

# Ambiguity Attitudes in a Large Representative Sample

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Using a theorem showing that matching probabilities of ambiguous events can capture ambiguity attitudes, we introduce a tractable method for measuring ambiguity attitudes and apply it in a large representative sample. In addition to ambiguity aversion, we confirm an ambiguity component recently found in laboratory studies: a-insensitivity, the tendency to treat subjective likelihoods as 50-50, thus overweighting extreme events. Our ambiguity measurements are associated with real economic decisions; specifically, a-insensitivity is negatively related to stock market participation. Ambiguity aversion is also negatively related to stock market participation, but only for subjects who perceive stock returns as highly ambiguous.

*Keywords:* ambiguity aversion; uncertainty; portfolio choice; Knightian uncertainty; nonexpected utility; reference dependence; stock market participation; nonparticipation

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## 1. Introduction

This paper introduces a tractable method for measuring ambiguity attitudes, requiring only three indifference curves and an average of five minutes per subject. The method is based on a theorem that shows that it is surprisingly simple to capture ambiguity attitudes through matching probabilities. The matching probability of an ambiguous event is the objective probability at which, for a given prize, a subject is indifferent between betting on the ambiguous event versus betting on the objective probability. Our method makes it possible to measure the ambiguity attitudes of the general population.

Despite the economic relevance of ambiguity (unknown probabilities), few empirical papers have studied the ambiguity attitudes of the general population, and there is little direct evidence on the relation between ambiguity attitudes and actual economic decisions. We obtain quantitative measurements of ambiguity attitudes for a large representative sample, paying €7,650 in real incentives to the subjects. We show that these measurements, despite the simplicity of their experimental elicitation, are reliable enough to correlate with the subjects' actual (nonexperimental) economic decisions.

Besides the well-known aversion to ambiguity, we find another relevant component of ambiguity attitudes: a-insensitivity (ambiguity-generated likelihood insensitivity). A-insensitivity implies that people do not sufficiently discriminate between different levels of ambiguity, transforming subjective likelihoods toward 50-50. This leads to ambiguity seeking for low likelihoods and reinforces ambiguity aversion for high likelihoods. As a result, people overweight extreme events, both favorable and unfavorable.

A-insensitivity reinforces risk seeking for long shots. With unknown probabilities, even more than with known probabilities, people (over) value unlikely big gains (Ellsberg 2001, p. 203). A-insensitivity implies insufficient sensitivity to regular signals, but oversensitivity to signals affecting the tails of distributions. Similarly, it undervalues preventive measures that reduce uncertainty without eliminating it, while overvaluing the complete elimination of uncertainty. Several studies have demonstrated the psychological plausibility of a-insensitivity (Fox and Tversky 1998, Hogarth and Einhorn 1990, Maafi 2011, Wu and Gonzalez 1999), and many recent experimental studies have confirmed a-insensitivity among students in laboratory experiments (surveyed by Trautmann and van de Kuilen 2015). Our study is the first to investigate and demonstrate this component of ambiguity

attitudes among the general population, rather than in a sample of students only.

Using the results from our elicitation procedure, we test whether ambiguity attitudes can help to explain the nonparticipation puzzle: many households do not participate in the stock market, which cannot be explained by standard portfolio choice models (§6.1). We find that a-insensitivity contributes to the explanation: in our sample, subjects with higher a-insensitivity are less likely to own stocks. The effect of ambiguity aversion is weaker: it has a negative relation with stock market participation only for those subjects who feel incompetent about investments.<sup>1</sup> We discuss how these results are consistent with recent experimental evidence on reference dependence for decisions under ambiguity. All results are robust to controlling for education, financial assets, income, age, family structure, risk aversion, trust, and financial literacy.

There has been an extensive debate in the literature about the validity of hypothetical choice. The findings are mixed. On the one hand, many studies found no significant differences between hypothetical choice and real incentives, suggesting that hypothetical choice can serve as a valid substitute. On the other hand, many other studies did find differences, invalidating hypothetical choice. There is no universal rule, and the validity of hypothetical choice depends on the context. Camerer and Hogarth (1999) and Hertwig and Ortmann (2001) provide surveys of this literature. Usually, hypothetical choice works well for sophisticated subjects with simple stimuli that take no effort from the subjects. Its only drawback then is noisier data, without systematic differences. Because implementing real incentives for large representative samples is complex,<sup>2</sup> we investigate the validity of hypothetical choice for our sample regarding ambiguity measurement. Noussair et al. (2014) and von Gaudecker et al. (2011) investigated this question for risky choices and for samples similar to ours and found no differences. Unfortunately, for our more complex ambiguous stimuli, we do find differences, and hypothetical choice does not perform well. Real incentives seem to be desirable for ambiguity and nonacademic subjects.

This paper proceeds as follows. Section 2 presents the innovative discovery of Chew and Sagi (2008) that subjective probabilities can accommodate the Ellsberg paradox. The section then introduces matching probabilities and explains informally how they capture ambiguity attitudes. Section 3 provides a formal

analysis. Theorem 3.1 proves that matching probabilities directly capture ambiguity attitudes without any need to measure utility or probability weighting (risk attitudes). Section 3 also shows how matching probabilities can measure the two indexes of source preference and source sensitivity of Abdellaoui et al. (2011). Section 4 presents our measurement method, which is our main contribution. Section 5 summarizes the ambiguity attitudes of the general public, confirming a-insensitivity. Section 6 shows that our indexes of ambiguity attitudes are significantly related to the subjects' actual stock market participation decisions. Section 7 summarizes the simple recipe that other researchers can follow to use our method, and concludes.

## 2. Subjective Probabilities for Ambiguity and Matching Probabilities

Following traditions in the field, we use the classical Ellsberg (Ellsberg 1961, 2001) urn paradox to measure ambiguity attitudes. Figure 1 shows an example of the choices presented to our subjects in the experiment.  $Y_K$  denotes the event of a yellow<sup>3</sup> ball drawn from the known urn  $K$ ,  $P_U$  denotes the event of a purple ball drawn from the unknown urn  $U$ , and  $P_K$  and  $Y_U$  are defined similarly. *Prospects* are event-contingent payments, also called acts or gambles in the literature. For example,  $15_{Y_K}0$  denotes the prospect yielding €15 if event  $Y_K$  occurs, and €0 otherwise. The prevailing preferences in Figure 1 are

$$15_{Y_K}0 > 15_{Y_U}0 \tag{1}$$

and

$$15_{P_K}0 > 15_{P_U}0. \tag{2}$$

That is, people prefer to gamble on the known urn rather than on the unknown urn (ambiguity aversion), regardless of the color. These preferences violate classical decision models using subjective probabilities, such as expected utility and some generalizations (Machina and Schmeidler 1992). The following contradiction follows for such models. Here  $P(\cdot)$  denotes probability; the symbols  $P_K$  and  $P_U$  refer to events of a purple ball drawn.

$$P(Y_K) > P(Y_U), \tag{3}$$

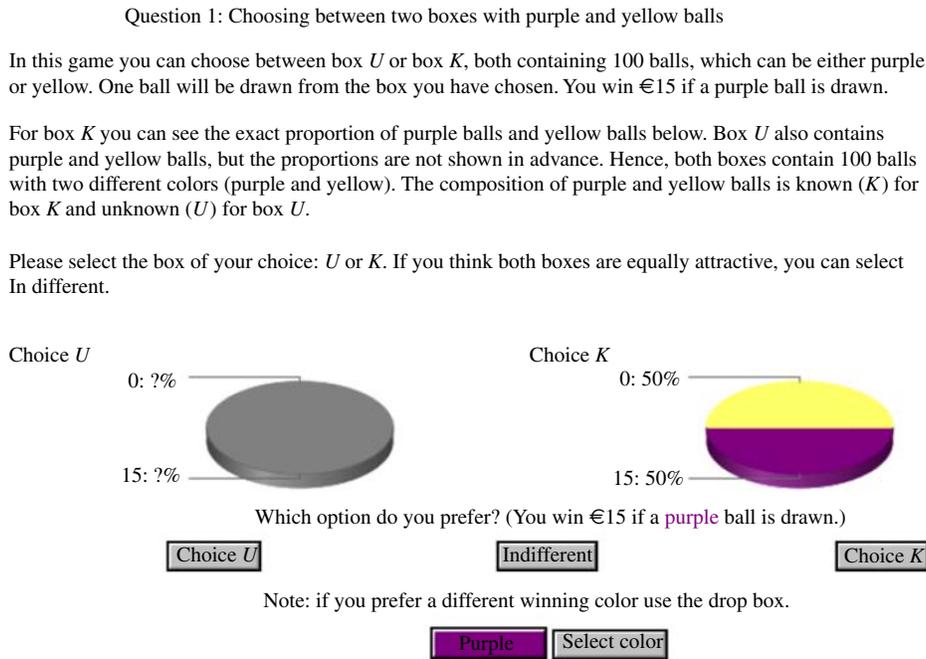
$$\frac{P(P_K)}{1} + > \frac{P(P_U)}{1} + : \text{Contradiction.} \tag{4}$$

<sup>1</sup> This result is consistent with the competence hypothesis of de Lara Resende and Wu (2010), Fox and Weber (2002), Heath and Tversky (1991), Kilka and Weber (2001), and Smith (1969, p. 325).

<sup>2</sup> Hence consumer surveys usually use hypothetical choice (Barsky et al. 1997, Dohmen et al. 2011).

<sup>3</sup> We did not use the classical Ellsberg colors red and black, because the color-blind often cannot distinguish red from other colors. Our survey used the Dutch term for “box” instead of “urn” because it sounds more natural in Dutch.

Figure 1 Screenshot of Choice Presented to Subjects



The common conclusion has been that subjective probabilities cannot accommodate Ellsberg’s paradox. However, Chew and Sagi (2006, 2008) provided a subtle new insight. They showed that subjective probabilities can still be used by relaxing an implicit assumption in the above reasoning, as follows.<sup>4</sup> First, we treat the urns  $K$  and  $U$  as two different sources of uncertainty. A *source of uncertainty* is a group of events generated by the same random mechanism (Tversky and Fox 1995). Thus uncertainty about the Dow Jones index concerns a different source of uncertainty than uncertainty about the Nikkei index. Second, we allow for different decision attitudes toward probabilities of different sources. We can then assign a probability of 0.5 not only to  $Y_K$  and  $P_K$  but also (subjectively) to  $Y_U$  and  $P_U$ . Because of symmetry (and an exchangeability condition of Chew and Sagi), these probabilities would be the subjective probabilities used by an ambiguity-neutral decision maker, which is why we call them *ambiguity neutral*, or *a-neutral* for short.

Using the second point above, a decision maker can weigh the probabilities of source  $U$  differently than those of source  $K$ . Equations (3) and (4) implicitly assumed identical weightings. Ambiguity aversion can be captured by underweighting the a-neutral probabilities 0.5 of events  $Y_U$  and  $P_U$  relative to the objective probabilities 0.5 of events  $Y_K$  and  $P_K$ . Thus, this accommodates Ellsberg’s paradox: people prefer

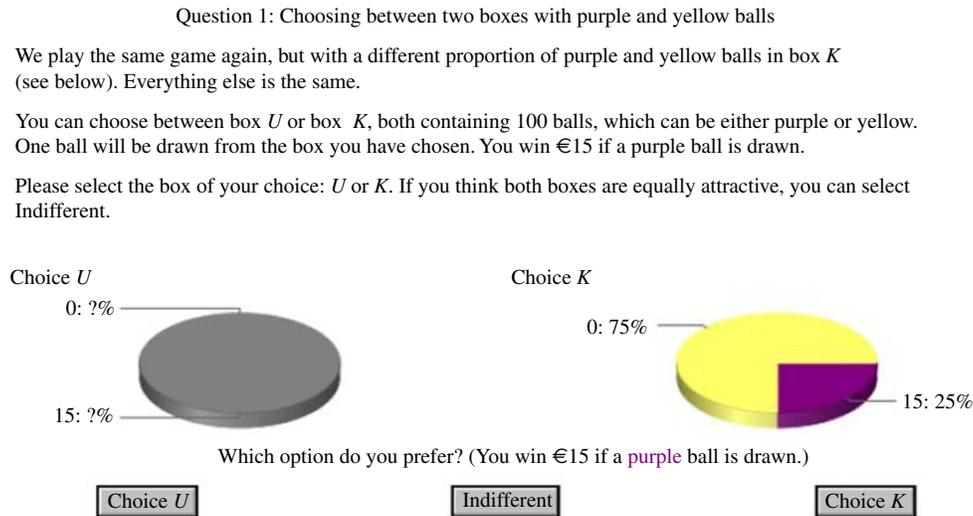
an objective probability of 0.5 because they underweight the (a-neutral) subjective probability of 0.5 because of ambiguity. Equations (3) and (4) hold with equality, but the preferences in Equations (1) and (2) still hold.

Chew and Sagi (2008) used general and betweenness decision models. We instead use the source method of Abdellaoui et al. (2011), which comprises most of the popular decision models of ambiguity today, including prospect theory (Tversky and Kahneman 1992), Choquet expected utility (Schmeidler 1989), and multiple priors (Gilboa and Schmeidler 1989). Section 3 gives formal definitions. The model is tractable and works well empirically. Kothiyal et al. (2013) showed that the source method better predicts choice under ambiguity than other currently popular models of ambiguity.

Based on the source method, we measure ambiguity attitudes using matching probabilities, which are measured with a bisection procedure. If, in Figure 1, the subject selected “Choice  $K$ ,” then Choice  $K$  was made less attractive and the subject was presented with Figure 2. If the subject selected Choice  $K$  again, then Choice  $K$  was once again made less attractive. If, instead, the subject selected the unknown urn, Choice  $U$ , then Choice  $K$  was made more attractive. This process continued until the subject selected “Indifferent.”<sup>5</sup>

<sup>4</sup> Multistage accommodations of Ellsberg’s paradox also use probabilities, but in unconventional manners, for instance, by violating the multiplication rule of conditioning (Halevy 2007, Klibanoff et al. 2005, Nau 2006, Yates and Zukowski 1976).

<sup>5</sup> Alternatively, the subject could reach the maximum number of six iterations without choosing indifference, in which case we took the average of the remaining upper and lower bound.

Figure 2 Screenshot After Choice  $K$  in Figure 1 (Now  $K$  is Worse)

We thus kept urn  $U$  fixed, but changed the number of purple balls in  $K$  until we found the number  $X$  of purple balls in urn  $K$  that made the subject indifferent. That is, we found the number of purple balls in urn  $K$  such that  $15_{P_K}0 \sim 15_{P_U}0$ . Details are in Appendix A. Then we call  $X/100$  the *matching probability* of the a-neutral probability 0.5 of  $P_U$  and write

$$m(0.5) = X/100. \quad (5)$$

Under ambiguity aversion,  $m(0.5) < 0.5$ . The value  $m(0.5)$  depends on the source of ambiguity, the source being  $U$  in this case.

We can similarly measure  $m(p)$  for ambiguous Ellsberg urn probabilities other than  $p = 0.5$ . In our experiment, we also consider a known urn containing 10 colors with 10 balls of each color versus an unknown urn containing 100 balls of 10 colors in unknown proportions. We use these urns to measure the matching probabilities  $m(0.1)$  and  $m(0.9)$ .

To measure  $m(0.1)$  we consider gambles in which the subject wins a prize if the randomly selected ball is of one particular color. For example,  $m(0.1) = 0.14$  means that the subject is indifferent between gambling on one color from the known urn with 14 of the 100 balls in the known urn of that color versus gambling on one color from the unknown 10-color urn. This matching probability would imply ambiguity seeking, with the a-neutral probability 0.1 preferred to the objective probability 0.1. To measure  $m(0.9)$  we consider gambles in which the subject wins a prize provided that the randomly selected ball is *not* of one particular color. For example,  $m(0.9) = 0.8$  means that the subject is indifferent between gambling on 80 of

the 100 balls in the known urn versus gambling on 9 colors from the unknown 10-color urn.<sup>6</sup>

Although most current theoretical papers generally assume universal ambiguity aversion, as early as 1962 Ellsberg predicted prevailing ambiguity seeking for unlikely events, such as 1 color drawn from the unknown 10-color urn (Ellsberg 2001, p. 203). Many empirical studies have confirmed Ellsberg's prediction (reviewed by Trautmann and van de Kuilen 2015), although only in laboratory studies, and the current paper is the first to test it outside the laboratory. The follow-up study Dimmock et al. (2015) confirmed it for a large representative sample from the United States. Binmore et al. (2012) and Charness et al. (2013) recently cast further doubt on the universality of ambiguity aversion.

Symmetry of the colors in the unknown 10-color urn implies that the exchangeability condition of Chew and Sagi (2008) holds (i.e., the a-neutral probability of each color in the 10-color urn is  $1/10$ ). We further assume that the unknown 2-color urn and the unknown 10-color urn can be treated as one source with the same  $m$  function. That is, gambling on 5 colors from the unknown 10-color urn is equivalent to gambling on 1 color from the unknown 2-color urn. Given that a similar mechanism underlies the two unknown urns, this assumption is reasonable. This assumption is also necessary for tractability. The dependence of preferences on sources of uncertainty, with domestic stocks treated differently than foreign stocks, for instance, is an empirical fact that every ambiguity theory must accommodate. However, these theories, including the source method, can only be

<sup>6</sup> More precisely, then 20 balls in the known urn have the one losing color, and the other 80 balls have the nine winning colors. In our experiment we avoid the words "lose" or "losing."

useful if dependence on sources is not too general. This can be compared with theories of the utility of commodities, which can only be useful if dependence on commodities is not too general.

### 3. Indexes of Ambiguity and Their Foundation

This section presents the indexes of ambiguity that we use and gives a theoretical foundation.

#### 3.1. Indexes of Ambiguity Attitudes Derived from Matching Probabilities

Jaffray (1989, Equation (10)) and Kahn and Sarin (1988) used the following indexes of ambiguity attitudes (AA indexes), which we term *event-specific indexes*. Each  $AA_p$  shows the level of “local” ambiguity aversion for an Ellsberg urn event with a-neutral probability  $p$ .

$$AA_{0.1} = 0.1 - m(0.1), \quad (6)$$

$$AA_{0.5} = 0.5 - m(0.5), \quad (7)$$

$$AA_{0.9} = 0.9 - m(0.9). \quad (8)$$

Ambiguity aversion implies that these indexes have positive values, with matching probabilities below a-neutral probabilities. A-insensitivity corresponds with a positive value of  $AA_{0.9}$  and a negative value of  $AA_{0.1}$ .

We now show how the global ambiguity attitude indexes of Abdellaoui et al. (2011) can be measured using our matching probabilities (Figure 3). First, on the open interval  $(0, 1)$  we find the best-fitting line between  $m(p)$  and  $p$  (in terms of quadratic distance).<sup>7</sup> Say this line is

$$p \mapsto c + sp \quad (9)$$

with  $c$  the intercept and  $s$  the slope. We define

$$a_{s_0} = 1 - s \text{ is the index of } a(\text{mbiguity-generated likelihood}) - \text{insensitivity} \quad (10)$$

and

$$b_{s_0} = 1 - s - 2c \text{ is the index of } \text{ambiguity aversion}. \quad (11)$$

Let  $d = 1 - c - s$  be the distance of the regression line from 1 at  $p = 1$ . Then index  $b_{s_0}$  can be written as  $b_{s_0} = d - c$ . Parameter  $b_{s_0}$  is related to elevation of the weighting function, and parameter  $a_{s_0}$  is related to curvature. Gonzalez and Wu (1999) provide a clear discussion of elevation and curvature, however, in the context of decision under risk and risk attitudes. Our

indexes concern ambiguity attitudes, capturing how ambiguity deviates from risk (see Theorem 3.1).

Figure 3 illustrates these global ambiguity attitude indexes. Index  $b_{s_0}$  is inversely related to the average height of the curve and, thus, it is a global index of ambiguity aversion. Index  $a_{s_0}$  is an index of the flatness of the curve in the interior of its domain. It reflects a lack of discrimination of intermediate levels of likelihood, or a tendency to transform all a-neutral probabilities to 50%.

Figure 3 displays possible shapes of matching functions  $m(p)$ , illustrating the joint effects of ambiguity aversion and insensitivity. The  $x$ -axis displays the a-neutral probabilities  $p$  and the  $y$ -axis displays  $m(p)$ . The indexes, symbols, text, and lines in the figures will be explained later and can be ignored for now. At present, we consider only the bold curves, designating matching probabilities. Figure 3(a) displays ambiguity neutrality, with matching probabilities equal to the a-neutral probabilities. Figure 3(b) displays universal ambiguity aversion, with all a-neutral probabilities (for gains) matched by smaller objective probabilities. Figure 3(c) displays a-insensitivity, with all matching probabilities moved toward 50-50. Figure 3(d) displays the prevailing empirical pattern (Trautmann and van de Kuilen 2015; Wakker 2010, §10.4.2), combining ambiguity aversion and a-insensitivity. Note how the lines in Figures 3(c) and (d) have the same slope and hence the same a-insensitivity, but the line in Figure 3(d) is lower, which enhances ambiguity aversion.

#### 3.2. Decision-Theoretic Foundation for Matching Probabilities

This subsection is not essential for readers focused on empirical implications and willing to accept our ambiguity indexes at face value. The subsection is essential, however, for a decision-theoretic justification of our method, which is provided here. A prospect  $\alpha_E\beta$  yields outcome  $\alpha$  if event  $E$  occurs and outcome  $\beta$  otherwise. Outcomes designate money and are nonnegative.  $E$  is an uncertain event, such as event  $Y_U$ , and the decision maker is uncertain whether the outcome of prospect  $\alpha_E\beta$  will be  $\alpha$  or  $\beta$ . Using the tractable specification provided by the source method of Abdellaoui et al. (2011), for  $\alpha \geq \beta$ , the prospect is evaluated by

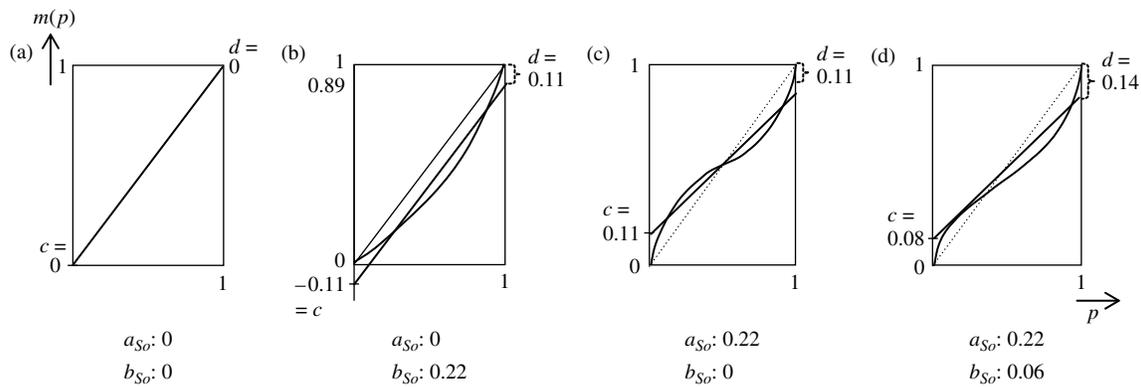
$$w_{s_0}(P(E))U(\alpha) + (1 - w_{s_0}(P(E)))U(\beta). \quad (12)$$

Here  $U$  denotes utility<sup>8</sup> ( $U(0) = 0$ ), and  $P$  denotes a subjective probability measure, justified by the conditions of Chew and Sagi (2006). As mentioned earlier, because an ambiguity-neutral decision maker would treat these subjective probabilities as objective probabilities, we call them *ambiguity-neutral*, or *a-neutral* for

<sup>7</sup> The fitted line should not be interpreted as a statistical estimation, but merely as a mathematical recoding of data per subject, with no statistical claim made.

<sup>8</sup> No confusion with the symbol  $U$  for unknown urn (used in our stimuli) will arise.

Figure 3 Quantitative Indexes of Ambiguity Aversion ( $b_{S_0}$ ) and A-insensitivity ( $a_{S_0}$ )



short. The source function  $w_{S_0}: [0, 1] \rightarrow [0, 1]$  weights the a-neutral probabilities and is strictly increasing between its fixed points 0 and 1. Low values of  $w_{S_0}$  imply low weights for the best outcome, designating pessimism. The subscript  $S_0$  indicates that  $w$  depends on the source of uncertainty. For example,  $w$  can be different for the known versus the unknown urn.

We follow the convention of dropping the subscript  $S_0$  if the source concerns known objective probabilities. We use the term risk for this case. Hence  $w$  denotes the probability weighting function for risk. For risk, we also write  $\alpha_p\beta$  instead of  $\alpha_E\beta$ , with  $p$  the objective probability of event  $E$ . Using this notation, the matching probability  $P(E) = p$  is formally defined by  $\alpha_p\beta \sim \alpha_E\beta$  for some  $\alpha > \beta$ , which then holds for all  $\alpha > \beta$ .

It is convenient to define a function  $m_{S_0}(p) = w^{-1}w_{S_0}$ , so that we can write

$$w_{S_0}(p) = w(m_{S_0}(p)). \tag{13}$$

The function  $m_{S_0}$  captures the difference in weighting between unknown and known probabilities. That is,  $m_{S_0}$  captures the ambiguity attitude. Hence it is called the ambiguity function.

It may appear difficult to measure the function  $m_{S_0}$ . Seemingly we must measure utility  $U$ , probability weighting for risk  $w$ , and weighting for uncertainty  $w_{S_0}$ , as done by Abdellaoui et al. (2011). Fortunately, the following result provides a convenient shortcut, which is the basis of our measurement method. We present the proof in the main text because it is simple and clarifying.

**THEOREM 3.1.** Under Equation (12), the matching probability is the ambiguity function.

**PROOF.** Assume, for  $\alpha > \beta$ , that  $\alpha_E\beta \sim \alpha_q\beta$ , implying that  $q$  is the matching probability of event  $E$  and of a-neutral probability  $P(E)$ . Then

$$\begin{aligned} w(m_{S_0}(P(E)))(U(\alpha) - U(\beta)) &= w(q)(U(\alpha) - U(\beta)), \\ w(m_{S_0}(P(E))) &= w(q), \\ m_{S_0}(P(E)) &= q. \quad \square \end{aligned}$$

That is, the ambiguous prospect  $\alpha_E\beta$  is equivalent to the risky prospect  $\alpha_{m_{S_0}(E)}\beta$ , yielding  $\alpha$  with objective probability  $m_{S_0}(E)$  and yielding  $\beta$  otherwise. The novelty of Theorem 3.1 is that we can immediately measure the ambiguity function from the matching probabilities described in §2, with no need to measure utility or probability weighting. Wakker (2010, Example 11.2.2) suggested that matching probabilities can be convenient for analyzing ambiguity attitudes.

Theorem 3.1 gives a preference foundation for the use of matching probabilities to measure ambiguity attitudes. They were used heuristically in several studies in the literature. The closest result in the literature is in Baillon et al. (2012), who obtained a similar decomposition for the special case of interval uncertainty. They heuristically used interval midpoint probabilities instead of the behaviorally derived a-neutral probabilities of the source method. Wakker (2004) showed that the function  $w^{-1}W$  satisfies bounded subadditivity if and only if preferences do. Tversky and Wakker (1995) provided theoretical preference conditions for comparing different sources (and persons) regarding insensitivity and aversion. Note that, unlike the a-neutral probability measure  $P_{S_0}(E)$ , the matching probability function  $m_{S_0}(E)$  is generally non-additive:  $m_{S_0}(E)$  captures the additional deviation from expected utility due to ambiguity, whereas  $w$  captures the deviation from expected utility due to risk.

## 4. Experimental Design

### 4.1. Sample and Incentives

The data source for this study is a cross section taken from the Longitudinal Internet Studies for the Social Sciences (LISS), a representative household survey conducted by CentERdata at Tilburg University in the Netherlands. To limit sample selection bias, LISS provides recruited households with free computers and Internet access if necessary. To encourage participation and retention, subjects are paid for each survey they complete. Knoef and de Vos (2009) showed that the LISS panel is generally representative of the Dutch

population. (See <http://www.lissdata.nl/lissdata/> for more information.) The experiment is computer-based and subjects can participate from their homes. Web Appendix WA.1 (available as supplemental material at <http://dx.doi.org/10.1287/mnsc.2015.2198>) gives further details. Half of the sample had one of their choices randomly selected and played for a possible real reward of €15, whereas the other half played for hypothetical rewards only. In total, we paid €7,650 in real incentives. The long-term relationship between the subjects and LISS makes the incentives credible and also serves to reduce suspicion.

Hypothetical choice is usually of lower quality and noisier (Camerer and Hogarth 1999, Hertwig and Ortmann 2001). In our sample, there was a bias toward ambiguity seeking in the hypothetical choice group (average  $b = 0.12$  for real rewards, versus  $b = 0.07$  for hypothetical;  $p = 0.01$ ; see the web appendix for more details). This bias was driven almost entirely by subjects with low education under hypothetical choice, a group of subjects not present in most academic laboratory experiments. The results also suggest that, for hypothetical choice, ambiguity aversion was confounded with trust, with less trusting subjects being more ambiguity averse. In contrast, with real rewards, the relation between trust and ambiguity aversion was insignificant, probably because these subjects were told clearly that LISS was responsible for calculating all prizes with funds provided by the research team, reducing suspicion. Finally, when explaining economic decisions, the ambiguity measurements in the hypothetical group indeed rarely reached significance. Thus, for measuring ambiguity attitudes for nonacademic subjects, hypothetical choice does not work well, unfortunately. Hence we only report the results of the real incentive group in the main text. The results of hypothetical choices are in Web Appendix E. We also limit our sample to the subjects with complete data on financial assets and other variables, resulting in a final sample of 666 subjects. There are no differences between the ambiguity attitudes of the subjects with complete and those with incomplete data.

#### 4.2. Stimuli

The stimuli were explained in §2, and Appendix A provides further details. The default winning color was purple, but to avoid suspicion (Pulford 2009; Zeckhauser 1986, p. S445) subjects could select a different winning color in the first round of each question. The long-term relationship between the subjects and LISS also serves to reduce suspicion. We first tested our method in a pilot experiment with students, with satisfactory results (Web Appendix B).

To test the consistency of the subjects' choices, our program generated two check questions following the three measurements of matching probabilities. The check questions were generated by taking each subject's elicited matching probability from the two-color question (a-neutral probability of winning of 0.5) and increasing (decreasing) this value by 20%. Inconsistency results if a subject preferred the ambiguous prospect in the first check question, and/or the unambiguous prospect in the second check question.

#### 4.3. Demographic Variables

The LISS panel contains information about demographic characteristics, income, and asset ownership, summarized in Table 1. In our ambiguity survey we also measured stock market participation, risk aversion (using a method based on Tanaka et al. 2010), trust (using the questions of Guiso et al. 2008), and financial literacy (van Rooij et al. 2011). Web Appendix A provides details regarding these questions.

**Table 1** Summary Statistics

Variable	All	Nonparticipants	Stock market participants
<i>Stock Market Participant</i>	20.4%	0.0	100.0
<i>Private Business Owner</i>	5.9%	4.3	11.8
<i>Total Financial Assets</i>	50,153	33,485	115,112
<i>Income</i>	3,816	3,567	4,787
<i>Age</i>	50.3	49.4	53.4
<i>Female</i>	52.7%	57.2	35.3
<i>Household Size</i>	2.5	2.5	2.4
<i>Live with Partner</i>	72.4%	71.3	76.5
<i>Education</i>			
<i>Low</i>	9.0%	10.0	5.1
<i>Intermediate/Low</i>	28.7%	31.5	17.6
<i>Intermediate/High</i>	10.2%	10.2	10.3
<i>Vocational 1</i>	19.4%	19.6	18.4
<i>Vocational 2</i>	23.4%	21.9	29.4
<i>University</i>	9.3%	6.8	19.1
<i>Risk Aversion</i>	0.14	0.13	0.18
<i>Trust</i>	6.01	5.85	6.63
<i>Financial Literacy</i>	0.14	-0.01	0.72
<i>Don't Know Returns</i>	24.5%	28.3	9.6

*Notes.* *Income* and *Total Financial Assets* are reported at the household level. All other variables are reported at the individual level. The first two variables are dummy variables: *Stock Market Participant* indicates ownership of publicly traded stocks or equity mutual funds; *Private Business Owner* indicates ownership in a private firm. *Total Financial Assets* is the sum of bank accounts, investments, insurance, loans made to others, and other financial assets. *Income* is gross family income in euros per month. *Risk Aversion* is the constant relative risk aversion (CRRA) coefficient derived from certainty equivalents. *Trust* is the response to a question asking whether others can be trusted (0–10 scale); high values indicate greater trust. *Financial Literacy* is a factor extracted from questions measuring financial knowledge, following van Rooij et al. (2011); high values indicate greater knowledge. *Don't Know Returns* is a dummy variable for individuals who answer "don't know" to a question about historical asset returns. See Web Appendix A for detailed definitions.

## 5. Results on Ambiguity Attitudes and Demographic Variables

### 5.1. Matching Probabilities, Ambiguity Aversion, and A-insensitivity

For each question, only approximately 1% of the subjects chose to switch to a different winning color. This confirms that suspicion played little role. It also shows that subjects had no preferences for particular colors; i.e., they considered the colors to be exchangeable in the sense of Chew and Sagi (2006), supporting our assumption that the colors have equal a-neutral probabilities.

Table 2 shows the proportion of subjects whose first-round choices revealed ambiguity-averse, ambiguity-seeking, and ambiguity-neutral attitudes at a-neutral probabilities,  $p$ , of 0.10, 0.50, and 0.90. For example, in the first round of Question 1 (Figure 1), 68.3% of the subjects prefer a known 50% probability of winning (urn  $K$ ) instead of urn  $U$  with unknown proportions. Hence,  $m(0.5) < 0.5$  for the majority of respondents. A  $\chi^2$  test confirms that, for this question, ambiguity aversion is more common than ambiguity seeking ( $p$ -value  $< 0.01$ ). Also for the high-likelihood event ( $p = 0.9$ ; Figure A.2), the majority (53.2%) is ambiguity averse, and this exceeds the proportion that is ambiguity seeking ( $p$ -value  $< 0.01$ ). For the low-likelihood event ( $p = 0.1$ ; Figure A.1), the modal subject is ambiguity seeking (49.4%), which exceeds the 33.5% who are ambiguity averse ( $p$ -value  $< 0.01$ ). These results are consistent with a-insensitivity and show that universal ambiguity aversion does not hold.

Table 3 summarizes the more precise matching probability estimates obtained after six rounds of bisection (§2). The average matching probabilities  $m(0.5) = 0.40$  and  $m(0.9) = 0.69$  confirm ambiguity aversion for moderate and high likelihoods. In both cases we reject ambiguity seeking or neutrality ( $m(p) \geq p$ ) in favor of ambiguity aversion ( $m(p) < p$ ), with  $p$ -value  $< 0.01$  in a two-sided  $t$ -test. The average matching probability  $m(0.1) = 0.22$  implies ambiguity seeking for the low-likelihood event ( $p$ -value  $< 0.01$ ).

**Table 2** Ambiguity Attitudes Revealed by First Round Choices

A-neutral probability $p$	0.10	0.50	0.90
Ambiguity averse	33.5%	68.3	53.2
Ambiguity seeking	49.4%	22.1	35.3
Ambiguity neutral	17.1%	9.6	11.6

*Notes.* This table shows the frequency distribution of subjects with ambiguity averse, seeking, and neutral attitudes at a-neutral probabilities  $p$  of 0.10, 0.50, and 0.90. For example, we offer subjects the choice between a known urn ( $K$ ) with 50 yellow balls and 50 purple balls, and an unknown urn ( $U$ ) with yellow and purple balls in unknown proportions. A preference for urn  $K$  ( $U$ ) reveals ambiguity aversion (ambiguity seeking) at  $p = 0.50$ . Indifference implies ambiguity neutrality.

**Table 3** Statistics of the Ambiguity Attitude Indexes

Variable	Mean	Median	SD	Min	Max
Matching Probability $m(0.1)$	0.22	0.10	0.25	0.01	0.99
Matching Probability $m(0.5)$	0.40	0.39	0.24	0.02	0.98
Matching Probability $m(0.9)$	0.69	0.89	0.33	0.01	0.99
$AA_{0.1}$	-0.12	0.00	0.25	-0.89	0.09
$AA_{0.5}$	0.10	0.11	0.24	-0.48	0.48
$AA_{0.9}$	0.21	0.01	0.33	-0.09	0.89
Index $b$ (ambiguity aversion)	0.12	0.09	0.41	-0.97	0.97
Index $a$ (a-insensitivity)	0.41	0.29	0.44	-0.22	2.21

*Notes.* Rows 1–3 show the matching probabilities for the three ambiguity questions ( $m(0.1)$ ,  $m(0.5)$ , and  $m(0.9)$ ). Rows 4–6 show the three indexes of ambiguity attitudes based on the differences between the objective and matching probabilities:  $AA_{0.1}$ , Equation (6);  $AA_{0.5}$ , Equation (7); and  $AA_{0.9}$ , Equation (8). The last two rows show the overall indexes of ambiguity attitudes: Index  $b$ , Equation (11) (ambiguity aversion); and Index  $a$ , Equation (10) (a-insensitivity).

These results again confirm a-insensitivity. The average indexes  $b = 0.12$  and  $a = 0.41$  are positive ( $p$ -value  $< 0.01$ ) and show ambiguity aversion and a-insensitivity, respectively.

All our findings are consistent with the prevailing empirical findings in the literature (see §1). In general, ambiguity attitudes display characteristics similar to risk attitudes, but to a stronger extent (Maafi 2011; Wakker 2010, §10.4). For example, a-insensitivity is the analog of the inverse-S-shaped probability weighting that is commonly found for risk (Wakker 2010, §7.1) and that also entails a global drift of decision weights toward 50-50 (Wu and Gonzalez 1998).

To statistically confirm that ambiguity attitudes are best explained by two components, specifically, ambiguity aversion and a-insensitivity, Table 4 shows a principal component analysis of the event-specific ambiguity indexes  $AA_{0.1}$ ,  $AA_{0.5}$ , and  $AA_{0.9}$ . The three ambiguity indexes have approximately equal loadings on the first component: this component captures ambiguity aversion/seeking, the general tendency to underweight/overweight all uncertain events. This component explains 58% of the variance in the three indexes. Turning to the second component,  $AA_{0.1}$  loads negatively on this component (-0.62) and  $AA_{0.9}$  loads positively (0.78), whereas the loading of  $AA_{0.5}$  is close to 0. Thus, the second component corresponds with high values of  $m(0.1)$  ( $\approx c$  in Equation (9)) and low values of  $m(0.9)$  ( $\approx 1 - c - s$  in Equation (9)), that is, with low values of  $s$ , and it captures a-insensitivity (Equation (10)). It explains 25% of the variance in the three indexes. Together the first two components account for 83% of the variance in the decisions of the subjects.

The principal component analysis justifies the use of our indexes  $a$  and  $b$ , because they are very close to the first two principal components: the correlations between index  $b$  and the first principal component,

**Table 4** Principal Component Analysis of the Ambiguity Attitude Indexes

Variable	1st comp.	2nd comp.	3rd comp.
$AA_{0.1}$	0.57	-0.62	0.54
$AA_{0.5}$	0.63	-0.10	-0.77
$AA_{0.9}$	0.53	0.78	0.33
Eigenvalue of the component	1.74	0.74	0.52
Proportion of variance explained	0.58	0.25	0.17

Notes. This table shows a principal component analysis of the event-specific ambiguity indexes:  $AA_{0.1}$ , Equation (6);  $AA_{0.5}$ , Equation (7); and  $AA_{0.9}$ , Equation (8). Rows 1–3 show the loadings of the indexes on the three components. Row 4 displays the eigenvalue of the component and row 5 shows the percentage of variance explained.

and between index  $a$  and the second principal component, are both 0.99. The components and indexes capture two independent aspects of ambiguity attitudes, while explaining nearly all cross-sectional variance of decisions under ambiguity.

### 5.2. Inconsistencies

For the first check question (matching probability  $m(0.5)$  increased by 20%), 19.7% of the subjects chose the unknown urn, implying inconsistency. For the second check question (matching probability  $m(0.5)$  decreased by 20%), 34.1% chose the known urn, implying inconsistency. Approximately 11% of the subjects choose “Indifferent” rather than directly contradicting their earlier choice.<sup>9</sup> Overall, however, the responses to the check questions are related to their preceding counterparts ( $p$ -value < 0.001;  $\chi^2$  test).

Laboratory studies with students also commonly find similar rates of inconsistencies (Harless and Camerer 1994, p. 1263). Duersch et al. (2013) explicitly tested the consistency and stability of ambiguity preferences, using a different elicitation method based on the Ellsberg one-urn problem with three colors of balls. When the student subjects in their experiment were offered the same pair-wise choice twice within 10 minutes, 29% of the choices were inconsistent, similar to the rate we find in our survey.

In the remainder of the paper we deal with inconsistencies by checking whether the full sample results are robust when we exclude subjects who made errors on the check questions. These robustness tests are in Appendix B and Web Appendix C. We find that our main results either remain the same or become stronger after excluding inconsistent subjects.

### 5.3. Demographic Variables

Because stock market participation is our key dependent variable in §6, Table 1 separately summarizes the

<sup>9</sup> Unfortunately, there was a coding error in the implementation of the survey. For the first and second check questions, 28.2% and 36.2% of the subjects, respectively, were presented with a choice that was too similar to their initial choice. The inconsistencies are significantly higher for the subjects that received these erroneous check questions.

**Table 5** Correlations Between Ambiguity Attitude Indexes

Variable	(1) $b$	(2) $a$	(3) $AA_{0.1}$	(4) $AA_{0.5}$	(5) $AA_{0.9}$
(1) <i>Index b</i> (ambiguity aversion)	1				
(2) <i>Index a</i> (a-insensitivity)	0.22	1			
(3) $AA_{0.1}$	0.72	-0.46	1		
(4) $AA_{0.5}$	0.77	<i>0.04</i>	0.45	1	
(5) $AA_{0.9}$	0.79	0.73	0.27	0.39	1

Notes. The variables are defined in Table 3. Correlations that are *not* significant at the 0.10 level are italicized.

characteristics of nonparticipants and the 20.4% that do participate in the stock market.<sup>10</sup>

### 5.4. Relations Between Ambiguity Attitudes and Demographic Variables

Table 5 shows that the two ambiguity attitude indexes are positively correlated ( $\rho = 0.22$ ;  $p$ -value < 0.001). This is consistent with both indexes being related to irrationality (i.e., deviations from expected utility). The correlation is small, however, confirming that they capture different components of ambiguity attitudes.

To explore how the ambiguity attitude indexes relate to economic and demographic characteristics, Table 6 regresses the indexes on the full set of control variables for the stock market participation regressions in §6. Here and throughout, \*, \*\*, and \*\*\* indicate significance at the 0.10, 0.05, and 0.01 levels, respectively. These levels are common in field studies and in finance, because data, even if finding large effect sizes, are noisier than, for instance, in most controlled laboratory experiments in psychology. To be consistent with subsequent regressions, we standardize all nonbinary variables. The indexes have little relation with age, education, financial assets, and income. Risk aversion is negatively related to the indexes  $a$  and  $b$ . Higher financial literacy is associated with lower a-insensitivity, but the size of the effect is low (partial correlation = -0.097). The adjusted  $R^2$  values show that the control variables jointly explain only between 0.2% and 5.4% of the variance in ambiguity attitudes. Hence, the ambiguity variables are clearly not a proxy for low education or financial illiteracy, but contain independent empirical information. These results are consistent with the work of Sutter et al. (2013), who also found only weak relations between ambiguity attitudes and demographic variables.

<sup>10</sup> Our stock market participation variable is equal to 1 if the subject directly owns stocks or equity mutual funds, and 0 otherwise. We do not have data on indirect stock holdings in employer-provided pension funds. The Dutch pension system is overwhelmingly based on defined benefit plans and thus the stock participation that we measure represents active decisions by the subject rather than passive effects due to variation in employers’ pension offerings.

**Table 6** Regressions for Demographic Predictors of Ambiguity Attitudes

	Index $b$	Index $a$	$AA_{0.1}$	$AA_{0.5}$	$AA_{0.9}$
<i>Risk Aversion</i>	−0.159*** [3.30]	−0.127*** [3.09]	−0.059 [1.38]	−0.100** [2.12]	−0.184*** [4.01]
<i>Trust</i>	−0.031 [0.73]	0.010 [0.23]	−0.024 [0.56]	−0.047 [1.06]	−0.007 [0.18]
<i>Financial Literacy</i>	0.031 [0.65]	−0.112** [2.23]	0.106** [2.06]	0.025 [0.49]	−0.040 [0.86]
<i>Don't Know Returns</i>	0.128 [1.28]	−0.182** [1.98]	0.267*** [2.84]	0.041 [0.41]	0.008 [0.08]
<i>Total Financial Assets</i>	0.067 [0.70]	−0.138 [1.60]	0.129 [1.37]	0.109 [1.10]	−0.05 [0.54]
<i>Total Financial Assets</i> <sup>2</sup>	−0.085 [1.03]	0.038 [0.51]	−0.073 [0.94]	−0.125 [1.48]	−0.014 [0.18]
<i>Income</i>	−0.039 [0.27]	−0.053 [0.36]	0.015 [0.11]	−0.055 [0.39]	−0.046 [0.30]
<i>Income</i> <sup>2</sup>	0.077 [0.57]	0.070 [0.54]	0.008 [0.08]	0.079 [0.64]	0.083 [0.57]
<i>Age</i>	−0.034 [0.18]	0.325* [1.73]	−0.284 [1.63]	0.025 [0.12]	0.135 [0.73]
<i>Age</i> <sup>2</sup>	0.001 [0.00]	−0.262 [1.34]	0.203 [1.08]	−0.037 [0.17]	−0.129 [0.68]
<i>Female</i>	0.032 [0.42]	−0.034 [0.42]	0.007 [0.10]	0.119 [1.49]	−0.031 [0.38]
<i>Household Size</i>	0.038 [0.99]	−0.015 [0.44]	0.032 [0.89]	0.054 [1.32]	0.009 [0.23]
<i>Live with Partner</i>	−0.074 [0.61]	0.092 [0.77]	−0.087 [0.74]	−0.146 [1.21]	0.034 [0.27]
<i>Education</i> (joint $p$ -value)	0.026**	0.149	0.001***	0.356	0.068*
Adjusted $R^2$	0.022	0.054	0.032	0.002	0.036
No. of observations	666	666	666	666	666

*Notes.* The dependent variables are defined in Table 3. The independent variables are defined in Table 1. The education controls are five dummy variables for highest level of education achieved (base category is primary school). The regressions include constants but these are not displayed for brevity's sake. The  $t$ -statistics are calculated using standard errors clustered by household.

\*, \*\*, and \*\*\* indicate significance at the 10%, 5%, and 1% levels, respectively.

## 6. Ambiguity Attitudes and Household Portfolio Choice

To investigate the validity of our measures of ambiguity attitudes and their importance for other sources of uncertainty, we now test whether our measures are associated with financial decisions.

### 6.1. Background and Predictions on the Nonparticipation Puzzle

The nonparticipation puzzle refers to the empirical fact that many households do not participate in the stock market, whereas portfolio choice models with standard preferences predict that they should, given the historical returns on equities (Heaton and Lucas 1997). Further, the empirical evidence shows that economic frictions, such as participation costs, cannot explain a large fraction of nonparticipation (Andersen and Nielsen 2011).

Several theoretical papers have shown that ambiguity aversion can plausibly explain part of the participation puzzle (Bossaerts et al. 2010, Cao et al.

2005, Easley and O'Hara 2009). Nevertheless, there have been few nonexperimental, empirical studies of ambiguity aversion and stock market participation. Although it was not their main focus, Guiso et al. (2008) control for ambiguity aversion, measured using hypothetical choices over compound lotteries. We elicited ambiguity attitudes directly using real incentives and using a behaviorally founded model that has been justified psychologically and descriptively.

The present theoretical literature predicts a negative relation between ambiguity aversion (our index  $b_{So}$ ) and stock market participation (Bossaerts et al. 2010, Dow and Werlang 1992). The distribution of future stock market returns is unknown, with the historical data providing only rough guidance.<sup>11</sup> Further, an unambiguous alternative is available: (insured) bank deposits and government bonds provide known

<sup>11</sup> For empirical studies demonstrating that expected equity returns are ambiguous, and that this ambiguity is priced, see Anderson et al. (2009), Liu et al. (2005), and Pan (2002).

returns. Hence, in theoretical models of stock participation, higher ambiguity aversion reduces stock market participation.

Several studies have found that perceived competence is an important psychological factor underlying ambiguity perception (de Lara Resende and Wu 2010; Fox and Weber 2002; Heath and Tversky 1991; Kilka and Weber 2001; Smith 1969, p. 325). We therefore expect that ambiguity aversion will have a stronger effect on stock market participation for subjects who perceive themselves as less competent in financial matters.

A-insensitivity (index  $a_{s_0}$ ) implies an extremity orientedness, where the best and worst outcomes are overweighted, which is amplified by their ambiguity. The distribution of aggregate stock returns is negatively skewed (Albuquerque 2012, Duan and Zhang 2014). Further, option prices suggest that investors expect extreme negative returns, or disasters, to occur more frequently than is historically observed (Chang et al. 2013, Liu et al. 2005, Pan 2002). Hence, the effect of overweighting will be stronger for the worst than for the best outcomes (further amplified by loss aversion), and we therefore expect that a-insensitivity will be negatively associated with stock market participation. It will similarly be negatively associated with private business ownership. The model and simulations reported in Web Appendix D support this prediction.

## 6.2. Results on Ambiguity Attitudes and Stock Market Participation

Table 7 shows the results of logit regressions with stock market participation as the dependent variable. In all specifications, we control for financial assets, income, age, and the squared values of these three variables, and education, gender, household size, and family composition. In the interest of brevity, the paper does not report the coefficients for most control variables (see Web Appendix Table A.1). To facilitate the interpretation of results, we standardize all nonbinary variables and report marginal effects. That is, the reported marginal effects show the (absolute) change in the probability of stock market participation given a change of one standard deviation in the independent variable (or a change from 0 to 1 for dummy variables).<sup>12</sup> Column (1) of Table 7 shows:

- (i) the coefficient for ambiguity aversion is not significant in the whole population; and
- (ii) the coefficient for a-insensitivity has a significant negative relation with stock market participation.

<sup>12</sup> Because the 666 subjects belong to 600 distinct households, in all regressions we cluster the standard errors by household to avoid overstating significance due to within-household correlations.

**Table 7** Ambiguity Attitudes and Stock Market Participation

	(1)	(2)	(3)	(4)
<i>Index b</i> (ambiguity aversion)	0.009 [0.55]	0.011 [0.65]		
<i>Index a</i> (a-insensitivity)	-0.033** [2.16]	-0.028* [1.82]		
$AA_{0.1}$			0.039** [2.10]	0.036* [1.79]
$AA_{0.5}$			-0.015 [0.80]	-0.012 [0.59]
$AA_{0.9}$			-0.018 [1.12]	-0.014 [0.86]
<i>Risk Aversion</i>		-0.002 [0.13]		-0.002 [0.12]
<i>Trust</i>		0.026* [1.83]		0.026* [1.78]
<i>Financial Literacy</i>		0.077*** [3.62]		0.077*** [3.62]
Controls and constant	Yes	Yes	Yes	Yes
Pseudo- $R^2$	0.194	0.222	0.196	0.223
No. of observations	666	666	666	666

*Notes.* This table shows logit regressions with stock market participation as the dependent variable. The ambiguity attitude variables are defined in Table 3: *Index b*, Equation (11) (ambiguity aversion); *Index a*, Equation (10) (a-insensitivity);  $AA_{0.1}$ , Equation (6);  $AA_{0.5}$ , Equation (7); and  $AA_{0.9}$ , Equation (8). The other independent variables are defined in Table 1. The regressions include a constant and a full set of control variables, but the coefficients are not displayed for brevity's sake (see Web Appendix A). The *t*-statistics are calculated using standard errors clustered by household.

\*, \*\*, and \*\*\* indicate significance at the 10%, 5%, and 1% levels, respectively.

We discuss these findings in §6.3. Finding (ii) is confirmed by the results in column (3), which considers the three event-specific ambiguity attitude variables: nonparticipants are more likely to overweight low a-neutral probabilities. This result is consistent with Liu et al. (2005) and Pan (2002), who argue that rare, low-likelihood disasters affect investment decisions.

Column (2) of Table 7 includes additional control variables. In this specification, the coefficient of a-insensitivity is significant at the level  $p = 0.07$ . Despite the lower statistical significance, the implied economic magnitude of the effect of a-insensitivity is large. The results in column (2) imply that a change of one standard deviation in index *a* is associated with a 2.8 percentage point change in the probability of stock market participation. This is a change of 13.7% relative to the mean participation rate, and it is equivalent to the economic effect of a €23,000 change in financial assets. As in column (2), the results in column (4) are significant at the level  $p = 0.07$ , but the implied economic magnitude of the result is large. The results in column (4) imply that a change of one standard deviation in  $AA_{0.1}$  is associated with a 3.6 percentage point change in the probability of stock market participation.

Additional tests, in Appendix B, rule out alternative interpretations of our findings, such as alternatives based on risk aversion, trust, optimism, education, quantitative skill, and financial literacy. In unreported

results, we ran the regressions without control variables. In all cases, the significances of the ambiguity variables were maintained and were usually stronger. Table B.1 in Appendix B also shows results for the subsample of respondents who did not make errors on the check questions: the marginal effects are larger for this subsample.

To investigate competence effects we asked subjects which asset class provides the best long-term average return: stocks, bonds, savings accounts, or “don’t know.” Subjects who chose “don’t know” feel less sure and less competent than the others. Note that subjects who answer incorrectly (bonds or savings accounts) are probably more incompetent but may not perceive themselves as incompetent. Following prior studies of the competence effect (Heath and Tversky 1991, de Lara Resende and Wu 2010), we focus on perceived competence (self-assessed), rather than actual competence (correct answers). Hence we use “don’t know” answers as a proxy for perceived incompetence. Details are in Web Appendix A (see question 2).

Table 8 includes interaction terms between the ambiguity attitude indexes and the dummy for “don’t know” returns. Ambiguity aversion does have a negative effect on stock market participation for subjects who feel incompetent.<sup>13</sup> Subjects whose ambiguity aversion is one standard deviation above the mean and who do not know which asset class tends to give the highest return over longer periods of time are 10.3 percentage points less likely to participate in the stock market. Ambiguity aversion has no relation with stock market participation for subjects who chose stocks, bonds, or deposits as the asset that normally has the highest long-term return.

The interaction between a-insensitivity (index *a*) and perceived incompetence is not significant. A-insensitivity affects the decision weights of extreme outcomes, whereas the question about asset returns is framed in terms of the general tendency of returns. Thus this question is more related to the center of the distribution (mean or median) and does not speak to the extremes.<sup>14</sup>

<sup>13</sup> Note that we control for the direct effect of “don’t know” and financial literacy in the regressions and, hence, the interaction term does not simply measure a lack of financial knowledge; instead, it measures the combined effect of ambiguity aversion and perceived incompetence.

<sup>14</sup> In results reported in Web Appendix F, we interact the ambiguity attitude indexes and an indicator variable for subjects who gave an incorrect answer to the asset return question. An incorrect answer identifies subjects who lack financial knowledge but are nevertheless confident of their knowledge. These interaction terms are not significant, which supports our interpretation that it is the interaction of ambiguity attitudes with perceived incompetence, rather than with actual competence, which is relevant.

**Table 8** Ambiguity Attitudes, Perceived Incompetence, and Stock Market Participation

	(1)	(2)	(3)	(4)
<i>Index b</i> (ambiguity aversion)	0.025 [1.35]	0.025 [1.29]		
<i>Index b</i> × <i>Don't Know</i>	-0.102*** [3.27]	-0.103*** [3.21]		
<i>Index a</i> (a-insensitivity)	-0.036** [2.20]	-0.033** [1.96]		
<i>Index a</i> × <i>Don't Know</i>	0.005 [0.13]	0.019 [0.41]		
<i>AA</i> <sub>0.1</sub>			0.056*** [2.79]	0.052** [2.49]
<i>AA</i> <sub>0.1</sub> × <i>Don't Know</i>			-0.094** [2.04]	-0.105** [2.00]
<i>AA</i> <sub>0.5</sub>			-0.020 [1.00]	-0.018 [0.83]
<i>AA</i> <sub>0.5</sub> × <i>Don't Know</i>			0.043 [0.84]	0.044 [0.81]
<i>AA</i> <sub>0.9</sub>			-0.008 [0.50]	-0.006 [0.35]
<i>AA</i> <sub>0.9</sub> × <i>Don't Know</i>			-0.086* [1.77]	-0.070 [1.42]
<i>Don't Know Returns</i>	-0.108** [2.37]	-0.070 [1.51]	-0.115** [2.36]	-0.076 [1.56]
<i>Risk Aversion</i>		0.001 [0.03]		0.001 [0.07]
<i>Trust</i>		0.027* [1.89]		0.026* [1.82]
<i>Financial Literacy</i>		0.068*** [3.12]		0.067*** [3.09]
Controls and constant	Yes	Yes	Yes	Yes
Pseudo- <i>R</i> <sup>2</sup>	0.210	0.232	0.217	0.238
No. of observations	666	666	666	666

*Notes.* This table shows logit regressions with stock market participation as the dependent variable. The regressions include interaction terms of the ambiguity attitude variables with “don’t know returns” (see question 2 in Web Appendix A.2). All other variables are the same as in Table 7. A full set of control variables and a constant are included but are not shown to save space. The *t*-statistics are calculated using standard errors clustered by household.

\*, \*\*, and \*\*\* indicate significance at the 10%, 5%, and 1% levels, respectively.

### 6.3. Discussion

Ambiguity aversion had some effect on stock market participation, but the effect is not very strong and is limited to a subgroup. Despite the weak relation, our variables are valid measures of ambiguity aversion for several reasons. First, our estimates of ambiguity aversion are similar to those found in laboratory studies. Second, we find significant results for a-insensitivity, which is measured using the same set of questions. Also, the standard errors on the coefficients for ambiguity aversion and a-insensitivity in Table 7 are nearly identical. Thus our results are not simply due to a lack of power. This is further supported by the significant interactions between ambiguity aversion and perceived incompetence, which have the coefficient signs predicted by the competence hypothesis.

Although it has not been studied in prior theoretical studies, a-insensitivity is significantly related to stock market participation. Noussair et al. (2014) found that prudence, which is empirically similar to insensitivity for risk, is related to financial decisions for a representative sample of Dutch households, supporting our finding. In a follow-up study in a representative U.S. sample, Dimmock et al. (2016) found a stronger negative effect of ambiguity aversion, reaching significance in the full sample of 3,258 respondents. Baillon et al. (2013) and Dimmock et al. (2015) showed that our index of a-insensitivity amounts to a perceived level of ambiguity in the alpha maxmin model.

So far we have implicitly assumed that ambiguity attitudes measured with Ellsberg urns can be used as a proxy for ambiguity attitudes toward stock returns. For the source method to be tractable, source dependence should not be too general and there should be some consistency across different sources. Our results support this tractability. As an additional test of whether ambiguity attitudes elicited for one source of uncertainty (Ellsberg urns) are associated with economic choices concerning a different source of uncertainty, we also test the relation between ambiguity attitudes and private business ownership. The returns to private business ownership are highly ambiguous (Moskowitz and Vissing-Jorgensen 2002, p. 745). Table 9 shows that there is a negative relation between a-insensitivity (index *a*) and private business ownership. The implied magnitude is highly economically significant. Although the majority of private equity owners do not own publicly traded equities, we find similar results for both dependent variables.

The weak effect of ambiguity aversion that we found may be explained by reference dependence, distinguishing between gains and losses. Reference dependence is a central concept in prospect theory for risk, but it has not yet been incorporated in the currently popular ambiguity theories. Stock market participation typically involves both gains and losses and, hence, reference dependence is relevant, as emphasized by De Giorgi and Post (2011) and Trautmann and van de Kuilen (2015, §3). Reference dependence and its implied reflections of loss attitudes relative to gain attitudes reverse the motivational<sup>15</sup> effects of ambiguity aversion, in the same way as it is known to reverse risk aversion (Baucells and Villasís 2010; de Lara Resende and Wu 2010; Dimmock et al. 2015; Du and Budescu 2005; Heath and Tversky 1991; Henderson 2012; Markle et al. 2014; Trautmann and van de Kuilen 2015; Wakker 2010, §12.7). Reference dependence thus weakens the effects of ambiguity aversion. However, reflection

<sup>15</sup> We use the terms “motivational” (goal-oriented) and “cognitive” following the psychology literature.

**Table 9** Ambiguity Attitudes and Private Business Ownership

	Private business	
	(1)	(2)
<i>Index b</i> (ambiguity aversion)	−0.006 [0.52]	
<i>Index a</i> (a-insensitivity)	−0.020** [1.97]	
<i>AA</i> <sub>0.1</sub>		0.012 [1.04]
<i>AA</i> <sub>0.5</sub>		−0.003 [0.27]
<i>AA</i> <sub>0.9</sub>		−0.021* [1.90]
<i>Risk Aversion</i>	−0.005 [0.62]	−0.005 [0.61]
<i>Trust</i>	0.001 [0.04]	0.001 [0.04]
<i>Financial Literacy</i>	0.030** [2.54]	0.030** [2.54]
Controls and constant	Yes	Yes
Pseudo- <i>R</i> <sup>2</sup>	0.176	0.176
No. of observations	666	666

*Notes.* This table shows logit regressions. The dependent variable indicates ownership (1) or not (0) of equity in a private business. The independent variables are the same as in Table 7. A full set of control variables and a constant are included, but are not shown to save space. The *t*-statistics are calculated using standard errors clustered by household.

\* and \*\* indicate significance at the 10% and 5%, levels, respectively.

does not affect the cognitive effects of a-insensitivity, just as reflection does not affect the inverse-S shape of probability weighting for risk (Abdellaoui et al. 2005; Baillon and Bleichrodt 2015; de Lara Resende and Wu 2010, p. 127; Hogarth and Einhorn 1990; Trautmann and van de Kuilen 2015, §3). Thus it does not weaken the effects of a-insensitivity. Web Appendix D presents simulations based on reference-dependent prospect theory for ambiguity that are consistent with all of our findings, supporting the relevance of reference dependence for ambiguity. For risk, He and Zhou (2011) provided analytical results.

Even in laboratory studies it is difficult to implement actual, rather than hypothetical, losses (de Lara Resende and Wu 2010, p. 119), and the difficulties are greater in a household survey. Our study has demonstrated the importance of real incentives for measuring the ambiguity attitudes of nonacademic subjects. A promising direction for future empirical research, to improve the predictive power of ambiguity theories, therefore concerns the development of simple, incentive-compatible methods to measure ambiguity attitudes for losses that can be applied in the general population. So far, only some studies have measured ambiguity attitudes toward losses for students in laboratories (reviewed by Trautmann and van de Kuilen 2015). Dimmock et al. (2015) measured

them for a representative sample from the United States.

## 7. Conclusion

This paper provides a simple method for measuring ambiguity attitudes and deriving ambiguity indexes, using only three matching probabilities that can be elicited from subjects in five minutes on average. Thus researchers interested in measuring the ambiguity attitudes of individuals need only follow our simple recipe: First, measure matching probabilities  $m(0.5)$  for a-neutral probabilities 0.5 (Figures 1 and 2), and their analogs  $m(0.1)$  (Figure A.1) and  $m(0.9)$  (Figure A.2). Second, calculate the indexes from Equations (10) and (11). We show, in Theorem 3.1, that this method is theoretically justified.

We apply our method to a large representative sample and find there are two independent factors underlying ambiguity attitudes. Besides ambiguity aversion, a-insensitivity (failing to distinguish between different levels of uncertainty) is prevalent. Thus earlier findings of a-insensitivity with students in laboratories are now confirmed in the general population.

Our measures can predict actual stock market participation. This paper provides the first direct empirical evidence that ambiguity attitudes affect stock market participation for the general population. Even after controlling for variables previously used in the literature, the new tools of decision under ambiguity provide explanatory power for investor behavior in financial markets, in particular the nonparticipation puzzle. A-insensitivity has a negative relation with participation. Ambiguity aversion reaches significance only for those subjects who felt incompetent about investments. Reference dependence may provide further insights, and developing proper measurement methods of ambiguity attitudes for losses is an important topic for future research.

### Supplemental Material

Supplemental material to this paper is available at <http://dx.doi.org/10.1287/mnsc.2015.2198>.

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### Appendix A. Matching Probabilities for Ambiguity Attitudes

Following a practice question (whose results are not used in our analyses), we asked subjects three sets of questions

that involved choices between an ambiguous and an unambiguous prospect (using the neutral term option), starting with Figure 1.

The second question was similar to the first, but now both urns contained 10 different colors of balls. Figure A.1 shows the initial choice in the first round of the second question, and Figure A.2 shows the initial choice in the first round of the third question.

In adaptive questions, where answers to some questions determine subsequent questions, subjects may answer strategically (Harrison 1986). In our experiment, this is unlikely. First, our subjects are less sophisticated than students. Second, it would primarily have happened in the end (only after discovery), at the 0.9 probability event, where it would increase ambiguity seeking. However, here we found strong ambiguity aversion.

### Appendix B. Alternative Explanations

Tables 7 and 8 show a relation between the elicited ambiguity attitudes and stock market participation. A potential concern, however, is that the elicited ambiguity attitudes inadvertently measure some other concept.

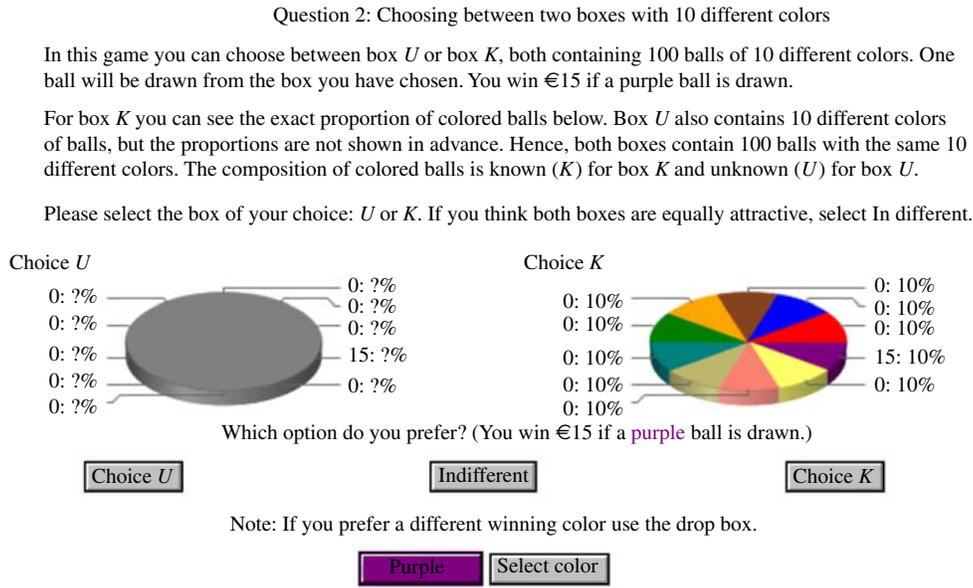
#### B.1. Risk Aversion

Although we measure ambiguity attitudes relative to risky attitudes (ambiguity is the difference between general uncertainty and risk) and the two concepts are conceptually distinct, they may still be statistically related (Abdellaoui et al. 2011, Figures 12 and 13; Bossaerts et al. 2010; our Table 6). Controlling for risk aversion, however, does not alter the significance of our ambiguity attitudes. Table 7 shows that individuals who overweight low a-neutral probabilities have lower participation rates, which is directionally inconsistent with the possibility that our ambiguity indexes inadvertently measure risk aversion. The interaction terms in Table 8 show that, consistent with our predictions, ambiguity aversion has a negative relation with stock market participation for individuals who do not know long-term returns of various asset classes. There is no clear reason to expect risk aversion to interact with the perceived financial competence (in unreported results we find no interaction between risk aversion and the *Don't Know* variable). Hence this result provides indirect evidence that the ambiguity attitude indexes do not inadvertently measure risk aversion.

It is not surprising that risk aversion had little effect on stock market participation in our study. Nonparticipation is a puzzle precisely because it cannot be explained by risk aversion. Most studies using household survey data find no relation between risk aversion and participation: see, for example, Guiso et al. (2008), van Rooij et al. (2011), and Noussair et al. (2014).<sup>16</sup> A potential reason is that risk aversion is usually measured using gains. However, stock returns involve losses where risk attitudes can be different and even opposite to those for gains, as predicted by prospect theory.

<sup>16</sup> We use a measure of pure decision-theoretic risk aversion. Some studies, e.g., Puri and Robinson (2007), used questions specifically about stock market risk, but this may create a mechanical relation (see Web Appendix A.5).

Figure A.1 Screenshot for a-Neutral Probability 0.1



**B.2. Trust**

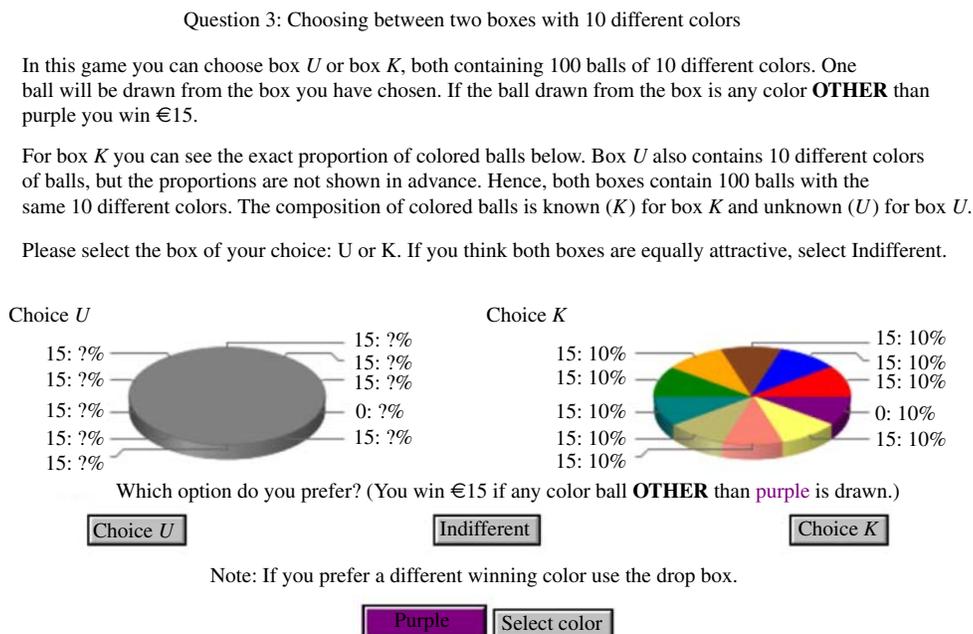
Guiso et al. (2008) show that trust is positively associated with stock market participation. They argue that trust and ambiguity attitudes are distinct, but there is some conceptual similarity. Although the results in Tables 7 and 8 are robust to controlling for trust, conceivably ambiguity aversion could arise if subjects assume that ambiguous situations are biased (distrust). However, this is unlikely to drive our results. First, subjects tend to overweight low likelihoods, whereas distrust would predict underweighting. Second, the correlations between trust and the ambiguity attitudes are low. Finally, the elicitation procedure

avoided suspicion (see §4), as did the long-term relationship with LISS.

**B.3. Optimism**

Puri and Robinson (2007) show that optimistic subjects have higher stock market participation and more favorable economic expectations. The LISS panel does not contain their questions, but it does measure economic expectations for 347 of our subjects: none of the economic expectation variables are significantly correlated with our ambiguity attitude indexes. Further, our indexes are not significantly correlated with questions on depression, positive attitudes, or life satisfaction.

Figure A.2 Screenshot for a-Neutral Probability 0.9



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**Table B.1** Ambiguity Attitudes and Stock Market Participation: Subsamples

	Tertiary education		Questions clear		Not inconsistent	
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Index b</i> (ambiguity aversion)	0.027 [1.05]	0.045 [1.55]	0.009 [0.44]	0.021 [0.94]	0.038 [1.11]	0.049 [1.28]
<i>Index b</i> × <i>Don't Know</i>		−0.166*** [2.79]		−0.085** [2.35]		−0.016 [0.17]
<i>Index a</i> (a-insensitivity)	−0.040 [1.59]	−0.053** [1.99]	−0.040** [2.24]	−0.046** [2.38]	−0.054* [1.73]	−0.059* [1.79]
<i>Index a</i> × <i>Don't Know</i>		0.152* [1.72]		0.033 [0.65]		0.003 [0.03]
<i>Don't Know Returns</i>		−0.026 [0.37]		−0.059 [1.20]		−0.139* [1.68]
<i>Risk Aversion</i>	−0.015 [0.72]	−0.015 [0.72]	−0.001 [0.07]	−0.001 [0.08]	0.015 [0.62]	0.017 [0.68]
<i>Trust</i>	0.029 [1.26]	0.029 [1.30]	0.023 [1.51]	0.025 [1.63]	−0.003 [0.15]	−0.002 [0.09]
<i>Financial Literacy</i>	0.122*** [3.52]	0.118*** [3.51]	0.058** [2.38]	0.049* [1.96]	0.068** [2.36]	0.046 [1.42]
Controls and constant	Yes	Yes	Yes	Yes	Yes	Yes
Pseudo- $R^2$	0.246	0.261	0.230	0.238	0.293	0.307
No. of observations	347	347	540	540	330	330

*Notes.* This table shows logit regressions with stock market participation as the dependent variable. Columns (2), (4), and (6) include interaction terms between the *Don't Know* dummy and the ambiguity indexes. The independent variables are the same as in Table 7. The subsamples are limited to tertiary education (only subjects who completed some form of tertiary education), Questions clear (only subjects who stated that the ambiguity attitude questions were clear or very clear), and Not inconsistent (only subjects who did not violate their earlier choices on the check questions). The regressions include a constant and a full set of controls, but the coefficients are not displayed for brevity's sake. The *t*-statistics are calculated using standard errors clustered by household.

\*, \*\*, and \*\*\* indicate significance at the 10%, 5%, and 1% levels, respectively.

#### B.4. Education, Quantitative Skill, and Financial Literacy

Table 6 shows that education has little explanatory power for the ambiguity indexes, suggesting that the indexes are not proxies for low quantitative skills. We found similar

results in our pilot experiment on a sample of upper year university students who had already completed courses in statistics, calculus, finance, and economics. Financial literacy, although related to stock market participation, does not eliminate the significance of ambiguity attitudes, further suggesting that quantitative skills do not explain ambiguity attitudes.

We further explore this possibility in Table B.1 by excluding subjects with low levels of education or who found the elicitation procedure confusing. In columns (1) and (2), the sample is restricted to subjects who have completed some form of tertiary education. In columns (3) and (4), the sample is restricted to subjects who, at the end of our survey module, stated that the ambiguity elicitation questions were either "clear" or "very clear." In columns (5) and (6), the sample is restricted to subjects whose answers to the check questions did not violate their earlier choices. The results are similar to the full sample.

As a further test of whether the ambiguity attitude indexes measure a lack of sophistication or unfamiliarity with financial decision making, Table B.2 estimates a logit model in which the dependent variable is 1 if the subject has a bank account. Presumably, subjects without a bank account are relatively unsophisticated or lack financial expertise. The coefficients on the ambiguity indexes are all insignificant.

**Table B.2** Ambiguity Attitudes and Bank Account Ownership

	Bank account	
	(1)	(2)
<i>Index b</i> (ambiguity aversion)	−0.009 [0.95]	
<i>Index a</i> (a-insensitivity)	−0.015 [1.45]	
$AA_{0.1}$		0.008 [0.66]
$AA_{0.5}$		−0.005 [0.42]
$AA_{0.9}$		−0.018 [1.58]
<i>Risk Aversion</i>	−0.009 [0.74]	−0.009 [0.74]
<i>Trust</i>	0.006 [0.58]	0.005 [0.57]
<i>Financial Literacy</i>	0.009 [0.77]	0.009 [0.77]
Controls and constant	Yes	Yes
Pseudo- $R^2$	0.117	0.118
No. of observations	666	666

*Notes.* This table shows logit regressions. The dependent variable indicates ownership of a bank account. The independent variables are the same as in Table 1. The regressions include controls and a constant, but these are not displayed for brevity's sake. The *t*-statistics are calculated using standard errors clustered by household.

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