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An Axiomatization of Cumulative Prospect Theory for Decision Under Risk

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Abstract

Cumulative prospect theory was introduced by Tversky and Kahneman so as to combine the empirical realism of their original prospect theory with the theoretical advantages of Quiggin's rank-dependent utility. Preference axiomatizations were provided in several papers. All those axiomatizations, however, only consider decision under uncertainty. No axiomatization has been provided as yet for decision under risk, i.e., the case in which given probabilities are transformed. Providing the latter is the purpose of this note. The resulting axiomatization is considerably simpler than that for uncertainty.

Key words: prospect theory, rank-dependent utility, rank-dependence, sign-dependence, comonotonicity, tradeoff consistency

JEL Classification: D81

Prospect theory (Kahneman and Tversky, 1979) was one of the first models for decision under risk that permitted descriptive deviations from rationality and achieved theoretical tractability at the same time. However, a difficulty in its method for transforming probabilities is that violations of stochastic dominance are implied (Fishburn, 1978; Kahneman and Tversky, 1979, pp. 283/284). The problem has been solved by Quiggin's (1981) rank-dependent utility. Tversky and Kahneman (1992) invoke Quiggin's idea and thus combine the descriptive advantages of original prospect theory with the theoretical advantages of rank-dependent utility. Their theory provides one of the most promising nonexpected utility models presently available.

An additional advantage of cumulative prospect theory as compared to original prospect theory is that it can also be applied to uncertainty, i.e., the case in which probabilities of events are not given. Preference axioms have been provided by Tversky and Kahneman (1992), Wakker and Tversky (1993), and, in a somewhat different setup, by Luce and Fishburn (1991). The functional of cumulative prospect theory already appeared in Starmer and Sugden (1989, Appendix), but they did not provide a preference axiomatization.

Although the theory is meant to apply to both uncertainty and risk, preference axiomatizations have hitherto been provided only for decision under uncertainty. There is no preference axiomatization of cumulative prospect theory available for risk yet, i.e., the case where probabilities are given and are to be transformed. Filling that gap is the purpose of this note. The result for risk turns out to be considerably simpler than that for uncertainty. We hope that the simplicity will contribute to a better accessibility of cumulative prospect theory.

1. Basic concepts

This section provides the basic definitions of decision under risk and cumulative prospect theory. X is a set of *outcomes*. Readers only interested in monetary outcomes can assume henceforth that $X = \mathbb{R}$. The rest of this note can be read, and is most easily accessible, under that assumption. It can also be assumed that X is a convex subset of $\mathbb{R}^m$ (commodity bundles), a case that is covered as well by the following analysis. In general, X is a "connected topological space" (for topological definitions, see for instance Dugundji (1996)). The *status quo* is a given element of X, denoted by 0; we use this notation also for general outcomes. Throughout this note the status quo is fixed.

Prospects are finitely supported probability distributions over X, i.e., they assign probability 1 to a finite subset of X. They are denoted by $(x_1, p_1; \ldots; x_n, p_n)$ etc., yielding outcome x_j with probability p_j; the probabilities are nonnegative and sum to one.

Let $\succcurlyeq$ denote a preference relation over prospects. The notation $\succ$, $\sim$, $\preccurlyeq$, and $\prec$ is as usual. Outcomes are identified with degenerate ("riskless") prospects. The symbol $\succcurlyeq$ is also used for the preference relation over outcomes derived through degenerate prospects. An outcome $x \succ 0$ is a *gain* and an outcome $x \prec 0$ is a *loss*. For real-valued outcomes, this note is easiest to read if $\succcurlyeq$ is assumed to agree with the natural ordering $\geq$ on the outcomes. As a notational convention, outcomes are always assumed rank-ordered, i.e., in the notation $(x_1, p_1; \ldots; x_n, p_n)$ it is implicitly understood that $x_1 \succcurlyeq \cdots \succcurlyeq x_n$.

A function V *represents* $\succcurlyeq$ if, for all prospects P, Q, $P \succcurlyeq Q$ if and only if $V(P) \geq V(Q)$; then $\succcurlyeq$ is a *weak order*, i.e., it is *complete* ($P \succcurlyeq Q$ *or* $Q \succcurlyeq P$ for all prospects P and Q) and transitive. $\succcurlyeq$ is *continuous* if the sets

$$\{(x_1, \ldots, x_n) \in X^n : (x_1, p_1; \ldots; x_n, p_n) \succcurlyeq (y_1, p_1; \ldots; y_n, p_n)\} \quad \text{and}$$

$$\{(x_1, \ldots, x_n) \in X^n : (x_1, p_1; \ldots; x_n, p_n) \preccurlyeq (y_1, p_1; \ldots; y_n, p_n)\}$$

are closed sets for every natural n, fixed n-tuple of probabilities $(p_1, \ldots, p_n)$, and prospect $(y_1, p_1; \ldots; y_n, p_n)$. Closed refers to the "product topology," i.e., it generates the natural continuity in $\mathbb{R}^n$. Stochastic dominance is defined with respect to the preference order over outcomes. That is, $\succcurlyeq$ satisfies *stochastic dominance* if $(x_1, p_1; \ldots; x_n, p_n) \succcurlyeq (y_1, p_1; \ldots; y_n, p_n)$ whenever $x_j \succcurlyeq y_j$ for all j with a strict preference if $x_j \succ y_j$ for at least one j with $p_j > 0$. This definition coincides with the traditional definition if outcomes are real-valued and the preference relation over outcomes agrees with the natural ordering.

A *weighting function* w is a strictly increasing function from $[0,1]$ to $[0,1]$ with $w(0) = 0$ and $w(1) = 1$. *Rank-dependent utility* holds if there exist a function $v: X \to \mathbb{R}$ and a weighting function w such that $\succcurlyeq$ is represented by

$$(x_1, p_1; \ldots; x_n, p_n) \to \sum_{j=1}^{n} \pi_j v(x_j) \tag{1.1}$$

where

$$\pi_j = w\left(\sum_{i=1}^{j} p_i\right) - w\left(\sum_{i=1}^{j-1} p_i\right)$$

for all j. We follow the common convention that

$$\sum_{i=1}^{0} p_i = 0$$

The π_j's are *decision weights*. Following Tversky and Kahneman (1992), v is called the *value function*. It is well-known that rank-dependent utility satisfies stochastic dominance.

For any prospect P, we define P^+ and P^- in the usual way, i.e., for P^+ all loss outcomes of P are replaced by the status quo and for P^- all gain outcomes of P are replaced by the status quo. The *dual* $\hat{w}$ of a weighting function w is defined by $\hat{w}(p) = 1 - w(1-p)$. *Cumulative prospect theory* (*CPT*) holds if there exist two weighting functions w^+ and w^- and a value function $v: X \to \mathbb{R}$ with $v(0) = 0$ such that $\succcurlyeq$ is represented by the function

CPT(P) =

the rank-dependent utility of P^+ with respect to w^+

+

the rank-dependent utility of P^- with respect to the dual of w^-.

Cumulative prospect theory also satisfies stochastic dominance. The CPT functional can be written in the form of (1.1) with the *decision weights* π_j defined as follows. Let $0 \le k \le n$ be such that $x_1 \succcurlyeq \cdots \succcurlyeq x_k \succcurlyeq 0 \succcurlyeq x_{k+1} \succcurlyeq \cdots \succcurlyeq x_n$. There can be several such k, in which case the particular choice of k is immaterial because the decision weight assigned to $v(0) = 0$ does not matter. Note that $k = n$

if all outcomes are gains and $k = 0$ if they are all losses. Now

$$\pi_j = w^+\left(\sum_{i=1}^{j} p_i\right) - w^+\left(\sum_{i=1}^{j-1} p_i\right) \quad \text{for all } j \leq k \qquad \text{and} \tag{1.2}$$

$$\pi_j = w^-\left(\sum_{i=j}^{n} p_i\right) - w^-\left(\sum_{i=j+1}^{n} p_i\right) \quad \text{for all } j > k. \tag{1.3}$$

2. The preference axiomatization of cumulative prospect theory for risk

For a prospect $P = (x_1, p_1; \ldots; x_n, p_n)$ and an outcome α we define $\alpha_j P$ as the prospect resulting from P by replacing x_j by α, i.e.,

$$\alpha_j P = (x_1, p_1; \ldots; x_{j-1}, p_{j-1}; \boldsymbol{\alpha}, \mathbf{p_j}; x_{j+1}, p_{j+1}; \ldots; x_n, p_n).$$

It is understood throughout this note and without further mentioning that $x_{j-1} \succcurlyeq \alpha \succcurlyeq x_{j+1}$ in this notation. This implicit convention agrees with the rank-ordered notation of prospects adopted throughout this note. When we write $\alpha_j P$, the notation $(x_1, p_1; \ldots; x_n, p_n)$ for P must be understood.

The following relations can be used to elicit the ordering of value differences empirically. This note uses them for theoretical purposes. We write $[\alpha; \beta] \succ^* [\gamma; \delta]$, or $\alpha\beta \succ^* \gamma\delta$ for short, if the four outcomes are all gains or all losses and there exist prospects $P = (x_1, p_1; \ldots; x_n, p_n)$ and $Q = (y_1, p_1; \ldots; y_n, p_n)$ and an index j with $p_j > 0$ such that

$$\alpha_j P \succcurlyeq \beta_j Q \qquad \text{and} \tag{2.1}$$

$$\gamma_j P \prec \delta_j Q. \tag{2.2}$$

Note that P and Q should have the same probability tuple $p_1, \ldots p_n$. We write $\succcurlyeq^*$ instead of $\succ^*$ if in the lower preference we have $\preccurlyeq$ instead of $\prec$. As an illustration, consider the preferences

$$\left(90, \frac{1}{3}; \mathbf{1}, \frac{2}{3}\right) \succcurlyeq \left(95, \frac{1}{3}; \mathbf{0}, \frac{2}{3}\right)$$

$$\left(90, \frac{1}{3}; \mathbf{9}, \frac{2}{3}\right) \prec \left(95, \frac{1}{3}; \mathbf{8}, \frac{2}{3}\right).$$

Then $[1; 0] \succ^* [9; 8]$. The preferences suggest that the tradeoff of receiving \$1 instead of \$0 outweighs something that the tradeoff of receiving \$9 instead of \$8 does not outweigh.

The following lemma shows that the * relations elicit orderings of value differences under cumulative prospect theory. Note that the rank-dependence of cumulative prospect theory is accommodated by the rank-ordered notation of outcomes in prospects. Hence this notation is crucial for the meaning of the above * relations and the axioms derived from them later. Sign-dependence is accommodated by the requirement that all outcomes should have the same sign. Because the proof of the lemma may be clarifying, it is given in the main text.

Lemma 2.1. *Under CPT*,

$$\alpha\beta \succ^* \gamma\delta \Rightarrow v(\alpha) - v(\beta) > v(\gamma) - v(\delta), \tag{2.3}$$

$$\alpha\beta \succcurlyeq^* \gamma\delta \Rightarrow v(\alpha) - v(\beta) \geq v(\gamma) - v(\delta). \tag{2.4}$$

Proof: Consider (2.3). We substitute the CPT formula described in (1.1), (1.2), and (1.3), in (2.1) and (2.2). The decision weights for $\alpha_j P$ and $\gamma_j P$ are the same and are written as π_i, $i = 1, \ldots, n$, and the decision weights for $\beta_j Q$ and $\delta_j Q$ are also the same and are written as λ_i, $i = 1, \ldots, n$. The π_i's may differ from the λ_i's because of different signs of outcomes. The jth outcomes in all lotteries have the same sign however, and therefore $\pi_j = \lambda_j$, given by (1.2) if the outcomes are positive and by (1.3) if they are negative. This equality is crucial and is emphasized by the notation $\mu = \pi_j = \lambda_j$. Substituting CPT in the two preferences in (2.1) and (2.2) implies:

$$\sum_{i \neq j} \pi_i v(x_i) + \mu v(\alpha) \geq \sum_{i \neq j} \lambda_i v(y_i) + \mu v(\beta); \qquad \text{hence}$$

$$\mu(v(\alpha) - v(\beta)) \geq \sum_{i \neq j} \lambda_i v(y_i) - \sum_{i \neq j} \pi_i v(x_i)$$

and

$$\sum_{i \neq j} \pi_i v(x_i) + \mu v(\gamma) < \sum_{i \neq j} \lambda_i v(y_i) + \mu v(\delta); \qquad \text{hence}$$

$$\mu(v(\gamma) - v(\delta)) < \sum_{i \neq j} \lambda_i v(y_i) - \sum_{i \neq j} \pi_i v(x_i).$$

The second and fourth inequalities have the same right-hand side. Hence $\mu(v(\alpha) - v(\beta)) > \mu(v(\gamma) - v(\delta))$. μ is positive and can be dropped. The consequent inequality in the lemma follows. The second implication in the lemma is derived similarly (where again μ can be dropped in the end because p_j is positive and the weighting functions are strictly increasing, implying that μ is positive). □

In Wakker and Deneffe (1996), sequences $x^0, x^1, \ldots$ were elicited experimentally such that $[x^{j+1}; x^j] \sim^* [x^1; x^0]$ for all j ($\sim^*$ means both $\succcurlyeq^*$ and $\preccurlyeq^*$). This implies, by the above lemma, that such sequences of outcomes are equally-spaced in v-units. In this manner, v can be elicited empirically. Other empirical investigations using the * relations are Bouzit and Gleyses (1996), Fennema and van Assen (1998), Abdellaoui (1998), and Bleichrodt and Luis Pinto (1998a,b).

The following condition is necessary for CPT so as to avoid contradictory inequalities of value differences. In Wakker and Tversky (1993) the analogous condition was called sign-comonotonic tradeoff consistency, to distinguish it from other similar conditions. Here we omit qualifying adjectives because there are no other similar conditions in this note.

Definition 2.2. *Tradeoff consistency* holds if there are no outcomes $\alpha, \beta, \gamma, \delta$ such that both $\alpha\beta \succ^* \gamma\delta$ and $\gamma\delta \succcurlyeq^* \alpha\beta$.

In words, tradeoff consistency requires that utility elicitations should not run into contradictions. The condition turns out to be not only necessary but also sufficient for cumulative prospect theory, given some natural conditions.

Theorem 2.3. *The following two statements are equivalent*:

(i) *Cumulative Prospect Theory holds with a continuous value function.*
(ii) $\succcurlyeq$ *satisfies the following conditions*:
   (a) *Weak ordering.*
   (b) *Continuity.*
   (c) *Stochastic dominance.*
   (d) *Tradeoff consistency.*

In Statement (i), both weighting functions are uniquely determined and the value function is unique up to multiplication by a positive factor. □

If $X = \mathbb{R}$ and the preferences $\succcurlyeq$ over outcomes agree with the natural ordering of real numbers, then our stochastic dominance agrees with common terminology and the value function v in Statement (i) is strictly increasing. The special case of rank-dependent utility, i.e., the case in which w^- is the dual of w^+ so that there is no sign-dependence, can be axiomatized by a tradeoff consistency condition of * relations defined as in (2.1) and (2.2) but without the restriction that the outcomes $\alpha, \beta, \gamma, \delta$ should be of the same sign. This result has been proved by Wakker (1994). Continuity of the probability transformations in Theorem 2.3 can be axiomatized as in Wakker (1994, Theorem 25d).

This note has axiomatized cumulative prospect theory for decision under risk by combining results known before in the literature. It is motivated by the absence of an axiomatization for risk from Tversky and Kahneman (1992), Wakker and Tversky (1993), and other works.

Appendix

Proof of Theorem 2.3. For necessity of the preference conditions in Statement (ii), weak ordering and continuity follow routinely. Stochastic dominance (defined with respect to the preference ordering over outcomes) follows because all decision weights belonging to positive probabilities are positive which, in turn, follows from strict increasingness of the weighting function. Tradeoff consistency, finally, follows from Lemma 2.1. In the rest of this proof, Statement (ii) is assumed and (i) and the uniqueness results are derived. A detailed self-contained derivation would take much space. Hence, instead, first an informal sketch of the proof is given. Next a complete derivation is given that, however, leans heavily upon Wakker and Tversky's (1993) analysis and definitions.

Informal sketch of the proof. Fix an n-tuple of positive probabilities $p_1, \ldots, p_n$ for $n \geq 3$, and take some $0 \leq k \leq n$. For now, restrict attention to the set of prospects $(x_1, p_1; \ldots; x_n, p_n)$ with $x_1 \succcurlyeq \cdots \succcurlyeq x_k \succcurlyeq 0 \succcurlyeq x_{k+1} \succcurlyeq \cdots \succcurlyeq x_n$. On this set, the *sure-thing principle* is satisfied, i.e., $\alpha_j P \succcurlyeq \alpha_j Q$ implies $\beta_j P \succcurlyeq \beta_j Q$. The principle follows because, for any prospect R, reflexivity implies $\beta_j R \succcurlyeq \beta_j R$ and $\alpha_j R \preccurlyeq \alpha_j R$, and thus the obvious $\beta\beta \succcurlyeq^* \alpha\alpha$ (note that, due to the set of prospects considered, α and β are of the same sign). Now $\alpha_j P \succcurlyeq \alpha_j Q$ and $\beta_j P \prec \beta_j Q$ would imply $\alpha\alpha \succ^* \beta\beta$, i.e., a violation of tradeoff consistency. Hence the sure-thing principle must hold on the set of prospects.

From the sure-thing principle an additive representation $(x_1, p_1; \ldots; x_n, p_n) \mapsto V_1(x_1) + \cdots + V_n(x_n)$ is derived on the set of prospects now under consideration. This set is not a full Cartesian product (it is a product of two "comonotonic" subsets of Cartesian products), but still an additive representation can be obtained by adapting classical results for full Cartesian products to the subset now considered (Chateauneuf and Wakker 1993, Theorem C.6).

Set $V_j(0) = 0$ for all j. By an analogue of Lemma 2.1, $\alpha\beta \succ^* \gamma\delta$ implies $V_j(\alpha) - V_j(\beta) > V_j(\gamma) - V_j(\delta)$ for the index j as in (2.1). Tradeoff consistency implies that the V_j's all order differences in the same manner. Hence they are proportional and positive weights $\pi_1, \ldots, \pi_n$, summing to one, can be found such that $V_j = \pi_j v$ for all j, with $v = V_1 + \cdots + V_n$. The π_j's are the decision weights belonging to the p_j's.

The representations can be fit together for the various choices of probability n-tuples $p_1, \ldots, p_n$ and indexes k. The weighting functions w^+ and w^- can be recovered from the decision weights, e.g., for a probability n-tuple $p_1, \ldots, p_n$ and $j \leq k$, $w^+(p_1 + \cdots + p_j) = \pi_1 + \ldots + \pi_j$.

Detailed proof. Define $S = [0, 1]$. S will play a role as *state space* in decision under uncertainty, and is endowed with the regular Borel sigma-algebra containing all intervals etc. We consider the set of *simple acts*, i.e., functions $(x_1, A_1; \ldots; x_n, A_n)$ assigning x_j to A_j for each j where $A_1, \ldots, A_n$ is a measurable partition of S. Each such act generates a probability distribution over X equal to the prospect

$(x_{\rho_1}, p_{\rho_1}; \ldots; x_{\rho_n}, p_{\rho_n})$. Here $(\rho_1, \ldots, \rho_n)$ is a permutation on $\{1, \ldots, n\}$ such that $x_{\rho_1} \succcurlyeq \cdots \succcurlyeq x_{\rho_n}$ and p_{ρ_j} is equal to the Lebesgue measure of A_{ρ_j} for each j (the Lebesgue measure assigns to each interval its length). Thus, the preference relation $\succcurlyeq$ over prospects generates a preference relation $\hat{\succcurlyeq}$ over the simple acts in the obvious manner.

Consider any fixed partition $(A_1; \ldots; A_n)$ with $n \geq 3$ and all sets A_j nonnull (positive Lebesgue measure), and consider the set of all simple acts $(x_1, A_1; \ldots; x_n, A_n)$. The conditions of Proposition 8.2 and Statement (ii) in Theorem 6.3 of Wakker and Tversky (1993) will be derived on this set of acts. Weak ordering is immediate. Continuity of preference follows because ("outcome-wise") convergence of acts on this set implies ("outcome-wise") convergence of the belonging prospects, first within every "comonotonic" subset of acts, next in general by considering various comonotonic subsequences. The monotonicity condition of Proposition 8.2 follows immediately from stochastic dominance and the A_j's being nonnull. Finally, "weak sign-comonotonic tradeoff consistency" is discussed.

Weak sign-comonotonic tradeoff consistency means that the tradeoff consistency principle holds within every subset of acts $(x_{\rho_1}, A_{\rho_1}; \ldots; x_{\rho_n}, A_{\rho_n})$ with $x_{\rho_1} \succcurlyeq \cdots \succcurlyeq x_{\rho_k} \succcurlyeq 0 \succcurlyeq x_{\rho_{k+1}} \succcurlyeq \cdots \succcurlyeq x_{\rho_n}$ for some index k and permutation ρ. The condition immediately follows because within each such subset of acts it is equivalent to our tradeoff consistency. Hence Proposition 8.2 of Wakker and Tversky (1993) can be applied and a CPT representation for uncertainty in the sense of Wakker and Tversky (1993) results on the whole subset of acts $(x_1, A_1; \ldots; x_n, A_n)$.

Similarly, a CPT representation can be derived on every subset of acts of the form $(y_1, B_1; \ldots; y_m, B_m)$, for any partition $(B_1, \ldots, B_m)$ with $m \geq 3$ and with nonnull B_j. The CPT representations for two different partitions can be fitted together (same value function and same nonadditive measures) by considering the partition that is the common refinement of the two partitions. Thus, a CPT representation on the set of all simple acts on S results. (Two-valued acts $(y_1, B_1; y_2, B_2)$ are obviously included by taking a refinement of the partition (B_1, B_2).)

The CPT representation on simple acts involves nonadditive measures W^+ and W^-. We define $w^+(p) = W^+([0, p[)$ and $w^-(q) = W^-([1-q, 1[)$. By stochastic dominance, w^+ and w^- are strictly increasing. The restriction of the CPT representation over acts to the simple acts that are nondecreasing functions on S suffices to yield the CPT representation over all prospects. It can, but need not, be derived from stochastic dominance that all acts that generate the same probability distribution over outcomes are equivalent. The uniqueness results follow from those in Wakker and Tversky (1993). □

Note

1. This note was written while the second author visited CERMSEM of the Dept. of Mathematics and Computer Science, University of Paris I-Panthéon-Sorbonne.

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