

CORRECTION OF FOOTNOTE 4 of Fennema & Wakker (1994)

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Bleichrodt (1994) pointed out a mistake in the proof in Footnote 4 of Fennema & Wakker (1994). The one-before-the last line writes “smaller” where it should be “larger.” Hence the proof in the footnote does not work. The result to be proved there, that under concavity of the probability weighting function w in the RDU-model with linear utility the single-prize lottery $(p, M; (1 - p), 0)$ has a higher RDU-value than the two-prize lottery $(q, M; r, m; (1 - q - r), 0)$, does nevertheless hold true. An alternative proof is given below:

This can be shown as follows. Rewrite the single prize lottery as $(M, q; M, r'; 0, 1 - q - r')$, where $r' = p - q$ and, since $M > m$, $r' < r$. By equality of expected value, $Mr' = mr$. The average derivative of w over the interval $[q, q + r']$ is $\frac{w(q+r')-w(q)}{r'}$, and over the interval $[q, q + r]$ it is $\frac{w(q+r)-w(q)}{r}$. By concavity of w and the inequality $r' < r$, the former is larger than the latter, i.e.

$$\frac{w(q + r') - w(q)}{r'} > \frac{w(q + r) - w(q)}{r}.$$

Multiplying by $Mr' = mr$ yields

$$\frac{w(q + r') - w(q)}{r'} Mr' > \frac{w(q + r) - w(q)}{r} mr,$$

i.e.

$$(w(q + r') - w(q))M > (w(q + r) - w(q))m.$$

Adding up $w(q)M$ yields

$$w(q)M + (w(q + r') - w(q))M > w(q)M + (w(q + r) - w(q))m.$$

The left-hand side is the RDU value of the single prize lottery, the right-hand side is the RDU value of the two prize lottery.

REFERENCES

- Bleichrodt, H. (1994), Personal Communication.
Fennema, H. & P.P. Wakker (1994), “An Explanation and Characterization for the Buying of Lotteries,” in Sixtos Rios (Ed.), *Decision Theory and Decision Analysis: Trends and Challenges*, Kluwer Academic Publishers, Dordrecht, 163–175.