Addition to

Online Appendix to

"Source Theory: A Tractable and Positive Ambiguity Theory"

Aurélien Baillon, Han Bleichrodt, Chen Li, & Peter P. Wakker

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In the Online Appendix, the first part, the Addition to the Proof of Observation 1, is not well organized. Strict increasingness of w is used (in the assumption in that proof that $K(B) = p$ and in the construction of the A_i (with $W(A_i) = W(A - B)$) before it is proved. I, therefore, derive strict increasingness of w first and independently here.

LEMMA 1. w is strictly increasing at $p = 0$; i.e., $w(p) > 0$ for all $p > 0$ in RK.

PROOF. For contradiction, assume $w(q) = 0$ for some $q > 0$ in RK. Then $w(p) = 0$ for all $0 \leq p \leq q$ and every risky event R with $K(R) \leq q$ is null. Every nonnull event has K value exceeding q. By convex-rangedness and induction we can define a nested infinite sequence of events E_i with $W(E_i) = 1 - 1/i$. We get an infinite sequence of disjoint nonnull events $E_i - E_{i-1}$ each with K value exceeding $q > 0$, which cannot be. A contradiction has resulted. □

LEMMA 2. w is strictly increasing on all RK.

PROOF. For contradiction, assume $w(p) = w(q)$ for some $p < q$ in RK. By Lemma 1, $w(q) > 0$. Let Q be an event with $K(Q) = q$. For $R \subset Q$, if $K(R) \leq q - p$ then $K(Q - R) \geq p$ so that $W(Q - R) = W(Q)$, implying that R is null. Every nonnull subset of Q has K value > $q - p$. By convex-rangedness and induction we can define a nested infinite sequence of events E_i with $W(E_i) = W(Q) - 1/i$. We get an infinite sequence of disjoint nonnull events $E_i - E_{i-1}$ each with K value exceeding $q - p > 0$, which cannot be. A contradiction has resulted. □

Now the Addition to the Proof of Observation 1 in the Online Appendix can be used.