise, banks cannot pay the higher tax and the required return of equity holders. The higher borrowing rate for entrepreneurs results in a smaller number of initiated projects, and may thus not be socially optimal. We model this trade-off as follows.

The total amount of accepted loans depends on the mean return on assets μ^* as

$$Q(\mu^*) = a - b(\mu^* - 1), \quad (11)$$

where a and b are some positive constants. Since Q is the aggregate amount of assets in the economy, the number of banks equals $n = Q(\mu^*)/x$.⁸ By assumption, μ^* corresponds to a unique (x^*, y^*) . In other words, we consider equilibria in which banks make identical management decisions on size and capital. The banks differ ex-post as they are exposed to bank-specific shocks that are correlated in a way that we will specify in section 4.3.⁹

Under a fixed number of banks (n) the equilibrium μ^* is the minimal mean asset return under the additional constraint $x = Q(\mu^*(x, y))/n$. Thus, size would no longer be part of a bank's decision making. Consequently, our main result that policies on capital are most important remains unaffected. Importantly, a fixed n violates the free entry assumption in our long-run model with perfect competition. Therefore, we focus on the case with n endogenous.

Entrepreneurs only initiate a project if the expected return exceeds the offered loan rate. They

⁸We avoid clutter by allowing for non-integer n . This does not affect our main results.

⁹Ex-ante symmetric setups are also in Ibragimov et al. [17], Repullo and Suarez [25], and De Nicolò et al. [11], among others.

earn on average a consumer surplus from the project:

$$CS = \int_{\mu^*}^{1+a/b} Q(\tilde{\mu}) d\tilde{\mu} = \frac{1}{2b} Q(\mu^*)^2. \quad (12)$$

The financiers do not earn a surplus by the risk neutrality of the equity holders and debt holders.

The government surplus follows from multiplying the surplus in (10) by the number of banks,

$$GS = Q(\mu^*) r_\tau y. \quad (13)$$

The total surplus on loans, TS , is the sum of (12) and (13):

$$TS = Q(\mu^*) \left(\frac{1}{2b} Q(\mu^*) + r_\tau y \right) \quad (14)$$

We have four straightforward observations. First, perfect competition among banks erodes the surplus of banks such that the entrepreneurs (CS) and the government (GS) share the total surplus on bank loans.

Second, the size x is absent in (14). In other words, for given capital ratio, the size has only an indirect effect on TS through μ^* . It follows from $Q'(\mu^*) < 0$ in (14) that $\partial TS(\mu^*, y)/\partial \mu^* < 0$. This indicates that maximizing TS conditional on y is the same as minimizing μ^* conditional on y . Therefore, the optimal size function $x^{TS}(y)$ for total surplus is the same function as $x^c(y)$ of the competitive equilibrium in Proposition 3.

Third, since the competitive equilibrium minimizes the mean asset return μ^* (see section 3) such that $\mu_x^c = \mu_y^c = 0$,

$$\left. \frac{\partial Q(\mu^*(x, y))}{\partial x} \right|_{(x, y) = (x^c, y^c)} = \left. \frac{\partial Q(\mu^*(x, y))}{\partial y} \right|_{(x, y) = (x^c, y^c)} = 0.$$

Consequently, we obtain from (14)

$$\left. \frac{\partial TS(x, y)}{\partial y} \right|_{(x, y) = (x^c, y^c)} = Q(\mu^c) r_\tau > 0.$$

Thus, the competitive equilibrium does not maximize the total surplus, but the total amount of

loans in the economy, $Q(\mu^*)$. The total surplus TS is strictly increasing in the capital ratio y at the competitive equilibrium.

Fourth, equation (14) does *not* imply that full equity financing ($y = 1$) maximizes the total surplus. In that case, the required return μ exceeds the competitive return μ^c , which may lead to a lower TS .

4.2. Systemic risk

The total surplus analysis neglects the cost of replacing defaulting banks. In particular, it neglects the nonlinear effect of a systemic crisis, i.e., having multiple defaults simultaneously. We define such a systemic crisis as the event where the amount of unpaid debt exceeds some fraction δ_S of total assets. This is similar to the approach in Ibragimov et al. [17].¹⁰

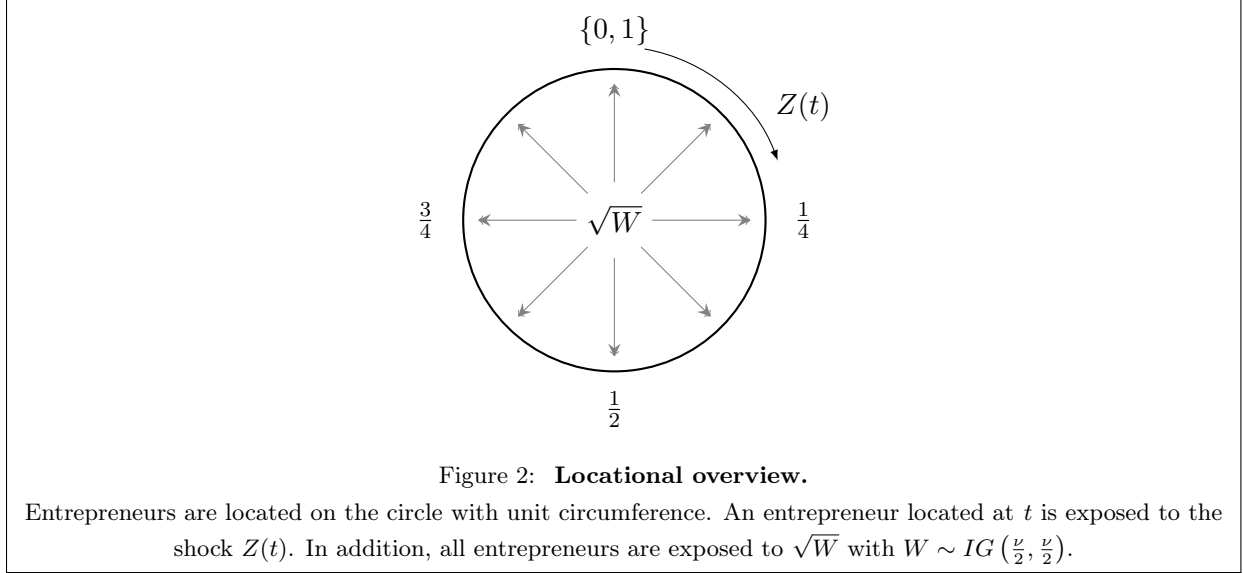
Let $PS(x, y)$ represent the probability of a systemic crisis given the size x and capital ratio y of the individual banks $PS(x, y) = \mathbb{P}\left(\frac{\text{unpaid debt}}{\text{total assets in the economy}} > \delta_S\right)$. Define $\lambda > 0$ as the loss due to a systemic crisis as a fraction of the total surplus on banking operations (TS). We define social welfare as

$$SW(x, y) = (1 - \lambda PS(x, y)) TS(x, y). \quad (15)$$

Thus, a social planner needs to balance the total surplus with the probability of a systemic crisis. The risk preference against a systemic crisis is more pronounced if the parameter λ is high.

The bank size and capital ratio that maximize the total surplus does not correspond to the social optimum, i.e., it does not maximize social welfare. Since the probability on a systemic crisis increases in the default probability, the PD that maximizes total surplus TS is too high for maximizing social welfare. Hence, it is beneficial to have larger banks by their lower PD . However, having larger banks implies that a smaller number of banks suffices to finance a given proportion of loans to the entrepreneurs. Consequently, a smaller number of defaults can cause a systemic crisis when banks are larger. These two counterbalancing effects determine the optimal bank size that maximizes social welfare.

¹⁰Recall that banks are ex-ante identical, but differ ex-post by the bank-specific shocks. Thus, some banks may default whilst others may not.



4.3. The joint distribution of banks' asset values

Recall that we model the gross return per unit of assets of a single bank i by the random variable $R_A = \mu + \sigma X = \mu^* + Y_i$, where μ^* is the equilibrium mean asset return conditional on size x and capital ratio y , while $Y_i = \sigma(x, \mu^*)X_i$ is random noise with mean zero. For a banking system consisting of n banks, we model the joint distribution of (Y_1, \dots, Y_n) as follows. We assume a symmetric banking system where each of the n banks has a market share $1/n$. Suppose there exists an infinite number of entrepreneurs uniformly spread on a circle with unit circumference. The entrepreneurs t and u are parameterized by $t, u \in [0, 1]$ with distance

$$d(t, u) = \min(|u - t|, 1 - |u - t|). \quad (16)$$

The investment project of entrepreneur t is exposed to the shock $Z(t) \sim N(0, 1)$. The dependence between the shocks of entrepreneurs t and u is given by $\text{Cov}(Z(t), Z(u)) = \rho^{d(t, u)}$.¹¹ Thus, the returns of two projects are at least $\sqrt{\rho}$ and more correlated if the projects are closer to each other. Figure 2 illustrates the model.

In practice, asset backed securities are collateralized by assets having a similar risk profile, such

¹¹~~Ibragimov et al. [17] derive this result in a discrete setting with a large number m of entrepreneurs. The shocks to projects of two adjacent entrepreneurs follow an AR(1) process $x_t = \rho_m x_{t-1} + w$ with $w \sim N(0, 1 - \rho_m^2)$. We present the limiting case where $m \rightarrow \infty$. Notice that $Z(t)$ is not a Wiener process because of the circular condition $Z(0) = Z(1)$. Indeed $\text{Cov}(Z(t), Z(u)) \neq \min(t, u)$ which would be true for a Wiener process.~~

as mortgages within the same region. This motivates us to assume that banks prefer adjacent projects. Indeed, such preferences minimize the distance cost. Bank i invests in the projects of the entrepreneurs located on the interval $[\frac{i-1}{n}, \frac{i}{n}]$. The shock distribution per unit of assets of bank i is given by

$$Y_i = \hat{\sigma} \sqrt{W} n \int_{t=(i-1)/n}^{i/n} Z(t) dt, \quad (17)$$

where the factor $\hat{\sigma}$ is a deterministic scaling factor ~~for the standard deviation of the common shock~~. The random variable W represents a common stochastic scaling factor that affects all Y_i simultaneously. Here, we assume $W \sim IG(\frac{\nu}{2}, \frac{\nu}{2})$ follows an inverse gamma distribution with $\nu > 2$.

Using well-known properties of the t -distribution (see, e.g., McNeil et al. [20]), we have that the total shock (Y_1, \dots, Y_n) follow a scaled multivariate Student $t(\nu)$ -distribution with

$$\text{Var}(Y_i) = \frac{2\nu\hat{\sigma}^2 n^2}{\nu-2} \frac{\rho^{1/n} - 1 - \frac{1}{n} \ln(\rho)}{\ln^2(\rho)} \quad \text{Cov}(Y_1, Y_i) = \frac{\nu}{\nu-2} \hat{\sigma}^2 \text{Cov}(Z_1, Z_i) \quad (18)$$

where the covariance of the bank-specific shocks (excluding the common shock W) is

$$\text{Cov}(Z_1, Z_{1+i}) = \begin{cases} \rho^{(i-1)/n} n^2 \left(\frac{\rho^{1/n}-1}{\ln(\rho)} \right)^2 & \frac{i+1}{n} \leq \frac{1}{2} \\ \frac{n^2}{\ln^2(\rho)} f(i, n, \rho) & \frac{i}{n} \leq \frac{1}{2} \leq \frac{i+1}{n} \\ \frac{n^2}{\ln^2(\rho)} f(n-i, n, \rho) & \frac{i-1}{n} \leq \frac{1}{2} \leq \frac{i}{n} \\ \rho^{1-(i+1)/n} n^2 \left(\frac{\rho^{1/n}-1}{\ln(\rho)} \right)^2 & \frac{1}{2} \leq \frac{i-1}{n} \end{cases}$$

with $f(i, n, \rho) = \rho^{\frac{1}{2}} \ln(\rho) \left(\frac{2(i+1)}{n} - 1 \right) + \sqrt[n]{\rho}^{i-1} - 2\sqrt[n]{\rho}^i + \rho^{1-(i+1)/n}$. The derivations of (18) is in the appendix. It is straightforward to generalize the previous results to an arbitrary pair of banks by normalizing the project locations.

5. Calibration

Based on the model in section 2 and 4, it is difficult to obtain analytical results for the impact of the size and the capital ratio on social welfare. Therefore, we investigate this impact with a calibration study. First, we discuss the implementation, then we present the numerical results.

5.1. Implementation

We assume that the operational cost function has the form $c(x) = c_0 + c_1x + c_2x^2$ with $c_0, c_2 \geq 0$. This implies the convex average cost function $\bar{c}(x) = \frac{c_0}{x} + c_1 + c_2x$. As discussed in section 4.2, we assume that a systemic crisis occurs if the expected total loss on debt is at least $\delta_S Q(\mu^*)$, where $\delta_S \in [0, 1]$ is a threshold parameter. Since each default results in a dead weight loss of $D\eta$, a systemic crisis occurs if the number of defaults exceeds $n_D := \delta_S Q / (D\eta)$:

$$PS(x, y) = \mathbb{P}\left(\# \text{defaults} \geq \frac{\delta_S Q(\mu^*(x, y))}{(1 - y)x\eta}\right) \quad (19)$$

If the threshold $\delta_S Q / (D\eta)$ is non-integer, we linearly interpolate the probability on a systemic crisis $PS(x, y)$ by weighting the probabilities based on the thresholds equal to the two nearest integers.

Table 4 presents the baseline parameter values. The rationale behind the chosen values is as follows. The risk free interest rate is set at 0.1 such that our model represents a period of about 5 years. Our tax rate should be read as the joint effect of the corporate tax rate and the tax on interest income. Based on the estimates in Graham [15], Strebulaev [27] chooses the marginal rate on interest income 22.9 percentage points higher than the marginal rate on dividends. In our model, dividends are implicit in r_E , which is a total return that includes dividend distributions. Using a corporate tax rate of 0.35, we choose the effective corporate tax parameter equal to $\tau = 1 - (1 - 0.35)(1 - 0.229) \approx 0.5$.

r_f	τ	η	ν	$\hat{\sigma}$	c_0	c_1	c_2	ρ	a	b	δ_S	λ
0.1	0.5	0.1	4	0.05	10^{-3}	0	10^{-3}	0.5	25	25	0.1	1

Table 4: **Baseline parameters.**

This table reports baseline parameters of the risk free rate r_f , the tax rate τ , the bankruptcy cost η , the tail index ν of the shock distribution, the uncertainty $\hat{\sigma}$, the cost function parameters c_0 , c_1 , and c_2 , the asset dependence ρ , the parameters a and b in the asset demand function $Q(\mu^*) = a - b(\mu^* - 1)$, the threshold parameter δ_S for a systemic crisis, and the fraction λ of total surplus lost in case of a systemic crisis.

The bankruptcy cost parameter is set at $\eta = 0.1$ to match the value in De Nicolò et al. [11]. In addition, it satisfies $\eta = r_f (1/\bar{\tau} - 1)$ which is the border case in Propositions 4 and 5. The tail index ν is set at 4 to match the estimates from the stock indices in Poon et al. [24]. This estimate is also the middle of the three tail indices studied in Ibragimov et al. [17]. The scaling factor of the volatility $\hat{\sigma}$ is half of the riskfree rate, which gives reasonable default probabilities and matches

the volatility-risk free rate ratio in De Nicolò et al. [11]. Setting $c_0 = c_2$ ensures an average cost minimizing size at one.

By setting $\rho = 0.5$, the asset portfolios of two banks with maximal distance, i.e., opposite to each other on the unit circle, still have a high correlation between 0.85 and 0.96 if the number of banks is between 5 and 20. This high correlation represents the increased dependence during crisis events. The values for the parameters $a = 25$ and $b = 25$ of the demand function imply that the number of banks Q/x is less than 25 if all banks minimize operational costs at $x = 1$. The number of banks is about 20 if the excess return $\mu^* - R_f$ on assets equals the risk free rate r_f . We set the systemic crisis threshold δ_S at 0.10. That is, 10% of the banks in distress represents a systemic crisis. By setting $\lambda = 1$, the surplus on banking operations vanishes in case of a systemic crisis.

5.2. Numerical results: three optima

We run the optimization procedure on a two-dimensional grid with intervals of 0.005. The first dimension corresponds to size and runs from 0.8 to 1.2. The second dimension represents the capital ratio y and runs from 5% to 40%.¹²

Table 5 reports the optimal results in the baseline setup. ~~First, we consider the size in each optimum.~~ The competitive size is 5% larger than the size of one that minimizes the average cost. In the total surplus optimum, the optimal size is close to one because larger banks crowd out other banks in financing projects. Having fewer banks would be suboptimal from a social point of view. The social welfare optimum takes systemic risk into account. This reduces the optimal size even further because more banks corresponds to a more stable system. Although the socially optimal size is smaller than in the total surplus optimum, the number of banks is also lower. This stems from a stronger preference for profitable projects to increase the mean return on assets. The higher return reduces default probabilities, and hence systemic risk.

Next, we consider the capital ratio. The capital ratio of the total surplus optimum is substantially higher than the capital ratio in the competitive equilibrium. Since equity is not tax deductible, the total surplus is higher for higher capital ratios. This result coincides with the direct impact of y on TS in (14). By taking into account the additional effect of the probability of a systemic crisis, the

¹²Our computations would extend the grid if the optimum is at a boundary point at the initial grid.

		<i>CE</i>	<i>TS</i>	<i>SW</i>
x	size	1.050	1.005	1.000
$y(\%)$	capital ratio	10	24	37.5
n	number of banks	21.1	21.7	21.5
$PD(\%)$	probability of default	3.8	0.22	0.04
$PS(\%)$	probability of systemic crisis	7.8	0.46	0.08
$r_A(\%)$	net mean asset return	11.5	12.6	14.0
$r_c(\%)$	net contracted interest rate	10.0	10.1	10.1
TS	total surplus on bank operations	10.0	10.1	10.1
SW	social welfare on bank operations	9.2	10.0	10.1

Table 5: **The three optimums in the baseline model.**

This table reports several variables at the three different optimums. CE is the competitive equilibrium ($\min_{x,y} \mu^*(x,y)$), TS is the total surplus optimum ($\max_{x,y} TS$), and SW is the social welfare optimum ($\max_{x,y} SW$). The parameter values are in Table 4.

capital ratio for maximal social welfare increases further in order to reduce default probabilities. Notably, a higher capital ratio in the three optima is associated with a smaller size.

When moving from the optimum of CE to TS and from TS to SW shows that the probability of default PD and systemic risk PS are both decreasing. This ordering corresponds to the different weight each equilibrium attaches to the stability of the system. The net contracted interest rate r_c is close to the riskfree rate r_f of 10%. Therefore, including a deposit insurance scheme in our model would not materially affect our results since r_c cannot decrease below r_f . The welfare measures TS and SW in the two bottom rows of Table 5 show that the competitive equilibrium has a lower social welfare due to systemic risk effects. However, by comparing the TS and SW optimum we find that maximizing total surplus increases capital ratios sufficiently such that systemic risk is substantially reduced, even though systemic risk is then not explicitly taken into account.

5.3. Policy implications

We consider policies that limit size below the size of the competitive equilibrium, 1.05. Similarly, we study capital ratio requirements imposing a ratio above the capital ratio of the competitive equilibrium (10.0%). The size and the capital ratio in the social welfare optimum, respectively 1.00 and 37.5%, are natural choices for the other bound of such policies.

The results are shown in Figure 3. The top plots present for each of the three optima the capital ratio that maximizes social welfare and the corresponding level of social welfare. The size varies between the competitive equilibrium size and the size for optimal social welfare. The horizontal

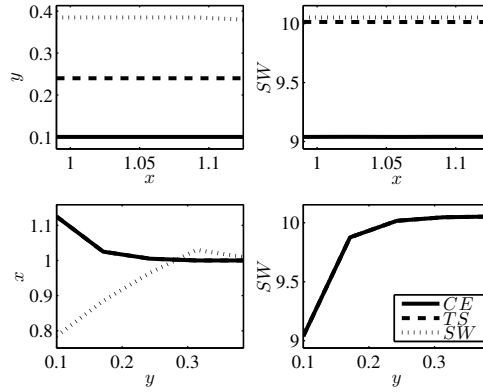


Figure 3: **Size and capital restrictions.**

The figure plots the optimal size x , the optimal capital ratio y and social welfare SW as a function of size and capital. CE is the competitive equilibrium ($\min_{x,y} \mu^*(x,y)$), TS is the total surplus optimum ($\max_{x,y} TS$), and SW is the social welfare optimum ($\max_{x,y} SW$). The solid line and dashed line coincide in the two bottom plots with a restriction on capital since $x^c(y) = x^{TS}(y)$. Parameter values are in Table 4.

lines in the figure suggest that a policy which restricts size is an ineffective policy. More specifically, the optimal capital ratio for banks and society do both not change under different size restrictions. This result is in line with Proposition 4 although the model assumptions differ from the conditions in this proposition. The condition $\eta = r_\tau$ leads to the result that the competitive capital ratio is insensitive to size. The level of social welfare appears to be insensitive to size when optimizing social welfare. Apparently, the effect of the number of banks on social welfare is small in the model.

By varying the imposed capital ratio y in each of the three optima, the bottom plots in Figure 3 show the size that maximizes social welfare and the corresponding level of social welfare. The optimal size is shown to be sensitive to the capital ratio. As such, policies that restrict the capital ratio do affect the optimal size. Since $x^c(y) = x^{TS}(y)$, differences in the optimal competitive size and the optimal size for social welfare are explained by differences in systemic risk rather than in total surplus.

With a low capital requirement, banks prefer a larger size than socially optimal (left bottom plot). The intuition is that the default probability is high under a low capital requirement. Then, it is important for individual banks to utilize the diversification benefits. By contrast, it is for the social optimum necessary to have a large number of banks to prevent a systemic crisis. This leads to smaller banks in the social optimum. As the capital requirement becomes more restrictive, i.e., a higher capital ratio closer to the social optimum, the difference between the size of the two optima

shrinks. Still, social welfare improves by a stricter capital requirement.

We highlight the following two observations from Figure 3. First, a policy that raises capital ratios is more effective than a policy that restricts size. Second, it is not necessary to raise the capital requirement to the socially optimal level. In our calibration, a policy with a capital ratio at 20% instead of the social optimal 37.5% suffices to raise social welfare close to its optimal value.

5.4. Sensitivity analysis

We check the robustness of our results by calibrating our model with different parameter choices. This serves simultaneously as an investigation on the impact of alternative policies.

5.4.1. Asset dependence

It is generally acknowledged that correlations are higher during crisis periods. Instead of the baseline choice $\rho = 0.5$, we repeat the exercise using a low correlation ($\rho = 0.2$) and a high correlation ($\rho = 0.8$). Table 6 shows that the correlation has only a clear impact on the size in the competitive equilibrium. The difference between the competitive size and the socially optimal size is most pronounced in a prosperous economy where correlations are low. The left plot in Figure 4 shows this effect for a wider range of ρ .

The left panel in Figure 4 indicates that a policy that restricts size is potentially helpful for $\rho = 0.2$. The more substantial diversification benefits with $\rho = 0.2$ provokes banks to diversify more by growing large. In doing so, they neglect the externality on the smaller number of banks in the economy. Consequently, if the correlation ρ is low, the loss in social welfare at the competitive equilibrium is higher and the level of social welfare is lower (right plot). Nonetheless, a size restriction leads to a suboptimal capital ratio and hence a suboptimal social welfare (not plotted).

The middle plot suggests a general opportunity for policy interventions on the capital ratio. That is, the state of the economy as measured by the asset dependence ρ has hardly any effect on the optimal capital ratio and the welfare gain of a policy that restricts the capital ratio. Our qualitative conclusions regarding policies that restrict size or capital ratios remain unchanged.

5.4.2. Policy parameters

Figure 5 reports the impact of the parameters $\hat{\sigma}$, η , r_f and τ at the optima. The discussion focuses on the competitive equilibrium (solid line) and the social optimum (dotted line). First,

	$\rho = 0.2$			$\rho = 0.8$		
	<i>CE</i>	<i>TS</i>	<i>SW</i>	<i>CE</i>	<i>TS</i>	<i>SW</i>
x	1.125	1.005	0.990	1.015	1.000	1.000
$y(\%)$	10	24	38.5	10	24	36
n	19.7	21.7	21.7	21.8	21.8	21.5
$PD(\%)$	3.6	0.21	0.04	3.9	0.22	0.05
$PS(\%)$	9.6	0.54	0.08	6.1	0.36	0.08
$r_A(\%)$	11.5	12.6	14.1	11.5	12.6	13.8
$r_c(\%)$	10.6	10.0	10.0	10.6	10.1	10.0
TS	10.0	10.1	10.1	10.0	10.1	10.1
SW	9.1	10.0	10.1	9.4	10.0	10.1

Table 6: **Sensitivity to asset dependence.**

This table reports several variables at the three different optimums. Except for the asset dependence ρ , the parameter values are as in Table 4. The definition of CE , TS , SW , and the variables in the rows is in Table 5.

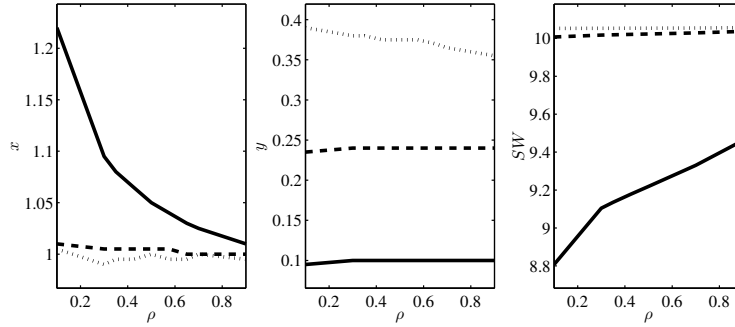


Figure 4: **Sensitivity plots of the asset dependence.**

The figure presents the optimal size x , the optimal capital ratio y , and social welfare SW for several values of the asset dependence parameter ρ . The solid line is for the competitive equilibrium ($\min_{x,y} \mu^*(x,y)$), the dashed line is for maximal total surplus ($\max_{x,y} TS$), and the dotted line is for maximal social welfare ($\max_{x,y} SW$). Baseline parameter values are in Table 4.

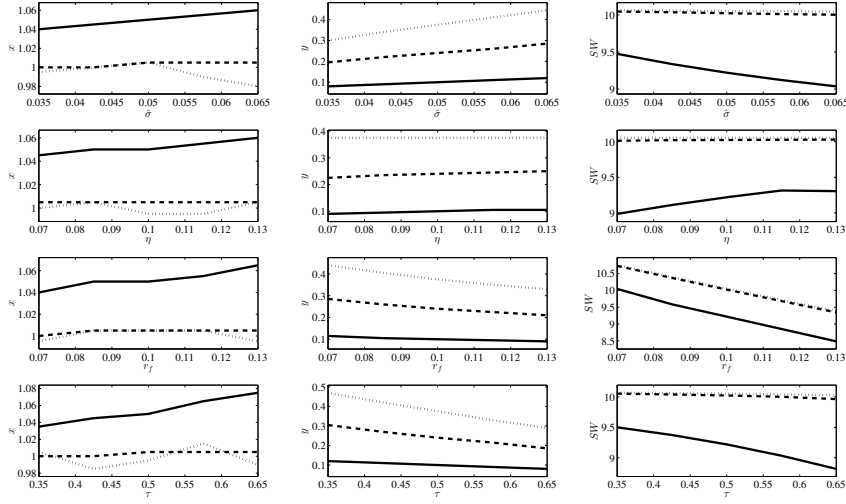


Figure 5: **Sensitivity plots.**

This figure presents the optimal size x , the optimal capital ratio y , and social welfare SW by varying the uncertainty $\hat{\sigma}$, the bankruptcy cost η , the risk free rate r_f , or the tax rate τ . The solid line is for the competitive equilibrium ($\min_{x,y} \mu^*(x,y)$), the dashed line is for maximal total surplus ($\max_{x,y} TS$), and the dotted line is for maximal social welfare ($\max_{x,y} SW$). Baseline parameter values are in Table 4.

the result that banks tend to prefer a larger size and a lower capital ratio than socially optimal is robust for different parameter settings. Thus, policies on size and capital have some welfare enhancing effect.

Second, focusing on the uncertainty parameter $\hat{\sigma}$ (the top row), the competitive size increases with the uncertainty which is in contrast to Proposition 5. The reason is that the fat-tailed shocks considered here have a different impact on the probability of default than the uniform shocks considered in the proposition. The welfare loss at the competitive equilibrium is maximal for a high uncertainty (right plot), because banks overdiversify to overcome the high uncertainty. The policy implications ~~in an economy~~ with a high uncertainty are similar to the low correlation case (Figure 4). That is, a general need for capital policies regardless of the business cycle.

Third, a low bankruptcy cost η corresponds to the highest welfare loss in the competitive equilibrium. For any bankruptcy cost, raising capital requirements improves social welfare. Hence, bankruptcy costs do not play a key role in policy discussions.

Fourth, the impact of the risk-free rate r_f is similar to the analytical results in Table 3. When r_f is low, competitive banks are smaller and hold more capital. A low r_f moves the focus from

minimizing the financing costs towards minimizing the average operational cost. In addition, a low r_f reduces the tax benefit of debt financing relative to equity financing. As a consequence, banks prefer a smaller size and hold somewhat more capital. Society prefers even smaller and better capitalized banks in order to reduce the probability on a systemic crisis. In general, the welfare loss at the competitive equilibrium does not depend on r_f since welfare decreases in both optimums with a similar magnitude. In our risk-neutral setting, this financing cost r_f is the only parameter that materially affects the optimal level of social welfare. The intuition is that a higher financing cost for banks results in a higher required asset return, which results in a drop in the number of financed projects and hence a lower social welfare. The difference between the socially optimal capital ratio and the competitive capital ratio is maximal when r_f is low. This indicates that when monetary policy is loosening, it is important to tighten prudential policy.

Lastly, concerning the tax rate τ , a high tax leads to a large welfare loss. The higher marginal tax rate amplifies the effect of the lower capital ratio on the surplus in (10), and thus social welfare. Therefore, a policy on the capital ratio is most effective when taxes are high.

5.5. Summary

Overall, we draw the following robust conclusions from our calibration.

- (i) Size preferences of banks and society are more aligned than capital preferences. Consequently, policy should target at capital ratios. A capital policy may endogenously lead to smaller bank sizes which makes a policy aiming at smaller banks obsolete.
- (ii) Optimal social welfare is approximately obtained when maximizing total surplus. In conjunction with the fact that $x^c(y) = x^{TS}(y)$, a capital requirement is effective in optimizing social welfare. Raising the capital requirements close to, but not necessarily equal to, the social optimum is sufficient to align banks' incentives close to maximizing social welfare.
- (iii) Policy interventions are particularly helpful if the correlation ρ is low, the uncertainty $\hat{\sigma}$ is high, the bankruptcy cost (or risk aversion) η is low, and taxes τ are high. This corresponds to periods with major innovations and stringent tax regimes.
- (iv) The difference between the socially optimal capital ratio and the competitive capital ratio is maximal when r_f is low. Thus, a less stringent monetary policy should be accompanied with

a more stringent prudential policy.

- (v) Although the optimal level of social welfare is constant in most of the parameters, the optimal capital requirement to attain this level may change substantially when the parameters change.

This suggests a contingent policy on capital requirements.

6. Conclusion

We have studied the joint effect of bank size, capital structure, and asset dependence on social welfare. Our analysis takes into account the interaction between size and capital. Namely, higher capital requirements simultaneously result in larger banks if bankruptcy costs are low, and financing costs and taxes are high. We studied policies on size, capital as well as other policy parameters.

Without any regulation, banks take a suboptimal size-capital decision. Banks tend to hold insufficient capital while the size may be too low or too high. Our main conclusion is that capital requirements are more effective than policy measures on size. Capital requirements may have an additional channel to limit systemic risk through its effect on size. This effect is relevant in economies where dependencies are low, uncertainty is high, bankruptcy costs is low, and taxes are high.

Our study contributes to the policy debate in at least the following two directions. First, our results support the countercyclical capital buffer in Basel III. Building up capital during a good economy (low asset dependence) is more effective in improving social welfare. Instead, a strict capital requirement in bad times with high asset dependence is not particularly effective. Second, our result illustrates the joint welfare effect of monetary policy and prudential policy. The financing cost is the only parameter that materially affects the optimal level of social welfare. It indicates that a less stringent monetary policy should be accompanied with a more stringent prudential policy.

Appendix

Appendix A. Proofs

Proof of Proposition 1

Risk neutrality ($\mathbb{E}[R_D] = R_f$), together with $D = x(1 - y)$ and (1) implies (5). Applying (5) to (2) yields $\mathbb{E}[E_{BT}] = x [\mu - \bar{c}(x) - (R_f + \eta PD)(1 - y)]$. Consequently, the expected after-tax return on

the bank's equity equals

$$\mathbb{E}[r_E] = \frac{\bar{\tau} (\mathbb{E}[E_{BT}] - E)}{E} = \frac{\bar{\tau}}{y} (\mu - \bar{c}(x) - 1 - (r_f + \eta PD)(1 - y))$$

Equation (3) follows now from the risk neutrality condition $\mathbb{E}[r_E] = r_f$.

Next, we prove the remaining equilibrium equation (4). Define the threshold

$$h(\mu, R_c; x) = \frac{\bar{c}(x) + R_c(1 - y) - \mu}{\sigma(x, \mu)} \quad (\text{A.1})$$

such that a default corresponds to the event $\{X < h\}$. In the following derivation, we suppress the fixed argument x for convenience. By using $R_A = \mu^* + \sigma^* X$ in the banking equilibrium, we rewrite (5) and (3) as respectively

$$\mathbb{E}[\min(X, h)] = \frac{\bar{c}(x) - \mu^* + (1 - y) + (r_f + \eta \mathbb{P}(X < h))(1 - y)}{\sigma(x, \mu^*)} \quad (\text{A.2})$$

$$0 = \frac{\bar{c}(x) - \mu^* + 1 + \frac{r_f}{\bar{\tau}} y + (r_f + \eta \mathbb{P}(X < h))(1 - y)}{\sigma(x, \mu^*)} \quad (\text{A.3})$$

Subtracting (A.3) from (A.2) results in

$$\mathbb{E}[\min(X, h)] = -\frac{y}{\sigma(x, \mu^*)} \left(1 + \frac{r_f}{\bar{\tau}}\right) \quad (\text{A.4})$$

Equation (4) follows by noting that the equilibrium mean asset return μ^* solves the equilibrium equation (A.4), and $F^{-1}(PD)$ is another expression for the default threshold h . \square

Proof of Proposition 2

We find conditions for existence and uniqueness of the banking equilibrium (R_c^*, μ^*, PD^*) . Following the proof of Proposition 1, h refers to the default threshold for X . Given μ^* and x , the function $h : \mathbb{R} \rightarrow \mathbb{R}$ in (A.1) is a bijection of the gross contracted interest rate on debt (R_c) to the default threshold h . As such, we find conditions for existence and uniqueness of the equilibrium (h^*, μ^*) .

Existence: By (A.4), an equilibrium necessarily satisfies $-\frac{1+r_f/\bar{\tau}}{\sigma(x, \mu^*)} y \leq h(\mu^*) \leq \sup(X)$. By $\sigma_2 \geq 0$, it suffices for existence to have $-\frac{1+r_f/\bar{\tau}}{\sigma(x, \mu^*)} y \geq \inf(X)$. Hence, (7) is sufficient since $y \leq 1$.

Next, we derive the sufficient condition for existence in (8). Since $\sigma_2 \equiv 0$, we have $\tilde{\sigma}(x) := \sigma(x, \mu(x, y))$ such that $\tilde{\sigma}(x)$ is not affected by R_c^* through μ . Substituting $R_A = \mu + \sigma X$ and (3) into (5), we get

$$\mathbb{E} \left[\min \left(\tilde{\sigma}(x)X + \frac{r_f}{\bar{r}}y, (1-y)(R_c^* - r_f - \eta PD) - 1 \right) \right] = -y \quad (\text{A.5})$$

For $R_c^* = 0$, the right hand side (RHS) exceeds the left hand side (LHS) since

$$\text{LHS}|_{R_c^*=0} < -(1-y)(r_f + \eta PD) - 1 < -1 < -y = \text{RHS}|_{R_c^*=0}$$

while for $R_c^* \rightarrow \infty$ the LHS exceeds the RHS. The continuity in R_c^* of the LHS of (A.5) implies that (A.5) has a solution in R_c^* . This implies the sufficient condition in (8).

Uniqueness: Under (8), the RHS of (4) is constant which pins down PD^* uniquely. Then, μ^* and R_c^* are also unique by (3) and (5), respectively. \square

Proof of Proposition 3

The competitive equilibrium is the special case of the banking equilibrium where the mean asset return μ^* is minimal. The corresponding variables are denoted with a superscript c .

- (i) We find from (6), $\mu_1^c = \mu_2^c = 0$, $\sigma_1(x, \mu) < 0$ and $\sigma(x, \mu) > 0$, the partial derivatives of $PD^*(x, y)$ at the competitive equilibrium:

$$PD_1^c = (g')^c \frac{\sigma_1(x, \mu^c) + \sigma_2(x, \mu^c)\mu_1^c}{(1 + r_f/\bar{r})y^c} < 0 \quad (\text{A.6})$$

$$PD_2^c = -(g')^c \frac{\sigma(x^c, \mu^c)/y^c + \sigma_2(x^c, \mu^c)\mu_2^c}{(1 + r_f/\bar{r})y^c} < 0 \quad (\text{A.7})$$

The inequalities in (A.6) and (A.7) imply under (C)

$$PD_x^c = PD_1^c + \frac{dx^c(y^c)}{dy} PD_2^c < 0 \quad PD_y^c = \frac{dy^c(x^c)}{dx} PD_1^c + PD_2^c < 0$$

For the operational cost function $\bar{c}(x)$, it immediately follows that $\frac{\partial}{\partial y} \bar{c}(x) = 0$. Taking the partial derivative of (3) to x and using $\mu_1^c = 0$ and (A.6) gives $\bar{c}'(x^c) > 0$. Thus, (C) is

equivalent to

$$\frac{d}{dy} \bar{c}(x^c(y)) = \bar{c}'(x^c) \frac{dx^c(y^c)}{dy} > 0$$

(ii) The claim on the derivatives in case of $\sigma_1^c = 0$ is straightforward from the representation of PD in (4) in Proposition 1.

(iii) Because $\mu^c = \mu^*(x^c, y^c)$ has a positive definite Hessian matrix at (x^c, y^c) :

$$\mu_{11}^c \mu_{22}^c - (\mu_{12}^c)^2 > 0 \quad \mu_{11}^c > 0 \quad \mu_{22}^c > 0 \quad (\text{A.8})$$

It follows from the definition of $y^c(x)$ that $\mu_2^*(x, y^c(x)) = 0$ for all x . Differentiating to x ,

$$\frac{d}{dx} \mu_2^*(x, y^c(x)) = \mu_{12}^*(x, y^c(x)) + \mu_{22}^*(x, y^c(x)) \frac{dy^c}{dx} = 0$$

Evaluating this expression at $x = x^c$ gives $\frac{dy^c(x^c)}{dx} = -\frac{\mu_{12}^c}{\mu_{22}^c}$. Similarly, $\mu_1^*(x^c(y), y) \equiv 0$ leads to $\frac{dx^c(y^c)}{dy} = -\frac{\mu_{12}^c}{\mu_{11}^c}$. The cross partial derivative of $\mu(x, y)$ at μ^c follows from (3):

$$\mu_{12}^c = [(1 - y^c) PD_{12}^c - PD_1^c] \eta \quad (\text{A.9})$$

Together with the fact that $\eta > 0$, the inequality in (9) is equivalent to $\mu_{12}^c < 0$. Further from (A.8) and the expressions for $\frac{dy^c(x^c)}{dx}$ and $\frac{dx^c(y^c)}{dy}$, this is equivalent to condition (C). \square

Proof of Proposition 4 and 5

We combine the proofs of Proposition 4 and 5 because of the similar structure of the derivations. Existence and uniqueness of the banking equilibrium follows from $\sigma_2(x, \mu) \equiv 0$ in Proposition 2. The cdf of X is $F(x) = \frac{1}{2\sqrt{3}} (x + \sqrt{3})$ when $-\sqrt{3} \leq x \leq \sqrt{3}$. We define for $PD^* \in [0, 1]$, the inverse cdf $F^{-1}(PD^*) = \sqrt{3} (2PD^* - 1)$. Using $\mathbb{P}(X < F^{-1}(PD^*)) = PD^*$,

$$\mathbb{E}[\min(X, F^{-1}(PD^*))] = -\sqrt{3} (PD^* - 1)^2$$

Substitution of (4) and $PD^* \leq 1$ yields for the banking equilibrium of a given (size, capital)-pair

(x, y) :

$$PD^*(x, y) = 1 - \sqrt{\frac{(1 + r_f/\bar{\tau})y}{\sigma_0^2 x^{-2\delta}}} = 1 - w(x)\sqrt{y} \quad (\text{A.10})$$

where we have defined for brevity the positive and increasing function

$$w(x) := \sqrt{1 + \frac{r_f}{\bar{\tau}} \frac{x^\delta}{\sigma_0}} \quad (\text{A.11})$$

with, as before, $r_f \geq 0$, $\bar{\tau} \in (0, 1]$, $\sigma_0 > 0$ and $0 < \delta \leq \frac{1}{4}$. The partial derivatives up to the second order of PD^* in (A.10) are

$$\begin{aligned} PD_1^*(x, y) &= -w'(x)\sqrt{y} \leq 0 & PD_{11}^*(x, y) &= -w''(x)\sqrt{y} & PD_{12}^*(x, y) &= -\frac{w'(x)}{2\sqrt{y}} \\ PD_2^*(x, y) &= -\frac{w(x)}{2\sqrt{y}} \leq 0 & PD_{22}^*(x, y) &= \frac{w(x)}{4y\sqrt{y}}. \end{aligned}$$

Differentiating (3) leads to

$$\begin{aligned} \mu_1^*(x, y) &= \bar{c}'(x) - \eta w'(x)\sqrt{y}(1 - y) & \mu_{11}^*(x, y) &= \bar{c}''(x) - \eta w''(x)\sqrt{y}(1 - y) \\ \mu_2^*(x, y) &= -\eta \left[\frac{1}{2\sqrt{y}} w(x) - \frac{3}{2} \sqrt{y} w(x) + 1 \right] + r_f \left[\frac{1}{\bar{\tau}} - 1 \right] & \mu_{22}^*(x, y) &= \eta w(x) \left(\frac{1 + 3y}{4y\sqrt{y}} \right) \\ & & \mu_{12}^*(x, y) &= \frac{1}{2} \eta w'(x) \frac{1}{\sqrt{y}} (3y - 1) \end{aligned}$$

The results above are for the banking equilibrium of any given (x, y) . Next, we apply these results to the competitive equilibrium (x^c, y^c) , which is the specific banking equilibrium where the mean asset return $\mu^*(x, y)$ is minimal. ~~As before, we denote the corresponding variables with a superscript c . The first and second order derivative of $w(x)$ at x^c are denoted by w_1^c and w_{11}^c , respectively. We adopt a similar notation for \bar{c} , the average cost function $\bar{c}(x)$ at x^c .~~

The inequality (9) in Proposition 3 holds if and only if $y^c < \frac{1}{3}$. To see this, for $y^c < \frac{1}{3}$ we have $(1 - y^c) PD_{12}^c = \left(1 - \frac{1}{y^c}\right) \frac{w_1^c \sqrt{y^c}}{2} < -w_1^c \sqrt{y^c} = PD_1^c$. Thus, by Proposition 3(ii), the capital ratio increases in the size ($y_x^c > 0$) and the size increases in the capital ratio if and only if $y^c < \frac{1}{3}$.

In line with our technical assumption, the choice $\bar{c}(x) = x^\gamma$ with $\gamma > 1$ leads to properly defined functions $x^c(y)$ and $y^c(x)$ near the competitive equilibrium. More specifically, (i) μ^* is convex in x along $(x, y^c(x))$, and (ii) μ^* is convex in y along $(x^c(y), y)$. Statement (i) follows from substituting

the expression for $\mu_1^c = 0$ into the expression for μ_{11}^* with the imposed restriction $\delta < \gamma$, which gives $\mu_{11}^c = \gamma(\gamma - 1)x^{\gamma-2} - \gamma(\delta - 1)x^{\gamma-2} > 0$. Statement (ii) is straightforward from μ_{22}^* . This confirms that $x^c(y)$ and $y^c(x)$ lead to the unique minimum of μ^* conditional on y and x , respectively.

Next, we find the unique pair (x^c, y^c) of the competitive equilibrium. The size $x^c(y)$ minimizes μ given the capital ratio y . From $\mu_1(x^c(y), y) = 0$, and (A.11),

$$x^c(y) = \left(\frac{\eta \tilde{R}_f \delta}{\gamma \sigma_0} \sqrt{y} (1 - y) \right)^{1/(\gamma - \delta)}. \quad (\text{A.12})$$

The function $x^c(y)$ has an inverted U-shape for the optimal size x as a function of the capital ratio y . This optimal size is zero if $y = 0$ or $y = 1$, and maximal at $y = \frac{1}{3}$, i.e., $0 \leq x^c(y) \leq \left(\frac{2\eta \tilde{R}_f \delta}{3\sqrt{3}\gamma\sigma_0} \right)^{1/(\gamma - \delta)} := x_{\max}$. The capital ratio $y^c(x)$ minimizes μ^* given the size x . From $\mu_2(x, y^c(x)) = 0$, (A.11) and the expression for μ_{11}^* , we obtain

$$x^\delta = \frac{2(\eta - r_\tau)\sigma_0}{\eta \tilde{R}_f \left(3\sqrt{y^c(x)} - \frac{1}{\sqrt{y^c(x)}} \right)} \quad y^c(x) \neq \frac{1}{3} \quad (\text{A.13})$$

Equation (A.13) defines $y^c(x)$ implicitly. The denominator always increases with $y^c(x)$ and switches sign at $y^c(x) = \frac{1}{3}$, which implies $\lim_{x \rightarrow \infty} y^c(x) = \frac{1}{3}$. This corresponds to $\mu_{12} = 0$ and $x_y^c = y_x^c = 0$. As a consequence, the sign of the numerator in (A.13) indicates whether the size and the capital ratio move in the same direction. We distinguish three cases:

(i) $\eta > r_\tau$:

The positive numerator in (A.13) and $x^\delta \geq 0$ imply $y^c(x) > \frac{1}{3}$, and $y^c(x)$ decreases in x . Capital ratios $y^c(x) \leq \frac{1}{3}$ are excluded because no $x \geq 0$ corresponds to such $y^c(x)$. Thus, $y^c(x)$ is a decreasing function with $y^c \left(\left(\frac{(\eta - r_\tau)\sigma_0}{\eta \tilde{R}_f} \right)^{1/\delta} \right) = 1$ and asymptotic lower bound $\lim_{x \rightarrow \infty} y^c(x) = \frac{1}{3}$. From $y^c > \frac{1}{3}$, we know that $y_x^c < 0$ and $x_y^c < 0$ hold.

(ii) $\eta = r_\tau$: The limit $\eta \rightarrow r_\tau$, $\{(x, y^c(x)) : x \geq 0\}$ is a horizontal line with $y^c(x) \equiv \frac{1}{3}$ and $y_x^c \equiv 0$.

(iii) $\eta < r_\tau$:

The negative numerator in (A.13) and $x^\delta \geq 0$ give $y^c(x) < \frac{1}{3}$ and $y^c(x)$ increases in x . Here, capital ratios $y^c(x) \geq \frac{1}{3}$ are excluded as no $x \geq 0$ corresponds to such $y^c(x)$. Now, y^c is an increasing function with asymptotic bounds $\lim_{x \downarrow 0} y^c(x) = 0$ and $\lim_{x \rightarrow \infty} y^c(x) = \frac{1}{3}$. From

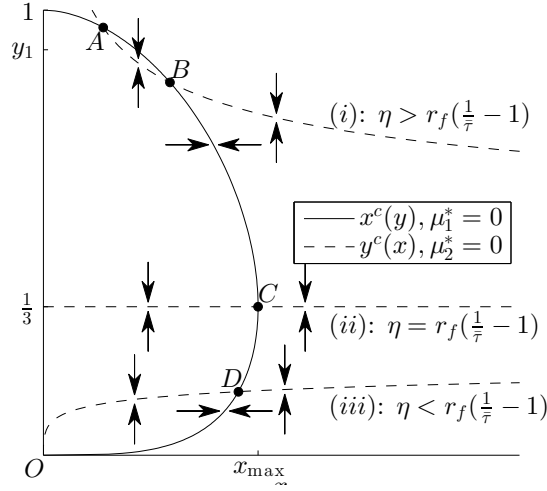


Figure A.6:

Phase diagram of $x^c(y)$ and the three possible shapes for $y^c(x)$.

The figure shows the optimal size curve $x^c(y)$ and three possible shapes for $y^c(x)$. The parameters y_1 and x_{\max} are defined in the main text. The curves are defined in (A.12) and (A.13). The shape of $y^c(x)$ is determined by the bankruptcy cost η , the risk free rate r_f , and the tax rate $\tau = 1 - \bar{\tau}$. Horizontal and vertical arrows indicate changes towards the optimal size $x^c(y)$ and the optimal capital ratio $y^c(x)$, respectively.

$y^c < \frac{1}{3}$, we obtain $y_x^c > 0$ and $x_y^c > 0$.

The phase diagram in Figure A.6 shows an example for each of the three possible cases. The competitive equilibrium (x^c, y^c) is determined by the cutting points of $x^c(y)$ and $y^c(x)$. In case (i), there are at most two intersection points (the unstable equilibrium A and the stable equilibrium B in Figure A.6), while there exists a unique intersection point in cases (ii) and (iii), the points C and D , respectively. We can prove this formally by studying the analytical properties of the cutting points of the curves $x^c(y)$ and $y^c(x)$, in respectively (A.12) and (A.13). The details are upon request. Hence, the competitive equilibrium is unique in each of the three cases.

Next, we express the optimal solution (x^c, y^c) in terms of the parameters. Substitution of (A.12) in (A.13) and using the positive sign of x^δ gives

$$\left| 3\sqrt{y^c} - \frac{1}{\sqrt{y^c}} \right| \left(\frac{\eta \tilde{R}_f \delta}{\gamma \sigma_0} \sqrt{y^c} (1 - y^c) \right)^{\delta/(\gamma - \delta)} = \frac{\sigma_0}{\tilde{R}_f} \left| 2 - \frac{2r_\tau}{\eta} \right| \quad (\text{A.14})$$

Define $\zeta := \gamma/\delta > 4$ which gives for any $y^c \in (0, 1) \setminus \{\frac{1}{3}\}$,

$$\sqrt{y^c} (1 - y^c) \left| 3\sqrt{y^c} - \frac{1}{\sqrt{y^c}} \right|^{\zeta-1} = \frac{\zeta}{\eta} \frac{\sigma_0^\zeta}{(1 + \frac{r_f}{\bar{\tau}})^{\zeta/2}} \left| 2 - \frac{2r_\tau}{\eta} \right|^{\zeta-1} \quad (\text{A.15})$$

Define $\phi(y; \zeta)$ as the LHS of (A.15) and $\psi(\eta; r_f, \sigma_0, \bar{\tau}, \zeta)$ as the RHS. The competitive capital ratio must satisfy $\phi(y^c) = \psi$. As discussed before, $y^c - \frac{1}{3}$ has the same sign as $\eta - r_\tau$. It follows that there exists at most two solutions for the equation $\phi(y) = \psi$. When two solutions exist, which may occur if $\eta > r_\tau$, we have shown that the smallest solution on $(\frac{1}{3}, 1)$ corresponds to the global minimum (x^c, y^c) of $\mu^*(x, y)$. To find an upper bound for this solution, we derive an explicit expression for the point $y_1 \in (\frac{1}{3}, 1)$ that satisfies $\phi'(y_1) = 0$.

To formally study the shape and derivative of ϕ , it is convenient to consider the monotonic transform $\tilde{\phi}(y) := \ln(\phi(y; \zeta))$. The derivatives of $\tilde{\phi}(y)$ and $\phi(y)$ have the same sign if $\phi(y) > 0$, i.e. if $y \neq \frac{1}{3}$. Hence, y_1 is also a zero of $\tilde{\phi}'$. For $y^c > \frac{1}{3}$, $\tilde{\phi}'(y^c) > 0$ is equivalent to

$$h(y^c) := 3(\zeta + 2)(y^c)^2 - 2(\zeta + 2)y^c + 2 - \zeta < 0$$

Similarly, if $y^c < \frac{1}{3}$ then $\tilde{\phi}'$ is positive if and only if $h(y^c) > 0$. The zeros of h and so ϕ' are at

$$y_0 = \frac{1}{3} - \frac{2}{3}\sqrt{\frac{\zeta-1}{\zeta+2}} < 0 \quad y_1 = \frac{1}{3} + \frac{2}{3}\sqrt{\frac{\zeta-1}{\zeta+2}} < 1 \quad (\text{A.16})$$

The function h satisfies $h(0) = 2 - \zeta < 0$ and $h(1) = 4$. It has a global minimum $h(\frac{1}{3}) = \frac{4}{3}(1 - \zeta) < 0$. Considering $y^c \in [0, 1]$, the function h is only positive at the interval $(y_1, 1]$. Hence, the function ϕ decreases on the interval $I_0 := (0, \frac{1}{3}) \cup (y_1, 1]$, and increases on $I_1 := (\frac{1}{3}, y_1)$. It is straightforward that $\phi(0; \zeta) \rightarrow \infty$, $\phi(y_1; \zeta) = \sqrt{y_1}(1 - y_1) \left(3\sqrt{y_1} - \frac{1}{\sqrt{y_1}} \right)^{\zeta-1} > 0$, $\phi(\frac{1}{3}; \zeta) = 0$, and $\phi(1; \zeta) = 0$. The preceding results imply for the competitive equilibrium (x^c, y^c) with minimal $\mu^*(x^c, y^c) = \mu^c$:

- (i) $\eta \leq r_\tau$: a unique equilibrium with $y^c \leq \frac{1}{3}$ and $x^c \leq x_{\max}$
- (ii) $\eta > r_\tau$ and $\phi(y_1) \geq \psi(\eta; r_f, \sigma_0, \bar{\tau}, \zeta)$: a unique equilibrium with $y^c \in (\frac{1}{3}, y_1]$ and $x^c < x_{\max}$
- (iii) $\eta > r_\tau$ and $\phi(y_1) < \psi(\eta; r_f, \sigma_0, \bar{\tau}, \zeta)$: a boundary solution $(x^c, y^c) = (0, 1)$

where $x_{\max} = \left(\frac{2\eta\bar{R}_f}{3\sqrt{3}\sigma_0\zeta} \right)^{1/(\gamma-\delta)}$ and $y_1 = \frac{1}{3} + \frac{2}{3}\sqrt{\frac{\zeta-1}{2+\zeta}}$. To guarantee an interior solution in case

(iii), we derive a sufficient condition which ensures $\phi(y_1) \geq \psi(\eta; r_f, \sigma_0, \bar{\tau}, \zeta)$. Define $\tilde{\eta} := \frac{1}{\eta}$, $a := 2 \left(\frac{\zeta \sigma_0^\zeta}{(1+r_f/\bar{\tau})^{\zeta/2}} \right)^{\frac{1}{\zeta-1}} > 0$, and $r(\tilde{\eta}) := \psi^{\frac{1}{\zeta-1}} = a \tilde{\eta}^{\frac{1}{\zeta-1}} (1 - r_\tau \tilde{\eta})$. As we are considering case (iii), it follows that $\tilde{\eta} < 1/r_\tau$ holds. The condition $\phi(y_1) \geq \psi$ is thus equivalent to

$$\phi(y_1)^{\frac{1}{\zeta-1}} \geq r(\tilde{\eta}). \quad (\text{A.17})$$

It is straightforward to show that $r(0) = r\left(\frac{1}{r_\tau}\right) = r'\left(\frac{1}{r_\tau}\right) = 0$ and $r''(\tilde{\eta}) < 0$. Thus, the function r attains its maximum on $[0, \frac{1}{r_\tau}]$ at $\frac{1}{\zeta r_\tau}$. Thus, (A.17) holds with equality for some $\tilde{\eta}$ if and only if

$$\max_{\tilde{\eta}} r(\tilde{\eta}) = r\left(\frac{1}{\zeta r_\tau}\right) \geq \phi(y_1)^{\frac{1}{\zeta-1}}. \quad (\text{A.18})$$

If condition (A.18) holds strictly then two distinct $\tilde{\eta}$ at $\left(0, \frac{1}{r_\tau}\right)$ solve $r(\tilde{\eta}) = \phi(y_1)^{\frac{1}{\zeta-1}}$. One solution $\tilde{\eta}_0 \in \left(0, \frac{1}{\zeta r_\tau}\right)$ and another solution $\tilde{\eta}_1 \in \left(\frac{1}{\zeta r_\tau}, \frac{1}{r_\tau}\right)$. In addition, $r(\tilde{\eta}) > \phi(y_1)^{\frac{1}{\zeta-1}}$ is equivalent to $\tilde{\eta} \in (\tilde{\eta}_0, \tilde{\eta}_1)$ because $r'' < 0$. Then, no equilibrium exists on the interval $(\tilde{\eta}_0, \tilde{\eta}_1)$ between the two solutions. For such $\tilde{\eta}$, we obtain the boundary solution $(x^c, y^c) = (0, 1)$. Thus, an interior equilibrium exists if and only if $\tilde{\eta} = \frac{1}{\eta} \in [0, \tilde{\eta}_0] \cup [\tilde{\eta}_1, 1/r_\tau]$.

Let $\tilde{\eta}_0^L = \phi(y_1) a^{1-\zeta}$ and $\tilde{\eta}_1^U = \frac{1}{r_\tau} \left(1 - \frac{1}{a} [r_\tau \phi(y_1)]^{\frac{1}{\zeta-1}}\right)$. One can verify that $\tilde{\eta}_0^L \leq \tilde{\eta}_0 \leq \tilde{\eta}_1^U$ and all $\tilde{\eta} = \tilde{\eta}_0$ satisfy (A.17). Provided (A.18) holds, a sufficient condition for $\phi(y_1) \geq \psi$ is $\tilde{\eta} \notin [\tilde{\eta}_0, \tilde{\eta}_1] \subset [\tilde{\eta}_0^L, \tilde{\eta}_1^U]$. In terms of $\eta = 1/\tilde{\eta}$, it follows that a sufficient condition is $\eta \notin [\eta_a, \eta_b]$ where

$$\eta_a := \frac{1}{\tilde{\eta}_1^U} = \frac{r_\tau}{1 - \frac{1}{2} \left(\frac{1}{\zeta} \phi(y_1) \tilde{R}_f^\zeta \sigma_0^{-\zeta} r_\tau \right)^{\frac{1}{\zeta-1}}} \quad \eta_b := \frac{1}{\tilde{\eta}_0^L} = \frac{(2\sigma_0)^\zeta \zeta}{2\phi(y_1) (1 + r_f/\bar{\tau})^{\zeta/2}}$$

This proves that $\eta \notin [\eta_a, \eta_b]$ is sufficient to guarantee the existence of an interior equilibrium.

The bound η_a exceeds $r_\tau = r_f \left(\frac{1}{\bar{\tau}} - 1\right)$. This means for the corresponding equilibrium capital ratio $y^c \in \left(\frac{1}{3}, 1\right]$ where the restriction $\phi(y^c) \geq \psi$ may bind (case (iii)). Using (A.15), the bound η_b exceeds ζr_τ . This proves Proposition 4 except for the sign of the derivative of PD . We find the latter jointly with the comparative statics in Proposition 5.

To study the marginal effect of the parameters σ_0^ζ , η , r_f , and $\bar{\tau}$ on y^c , use $\tilde{\phi}(y^c) := \ln(\phi(y^c; \zeta))$ and $\tilde{\psi}(\eta; r_f, \sigma_0, \bar{\tau}, \zeta) := \ln(\psi(\eta; r_f, \sigma_0, \bar{\tau}, \zeta))$. The identity $\tilde{\phi}(y^c) = \tilde{\psi}$ from (A.15) implies for any

parameter $p \in \{\sigma_0, \eta, r_f, \bar{\tau}\}$

$$\frac{dy^c}{dp} = \left(\frac{d\tilde{\phi}(y^c)}{dy} \right)^{-1} \frac{d\tilde{\psi}}{dp} = \frac{\tilde{\psi}_p}{\tilde{\phi}_y^c} \quad (\text{A.19})$$

As argued after (A.16), $\tilde{\phi}_y(y^c) < 0$ is equivalent to $y^c < \frac{1}{3}$, which is the same as $\eta < r_\tau = r_f(1/\bar{\tau} - 1)$. Therefore, $\tilde{\phi}_y(y^c) < 0$ if $\eta < r_\tau$, and $\tilde{\phi}_y(y^c) > 0$ if $\eta > r_\tau$.

For the total effect on y^c , assume an interior minimum of $\mu^*(x, y)$, i.e., $\phi(y_1) \geq \psi$ if $\eta > r_\tau$.

- (i) σ_0 : Because $\tilde{\psi}_{\sigma_0} > 0$, we find from (A.19) that the capital ratio y^c increases with the volatility scalar σ_0 if and only if $\eta > r_\tau$.
- (ii) η : Differentiation to η yields $\frac{d\tilde{\psi}}{d\eta} = \frac{1}{\eta} \frac{\eta - \zeta r_\tau}{r_\tau - \eta}$. Since $\zeta \geq 4$, $\tilde{\psi}_\eta > 0$ holds if and only if $r_\tau < \eta < \zeta r_\tau$. This indicates $y_\eta^c = \tilde{\psi}_\eta / \tilde{\phi}_y > 0$ if and only if $\eta \in [0, \zeta r_\tau)$.
- (iii) r_f : Differentiating $\tilde{\psi}$ to r_f gives $\frac{d\tilde{\psi}}{dr_f} = -\frac{1}{2} \frac{\zeta}{r_f + \bar{\tau}} - \frac{(\zeta - 1)(1/\bar{\tau} - 1)}{\eta - r_\tau}$. The denominator of the second term changes sign at $\eta = r_\tau$. Thus, $\tilde{\psi}_{r_f} < 0$ if $\eta > r_\tau$. From $\tilde{\psi}_{r_f} > 0$ for $\eta = 0$ and $\frac{d}{d\eta} \tilde{\psi}_{r_f} > 0$ for $\eta \in [0, r_\tau)$, it follows that $\tilde{\psi}_{r_f} > 0$ for all $\eta \in [0, r_\tau)$. Now (A.19) gives that the capital ratio decreases with r_f : $y_{r_f}^c = \tilde{\psi}_{r_f} / \tilde{\phi}_y < 0$ for all η .
- (iv) τ : Differentiating $\tilde{\psi}$ to $\frac{1}{\bar{\tau}}$ with $\bar{\tau} = 1 - \tau$ gives $\frac{d\tilde{\psi}}{d(1/\bar{\tau})} = -\frac{1}{2} \frac{\zeta r_f \bar{\tau}}{\bar{\tau} + r_f} - \frac{(\zeta - 1)r_f}{\eta - r_\tau}$. The denominator of the second term changes sign at $\eta = r_\tau$. Along similar lines as $\tilde{\psi}_{r_f}$, it can be shown that $\tilde{\psi}_{1/\bar{\tau}} < 0$ if $\eta > r_\tau$, and $\tilde{\psi}_{1/\bar{\tau}} > 0$ if $\eta < r_\tau$. This gives that for all η the capital ratio decreases with $\frac{1}{\bar{\tau}}$, which gives $y_\tau^c < 0$.

Collecting the previous results gives the sensitivity results of y^c . The sensitivity results for x^c follow from a similar analysis.

Next, we consider the effects on PD^c . Regardless of $\bar{c}(x)$, the sign of the total derivative of PD^c to x is always negative for the chosen functional form of $w(x)$:

$$PD_x^c = PD_1^c + \frac{dy^c(x)}{dx} PD_2^c = -w'(x^c) \sqrt{y^c} + \frac{\frac{1}{2} \eta w'(x^c) (3y^c - 1) / \sqrt{y^c} w(x^c)}{\eta w(x^c) \left(\frac{1+3y^c}{4y^c} \right) / \sqrt{y^c}} \frac{w(x^c)}{2\sqrt{y^c}} = -\frac{2w_1^c \sqrt{y^c}}{3y + 1} < 0$$

~~In other words, a possible positive indirect effect through capital is dominated by the direct effect of size on PD .~~

The total derivative of PD to the capital ratio y does depend on \bar{c} through ζ . Notice first from (A.10), (A.11), and (A.12) that in any banking equilibrium $(x^c(y), y)$:

$$PD^*(x^c(y), y) = 1 - \frac{\tilde{R}_f}{\sigma_0} \sqrt{y} x^c(y)^\delta = 1 - \left(\frac{\tilde{R}_f}{\sigma_0} \sqrt{y} \right)^{\zeta/(\zeta-1)} \left(\frac{\eta}{\zeta} (1-y) \right)^{1/(\zeta-1)} \quad (\text{A.20})$$

We obtain at the competitive equilibrium that

- (i) $PD_y^c < 0$ if and only if $y^c \in \left[0, \frac{\zeta}{2+\zeta}\right]$.
- (ii) $PD_{\sigma_0}^c > 0$ if (a) $\eta \leq r_\tau$, or (b) $y^c \in \left[\frac{\zeta}{2+\zeta}, y_1\right]$.
- (iii) $PD_\eta^c < 0$ if (a) $y^c \in \left[0, \frac{\zeta}{2+\zeta}\right]$ and $\eta < \zeta r_\tau$, or (b) $y^c \in \left[\frac{\zeta}{2+\zeta}, y_1\right]$ and $\eta > \zeta r_\tau$.
- (iv) $PD_{r_f}^c < 0$ if $y^c \in \left[\frac{\zeta}{2+\zeta}, y_1\right]$.
- (v) $PD_{\bar{r}}^c > 0$ if $y^c \in \left[\frac{\zeta}{2+\zeta}, y_1\right]$.

The sign of the partial derivatives is ambiguous outside these intervals. Using (A.15) and $\frac{\zeta}{2+\zeta} < y_1$, the condition $y^c \in \left[0, \frac{\zeta}{2+\zeta}\right]$ is equivalent to $\phi\left(\frac{\zeta}{\zeta+2}\right) \geq \psi$. Rewriting the latter inequality gives $\sigma_0 \leq \tilde{\sigma}$ with $\tilde{\sigma} := \left[\phi\left(\frac{\zeta}{\zeta+2}\right) \frac{\eta}{\zeta} \left| 2 - \frac{2r_\tau}{\eta} \right|^{1-\zeta} \right]^{1/\zeta} \tilde{R}_f$. \square

Proof of equation (18): Using (16) and (17), the shock $Y_i \sim c(s) t(\nu)$ of a bank with market share $s := \frac{1}{n}$ follows a scaled univariate Student $t(\nu)$ -distribution with scaling factor

$$c(s) = \hat{\sigma} \frac{1}{s} \left(\int_{t=(i-1)s}^{is} \int_{u=(i-1)s}^{is} \rho^{d(t,u)} dt du \right)^{1/2} = \frac{\hat{\sigma}}{\ln(\rho^s)} \sqrt{2(\rho^s - 1 - \ln(\rho^s))}. \quad (\text{A.21})$$

The standard deviation $\sigma_Y = \sqrt{\frac{\nu}{\nu-2}} c(s)$ is a function of size x and the mean asset return μ^* since $s(x, \mu^*) = x/Q(\mu^*)$. We thus write $\sigma_Y(s)$ and $\sigma_Y(x, \mu)$ interchangeably to refer to the standard deviation of Y . At the competitive equilibrium, the capital ratio has no effect on the market share $s^c = s(x^c, y^c)$ since $\frac{\partial s^c}{\partial y} = \frac{\partial s^c}{\partial \mu} \frac{\partial \mu^c}{\partial y} = 0$.

The bank-specific shock for a bank that invests in projects on the interval $[0, s]$ with $s \leq \frac{1}{2}$ is

$$\text{Var}(Z) = \frac{1}{s^2} \int_{u=0}^s \int_{t=0}^s \text{Cov}(Z(t), Z(u)) dt du = \frac{2(\rho^s - 1 - \ln(\rho^s))}{\ln^2(\rho^s)}$$

The total shock $Y = \frac{1}{s} \int_{t=0}^s Y(t) dt = \frac{\hat{\sigma}\sqrt{W}}{s} \int_{t=0}^s Z(t) dt$ that includes the macro shock is a scaled univariate Student $t(\nu)$ -distribution with $\text{Var}(Y) = \hat{\sigma}^2 \mathbb{E}[W] \text{Var}(Z)$.

For the expression of the covariance in (18), it is convenient to consider banks 1 and $1+i$. Similar to the derivation of $\text{Var}(Y)$, the covariance of the shocks, including the macro shock, is $\text{Cov}(Y_1, Y_{1+i}) = \frac{\nu}{\nu-2} \hat{\sigma}^2 \text{Cov}(Z_1, Z_{1+i})$. By denoting $s = \frac{1}{n}$, the covariance of the bank-specific shocks, excluding the macro-shock W , is

$$\begin{aligned} \text{Cov}(Z_1, Z_{1+i}) &= \frac{1}{s^2} \int_{u=is}^{(i+1)s} \int_{t=0}^s \text{Cov}(Z(t), Z(u)) dt du \\ &= \frac{1}{s^2} \left[\int_{u=is}^{(i+1)s} \int_{t=\min([u-\frac{1}{2}]^+, s)}^s \rho^{u-t} dt + \int_{t=0}^{\min([u-\frac{1}{2}]^+, s)} \rho^{1-(u-t)} dt du \right] \quad (\text{A.22}) \end{aligned}$$

We use (A.22) to work out three different cases.

- (i) If $(i+1)s \leq \frac{1}{2}$, then all projects of banks 1 and $1+i$ are on $[0, \frac{1}{2}]$ such that the shortest distance between projects t and u is $u-t$:

$$\text{Cov}(Z_1, Z_{1+i}) = \frac{1 - \rho^{-s}}{s^2 \ln(\rho)} \int_{u=is}^{(i+1)s} \rho^u du = \frac{\rho^{(i-1)s} (1 - \rho^s)^2}{\ln^2(\rho^s)}$$

- (ii) If $is \geq \frac{1}{2} + s$, each project t of bank 1 is on $[0, s]$ while each project u of bank i is on $[s + \frac{1}{2}, 1]$.

The shortest distance is then $t + 1 - u$:

$$\text{Cov}(Z_1, Z_{1+i}) = \frac{1}{s^2} \frac{\rho(\rho^s - 1)}{\ln(\rho)} \int_{u=is}^{(i+1)s} \rho^{-u} du = \frac{\rho^{1-(i+1)s} (1 - \rho^s)^2}{\ln^2(\rho^s)}$$

- (iii) In the intermediate case, the projects of bank $1+i$ cover either $\frac{1}{2}$ or $\frac{1}{2} + s$. If the projects of bank $1+i$ cover $\frac{1}{2}$, then $is \leq \frac{1}{2} < (i+1)s \leq \frac{1}{2} + s$. Starting from (A.22),

$$\begin{aligned} &s^2 \text{Cov}(Z_1, Z_{1+i}) \\ &= \int_{u=is}^{1/2} \left[\int_{t=0}^0 \rho^{t+(1-u)} dt + \int_{t=0}^s \rho^{u-t} dt \right] du + \int_{u=\frac{1}{2}}^{(i+1)s} \left[\int_{t=0}^{u-\frac{1}{2}} \rho^{t+(1-u)} dt + \int_{t=u-\frac{1}{2}}^s \rho^{u-t} dt \right] du \\ &= \frac{1}{\ln^2(\rho)} \left[\rho^{\frac{1}{2}} \ln(\rho) (2s(1+i) - 1) + \rho^{(i-1)s} - 2\rho^{is} + \rho^{1-(i+1)s} \right] \quad (\text{A.23}) \end{aligned}$$

If the projects of bank $1+i$ cover $\frac{1}{2} + s$, then let $\tilde{i} := \frac{1}{s} - i$, and due to symmetry, the expression

for the covariance of Z_1 and Z_{1+i} follows by substituting $i = \frac{1}{s} - \tilde{i}$ for i in (A.23), and $s = 1/n$.

In all cases, the correlation decreases by a factor ρ^{is} when i moves away from $[0, s]$. \square

Appendix B. Estimation procedure

We adopt the following procedure to find for a given pair (x, y) the corresponding equilibrium (PD^*, μ^*, R_c^*) . The scaled Student $t(\nu)$ -distribution $Y = \sigma(s, y)X$ with density g and cdf G has a default threshold equal to $G^{-1}(PD)$. For the conditional loss of a standard, i.e., unscaled, Student $t(\nu)$ -distribution T (eq. (2.27) in McNeil et al. [20])

$$\mathbb{E}[T \mid T < F_T^{-1}(PD)] = -\frac{f_T(F_T^{-1}(PD))}{PD} \left(\frac{\nu + (F_T^{-1}(PD))^2}{\nu - 1} \right) \quad (\text{B.1})$$

It follows from some rewriting of (4) and (B.1) that

$$-f_T(F_T^{-1}(PD)) \left(\frac{\nu + (F_T^{-1}(PD))^2}{\nu - 1} \right) + (1 - PD) F_T^{-1}(PD) = -\frac{y}{\sigma(s, \mu)} \sqrt{\frac{\nu}{\nu - 2}} \left(1 + \frac{rf}{\bar{\tau}} \right) \quad (\text{B.2})$$

An iterative procedure finds for given x and y the equilibrium $(PD^*(x, y), \mu^*(x, y), R_c^*(x, y))$:

- (i) Initialize μ with (3) and $PD^* = \frac{1}{2}$.
- (ii) Compute σ in (18) using $s = x/Q(x, \mu)$
- (iii) Find the unique PD that solves (B.2)
- (iv) Compute μ in (3)
- (v) Stop if some stopping criterium is satisfied, otherwise go to (ii).

From $PD = \mathbb{P}(R_A x < c(x) + R_c D)$, we obtain $R_c^*(x, y) = \frac{1}{1-y} \left(\mu^* - \bar{c}(x) + \sigma(s, \mu^*) \sqrt{\frac{\nu-2}{\nu}} F_T^{-1}(PD^*) \right)$.

To minimize μ , we run over a grid of (x, y) by moving in a direction where the concave μ decreases.

The social welfare $SW(x, y)$ in (15) follows from the total surplus in (14), and PS in (19). The probability PS is based on the expressions in (18), and (19).

References

- Acharya, V. V. (2009). A theory of systemic risk and design of prudential bank regulation. *Journal of Financial Stability*, 5(3):224–255.
- Allen, F., Carletti, E., and Marquez, R. (2014). Deposits and bank capital structure. *Working paper*.
- Allen, L. and Rai, A. (1996). Operational efficiency in banking: An international comparison. *Journal of Banking & Finance*, 20(4):655–672.
- Angeloni, I. and Faia, E. (2013). Capital regulation and monetary policy with fragile banks. *Journal of Monetary Economics*, 60(3):311–324.
- Boyd, J. H. and Heitz, A. (2016). The social costs and benefits of too-big-to-fail banks: A “bounding” exercise. *Journal of Banking and Finance*, 68:251–265.
- Bradley, M., Jarrell, G. A., and Kim, E. (1984). On the existence of an optimal capital structure: Theory and evidence. *Journal of Finance*, 39(3):857–878.
- Brennan, M. J. and Schwartz, E. S. (1978). Corporate income taxes, valuation, and the problem of optimal capital structure. *Journal of Business*, 51(1):103–114.
- Cifuentes, R., Ferrucci, G., and Shin, H. S. (2005). Liquidity risk and contagion. *Journal of the European Economic Association*, 3(2-3):556–566.
- Davies, R. and Tracey, B. (2014). Too big to be efficient? The impact of implicit subsidies on estimates of scale economies for banks. *Journal of Money, Credit and Banking*, 46(s1):219–253.
- De Bandt, O. and Hartmann, P. (2000). Systemic risk: A survey. *ECB Working Paper*, 35.
- De Nicolò, G., Gamba, A., and Lucchetta, M. (2014). Microprudential regulation in a dynamic model of banking. *Review of Financial Studies*, 27(7):2097–2138.
- Diamond, D. W. (1984). Financial intermediation and delegated monitoring. *Review of Economic Studies*, 51(3):393–414.

- Diamond, D. W. and Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, 91(3):401–419.
- Galati, G. and Moessner, R. (2013). Macroprudential policy – a literature review. *Journal of Economic Surveys*, 27(5):846–878.
- Graham, J. R. (1999). Do personal taxes affect corporate financing decisions? *Journal of Public Economics*, 73(2):147–185.
- Hughes, J. P. and Mester, L. J. (1993). A quality and risk-adjusted cost function for banks: Evidence on the "too-big-to-fail" doctrine. *Journal of Productivity Analysis*, 4(3):293–315.
- Ibragimov, R., Jaffee, D., and Walden, J. (2011). Diversification disasters. *Journal of Financial Economics*, 99(2):333–348.
- Kraus, A. and Litzenberger, R. H. (1973). A state-preference model of optimal financial leverage. *Journal of Finance*, 28(4):911–922.
- Laeven, L., Ratnovski, L., and Tong, H. (2014). Bank size and systemic risk. *IMF Staff Discussion Note*, 14/04.
- McNeil, A. J., Frey, R., and Embrechts, P. (2010). *Quantitative Risk Management: Concepts, techniques, and tools*. Princeton University Press.
- Miao, J. (2005). Optimal capital structure and industry dynamics. *Journal of Finance*, 60(6):2621–2659.
- Modigliani, F. and Miller, M. H. (1958). The cost of capital, corporation finance and the theory of investment. *American Economic review*, 48(3):261–297.
- Pais, A. and Stork, P. A. (2013). Bank size and systemic risk. *European Financial Management*, 19(3):429–451.
- Poon, S.-H., Rockinger, M., and Tawn, J. (2004). Extreme value dependence in financial markets: Diagnostics, models, and financial implications. *Review of Financial Studies*, 17(2):581–610.
- Repullo, R. and Suarez, J. (2013). The procyclical effects of bank capital regulation. *Review of Financial Studies*, 26(2):452–490.

- Santos, J. A. (2001). Bank capital regulation in contemporary banking theory: A review of the literature. *Financial Markets, Institutions & Instruments*, 10(2):41–84.
- Strebulaev, I. A. (2007). Do tests of capital structure theory mean what they say? *Journal of Finance*, 62(4):1747–1787.
- Titman, S. and Tsyplakov, S. (2007). A dynamic model of optimal capital structure. *Review of Finance*, 11(3):401–451.
- Wagner, W. (2010). Diversification at financial institutions and systemic crises. *Journal of Financial Intermediation*, 19(3):373–386.
- Wagner, W. (2011). Systemic liquidation risk and the diversity–diversification trade-off. *Journal of Finance*, 66(4):1141–1175.