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Supplementary information

Economists' views on the ergodicity problem

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Supplementary Information on:
"Economists' Views on the Ergodicity Problem"

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This paper provides supplementary information for Doctor, Wakker, & Wang's[1] criticism of Peters[2] (2019). Peters, jointly with collaborators, criticizes the expected utility (EU) model of economics and, based on that, all of economics. He claims that his expertise of ergodic theory, a subdiscipline of mathematics that considers particular kinds of intertemporal processes, can better explain economic phenomena than the current economic theories. We will show that his criticisms are unfounded.

1. Expected Utility Does not Need Parallel Universes

EU concerns choices between different probability distributions $(p_1 : x_1, \ldots, p_n : x_n)$ over (say) money, yielding $\$x_j$ with probability p_j. A utility function U assigns to each money amount its subjective value. Then the probability distribution is chosen with the highest EU value $\sum_{j=1}^{n} p_j U(x_j)$. This is what EU does, modeling risky choice making—no more and no less. Decisions can be single-person and one-time. Then averaging over persons or time plays no role. The EU value does not actually have to be realized or consumed in any sense. In general, there may not even exist any outcome yielding that particular utility level. However, Peters erroneously thinks that the EU value must actually be realized in some sense. He, thus, writes: "But I do not … harvest the average psychological consequences of the actions of my multiverse clones" (p. 2018). For unclear reasons, Peters[3] accepts averages as criteria for resolving uncertainty only if they can be taken as averages over time or over persons (“systems”):

> Of course, we are not a priori interested in such an average because we cannot realize the average payout over all possible states of the universe. Following the arguments of Boltzmann and Maxwell, this quantity is meaningful only in two cases.
> — The physical situation could be close to an ensemble of non-interacting systems which eventually share their resources. This would be the case if many participants took part in independent rounds of the lottery, with an agreement to share their payouts, which would be a problem in portfolio construction, and different from Bernoulli’s set-up.
> — The ensemble average could reflect the time-average performance of a single participant in the lottery.
> …
> …
> In general, to realize an average over the ensemble, ensemble members must exchange resources, but this is often impossible. (p. 4920)

Similar claims are in Peters & Gell-Mann ([4] abstract).

For the above probability distribution, only one x_j will actually be realized but which one is not known in advance. Thus, EU involves imagining, a priori, some outcomes that later may not have actually been received. This procedure involves imagining consequences that will never happen. But we do this every day, and such is the nature of every probabilistic decision. We do not need to believe in “parallel universes” or the existence of “multiverse clones,” contrary to Peters’[2] claims (below his equation 8). His claims probably derive from his misunderstanding that the EU level should be actually physically consumed in some sense. Famous justifications of EU[5,6,7] do not involve multiverses, or averages over persons or time.

2. Mathematical Similarity without Requiring Ergodicity

Peters[2] (p. 1218; see also Peters & Adamou[8] Theorem 1) shows that his alternative ergodic model has some mathematical properties similar to EU. He writes: “the equations that appear in the two frameworks can be very similar. … conceptually the two approaches couldn’t be more different.” For this similarity, not EU, but Peters’ theory needs an ergodicity assumption (mainly through his equation 8). This assumption concerns dynamic processes over time, entailing that averages over time and states are the same. But Peters criticizes EU (and then in one blow all of economics) for “implicitly” making the ergodicity assumption. His abstract writes: “It may therefore come as a surprise to learn that the prevailing formulations of economic theory — expected utility theory and its descendants — make an indiscriminate assumption of ergodicity." He also writes: "And it turns out a surprising reframing of economic theory follows directly from asking the core ergodicity question: … At a crucial place in the foundations of economics, it is assumed that the answer is always yes — a pernicious error" (p. 1216). Peters & Gell-Mann[4] write: "it assumes that expectation values reflect what happens over time," and make similar claims throughout their paper. Again, EU does not need any assumptions about any processes over time, let be that such processes satisfy any ergodicity condition. Only Peters' own theory needs such assumptions.

The mathematical isomorphism between intertemporal phenomena (including ergodic theory) and interpersonal phenomena (welfare theory), inter-event phenomena (risk/uncertainty) or, in general, any kind of multiattribute phenomena has often been noted. Thus, Keeney & Raiffa analyze intertemporal choice as a special case of their multiattribute utility ([9] Ch. 9). Wakker's[10] Appendix D shows an isomorphism of intertemporal choice (Interpretation D.3 there) with decision under uncertainty (D.1), interpersonal choice (D.2), and five other multiattribute examples.

3. Peters' Growth Rate Does not Fit Preferences

Peters'[2] model, based on growth rates, may predict preferences incorrectly. We first discuss this issue from a normative perspective. Peters claims that multiplicative growth processes should be evaluated by their growth rate (e.g., his p. 1220). Peters & Gell-Mann write: "the rate of change in wealth is an ergodic observable, and he who chooses wisely with respect to its expectation value also chooses wisely with respect to the time average" ([4]below their equation 5). They also write: "deep insight is gained by finding the right object to optimize—we suggest time-average growth" (Section V). Such claims can be criticized. The following example elaborates on the example in Doctor, Wakker, & Wang[1]. It shows that growth rate need not be a good optimality criterion even for a multiplicative growth process.

EXAMPLE 1. Assume a wealth level of \$10,000 and a process of three rounds. Under growth process A, in every round, with certainty, wealth is multiplied by a factor

10^{-3}. Thus, at the end of the three rounds, one is sure to be left with less than half a cent.

Under growth process B, with probability 10^{-4} wealth is multiplied by $10^{-200,000}$, and with probability $1 - 10^{-4}$ wealth is multiplied by 10. Here, with a probability exceeding 0.999 one ends up with $10,000,000, and with a probability less than 0.001 one ends up with essentially 0 or maybe some more. Growth process A has a higher average growth rate than growth process B, but still virtually everyone will prefer B.

Replacing probability 10^{-4} above by probability 10^{-m} with m large enough, and next replacing the factor $10^{-200,000}$ by a factor 10^{-n} with n large enough (much exceeding m), one can create similar examples for any starting wealth level and any finite number of rounds, with results as extreme as desired. □

As for empirical findings, many economic papers have investigated human perceptions of exponential growth processes. For example, the exponential growth bias has often been discussed. It refers to people's tendency to underestimate exponential growth, and has been studied both theoretically[11] and empirically[12]. As a timely subject, it also underlies people's underestimation of the potential spread of the COVID-19 coronavirus.

A more fundamental problem in dynamic decisions is that we do not just maximize our entire wealth at the end of our life, but intermediate consumption patterns virtually always play a role. We seek for optimal consumption patterns over all the months of our life. For instance, we want to avoid large temporary downward fluctuations of income and spend more money when raising children. This optimization is too complex to be captured by one simple multiplicative growth factor, contrary to suggestions of Peters' ergodic economics. For nonquantitative outcomes, growth rates cannot even be defined. Dynamic questions as discussed here are central, for instance, in economic growth theory[13] and in life-cycle consumption theory[14]. There, dynamic criteria more refined than growth rates are examined in detail. The Journal of Economic Dynamics & Control focuses on such topics.

4. Confusing Ubiquity and Explanatory Power

Peters suggests that economists should primarily study intertemporal processes, the topic of ergodic theory. For example, he suggests that risk attitudes and risky variance are not important and that interpersonal variations are not important[a,15], and then, in one blow, that neither is any economic theory[b]. He writes: “If we pay close attention to the ergodicity problem, natural solutions emerge. We therefore have reason to be optimistic about the future of economic theory” (p. 2016). “For that to make any sense in the context of individuals making financial decisions, an ergodic observable had to be created. Expected utility theory — unknowingly, because ergodicity hadn’t been invented — did just that. But because of the lack of conceptual clarity, the entire field of economics drifted in a direction that places too much emphasis on psychology” (p. 1219). And: “Placing considerations of time and ergodicity centre stage, we will arrive at a clear interpretation both of discounting and of utility theory, without appealing to subjective psychology or indeed other forms of personalization” (pp. 1216-1217). And: “Perhaps people aren’t so different, but their circumstances are” (p. 1218). Throughout, Peters & Gell-Mann claim that EU should consider intertemporal aspects, and that those are the most relevant aspects[4].

Time is, indeed, ubiquitous, but so are risk, other people, our neural processes, laws of physics, and so on. Although there are situations where development over time is the most central aspect, in many other situations other aspects are more central, such as risky variations, interpersonal interactions, or tradeoffs between different goods or criteria. Nevertheless, in many economic subfields, dynamics and growth over time are central. But then they are given that central role, with an additive or multiplicative growth process if appropriate, and usually without the ergodicity assumption, e.g., throughout finance. There, besides (logarithmic) returns (≈ growth rate), volatility is of central importance. Other fields are discussed at the end of §3.

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[a] Thousands of economic studies found interpersonal variations (e.g., a study[15] with 2939 subjects from 30 countries) and used them to explain behaviors (undersaving, smoking, etc.).

[b] Economics, like any discipline, studies many topics, and in most neither risk nor time is central. Such topics include market prices making supply and demand agree, fair and efficient divisions among people, competing versus completing commodities, inefficiencies in prisoner dilemmas, good auction systems, micro-economic explanations of macro-economic phenomena, and so on.

Peters' claims that, because of the ubiquity of time, we should always study intertemporal growth. Similarly, a risk theorist can claim that we always face uncertainties and, therefore, we should always study risk theories. A physicist can claim that everything consists of molecules, so we should always study physics. A neuroscientist can claim that all decisions come from our brains and we should always study neuroscience. These claims are all as invalid as is Peters' claim. In the annotated bibliography Wakker (2020)[16], the keyword "own small expertise = meaning of life" gives references to other authors falling victim to this ubiquity fallacy.

5. Falsifications of EU Are Well-Known

Building on Peters & Gell-Mann[4], Meder et al.[17] incorrectly apply static EU to a dynamic context (see the Appendix) in their experiment. Peters' suggestion that all of economics would make this mistake is unfounded. Peters cites Meder et al.'s[17] experiment for finding that two different utility measurements gave inconsistent utility functions under EU.[c] This would falsify EU. However, this experiment has a number of problems (see Appendix). But even a flawless falsification of EU would not affect economics because such falsifications have long been known[18]. Because of this and numerous other falsifications of EU, most famously the paradox of Economics' (1988) Nobel Memorial Prize winner Allais[19], many economists prefer to use generalizations of EU, using insights from psychology. The most popular, prospect theory, was introduced by two psychologists: Kahneman & Tversky[20], awarded part of the Nobel Memorial Prize (2002). Peters' claim that Meder et al.[17] would falsify EU, even if correct, would provide no valid criticism of economics.

Many other falsifications of classical models beyond risky or intertemporal decisions have been found[21,22]. This led to the new behavioral approach, which was awarded the Nobel Memorial Prize (2017)[23]. Its main novelty is to use many insights from psychology. Peters deviates much from this approach. For example, Peters writes: "But because of the lack of conceptual clarity, the entire field of economics drifted in a direction that places too much emphasis on psychology" (p. 1219).

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[c] Meder et al.'s[17] equation (10) for prospect theory is incorrect. See the Appendix.

6. Conclusion

Like physics, economics is a big field, and many questions of great variety are studied. Risk theory and, within that, expected utility theory (EU), is only a small subpart. Peters' suggestion that, by criticizing EU, he has refuted all of economics is absurd. Further, his criticisms of EU are flawed. EU does not need to assume parallel universes nor ergodicity. Peters suggests growth rate as an almost universal decision criterion, which is both normatively and empirically flawed. Finally, ubiquity of time does not imply that everyone should always study it or that all questions in economics can be resolved using ergodic theory.

Appendix. A Discussion of Meder et al.'s[17] Experiment

Meder et al. carried out an experiment to test EU and prospect theory against Peters' ergodic theory.[17] We note that it was in a preliminary stage and its authors were still in the process of reconsidering. We discuss it here because Peters paid much attention to this study. There were two within-subject treatments. At the beginning of each treatment, subjects were endowed with about \$150 (≈ 1000 Danish Kroner, which was possibly changed during a passive session). There were 312 rounds in which each subject had to choose between two fifty-fifty probability distributions over intermediate outcomes (our term). In treatment^{+}, the intermediate outcomes of a probability distribution were $Y + s_1$ or $Y + s_2$, where Y was the total money accumulated in the preceding round, and $s_1 > 0$ and $s_2 < 0$ varied per round. In treatment$^{\times}$, the intermediate outcomes of a probability distribution were $Y \times s_1$ or $Y \times s_2$, where Y was the total money accumulated in the preceding round, and $s_1 > 1$ and $0 < s_2 < 1$ varied per round. That is, there were additive changes in the first treatment and there were multiplicative changes in the second treatment.

At the end of each treatment, 10 of the 312 rounds were randomly chosen and implemented, and the resulting final outcome (over the two treatments; our term) was paid to the subject. One modification: sequences of choices and resolutions of uncertainty ending in a final outcome outside the domain [0, 560] (4000 Danish Kroner ≈ \$560) were replaced by alternative choices and resolutions. Hence, the growth processes are not fully additive or multiplicative, and the changes of wealth in different rounds are not fully stochastically independent, which complicates the analysis.

Meder et al.[17] applied expected utility and prospect theory in a way that we call static: they applied EU and PT to each choice in each round separately, as if it was the only choice made and as if intermediate outcomes were actually received. This static analysis is incorrect. The intermediate outcomes are not outcomes received and consumed by subjects. Instead, they are only intermediate values playing a complex role in determining the final outcome, which is the only outcome received by the subject. A dynamic analysis should have been used, explained next. This means applying EU to terminal wealth, which in this experiment concerns final outcomes. PT is to be applied to perceived changes in wealth, which in this experiment also refers to final outcomes. It was made very clear to the subjects that the final outcome was the only one they would receive, and that the intermediate outcomes were only components to determine the real, final, outcome.

For a normative analysis of EU with full information about the experimental procedure, a sophisticated probability calculus should be carried out to determine for each sequence of 312 choices what probability distribution over final outcomes is degenerated by that combination of choices. As explained before, this is a probability distribution over the interval [0, 560]. Then, the combination of choices with the highest EU is chosen.

For a descriptive analysis using EU or PT that seeks to describe what subjects actually did in the experiment (the case of interest here), it is unrealistic to assume that subjects can determine the probability distributions over final outcomes that are generated by their choices. This would be too cognitively demanding under full prior information, and in reality is impossible because subjects do not know precisely what stimuli are to come in advance. Hence, subjects face a situation of unknown probabilities, often called choice under ambiguity in economics. They may (as-if) have considered sets of possible probability distributions over [0, 560], and used maxmin expected utility[24], or any other of the modern ambiguity theories, which we will not elaborate on here[25]. Treatment$^{\times}$ is much harder to assess than treatment^{+} (see the aforementioned exponential growth bias), therefore carrying more ambiguity. Hence, ambiguity aversion (risk aversion for unknown probabilities) will be considerably greater there. This can explain the greater aversion found in treatment$^{\times}$, and confounds the claims on utility curvature by Meder et al.[17] and Peters ([2]p. 1219). Further, increased complexity of probability calculus in itself increases aversion, again, irrespective of utility curvature.[26] Thus, the strong focus of economics on psychology and the full consideration of dynamics, as properly done in any economic analysis, leads to a qualitative explanation of the differences found. Peters was overly negative when writing: "the strong focus on psychology and lack of consideration for dynamics, prevalent in expected utility theory, corresponds to the belief that the difference between the red and blue curves is spurious" (p. 1220). Because both dynamics and uncertainty are central in Meder et al.'s stimuli, any model focusing on only one of these two will be deficient. Ergodic economics and expected utility should not compete but should collaborate here.

Note that equation (10) in Meder et al.[17] does not capture prospect theory because probability weighting is omitted. This equation (10) is in fact a special case of EU with a different, three-parameter, utility function. Even if all probabilities are 0.5, then still probability weighting is crucial and does not cancel, contrary to some claims to the opposite. There is more risk aversion as $w(0.50)$ is smaller, which should be corrected for before estimating utility. For instance, with, for simplicity, linear utility, the lottery $(0.5{:}\,90, 0.5{:}\,10)$ is evaluated by $w(0.5) \times 90 + \big(1 - w(0.5)\big) \times 10$, and the lottery $(0.5{:}\,60, 0.5{:}\,40)$ is evaluated by $w(0.5) \times 60 + \big(1 - w(0.5)\big) \times 40$ (equation 2[20]; p. 301[27]; Appendix 9.8[10]). There is a strict preference for $(0.5{:}\,90, 0.5{:}\,10)$ over $(0.5{:}\,60, 0.5{:}\,40)$ if $w(0.5) > 0.5$, but the preference reverses if $w(0.5) < 0.5$. This shows that probability weighting w cannot be ignored even if all probabilities are 0.5. Further, the total money accumulated up to that point, Y, cannot be ignored in the utility calculations. Note that loss aversion plays no role in this experiment because final outcomes can never be losses.

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