Journal of Behavioral Decision Making, Vol. 1, 169–175 (1988)

Nonexpected Utility as Aversion of Information

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ABSTRACT
This paper argues that a violation of the independence condition, used by von Neumann and Morgenstern to characterize expected utility maximization, will always lead to the existence of choice situations in which information-aversion is exhibited.

KEY WORDS Sure-thing principle Von Neumann Morgenstern-independence (Non)expected utility

1. INTRODUCTION

Von Neumann and Morgenstern (1947, 1953) introduced the 'independence condition' for characterizing the maximization of expected utility for decision making under risk. This condition has been criticized ever since, firstly because empirical research shows systematic violations of the condition by persons in experimental and actual decision situations, secondly because introspection has led many scientists to dispute the normative appeal of the condition. Opponents of the condition have come to unify alternative theories under the provocative heading of 'nonexpected utility' (see for instance Karni and Machina, 1987).

A systematic discussion of the independence condition is given by McClennen (1983). He classifies arguments which have been provided in the literature for the appropriateness of the independence condition, and opposes to each of them. To the four arguments discussed by McClennen we add a fifth argument which is a strengthening of the third argument. We thus hope to contribute to the further understanding of the independence condition. This paper considers the context of decision making under risk, with given probabilities; it can be reformulated in a completely straightforward way for decision making under uncertainty, with the 'sure-thing principle' as an analog of independence.

2. THE FOUR DEFENCES

In decision making under risk one deals with preferences over 'lotteries', where in a formal set-up lotteries are modelled as probability distributions over 'prizes'. The kind of prizes that the lotteries yield is immaterial for the present analysis; for instance, prizes may be amounts of money. We shall assume throughout that every pair of lotteries is comparable, and that preferences are transitive, i.e. if a first lottery is better than a second lottery, and the second lottery is better than a third, then the first lottery should be better than the third. For the other conditions mixtures of lotteries will be used. Let there be

0894-3257/88/030169-07$05.00

Received 1 July 1987
Final Revision 28 March 1988

given a 'mixing event', with some known probability, say 1/6. For example, the mixing event may concern the result of a throw of a dice. The 'mixture' which yields some lottery if the mixing event obtains, and another if the mixing event does not obtain, is a 'mixture' of the first and second lottery (say with probabilities 1/6 and 5/6), and is a lottery again. We shall in the sequel use the approach of Jensen (1967) which differs only in details from the approach of von Neumann and Morgenstern. The continuity condition of Jensen requires that, if a first lottery is better than a second, then for any 'unfavorable' lottery, no matter how unfavorable this lottery is, a positive probability threshold can be found such that: the 'distortion' of the first lottery, obtained by mixing the first lottery with the unfavorable lottery, remains close enough to the first lottery to be better than the second lottery, as long as the probability with which the (distortion=) mixture will result in the unfavorable lottery is smaller than the threshold. Let us now turn to the main topic of this paper, the independence condition. As Jensen shows, independence together with the other conditions mentioned so far (all lotteries comparable; transitivity; continuity) are necessary and sufficient for the existence of a utility function on the prizes such that the decision maker maximizes expected utility with respect to this utility function.

The independence condition says that if a first mixture (O4 in Exhibit 1) of a first and a 'fixed' lottery is changed into a second mixture (O5 in Exhibit 1) by replacing the first lottery by a second lottery, then this change is an improvement (so in □2 Exhibit 1 the way down to O5 is preferred to the way up to O4) if and only if the second lottery is better than the first lottery. McClennen (1983) considers four defences of the independence condition, and opposes to each of them. We shall give major attention to the third defence. *We shall assume throughout the sequel that the second lottery is considered better than the first.*

The first defence is criticized by McClennen for begging the question, and will not be repeated here. The second defence states that the independence condition may be considered as a complementarity condition, such as occurs in the evaluation of economic goods in consumer theory. The condition will rarely be appropriate in the evaluation of economic goods, because there will be interaction between the goods, and it is appropriate to reckon with this interaction. In the evaluation of lotteries there is no such interaction because the mixing event and its complement are mutually exclusive. McClennen argues that the absence, in decision making under risk, of one objection against complementarity as held in consumer theory, does not automatically imply a justification of independence for decision making under risk. Let us delay consideration of the third defence for a while, and mention the fourth defence described by McClennen. This concerns a monotonicity idea. Suppose that mixtures are as above and in Exhibit 1. Then the second mixture gives at least as good a lottery as the first mixture, both if the mixing event obtains and if it does not. So by a monotonicity idea the second mixture should be at least as good as the first. McClennen argues that this is not as self-evident as it may seem (we roughly, reformulate, for our lottery set-up) since, for one thing, there may be many different ways to consider a mixture such as the first mixture above as a mixture of lotteries. For brevity, we do not give a full explication and comments on this part of McClennen's work. Instead we turn to the third defence in his paper. It is a commutativity (McClennen uses the term 'commutivity') argument. Roughly, it entails that the order of choice of the decision maker and nature may be reversed.

Let the *ex-ante* choice situation (□2 in Exhibit 1) be the situation where a decision maker must choose between the first and the second mixture as above, without knowing whether or not the mixing event will actually obtain; for example, the decision maker must make (his or) her choice *before* chance has decided whether the mixing event will obtain. Let the *ex-post* choice situation (O3 in Exhibit 1) be the situation where the decision maker will be informed about whether the mixing event obtains before she has to choose; so in any case, the decision maker will have to make her choice *after* chance has decided whether or not the mixing event obtains. We shall present the commutativity argument in three steps.

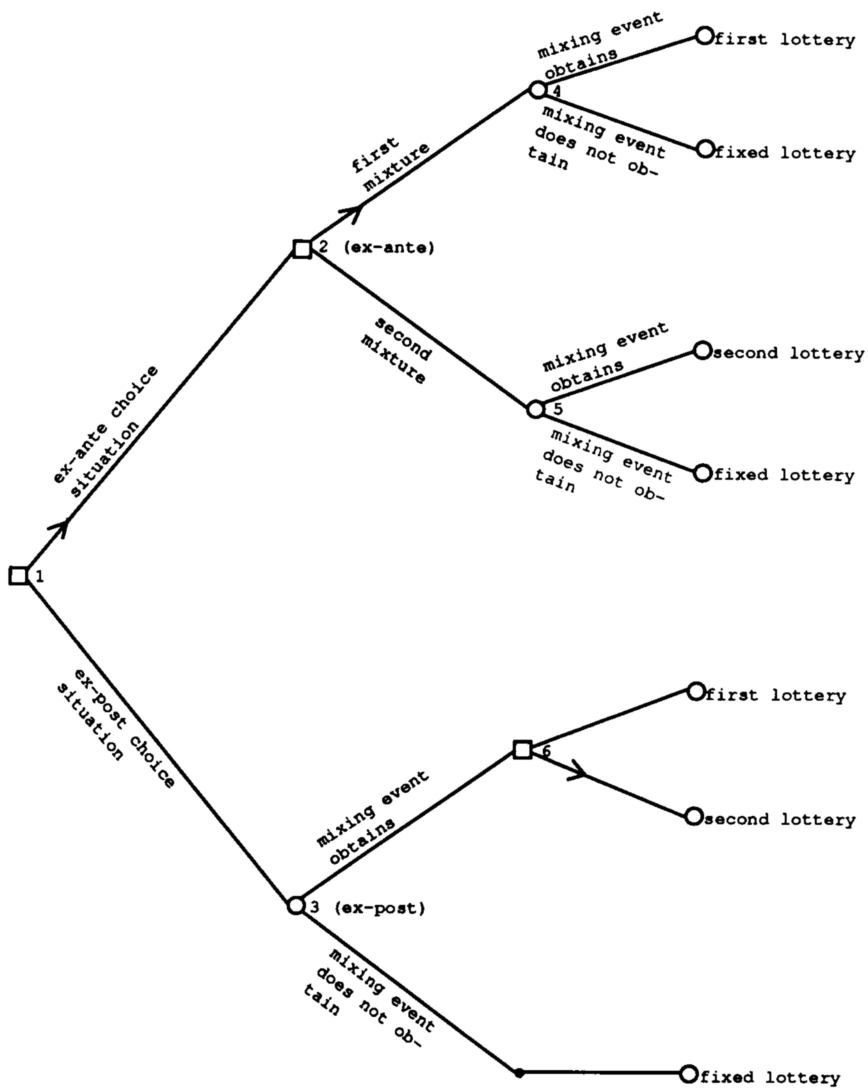

Exhibit 1. In □-points the decision maker must choose; in O-points chance chooses. In □ 1 a choice must be made between the ex-post and ex-ante choice situation. Arrows indicate preferences of a decision maker who violates independence.

The *first step* of the commutativity argument says that it should be immaterial to the decision maker if she has to choose before (*ex-ante*) or after (*ex-post*) the choice of chance concerning whether or not the mixing event obtains. The argumentation is that in each case she has the same two 'strategies' at her disposal. The first strategy will result in the fixed lottery if the mixing event does not obtain, and in the first lottery if the mixing event does obtain. It is accomplished in the ex-ante choice situation by simply choosing the first mixture; in the ex-post choice situation it results from the plan to choose the first lottery if the mixing event does obtain. (The availability of this 'plan' is questionable and is related to 'dynamic consistency'; a further comment on it will be given in Section 4.) The second strategy will, in analogous ways, result in the fixed lottery if the mixing event does not obtain and in the second lottery if the mixing event does obtain. The *second step* of the commutativity argument says that in the ex-post choice situation the decision maker will choose the second strategy as described above, resulting in the second lottery if the mixing event obtains. The argument is that, in the ex-post choice situation, if the mixing event obtains then the decision maker will know this, so in fact will have to choose between the first and second lottery; she will then choose the preferred second lottery. If the mixing event does not obtain, the decision maker will receive the fixed lottery and will have nothing to choose. So in the ex-post choice situation the decision maker will receive the same as the second mixture would have given her: the second lottery if the mixing event obtains, the fixed lottery if not. So, and this is the *third step* of the commutativity argument, in the ex-ante choice situation, the decision maker should act in accordance with the ex-post choice situation and also prefer the second strategy, i.e. the second mixture. This is also what the independence condition requires. Allais (1953, 1979) introduced the terms 'ex-ante' and 'ex-post'. He argued that the difference between the two is relevant, for instance because of ex-ante considerations related to entire probability distributions which are no longer relevant in the ex-post choice situation. Thus Allais disagrees with the commutativity argument. McClennen supports Allais' views.

Let us finally mention that in Karni and Safra (1986) it is shown that in the context of English auctions the independence condition is equivalent to the condition of 'dynamic consistency'. This roughly means that a bidder does not in advance prefer a bidding plan that she knows she will not follow when the actual bidding takes place.

3. A FIFTH DEFENCE

The defence of independence that we shall now propose resembles the third defence described above. It is a well-known fact that under the assumption of independence (thus, under the assumption of expected utility maximization, taking the other conditions for granted) it is never bad, i.e. will never decrease expected utility, to receive additional information. We claim the reverse implication: if someone considers it rational to violate the independence condition, (he or) she will sometimes prefer to decline new information; she will not do this for irrational reasons, such as fear of the truth, but will consider it fully rational.

Let lotteries and mixtures again be as in Exhibit 1. We suppose that the decision maker violates the independence condition, and prefers the second lottery to the first, but prefers the first mixture to the second. Violation of independence also occurs if one of these two preferences is replaced by indifference. This case is analogous, but somewhat more tedious to formulate, hence it will not be considered. Further, any violation of independence can be modelled through preferences as supposed here. We introduce a new decision problem, a two-stage choice situation, as in □1. Again, the decision maker will be faced with the first and second mixture. But now she must first make another choice: she must decide whether or not to receive, before choosing between the mixtures, the information about whether the mixing event obtains or not; this information will be free of charge. So she must choose between the ex-ante or the ex-post choice situation. In Exhibit 1 she will not be indifferent, contrary to what the commutativity principle requires. She knows that if she chooses to receive the information, this will bring her into the ex-post choice situation, and she will end up with the fixed lottery if the mixing event does not obtain; if the mixing event does obtain, then she will know this and have the choice between the first and second lottery, in which case the preferred second lottery will be chosen. So choosing to receive the information is identical with choosing the second mixture.

If she chooses not to receive the information, then she will be in the ex-ante choice situation and she will choose the preferred first mixture. So choosing not to receive the information is identical with choosing the first mixture. The decision maker, preferring the first mixture, will thus choose not to receive the information. *So if a decision maker violates the independence condition of von Neumann and Morgenstern, then not only will she violate the commutativity principle by not being indifferent between ex-post and ex-ante choice situations, but furthermore she will strictly prefer the ex-ante choice situation, i.e. she will strictly prefer it not to receive additional information.*

In practical situations there may be many good reasons why one would not want to receive information. For example, one may want to avoid responsibility, one may fear unfavorable information and subsequent 'irrationality from fear', one may dislike the effort of accepting and storing information; etc. We think, however, that these are reasons which fall outside the scope of the formal analysis of choices as we carried out above. If such reasons are relevant in a practical situation, then they should explicitly be incorporated into the model at appropriate places. We advocate the view that reasons as above in this paragraph should lead to more complicated modellings of actual decision situations, rather than to abandonment of independence, and thus to abandonment of the principle that receipt of information, free of charge, is never bad.

4. POSSIBLE OBJECTIONS AGAINST OUR ARGUMENT

In this section we shall consider some possible objections against our argument set forth the previous section. This will cause us to make explicit some assumptions underlying the argument.

The first objection may be that if the decision maker chooses to receive additional information, i.e. to be in the ex-post choice situation O3, then receipt of this additional information may change the situation in ways not analysed by us. To defend our analysis against this objection we must assume that the receipt of the information satisfies a *ceteris paribus* condition. The receipt of the information should not affect other relevant aspects of the situation. That is why the information should be free of charge. Further, it should not change the needs and desires of the decision maker, or her beliefs, apart from what the new information entails. We interpret the assumption that receipt of information does not alter utilities, and induces the replacement of probabilities by conditional probabilities given the content of the new information, as formal reflections of this *ceteris paribus* condition. In actual practice the satisfaction of this *ceteris paribus* condition is far from trivial. Indeed, its nontriviality has sometimes been overlooked, and complicates the applicability of the formula of Bayes. Explicit attention and further references to it are given in Diaconis and Zabell (1982, beginning of section 1.2), Shafer (1985, the beginning of section 2), and Eells (1987, p. 174). The most famous reference is given in Ramsey (1931, p. 180). It also underlies the 'well-known fact' mentioned at the beginning of the previous section.

For the second objection, let us reconsider the commutativity argument as presented above. In step 1, it was assumed that the decision maker had the first and second strategies available as described, through an advance 'plan'. However, the assumption that a decision maker will act in accordance with advance planning comes down to the assumption of dynamic consistency. Apparently the assumption of dynamic consistency is implicit in (our presentation of) the commutativity argument. Thus a second objection against our argument as presented in the previous section, may be that our argument also implicitly assumes dynamic consistency. In defence against this objection, let us emphasize that we assumed that in □6, i.e. the case where the decision maker has chosen to receive the information, and where furthermore, the mixing event has obtained, the decision maker chooses the second lottery simply because the second lottery at that moment is preferred to the first lottery. And this she could foresee at □1. Nowhere was there an assumption that at □1 she would also agree with that choice of the second lottery at □6. Perhaps she would, when able to determine and lay down already at □1 her choice at the point □6, have preferred to lay down the choice of the first lottery at □6. But she simply has no way available to lay down in □1 her choice at □6. The assumption that she would always agree beforehand with choices she could foresee herself making in the future would indeed come down to the assumption of dynamic consistency. In fact, dynamic consistency is more or less equivalent to independence (see Karni and Safra, 1986). Thus, had this assumption been implicit in our argument, it would have deprived our argument of all force.

A third objection against our argument may be that our analysis is incorrect because it ignores phenomena on a 'meta-level', and that these phenomena, rather than independence, account for the appropriateness of preference for receiving the information at □1. After receipt of the information the situation seems simpler to analyse, because there remain fewer cases to reckon with during the analysis, especially if the mixing event does not obtain. According to the third objection, it is because of this (meta-) argument that the receipt of information should be preferred. Such meta-arguments depend upon the way the decision situations are presented and may, under different presentations, very well induce violations of independence (e.g. in the well-known Allais paradox). Thus, according to this third objection there is no reason yet to rely upon independence. To defend our argument against this objection the idealized assumption must be made that there is not any dislike (or like) in analyzing the decision situation, so that meta-arguments as above are ruled out. This idealized assumption may be placed under the *ceteris paribus* condition about the receipt of the information: this receipt should not make the desirability (utility) of prizes higher, for there having been carried out less complicated analyses.

When looking at the defences against the above three possible objections one may feel that our argumentation is circular. Whenever there is a violation of independence, we will say that a *ceteris paribus* condition has been violated, e.g. about the receipt of information. In this vein one may interpret this paper not so much as a real defence and argument for independence, but rather as a proposal about conditions to impose on some primitives of decision theory, and an intended contribution to the settling of the paradigm of decision theory.

5. CONCLUSION

If a decision maker satisfies the independence condition and maximizes expected utility, then new information will never be declined. This paper shows a reversed implication: if a decision maker never declines additional information, if free of charge, then the decision maker must satisfy the independence condition. We would like to see it as a desideratum for normative decision theories that receipt of new information will always be appreciated. Hence we hope that the result of this paper will serve as an argument in favour of the normative appropriateness of the independence condition.

ACKNOWLEDGEMENT

The first and second objections above were put forward by referees. Several obscurities in an earlier version of the paper were removed on the basis of comments of Th. Bezembinder. The research was made possible by a fellowship of the Royal Netherlands Academy of Arts and Sciences.

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