

## Nash was a first to axiomatize expected utility

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**Abstract** Nash is famous for many inventions, but it is less known that he, simultaneously with Marschak, also was the first to axiomatize expected utility for risk. In particular, these authors were the first to state the independence condition, a condition that should have been but was not stated by von Neumann and Morgenstern. Marschak’s paper resulted from interactions with several people at the Cowles Commission. We document unique letters and personal communications with Nash, Samuelson, Arrow, Dalkey, and others, making plausible that Nash made his discovery independently from the others.

**Keywords** John Nash · Expected utility · Independence preference axiom · Foundations of decision theory · History of decision theory

Many contributions by John Nash in different fields have been well known and widely appreciated. What has not been acknowledged is that, in his celebrated article “The Bargaining Problem,” [Nash \(1950\)](#) was also the first to axiomatize expected utility under risk, simultaneously with and independently of [Marschak \(1950\)](#). Thus, they laid the foundations for decision theory for risk. In particular, they were the first to correctly define the independence preference axiom, which plays a central role in the empirical and normative debates about expected utility and its generalizations, central topics for this journal. [Fishburn and Wakker \(1995\)](#) and [Moscati \(2016\)](#) discuss the

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general history of independence and related preference conditions. This note focuses on the (shared) priority of Nash and the independence of his discovery.

Independence means that two indifferences  $P \sim P'$  and  $Q \sim Q'$  between lotteries imply the indifference  $\lambda P + (1 - \lambda)Q \sim \lambda P' + (1 - \lambda)Q'$  between probabilistic mixtures of lotteries. It is necessary for transferring the mixture operation from lotteries to indifference classes of lotteries. The latter was done by von Neumann and Morgenstern (1944), but they did not state the independence axiom, and therefore their axiomatization was incomplete. They needed the expected utility model to evaluate mixed strategies in their foundation of game theory. Only after the clarifications by Nash and Marschak could this aspect of the foundations of game theory be understood. For applications to game theory of the non-expected utility models introduced since the 1980s, see Chen and Neilson (1999), Haller (2000), and many others.

In a series of unpublished working papers and conference papers appeared between July 1948 and December 1949, Marschak, then a senior researcher at the Cowles Commission in Chicago, Herman Rubin, a junior researcher at Cowles and a collaborator of Marschak, and Norman Dalkey, a researcher at the RAND Corporation in Santa Monica, pointed out the omission of von Neumann and Morgenstern (Marschak 1948a, b, 1949a, b; Rubin 1949a, b; Dalkey 1949). Because Marschak (1950) is a publication with early working paper versions of 1948 and 1949, we feel that Marschak deserves priority over Rubin and Dalkey. Marschak's (1950) acknowledgment, as well as witnesses from those days such as Dalkey himself, Kenneth Arrow and Paul Samuelson (documented by Bleichrodt et al. 2015), indicate that researchers at the Cowles Commission and the RAND Corporation interacted much and shared their ideas. They did not assign much importance to settling priority of the discovery of independence. We know now, in retrospect, how important this condition is to distinguish theories.

Nash (1950) and Marschak (1950) appeared side by side in the April 1950 issue of *Econometrica*. In the introductory section of his article, Nash (1950, pp. 156–157) put forward a series of assumptions that warrant expected utility theory. In particular, Nash's assumption 5 is the independence preference axiom, and coincides with the postulate Marschak (1949b, 1950) had called Postulate IV<sub>2</sub>. Like Marschak, Rubin or Dalkey, Nash did not assign much importance to the independence assumption, nor claimed any originality or priority for it. He simply needed expected utility and just wrote it down, apparently unaware that it was new and that he was correcting von Neumann and Morgenstern (Nash 1998). In effect, we argue, unlike researchers at Cowles and RAND, Nash discovered the independence idea on his own, and notably without interactions with Marschak, Rubin or Dalkey. He used to work on his own, and could develop whatever he needed.

Various types of evidence support the independence of Nash's invention. The main ideas for the bargaining problem article occurred to Nash in spring 1948 when, as an undergraduate student in mathematics at the Carnegie Institute of Technology, he attended the only course in economics he ever attended, namely an elective course on international trade given by Bert Hoselitz (Nash 1994; Nasar 1998). Nash acquired the analytical tools to suitably model the bargaining problem only after beginning his Ph.D. in mathematics at Princeton in September 1948, and becoming acquainted with von Neumann and Morgenstern's *Theory of Games*. He wrote the bargaining problem paper in spring 1949, during his second semester at Princeton (Nasar 1998), and in

it he cited as only reference *Theory of Games*. He also thanked von Neumann and Morgenstern, and only them, for reading the original version of the paper and giving him some advice on how to present the ideas contained in it. When the paper was published in April 1950, Nash was still a Ph.D. student. At that time, he was in fact completing his dissertation on “Non-Cooperative Games”, which he would defend on May 29, 1950 (Nash 1948–2002).

The above-described genesis of the 1950 article suggests that the gifted Ph.D. student Nash wrote it much in isolation, interacting only with von Neumann and Morgenstern. With regard to the unpublished working papers of Marschak, Rubin and Dalkey, it is highly unlikely that Nash was aware of them. These unpublished works circulated at Cowles and RAND, but not in Princeton. The online catalogues of the libraries of Princeton University and the Institute for Advanced Studies, in fact, do not display any trace of them (search carried out on September 24, 2015). Nash did spend his first research period at RAND in 1950. But this was only in summer 1950, that is, some months after the publication of the bargaining problem article (Nash 1948–2002).

The independence of Nash’s discovery is further supported by the recollections of Samuelson, who in 1950 was one of the major players in the debate on expected utility and the independence axiom (Samuelson 1950a, b, c). According to Samuelson (1992)<sup>1</sup>: “I now have no memory of 1950 Nash as being in the ‘independence’ act. [...] Nash was a loner and I know of no ‘influences’ on him.”

We conclude that Nash shares priority with Marschak on axiomatizing expected utility and thus on opening the rich field of axiomatizations for decision under risk. This intellectual contribution of Nash deserves wide recognition.

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<sup>1</sup> Bleichrodt et al. (2015) documents this and other unique letters and personal communications.

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