Web Appendix

of

Han Bleichrodt, Rogier J.D. Potter van Loon, Kirsten I.M. Rohde, & Peter P. Wakker

(2013) "A Criticism of Doyle's Survey of Time Preference: A Correction on the CRDI and CADI Families," *Judgment and Decision Making* 8, 630–631

October, 2013

The main text showed how Doyle's Eq. 33 was based on the incorrect Eq. 4. We show the same here for Doyle's Eqs. 31 and 32 (Appendix A) and Eqs. 34, 35, and 36 (Appendix B). We reproduce the incorrect Eq. 4 from the main text:

$$D(0) = 1. \tag{4}$$

We again assume $U(x) = x$ throughout, following Doyle. As in our paper and in Bleichrodt, Rohde, and Wakker (2009; BRW henceforth), we write *ln* for the natural logarithm instead of Doyle's log. As in the main text, (T:F) denotes receiving $F > 0 at time $T > 0$.

Appendix A. Reproducing Doyle's results for CRDI$^+$ & CRDI^0

We show how Doyle's analysis in his §3.6.2 essentially uses the incorrect Eq. 4 for Eqs. 1 ($\psi > 0$) and 2 ($\psi = 0$). The case of Eq. 3 ($\psi < 0$) was analyzed in the appendix of the paper. We treat Eqs. 1 and 2 separately.

CASE 1: Eq. 1 ($\psi > 0$)

We first consider Eq. 1, concerning $\psi > 0$. Assume that P is the present value of (T:F), which indeed exists for $\psi > 0$:

$$(0{:}P) \sim (T{:}F). \tag{A.1}$$

It implies, assuming the incorrect Eq. 4:

$$P = F.\beta.\exp(-\rho T^{\Psi}). \qquad (A.2^{+})$$

Doyle's Eq. 31, i.e. Eq. A.3^{+} below, now follows:

LEMMA A.1^{+}. For Eq. 1 with Eq. 4, Eq. A.1 implies

$$\rho = \frac{ln(\beta F/P)}{T^{\Psi}} . \qquad (A.3^{+})$$

PROOF. Consider the following rewritings of Eq. A.2^{+}:

$$\frac{\beta F}{P} = \frac{1}{\exp(-\rho T^{\Psi})} = \exp(\rho T^{\Psi});$$

$$ln(\frac{\beta F}{P}) = \rho T^{\Psi};$$

$$\frac{ln(\beta F/P)}{T^{\Psi}} = \rho.$$ □

Eq.4, and the above results, are correct if $\beta = 1$. See the end of the appendix in the main text.

CASE 2. Eq. 2 ($\psi = 0$)

We next consider Eq. 2, concerning $\psi = 0$. Again assume Eq. A.1, implied by the incorrect Eq. 4.[1] Eq. 4 implies:

$$P = F.\beta.T^{-\rho}. \qquad (A.2^{0})$$

Doyle's Eq. 32, i.e. Eq. A.3^{0} below, now follows:

LEMMA A.1^{0}. For Eq. 2 with Eq. 4, Eq. A.1 implies

$$\rho = \frac{ln(\beta F/P)}{ln(T)} . \qquad (A.3^{0})$$

PROOF. Consider the following rewritings of Eq. A.2^{0}:

---

[1] In reality, a present value does not exist for $\psi = 0$.

$$\frac{\beta F}{P} = \frac{1}{T^{-\rho}} = T^{\rho};$$

$$ln(\frac{\beta F}{P}) = \rho ln(T);$$

$$\frac{ln(\beta F/P)}{ln(T)} = \rho. \qquad \square$$

CASE 3. Eq. 3 ($\psi < 0$)

See the appendix in the paper.

Appendix B. Reproducing Doyle's results for the CADI family

The CADI family is defined, with parameters $\beta > 0$, $\sigma > 0$, and η, by[2]

If $\eta > 0$, then $D(T) = \beta.\exp(-\sigma e^{\eta T})$ for $T \in \mathbb{R}$; (B.1)

If $\eta = 0$, then $D(T) = \beta.\exp(-\sigma T)$ for $T \in \mathbb{R}$; (B.2)

If $\eta < 0$, then $D(T) = \beta.\exp(\sigma e^{\eta T})$ for $T \in \mathbb{R}$. (B.3)

Unlike CRDI, CADI is defined for all $T \in \mathbb{R}$ regardless of its parameter values.

We show how Doyle's analysis in his §3.6.3 essentially uses the incorrect Eq. 4. We again assume the present value P of Eq. A.1. Unlike with the CRDI family, for the CADI family a present value P always exists, and Eq. A.1 can be satisfied for each of the Eqs. B.1, B.2, and B.3. We consider the three cases of η separately.

CASE 1. Eq. B.1 ($\eta > 0$)

Eq A.1 implies, assuming the incorrect Eq. 4:

---

[2] BRW, p. 31, use the following notation: $\varphi = D$, $t = T$, $k = \beta$, $r = \sigma$ and $c = -\eta$.

$$P = F.\beta.\exp(-\sigma e^{\eta T}). \qquad (B.4^{+})$$

Doyle's Eq. 34, i.e. Eq. $B.5^{+}$ below, now follows:

LEMMA $B.1^{+}$. For Eq. B.1 with Eq. 4, Eq. A.1 implies

$$\sigma = \frac{ln(\beta F/P)}{e^{\eta T}} . \qquad (B.5^{+})$$

PROOF. Consider the following rewritings of Eq. $B.4^{+}$:

$$\frac{\beta F}{P} = \frac{1}{\exp(-\sigma e^{\eta T})} = \exp(\sigma e^{\eta T});$$

$$ln(\frac{\beta F}{P}) = \sigma e^{\eta T};$$

$$\frac{ln(\beta F/P)}{e^{\eta T}} = \sigma.$$

□

CASE 2. Eq. B.2 ($\eta = 0$)

Eq A.1 implies, assuming the incorrect Eq. 4:

$$P = F.\beta.\exp(-\sigma T). \qquad (B.4^{0})$$

Doyle's Eq. 35, i.e. Eq. $B.5^{0}$ below, now follows:

LEMMA $B.1^{0}$. For Eq. B.2 with Eq. 4, Eq. A.1 implies

$$\sigma = \frac{ln(\beta F/P)}{T} . \qquad (B.5^{0})$$

PROOF. Consider the following rewritings of Eq. $B.4^{0}$:

$$\frac{\beta F}{P} = \frac{1}{\exp(-\sigma T)} = \exp(\sigma T);$$

$$ln(\frac{\beta F}{P}) = \sigma T;$$

$$\frac{ln(\beta F/P)}{T} = \sigma.$$

□

CASE E 3. Eq. B.3 ($\eta < 0$)

Eq A.1 implies, assuming the incorrect Eq. 4:

$$P = F.\beta.\exp(\sigma e^{\eta T}). \qquad (B.4^{-})$$

Doyle's Eq. 36, i.e. Eq. B.5⁻ below, now follows:

LEMMA B.1⁻. For Eq. B.3 with Eq. 4, Eq. A.1 implies

$$\sigma = \frac{ln(\beta F/P)}{-e^{\eta T}} . \qquad (B.5^{-})$$

PROOF. Consider the following rewritings of Eq. B.4⁻:

$$\frac{\beta F}{P} = \frac{1}{\exp(\sigma e^{\eta T})} = \exp(-\sigma e^{\eta T});$$

$$ln(\frac{\beta F}{P}) = -\sigma e^{\eta T};$$

$$\frac{ln(\beta F/P)}{-e^{\eta T}} = \sigma.$$

□