The Midweight Method to Measure Attitudes Toward Risk and Ambiguity

Gijs van de Kuilen

Department of Economics, Tilburg University, 5000 LE Tilburg, The Netherlands, g.v.d.kuilen@uvt.nl

Peter P. Wakker

Econometric Institute, Erasmus University, 3000 DR Rotterdam, The Netherlands, wakker@ese.eur.nl

This paper introduces a parameter-free method for measuring the weighting functions of prospect theory and rank-dependent utility. These weighting functions capture risk attitudes, subjective beliefs, and ambiguity attitudes. Our method, called the midweight method, is based on a convenient way to obtain midpoints in the weighting function scale. It can be used both for risk (known probabilities) and for uncertainty (unknown probabilities). The resulting integrated treatment of risk and uncertainty is particularly useful for measuring ambiguity, i.e., the difference between uncertainty and risk. Compared to existing methods to measure weighting functions and attitudes toward uncertainty and ambiguity, our method is more efficient and can accommodate violations of expected utility under risk. An experiment demonstrates the tractability of our method, yielding plausible results such as ambiguity aversion for moderate and high likelihoods but ambiguity seeking for low likelihoods, as predicted by Ellsberg.

1. Introduction

Decision under risk relates to cases where known statistical probabilities are available for random events. Since Keynes (1921) and Knight (1921), it has been understood that such probabilities are often not available. Savage (1954) proposed expected utility with subjective instead of objective probabilities for such cases. Then prospects are evaluated by their (subjective) probability weighted average utility. However, Allais (1953) showed that people often do not weight probabilities linearly, which violates expected utility. Ellsberg (1961) put forward a more fundamental problem: often people cannot assign probabilities, not even subjective ones, to random events (ambiguity). For a long time, researchers did not know how to model decision making under ambiguity. Only since Gilboa (1987), Gilboa and Schmeidler (1989), and Schmeidler (1989) have models become available that can handle ambiguity. Tversky and Kahneman's (1992) prospect theory added descriptive improvements to these models.

Virtually all nonexpected utility models use weighting functions that generalize probabilities by relaxing the additivity requirement. That is, the weight of the union of two disjoint events need not be the sum of their separate weights. Obviously, the increased flexibility comes at a price: eliciting nonadditive weighting functions requires more work. Whereas for risk there have been many papers measuring weighting functions, for ambiguity there have been only

a few (§8). This paper introduces a new method, the midweight method, to measure weighting functions, both for risk and for ambiguity. This method is more efficient than previous measurement methods because it minimizes the need to measure utility and focuses on the weighting function. Our method is nonparametric in the sense that it does not make any prior assumption about the form of utility or weighting functions. Hence, any empirical shape can be detected. Especially for ambiguity, which is hard to model but prevails in practice (Greenspan 2004), it is desirable that tractable measurement methods become available.

This paper proceeds as follows. Section 2 briefly presents prospect theory. Section 3 introduces the midweight method, first for risk, then for uncertainty. An empirical measurement of the weighting function for risk is in §4. Section 5 applies the midweight method to measure general uncertainty attitudes, and §6 applies the method to measure ambiguity attitudes based on the Abdellaoui et al. (2011) source method. Predictions derived from our measurements are tested in §7. Discussions and conclusions are in §§8–10. Throughout this paper, we first present results for risk and then extend them to uncertainty. In this way, we make this paper accessible to readers unfamiliar with the relatively new models of uncertainty and ambiguity. Those readers interested only in our method for risk can skip all texts on ambiguity without loss of continuity.

2. Prospect Theory for Risk and for Uncertainty

Outcomes are monetary, with $\mathbb{R}^+$ the outcome set. For simplicity, we do not consider losses (negative outcomes). Our method can be applied to losses exactly as it will be to gains. Because the midweight method requires no more than three distinct outcomes, we focus on this case in this theoretical exposition. For discussions and motivations of the following theories, see Wakker (2010).

We first consider decision under risk. We use Tversky and Kahneman's (1992) prospect theory, which coincides with Quiggin's (1981) rank-dependent utility because we only consider gains. It is an improved version of Kahneman and Tversky's (1979) original prospect theory because it corrects a theoretical problem of probability weighting and allows more than two nonzero outcomes. A *prospect* $(p_1{:}x_1, p_2{:}x_2, p_3{:}x_3)$ yields x_j with probability p_j, $j = 1, 2, 3$. The p_j's are nonnegative and sum to 1. The prospect is evaluated by

$$\text{(for } x_1 \geq x_2 \geq x_3)\text{: } w(p_1)U(x_1)$$
$$+ (w(p_1 + p_2) - w(p_1))U(x_2) + (1 - w(p_1 + p_2))U(x_3). \quad (1)$$

Here U denotes *utility*, which is continuous and strictly increasing. The (*probability*) *weighting function* w maps $[0, 1]$ to $[0, 1]$ and is strictly increasing and continuous, with $w(0) = 0$ and $w(1) = 1$. In what follows, $x_p y$ denotes the two-outcome prospect yielding x with probability p and y with probability $1 - p$.

We now turn to decision under uncertainty. The major improvement of Tversky and Kahneman's (1992) prospect theory relative to the 1979 version was that the new theory could handle not only risk but also the more general and more important context of uncertainty (which includes ambiguity). We will use this extension in our study, where it coincides with Gilboa's (1987) and Schmeidler's (1989) rank-dependent utility (also called Choquet expected utility) because no losses are involved. Under uncertainty, prospects assign outcomes to uncertain events of which the probabilities need not be known. In our experiment, the uncertain events concerned the average temperature in the Dutch city of Eindhoven 11 days ahead. $(E_1{:}x_1, E_2{:}x_2, E_3{:}x_3)$ denotes the *prospect* yielding x_j if E_j obtains, where the E_j's denote three temperature intervals, or unions of temperature intervals. It is always understood that the E_j's are exhaustive and mutually exclusive. Our subjects were not provided with the historical frequencies of these events. Statistics of the past, even if available, would not have eliminated all ambiguity because of changed circumstances today, due to global warming, for example. We denote by $x_E y$ the prospect yielding x under event E and y otherwise.

We use utility U as before, but instead of the weighting function w for probabilities, we use a function W defined on events. For reasons explained later, W is called an *event weighting function*, or *weighting function* for short. W assigns weight 0 to the vacuous event and weight 1 to the universal event, and $A \supset B$ implies $W(A) \geq W(B)$. W shares these properties with probability measures. However, $W(A \cup B) \neq W(A) + W(B)$ may hold for disjoint events A, B, violating additivity, and this is where W generalizes probability measures. A prospect $(E_1{:}x_1, E_2{:}x_2, E_3{:}x_3)$ is evaluated by

$$\text{(for } x_1 \geq x_2 \geq x_3)\text{: } W(E_1)U(x_1)$$
$$+ (W(E_1 \cup E_2) - W(E_1))U(x_2)$$
$$+ (1 - W(E_1 \cup E_2))U(x_3). \quad (2)$$

Risk can be considered to be the special case of uncertainty where probabilities p_j are given for the events E_j, and $W(E_j) = w(p_j)$. So as to clarify this point, we use the same terms for risk and uncertainty whenever no confusion arises.

Convexity of w can be defined as

$$w(a + b) - w(b) \leq w(a + b + i) - w(b + i)$$
$$\text{for all nonnegative } a, b, \text{ and } i. \quad (3)$$

It is naturally extended to uncertainty, with W *convex* if

$$W(A \cup B) - W(B) \leq W(A \cup B \cup I) - W(B \cup I)$$
$$\text{for all disjoint events } A, B, \text{ and } I. \quad (4)$$

Concavity is defined by reversing the inequality signs. Our terminology is consistent in the following sense: If W is a transform $w(P)$ of a probability measure P, then under some richness assumptions, convexity (concavity) of W is equivalent to convexity (concavity) of w (Wakker 2010, Table 10.9.1). In the domain investigated in our study, we test the often found *inverse-S shape* of weighting functions by testing concavity for unlikely events and convexity for events of moderate and high likelihood.

Assuming zero decision weight (and probability) for single temperature values, it is immaterial how we take openness and closedness of intervals. For convenience, we usually take intervals left-closed and right-open.

3. The Midweight Method Defined

The midweight method, which measures midpoints in the weighting scale, starts by measuring a utility midpoint. To this end we elicit

$$x_{2_p}y \sim x_{1_p}Y \quad \text{and} \quad x_{1_p}y \sim x_{0_p}Y \quad \text{for risk, and}$$
$$x_{2_E}y \sim x_{1_E}Y \quad \text{and} \quad x_{1_E}y \sim x_{0_E}Y \quad \text{for uncertainty,} \quad (5)$$

with $x_2 > x_1 > x_0 > Y > y$ (as in Wakker and Deneffe's 1996 tradeoff method). Then, with $0 < \pi = w(p)$

Figure 1   Distributing d's Weight Evenly over the Upper and Lower Branch
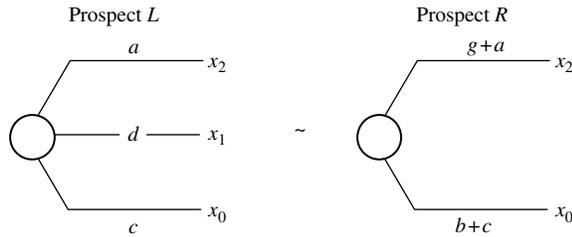
or $0 < \pi = W(E)$,

$$\pi(U(x_2) - U(x_1)) = (1-\pi)(U(Y) - U(y))$$
$$= \pi(U(x_1) - U(x_0)), \quad \text{which implies}$$
$$U(x_2) - U(x_1) = U(x_1) - U(x_0). \qquad (6)$$

That is, x_1 is the utility midpoint of x_2 and x_0. These outcomes will be used throughout what follows, and from here on the preference domain will be restricted to prospects that use only these three outcomes (called the *probability triangle* of x_0, x_1, and x_2 for risk).

We first present the midweight method for risk. For any probability a and larger probability $d + a$ we find their w-midpoint probability $g + a$, with $0 < g < d$. We start from the left prospect $L = (a{:}x_2, d{:}x_1, c{:}x_0)$ in Figure 1, with x_0, x_1, x_2 as in Equations (5) and (6) for risk. Here d, the probability mass of x_1 in the left prospect, is divided (this is what d refers to) over the other outcomes to yield the equivalent right prospect R. g is moved to the high outcome x_2, and the remainder $b = d - g$ is moved to the low outcome x_0.

Because the proof of the following theorem is instructive, it is given in the main text.

THEOREM 1. *The indifference in Figure 1 implies that*

$$w(g + a) = \frac{w(a) + w(d + a)}{2}$$

whenever $U(x_2) - U(x_1) = U(x_1) - U(x_0) > 0$.

PROOF. Figure 2 depicts the decision weights to be derived. The move of g probability mass from outcome x_1 up to outcome x_2 increases the prospect theory value by $\delta_{12} \cdot (U(x_2) - U(x_1))$, where δ_{12} is the extra decision weight for the upper branch, $w(g + a) - w(a)$ (the lower $*$ in Figure 2). The move of b probability mass from outcome x_1 down to outcome x_0 decreases the prospect theory value by $\delta_{10} \cdot (U(x_1) - U(x_0))$, where δ_{10} is the extra decision weight for the lower branch, i.e., $(1 - w(g + a)) - (1 - w(d + a)) = w(d + a) - w(g + a)$ (the upper $*$ in Figure 2). Dropping the equal utility differences, $w(g + a) - w(a) = w(d + a) - w(g + a)$ must hold so as to preserve indifference. The theorem follows. $\square$

Our approach is general in the sense that the weight midpoint between any two probabilities can be measured directly. The only richness of outcomes needed is that a utility midpoint exists for at least one pair of outcomes. With a method available to measure midpoints of the weighting function, we can measure the weighting function to any desired degree of precision. For example, we can start with $p = 0$ and $q = 1$ to find $w^{-1}(1/2)$, i.e., the probability corresponding to weight $1/2$. Then we use $p = 0$ and $q = w^{-1}(1/2)$ to find $w^{-1}(1/4)$, and so on.

The midweight method can be applied to uncertainty in a way very analogous to risk, as is explained next. For any event A and a larger event $D \cup A$

Figure 2   Decision Weights for Prospects of Figure 1
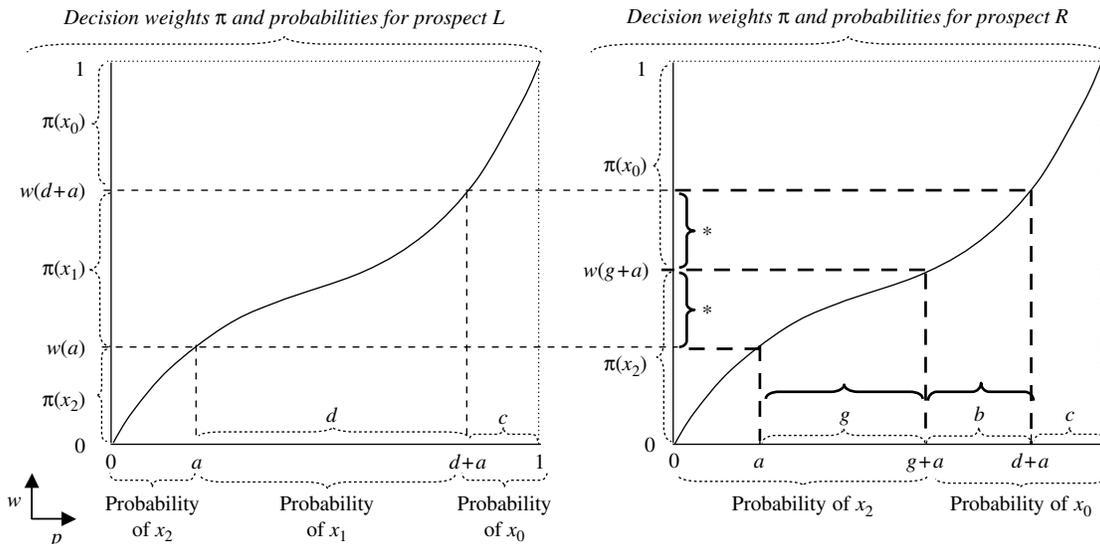
* Decision weight moved from $U(x_1)$ to $U(x_2)$ = decision weight moved from $U(x_1)$ to $U(x_0)$; g: probability moved from x_1 to x_2; b: probability moved from x_1 to x_0.

Figure 3 Distributing D's Weight Evenly over the Upper and Lower Branch
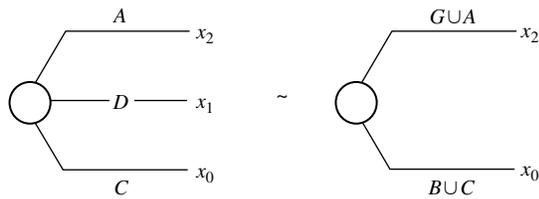
a W-midpoint event $G \cup A$ ($G \subset D$) can be determined by eliciting indifference between the prospects $(A{:}x_2, D{:}x_1, C{:}x_0)$ and $x_{2\,{}_{G \cup A}}x_0$ as in Figure 3.

Theorem 2. *The indifference in Figure 3 implies that*

$$W(G \cup A) = \frac{W(A) + W(D \cup A)}{2}$$

whenever $U(x_2) - U(x_1) = U(x_1) - U(x_0) > 0$.

Proof. The proof is similar to that for risk, with the value increase $(W(G \cup A) - W(A))(U(x_2) - U(x_1))$ of the right prospect equal to its value decrease $(W(D \cup A) - W(G \cup A))(U(x_1) - U(x_0))$, implying the theorem. □

A midpoint event $G \cup A$ as just constructed exists for all events A and $D \cup A$ if the event space is sufficiently rich (such as a continuum), as, for instance, in Gilboa's (1987) preference foundation of rank-dependent utility.

4. Direct Measurement of the Weighting Function for Risk

This section describes an experiment measuring the weighting function for risk.

Subjects. $N = 78$ undergraduate students from a wide range of disciplines were recruited at the University of Amsterdam. They were self-selected from a mailing list of about 400 people. Fourteen subjects were excluded from the analysis because they gave erratic or heuristic answers such as always choosing the left prospect or always choosing the right prospect (details are in Online Appendix F, provided in the e-companion[1]). The practice choices also served to detect such erratic and heuristic answers. These subjects apparently did not understand the choices or did not think about them seriously. In future studies, individual interviews could be used to reduce such misunderstandings. The following analysis is based on the remaining 64 subjects (26 female; median age 21). Including the excluded subjects would not have altered the results presented hereafter.

[1] An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

Procedure. Subjects were seated in front of personal computers in seven different sessions with approximately 11 subjects per session. After receiving experimental instructions (see Appendix B), subjects were given the experimental questions. Subjects were asked two practice choice questions to familiarize them with the experimental procedure.

Stimuli: General. In each question subjects chose between a prospect L (left) and a prospect R (right). Both prospects yielded prizes depending on the outcome of a roll with two 10-sided dice, each determining one digit of a random number below 100. Prospects were framed as in Figure 4. Subjects indicated their choice by clicking on the appropriate button. They were encouraged to answer at their own pace. The position of each prospect was counterbalanced among subjects.

Measuring Utility. We set $x_0 = 60$ and obtained values x_1 and x_2 to generate indifferences, in the order of elicitation:

$$\mathbf{x_1}_{0.25}30 \sim 60_{0.25}40 \quad \text{and} \quad \mathbf{x_2}_{0.25}30 \sim x_{1\,0.25}40. \quad (7)$$

(The values that were elicited are shown in bold.) Then under prospect theory, x_1 is the utility midpoint of x_0 and x_2 (Equation (6)). Because all further measurements in the experiment depended on the values x_1 and x_2, these values were elicited twice, and the average of the two elicitations was used as input in the rest of the experiment so as to reduce noise. Throughout this paper, indifferences are obtained using a bisection choice method. Bisection methods, although time consuming, give more consistent results than direct matching (Bardsley and Moffat 2009, Bostic et al. 1990, Noussair et al. 2004). Our method is similar to Abdellaoui's (2000), with five iteration steps, and is explained in detail in Appendix A.

Measuring Probability Weighting for Risk. Using the midweight method, we elicited five probabilities: $w^{-1}(1/8)$, $w^{-1}(2/8)$, $w^{-1}(4/8)$, $w^{-1}(6/8)$, and $w^{-1}(7/8)$. We framed the prospects as in Figure 4. All left prospects used in the experiment are special cases of prospect L in Figure 1 with at least one probability 0, so that at most two branches remain.

The midweight method concerns indifference between prospect $L = (a{:}x_2, d{:}x_1, c{:}x_0)$ and prospect $R = x_{2\,{}_{g+a}}x_0$ which, as shown in §3, implies that probability $g + a$ is the weight midpoint between probability a and probability $d + a$. For example, to obtain $w^{-1}(4/8)$, the weight midpoint between 0 and 1, we take $a = 0$, $c = 0$, and $d = 1$ in Figure 1, yielding the left panel of Figure 5. Now prospect L is the degenerate prospect yielding x_1 with certainty. Figure 5 lists the indifferences elicited to obtain the probabilities $w^{-1}(1/8)$, $w^{-1}(2/8)$, $w^{-1}(4/8)$, $w^{-1}(6/8)$, and $w^{-1}(7/8)$. In general, to find the g's to generate the

Figure 4   **Framing of the Prospect Pairs**

Prospect L		
Roll	Probability	Prize
1 to p	$p\%$	€x_{i-1}
$p+1$ to 100	$(100-p)\%$	€Y

Prospect R		
Roll	Probability	Prize
1 to p	$p\%$	€x_i
$p+1$ to 100	$(100-p)\%$	€y

required indifferences, we used a bisection method as in the outcome part of the experiment, explained in Appendix A. To test for order effects, for one group we elicited $w^{-1}(2/8)$ and $w^{-1}(1/8)$ before $w^{-1}(6/8)$ and $w^{-1}(7/8)$, and for the other group we did it the other way around.

Motivating Subjects. We used a variation of the random incentive system, the almost exclusively used real incentive system for individual choice experiments today. For each of the seven sessions, there were as many envelopes as subjects, with one envelope containing a blue card and all others containing a white card. Each subject was asked to choose an envelope. Only the subject who had the blue card could play for real. For this subject, one choice question was again selected randomly and the chosen prospect in that choice question was played out for real, with the subject paid according to the prospect chosen and the outcome that resulted from playing out this prospect. All other subjects in a particular session, who had chosen a white card, received a fixed payment of €5. The possible monetary outcomes of the prospects used during the experiment ranged from €30 to approximately €250. All payments were done privately, immediately at the end of the experiment. The average payment under real play was €77.57, so that the total reward per subject was approximately €11.60, and it took subjects about 20 minutes to complete the experiment. Armantier (2006) and Harrison et al. (2007, footnote 16) compared this version of

the random incentive system, where only some subjects are paid for real, to the more popular rewarding scheme where all subjects are paid for real, and found no difference. These papers considered static choice, as in our paper. Baltussen et al. (2010) did find a difference for dynamic decision making.

Further Stimuli. Our questions were chained. It is well known that chaining can give incentives for not truthfully answering questions (Harrison 1986). To check whether subjects had been aware of this possibility, we asked two *strategy-check questions*: "Was there any special reason for you to specially choose left more often, or specially choose right more often?" and "Can you state briefly which method you used to determine your choice?" These questions were asked in a questionnaire at the end of the experiment, with further questions about age, field of study, and gender. Further discussion is in §9.

Results: Utility. The first measurement of outcome $x_1(x_2)$ did not differ significantly from its second measurement (Wilcoxon signed-rank tests, $z = 1.23$, $p = 0.2$ and $z = -1.48$, $p = 0.14$). We therefore take averages of the two measurements in the following analyses. We had also used those averages for the stimuli in the experiment that used x_1 and x_2 as inputs.

The median values of x_1 and x_2 are 92.25 and 123, respectively, which together with $x_0 = 60$ suggests linear utility. The deviation from linearity is not significant (Wilcoxon signed-rank test, $z = 0.89$, $p = 0.38$),

Figure 5   **Indifferences to Elicit** $w^{-1}(j/8)$
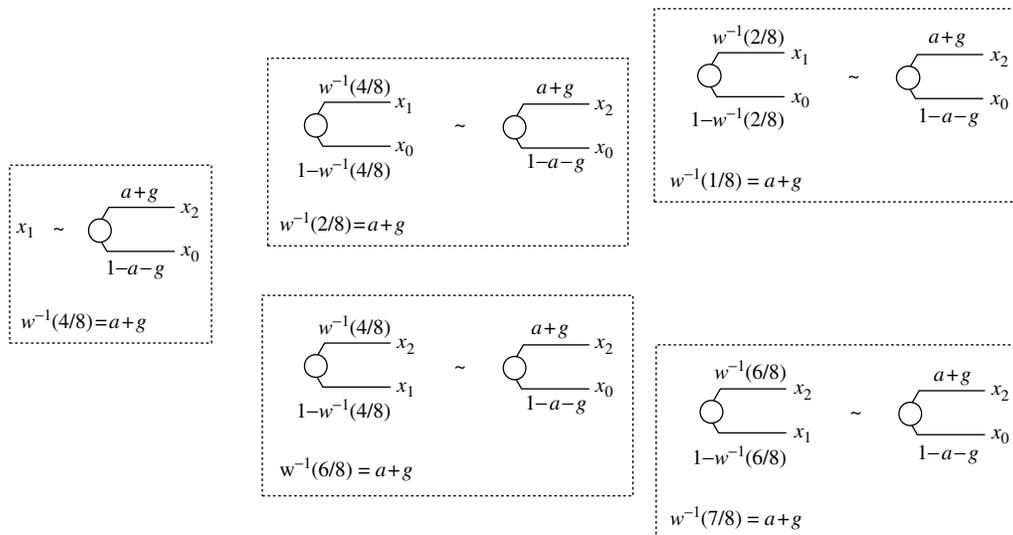
Table 1   Counts of $w^{-1}(p) - p > 0$ and $w^{-1}(p) - p < 0$

$w^{-1}(p) - p$	>0	<0
$p = 1/8$	49**	15
$p = 2/8$	48**	16
$p = 4/8$	44**	20
$p = 6/8$	44**	18
$p = 7/8$	41	23

** Denotes significance at the 1% level using a two-tailed Wilcoxon signed-rank test.

in agreement with the common hypothesis that utility is approximately linear for moderate amounts of money (Rabin 2000). At the individual level, 26 (32) out of 64 subjects exhibited a concave (convex) utility function. This result is robust for gender and field of study.

Results: Probability Weighting. Because there was no order effect for decision weights, we pooled the data. Figure 6 displays the median weighting function. Means were similar to medians, and standard deviations were approximately 0.2. The median weighting function is convex (pessimistic).

Table 1 confirms that subjects did not process probabilities linearly, but mostly underweighted them. The probabilities $w^{-1}(\pi)$ all differ significantly from their corresponding weights π except for $w^{-1}(7/8)$.

We used Bleichrodt and Pinto's (2000) classification system of individual weighting functions of subjects. In short, we considered *slope differences*, i.e., changes in the average slope of the probability weighting function between two adjacent probability intervals. A weighting function was classified as exhibiting lower (upper) subadditivity if the slope difference on the first (last) two intervals of the weighting function was negative (positive), suggesting concavity (convexity) there. A subject was classified as inverse-S in

Figure 6   Median Weighting Function

case of both lower and upper subadditivity, as concave if three slope differences were negative and the subject did not exhibit upper subadditivity, and as convex if three slope differences were positive and the subject did not exhibit lower subadditivity. Based on this classification, 20.13% of the weighting functions exhibited lower subadditivity and 43.75% exhibited upper subadditivity. Their intersection, inverse-S, occurred for 10.49% of the subjects. 23.44% of the weighting functions were concave, 53.13% were convex, 0% was linear, and 12.49% remained unclassified.

All the above analyses were nonparametric. We also estimated Prelec's (1998) two-parameter compound invariance weighting function for every subject by minimizing the sum of squared residuals. This weighting function is given by

$$w(p) = e^{-\beta(-\ln p)^\alpha} \tag{8}$$

where α captures likelihood insensitivity (i.e., the degree to which behavior is sensitive toward changes in likelihood), and β (which is the power) captures the degree of optimism or pessimism. The median values of α and β were 1.145 and 1.578, whereas the values of α and β based on median data were $\alpha = 1.054$ and $\beta = 1.763$. The former weighting function is depicted in Figure 6 and, obviously, accommodates the prevailing convexity. Further results, including individual results, are in the online appendix, provided in the e-companion.

Results: Strategy-Check Questions. In the strategy-check questions, none of the subjects revealed awareness of the chained nature of the questions, or an attempt to strategically exploit this chaining. Twenty-five subjects indicated a combination of (expected or maximal) value and safety, five maximized expected value, and four maximized highest value. Various other reasons were given for the choices made.
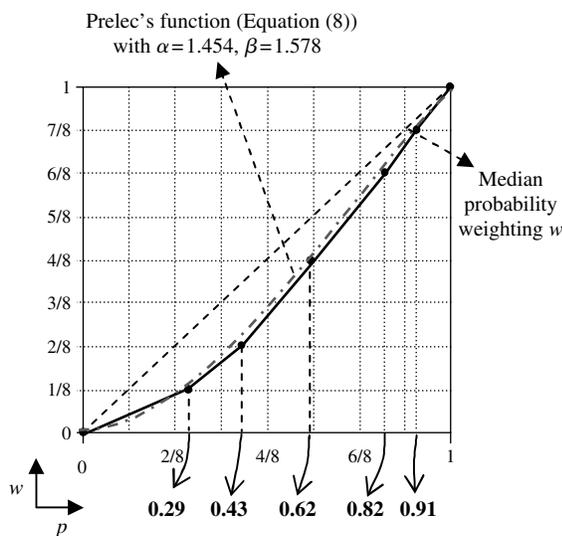
5. Direct Measurement of the Weighting Function for Uncertainty

This section describes an experiment measuring the weighting function for uncertainty.

Subjects. $N = 44$ undergraduate economics students were recruited at Tilburg University by an online recruitment system. The experiment was held on September 11, 2008. Three subjects were excluded from the data set because they gave erratic answers, such as always preferring the left prospect or always preferring the right one (details are as with risk; see Online Appendix F). The following analysis is based on the remaining 41 subjects (21 female; median age 20). Including the three excluded subjects would not have altered the results presented hereafter.

Procedure. Subjects were seated in front of personal computers in four different sessions with about 11 subjects per session. They were asked two practice

Figure 7   Elicited Indifferences for W
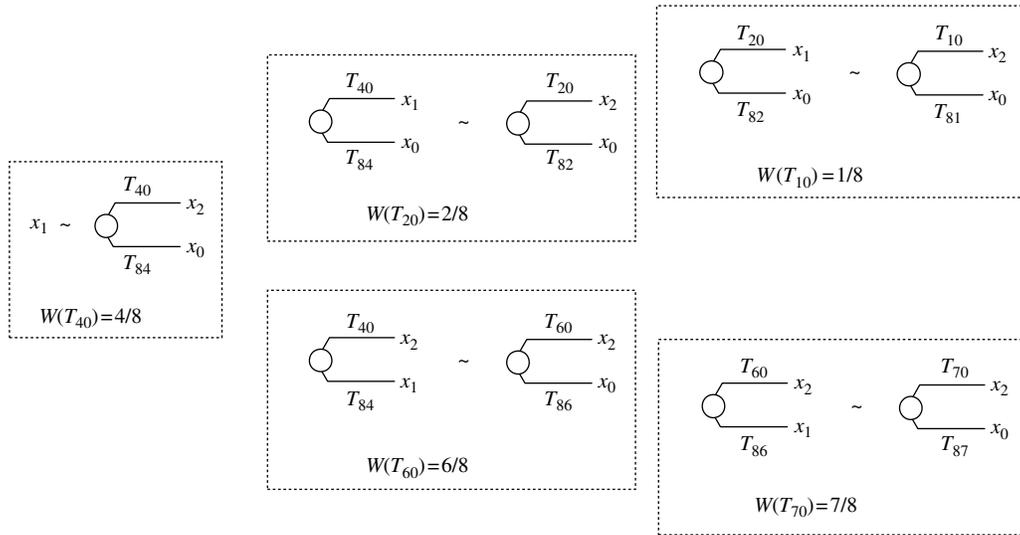
choice questions to familiarize them with the experimental procedure. In each question, they chose between a prospect L (left) and R (right) by clicking on the corresponding button. They were encouraged to answer the questions at their own pace.

Stimuli. Prospects yielded prizes depending on the mean temperature (°C) in Eindhoven 11 days after the experiment, as measured by the Royal Dutch Meteorological Institute (KNMI). Prospects were framed in a way similar to the risk experiment.

Measuring Utility. As for risk (Equation (7)), we set $x_0 = 60$ and then elicited indifferences:

$$\mathbf{x}_{1_E}30 \sim 60_E40 \quad \text{and} \quad \mathbf{x}_{2_E}30 \sim x_{1_E}40, \tag{9}$$

but now we used event E of mean temperature exceeding 15.7°C rather than a probability of 0.25. Again, x_1 and x_2 were elicited twice, their average was taken, and x_1 is the U midpoint of x_0 and x_2.

We then measured the W value of events $[t, \rightarrow)$ (temperature exceeding t). The temperatures measured were, in the order of elicitation, t_4, t_6, t_2, t_7, and t_1, satisfying

$$W[t_i, \rightarrow) = i/8. \tag{10}$$

See Figure 7. Obviously, t_i decreases in i. T_{ij} denotes $[t_i, t_j)$ for $t_i < t_j$ $(i > j)$; see Figure 8. We write $t_0 = \infty$ and $t_8 = -\infty$. This notation is natural. For example, $W[t_0, \rightarrow) = 0/8 = 0$ and $W[t_8, \rightarrow) = 8/8 = 1$, as in Equation (10). $T_{i0} = [t_i, \rightarrow)$. A bisection choice method

Figure 8   The t_i's
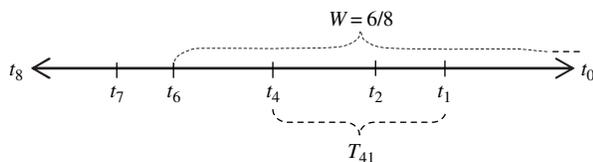
was again used to obtain indifferences between prospects. We used at most five iteration steps, stopping if the interval obtained was not broader than half a degree, and took its midpoint as the elicited indifference temperature t_i. Thus, a precision of a quarter degree results.

Subjects were informed that the average temperature in Eindhoven during the past 50 years had never been below 8.8°C or above 20.4°C. They were told that the average temperature could be assumed to be in [7.2°C, 22°C], and this interval was the starting indifference interval containing t_4.

Motivating Subjects. This was done the same way as under risk, with a random incentive system, white and blue cards, and a show-up fee of €7.50. For each group, the subject who selected the blue card was invited to collect the possible prize at any day after the uncertainty about the temperature had been resolved.

Results: Utility. Again, the first measurement of outcome $x_1(x_2)$ did not differ significantly from the second measurement (Wilcoxon signed-rank tests, $z = 1.03$, $p = 0.30$ and $z = -1.42$, $p = 0.15$). The median values of x_1 and x_2 were 77.25 and 91.50, respectively, which, together with $x_0 = 60$, suggests linear utility on average (Wilcoxon signed-rank test, $z = 1.48$, $p = 0.14$). Because the subjective likelihoods and subjective weightings may be different here than under risk, the values x_1 and x_2 can be expected to be different too; they were lower.[2] However, the absolute

---

[2] The historical probability of event E, based on data from the past 50 years, was 0.25, which is the same probability as used under risk. The subjects were not informed about such historical data. The median subjective probability of E (explained later) was 0.38. The difference in (subjective) probabilities and weights does not affect the validity of our experiment.

Figure 9 Median $W(t, \rightarrow)$

Table 2 Observed Qualitative Preferences

W	W concerns unlikely events	W concerns likely events
Additive	$75_{T_{10}}0 \sim 75_{T_{21}}0$	$0_{T_{87}}75 \sim 0_{T_{76}}75$
Convex	$75_{T_{10}}0 \succeq 75_{T_{21}}0$ (34%)	$0_{T_{87}}75 \succeq 0_{T_{76}}75$ (44%)
Concave	$75_{T_{10}}0 \preceq 75_{T_{21}}0$ (66%)*	$0_{T_{87}}75 \preceq 0_{T_{76}}75$ (56%)
Inverse-S	$75_{T_{10}}0 \preceq 75_{T_{21}}0$ (66%)*	$0_{T_{87}}75 \succeq 0_{T_{76}}75$ (44%)

*$p < 0.05$ (a two-sided Wilcoxon signed rank test with H_0: percentage is 50%).

For unlikely events we find significantly more concavity than convexity, rejecting additivity and agreeing with inverse-S. For likely events the deviations from additivity were not significant.

Discussion. The values $W[t, \rightarrow)$ suffice to evaluate all prospects with outcomes increasing in temperature.[3] To evaluate other prospects, more measurements of W are needed. For example, for prospects with outcomes decreasing in temperature, we need to measure values $W(\leftarrow, t)$. In the absence of additivity, $W(\leftarrow, t)$ cannot be inferred from $W[t, \rightarrow)$ as just measured because these two values need not sum to 1. In general, to evaluate a prospect f, we have to measure W at all events $\{t: f(t) \geq \alpha\}$ for all outcomes α of the prospect. This added complexity is, as always, the price we pay for working with a more general model.

In general, the family of nonadditive measures is large, and often special subfamilies are considered so as to increase tractability (Grabisch et al. 2008). In the next section, we consider a special subfamily, put forward by Abdellaoui et al. (2011). Based on Tversky and Fox's (1995) ideas, Abdellaoui et al. (2011) distinguished between different sources of uncertainty. A *source* (*of uncertainty*) is a group of events that are generated by the same random mechanism. In our study, the two rolls of the 10-sided die, used to generate risk, constitute one source of uncertainty. The temperature in Eindhoven is another source of uncertainty. Abdellaoui et al. (2011) considered cases where within each source (generic notation So) there exist subjective probabilities P_{So}, and for each source the weighting function W is a transform $w_{So}(P_{So})$ of those subjective probabilities. The transformation w_{So} depends on the source and is called a *source function*. As explained by Abdellaoui et al. (2011), the source function captures all deviations from expected utility, comprising both risk attitudes beyond utility curvature and deviations from risk due to ambiguity. Probabilistic sophistication within one source characterizes a uniform degree of ambiguity (Wakker 2008) for that source, not absence of ambiguity as has sometimes been claimed (Epstein and Zhang 2001). In the next

size of the x's (or of their differences) is immaterial because only their equal spacedness in utility matters for our analysis. At the individual level, 13 (21) out of 41 subjects exhibited a concave (convex) utility function. This result is robust for gender and field of study.

Results: W. The median t_i values are $t_1 = 19.75$, $t_2 = 16.85$, $t_4 = 13.00$, $t_6 = 10.96$, and $t_7 = 9.70$, with means very similar and standard deviations approximately 2.5. Figure 9 depicts the graph assigning the median $W(t, \rightarrow)$ to every temperature t.

Direct Tests of Properties of W. If we obtain enough quantitative measurements of the weighting function, then we can verify its properties such as additivity, convexity, and concavity. It is also possible to test such properties directly from qualitative preferences. Table 2 presents preferences that we observed through direct choices in the experiment (not allowing for indifferences) and the way in which they corroborate various properties of W. (We added the second row, regarding "additive," in the table for clarity.) For example, with $U(0) = 0$, the value of $75_{T_{10}}0$ in the middle column is $W(T_{10})U(75)$, with W applied to the unlikely event T_{10}. The value of $0_{T_{87}}75$ in the right column is $W(T_{70})U(75)$, with W applied to the likely event T_{70}.

PROOF FOR TABLE 2. We derive results for convexity of W. The other results are similar.

$75_{T_{21}}0 \preceq 75_{T_{10}}0 \Rightarrow W(T_{21}) \leq W(T_{10}) = 1/8 = W(T_{20}) - W(T_{10})$. Then T_{21} adds less weight to the vacuous event (which has weight zero) than to event T_{10}, to which it adds weight $1/8$ because it augments the weight $W(T_{10}) = 1/8$ to $W(T_{20}) = 2/8$ there. This corroborates convexity of W.

$0_{T_{76}}75 \preceq 0_{T_{87}}75 \Rightarrow W(T_{87} \cup T_{60}) \leq W(T_{70})$ shows that T_{87} adds less than $1/8$ weight to T_{60}, which is what it adds to its complement T_{70}. Again, the marginal W contribution of T_{87} to the larger T_{70} is larger than it is to the smaller T_{60}, corroborating convexity of W. □

[3] This can be inferred from Equation (2). It holds for general prospects f, as can be inferred from the general prospect (= rank-dependent) theory formula $\int_{\mathbb{R}^+} W(f^{-1}(U^{-1}[\alpha, \rightarrow))) \, d\alpha$.

section, we analyze the uncertain source concerning the temperature in Eindhoven using the Abdellaoui et al. (2011) method.

6. Using Subjective Probabilities to Measure Ambiguity

This section shows how the midweight method can simplify the measurement of the Abdellaoui et al. (2011) source functions. We assume that probabilistic sophistication holds with a subjective probability measure P (depending on the subject) for the temperature in Eindhoven. For each temperature event E, $W(E) = w_t(P(E))$ with w_t the Eindhoven temperature source function.

The measurement of W can now be simplified considerably. Thus this section, in combination with the previous one, provides a complete measurement of W. First, we measure the subjective probability measure P, something that also has to be done under Bayesian expected utility. Next, W is plotted as a function of P for the events as considered in the previous section, yielding the source function w_t. Then, the whole weighting function $W = w_t(P)$ is determined, also for events not considered in the previous section. Now all prospects can be evaluated, including those whose outcomes do not increase in temperature. With W and w_t entirely determined, we can, obviously, also investigate all their properties. For example, expected utility holds if and only if W equals P; i.e., if and only if the source function w_t is linear.

To measure P note that with $x > 0$ and A and B temperature events, we have the following implication:

$$x_A 0 \sim x_B 0 \quad \Rightarrow \quad w_t(P(A))U(x) = w_t(P(B))U(x)$$
$$\Rightarrow \quad P(A) = P(B). \tag{11}$$

Figure 11 The s_i's

Events A and B as in Equation (11) are called *equally likely*. Observations of equal likelihood can be used to measure P (Savage 1954). More specifically, we will use the Abdellaoui et al. (2011) method for eliciting subjective probabilities.

Stimuli. In the order of elicitation, we measured temperatures s_4, s_6, s_2, s_7, and s_1, such that the indifferences in Figure 10 hold, with the notation $s_0 = \infty$, $s_8 = -\infty$, and $S_{ij} = [s_i, s_j)$ for $i > j$. Then $P(s_i, \rightarrow) = i/8$ for all i, so that the notation is similar to that for the t_i's in preceding sections. See Figure 11. The measurement procedure of indifference was the same as in §4. Under expected utility, $s_j = t_j$ for all j.

Results: Subjective Probabilities. Figure 12 displays the subjective probability distribution resulting from the median s_i's that we observed, together with the historical probability distribution from the past 50 years regarding the mean temperature on September 22. Our subjects generally considered high temperatures more likely than they were in the past, possibly because of global warming (see Online Appendix F).

Results: Source Function. Figure 13 displays the median source function w_t. (For comparison, it also reproduces w of §4.) We used linear interpolation in the t_i scale to fit domains. The source function displays an inverse-S shape with an intersection with the diagonal at about 0.3, which is confirmed by the

Figure 10 Indifferences to Elicit Subjective Probabilities

Figure 12 **Median** $P[t, \rightarrow)$ **and Historical (Decumulative) Distribution**

Table 3 **Summary Statistics for** T-**Events**

	W	Mean P	Median P	Standard deviation P
$P(T_{10})$	1/8 (0.125)*	0.13	0.08	0.14
$P(T_{20})$	2/8 (0.25)	0.31	0.23	0.22
$P(T_{40})$	4/8 (0.50)**	0.64	0.69	0.24
$P(T_{60})$	6/8 (0.75)**	0.84	0.90	0.16
$P(T_{70})$	7/8 (0.875)**	0.92	0.95	0.08

Note. Asterisks indicate significant difference with P; * $p < 0.05$; ** $p < 0.001$.

Both α and β are significantly different between risk and uncertainty based on Mann-Whitney tests (α: $z = 4.606$, $p < 0.001$; β: $z = 2.358$, $p = 0.02$). Individual results are in Online Appendix C, provided in the e-companion.

Discussion of Results and Ambiguity Attitudes. The significant differences between the s_i's and the t_i's provide yet another falsification of expected utility. Relative to measurements under expected utility, the Abdellaoui et al. (2011) method requires the measurement of one additional curve per source. We emphasize that w_t concerns the attitude toward uncertainty including ambiguity rather than (merely) attitudes toward risk.

The difference between w_t and w (the probability weighting function for risk as measured in §4) reflects ambiguity. We can only compare between subjects here. Within-subject comparisons can obviously be obtained by carrying out both measurements of §§4 and 5 within individuals. For brevity, we have not carried out such a task here and leave it to future studies. Under universal ambiguity aversion, w_t would be below w everywhere, but this is clearly not the case. Instead, w_t is more inverse-S shaped than is w, implying ambiguity aversion for events of moderate and high likelihood, but ambiguity seeking for unlikely events.[4] It shows that modeling ambiguity attitudes through one single number to reflect a universal degree of ambiguity aversion is crude and that degrees of inverse-S shapedness also deserve attention.

values reported in Table 3. The differences between the W and P are always significant, both by t-tests and by Wilcoxon tests, rejecting expected utility, except for t_2 (which determines T_{20}). The latter is no surprise because t_2 is near the expected intersection point where overweighting changes into underweighting.

Again, we estimated Prelec's (1998) two-parameter weighting function (Equation (8)) for every individual by minimizing the sum of squared residuals. The median values of α and β were 0.684 and 1.208, respectively, whereas the values of α and β based on the median data were 0.622 and 1.166. The former weighting function is depicted in Figure 13 and, obviously, accommodates the prevailing inverse-S pattern.

Figure 13 **Source Functions**

7. A Predictive Exercise for Uncertainty[5]

The two practice questions of the experiment for uncertainty entailed a test of the common consequence effect. Whereas this important effect has been extensively tested for risk, tests for uncertainty are virtually absent. We are only aware of MacCrimmon and

---

[4] This phenomenon is in agreement with claims and findings by Curley and Yates (1989), Tversky and Fox (1995), Abdellaoui et al. (2005), Kahn and Sarin (1988, p. 270), Kahneman and Tversky (1979, p. 281), Kilka and Weber (2001), and Weber (1994), and was predicted by Ellsberg (2001) himself.

[5] Adding this section was suggested by two editors.

Larsson (1979, pp. 364–365), Tversky and Kahneman (1992), and Wu and Gonzalez (1999). Given the current interest in uncertainty, it is worth investigating this effect in more detail. After a first qualitative inspection of the data, we will see to what extent the quantitative estimations of the preceding sections predict the choices observed.

We denote events $L = (-\infty, 12.5)$ (mean temperature in Eindhoven below 12.5°C on September 22, 2008), $M = [12.5, 17)$, and $H = [17, \infty)$. Because all prospects in this section refer to these events, we suppress them and write (x_1, x_2, x_3) instead of $(L{:}x_1, M{:}Nx_2, H{:}x_3)$. Subjects were first asked to choose between the prospects

$$S_1 = (0, 50, 50) \quad \text{versus} \quad R_1 = (0, 0, 150)$$

and then to choose between the prospects

$$S_2 = 50 = (50, 50, 50) \quad \text{versus} \quad R_2 = (50, 0, 150).$$

The only difference between the two choice situations is that the common outcome 0 under event L in the first choice pair has been replaced by the common outcome 50 in the second choice pair. Under expected utility, choice should not be affected by such a replacement and, hence, only the choice patterns $S_1 S_2$ and $R_1 R_2$ are possible. We observed the following choice patterns: $S_1 S_2$: 26.83%; $R_1 R_2$: 4.88%; $S_1 R_2$: 24.39%; $R_1 S_2$: 43.90%. Thus, in the first choice situation, the two prospects are, on average, about equally preferred $(26.83\% + 24.39\% = 51.22\%$ chose the safer $S_1)$, but in the second there is a significant majority preference for S_2 $(26.83\% + 43.90\% = 70.73\%; p = 0.008)$. All tests in this section are two-sided Wilcoxon signed-rank tests. A significant majority of subjects violate EU (68.29% chose $R_1 S_1$ or $S_1 R_2$; $p = 0.02$). The violation $R_1 S_2$ significantly deviates from 25% $(p = 0.04)$ whereas the other violation $(S_1 R_2)$ does not. These qualitative results all agree with the usual findings in common consequence tests (mostly done for risk), confirming the certainty effect. The findings agree both with pessimistic and with inverse-S weighting functions (Wakker 2010, §10.4.3).

We next inspect the predictions resulting from the quantitative estimations of the preceding sections. As explained in §5, the values of $W[t, \rightarrow)$ observed there do not suffice to evaluate prospect R_2 because the outcomes of R_2 are not always increasing in temperature. Hence, additional measurements of W with other rankings of events are needed to predict the choices considered.

The source method of §6 does give complete information about W and can be used to predict the choices considered. To this effect, we used linear interpolation on the belief data to obtain the subjective probabilities $P(L)$, $P(M)$, and $P(H)$. Their means are

Table 4   **Predictions for the Four Choice Patterns**

	Prediction (%)			
	$S_1 S_2$	$R_1 R_2$	$S_1 R_2$	$R_1 S_2$
$S_1 S_2$ ($N = 11$)	63.64[a]	18.18	0	18.18
$R_1 R_2$ ($N = 2$)	50	50	0	0
$S_1 R_2$ ($N = 10$)	40	0	50	10
$R_1 S_2$ ($N = 18$)	38.89	5.56	0	55.56

[a] To clarify: Of the 11 subjects choosing $S_1 S_2$, 63.64% was predicted to do so.

$P(M) = 0.51$, $P(H) = 0.24$, and $P(L) = 0.25$. Hence, the mean of $P(L \cup H)$, for instance, is 0.49. We also used linear interpolation for the elicited w_t-function, with resulting means $w_t(P(H)) = 0.25$, $w_t(P(L \cup H)) = 0.40$, and $w_t(P(M \cup H)) = 0.62$. We felt that fitting a power utility function was more appropriate than using linear interpolation because the stimuli considered in this section involve outcomes below the interval where we measured utility, including the outcome 0. On larger domains, U will deviate more from linearity. We hence fitted power utility at the individual level, on the basis of the data obtained in the first part of the experiment, to obtain $U(€0)$, $U(€50)$, and $U(€150)$ for each subject.[6]

Table 4 gives the percentages of choice patterns predicted by the quantitative estimations from the source method. It does so for the four groups of subjects with revealed choice patterns.

For each group, our quantitative estimations predicted correctly in a (weak) majority of cases, considerably exceeding the 25% that would result under random choice. The percentages reported in Table 4 also suggest that the usual common consequence violation $R_1 S_2$ is not only due to noise but also has a systematic part. The unusual violation $S_1 R_2$ is mostly due to noise.

8. Other Measurements in the Literature

In parametric fittings, parametric families of utility and weighting functions are assumed beforehand. The parameters in these families are then chosen so as to minimize the distance to the data, possibly through likelihood functions in probabilistic choice models. In nonparametric measurements, all utility and weighting functions are possible, and their values at particular outcomes and probabilities or events are directly inferred from data. The latter is usually

---

[6] To investigate the predictions about weighting functions without need to estimate utility, we could have adjusted, say, the highest outcome 150 of R_1 and R_2 to obtain $R_1 \sim S_1$ for each individual. Then the preference between R_2 and S_2 would immediately correspond with whether W is convex, concave, or additive for the events considered (Wakker 2010, §7.4), irrespective of utility.

based on deterministic models. Deviations from the deterministic models are subsequently detected using cross-checks and statistical analyses. An intermediate measurement technique does use data fitting but does not commit to any parametric family and instead takes every utility value at every outcome and every weight of every probability or event as a parameter. This section mainly focuses on nonparametric fittings, whereas the next section discusses their pros and cons. There have been many parametric fittings of weighting functions for risk, referenced in Online Appendix G, provided in the e-companion.

8.1. Nonparametric Measurements of Weighting Functions for Risk

Gonzalez and Wu (1999) and Stott (2006) used the intermediate form of data fitting just explained, requiring much data per subject but yielding reliable results in return. Abdellaoui (2000) and Bleichrodt and Pinto (2000) provided two more tractable methods for eliciting probability weighting functions nonparametrically. As with all other measurements used before, but unlike our midweighting method, these methods need a detailed measurement of utility. From n observed indifferences we obtain $n - 2$ data points of the weighting function (plus 1 data point of utility), whereas Abdellaoui (2000) and Bleichrodt and Pinto (2000), for instance, would obtain only $(n - 1)/2$ data points of probability weighting (plus $(n - 1)/2$ data points of utility).

8.2. Measurements of Weighting Functions for Uncertainty

We focus on studies that considered more than one event (and its complement). The only parametric fittings for weighting functions under uncertainty that we are aware of are in Ahn et al. (2009), Andersen et al. (2007), and Hey et al. (2010). These works estimated weighting functions jointly with utility functions. Regarding nonparametric measurements, we are only aware of measurements by Diecidue et al. (2007) and Kilka and Weber (2001), who assumed linear utility; Mangelsdorff and Weber (1994), who assumed expected utility for risk; Abdellaoui et al. (2005), who adapted the methods of Abdellaoui (2000) and Bleichrodt and Pinto (2000) to uncertainty; and, finally, Abdellaoui et al. (2011), Fox et al. (1996), Fox and Tversky (1998), and Tversky and Fox (1995), who carried out complex measurements that included detailed measurements of utility functions. Our method is more efficient for the purpose of measuring weighting functions because it can do so while minimizing the need to measure utility. This is useful for studies that focus on the weighting function, being the new component of risk and ambiguity beyond expected utility. For the purpose of actually describing and predicting decisions, measurements of utility as in the above studies are needed after all. Here an advantage of our method is that it can minimize the dependence of weighting function measurements, and their errors, upon (errors in) utility measurements. There have also been some studies that used direct judgments of subjective probabilities (Einhorn and Hogarth 1985, Hogarth and Einhorn 1990, Wu and Gonzalez 1999), based on introspection and not on revealed preference. This paper has focused on revealed-preference based methods.

8.3. Measuring Endogenous Midpoints

We used Wakker and Deneffe's (1996) tradeoff measurement technique to obtain utility midpoints derived endogenously from preference. Abdellaoui et al. (2007); Fishburn and Edwards (1997, Axiom 8); Harvey (1986, tradeoffs midvalues above Equation (4)); and Köbberling and Wakker (2003, p. 408) similarly used this method. Abdellaoui et al. (2007) next obtained a probability q with $w(q) = 0.5$ through what amounts to a degenerate version of Figure 1 with $c = 1$ and $a = 0$. Finally, they used this probability to efficiently measure utility midpoints in general. Their approach can, like our approach, be interpreted as a special case of Blavatskyy's (2006) general procedure. Abdellaoui (2002), Abdellaoui and Munier (1999), Abdellaoui and Wakker (2005), and Wu et al. (2005) used a tradeoff method in the probability dimension, dual to Wakker and Deneffe's (1996) tradeoff method in the outcome dimension, to measure equalities of probability weighting differences, independent of utility. This can, in particular, be used to measure probability weighting midpoints but is less flexible than our method.

Vind (1991, p. 134; 2003, §IV.2, above Theorem IV.2.1) proposed an alternative method for obtaining endogenous utility midpoints under expected utility and, more generally, under state-dependent expected utility (from which he derived what he called a mean groupoid operation). He showed that y is the utility midpoint between x and z if the following indifferences hold:

$$x \sim x_{1_q}x_2, \quad z \sim z_{1_q}z_2, \quad \text{and} \quad x_{1_q}z_2 \sim z_{1_q}x_2 \sim y. \quad (12)$$

His method holds under prospect theory if we add the requirement that $x_1 > x_2$, $x_1 > z_2$, $z_1 > z_2$, and $z_1 > x_2$, assuming only gains.

Ghirardato et al. (2003, Definition 4) proposed another method to derive utility midpoints endogenously from preferences. They showed that β is the utility midpoint between α and γ under prospect theory if the following indifferences hold:

$$\alpha_q\gamma \sim x_qy, \quad x \sim \alpha_q\beta, \quad \text{and} \quad y \sim \beta_q\gamma \quad (13)$$

with $\alpha > \beta > \gamma$.

With β a utility midpoint between α and γ, the tradeoff method has γ as dependent variable and α and β as independent variables, whereas the other two methods have β as dependent variable and α and γ as independent variables. In the former case, the experimenter has no control over the range (α, γ), which is a drawback of the tradeoff method. We still preferred this method because it requires fewer indifferences to be measured and is easier to implement experimentally.

9. General Discussion

9.1. Inverse-S and Convex Weighting Functions
Empirical studies have found that individual weighting functions are mostly convex or inverse-S shaped, with the latter shape prevailing. Thus, the majority of studies found that a majority of weighting functions exhibited the inverse-S shape. We are aware of some 50 such references (Online Appendix G). Yet the finding is not universal; several studies not only found convex weighting functions for some of their subjects but also even for a majority, as we did for risk.[7] Several other studies found other empirical evidence against inverse-S.[8]

Thus, although we believe that inverse-S is the prevailing phenomenon, it is certainly not universal. It is not clear at this stage why different studies have conflicting results. Much about weighting functions remains yet to be discovered. Our findings and literature search suggest once more that probability weighting, more than utility, is a volatile phenomenon, with results depending on framing and ways of measurement and with no phenomena holding in great generality. As a first, admittedly after-the-fact, explanation, our method may have suppressed inverse-S somewhat because it keeps outcomes fixed and focuses on uncertainty, enhancing sensitivity toward uncertainty. Inverse-S entails insensitivity toward uncertainty. For risk, this enhancing of sensitivity may have been strong enough to suppress the inverse-S shape. Because inverse-S is more pronounced for unknown probabilities, it may still have shown up for those. The effect discussed here could be reduced by not sequencing all questions for finding one indifference successively but interspersing them with other questions (Abdellaoui 2000, p. 1504). Our

restriction to prospects at the boundary of the probability triangle may also have contributed to the extra observed pessimism.

As a second after-the-fact explanation, statistical regression to the mean may have enhanced inverse-S shapes in many studies. For example, in a direct measurement of the decision weight assigned to probability 0.1 under expected utility with noise, underestimation due to error can never be more than 0.1, but overestimation can be as much as 0.9. Average measurements of decision weights can then exceed 0.1 merely because of this error structure. The extent to which different measurements are susceptible to regression to the mean is a topic for future research. In our measurements of midpoints, there is always as much space for overestimation as for underestimation. Thus, the absence of regression to the mean in our design may further explain why we find fewer inverse-S shapes.

9.2. Parametric and Nonparametric Measurements
Advantages of parametric measurement techniques are that these can be applied to virtually any data set and that they smooth errors in the data. A drawback is that they require prior commitment to particular parametric families. These impose particular shapes of the weighting function that may not hold in reality and give no insights into the prevalence of alternative shapes. Some examples are Hey and Orme (1994) and Harless and Camerer (1994), who used power functions, excluding inverse-S shapes, and Donkers et al. (2001), who committed to inverse-S shapes, excluding other shapes. Often their findings crucially depend on the parametric family chosen (Blavatskyy and Pogrebna 2011; Harrison 2006, p. 61; Loomes et al. 2002). Another drawback is that these methods are often subject to colinearity effects, where utility and the weighting functions have similar effects and cannot be reliably separated, with errors in one generating errors in the other (Stott 2006, pp. 112, 121).

An obvious advantage of nonparametric measurements is that they need no prior commitment to any shape and that they will uncover true patterns and phenomena irrespective of what those are. They also make clear to what extent utility and weighting functions concern different phenomena, explaining different parts of the variance in data, and to what extent they overlap. Furthermore, they show how probability weighting and utility are related to decisions in a transparent manner. Hence they can be used in interactive measurement sessions, as common in prescriptive applications.

9.3. The Richness of Ambiguity
To date, most studies of ambiguity have only measured a single number for one event per person,

---

[7] See Goeree et al. (2002), Jullien and Salanié (2000), Kühberger et al. (1999, p. 217), Li et al. (2009), Mosteller and Nogee (1951, in their student population), and Qiu and Steiger (2011).

[8] See Barron and Erev (2003), Bearden et al. (2007); Birnbaum (2008, in particular pp. 484–486, and the many references to his preceding studies), Bleichrodt (2001), Fatas et al. (2007), Goeree et al. (2003), Hartinger (1999), Henrich and Mcelreat (2002), Humphrey and Verschoor (2004), Kunreuther and Pauly (2003), Loomes (1991), Loomes et al. (2002), Luce (1996), and Stott (2006).

intended to measure a universal aversion toward ambiguity of that person. Abdellaoui et al. (2011) showed that the more general source functions can capture the richness of ambiguity and uncertainty attitudes in a tractable manner. We have shown how source functions can be measured more efficiently using the midweight method. Our experiments confirm the Abdellaoui et al. (2011) finding that people are ambiguity averse for events of moderate and high likelihood but are, on the contrary, ambiguity seeking for unlikely events. This pattern of ambiguity attitudes was suggested before by Ellsberg (2001, pp. 203, 206).

9.4. Chaining

The values x_1, x_2, and $w^{-1}(p)$ that were elicited from subjects returned as inputs in later questions (chaining). Bisection also involves chaining. It is well known that subjects can exploit chaining by not answering at particular questions truthfully so as to improve stimuli in future questions. Such a distortion is unlikely to have arisen in our experiment. It is difficult for subjects to notice that their answer to one question will influence future stimuli. For example, we did not directly ask for the indifference values used in future questions but derived indifference values indirectly as midpoints between values used in choices, so that subjects had not seen the indifference values before and in this way could not recognize them. In addition, to exploit chaining, subjects must understand the presence of chaining and also how future questions will depend on current answers, which is very hard for subjects. Finally, our strategy-check questions revealed no strategic exploitation of chaining. We carefully formulated our instructions (end of Appendix B) in order to avoid deception.

Another complication of chained questions is that they generate error propagation. Blavatskyy (2006) described the general procedure of starting with measurements in one dimension, then using these to obtain measurements in the other dimension, possibly using the latter again to obtain more refined measurements in the first dimension, and so on. He examined general efficiency principles regarding error propagation of such general procedures.

To investigate the effects of error propagation, we performed a simulation study similar to Bleichrodt and Pinto (2000). We assumed that true utility differences were multiplied by an error factor $1 + e$, with e normally distributed with mean zero and standard deviation 0.05. We assumed the same proportional error in w^{-1}. We further assumed prospect theory with power utility $U(x) = x^{0.88}$ and weighting function $w(p) = e^{-0.93(-\ln p)^{0.85}}$ (found by Abdellaoui et al. 2011, Figure 3). We performed 1,000 simulations and found that propagation of error is not a

big problem. For each probability $w^{-1}(j/8)$ measured, the standard deviation was less than 0.05, the selected size of the response errors. Other simulation studies of error propagation in setups similar to ours were reported by Abdellaoui et al. (2005, p. 1394), Bleichrodt et al. (2010, p. 164), and Bleichrodt and Pinto (2000, p. 1495). These studies also found that the effects of error propagation are small.

In the experiment we used the midweight method to measure the weighting function over its whole domain. The method can also be used to investigate the local curvature of the weighting function. For example, if we want to know whether the weighting function is convex on a particular domain $[a, c)$, then we can use our method to find the w-midpoint q between a and c, then the w-midpoint between a and q, and so on. In this manner we obtain local tests of convexity on $[a, c)$.

10. Conclusion

We have introduced a new method for measuring functions that weigh risk and uncertainty. It is almost twice as efficient as methods that have been used before because it minimizes the required measurements of utility. Experiments have demonstrated the feasibility of our method for both risk and uncertainty. Our method serves well to study ambiguity because it can be used for risk and uncertainty in the same way, resulting in good measurements of ambiguity (the difference between uncertainty and risk). We can now study deviations from expected utility, including ambiguity attitudes, while almost entirely skipping the measurement of utility.

11. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

Appendix A. Bisection to Measure Indifference
The bisection method to find g to generate an indifference $(a{:}x_2, d{:}x_1, c{:}x_0) \sim x_{2_{g+a}}x_0$ as in Figure 1 proceeded as follows. We iteratively narrowed down so-called indifference intervals containing $g + a$, as follows. The first indifference interval $[b^1, u^1]$ was $[a, d + a]$, i.e., the interval of which the weighting-midpoint was to be found.[9] By stochastic

---

[9] The first indifference interval is, thus, $[0, 1]$ for $w^{-1}(4/8)$, $[0, w^{-1}(4/8))$ for $w^{-1}(2/8)$, $[w^{-1}(4/8), 1]$ for $w^{-1}(6/8)$, $[0, w^{-1}(2/8))$ for $w^{-1}(1/8)$, and $[w^{-1}(6/8), 1]$ for $w^{-1}(7/8)$.

Figure A.1   Bisection Method for Measuring $w^{-1}(0.5)$

Choice question	Indifference interval	Prospect L	Prospect R	Prospect chosen	Inference
1	$[b^1, u^1] = [0, 1]$	x_1	$0.50\ x_2$ / $0.50\ x_0$	L	$w^{-1}(0.5) > 0.50$
2	$[b^2, u^2] = [0.50, 1]$	x_1	$0.75\ x_2$ / $0.25\ x_0$	R	$w^{-1}(0.5) < 0.75$
3	$[b^3, u^3] = [0.50, 0.75)$	x_1	$0.63\ x_2$ / $0.37\ x_0$	R	$w^{-1}(0.5) < 0.63$
4	$[b^4, u^4] = [0.50, 0.63)$	x_1	$0.57\ x_2$ / $0.43\ x_0$	L	$w^{-1}(0.5) > 0.57$
5	$[b^5, u^5] = [0.57, 0.63)$	x_1	$0.60\ x_2$ / $0.40\ x_0$	L	$w^{-1}(0.5) > 0.60$
Conclusion	$[b^6, u^6] = [0.60, 0.63);$	x_1	$0.615\ x_2$ / $0.385\ x_0$	–	$w^{-1}(0.5) \approx 0.615$
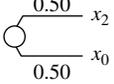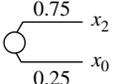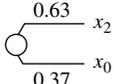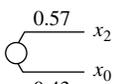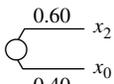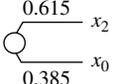
dominance, it indeed contains $g + a$. Each subject was first asked to make two practice choices between a particular prospect L and prospect $R = x_{2_{g^+ + a}} x_0$ (or $= x_{2_{g^- + a}} x_0$), where probability $g^+ + a$ ($g^- + a$) was set equal to the upper (lower) limit of the range of the first indifference interval of probability $g + a$ minus (plus) $1/100$. Then the iterative process started.

To construct the $(j + 1)$th indifference interval $[b^{j+1}, u^{j+1}]$ from the jth indifference interval $[b^j, u^j)$, we elicited whether the midpoint of $[b^j, u^j)$ was larger or smaller than $a + g$. To do so, we observed the choice between $(a{:}x_2, d{:}x_1, c{:}x_0)$ and $x_{2_{(b^j + u^j)/2}} x_0$. A right choice meant that the midpoint was larger than $g + a$ so that $g + a$ was contained in $[b^j, (b^j + u^j)/2)$, which was then defined as the $(j + 1)$th indifference interval $[b^{j+1}, u^{j+1})$. A left choice meant that the midpoint was smaller than $g + a$ so that $g + a$ was contained in $[(b^j + u^j)/2, u^j)$, which was then defined as the $(j + 1)$th indifference interval $[b^{j+1}, u^{j+1})$. We did five iteration steps like this, ending up with $[b^6, u^6)$, and took its midpoint as the elicited indifference probability $a + g$.[10]

---

[10] Because prospects yielded prizes depending on the result of a roll with two 10-sided dice, we only allowed values $j/100$ for probabilities. When a particular midpoint probability was not a value $j/100$, the computer took the closest value $j/100$ on the left of this value if the value was lower than half and on the right of this value if the value was higher than half. The order of elicitation was varied among subjects to reduce potential order effects. The order of elicitation was $w^{-1}(4/8)$, $w^{-1}(2/8)$, $w^{-1}(6/8)$, $w^{-1}(1/8)$, $w^{-1}(7/8)$

As an illustration, Figure A.1 replicates the bisection procedure followed to obtain the probability corresponding to the weight of 0.5. The particular pattern of answers depicted there, preferring the right prospect twice and the left prospect three times, was exhibited by six of our subjects. After the fifth iteration step, the midpoint of the last indifference interval was taken as the final indifference probability. Thus, individual indifference between the certain prospect (x_1) and the prospect $x_{2_{0.615}} x_0$ was inferred from the choices made by the six subjects whose choices are replicated in Figure A.1.

Appendix B. Experimental Instructions
[Instructions have been translated from Dutch into English.]

Welcome to this experiment. If you have a question while reading these instructions, please raise your hand. The experimenter will then come to your table to answer your question. This experiment will take about half an hour. We would like you to make a number of decisions during this experiment. Each time, you choose between what we call "prospects." Both prospects yield prizes depending on the roll of the two 10-sided dice similar to the ones that are on your table right now.

As you can see, one 10-sided die has the values 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, and the other has the values 00, 10, 20, 30, 40, 50, 60, 70, 80, and 90. If we code the sum of the

---

for some subjects, whereas for the others it was $w^{-1}(4/8)$, $w^{-1}(6/8)$, $w^{-1}(2/8)$, $w^{-1}(7/8)$, $w^{-1}(1/8)$.

roll "a 0 and a 00" as 100, then the sum of a roll with both 10-sided dice yields a random number from 1 up to 100.

The prospects from which you have to choose are called prospect L (left) and prospect R (right) and are presented in the following way:

Prospect L			Prospect R		
Roll	Probability	Prize	Roll	Probability	Prize
1 to 40	40%	€100	1 to 20	20%	€150
41 to 100	60%	€50	21 to 100	80%	€20

In the case depicted here, prospect L yields a prize of €100 if the sum of the roll with both 10-sided dice is 1 up to 40 and €50 if the sum of the roll is 41 up to 100. Similarly, prospect R yields a prize of €150 if the sum of a roll with both 10-sided dice is 1 up to 20, and prospect R yields a prize of €20 otherwise.

Both the prizes as well as the probabilities of yielding certain prizes can vary across decisions. We would like you to choose between prospect L and prospect R each time by clicking the corresponding button with the mouse.

You will receive €5 for your participation in this experiment. In addition, at the end of this experiment one subject will be selected at random, as follows. Each subject will randomly select a sealed envelope containing either a white or a blue card. Those with a white card receive €5 for their participation. For the subject with a blue card, one of the decisions will be randomly selected by rolling both 10-sided dice. Thereafter, the prize of the chosen prospect in the decision selected will be determined by rolling the two 10-sided dice again. The resulting prize, always more than €5, will be paid to the subject with the blue card.

There are no right or wrong answers in this experiment. The experiment exclusively concerns your own preferences. Those are what we are interested in. At every decision it is best for you, if it will be played for real, that you have chosen the prospect that you want most. If you select the envelope containing the blue card at the end of the experiment, the aforementioned decision may be selected at the end of the experiment. Then, the chosen prospect will be played out. Of course, you want that prospect to be your preferred prospect. If you have no further questions then you can now start with the experiment by clicking on the "Continue" button below.

References

Abdellaoui, M. 2000. Parameter-free elicitation of utility and probability weighting functions. *Management Sci.* **46** 1497–1512.

Abdellaoui, M. 2002. A genuine rank-dependent generalization of the von Neumann-Morgenstern expected utility theorem. *Econometrica* **70** 717–736.

Abdellaoui, M., B. R. Munier. 1999. How consistent are probability tradeoffs in individual preferences under risk? M. J. Machina, B. R. Munier, eds. *Beliefs, Interactions and Preferences in Decision-Making.* Kluwer Academic Publishers, Dordrecht, The Netherlands, 285–295.

Abdellaoui, M., P. P. Wakker. 2005. The likelihood method for decision under uncertainty. *Theory Decision* **58** 3–76.

Abdellaoui, M., F. Vossmann, M. Weber. 2005. Choice-based elicitation and decomposition of decision weights for gains and losses under uncertainty. *Management Sci.* **51** 1384–1399.

Abdellaoui, M., H. Bleichrodt, C. Paraschiv. 2007. Loss aversion under prospect theory: A parameter-free measurement. *Management Sci.* **53** 1659–1674.

Abdellaoui, M., A. Baillon, L. Placido, P. P. Wakker. 2011. The rich domain of uncertainty: Source functions and their experimental implementation. *Amer. Econom. Rev.* Forthcoming.

Ahn, D. S., S. Choi, D. Gale, S. Kariv. 2009. Estimating ambiguity aversion in a portfolio choice experiment. Working paper, Department of Economics, University of California, Los Angeles, Los Angeles.

Allais, M. 1953. Le comportement de l'homme rationnel devant le risque: Critique des postulats et axiomes de l'ecole Américaine. *Econometrica* **21** 503–546.

Andersen, S., G. W. Harrison, J. Fountain, E. E. Rutström. 2007. Elicitating beliefs: Theory and experiments. Working paper, Department of Economics, University of Central Florida, Orlando.

Armantier, O. 2006. Do wealth differences affect fairness considerations. *Internat. Econom. Rev.* **47** 391–429.

Baltussen, G., T. Post, M. J. van den Assem, P. P. Wakker. 2010. Random incentive systems in a dynamic choice experiment. Working paper, Department of Economics, Erasmus University, Rotterdam, The Netherlands.

Bardsley, N., P. G. Moffat. 2009. A meta-analysis of the preference reversal phenomenon. In preparation, School of Economics, University of East Anglia, Norwich, UK.

Barron, G., I. Erev. 2003. Small feedback-based decisions and their limited correspondence to description-based decisions. *J. Behav. Decision Making* **16** 215–233.

Bearden, J. N., T. S. Wallsten, C. R. Fox. 2007. Contrasting stochastic and support theory accounts of subadditivity. *J. Math. Psych.* **51** 229–241.

Birnbaum, M. H. 2008. Evaluation of the priority heuristic as a descriptive model of risky decision making: Comment on Brandstätter, Gigerenzer, and Hertwig (2006). *Psych. Rev.* **115** 253–260.

Blavatskyy, P. R. 2006. Error propagation in the elicitation of utility and probability weighting functions. *Theory Decision* **60** 315–334.

Blavatskyy, P., G. Pogrebna. 2011. Models of stochastic choice and decision theory: Why both are important for analyzing decisions. *J. Appl. Econometrics.* Forthcoming.

Bleichrodt, H. 2001. Probability weighting in choice under risk: An empirical test. *J. Risk Uncertainty* **23** 185–198.

Bleichrodt, H., J. L. Pinto. 2000. A parameter-free elicitation of the probability weighting function in medical decision analysis. *Management Sci.* **46** 1485–1496.

Bleichrodt, H., A. Cillo, E. Diecidue. 2010. A quantitative measurement of regret theory. *Management Sci.* **56** 161–175.

Bostic, R., R. J. Herrnstein, R. D. Luce. 1990. The effect on the preference-reversal phenomenon of using choice indifferences. *J. Econom. Behav. Organ.* **13** 193–212.

Curley, S. P., J. F. Yates. 1989. An empirical evaluation of descriptive models of ambiguity reactions in choice situations. *J. Math. Psych.* **33** 397–427.

Diecidue, E., P. P. Wakker, M. Zeelenberg. 2007. Eliciting decision weights by adapting de Finetti's betting-odds method to prospect theory. *J. Risk Uncertainty* **34** 179–199.

Donkers, A. C. D., B. Melenberg, A. H. O. van Soest. 2001. Estimating risk attitudes using lotteries: A large sample approach. *J. Risk Uncertainty* **22** 165–195.

Einhorn, H. J., R. M. Hogarth. 1985. Ambiguity and uncertainty in probabilistic inference. *Psych. Rev.* **92** 433–461.

Ellsberg, D. 1961. Risk, ambiguity and the Savage axioms. *Quart. J. Econom.* **75** 643–669.

Ellsberg, D. 2001. *Risk, Ambiguity and Decision.* Garland Publishers, New York. Original Ph.D. dissertation: Ellsberg, D. 1962. Risk, ambiguity and decision. Harvard University, Cambridge, MA.

Epstein, L. G., J. Zhang. 2001. Subjective probabilities on subjectively unambiguous events. *Econometrica* **69** 265–306.

Fatas, E., T. Neugebauer, P. Tamborero. 2007. How politicians make decisions: A political choice experiment. *J. Econom.* **92** 167–196.

Fishburn, P. C., W. Edwards. 1997. Discount-neutral utility models for denumerable time streams. *Theory Decision* **43** 139–166.

Fox, C. R., A. Tversky. 1998. A belief-based account of decision under uncertainty. *Management Sci.* **44** 879–895.

Fox, C. R., B. A. Rogers, A. Tversky. 1996. Options traders exhibit subadditive decision weights. *J. Risk Uncertainty* **13** 5–17.

Ghirardato, P., F. Maccheroni, M. Marinacci, M. Siniscalchi. 2003. A subjective spin on roulette wheels. *Econometrica* **71** 1897–1908.

Gilboa, I. 1987. Expected utility with purely subjective non-additive probabilities. *J. Math. Econom.* **16** 65–88.

Gilboa, I., D. Schmeidler. 1989. Maxmin expected utility with a non-unique prior. *J. Math. Econom.* **18** 141–153.

Goeree, J. K., C. A. Holt, T. R. Palfrey. 2002. Quantal response equilibrium and overbidding in private-value auctions. *J. Econom. Theory* **104** 247–272.

Goeree, J. K., C. A. Holt, T. R. Palfrey. 2003. Risk averse behavior in generalized matching pennies games. *Games Econom. Behav.* **45** 97–113.

Gonzalez, R., G. Wu. 1999. On the shape of the probability weighting function. *Cognitive Psych.* **38** 129–166.

Grabisch, M., I. Kojadinovic, P. Meyer. 2008. A review of methods for capacity identification in Choquet integral based multi-attribute utility theory: Applications of the Kappalab R package. *Eur. J. Oper. Res.* **186** 766–785.

Greenspan, A. 2004. Innovations and issues in monetary policy: The last fifteen years. *Amer. Econom. Rev., Papers Proc.* **94** 33–40.

Harless, D. W., C. F. Camerer. 1994. The predictive utility of generalized expected utility theories. *Econometrica* **62** 1251–1289.

Harrison, G. W. 1986. An experimental test for risk aversion. *Econom. Lett.* **21** 7–11.

Harrison, G. W. 2006. Hypothetical bias over uncertain outcomes. J. A. List, ed. *Using Experimental Methods in Environmental and Resource Economics*. Elgar, Northampton, UK, 41–69.

Harrison, G. W., M. I. Lau, E. E. Rutström. 2007. Estimating risk attitudes in Denmark: A field experiment. *Scandinavian J. Econom.* **109** 341–368.

Hartinger, A. 1999. Do generalized expected utility theories capture persisting properties of individual decision makers? *Acta Psych.* **102** 21–42.

Harvey, C. M. 1986. Value functions for infinite-period planning. *Management Sci.* **32** 1123–1139.

Henrich, J., R. Mcelreat. 2002. Are peasants risk-averse decision makers? *Current Anthropology* **43** 172–181.

Hey, J. D., C. Orme. 1994. Investigating generalizations of expected utility theory using experimental data. *Econometrica* **62** 1291–1326.

Hey, J. D., G. Lotito, A. Maffioletti. 2010. The descriptive and predictive adequacy of theories of decision making under uncertainty/ambiguity. *J. Risk Uncertainty* **41** 81–111.

Hogarth, R. M., H. J. Einhorn. 1990. Venture theory: A model of decision weights. *Management Sci.* **36** 780–803.

Humphrey, S. J., A. Verschoor. 2004. The probability weighting function: Experimental evidence from Uganda, India and Ethiopia. *Econom. Lett.* **84** 419–425.

Jullien, B., B. Salanié. 2000. Estimating preferences under risk: The case of racetrack bettors. *J. Political Econom.* **108** 503–530.

Kahn, B. E., R. K. Sarin. 1988. Modeling ambiguity in decisions under uncertainty. *J. Consumer Res.* **15** 265–272.

Kahneman, D., A. Tversky. 1979. Prospect theory: An analysis of decision under risk. *Econometrica* **47** 263–291.

Keynes, J. M. 1921. *A Treatise on Probability.* McMillan, London.

Kilka, M., M. Weber. 2001. What determines the shape of the probability weighting function under uncertainty? *Management Sci.* **47** 1712–1726.

Knight, F. H. 1921. *Risk, Uncertainty, and Profit.* Houghton Mifflin, New York.

Köbberling, V., P. P. Wakker. 2003. Preference foundations for non-expected utility: A generalized and simplified technique. *Math. Oper. Res.* **28** 395–423.

Kühberger, A., M. Schulte-Mecklenbeck, J. Perner. 1999. The effects of framing, reflection, probability, and payoff on risk preference in choice tasks. *Organ. Behav. Human Decision Processes* **78** 204–231.

Kunreuther, H. C., M. Pauly. 2003. Neglecting disaster: Why don't people insure against large losses? *J. Risk Uncertainty* **28** 5–21.

Li, L.-B., S.-H. He, S. Li, J.-H. Xu, L.-L. Rao. 2009. A closer look at the Russian roulette problem: A re-examination of the nonlinearity of the prospect theory's decision weight π. *Internat. J. Approximate Reasoning* **50** 515–520.

Loomes, G. 1991. Evidence of a new violation of the independence axiom. *J. Risk Uncertainty* **4** 92–109.

Loomes, G., P. G. Moffat, R. Sugden. 2002. A microeconometric test of alternative stochastic theories of risky choice. *J. Risk Uncertainty* **24** 103–130.

Luce, R. D. 1996. When four distinct ways to measure utility are the same. *J. Math. Psych.* **40** 297–317.

MacCrimmon, K. R., S. Larsson. 1979. Utility theory: Axioms versus "paradoxes." M. Allais, O. Hagen, eds. *Expected Utility Hypotheses and the Allais Paradox*. Reidel, Dordrecht, The Netherlands, 333–409.

Mangelsdorff, L., M. Weber. 1994. Testing Choquet expected utility. *J. Econom. Behav. Organ.* **25** 437–457.

Mosteller, F., P. Nogee. 1951. An experimental measurement of utility. *J. Political Econom.* **59** 371–404.

Noussair, C., S. Robbin, B. Ruffieux. 2004. Revealing consumers' willingness-to-pay: A comparison of the BDM mechanism and the Vickrey auction. *J. Econom. Psych.* **25** 725–741.

Prelec, D. 1998. The probability weighting function. *Econometrica* **66** 497–527.

Qiu, J., E.-M. Steiger. 2011. Understanding the two components of risk attitudes: An experimental analysis. *Management Sci.* **57** 193–199.

Quiggin, J. 1981. Risk perception and risk aversion among Australian farmers. *Australian J. Agricultural Econom.* **25** 160–169.

Rabin, M. 2000. Risk aversion and expected-utility theory: A calibration theorem. *Econometrica* **68** 1281–1292.

Savage, L. J. 1954. *The Foundations of Statistics*. Wiley, New York.

Schmeidler, D. 1989. Subjective probability and expected utility without additivity. *Econometrica* **57** 571–587.

Stott, H. P. 2006. Cumulative prospect theory's functional menagerie. *J. Risk Uncertainty* **32** 101–130.

Tversky, A., C. R. Fox. 1995. Weighing risk and uncertainty. *Psych. Rev.* **102** 269–283.

Tversky, A., D. Kahneman. 1992 Advances in prospect theory: Cumulative representation of uncertainty. *J. Risk Uncertainty* **5** 297–323.

Vind, K. 1991. Independent preferences. *J. Math. Econom.* **20** 119–135.

Vind, K. 2003. *Independence, Additivity, Uncertainty*. With contributions by B. Grodal. Springer, Berlin.

Wakker, P. P. 2008. Uncertainty. L. Blume, S. N. Durlauf, eds. *The New Palgrave: A Dictionary of Economics*. MacMillan Press, London, 6780–6791.

Wakker, P. P. 2010. *Prospect Theory for Risk and Ambiguity*. Cambridge University Press, Cambridge, UK.

Wakker, P. P., D. Deneffe. 1996. Eliciting von Neumann-Morgenstern utilities when probabilities are distorted or unknown. *Management Sci.* **42** 1131–1150.

Weber, E. U. 1994. From subjective probabilities to decision weights: The effects of asymmetric loss functions on the evaluation of uncertain outcomes and events. *Psych. Bull.* **115** 228–242.

Wu, G., R. Gonzalez. 1999. Nonlinear decision weights in choice under uncertainty. *Management Sci.* **45** 74–85.

Wu, G., J. Zhang, M. Abdellaoui. 2005. Testing prospect theories using probability tradeoff consistency. *J. Risk Uncertainty* **30** 107–131.