Electronic Supplementary Material of "Ambiguity and the Aggregation of Imprecise or Conflicting Beliefs"

Aurélien Baillon, Laure Cabantous, & Peter P. Wakker

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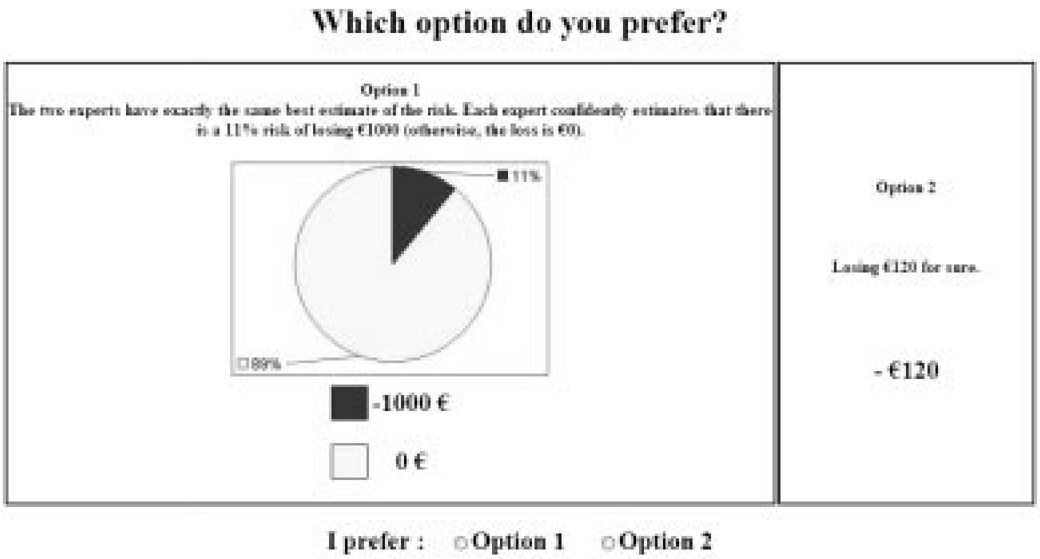
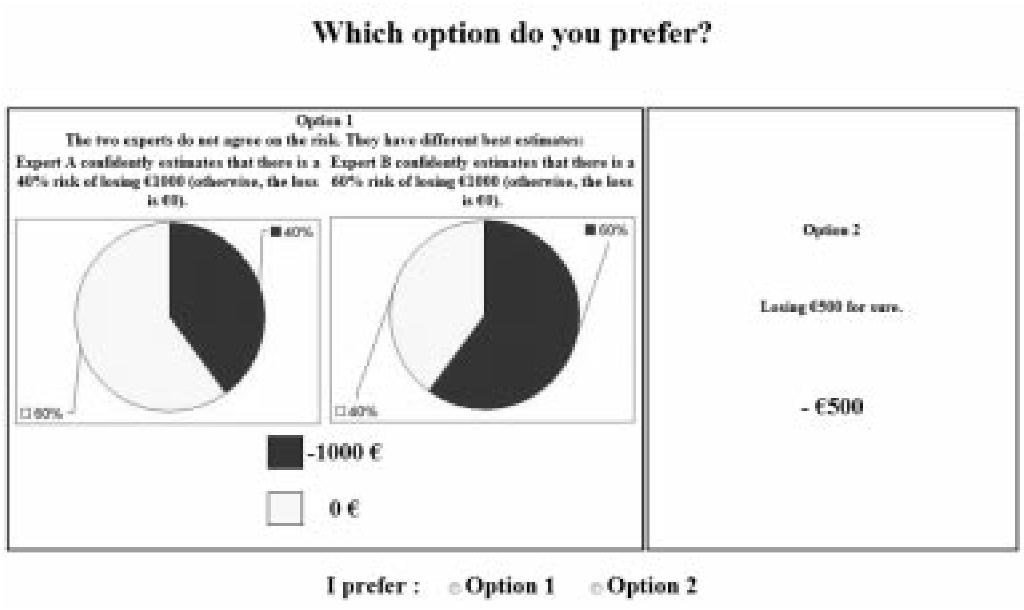
1. Experimental details for both experiments

Material used in the experiments to elicit certainty equivalents (CEs)



Screenshot A: A risky prospect vs. a sure amount



Screenshot B: A conflicting prospect vs. a sure amount

Figure E2: Screenshots of typical choice tasks (Experiments A and B)

To simplify the participants' task, the screenshots for risky, I and C decision contexts had exactly the same structure. Risky/I/C prospects (Option 1) were systematically displayed at the left-hand side and the sure loss (Option 2) was displayed at the right-hand side of the computer screen. Whatever the decision context, x (the high loss) was assigned to purple and y (the small loss) to yellow. There was absolutely no time pressure; the participants were given the time they needed and were encouraged to think carefully about the questions. The software allowed the participants to modify their answers if they wish, by going backward.

Certainty equivalent measurement

We developed computerized bisection software to estimate the CEs. Bisection does not require participants to state precise indifference values. It involves choices only, and generates more reliable data than direct matching (Bostic et al., 1990; Fischer et al. 1999; Noussair, Robbin, & Ruffieux 2004).

Each CE measurement started with a choice between the prospect considered and a sure loss equal to the expected value under the (midpoint) probability. A preference for the sure loss (the prospect) generated a higher (lower) sure loss in the next question. The new sure loss was the midpoint of the highest sure loss accepted up to that point (or the worst outcome of the prospect if no sure loss had been accepted) and the lowest sure loss rejected up to that point (or the best outcome of the prospect if no sure loss had been rejected).

Participants were asked to make choices until a sure loss resulted with a precision of ±1% of the difference between the two outcomes of the prospect, and this sure loss was taken as the CE. From 3 to 7 choices were usually required to estimate CEs. The precision was implemented as the stopping rule of the bisection process. For instance, if Option 1 involved outcomes 0 and 1000, the program stopped when the subjects had both rejected a sure loss but accepted another sure loss that was 20 lower. This bisection process thus stopped after 3 to 7 choices. Table E1 gives an example.

Question	Option 1	Option 2	Choice
1	$-1000_{\{0.4,0.6\}}0$	-500	Option 1
2	$-1000_{\{0.4,0.6\}}0$	-260	Option 2
3	$-1000_{\{0.4,0.6\}}0$	-380	Option 2
4	$-1000_{\{0.4,0.6\}}0$	-440	Option 1
5	$-1000_{\{0.4,0.6\}}0$	-420	Option 1
6	$-1000_{\{0.4,0.6\}}0$	-400	Option 2

Estimated Certainty Equivalent: ***−410***

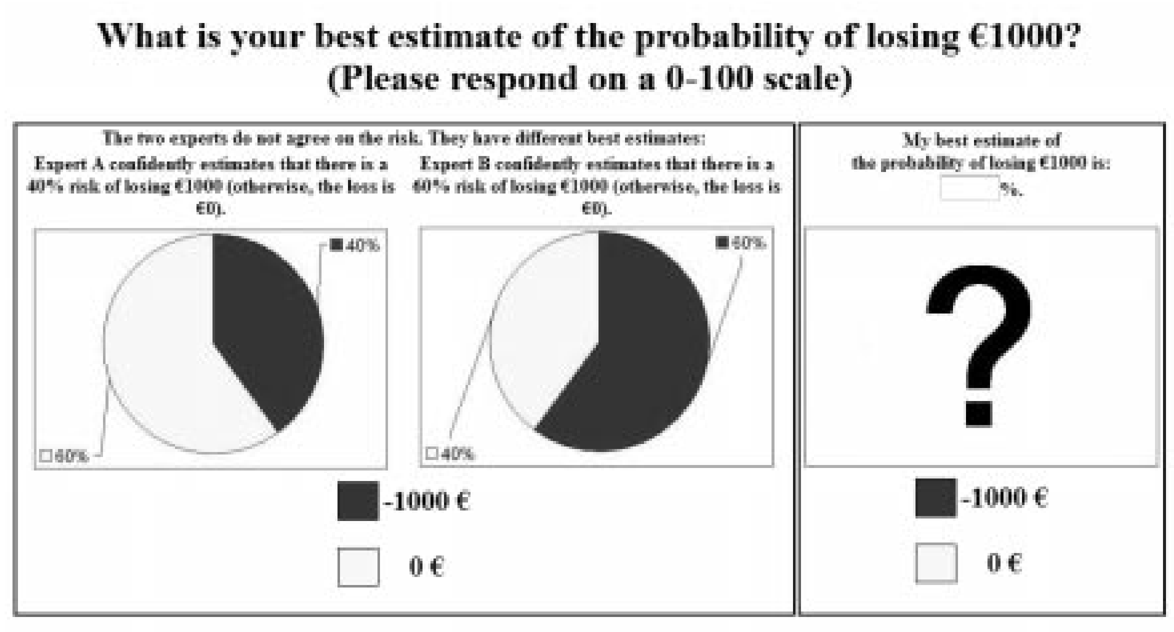
Table E1: Actual choices of Subject 6 in Experiment B for a C prospect

This elicitation process prevented subject from evaluating prospects higher (lower) than their best (worst) outcomes. It also ensured that a sure loss would not be rejected when a higher one had been already accepted (by not asking such a question). It did not prevent however, all violations of stochastic dominance (i.e., participants were not prevented to assign a lower certainty equivalent to $x_{p}y$ than to $x_{p'}y$ despite $p' > p$).

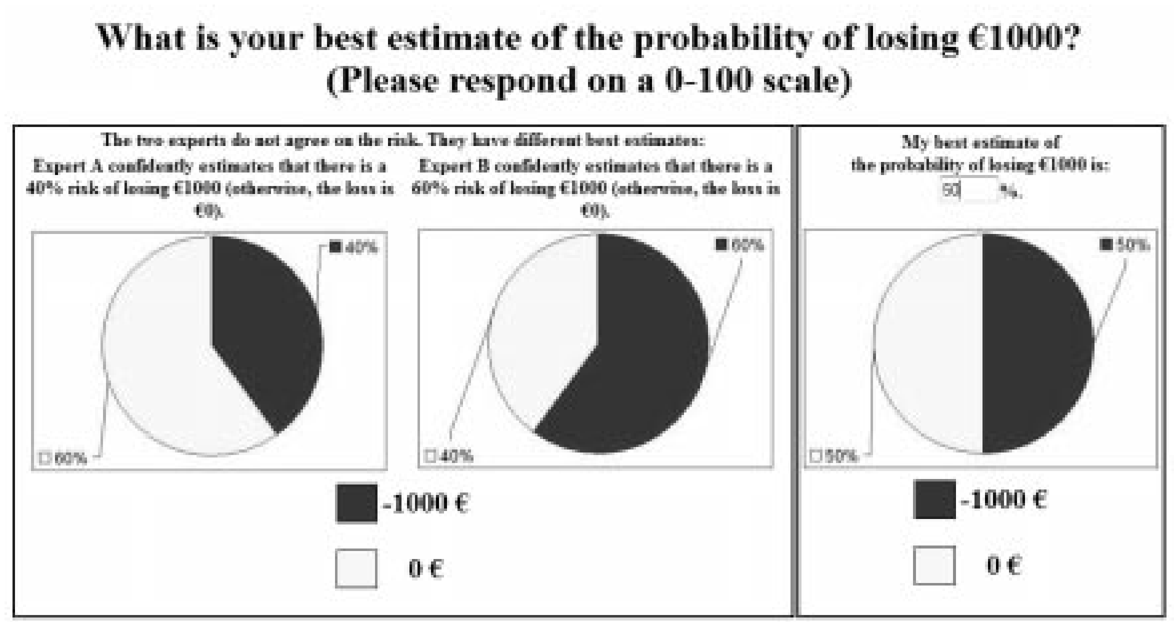
1. Experimental details of Experiment B

Material used to elicit judged beliefs

Figure E3 displays the successive screenshots used to elicit judged belief under C, before (Screenshot A) and after (Screenshot A) entering a judged belief.



Screenshot A: Initial screen



Screenshot B: Once a judged belief is typed in

Figure E3: Screenshot of a typical judgment task: A conflicting prospect (Experiment B)

2. Additional results for both experiments

Consistency checks

The article reports consistency checks. In the two experiments, we find that neither the source of uncertainty nor the question characteristics influences the consistency. In Experiment A, some choices are repeated twice as a consistency check. A Friedman test reveals that the consistency rate does not significantly depend on the source of uncertainty ($p = 0.34$). Similarly, a Cochran test for dichotomous data shows that it does not significantly depend on the question ($p = 0.08$). In Experiment B, several certainty equivalents (randomly selected) were elicited twice. An ANOVA shows that none of the source of uncertainty ($p = 0.15$), the midpoint probability ($p = 0.23$), the minimum loss ($p = 0.85$), and the maximum loss ($p = 0.68$) of the prospects, significantly influences the measurement errors.

Certainty equivalents

In Tables E2, E3, and E4, each line has 61 observations and the degree of freedom of each test is 60. EV means expected value and StD refers to standard deviation.

#		Mid-point	X	y	EV	Mean	Median	StD	*t*-test Mean vs.EV		95% Confidence Interval	
									t	p	Lower Bound	Upper Bound
Experiment A												
A1	R	0.1	–1000	0	–100	–146	–90	148	–2.41	0.02	–184	–108
A2	R	0.3	–1000	0	–300	–333	–290	134	–1.90	0.06	–367	–298
A3	R	0.5	–1000	0	–500	–483	–490	138	0.98	0.33	–518	–448
A4	R	0.7	–1000	0	–700	–633	–670	150	3.51	0.00	–671	–594
A5	R	0.9	–1000	0	–900	–767	–770	130	7.99	0.00	–800	–733
A6	R	0.5	–500	0	–250	–223	–215	65	3.23	0.00	–240	–207
A7	R	0.5	–500	–250	–375	–358	–373	32	4.12	0.00	–366	–350
A8	R	0.5	–750	–500	–625	–603	–608	35	5.06	0.00	–611	–594
A9	R	0.5	–1000	–500	–750	–701	–715	65	5.90	0.00	–717	–684
A10	R	0.5	–1000	–750	–875	–852	–863	36	4.99	0.00	–861	–843
A11	I	0.1	–1000	0	–100	–221	–210	146	–6.49	0.00	–259	–184
A12	I	0.3	–1000	0	–300	–346	–290	121	–2.96	0.00	–377	–315
A13	I	0.5	–1000	0	–500	–488	–490	120	0.76	0.45	–519	–458
A14	I	0.7	–1000	0	–700	–655	–650	105	3.34	0.00	–682	–628
A15	I	0.9	–1000	0	–900	–788	–790	107	8.13	0.00	–816	–761

A16	C	0.1	–1000	0	–100	–102	–90	76	–0.25	0.80	–122	–83
A17	C	0.3	–1000	0	–300	–333	–290	123	–2.09	0.04	–364	–301
A18	C	0.5	–1000	0	–500	–467	–450	156	1.63	0.11	–507	–427
A19	C	0.7	–1000	0	–700	–649	–670	135	2.98	0.00	–683	–614
A20	C	0.9	–1000	0	–900	–828	–850	102	5.53	0.00	–854	–802
Experiment B												
B1	R	0.1	–1000	0	–100	–135	–90	122	–2.26	0.03	–166	–104
B2	R	0.2	–1000	0	–200	–215	–190	131	–0.92	0.36	–249	–182
B3	R	0.4	–1000	0	–400	–409	–390	134	–0.51	0.61	–443	–374
B4	R	0.5	–1000	0	–500	–482	–490	168	0.85	0.40	–525	–439
B5	R	0.6	–1000	0	–600	–558	–590	156	2.10	0.04	–598	–518
B6	R	0.8	–1000	0	–800	–749	–790	128	3.10	0.00	–782	–716
B7	R	0.9	–1000	0	–900	–841	–870	113	4.06	0.00	–870	–813
B8	R	0.5	–500	0	–250	–244	–245	84	0.58	0.56	–265	–222
B9	R	0.5	–500	–250	–375	–350	–358	52	3.86	0.00	–363	–336
B10	R	0.5	–750	–500	–625	–611	–623	46	2.32	0.02	–623	–599
B11	R	0.5	–1000	–500	–750	–701	–715	95	4.01	0.00	–726	–677
B12	R	0.5	–1000	–750	–875	–841	–848	50	5.24	0.00	–854	–829
B13	I	0.1	–1000	0	–100	–173	–130	130	–4.41	0.00	–207	–140
B14	I	0.5	–1000	0	–500	–489	–490	141	0.58	0.56	–526	–453
B15	I	0.9	–1000	0	–900	–833	–870	134	3.88	0.00	–868	–799
B16	C	0.1	–1000	0	–100	–117	–90	100	–1.35	0.18	–143	–92
B17	C	0.5	–1000	0	–500	–510	–490	146	–0.52	0.61	–547	–472
B18	C	0.9	–1000	0	–900	–854	–890	128	2.82	0.01	–887	–821

Table E2: Certainty Equivalent of each Prospect

Risk, uncertainty, and ambiguity attitudes directly inferred from certainty equivalents.

The main text reported tests of risk and ambiguity attitudes using the parameters estimated through parametric fitting. We can also analyze risk and ambiguity attitudes by directly comparing CEs with expected values and other CEs. These tests are consistent with the results in the main text, and are reported next.

Experiment A

To test risk attitudes, we compare CEs with expected values for the risky prospects A1-A10 using *t*-tests. We find that people are mainly risk averse for low probabilities (A1: $p = 0.02$) and mainly risk seeking for moderate and high probabilities (A4-A10: $p < 0.01$). Risk neutrality cannot be rejected for the other two prospects. These results are consistent with the common findings for prospect theory for risk (Wakker 2010 §9.5). For the other prospects, both risk- and ambiguity attitudes play a role. We use the term uncertainty attitude for the combination of the two. We use midpoint probabilities for I and C to obtain (analogs of) expected value.

For I-ambiguity we find uncertainty aversion in the sense that CE < EV for low probabilities (A11-A12: $p < 0.01$) and uncertainty seeking for moderate and high probabilities (A14, A15: $p < 0.01$). Neutrality cannot be rejected for A13. These results confirm prospect theory's prediction that phenomena for risk similarly occur for ambiguity, even in an amplified manner (Kahn & Sarin 1988; Weber 1994). The latter prediction is less clearly confirmed for C-ambiguity. We only find uncertainty aversion for one low probability (A17: $p = 0.04$). We do find uncertainty seeking for moderate and high probabilities (A19, A20: $p < 0.01$). Uncertainty neutrality cannot be rejected for A16 and A18. These results show again that other factors play a role for C-ambiguity.

We can directly infer ambiguity attitudes from CEs by comparing the CEs elicited for I- or C-ambiguity (A-11-A15 and A16-A20) with those elicited under risk (A1-A5). Although this analysis will, obviously, not give exactly the same results and significance values as the analysis in the main text (which involves more data –we ignore A6-A10 here– and nonlinear transformations), most results, and all main phenomena described in the main text remain the same. First an ANOVA corrected for repeated measures (with the Greenhouse-Geisser correction) with two factors (the five probability levels and the two types of ambiguity plus risk) confirms that CEs depend on the probability level ($p < 0.01$), which is obvious, but they also depend on the type of ambiguity or risk ($p < 0.01$). The interaction term is also significant ($p < 0.01$), confirming that the degree of ambiguity aversion depends on how likely the occurrence of the bad outcome is. These results are the same as those obtained for matching probabilities in the main text.

Similarly as the ambiguity aversion index in the main text is an overall index, we can use CEs to analyze overall ambiguity aversion. Thus we compare, for all probability levels together, the CEs under the two types of ambiguity with those under risk (i.e., the main effect of the second factor of the ANOVA), to assess ambiguity aversion. The result can then be compared with the results derived from the ambiguity aversion index. This analysis shows that, overall, CEs under I-ambiguity are lower than those under risk ($p < 0.01$), which indicates ambiguity aversion under I-ambiguity. There is no difference between the effect of risk and the effect of C-ambiguity ($p = 1$), indicating ambiguity neutrality in C-ambiguity. CEs under I-ambiguity are lower than under C-ambiguity ($p < 0.01$), indicating more ambiguity aversion under I-ambiguity than under C-ambiguity. In other words, these results replicate the findings that the ambiguity aversion index is positive under I-ambiguity and higher under I- than under C-ambiguity.

Pairwise comparisons of CEs at each probability level are also instructive to observe the effect of a-insensitivity. Consistent with the a-insensitivity index being negative for C-ambiguity, CEs tend to be higher under C-ambiguity than under risk at $r = 0.1$ ($p = 0.06$) and lower at $r = 0.9$ ($p < 0.01$). This indeed reveals that CEs vary more under C-ambiguity than under risk when the midpoint probability varies, suggesting a-generated over-sensitivity for C-ambiguity. CEs, on the contrary, vary less under I-ambiguity than under risk when probabilities vary: they are significantly lower at 0.1 ($p = 0.01$) and not significantly different at 0.9 ($p = 0.72$). Again, this is consistent with the positive a-insensitivity index that we obtain for I-ambiguity. Finally, we report in the main text that $m_i(0.1) > m_c(0.1)$ ($p < 0.01$) and $m_i(0.9) < m_c(0.9)$ ($p < 0.01$). We similarly find that the CEs are lower under I- than under C-ambiguity at 0.1 ($p < 0.01$) and higher under I- than under C-ambiguity at 0.9 ($p = 0.04$). This inversion, which appears both for matching probabilities and CEs can be explained by more a-insensitivity under I- than under C-ambiguity (as we found using indexes defined from matching probabilities).

Experiment B

As in Experiment A, we compare CEs with expected values to investigate risk and uncertainty attitudes. For risk (B1-B10), we find risk aversion for small probabilities (B1: $p = 0.03$), but risk seeking for moderate and high probabilities (B5: $p = 0.04$; B10: $p = 0.02$; B6, B7, B9-B12: $p < 0.01$). Risk neutrality cannot be rejected for the other four prospects. These results are again consistent with what prospect theory predicts.

For the uncertainty attitude regarding I-ambiguity we find, as in Experiment A, uncertainty aversion for small probabilities (B13: $p < 0.01$), but uncertainty seeking for intermediary and high probabilities (B15: $p < 0.01$). Uncertainty neutrality cannot be rejected for B14. Again, these results are consistent with what prospect theory predicts.

For the uncertainty attitude regarding C-ambiguity we find, as in Experiment A, a less clear pattern. Uncertainty seeking for high probabilities (B18: $p < 0.01$) is still significant, but uncertainty neutrality cannot be rejected for B16 and B17.

Ambiguity attitudes can be analyzed by comparing the CEs under risk (B1, B4, B7) with the CEs obtained for I-ambiguity (B13, B 14, B15), and then with the CEs obtained for C-ambiguity (B16, B17, B18). As for matching probabilities, an ANOVA corrected for repeated measures (with the Greenhouse-Geisser correction) with two factors (the five probability levels and the two types of ambiguity plus risk) shows that the probability factor and the interaction term are significant ($p < 0.01$).

The effect of the type of ambiguity or risk (not distinguishing probability levels; i.e., studying the main effect of the first factor of the ANOVA) does not differ from one type to the others ($p > 0.4$ for all pairwise comparisons). This is consistent with the finding that the ambiguity aversion index is not significantly different from 0 for C-ambiguity and only marginally significant for I-ambiguity.

In the main text, we report a-insensitivity in I-ambiguity and more a-insensitivity in I- than in C-ambiguity. This is consistent with CEs in I-ambiguity being significantly lower than CEs under risk and C-ambiguity at midpoint 0.1 ($p = 0.04$ and $p < 0.01$) and not significantly different at midpoint 0.9 ($p = 1$ and $p = 0.4$). For low likelihoods, our subjects indeed disliked the I-ambiguity prospect more than the other prospects, but this difference disappears for higher likelihoods (and was even reversed, though not significantly, in CEs). Consequently, as for matching probabilities, the CEs at 0.1 and 0.9 tended to be closer to each other for I-ambiguity than they were for C-ambiguity or risk. This property for matching probability is what we defined as a-insensitivity.

As a conclusion, for both Experiment A and Experiment B, all phenomena observed for matching probabilities are consistent with those derived from the CEs. Therefore, the parametric family that we used to obtain matching probabilities does not influence the results. The CE analysis however is more complex, for instance involving multiple, simultaneous comparisons to observe a-insensitivity. This is why we decided to report the analysis in terms of matching probability in the main text.

Risk attitude

In both Experiment A and B, the mean estimate of the parameter of the utility function is significantly greater than 1, indicating concavity of the utility function. Although in the loss domain, individuals are supposed to have convex utility functions (Tversky and Kahneman, 1992), experimental studies in the loss domain have reported mixed attitudes. Some studies have reported convex utility function at least at the individual level, while others have reported linear (Abdellaoui Bleichrodt and L'Haridon, 2008) and concave utility functions (Abdellaoui, Bleichrodt and Paraschiv, 2007; Fennema and Van Assen, 1998; Etchart-Vincent, 2004). In our experiment, the convexity of the utility function can be due to the fact that the prospects are characterized by small amounts (between 0 and −1000) with respect to the above-mentioned studies. For such prospects, the impact of the decreasing marginal utility of money effect, which implies concavity of the utility function, is likely to be bigger than the impact of the diminishing sensitivity effect that implies convexity (Tversky and Kahneman, 1992).

In both experiments, the probability weighting function exhibits a small degree of elevation. This result is consistent with several studies in the loss domain showing that participants can consider that negative risky gambles are attractive (Abdellaoui, 2000; Etchart-Vincent, 2004). Concerning the curvature of the probability weighting function, the probability weighting function of Experiment A exhibits the usual inverse S-shape whereas it is convex in Experiment B. This last result is still common at the individual level.

Matching probabilities and judged beliefs

	Mid point	Mean	Median	StD	*t*-test Mean vs. Midpoint		95% Confidence Interval	
					t	p	Lower Bound	Upper Bound
Experiment A – Matching probabilities								
I	0.1	0.19	0.13	0.16	4.43	0.00	0.15	0.23
I	0.3	0.33	0.31	0.15	1.52	0.13	0.29	0.37
I	0.5	0.53	0.54	0.14	1.71	0.09	0.49	0.56
I	0.7	0.73	0.73	0.15	1.67	0.10	0.69	0.77
I	0.9	0.86	0.88	0.11	–2.87	0.01	0.83	0.89
C	0.1	0.06	0.04	0.07	–4.20	0.00	0.05	0.08
C	0.3	0.31	0.30	0.15	0.46	0.65	0.27	0.35
C	0.5	0.49	0.50	0.19	–0.30	0.76	0.44	0.54
C	0.7	0.73	0.75	0.13	1.92	0.06	0.70	0.77
C	0.9	0.90	0.92	0.08	0.37	0.71	0.88	0.92
Experiment B – Matching probabilities								
I	0.1	0.16	0.12	0.13	3.30	0.00	0.12	0.19
I	0.5	0.52	0.51	0.12	1.45	0.15	0.49	0.55
I	0.9	0.86	0.89	0.11	–2.57	0.01	0.84	0.89
C	0.1	0.10	0.07	0.10	–0.06	0.96	0.07	0.13
C	0.5	0.54	0.53	0.11	2.77	0.01	0.51	0.56
C	0.9	0.87	0.90	0.12	–1.72	0.09	0.84	0.90
Experiment B – Judged beliefs								
I	0.1	0.11	0.10	0.02	1.62	0.11	0.10	0.11
I	0.5	0.50	0.50	0.02	1.55	0.13	0.50	0.51
I	0.9	0.90	0.90	0.03	0.47	0.64	0.89	0.91
C	0.1	0.10	0.10	0.03	0.37	0.71	0.09	0.11
C	0.5	0.50	0.50	0.03	0.46	0.64	0.49	0.51

C	0.9	0.91	0.90	0.03	2.72	0.01	0.90	0.92

Indexes

	Index		Mean	Median	StD	*t*-test		95% Confidence Interval	
						t	p	Lower Bound	Upper Bound
Experiment A									
Matching probabilities	Ambiguity Aversion	I	0.06	0.06	0.15	2.94	0.00	0.02	0.10
		C	0.00	0.00	0.11	0.04	0.97	–0.03	0.03
	Insensitivity	I	0.13	0.06	0.27	3.84	0.00	0.06	0.20
		C	–0.05	–0.04	0.20	–2.08	0.04	–0.10	0.00
Experiment B									
Matching probabilities	Ambiguity Aversion	I	0.03	0.00	0.13	1.77	0.08	0.00	0.06
		C	0.01	0.00	0.13	0.46	0.64	–0.02	0.04
	Insensitivity	I	0.12	0.03	0.25	3.64	0.00	0.05	0.18
		C	0.03	0.00	0.21	1.16	0.25	–0.02	0.09
Judged beliefs	Ambiguity Aversion	I	0.01	0.00	0.04	1.49	0.14	0.00	0.02
		C	0.01	0.00	0.05	1.62	0.11	0.00	0.02
	Insensitivity	I	0.00	0.00	0.04	0.89	0.38	–0.01	0.01
		C	–0.01	0.00	0.06	–1.75	0.08	–0.03	0.00

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