MANAGEMENT SCIENCE
Vol. 56, No. 11, November 2010, pp. 2015–2030
ISSN 0025-1909 | EISSN 1526-5501 | 10 | 5611 | 2015
DOI 10.1287/mnsc.1100.1219

Time-Tradeoff Sequences for Analyzing Discounting and Time Inconsistency

Arthur E. Attema
Department of Health Policy and Management, Erasmus University, 3000 DR Rotterdam, The Netherlands,
attema@bmg.eur.nl

Han Bleichrodt, Kirsten I. M. Rohde, Peter P. Wakker
Erasmus School of Economics, Erasmus University, 3000 DR Rotterdam, The Netherlands
{bleichrodt@ese.eur.nl, rohde@ese.eur.nl, wakker@ese.eur.nl}

This paper introduces time-tradeoff (TTO) sequences as a general tool to analyze intertemporal choice. We give several applications. For empirical purposes, we can measure discount functions without requiring any measurement of or assumption about utility. We can quantitatively measure time inconsistencies and simplify their qualitative tests. TTO sequences can be administered and analyzed very easily, using only pencil and paper. For theoretical purposes, we use TTO sequences to axiomatize (quasi-)hyperbolic discount functions. We demonstrate the feasibility of measuring TTO sequences in an experiment, in which we tested the axiomatizations. Our findings suggest rejections of several currently popular discount functions and call for the development of new ones. It is especially desirable that such discount functions can accommodate increasing impatience.

Key words: time inconsistency; time preference; stationarity; increasing impatience; hyperbolic discounting
History: Received September 2, 2009; accepted May 16, 2010, by Teck-Hua Ho, decision analysis. Published online in *Articles in Advance* October 11, 2010.

1. Introduction

Time inconsistencies occur if agents deviate from choices they preferred a priori when offered the chance to revise at the moment of actual choice. Strotz (1956) was the first to analyze the implications of time inconsistencies. It has since been well understood that time inconsistencies lie at the heart of many anomalies (Laibson 1997, O'Donoghue and Rabin 2001). There is a close connection between time inconsistencies and violations of constant discounting. The latter violations have often been found and have led to the development of hyperbolic discounting (Frederick et al. 2002).

One difficulty with the general analysis of intertemporal choice is that two different subjective factors, time discounting and outcome utility, each play a role, and it may not be easy to disentangle them. Hence, most analyses of intertemporal discounting have simply assumed linear utility (reviewed by Takeuchi 2010, Table 1). Diminishing marginal utility will distort the findings of such analyses.

A second difficulty is that there are many empirical violations of the discounted utility model, the most widely used model today. These violations distort the results of the analyses based on this model. The most questionable assumption of discounted utility, extensively violated empirically, is intertemporal separability (Dolan and Kahneman 2008, p. 228; Kapteyn and Teppa 2003, p. C151; Prelec and Loewenstein 1991; Wathieu 1997). Discounted utility is still the most frequently used model because no tractable alternatives are available. We will also assume discounted utility, but will avoid or minimize the distortions because of its empirical violations.

We introduce time-tradeoff (TTO) sequences as a general tool to study intertemporal choice. In particular, TTO sequences resolve the two aforementioned difficulties. As regards the first difficulty, the interactions of time discounting and utility do not affect TTO sequences. With reference to the second difficulty, the violations of intertemporal separability do not affect TTO sequences either because the latter only concern the receipt of single outcomes. TTO sequences facilitate and generalize both theoretical and empirical studies. We next discuss some applications.

As we will show, TTO sequences allow us to measure the discount function up to its power without any interaction with utility. This means, as explained later, that we can estimate departures from constant discounting but not the discount rate itself. To measure the power and, thus, the complete discount function including the discount rate, we either need one extra data point regarding utility and no time separability, or one extra data point regarding time separability and no utility. The latter approach is the first one available in the literature that measures the discount function in an entirely utility-free manner. In §12, we discuss alternative approaches in the literature to resolve the problem of unknown utility. These always use separate measurements of utility and in this sense are not utility free.

We next address some other applications of TTO sequences for which no extra data points are required. The first is related to time inconsistency. Under the common assumption of homogenous time, hyperbolic discount functions accommodate time inconsistency, which leads to arbitrage opportunities. The degree to which time inconsistency can arise, and is empirically or normatively appropriate, is of central interest in the literature today. Epper et al. (2009, p. 2) wrote: "Recent research has not focused on the *magnitude* of observed discount rates, but rather on their hyperbolicity" [italics in original]. Prelec (2004) presented an important advance by introducing a theoretical measure of time inconsistency. One agent is more prone to time inconsistency and arbitrage than another if and only if the Pratt–Arrow measure of the logarithm of the discount function of the former always exceeds that of the latter. This result is the analog for intertemporal choice of the famous risk aversion measure of Pratt and Arrow for decision under risk.

Unfortunately, Prelec's measure seems to be complex to observe or analyze. In decision under risk with expected utility, where the Pratt–Arrow utility measure was introduced, utility is the only subjective factor and can readily be measured and analyzed. Risk premia provide a simple empirical criterion to test the Pratt–Arrow measure. In intertemporal choice, the discount function interacts with utility in seemingly inextricable manners. There is no analog to risk premia. Furthermore, even if we do succeed in measuring the discount function, logarithms and derivatives still need to be taken to determine Prelec's measure. Hence, this measure may seem to be difficult to use.

Surprisingly, TTO sequences immediately give Prelec's measure of time inconsistency. We can straightforwardly graph this measure, bypassing the measurements and calculations just described. In particular, no measurement or assumption regarding utility is needed. For example, we can immediately observe which agents are most prone to time inconsistencies, as we show in a representation theorem and in an experiment.

In a theoretical application of TTO sequences, we will give preference axiomatizations for a number of qualitative properties of discounting and for the currently popular discount models. We test these axiomatizations in an experiment. Most empirical studies have rejected constant discounting, but have not tested for possible failures of the alternative discount functions or for better fits of such functions. Exceptions are Keller and Strazzera (2002) and van der Pol and Cairns (2002), who both assumed linear utility. Other exceptions, based on utility measurements, are discussed in §12. We can measure and critically test the alternatives more efficiently by using TTO sequences. We also indicate an application to the models of Epper et al. (2009) and of Halevy (2008), where risk underlies intertemporal choice. TTO curves then immediately reveal pessimism and optimism (convex and concave probability weighting) of the underlying risk attitudes. By explicitly introducing risk, we can also measure the discount function of Baucells and Heukamp's (2009) PTT model that incorporates interactions between risk and time.

The questions used to elicit TTO sequences are easy to comprehend for subjects, fostering reliable data. In an experiment, we demonstrate the feasibility by measuring TTO sequences of 55 subjects. Our experimental findings lead to a number of suggestions for new models of intertemporal choice, in particular regarding discount functions that allow for increasing impatience. We also find violations of some popular hyperbolic discount functions.

This paper proceeds as follows. Section 2 gives elementary definitions of discounted utility. We introduce TTO sequences in §3. Section 4 examines how these sequences can be used for qualitative tests of impatience. This is followed by a section that considers TTO sequences and quantitative measurements of the degree of time inconsistency. Axiomatizations and tests of popular parametric families of discounting are discussed in §6. Section 7 shows how TTO sequences plus some minimal extra information can be used to measure the discount function. Sections 8 to 10 present an experiment. We outline our method, describe some results that can be inferred using only pencil and paper, and present detailed statistical tests. Sections 11 and 12 contain discussions, and §13 concludes.

2. Discounted Utility: Elementary Definitions

An *outcome stream* $(t_1\colon x_1, \ldots, t_m\colon x_m)$ yields *outcome* x_j at *time point* t_j for $j = 1, \ldots, m$ and nothing at other time points. We assume $t_j \geq 0$ for all j. Most of the results in this paper hold for general outcomes. The outcome set may for instance be a finite set of qualitative health states. However, for simplicity of presentation, we assume that outcomes are monetary and nonnegative, with the neutral outcome "nothing" equated with the 0 outcome. Time point $t = 0$ corresponds with the present. Throughout this paper, we assume *discounted utility* (DU), which here refers to general, possibly nonconstant, discounting. Outcome streams are evaluated by

$$\mathrm{DU}(t_1\colon x_1, \ldots, t_m\colon x_m) = \sum_{i=1}^{m} \varphi(t_i)U(x_i), \tag{1}$$

with φ the *discount function* and U the (instant) *utility function*. We assume (a) $\varphi(t) > 0$ for all t, (b) φ is strictly decreasing (*impatience*) and continuous, (c) $U(0) = 0$, and (d) U is strictly increasing and continuous. The length m of an outcome stream can be any natural number. For two outcome streams x and y we write $x \succeq y$ if $\mathrm{DU}(x) \geq \mathrm{DU}(y)$ ((*weak*) *preference*), $x \succ y$ if $\mathrm{DU}(x) > \mathrm{DU}(y)$ (*strict preference*), $x \sim y$ if $\mathrm{DU}(x) = \mathrm{DU}(y)$ (*indifference*), $x \preceq y$ if $\mathrm{DU}(x) \leq \mathrm{DU}(y)$, and $x \prec y$ if $\mathrm{DU}(x) < \mathrm{DU}(y)$.

As is well known, the functions φ and U in Equation (1) are *ratio scales*:

In Equation (1), φ can be replaced by φ/λ and U by U/λ' for any $\lambda > 0$ and $\lambda' > 0$ (2)

without affecting preference. No other replacement is possible. In the literature, a normalization $\varphi(0) = 1$ (taking $\lambda = \varphi(0)$ in Equation (2)) is often assumed, but it is more convenient for this paper not to commit to such a scaling.

The summation in Equation (1) implies intertemporal separability, the most questionable assumption of discounted utility. To depend on this assumption as little as possible, most of this paper will focus on outcome streams (t: x) with only one nonzero outcome, called *timed outcomes*. Fishburn and Rubinstein (1982) axiomatized the restriction of discounted utility to timed outcomes, showing which preference conditions regarding separability of time and outcomes are still required then.

Decreasing impatience holds if an indifference (s: β) $\sim$ (t: γ), with $s < t$ and $\beta < \gamma$, implies ($s + \varepsilon$: β) $\preceq$ ($t + \varepsilon$: γ) for all $\varepsilon > 0$.[1] Then a common delay (ε) increases the willingness to wait for the good outcome (γ). *Increasing impatience* holds if the weak preference in the implication is reversed. *Constant impatience*, or *stationarity*, holds if the weak preference is an indifference.[2]

3. Time-Tradeoff Sequences Defined

This section defines TTO sequences, and the TTO curves that can be derived, without yet giving motivations or applications. These will be given in following sections. A *time-tradeoff* (*TTO*) *sequence* is a sequence $t_0, \ldots, t_n$ of time points such that there exist two outcomes $\beta < \gamma$ with

$$\begin{aligned}
(t_0\colon \beta) &\sim (t_1\colon \gamma),\\
(t_1\colon \beta) &\sim (t_2\colon \gamma),\\
&\vdots\\
(t_{n-1}\colon \beta) &\sim (t_n\colon \gamma);
\end{aligned} \tag{3}$$

that is, each delay between two consecutive time points exactly offsets the same outcome improvement. Such a delay, $d_i = t_i - t_{i-1}$, is called the *willingness to wait* (*WTW*). Stationarity implies that the WTW is constant, so that the points $t_0, \ldots, t_n$ are equally spaced in time units. Increasing and decreasing impatience correspond with decreasing and increasing WTW, respectively. For a TTO sequence, we have

$$\frac{\varphi(t_0)}{\varphi(t_1)} = \frac{\varphi(t_1)}{\varphi(t_2)} = \cdots = \frac{\varphi(t_{n-1})}{\varphi(t_n)} = \frac{U(\gamma)}{U(\beta)}.$$

Hence,

$$\ln(\varphi(t_0)) - \ln(\varphi(t_1)) = \ln(\varphi(t_1)) - \ln(\varphi(t_2)) = \cdots = \ln(\varphi(t_{n-1})) - \ln(\varphi(t_n)). \tag{4}$$

A TTO sequence, even when not equally spaced in time units, is equally spaced in $\ln(\varphi)$ units. The outcomes β and γ serve as a gauge to peg out the time sequence, providing the same gauge at every new elicitation. This may not only hold algebraically, but also psychologically in the minds of decision makers, which enhances the validity of empirical measurements. TTO sequences adapt the standard sequences of Krantz et al. (1971, §1.2) to intertemporal choice.

A TTO sequence is also equally spaced in the units of any renormalization of $\ln(\varphi)$ that results from subtracting a constant and dividing by a positive constant:

$$\frac{\ln(\varphi(t)) - l}{r}. \tag{5}$$

We consider a convenient renormalization, being $\ln(\varphi)$ normalized at t_0 and t_n:

$$\tau_{t_0,\, t_n}(t) = \frac{\ln(\varphi(t)) - \ln(\varphi(t_n))}{\ln(\varphi(t_0)) - \ln(\varphi(t_n))}. \tag{6}$$

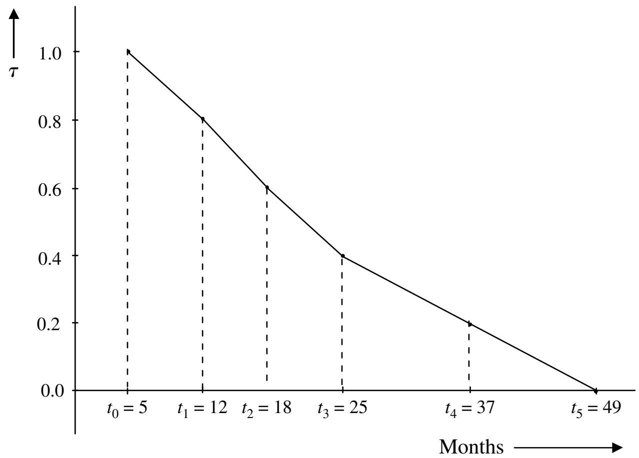
We call this function the *TTO curve* of the TTO sequence. Because $\tau_{t_0,\, t_n}$ is 1 at t_0 and 0 at t_n, with n equally big steps $\tau_{t_0,\, t_n}(t_{j-1}) - \tau_{t_0,\, t_n}(t_j)$ of size $1/n$ in between, this TTO curve is $1 - j/n$ at each point t_j. Figure 1 depicts some values of $\tau_{t_0,\, t_n}$ observed in an experiment reported later, reflecting the following indifferences of subject 38:

(05 months: €700) $\sim$ (12 months: €900),
(12 months: €700) $\sim$ (18 months: €900),
(18 months: €700) $\sim$ (25 months: €900),
(25 months: €700) $\sim$ (37 months: €900),
(37 months: €700) $\sim$ (49 months: €900);

[1] The letter s abbreviates "soon" or "short," γ abbreviates "good," and β abbreviates "bad."

[2] This paper will use stationarity only for timed outcomes, which is why we define it only for those.

Figure 1 TTO Curve τ_{t_0,t_5} of Subject 38



that is, $n = 5$, $t_0 = 5$, $t_1 = 12$, $t_2 = 18$, $t_3 = 25$, $t_4 = 37$, and $t_5 = 49$.

We can obtain a more refined TTO curve by taking γ and β closer to each other, so that the step size of the TTO sequence becomes smaller. We can cover a larger interval than $[t_0, t_n]$ by setting t_0 smaller (e.g., $t_0 = 0$, as done in some questions in the experiment) and by using more steps so that we attain a higher value t_n. Thus, using TTO sequences, we can measure normalizations of $\ln(\varphi)$ to any desired degree of precision and on any desired domain. In general, we use the term TTO curve for any renormalization of Equation (6) at points that may be different than t_0 or t_n, i.e., for any function in Equation (5). Using interpolation when required, we will assume that we have obtained the TTO curve τ to a sufficient degree of precision at all time points t within our domain of interest.

4. TTO Sequences and Qualitative Tests of Time Inconsistency

Qualitative tests of deviations from stationarity have been widely reported in the literature. They need no assumptions about utility and in this sense can be utility free (Takeuchi 2010). This section shows how TTO sequences can be used to facilitate such qualitative tests. Throughout this section, we use no information other than provided by the TTO curve.

In general, violations of stationarity (constant impatience) need not imply time inconsistency (Dasgupta and Maskin 2005, §I; Halevy 2009; Harvey 1995, p. 389; Thaler 1981). However, under an assumption of homogeneous time, entailing that we can use stopwatch time (we can always reset the clock at 0 at the time of choosing between outcome streams), the two conditions become equivalent. This homogeneity of time will be assumed throughout this paper. It is implicitly assumed in many papers, and is, for instance, standard practice in the literature on growth models. Violations of stationarity then do imply time inconsistency and vulnerability to arbitrage. We will return to this point in §5.

A TTO sequence readily identifies constant, increasing, or decreasing impatience through constant, decreasing, or increasing WTW (i.e., $t_{j+1} - t_j$). Constant, decreasing, or increasing WTW implies a linear, concave, or convex TTO curve, and this is not affected by normalizations. Hence, τ is the same as $\ln(\varphi)$ in this regard. We summarize our observations.

Observation 1. Stationarity implies linearity of the TTO curve and of $\ln(\varphi)$. Decreasing impatience implies convexity of the TTO curve and of $\ln(\varphi)$. Increasing impatience implies concavity of the TTO curve and of $\ln(\varphi)$. ◇

It can be seen that the implications in the observation can be reversed if we measure TTO curves in a sufficiently refined way and on as large a domain as we want.

Constant discounting holds if $\varphi(t) = \delta^t$ for a *discount factor* δ with $0 < \delta < 1$. Then delaying all nonzero outcomes in some outcome streams by a period ε implies that all discounted utilities are multiplied by the same factor δ^ε, so that the ordering of the outcome streams is not affected, and constant impatience indeed holds. It is well known that the reversed implication also holds under common assumptions, that is, constant impatience implies constant discounting (Koopmans 1960).

Nonconstant discounting may be caused by a nonlinear perception of time. Constant discounting is exponential in time units. In general, discounting is exponential in $\ln(\varphi)$ units. We can say that a constant discounter perceives time as it actually is ($\ln(\varphi)$ linear), whereas a nonconstant discounter perceives time according to a nonlinear $\ln(\varphi)$ (or τ) and then constantly discounts in terms of this perceived time. TTO curves τ directly measure this nonlinear time perception, with $\varphi(t) = e^{r\tau(t)}$ the discount function. In other words, TTO curves provide the time units in terms of which discounting is constant.

5. TTO Sequences and a Quantitative Measure of Time Inconsistency

This section shows how TTO sequences can be used to obtain a quantitative measure of time inconsistency, with a theorem justifying the procedure. This measure will show to what extent people are prone to arbitrage because of their time inconsistency. Again, we use no information other than provided by TTO curves.

5.1. Prelec's (2004) Domain of Arbitrage

Consider the following two indifferences, similar to Equation (3):

$$(s\colon \beta) \sim (t\colon \gamma) \quad \text{and} \quad (s+\sigma\colon \beta) \sim (t+\sigma+\varepsilon\colon \gamma)$$
$$\text{for } s < t,\ \beta < \gamma, \text{ and } \sigma > 0. \qquad (7)$$

For the special case of $s+\sigma=t$, the indifferences provide a TTO sequence $t_0=s$, $t_1=t=s+\sigma$, and $t_2=s+2\sigma+\varepsilon$. We have $\varepsilon>0$ under decreasing impatience, $\varepsilon=0$ under constant impatience, and $\varepsilon<0$ under increasing impatience. Thus, ε can be taken as an index of deviation from stationarity. For $\varepsilon>0$, we indeed obtain the following violations of stationarity:

$$(s\colon \beta)\succeq(t'\colon \gamma) \quad \text{and} \quad (s+\sigma\colon \beta)\preceq(t'+\sigma\colon \gamma) \quad \text{with at least one preference strict} \tag{8}$$

for all $t'\in[t,t+\varepsilon]$ and for no other t'. The interval $[t,t+\varepsilon]$ thus designates the domain of time inconsistency and arbitrage: At time zero, the person, when endowed with $(s+\sigma\colon\beta)$, is willing to exchange it for $(t'+\sigma\colon\gamma)$. When asked to reconsider at time point σ, the person (going by stopwatch time) now perceives the options as $(s\colon\beta)$ and $(t'\colon\gamma)$, and is willing to go back to the β option. The person is willing to pay small amounts for the two exchanges (small enough not to affect preference otherwise, and at least one positive). The person then ends up with the original endowment less some money, which entails arbitrage.[3]

For $\varepsilon<0$ in Equation (7), as is typical of increasing impatience, we have

$$(s\colon \beta)\preceq(t'\colon \gamma) \quad \text{and} \quad (s+\sigma\colon \beta)\succeq(t'+\sigma\colon \gamma) \quad \text{with at least one preference strict} \tag{9}$$

for all $t'\in[t+\varepsilon,t]$ and for no other t'. Now $[t+\varepsilon,t]$ is the domain of arbitrage.

5.2. Prelec's Comparisons of Domains of Arbitrage

Consider another preference relation $\succeq^*$, satisfying the assumptions of the preceding sections as does $\succeq$, with corresponding φ^*, U^*, τ^*.

Definition 1. The preference relation $\succeq^*$ exhibits *more decreasing impatience* than $\succeq$ if the equivalences for $\succeq$ in Equation (7) plus $(s\colon\beta^*)\sim^*(t\colon\gamma^*)$ imply $(s+\sigma\colon\beta^*)\preceq^*(t+\sigma+\varepsilon\colon\gamma^*)$. ◇

Prelec (2004) gave an equivalent definition. Under decreasing impatience for $\succeq$ and $\succeq^*$, Equation (7) and the condition of Definition 1 imply, for

$$(s\colon\beta^*)\sim^*(t\colon\gamma^*) \quad \text{and} \quad (s+\sigma\colon\beta^*)\sim^*(t+\sigma+\varepsilon^*\colon\gamma^*), \tag{10}$$

that either ε^* exceeds $\varepsilon>0$, or (even stronger) that such an ε^* does not exist. In the first case, the domain $[t,t+\varepsilon^*]$ of arbitrage for $\succeq^*$ exceeds the corresponding domain $[t,t+\varepsilon]$ for $\succeq$. In the second case, the domain for arbitrage for $\succeq^*$ is in fact $[t,t+\infty)$, as is readily verified, which obviously exceeds the corresponding domain $[t,t+\varepsilon]$ for $\succeq$.

Because, as discussed later, there is also interest in increasing impatience, we extend Definition 1.

Definition 2. The preference relation $\succeq^*$ exhibits *more increasing impatience* than $\succeq$ if the equivalences in Equation (7) plus $(s\colon\beta^*)\sim^*(t\colon\gamma^*)$ imply $(s+\sigma\colon\beta^*)\succeq^*(t+\sigma+\varepsilon\colon\gamma^*)$. ◇

Under increasing impatience, the arbitrage domain $[t+\varepsilon,t]$ is larger as the increase in impatience is larger.

5.3. TTO Curves to Identify Proneness to Arbitrage

As usual, τ^* is *more convex* than τ if there exists a convex transformation f such that $\tau^*(t)=f(\tau(t))$ for all t, which holds if and only if $-\tau^{*\prime\prime}/\tau^{*\prime}\geq-\tau''/\tau'$ everywhere on their domain. Remember that, whereas the Pratt–Arrow index is an index of concavity for increasing functions, for decreasing functions such as τ and τ^* it is an index of convexity rather than concavity. Analogously, τ^* is more concave than τ if there exists a concave transformation f such that $\tau^*(t)=f(\tau(t))$ for all t, which holds if and only if $\tau^{*\prime\prime}/\tau^{*\prime}\geq\tau''/\tau'$ everywhere on their domain.

The aforementioned comparative convexity and concavity definitions are not affected by a change in unit or level of τ. Hence, τ has the same degree of convexity as $\ln(\varphi)$:

$$-\frac{\tau''}{\tau'}=-\frac{\ln(\varphi)''}{\ln(\varphi)'}. \tag{11}$$

The following theorem adapts Prelec's (2004) Proposition 1, stating conditions in terms of TTO curves rather than of $\ln(\varphi)$. We also add results on increasing impatience.

Theorem 1. *Assume that $\succeq$ and $\succeq^*$ satisfy discounted utility (Equation (1)), with a TTO curve τ for $\succeq$ and a TTO curve τ^* for $\succeq^*$.*

(i) *$\succeq^*$ exhibits more decreasing impatience than $\succeq$ if and only if τ^* is more convex than τ.*

(ii) *$\succeq^*$ exhibits more increasing impatience than $\succeq$ if and only if τ^* is more concave than τ.* ◇

Theorem 1 demonstrates formally that the degree of convexity of a TTO curve determines the degree of decreasing impatience and, thus, the domain for arbitrage and the proneness to anomalies, in the sense of Prelec (2004). From a mathematical perspective, the move from Prelec's Proposition 1 to Theorem 1(i) is elementary, replacing the convexity of $\ln(\varphi)$ by the equivalent convexity of τ. From an empirical perspective, the move is crucial though, because τ is directly observable, whereas $\ln(\varphi)$ is not.

[3] Laibson (1997), O'Donoghue and Rabin (1999), Prelec (2004), Strotz (1956), and numerous others derived various similar choice anomalies from Equation (8). For example, a sophisticated person who is informed about the arbitrage possibility beforehand may avoid it, but then becomes vulnerable to commitments to dominated options, caused by lack of future self-control.

6. TTO Sequences to Axiomatize and Test Families of Discount Functions

Observation 1 demonstrated how TTO sequences can be used to test whether constant discounting holds. In this section, we investigate the theoretical axiomatization of alternative discount functions. The axioms will then be tested in an experiment. A popular function to capture decreasing impatience is the *quasi-hyperbolic discount* function (Laibson 1997). It is given by

$$\varphi(t)=1 \quad \text{if } t=0 \quad \text{and} \quad \varphi(t)=\beta\delta^t \quad \text{if } t>0, \tag{12}$$

for a constant $\beta \leq 1$ with, again, $0<\delta<1$ (allowing for discontinuity at $t=0$). Under quasi-hyperbolic discounting we have decreasing impatience at time point zero, and constant impatience thereafter. The following observation readily follows from substitution.

Observation 2. Quasi-hyperbolic discounting holds if and only if WTW $(t_{j+1}-t_j)$ for TTO sequences is constant with the only exception that WTW may be smaller than elsewhere at $t_j=0$. ◇

A more flexible model that captures decreasing impatience not only for the present but also for future time points is *generalized hyperbolic discounting* (Loewenstein and Prelec 1992). It is defined by

$$\varphi(t)=(1+ht)^{-r/h}, \tag{13}$$

with $h \geq 0$ and $r>0$. Here, h can be interpreted as an index of decreasing impatience. Stationarity with constant discounting e^{-rt} is the limiting case for $h \to 0$ (Loewenstein and Prelec 1992). This family incorporates several popular hyperbolic families other than quasi-hyperbolic discounting. Mazur (1987) and Harvey (1995) considered proportional discounting $(h=r)$, and Harvey (1986, Equation (7)) considered the special case $h=1$.

Rohde (2010) proposed the hyperbolic factor for analyzing generalized hyperbolic discounting. For a TTO sequence $t_0,\ldots,t_n$, the *hyperbolic factor* is defined as

$$\textit{hyperbolic factor}(i,j)=\frac{(t_j-t_i)-(t_{j-1}-t_{i-1})}{t_i(t_{j-1}-t_{i-1})-t_{i-1}(t_j-t_i)} \tag{14}$$

for all $j>i$. For one TTO sequence as in Equation (3) with $n=5$, $15(=5+4+3+2+1)$ hyperbolic factors can be calculated. The following result adapts Rohde's (2010) Theorems 8–10 to TTO sequences. Thus, we can readily calculate the hyperbolic factor for TTO sequences and can then test popular models of discounting.

Observation 3. For generalized hyperbolic discounting, $\varphi(t)=(1+ht)^{-r/h}$, the denominator in Equation (14) is always positive, and the hyperbolic factor is always equal to h, independent of i, j, and the TTO sequence considered. For constant discounting (stationarity), the hyperbolic factor is always zero. For quasi-hyperbolic discounting, the hyperbolic factor is nonnegative if $t_{i-1}=0$, and it is zero if $t_{i-1}>0$. ◇

In many respects, intertemporal attitudes and risk attitudes are substitutes for each other, and several papers have investigated their interactions (Prelec and Loewenstein 1991). Halevy (2008) introduced a theoretical model for risky intertemporal choice where time inconsistencies are generated by probability weighting. Epper et al. (2009) provided empirical evidence for such a model. TTO curves then measure the curvature of probability weighting, with the linearity, convexity, and concavity of the TTO curve corresponding with the linearity, convexity, and concavity of probability weighting, respectively. There is much empirical evidence that probability weighting is not always convex (Abdellaoui 2000, Bleichrodt and Pinto 2000, Camerer and Ho 1994), which will contribute to increasing impatience.

TTO sequences can be used to test the general discounted utility model (Equation (1)). This model requires that the TTO *curve* is independent of the outcomes β and γ.[4] Thus, outcome dependence of the TTO curve falsifies discounted utility. Then the model of Noor (2009), with time-dependent utility, may hold. Or Baucells and Heukamp's (2009) PTT model holds with outcome-dependent discounting, and with interactions between intertemporal and risky choice for timed outcomes. This was empirically tested by Baucells and Heukamp (2010). Using our notation, (p,t,x) denotes the receipt of outcome x at time t with probability p, with PTT value

$$w(pe^{-r(x)t})U(x).$$

Here U denotes utility as in our model, w is prospect theory's probability weighting function, and r is an outcome-dependent fade-rate function governing the exchange between risk and time. The PTT model can explain several interactions between outcomes, time, and risk found in the literature. TTO curves can be used to obtain equalities involving the w and r functions, where the U functions drop from these equations, and can thus facilitate the study of the PTT model.

7. Using TTO Sequences to Measure the Discount Function

This section briefly presents two ways in which TTO sequences can be used not only to measure time

[4] TTO sequences obviously depend on those outcomes, but the resulting curve should not. For example, if we measure one TTO sequence $t_0,\ldots,t_n$, and another TTO sequence $s_0,\ldots,s_m$, and $s_0=t_0$ and $s_1=t_3$, then $s_2=t_6$ should hold.

Figure 2 **Discount Function Using Figure 1 and Equation (16)**

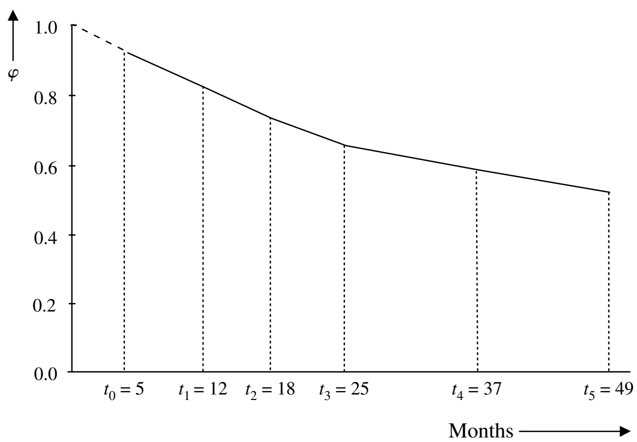
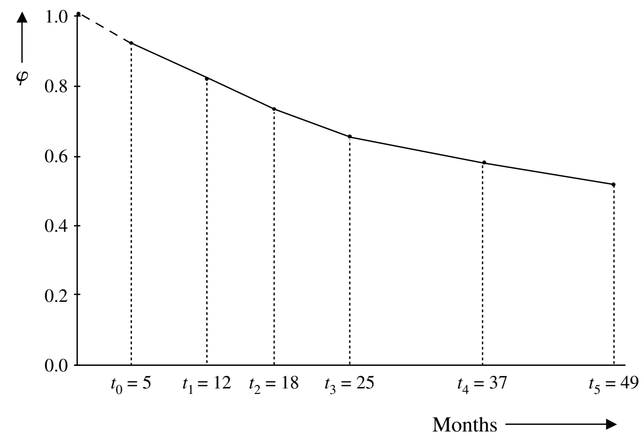


Figure 3 **Discount Function Using Figure 1 and Equation (18)**



inconsistencies, but also to measure the entire discount function. It can be skipped without loss of continuity. If we observe a TTO curve τ, we know that $\ln(\varphi) = r\tau + l$ for some l and $r > 0$, and that $\varphi = e^l e^{r\tau}$, but we do not know l and r. Not knowing l is no problem because φ is a ratio scale (Equation (2)) and l does not affect preference, so that it can be chosen arbitrarily. The parameter r, however, does affect preference between general outcome streams, and to obtain it we have to perform additional measurements. The unknown r (affecting the *absolute level* of discounting) cannot be inferred from preferences between timed outcomes, though, because for these $\varphi(t)^r U(x)^r$ generates the same preferences for every $r > 0$. Thus, timed outcomes and TTO curves (affecting *changes* in discounting as we will see later) give us $\ln(\varphi)$ and φ up to one empirically relevant parameter, the power r of the discount function φ. We now present two ways to obtain further data that, together with TTO curves, identify the power r of the discount function. Thus, we can obtain the entire discount function φ.

Method 1 (Measuring Discounting Using One Extra Utility Data Point). Assume that we have measured a TTO curve τ together with one data point of utility. For the latter say that, after some normalization $U(\beta) = 1$, we know one more value $U(\gamma)$, for some $\gamma > \beta > 0$. Then we find time points s and t to generate an indifference $(s\colon \beta) \sim (t\colon \gamma)$. Substituting DU (Equation (1)), setting $\lambda = 0$ in Equation (5), and taking logarithms implies

$$\varphi(\cdot) = e^{r\tau(\cdot)} \quad \text{with } r = \frac{\ln(U(\gamma)/U(\beta))}{\tau(s) - \tau(t)}. \tag{15}$$

This way we have identified r and, consequently, the complete discount function φ.

Figure 2 displays the discount function of subject 38 if we have the extra estimation

$$U(900)/U(700) = 1.12, \tag{16}$$

which is plausible under the diminishing marginal utility commonly assumed in empirical studies. This measurement was not entirely utility free because it needed the measurement of one utility point (Equation (16)). In return, it did not invoke any time separability. ◇

Method 2 (Utility-Free Measurement of Discounting). Assume that, besides the TTO curve τ (with $\varphi = e^{r\tau}$), we observe an indifference

$$(t\colon x, t'\colon x) \sim (s\colon x, s'\colon x) \quad \text{for } s < t < t' < s' \tag{17}$$

(where s stands for being spread out and t for being tight). Note that only one nonzero outcome x is involved here. Substituting DU (Equation (1)), setting $l = 0$ in Equation (5), and dropping the common factor $U(x)$, we obtain

$$e^{r\tau(t)} + e^{r\tau(t')} = e^{r\tau(s)} + e^{r\tau(s')}. \tag{18}$$

There exists a unique r that solves this equality for every quadruple $e^{\tau(t)}$, $e^{\tau(t')}$, $e^{\tau(s)}$, and $e^{\tau(s')}$.[5] Under discounted utility and the implied impatience, r must be positive, which can be seen to be equivalent to $\tau(t) + \tau(t') > \tau(s) + \tau(s')$ (corresponding with convexity of $e^{r\tau}$ in τ). If this inequality is violated, then DU is violated. In particular, violations of time separability will generate such violations.

Figure 3 depicts a discount function that we obtained for subject 38. We used the extra indifference

$$(12\colon 700, 16\colon 700) \sim (6\colon 700, 24\colon 700), \tag{19}$$

observed from this subject in a pilot experiment (added for the last subjects in the experiment), as a

[5] Equalities of this kind are often studied under expected utility with exponential utility $e^{r\tau}$. Equation (18) results if a 50/50 gamble with outcomes $\tau(t)$ and $\tau(t')$ is equivalent to one with outcomes $\tau(s)$ and $\tau(s')$. Including negative powers r for risk aversion and power $r = 0$ for risk neutrality, the exponential family covers any degree of risk aversion and can accommodate any quadruple $\tau(s) > \tau(t) > \tau(t') > \tau(s')$ in Equation (18).

version of Equation (17) to estimate r. The resulting power r was equal to 0.58. ◇

The two methods show how TTO curves, together with minimal extra data, provide measurements of discount functions. Given that the major empirical problems for DU occur for outcome streams with more than one outcome and that TTO curves are easy to measure, these curves provide a powerful tool for analyzing intertemporal choice. If only one extra data point of utility is needed, then we can make a special effort to obtain this data point as reliably as possible. This task will be easier and can be done more reliably than having to estimate the complete utility function (Abdellaoui et al. 2010, Andersen et al. 2008, Chapman 1996, Takeuchi 2010) or, worse, simply assuming that utility is linear as done mostly in the literature. If only one indifference based on time separability is used, then we can similarly make a special effort to obtain this indifference as reliably as possible. The latter, utility-free, approach is, obviously, not affected by errors in utility measurement.

Once TTO curves are available, the *possibility* to measure the entire discount function by measuring only one extra indifference, obviously, does not imply that we *recommend* using only limited data. The more indifferences available, the more reliable our conclusions, also when using TTO curves. The possibility to obtain discounting using only one extra indifference is only an extra option, showing that TTO curves provide inferences very efficiently.

Empirical tests of the procedures described in this section are left to future studies. This paper focuses on TTO sequences because they are sufficient to measure time inconsistencies.

8. Method of the Experiment

The next sections present an experiment demonstrating the feasibility of measuring TTO sequences to analyze intertemporal choice.

8.1. Participants

Fifty-five subjects took part, 28 male and 27 female. Thirty-one students were from Erasmus University Rotterdam, 21 of whom were from finance or economics and the others were from various other disciplines. The remaining 24 students were from Maastricht University (1 from economics, 1 from finance, and the rest from various other disciplines).

8.2. Motivating Subjects

Every subject received €10 for participation. All payoffs in the stimuli were hypothetical. This point is discussed in §11.

8.3. Procedure

The experiment was run by computer, and subjects were interviewed individually. On average, the task took 15 minutes per subject. We ran extensive pilots with 53 subjects to determine the appropriate setup.

Table 1 **Parameters for the Four TTO Sequences**

Sequence	t_0	β	γ
I	0 months	€700	€900
II	0 months	€2,800	€3,300
III	5 months	€700	€900
IV	0 months	€1,600	€1,900

Note. The outcomes β and γ and the initial time point t_0 are as in Equation (3).

We took one month as the unit of time. Subjects first went through a training phase in which they were asked to choose between €700 now (which is (0: 700)) and €900 in one month (which is (1: 900)), and between (0: 700) and (600: 900). Preferences (0: 700) $\prec$ (1: 900) and (0: 700) $\succ$ (600: 900) were mostly observed. These questions intended to instill the notion that a duration t should exist for which preferences switch. Then, in a training matching task, we asked for this switching value t to generate the indifference (0: 700) $\sim$ (t: 900), and then for the value t to generate the indifference (0: 2,800) $\sim$ (t: 3,300).

8.4. Stimuli

We elicited four TTO sequences for each subject (Table 1). Every sequence consisted of five steps ($n = 5$). All tasks were matching tasks, similar to the last task of the training phase.

The computer screen is given in the appendix (Figure A.1). The pilots suggested that a direct successive elicitation of the time points $t_1, \ldots, t_5$ of one TTO sequence could generate order effects. Hence, in the main experiment the elicitations of the four TTO sequences were interspersed: we first elicited t_1 for every TTO sequence, next t_2 for every TTO sequence, and so on.

8.5. Analysis

We performed all tests both parametrically and nonparametrically. These always gave similar results, and we report only the nonparametric tests.

8.6. Analysis of Group Averages

Changes in WTW indicate whether subjects satisfy constant, decreasing, or increasing impatience. We tested for constant WTW for each TTO sequence separately using a Friedman test.

Next, we tested equality for every single pair of consecutive WTWs (for d_i and d_{i-1}) using Wilcoxon tests. We also tested equality of WTW between the first questions of Sequence I ((0: 700) $\sim$ (t: 900)) and of Sequence III ((5: 700) $\sim$ (t: 900)). Because these concern the same outcomes, stationarity predicts WTW to be the same. We checked whether the temporal attitude suggested by this comparison is consistent with that suggested by comparisons within Sequence I;

that is, we checked whether the change in WTW from the first question of Sequence I to the first question of Sequence III has the same sign as the first change in WTW within Sequence I.

8.7. Analyses of Individual Data

A subject was classified as exhibiting increasing (constant, decreasing) impatience if at least 50% of the changes in WTW suggested so, where we considered all sequences together. A double classification as constant or increasing (50% constant and 50% increasing) was reclassified as increasing, with a similar procedure for decreasing. A double classification as increasing and decreasing was taken as unclassified, as were all other cases. We used these conservative criteria to reduce the effects of response error. Such a threshold of 50% has been used before in the literature (Abdellaoui 2000). We tested whether significantly more subjects were classified as increasingly or decreasingly impatient using Wilcoxon signed rank tests.

Next, we tested whether quasi-hyperbolic discounting holds. For every subject, we split all changes in WTW of all TTO sequences into two groups: the group containing all changes in WTW where the first time point was zero, and the group containing the rest. We chose the same 50% classification as before for both groups. Under quasi-hyperbolic discounting, the WTW should increase in the former group and be constant in the latter. We performed similar Wilcoxon tests as before.

We developed global heuristic measures of convexity of the TTO curve. For example, the normalized area above the measured TTO curve is a plausible index of convexity and of decreasing impatience. This area is a monotonic transform of the *decreasing-impatience* (*DI*) *index*, defined by

$$\text{DI index} = \sum_{i=1}^{n-1}\left(\frac{i}{n} - \tilde{t}_i\right),$$

$$\text{with } \tilde{t}_i \text{ the normalization of } t_i \ \left(\tilde{t}_i = \frac{t_i - t_n}{t_0 - t_n}\right). \quad (20)$$

The DI index bears some resemblance to the Gini index in inequality measurement.

For deviations from stationarity, absolute values of deviations from linearity are more relevant. Thus, we defined the *nonstationarity* (*NS*) *index* as

$$\text{NS index} = \sum_{i=1}^{n-1}\left|\frac{i}{n} - \tilde{t}_i\right|. \quad (21)$$

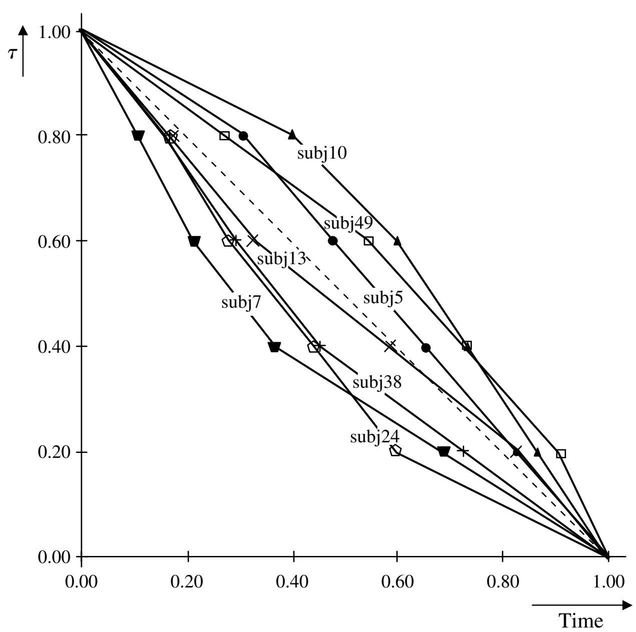
It provides an overall index of deviation from stationarity and proneness to inconsistencies without concern for the direction of the deviation. To the extent that stationarity is rational, the NS index could be interpreted as an irrationality index.

We calculated the DI and NS indexes for each subject. Hyperbolic factors were calculated as explained in §6. We compared the indexes of all subjects between sequences by means of Wilcoxon tests. To test for a possible special effect of the first questions, we also considered sequences with the first step left out. The DI index for these reduced sequences were computed as follows: DI index $= \sum_{i=1}^{3}(i/4 - \tilde{t}_{i+1})$, with $\tilde{t}_i$ the normalization of $\tilde{t}_i = (t_i - t_5)/(t_1 - t_5)$.

9. Results from Eyeballing the TTO Curves

Before presenting detailed statistical results, we present heuristic results that can immediately be inferred from eyeballing TTO curves. Figure 4 displays seven TTO curves, obtained from seven subjects, on normalized time intervals. We immediately see that the curve of subject 7 is more convex, implying more decreasing impatience compared to subject 38. By Theorem 1(i), subject 7 is more prone to time inconsistency and arbitrage than subject 38. Subject 24's curve is also below that of subject 38 everywhere, suggesting more decreasing impatience. Locally around 0.45, subject 38 exhibits more convexity though. Hence, the convexity ordering, although holding throughout most of the domain, does not hold everywhere. The curves of subjects 7 and 24 intersect, and there is no uniform ordering regarding their degree of nonstationarity over the whole interval $[t_0, t_5]$. There are several concave curves, exhibiting increasing rather than decreasing impatience. Theorem 1(ii) shows that subject 10 is more prone to time inconsistency than subject 5.

Figure 4 **TTO Curves τ of Several Subjects**



Some DI values are 0.63 (subject 7), 0.52 (subject 24), and 0.36 (subject 38). They suggest that, whereas there is no unambiguous ordering between subjects 7 and 24 as we saw before, subject 7 exhibits more decreasing impatience overall than subject 24. Similarly, subject 24 does so more than subject 38. Subjects 5, 10, and 49 exhibit increasing impatience. Accordingly, their DI indexes will be negative, being −0.26 (subject 5), −0.60 (subject 10), and −0.45 (subject 49). Overall, subject 10 exhibits more increasing impatience than subject 49, and subject 49 exhibits more increasing impatience than subject 5.

The DI index of subject 13 is 0.08, and this subject exhibits little decreasing or increasing impatience in an overall sense. Yet, this subject does deviate from stationarity. This is indicated by the NS index, which is 0.15 for this subject.

10. Results and Statistical Tests

10.1. Group Averages

Figure 5 gives the TTO curves constructed from the medians of the answers of all subjects. The curves suggest that subjects are initially increasingly impatient and constantly impatient thereafter. Statistical analyses confirm this pattern. The Friedman tests rejected constant WTW ($p < 0.01$) for all sequences. The null hypothesis of constant WTW is not rejected if the first WTW is excluded ($p > 0.20$ for all tests). Thus, our findings suggest that people satisfy stationarity for time points beyond a certain threshold. We can see that this threshold exceeds five months from the third sequence.

Figure 6 shows median WTWs. The vertical axes all have the same scale and give the WTW. For every sequence, the WTW drops initially and then remains more or less constant. This is confirmed by Wilcoxon tests, summarized in Table 2. The WTW decreases significantly in the first steps ($d_2 - d_1$) ($\alpha = 0.01$), suggesting increasing impatience. The WTW increases in the second step ($d_3 - d_2$) for Sequence III ($\alpha = 0.05$). No other changes are significant at $\alpha = 0.05$.

A Wilcoxon test shows that the first WTW of the third sequence is significantly lower ($p < 0.01$) than that of the first sequence. Thus, subjects are consistent between Sequences I and III.

Figure 5 TTO Curves τ for Median Answers of the Four TTO Sequences

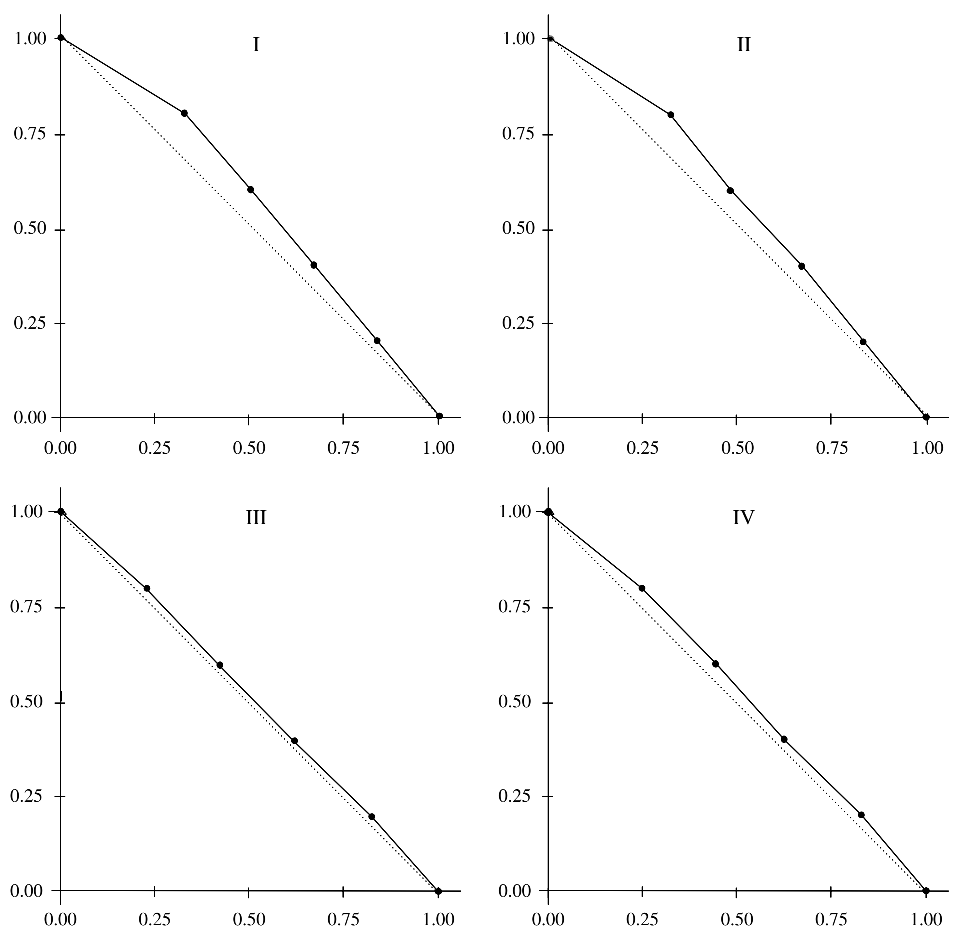
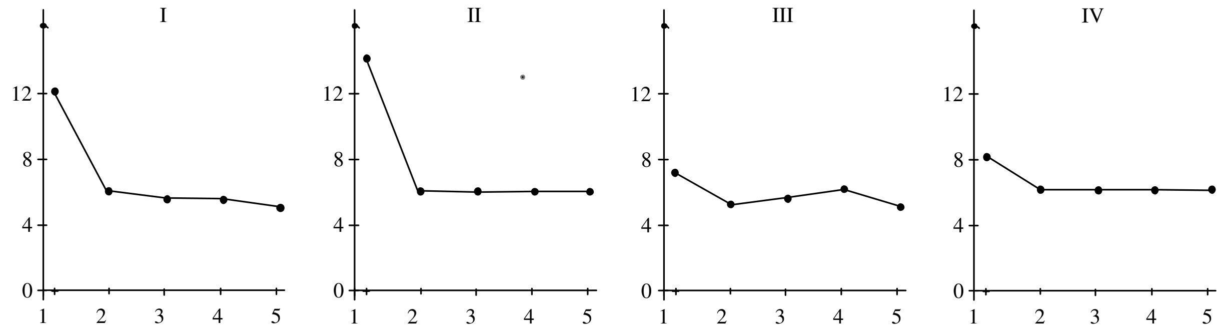


Figure 6 Median Willingness to Wait for Each Sequence



10.2. Individual Data

Individual data confirm the preceding findings. Subjects are increasingly impatient for time points close to zero and constantly impatient for later time points. The classification of subjects based on all sequences together yields 18 subjects exhibiting constant impatience, 3 exhibiting decreasing impatience, 10 exhibiting increasing impatience, and 24 not classified (Table 3). Thus, based on this classification, we cannot say much about the behavior of individual subjects. The Wilcoxon test shows, though, that there is more tendency toward increasing than toward decreasing impatience ($p = 0.052$). In the group of all questions with a first time point zero, 8 subjects show constant impatience, 3 show decreasing impatience, 36 show increasing impatience, and 8 subjects could not be classified. Most subjects indeed are increasingly impatient for time point zero, which is supported by the Wilcoxon test ($p = 0.000$). In the other group (first time point positive), 21 subjects exhibit constant impatience, 5 subjects exhibit decreasing impatience, 6 subjects exhibit increasing impatience, and 23 subjects could not be classified. It appears that most subjects exhibit constant impatience for time points not too close to zero.

Calculations of the hyperbolic factors revealed that positivity of the denominator in Equation (14), a necessary condition for generalized hyperbolic discounting (Observation 3), was widely violated for virtually all subjects in several questions. This provides evidence against generalized hyperbolic discounting, preventing us from calculating the hyperbolic factors in many cases. The medians of the DI indexes (regarding decreasing impatience) were significantly negative for all four sequences ($p < 0.01$) so that subjects were increasingly impatient overall. The medians of the DI index were −0.33, −0.28, −0.092, and −0.19, respectively. The third sequence had both a lower NS index and a lower absolute value of the DI index. This is probably caused by the fact that the third sequence starts closer to the threshold from whereon subjects satisfy constant impatience. The DI indexes of the reduced sequences, the sequences without the first steps, did not deviate significantly from zero, indicating that the increasing impatience found earlier is indeed due to the first step of every sequence.

Based on a Wilcoxon signed rank test, the DI index and the NS index were significantly different for every sequence ($p < 0.01$), where the NS index was always larger than minus the DI index. Because most indexes of decreasing impatience were negative, this finding implies that for most subjects the TTO curve τ intersected the curve of a linear TTO curve at least once. Most subjects were, therefore, not clearly increasingly impatient or clearly decreasingly impatient, but were a mixture of both.

There was no significant difference in the DI index and the NS index between Sequences I and II and between Sequences III and IV. For all other pairs of sequences, the sequence with the higher sequence number provided significantly higher DI indexes and significantly lower NS indexes than the ones before ($p < 0.01$ for all but one, $p < 0.05$ for all). Thus, subjects became less nonstationary and more decreasingly impatient or, equivalently, less increasingly impatient in later sequences. On average, men

Table 2 Wilcoxon Signed Rank Tests: Z (p-Value, Two-Tailed)

	WTW			
Sequence	$d_2 - d_1$	$d_3 - d_2$	$d_4 - d_3$	$d_5 - d_4$
I	−4.40 (0.000)	−0.51 (0.612)	−0.63 (0.531)	−0.34 (0.732)
II	−4.50 (0.000)	1.35 (0.176)	−0.93 (0.352)	−0.98 (0.329)
III	−3.39 (0.001)	2.00 (0.046)	−0.29 (0.769)	−0.95 (0.341)
IV	−3.19 (0.001)	1.03 (0.302)	−0.41 (0.681)	1.05 (0.293)

Table 3 Classification of Individuals

	Impatience			
Question	Constant	Decreasing	Increasing	Unclassified
All	18	3	10	24
Time point 0	8	3	36	8
Time point > 0	21	5	6	23

had higher DI indexes and lower NS indexes than women, except for the DI index in Sequence III, but the differences were usually not significant.

11. Discussion of the Experiment

11.1. Increasing Impatience

Our subjects satisfy increasing impatience for small delays, after which they satisfy constant impatience. Thus, we find a kind of reversed quasi-hyperbolic discounting, where impatience is constant after a certain threshold but is initially increasing rather than decreasing. Impatience continues to increase up to five months and is not constant immediately after the present. During the piloting, we were at first surprised by these results. Because caution is called for when applying a new method, we did additional piloting. Informal discussions with subjects indicated, however, that they understood the questions well and knew what they wanted to answer. These discussions supported our belief in our finding of increasing impatience. Many students indicated that they did not mind a delay right now, but after a long wait they disliked further delays more. This finding is contrary to what is commonly assumed in the literature, that subjects become more insensitive to delays over time.

Our finding of increasing impatience is consistent with several other studies (Carbone 2008, Gigliotti and Sopher 2004, Loewenstein 1987, Read et al. 2005, Rubinstein 2003, Sayman and Öncüler 2009, Scholten and Read 2006, Takeuchi 2010). Read et al. (2005) found that hyperbolic discounting is only observed when time is described in delay terms as opposed to calendar time terms. Rubinstein (2003) reported three experiments that provide evidence against constant or decreasing impatience. Bommier (2006) and Dasgupta and Maskin (2005) gave theoretical arguments for why increasing impatience can occur. Gollier and Zeckhauser (2005) showed that taking group averages increases decreasing impatience, so that impatience at the individual level is less decreasing than analyses based on group averages suggest. Baucells and Heukamp (2009) showed that increasing impatience can be generated by increasing relative risk aversion and violations of subproportionality of probability weighting. The interactions in their model suggest no universal pattern of increasing or decreasing impatience, but dependence on the particular outcomes and the (perceived) risk. Other studies found neither increasing nor decreasing impatience, so that stationarity was not rejected (Holcomb and Nelson 1992, Sopher and Sheth 2006).

11.2. Incentives

The importance of using real incentives rather than hypothetical choice is well recognized in the literature today. We nevertheless used hypothetical choice with a flat fee for participation for a number of reasons. For intertemporal choice, reliable future arrangements are difficult to implement for the experimenters but, more importantly, also for the subjects who face considerable transaction costs and reliability issues. The latter will distort the experiment. There have been some impressive studies in the literature that succeeded in implementing real incentives in intertemporal choice, but, with the exception of Andersen et al. (2008), they always involved relatively small time periods (Baucells and Heukamp 2010, Epper et al. 2009, Milkman et al. 2009, Takeuchi 2010, Tanaka et al. 2010). Here, genuine discounting will be low. The problem is aggravated because real incentives have to involve relatively small payments, in which case much of the discounting observed may be generated by transaction costs rather than by genuine discounting.

Although several studies have found differences between real and hypothetical choice (Hertwig and Ortmann 2001), the majority have concluded that the behavioral patterns are the same (Camerer and Hogarth 1999) for cognitively simple tasks such as those in our experiment. Ashraf et al. (2006) showed that hypothetical measurements of time preferences predicted actual decisions about savings commitments well. Note that there is no incentive for our subjects to please the experimenter one way or the other in the intertemporal choices in our experiment, unlike in experiments on social behavior.

11.3. Chaining

The measurement of TTO sequences is chained, which means that answers to one question are used as input in subsequent questions. A drawback is that order effects can occur. It is, however, unlikely that these would have caused the increasing impatience we found. The setup of the experiment made it difficult for subjects to notice that the questions were chained and that several of them together served to elicit sequences. Another drawback of chaining questions is that it leads to a propagation of errors, with an error in the first answer affecting the error in future answers. Propagations of errors in similar chained measurements have been analyzed by Bleichrodt and Pinto (2000) and Abdellaoui et al. (2005). Both studies concluded that the effect of error propagation on chained measurement was small. Depending on the error theory assumed, smaller errors may result when measuring a distance $[t_0, t_5]$ through five intermediate steps rather than in one step.

11.4. Matching in the Time Dimension

Many studies that provide evidence in favor of decreasing impatience elicit indifference values in the outcome dimension. They fix two time points and one outcome and elicit a second outcome that makes the subject indifferent between the two timed outcomes. We elicited indifference values in the time dimension. Delquié (1993) discussed the general relevance of the matching dimension. In daily life decisions, we often determine matching values in the time dimension, for example, when we think about maximum acceptable waiting times (for referee reports, medical treatments, salary raises, interests from savings, and so on). Because this paper is interested in properties of the discount function and not of the utility function, it is more natural to have subjects focus on this dimension. Ebert and Prelec (2007) found that generally subjects are insufficiently sensitive to the time dimension and that this generates decreasing impatience. Our design has countered this effect. Our method does not require richness in the outcome dimension and can be used with qualitative health outcomes for example. It naturally exploits the richness in the time dimension that is available anyhow. Takeuchi (2010) similarly had subjects focus on the time dimension, exploiting its richness.

Eliciting indifferences in the time dimension has not been very common in the experimental literature, but it has been used on a number of occasions, for instance, by Mazur (1987). He conducted experiments with pigeons instead of humans. Green et al. (1994) did similar experiments with humans. These studies exploited the richness of the time dimension as we did. They, however, still assumed linear utility.

11.5. Matching vs. Choice

We chose matching to directly elicit indifferences rather than deriving indifferences from choices. There have been many debates about the pros and cons of matching versus choice. One of the drawbacks of choice is that subjects more easily resort to noncompensatory heuristics than they do for matching (Brandstätter et al. 2006; Huber et al. 2001, p. 72; Montgomery 1983; Tversky 1972). Another drawback is that it takes more time and effort to obtain indifference values. A disadvantage of matching is that it generates biases of its own, with scale compatibility the most well-known one (Bostic et al. 1990, Huber et al. 2001, Tversky et al. 1988). A feature of TTO sequences that supports the use of matching questions is the robustness against such biases. For example, assume that a subject overweights the time dimension because of scale compatibility. We then have

$$\lambda\left[\ln(\varphi(t_{j-1})) - \ln(\varphi(t_j))\right] = \ln(U(\gamma)) - \ln(U(\beta)) \qquad (22)$$

with $\lambda > 1$ instead of $\lambda = 1$ as in Equation (4) and its proof. It is easily verified that all our inferences and applications of the TTO sequence remain correct. They continue to be equally spaced in $\ln(\varphi)$ units, which is all we used in our analyses.

Of course, biases different than those in Equation (22) can exist. Still, TTO sequences provide robustness against at least those parts of the biases that have the same effect on all questions. The main reason that we chose matching when eliciting TTO sequences is that it is considerably more tractable and efficient than choice. With its main bias neutralized, matching becomes an attractive option.

11.6. Topics for Future Investigation

Our findings suggest a number of new directions for the study of intertemporal choice. Virtually all existing models, including quasi-hyperbolic discounting and generalized hyperbolic discounting, assume universal decreasing or constant impatience, and have no clear extension to allow for increasing impatience. However, even if group averages satisfy decreasing impatience, there will still be individuals who exhibit increasing impatience, so that such functions are required for any data fitting at the individual level. For this reason, our test to discriminate which of the currently popular models fits the data better provided a very simple conclusion: they were all rejected. In particular, Rohde's (2010) hyperbolic factor, targeted to the currently popular families, was not defined for a large proportion of the answers given, which occurred at least once for virtually every subject. Hence, more general functions for discounting should be developed. This was a motivation for the functions introduced by Ebert and Prelec (2007) and Bleichrodt et al. (2009).

12. Further Discussions

12.1. Discussion of Other Studies That Corrected for Nonlinear Utility

We discuss some other studies that corrected for nonlinear intertemporal utility. Some papers provided a solution to the problem of unknown utility differently than we did, by measuring utility separately. Andersen et al. (2008) and Takeuchi (2010) did so by considering risky choices and estimating the von Neumann–Morgenstern utility function there, assuming expected utility.[6] Thus, they were the first to measure the discount function while reckoning with nonlinear utility. One drawback of their approach is that it is distorted by the empirical violations

[6] Takeuchi (2010) did so by measuring the probability at a high prize that is equivalent to an outcome considered, for several outcomes. After normalizing the utility of the high prize at 1 (with utility 0 for no prize), the probability then is the von Neumann–Morgenstern utility. This measurement method is known as the standard gamble method. Andersen et al. (2008) obtained von Neumann–Morgenstern utility through data fitting.

of expected utility that have been extensively documented (Starmer 2000, Tversky and Kahneman 1992).

A second drawback of the measurement of utility through risky choice is that it needs to assume that the cardinal risky utility function is a general cardinal function that can also be used for intertemporal choice, an assumption disputed since the ordinal revolution (Baumol 1958). Wakker (1994) argued for the plausibility of this assumption if empirically realistic nonexpected utility theories are used. This approach was adopted by Baucells and Heukamp (2009, 2010) and Epper et al. (2009). The latter used prospect theory to estimate utility from risky choice and then used this utility to measure discounting in intertemporal choice. Chapman (1996, Experiment 3) also used risky choices and expected utility to obtain information about utility for intertemporal choice, but did not carry out an extensive measurement. Abdellaoui et al. (2010) used risky options to measure intertemporal utility, but did not need to commit to expected utility or any other risk theory, or to the equation of cardinal risky utility with cardinal intertemporal utility. Scholten and Read (2010), finally, presented a model where tradeoffs as in §3 are central and utility can be time dependent, and used it to accommodate many deviations from discounted utility.

12.2. A Modification for the Health Domain

We use the term time-tradeoff sequence in analogy to the time-tradeoff method commonly used in the health domain (Gold et al. 1996). Here an indifference such as (living 10 years while being blind) $\sim$ (living 9 years in perfect health) is used to assess the relative utility of being blind, $U(\text{blind})/U(\text{perfect health})$, through the ratio $\varphi(9 \text{ years})/\varphi(10 \text{ years})$, where φ reflects a value of life duration, which is often taken linear for convenience. These health questions are of the same format as the questions used in our time-tradeoff sequences.

An interpretational difference between the time-tradeoff method from health and our method is that time reflects duration of experience in health questions, whereas in our study time reflects waiting time before receipt of an outcome. To express the two different interpretations of time, we may use the term waiting-time-tradeoff sequence, whereas the method in the health domain can then be called experience-time-tradeoff method. When no confusion will arise, it is easier to use the short term without interpretation expressed, which is what we do in our paper. The application of TTO sequences in the health domain is a topic for future study.

13. Conclusion

TTO sequences provide a new general tool for analyzing intertemporal choice, both for theoretical and for empirical purposes. They make it possible to (a) measure the discount function using only one utility observation and no time separability, or using only one time-separable choice and no utility observation or assumption about utility, (b) axiomatize qualitative properties of discount functions, (c) axiomatize parametric families of discount functions, (d) empirically test the axioms of those axiomatizations, and (e) readily quantify the degree of time inconsistency and measure it empirically (including Prelec's (2004) index).

Measurements and analyses of TTO sequences are so simple that they can be done using only paper and pencil, and no computer. This tractability makes the method well suited for obtaining exact quantitative measurements in field studies. Indexes of decreasing impatience (DI index) and of general time inconsistency can readily be calculated. They can be used in regressions to investigate the dependence of decreasing impatience and time inconsistency on demographic variables, for instance.

In our experiment, we find violations of some currently popular discount models, rendering support to recent findings that there is more increasing impatience than commonly believed. These results suggest that it is desirable to develop new discount models that are not completely restricted to decreasing impatience, but that allow for increasing impatience. We finally note that TTO sequences are easy both for subjects who generate them and for researchers who analyze them.

Acknowledgments

Arthur Attema's and Han Bleichrodt's research was made possible through grants from the Netherlands Organization for Scientific Research (NWO).

Appendix

Proof of Theorem 1. Because τ and τ^* are strictly decreasing functions, $\tau^*(t) = f(\tau(t))$ for a strictly increasing function f. Take any intervals $[d, c]$ and $[b, a]$ to the right of $[d, c]$ ($b > d$ and $a > c$) in the domain of f. Then $a = \tau(s)$, $b = \tau(t)$, $c = \tau(s+\sigma)$, and $d = \tau(t+\sigma+\varepsilon)$ for some $s < t$, $s+\sigma < t+\sigma+\varepsilon$, $\sigma > 0$, $\sigma+\varepsilon > 0$. Because the ranges of U and U^* contain nondegenerate intervals with 0 as a lower bound, there exist outcomes $\beta < \gamma$ with

$$(s\colon \beta) \sim (t\colon \gamma) \tag{23}$$

and outcomes $\beta^* < \gamma^*$ with

$$(s\colon \beta^*) \sim^* (t\colon \gamma^*). \tag{24}$$

(Here is where we crucially use continuity of utility.) Only the utility ratios $U(\beta)/U(\gamma)$ and $U^*(\beta^*)/U^*(\gamma^*)$ matter for all that follows and, hence, the particular choices of β, γ, β^*, and γ^* are immaterial for all that follows.

Figure A.1 Layout of the Computer Screen

We have equivalence of the following statements:

$$a-b=c-d;$$
$$\tau(s)-\tau(t)=\tau(s+\sigma)-\tau(t+\sigma+\varepsilon);$$
$$\ln\varphi(s)-\ln\varphi(t)=\ln\varphi(s+\sigma)-\ln\varphi(t+\sigma+\varepsilon);$$
$$\varphi(s)/\varphi(t)=\varphi(s+\sigma)/\varphi(t+\sigma+\varepsilon);$$
$$(s+\sigma\colon\beta)\sim(t+\sigma+\varepsilon\colon\gamma).$$

We also have logical equivalence of the following statements:

$$f(a)-f(b)\geq f(c)-f(d);$$
$$\tau^*(s)-\tau^*(t)\geq\tau^*(s+\sigma)-\tau^*(t+\sigma+\varepsilon);$$
$$\ln\varphi^*(s)-\ln\varphi^*(t)\geq\ln\varphi^*(s+\sigma)-\ln\varphi^*(t+\sigma+\varepsilon);$$
$$\varphi^*(s)/\varphi^*(t)\geq\varphi^*(s+\sigma)/\varphi^*(t+\sigma+\varepsilon);$$
$$(s+\sigma\colon\beta^*)\preceq(t+\sigma+\varepsilon\colon\gamma^*).$$

It is well known that f is convex if and only if for all a, b, c, and d as above $(a-b=c-d)$ we have $f(a)-f(b)\geq f(c)-f(d)$. As we have just demonstrated, this is, in view of Equations (23) and (24) and the independence of the choices β, γ, β^*, and γ^*, the same as the requirement that $(s+\sigma\colon\beta)\sim(t+\sigma+\varepsilon\colon\gamma)$ imply $(s+\sigma\colon\beta^*)\preceq(t+\sigma+\varepsilon\colon\gamma^*)$ for all s, t, σ, and ε as above; that is, convexity of f is equivalent to more decreasing impatience for $\succeq^*$ than for $\succeq$. Reversing inequalities and weak preferences shows that concavity of f is equivalent to more increasing impatience for $\succeq^*$ than for $\succeq$. □

References

Abdellaoui, M. 2000. Parameter-free elicitation of utilities and probability weighting functions. *Management Sci.* **46** 1497–1512.

Abdellaoui, M., A. E. Attema, H. Bleichrodt. 2010. Intertemporal tradeoffs for gains and losses: An experimental measurement of discounted utility. *Econom. J.* **120** 595–611.

Abdellaoui, M., F. Vossmann, M. Weber. 2005. Choice-based elicitation and decomposition of decision weights for gains and losses under uncertainty. *Management Sci.* **51** 1384–1399.

Andersen, S., G. W. Harrison, M. I. Lau, E. E. Rutstrom. 2008. Eliciting risk and time preferences. *Econometrica* **76** 583–618.

Ashraf, N., D. Karlan, W. Yin. 2006. Tying Odysseus to the mast: Evidence from a commitments savings product in the Phillippines. *Quart. J. Econom.* **121** 635–672.

Baucells, M., F. H. Heukamp. 2009. Probability and time tradeoff. Mimeo, IESE Business School, University of Navarra, Barcelona.

Baucells, M., F. H. Heukamp. 2010. Common ratio using delay. *Theory Decision* **68** 149–158.

Baumol, W. J. 1958. The cardinal utility which is ordinal. *Econom. J.* **68** 665–672.

Bleichrodt, H., J. L. Pinto. 2000. A parameter-free elicitation of the probability weighting function in medical decision analysis. *Management Sci.* **46** 1485–1496.

Bleichrodt, H., K. I. M. Rohde, P. P. Wakker. 2009. Non-hyperbolic time inconsistency. *Games Econom. Behav.* **66** 27–38.

Bommier, A. 2006. Uncertain lifetime and intertemporal choice: Risk aversion as a rationale for time discounting. *Internat. Econom. Rev.* **47** 1223–1246.

Bostic, R., R. J. Herrnstein, R. D. Luce. 1990. The effect on the preference-reversal phenomenon of using choice indifferences. *J. Econom. Behav. Organ.* **13** 193–212.

Brandstätter, E., G. Gigerenzer, R. Hertwig. 2006. The priority heuristic: Making choices without trade-offs. *Psych. Rev.* **113** 409–432.

Camerer, C. F., T.-H. Ho. 1994. Violations of the betweenness axiom and nonlinearity in probability. *J. Risk Uncertainty* **8** 167–196.

Camerer, C. F., R. M. Hogarth. 1999. The effects of financial incentives in experiments: A review and capital-labor-production framework. *J. Risk Uncertainty* **19** 7–42.

Carbone, E. 2008. Temptations and dynamic consistency. *Theory Decision* **64** 229–248.

Chapman, G. B. 1996. Temporal discounting and utility for health and money. *J. Experiment. Psych.: Learn., Memory, Cognition* **22** 771–791.

Dasgupta, P., E. Maskin. 2005. Uncertainty and hyperbolic discounting. *Amer. Econom. Rev.* **95** 1290–1299.

Delquié, P. 1993. Inconsistent trade-offs between attributes: New evidence in preference assessment biases. *Management Sci.* **39** 1382–1395.

Dolan, P., D. Kahneman. 2008. Interpretations of utility and their implications for the valuation of health. *Econom. J.* **118** 215–234.

Ebert, J. E. J., D. Prelec. 2007. The fragility of time: Time-insensitivity and valuation of the near and far future. *Management Sci.* **53** 1423–1438.

Epper, T., H. Fehr-Duda, A. Bruhin. 2009. Uncertainty breeds decreasing impatience: The role of risk preferences in time discounting. Mimeo, IED, ETH, Zürich.

Fishburn, P. C., A. Rubinstein. 1982. Time preference. *Internat. Econom. Rev.* **23** 677–694.

Frederick, S., G. F. Loewenstein, T. O'Donoghue. 2002. Time discounting and time preference: A critical review. *J. Econom. Literature* **40** 351–401.

Gigliotti, G., B. Sopher. 2004. Analysis of intertemporal choice: A new framework and experimental results. *Theory Decision* **55** 209–233.

Gold, M. R., J. E. Siegel, L. B. Russell, M. C. Weinstein. 1996. *Cost-Effectiveness in Health and Medicine.* Oxford University Press, New York.

Gollier, C., R. J. Zeckhauser. 2005. Aggregation of heterogeneous time preferences. *J. Political Econom.* **113** 878–896.

Green, L., N. Fristoe, J. Myerson. 1994. Temporal discounting and preference reversals in choice between delayed outcomes. *Psychonomic Bull. Rev.* **1** 383–389.

Halevy, Y. 2008. Strotz meets Allais: Diminishing impatience and the certainty effect. *Amer. Econom. Rev.* **98** 1145–1162.

Halevy, Y. 2009. Time consistency: Stationarity and time invariance. Mimeo, Department of Economics, University of British Columbia, Vancouver.

Harvey, C. M. 1986. Value functions for infinite-period planning. *Management Sci.* **32** 1123–1139.

Harvey, C. M. 1995. Proportional discounting of future costs and benefits. *Math. Oper. Res.* **20** 381–399.

Hertwig, R., A. Ortmann. 2001. Experimental practices in economics: A challenge for psychologists? *Behav. Brain Sci.* **24** 383–403.

Holcomb, J. H., P. S. Nelson. 1992. Another experimental look at individual time preference. *Rationality Soc.* **4** 199–220.

Huber, J., D. Ariely, G. Fischer. 2001. Expressing preference in a principal-agent task: A comparison of choice, rating, and matching. *Organ. Behav. Human Decision Processes* **87** 66–90.

Kapteyn, A., F. Teppa. 2003. Hypothetical intertemporal consumption choices. *Econom. J.* **113** C140–C152.

Keller, L. R., E. Strazzera. 2002. Examining predictive accuracy among discounting models. *J. Risk Uncertainty* **24** 143–160.

Koopmans, T. C. 1960. Stationary ordinal utility and impatience. *Econometrica* **28** 287–309.

Krantz, D. H., R. D. Luce, P. Suppes, A. Tversky. 1971. *Foundations of Measurement*, Vol. I: *Additive and Polynomial Representations*. Academic Press, New York.

Laibson, D. I. 1997. Golden eggs and hyperbolic discounting. *Quart. J. Econom.* **112** 443–477.

Loewenstein, G. F. 1987. Anticipation and the value of delayed consumption. *Econom. J.* **97** 666–684.

Loewenstein, G., D. Prelec. 1992. Anomalies in intertemporal choice: Evidence and an interpretation. *Quart. J. Econom.* **107** 573–597.

Mazur, J. E. 1987. An adjusting procedure for studying delayed reinforcement. M. L. Commons, J. E. Mazur, J. A. Nevin, H. Rachlin, eds. *Quantitative Analyses of Behavior*, Vol. 5. Lawrence Erlbaum, Hillsdale, NJ, 55–73.

Milkman, K. L., T. Rogers, M. H. Bazerman. 2009. Highbrow films gather dust: Time-inconsistent preferences and online DVD rentals. *Management Sci.* **55** 1047–1059.

Montgomery, H. 1983. Decision rules and the search for a dominance structure: Towards a process model of decision making. P. C. Humphreys, O. Svenson, A. Vari, eds. *Analyzing and Aiding Decision Processes*. North-Holland, Amsterdam, 343–369.

Noor, J. 2009. Hyperbolic discounting and the standard model: Eliciting discount functions. *J. Econom. Theory* **144** 2077–2083.

O'Donoghue, T., M. Rabin. 1999. Doing it now or later. *Amer. Econom. Rev.* **89** 103–124.

O'Donoghue, T., M. Rabin. 2001. Choice and procrastination. *Quart. J. Econom.* **116** 121–160.

Prelec, D. 2004. Decreasing impatience: A criterion for nonstationary time preference and "hyperbolic" discounting. *Scandinavian J. Econom.* **106** 511–532.

Prelec, D., G. F. Loewenstein. 1991. Decision making over time and under uncertainty: A common approach. *Management Sci.* **37** 770–786.

Read, D., S. Frederick, B. Orsel, J. Rahman. 2005. Four score and seven years from now: The date/delay effect in temporal discounting. *Management Sci.* **51** 1326–1335.

Rohde, K. I. M. 2010. The hyperbolic factor: A measure of time inconsistency. *J. Risk Uncertainty* **41** 125–140.

Rubinstein, A. 2003. Economics and psychology? The case of hyperbolic discounting. *Internat. Econom. Rev.* **44** 1207–1216.

Sayman, S., A. Öncüler. 2009. An investigation of time-inconsistency. *Management Sci.* **55** 470–482.

Scholten, M., D. Read. 2006. Discounting by intervals: A generalized model of intertemporal choice. *Management Sci.* **52** 1424–1436.

Scholten, M., D. Read. 2010. Intertemporal tradeoffs. *Psych. Rev.* Forthcoming.

Sopher, B., A. Sheth. 2006. A deeper look at hyperbolic discounting. *Theory Decision* **60** 219–255.

Starmer, C. 2000. Developments in non-expected utility theory: The hunt for a descriptive theory of choice under risk. *J. Econom. Literature* **38** 332–382.

Strotz, R. H. 1956. Myopia and inconsistency in dynamic utility maximization. *Rev. Econom. Stud.* **23**(3) 165–180.

Takeuchi, K. 2010. Non-parametric test of time consistency: Present bias and future bias. *Games Econom. Behav.* Forthcoming.

Tanaka, T., C. F. Camerer, Q. Nguyen. 2010. Risk and time preferences: Linking experimental and household survey data from Vietnam. *Amer. Econom. Rev.* **100** 557–571.

Thaler, R. H. 1981. Some empirical evidence of dynamic inconsistency. *Econom. Lett.* **8** 201–207.

Tversky, A. 1972. Elimination by aspects: A theory of choice. *Psych. Rev.* **79** 281–299.

Tversky, A., D. Kahneman. 1992. Advances in prospect theory: Cumulative representation of uncertainty. *J. Risk Uncertainty* **5** 297–323.

Tversky, A., S. Sattath, P. Slovic. 1988. Contingent weighting in judgment and choice. *Psych. Rev.* **95** 371–384.

van der Pol, M., J. Cairns. 2002. A comparison of the discounted utility model and hyperbolic discounting models in the case of social and private intertemporal preferences for health. *J. Econom. Behav. Organ.* **49** 79–96.

Wakker, P. P. 1994. Separating marginal utility and probabilistic risk aversion. *Theory Decision* **36** 1–44.

Wathieu, L. 1997. Habits and the anomalies in intertemporal choice. *Management Sci.* **43** 1552–1563.