# Ambiguity Theories Alternative to Prospect Theory

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*R&R*

Most ambiguity models today:

- theoretical; little attention for empirical findings;
- normatively motivated!?;
- focus on Ellsberg urns & ambiguity aversion (taken as rational!?);
- no insensitivity;
- me, being Bayesian (taking EU as normative), focuses on descriptive.

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Outline:

§1. The Anscombe-Aumann framework for decision under uncertainty;

§2. Multiple priors models;

§3. Multistage models with stages exogenous;

§4. Multistage models with stages endogenous (smooth model);

§5. Other ambiguity models;

§6. Applications of ambiguity models by “A-authors.”

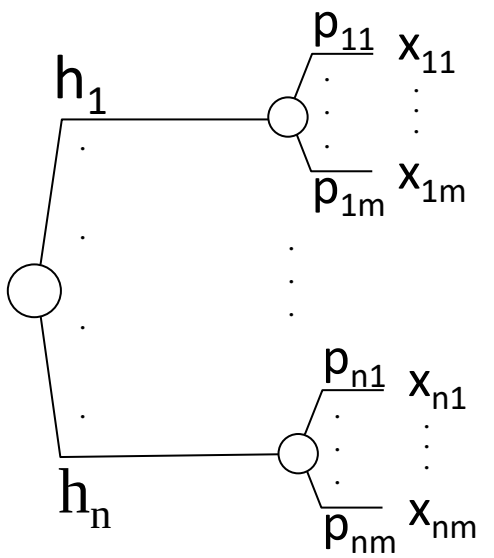
Popular framework for many ambiguity models today:
Anscombe-Aumann (1963) (AA).

Acts do not assign outcomes
to states of nature, but
probability distributions over “prizes”
(e.g., prize = money amount).

Is a two-stage approach:

1st stage:
horse
race

2nd stage:
roulette
wheel



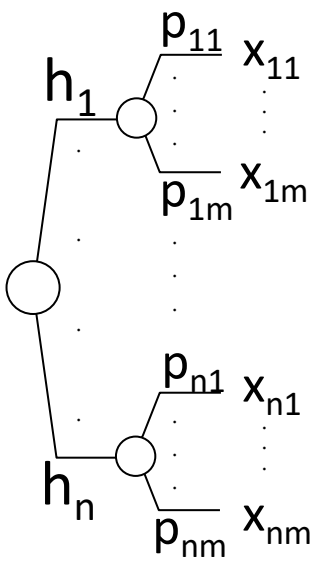

ambiguity;
our central
interest

auxiliary
structure;
facilitates
math<u>s</u>

1st stage (of central interest): ambiguous events
(e.g. horse race.)

2nd stage (only auxiliary/artificial): roulette wheel, generates probability distributions over money.

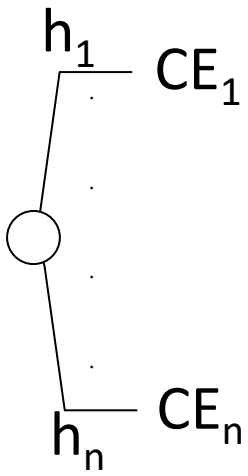
# AA evaluation of AA acts:




CE-substitution will be done, by EU (so, backward induc-tion); auxiliary.

→

Result of CE substitution:




Ambiguity-evaluation; our central interest.

Relative to our Structural Assumption 1.2.1 (Savage's uncertainty model):

Utilities of outcomes
are replaced by:
expected utilities of lotteries.

EU in 2nd stage is linear in probability.
Mathematically convenient!
AA gives "linear utility without linear utility."

This made AA popular.

# Two descriptive (& normative!?) problems for the auxiliary structure (2nd stage lotteries) in AA:

1. EU for risk questionable
   (Allais, Machina, prospect theory …).

   Many may defend EU for risk normatively!?

2. CE substitution (backward induction; "conse-quentialism") is very questionable for nonEU.

   Some defend backward induction normatively!?
   Natural under EU. Problematic under nonEU.
   Machina (1989): normative objections.

Others, criticizing backward induction in general under nonEU normatively:
Dominiak & Lefort 2011; Eichberger & Kelsey 1996; Gul & Pesendorfer 2005; Hayashi 2011; Karni & Safra 1990; Karni & Schmeidler 1991; Machina 1989; McClennen 1990; Ozdenoren & Peck 2008; Siniscalchi 2004.

Recently, leveled against AA: see keyword
**criticism of monotonicity in Anscombe-Aumann (1963) for ambiguity**
in
http://personal.eur.nl/wakker/refs/webrfrncs.docx

The following theories can all be defined equally well in AA framework as in Savage's. Following the literature, we do the former.

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In ambiguity, we don't know precisely the probability measure $P$ on $S$:
multiple priors models specify a set $\mathcal{P}$ of possible probability measures on $S$.
Then models can be defined:

Maxmin EU (Gilboa & Schmeidler 1989). Take subjective $U$ and subjective set $\mathcal{P}$:

$$x \mapsto inf_{P \in \mathcal{P}} EU(x) \, .$$

Model is pessimistic; "ambiguity-averse!?"

Maxmax EU: take subjective $U$ and subjective set $\mathcal{P}$:

$$x \mapsto sup_{P \in \mathcal{P}} EU(x) \, .$$

Model is optimistic; "ambiguity-seeking!?"

$\alpha$-maxmin expected utility (Hurwicz 1951; Jaffray 1994; Ghirardato et al. 2004):
take subjective $U$ and subjective set $\mathcal{P}$:

$$x \mapsto \alpha \times inf_{P \in \mathcal{P}} EU(x) + (1 - \alpha) \times sup_{P \in \mathcal{P}} \, EU(x)$$

Size of $\mathcal{P}$ is degree of ambiguity of info, and $\alpha$ captures attitude, aversion/seeking to ambiguity.

## Pros of multiple priors:

1. Set $\mathcal{P}$ fits well with natural way of speaking;
2. Easy to understand upon first acquaintance;
3. Requires no new mathematics.

## Cons:

1. Decision rules are crude;
2. Theory as such is too rich: there are "too many" sets $\mathcal{P}$;*
3. Endogenous (subjective) versus exogenous (objective) status of $\mathcal{P}$ is problematic.

* Special cases, e.g. $\varepsilon$-contamination, are considered.

## Generalizations:

the variational model (Maccheroni, Marinacci, & Rustichini 2006): take subjective U, subjective $\mathcal{P}$, and $c: \mathcal{P} \longrightarrow \mathbb{R}$ :

$$x \mapsto inf_{P \epsilon \mathcal{P}}(EU(x) + c(P))$$

$c$ function can serve to make some $P$'s less influential by setting $c(P)$ large, e.g. $c(P) = \infty$.

Special case & interpretation: see next slide.

Popular special case of variational model:

robust model (Hansen & Sargent 2001):

$c(P)$ is relative entropy (sort of distance) of $P$ with respect to some focal probability $Q$.

$Q$ is what you believe primarily.

But if another $P$ gives deviations so bad that it is much worse (by more than $c(P)$), then you go by $P$ rather than by $Q$.

Popular in statistics. They sell well in macroeconomics as

"model uncertainty."

Popular in *expert aggregation* and *climate change.*

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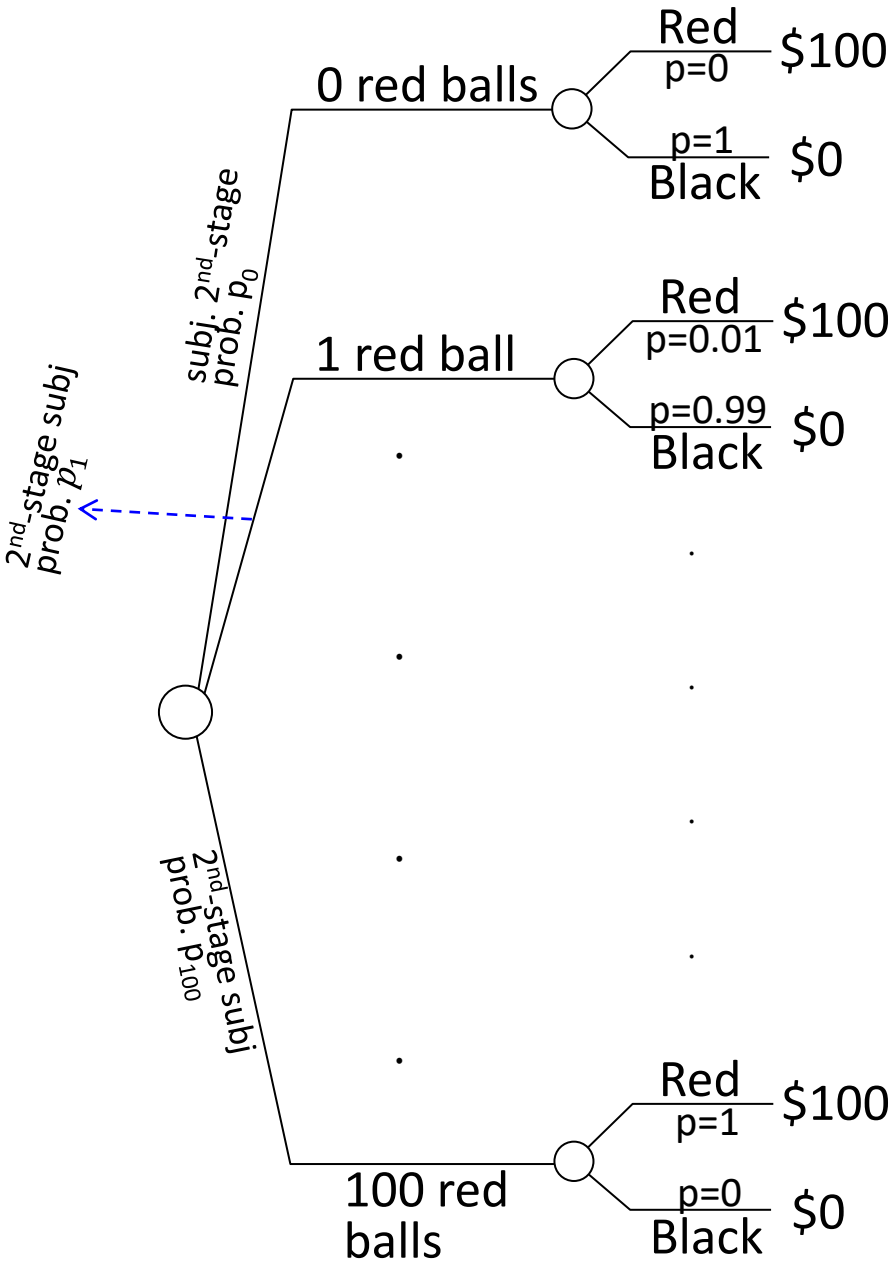
Not to be confused with two-stage of AA,
where 2nd stage is purely auxiliary/artificial add-on.
Here extra stage is essential part of ambiguity.

Imagine unknown Ellsberg urn:
100 balls, red/black, unknown proportion.
$100 if drawn ball red, $0 otherwise:
$100_R 0$.




Then what is the big deal here??

Is just $\sum_{i=0}^{n} p_i \times \frac{i}{100}$ probability at \$100 by multiplication rule (called reduction of compound lotteries, RCLA)????

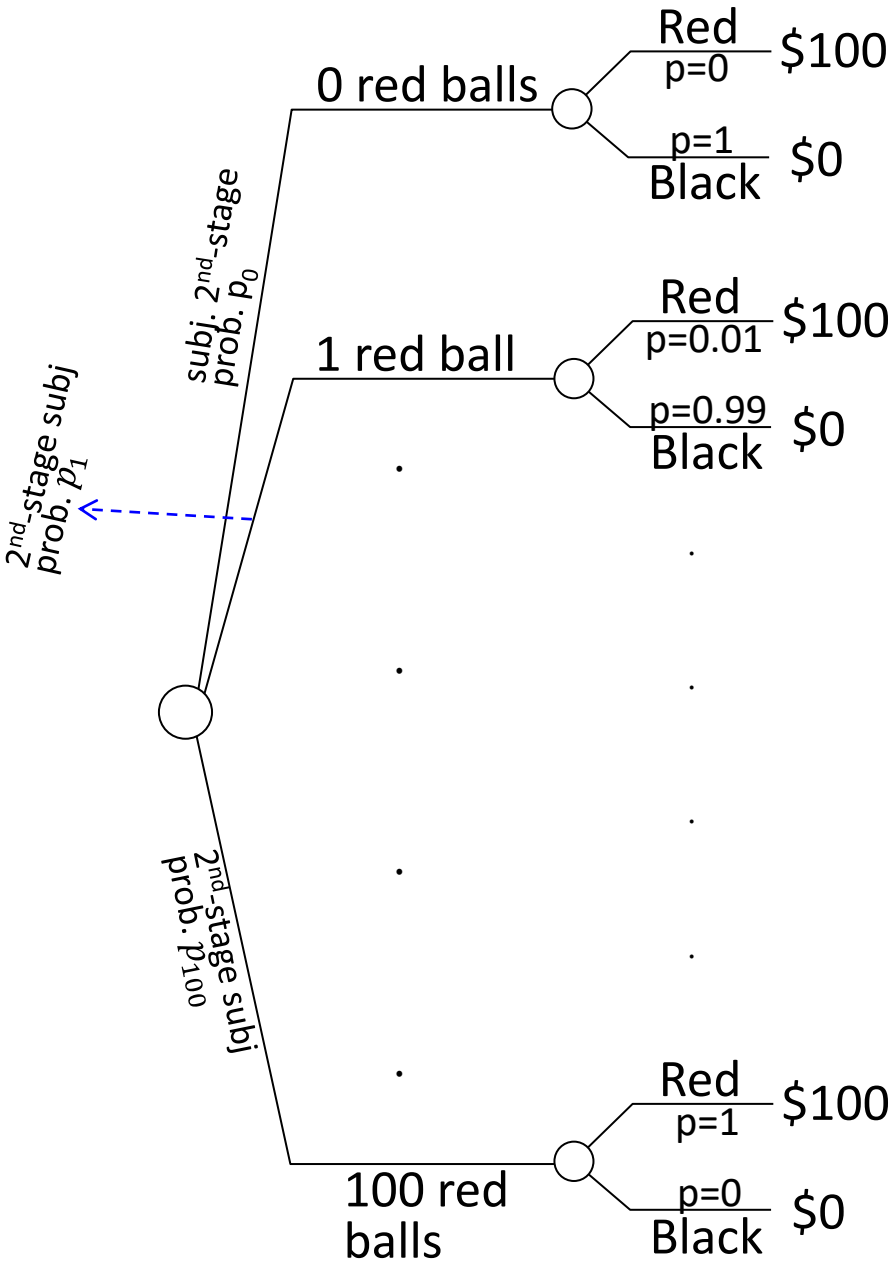
Well …

People give up RCLA!

Can then do backward induction with nonEU. Can get extra pessimism in 2nd stage: "ambiguity aversion."

Is old idea:

Becker & Brownson (1964), Yates & Zukowski (1976), Gärdenfors & Sahlin (1982), Segal (1987), Halevy (2007), Ergin & Gul (2009).




Remarkable version:

Use EU in both stages.

But …

with different utility function in two stages. Can take more concave U in 2nd stage for extra pessimism: "ambiguity aversion."

Analytically convenient!

Tversky & Kahneman (1975), Kreps & Porteus (1979; interpreted as time-attitude), Neilson (1993, 2010), Nau (2006).

Called recursive expected utility.

Pros:

1. Intuitive;
2. Much flexibility regarding models to use in the two stages;
3. The last version mentioned (two-stage EU): mathematically convenient. Need no new software.

Cons:

1. Exogenous two-stage setup to capture ambiguity rarely available in practice;
2. Backward induction questionable (as with AA);
3. 2-stage EU: modeling ambiguity through outcome-function is not homeomorphic (not psychological); this is not intuitive;
4. 2-stage EU: cannot capture insensitivity so descriptively problematic.

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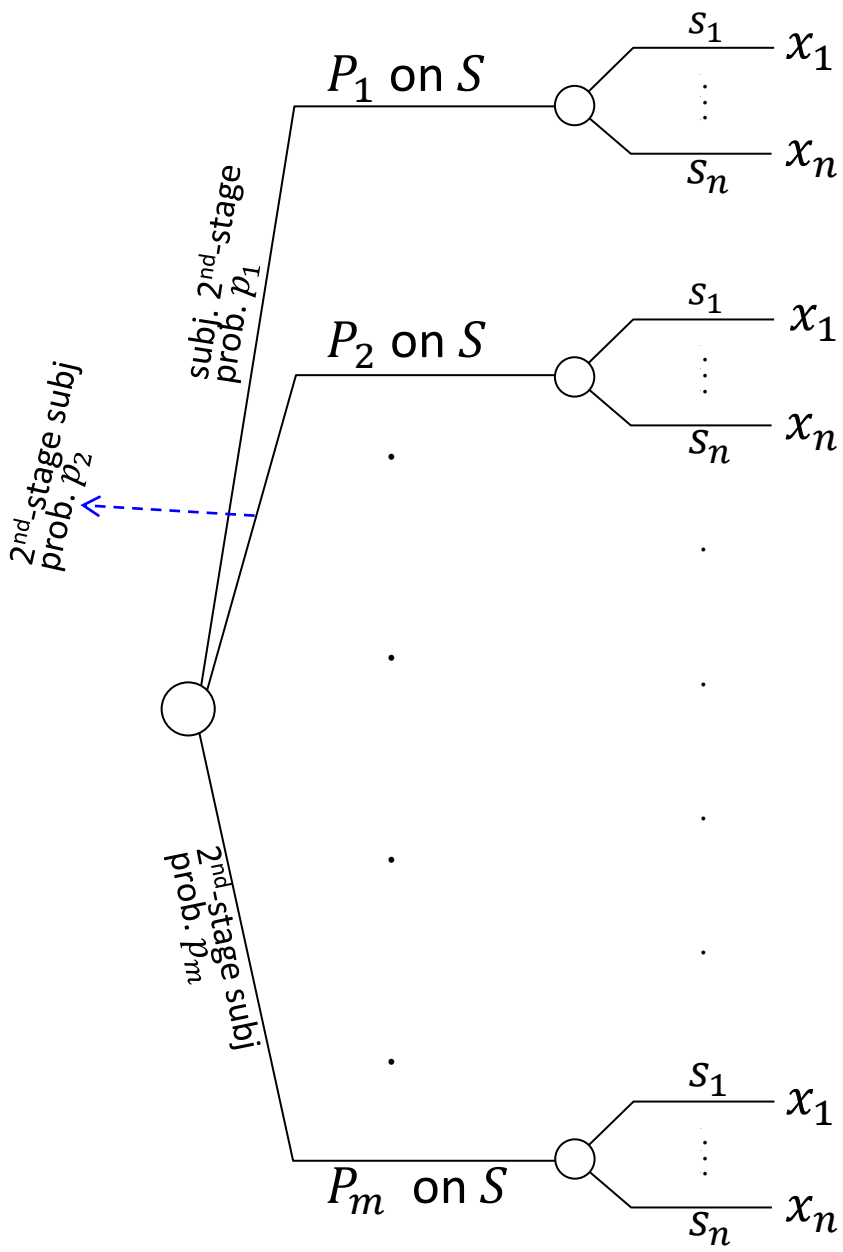


Now take two-stage setup endogenous.

Directly condition on $P_j$'s on $S$, without this being a "physically-defined" event.

Assign "2nd-stage" subjective probability $p_j$ to each $P_j$.

Do backward induction.

Violate RCLA.

Becker & Brownson (1964), Yates & Zukowski (1976), Gärdenfors & Sahlin (1982), Segal (1987), Halevy (2007), Ergin & Gul (2009).

This can be a general ambiguity theory!

But hard to observe …

Very general …

(Technical detail: then act on S in 2nd stage may not depend on stage, but be the same in all stages ...)

Very popular version:

smooth model (Klibanoff, Marinacci, Mukerji 2004).

Using EU in both stages.

Endogenous version of recursive EU.

# Discussion of smooth model

Pros:

(1) Is general ambiguity model.

(2) Mathematical convenience (EU + smoothness).

Cons:

(1) Those of exogenous recursive EU
(non-homeomorphic; not empirical: no insensitivity)

(2) Endogenous two-stage setup is unobservable and too general. In virtually all applications, people take it: … exogenous …

People often use smooth model nowadays (exogenous) because so convenient; awaiting more theory to come.

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- Multiple priors others:  Chateauneuf (1991); Gajdos, Hayashi, Tallon, & Vergnaud (2008);
- Variational alternatives: Chateauneuf & Faro (2009), Strzalecki (2011): multiplier;
- Vector expected utility: Siniscalchi (2009);
- 2-stage maxmin: Jaffray (1989); Olszewski (2007);
- Expected Uncertain $Ut^{\underline{v}}$ $Th^{\underline{v}}$ & Hurwicz expected utility Gul & Pesendorfer (2014, 2015)
- EU with uncertain probabilities: Izhakian (2017)

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# Applications of ambiguity theories with A-authors (2018)

Contract theory
Amarante, Massimiliano, Mario Ghossoub, & Edmund Phelps (2017) “Contracting on Ambiguous Prospects,” *Economic Journal* 127, 2241–2246.

General equilibrium theory:
Araujo, Aloisio, Alain Chateauneuf, Juan Pablo Gama, & Rodrigo Novinski (2018) “General Equilibrium with Uncertainty Loving Preferences,” *Econometrica* 86, 1859–1871.

Game theory:
Ahn, David S. (2007) “Hierarchies of Ambiguous Beliefs,” *Journal of Economic Theory* 136, 286–301.
Aryal, Gaurab & Ronald Stauber (2014) “Trembles in Extensive Games with Ambiguity Averse Players,” *Economic Theory* 57, 1–40.

Insurance:
Alary, David, Christian Gollier, & Nicolas Treich (2013) “The Effect of Ambiguity Aversion on Insurance and Self-Protection,” *Economic Journal* 123, 1188–1202.

Welfare theory:
Alon, Shiri & Gabrielle Gayer (2016) “Utilitarian Preferences with Multiple Priors,” *Econometrica* 84, 1181–1201.

Asset pricing:
Anderson, Evan W., Eric Ghysels, & Jennifer L. Juergens (2009) “The Impact of Risk and Uncertainty on Expected Returns,” *Journal of Financial Economics* 94, 233–263.

Health:
Attema, Arthur E., Han Bleichrodt, & Olivier L'Haridon (2018) “Ambiguity Preferences for Health,” *Health Economics* 27, 1699–1716.

Climate change:
Aydogan, Ilke, Loïc Berger, Valentina Bosetti, & Ning Liu (2018) “Three Layers of Uncertainty: An Experiment,” working paper.


The End