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A simple preference foundation of cumulative prospect theory with power utility

Peter P. Wakker[a,*], Horst Zank[b,1]

[a] CentER, Tilburg University, Tilburg, Netherlands

[b] Department of Quantitative Economics, Maastricht University, Maastricht, Netherlands

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Abstract

Most empirical studies of rank-dependent utility and cumulative prospect theory have assumed power utility functions, both for gains and for losses. As it turns out, a remarkably simple preference foundation is possible for such models: Tail independence (a weakening of comonotonic independence which underlies all rank-dependent models) together with constant proportional risk aversion suffice, in the presence of common assumptions (weak ordering, continuity, and first stochastic dominance), to imply these models. Thus, sign dependence, the different treatment of gains and losses, and the separation of decision weights and utility are obtained free of charge.

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1. Introduction

Under expected utility, attitudes to risk are modeled solely through the curvature of utility, i.e. the nonlinear sensitivity towards outcomes. A number of phenomena are, however, hard to explain in this manner. Examples are the classical Allais (1953) and Ellsberg (1961) paradoxes, the simultaneous

* Corresponding author. Current address: Faculty of Economics, CREED, Universiteit van Amsterdam, Roetersstraat 11, 1018 WB Amsterdam, Netherlands.

[1] Current address: School of Economic Studies, The University of Manchester, Oxford Road, Manchester M13 9PL, UK.

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existence of gambling and insurance (Friedman and Savage, 1948), and the equity premium puzzle (Mehra and Prescott, 1985). Alternative, nonexpected utility, models have been developed to accommodate these phenomena; for a survey, see Starmer (2000).

A popular nonexpected utility model is rank-dependent utility (RDU) (Quiggin, 1981; Schmeidler, 1989). The model received sound preference foundations, is mathematically tractable, and its empirical performance is promising. RDU adopts a useful concept in addition to the nonlinear evaluation of outcomes: a nonlinear weighting of probabilities, modeled through a probability transformation function. The classical paradoxes can be explained by probability transformation. Gambling and insurance are not only reconciled, but even have the same cause: the overweighting of small probabilities, for gains and losses, respectively.

Traditional rank-dependent models do not incorporate the important empirical phenomenon that, in most situations, agents do not perceive monetary outcomes as absolute wealth but as changes with respect to their status quo. In the last five decades, many authors have emphasized the importance of the status quo outcome in empirical decision making (Edwards, 1954; Harless and Camerer, 1994; Kahneman and Tversky, 1979; Markowitz, 1952; Tversky and Kahneman, 1991; Yaari, 1965). Agents are especially sensitive to losses, i.e. outcomes below the current status quo. This characteristic, known as loss aversion, is one of the major factors in human risk attitude and underlies much of the empirically observed risk aversion. Cumulative prospect theory (CPT) incorporates the different treatment of gains and losses (Luce and Fishburn, 1991; Tversky and Kahneman, 1992). The theory combines the theoretical soundness of RDU with the empirical realism of original prospect theory (Kahneman and Tversky, 1979).

Empirical studies of CPT have commonly assumed power utility. The implied infinite marginal utility at the origin reflects an extreme sensitivity of subjects towards changes near the status quo. Promising empirical results have been obtained. Fetherstonhaugh et al. (1997) and Stevens (1959) gave psychological explanations for the prevalence of power perception functions. For these reasons, the CPT model with power utility is generally assumed in parametric tests and is currently the most used nonexpected utility form. For a theoretical definition of loss aversion, the extreme derivatives of power utility at zero may cause some problems (Köbberling and Wakker, 2000).

As a price to pay for the empirical success of CPT, its model and preference foundation are more complex than they are for RDU. Both CPT and RDU invoke a comonotonic generalization of expected utility's independence condition, namely comonotonic independence (or, equivalently, tail independence). This preference condition does not entail the separation of utility and probability weighting that is characteristic of CPT and RDU. To obtain this separation, more complex preference conditions are commonly added. [2] Such preference conditions are more complex for CPT than for RDU because of the separate treatment of gains and losses (Luce and Fishburn, 1991; Tversky and Kahneman, 1992). Only when utility has been derived is constant proportional risk aversion [3] invoked so as to imply power utility (Tversky and Kahneman, 1992). The line of reasoning just sketched can be found in the traditional derivations of CPT.

CPT, and its predecessor prospect theory, are the most used models in empirical studies and applications of decision under risk, but have received little attention in theoretical economic studies (Wakker, 1998). The aim of this paper is to simplify the theoretical tractability of CPT by means of an appealing technique used before by Ebert (1988). Our analysis will show that CPT necessarily results from some natural preference conditions.

To obtain the desired simplification, we interchange two steps in the traditional line of reasoning, as follows. As in the traditional approach, we first invoke tail independence to obtain the rank-dependent additivily decomposable functional of Green and Jullien (1988), in which utility and probability weighting are not yet separated. With this functional obtained, we immediately invoke constant proportional risk aversion. This condition turns out to have a surprising extra merit at this early stage: It implies the separation of utility and probability weighting. That is, it implies the additional more complex preference conditions described before. These conditions can therefore simply be dropped. An additional surprise is that this approach, with constant proportional risk aversion imposed both on gains and on losses, leads to CPT without further modification. That is, the separate treatment of gains and losses (sign dependence), and their CPT aggregation, are likewise implied by constant proportional risk aversion when applied to gains and losses. This natural fit of CPT and constant proportional risk aversion can be attributed to the special role of the status quo in both. Finally, with utility established, constant proportional risk aversion characterizes power utility as it did in the traditional approach.

In summary, both the separation of utility and probability weighting and the different treatment of gains and losses are obtained free of charge under constant proportional risk aversion. We hope that this natural and elementary

[2] Axiom 4 of Quiggin (1982); weak event commutativity (Chew, 1989; Chew and Karni, 1994); projection independence (Segal, 1989); compound gambles and joint receipt (Luce and Fishburn, 1991); tradeoff consistency (Tversky and Kahneman, 1992; Wakker, 1994); weak multisymmetry (Gul, 1992, Assumption 2; Nakamura, 1995, Axiom 5); a linear utility and a quadratic probability transformation in Safra and Segal (1998, Theorem 3); comonotonic mixture independence (Chateauneuf, 1999); probability tradeoff consistency (Abdellaoui, 2002).

[3] Also known as constant relative risk aversion. We avoid the latter term because it can have different meanings, e.g. it can refer to interpersonal comparisons or to the risky/riskless utility distinction.

foundation of CPT will increase its interest for economic theory. An extension of our results to decision under uncertainty and to multiattribute outcomes will be provided by Zank (2000).

2. Definitions

Outcomes are monetary and $\mathbb{R}$ is the set of outcomes. A *lottery* $P = (p_1,x_1;\ldots;p_n,x_n)$ is a finite probability distribution over the set of outcomes, assigning probability p_j to outcome x_j, $j = 1,\ldots,n$. The probabilities p_j are nonnegative and sum to one. Lotteries are written in a rank-ordered form, i.e. it is implicitly assumed that the outcomes are rank-ordered ($x_1 \geqslant \cdots \geqslant x_n$) in the above notation.

Positive outcomes are *gains* and negative outcomes are *losses*. The *status quo* is the zero outcome. A lottery P can be decomposed into a *gain-part* P^+ and a *loss-part* P^-, where P^+ is the lottery P with all losses replaced by 0 and P^- is the lottery P with all gains replaced by 0. Luce and von Winterfeldt (1994) argued for the psychological plausibility of such a decomposition into a gain- and a loss-part. Domar and Musgrave (1944), Fishburn (1977), and Holthausen (1981) interpreted P^- as the risk part and P^+ as the return part of the lottery, using what they called a target outcome in the role of status quo.

A preference relation $\succcurlyeq$ is assumed over lotteries; the relations $\succ$, $\sim$, $\preccurlyeq$ and $\prec$ are defined as usual. V is a *representing function* or *representation* for $\succcurlyeq$ if it maps lotteries to the reals such that $P \succcurlyeq Q \Leftrightarrow V(P) \geqslant V(Q)$. If a representing function exists then $\succcurlyeq$ is a *weak order*, i.e. it is *complete* ($P \succcurlyeq Q$ or $P \preccurlyeq Q$ for all lotteries P,Q) and transitive.

Rank-dependent utility (RDU) holds if a representation exists of the form

$$(p_1,x_1;\ldots;p_n,x_n) \mapsto \sum_{j=1}^{n} \pi_j U(x_j), \tag{1}$$

explained next. U, the *utility function*, maps outcomes to the reals. The utility functions considered in this paper are all continuous and strictly increasing. The *decision weights* are defined as

$$\pi_j = w(p_1 + \cdots + p_j) - w(p_1 + \cdots + p_{j-1})$$

where $\pi_1 = w(p_1)$. w is a *probability transformation*, i.e. it is strictly increasing from $[0,1]$ to $[0,1]$ and satisfies $w(0) = 0$ and $w(1) = 1$. Continuity of w is left optional in our theorems and, hence, is not required in the definition.

Cumulative prospect theory (CPT) holds if the preference relation $\succcurlyeq$ can be represented by a CPT functional, defined next. Consider a lottery $P = (p_1,x_1;\ldots;p_n,x_n)$ and let

$$x_1 \geqslant \cdots \geqslant x_k \geqslant 0 > x_{k+1} \geqslant \cdots \geqslant x_n, \tag{2}$$

for some $k \in \{0,\dots,n\}$. The CPT value of the lottery is given by Eq. (1) with the following modifications. U is again the utility function, but now

$$U(0) = 0$$

is set. For the decision weights, we assume a probability transformation w^+ for gains and a probability transformation w^- for losses. For $j \leqslant k$ (gains), π_j is defined as in rank-dependent utility with the transformation function w^+, i.e.

$$\pi_j = w^+(p_1 + \cdots + p_j) - w^+(p_1 + \cdots + p_{j-1}).$$

For $j > k$ (losses), π_j is derived from w^- in a dual manner, i.e.

$$\pi_j = w^-(p_j + \cdots + p_n) - w^-(p_{j+1} + \cdots + p_n).$$

CPT can be written as the RDU value of the gain-part of the lottery with respect to a probability transformation w^+ plus a dual, obviously negative, RDU value of its loss-part with respect to a probability transformation w^-. The latter is dual because it would coincide with the regular RDU value if the dual of $w^-(p)$, defined by $1 - w^-(1-p)$, were used instead of w^-. Verification is left to the reader.

To summarize, the CPT-formula is

$$\begin{aligned}(p_1,x_1;\dots;p_n,x_n) \\ \mapsto \sum_{i=1}^{k} [w^+(p_1 + \cdots + p_i) - w^+(p_1 + \cdots + p_{i-1})]U(x_i) \\ + \sum_{j=k+1}^{n} [w^-(p_j + \cdots + p_n) - w^-(p_{j+1} + \cdots + p_n)]U(x_j). \end{aligned} \tag{3}$$

It follows that

$$\mathrm{CPT}(P) = \mathrm{CPT}(P^+) + \mathrm{CPT}(P^-) \tag{4}$$

where both terms on the right-hand side are RDU forms.

In general expected utility models with outcome set $\mathbb{R}_{++}$, constant proportional risk aversion can generate any utility function from the log/power family. In our model, however, utility must also be defined at zero, where the logarithm and negative powers are undefined. Hence, only the positive power functions are possible. A function $U : \mathbb{R} \to \mathbb{R}$ is from the *positive power family for gains* if

$$U(x) = \sigma^+ \cdot x^\alpha, \quad \text{with } \sigma^+ > 0 \text{ and } \alpha > 0, \text{ for all } x \geqslant 0, \tag{5}$$

and it is from the *positive power family for losses* if

$$U(x) = -\sigma^- \cdot |x|^\beta, \quad \text{with } \sigma^- > 0 \text{ and } \beta > 0, \text{ for all } x \leqslant 0. \tag{6}$$

The scale factors σ^+ and σ^- are positive so as to guarantee that the functions are strictly increasing. Because CPT requires $U(0)=0$, no location parameters have been added.

3. A preference characterization

The central condition in this paper is constant proportional risk aversion. It is usually studied when only gain outcomes are present. It then requires invariance of preference with respect to outcome multiplication by a common positive factor. On our domain with both gains and losses present, two extensions are conceivable. First, *strong constant proportional risk aversion* holds if preferences are invariant with respect to outcome multiplication by a common positive factor. That is, for all positive numbers μ,

$$(p_1,x_1;\ldots;p_n,x_n) \succcurlyeq (p_1,y_1;\ldots;p_n,y_n)$$
$$\Rightarrow (p_1,\mu\cdot x_1;\ldots;p_n,\mu\cdot x_n) \succcurlyeq (p_1,\mu\cdot y_1;\ldots;p_n,\mu\cdot y_n). \tag{7}$$

Second, (*weak*) *constant proportional risk aversion* holds if Eq. (7) is imposed only whenever either all outcomes $x_1,\ldots,x_n,y_1,\ldots,y_n$ are not losses or they all are not gains.

Strong constant proportional risk aversion is necessary for CPT with positive power utility and with the same powers $\alpha=\beta$ for gains and losses. Then the CPT functional is homogeneous of degree $\alpha=\beta$ and multiplying the outcomes by a positive constant does not affect the generated ordering of lotteries. (Weak) constant proportional risk aversion is implied by positive power utility but permits different powers $\alpha\neq\beta$ for gains and losses.

Empirical studies have found that utility functions for gains are mostly concave, utility functions for losses are probably mostly convex, and utility functions for losses are closer to linear than those for gains (Abdellaoui, 2000; Fennema and van Assen, 1998). These findings suggest that $0<\alpha<\beta<1$, in particular, therefore, $\alpha\neq\beta$ and strong proportional risk aversion does not hold. Indeed, if $(0.5,\$90;0.5,-\$6)\sim\$1$ then it seems plausible that $(0.5,\$90,000;0.5,-\$60,000)\prec\$10,000$. For these reasons, our main interest concerns constant proportional risk aversion with different powers for gains than for losses. Some empirical studies suggest that there are only weak relations between risk attitudes for gains and for losses at the individual level (Cohen et al. 1987), which adds to the desirability of independent variations of α and β.

The next preference condition weakens von Neumann–Morgenstern independence by imposing it only under a comonotonicity restriction. Such comonotonicity restrictions, introduced by Schmeidler (1989) (first version 1982), are essential in rank-dependent theories and are discussed by Diecidue and Wakker (2001) and Yaari (1987). In fact, the following preference condition weakens independence somewhat further by considering only maximal or minimal common outcomes. This version is particularly easy to formulate for decision under risk. We first define the probabilistic mixing of lotteries. Given two lotteries $P=(p_1,x_1;\ldots;p_n,x_n)$ and $Q=(q_1,y_1;\ldots;q_m,y_m)$, and

$\gamma \in [0,1]$, the *mixture* $\gamma P + (1-\gamma)Q$ is the lottery [4]

$$(\gamma p_1, x_1; \ldots; \gamma p_n, x_n; (1-\gamma)q_1, y_1; \ldots; (1-\gamma)q_m, y_m).$$

Definition 1. The preference relation satisfies *tail independence* if for all lotteries P, Q, C, C' and all $\gamma \in (0,1)$ the following holds:

$$\gamma P + (1-\gamma)C \succcurlyeq \gamma Q + (1-\gamma)C$$
$$\Leftrightarrow \quad \gamma P + (1-\gamma)C' \succcurlyeq \gamma Q + (1-\gamma)C'$$

whenever either all outcomes in the lotteries C, C' are at least as large as those in P, Q, or they are all at least as small.

The condition, extended to nonsimple lotteries, was introduced by Green and Jullien (1988), who called it ordinal independence. Von Neumann–Morgenstern independence is more restrictive because it does not impose restrictions on the outcomes of P, Q, C, C' and it also requires that the preferences should agree with those between the unmixed P and Q. Tail independence is implied by CPT, and is not affected by rank- and sign-dependence. The proof of this elementary result is presented in the main text because it clarifies the nature of the CPT form, in particular the way in which CPT combines the positive and negative part.

Lemma 2. CPT *implies tail independence.*

Proof. Take symbols as in Definition 1. The following equation is discussed for $s=+$, for $s=-$, and with s dropped. Note that the s superscript and mixing are compatible, e.g., $(\gamma P + (1-\gamma)C)^s = \gamma P^s + (1-\gamma)C^s$.

$$\mathrm{CPT}(\gamma P^s + (1-\gamma)C^s) - \mathrm{CPT}(\gamma Q^s + (1-\gamma)C^s)$$
$$= \mathrm{CPT}(\gamma P^s + (1-\gamma)C'^s) - \mathrm{CPT}(\gamma Q^s + (1-\gamma)C'^s).$$

Both for $s=+$ and for $s=-$, the outcomes of C^s and C'^s are either all rank-ordered above those of P^s and Q^s or all rank-ordered below, as they are without the superscript s. The preceding equality holds true for $s=+$ and $s=-$, CPT being an RDU form in both cases. (The $(1-\gamma)C^s$ and $(1-\gamma)C'^s$ parts of the RDU values cancel in both differences.) By summation (Eq. (4)), the equation also holds true if the superscript s is dropped. The equation implies, in particular, that the sign of the left-hand side is the same as the sign of the right-hand side, from which tail independence follows.

[4] For simplicity, we have not permuted the outcomes as would be required to maintain the rank-ordered notation of lotteries.

An alternative proof can be obtained by first showing that CPT is a special case of the functional of Green and Jullien (1988) and then invoking their derivation of the condition. $\square$

$\succcurlyeq$ satisfies *stochastic dominance* if $(p_1,x_1;\ldots;p_n,x_n) \succ (p_1,y_1;\ldots;p_n,y_n)$ whenever $x_j \geqslant y_j$ for all j and $x_j > y_j$ for at least one j with $p_j > 0$. The formulation used here is reminiscent of outcome monotonicity. On the domain of simple lotteries considered in this paper, the condition is equivalent to the more common formulations in terms of distribution functions.

The next condition imposes the common Euclidean continuity of $\mathbb{R}^n$ on n-outcome lotteries, for all n. That is, $\succcurlyeq$ satisfies *simple continuity* if, for any lottery $(p_1,x_1;\ldots;p_n,x_n)$, the sets $\{(y_1,\ldots,y_n) : y_1 \geqslant \cdots \geqslant y_n, (p_1,y_1;\ldots;p_n,y_n) \succcurlyeq (p_1,x_1;\ldots;p_n,x_n)\}$ and $\{(y_1,\ldots,y_n) : y_1 \geqslant \cdots \geqslant y_n, (p_1,y_1;\ldots;p_n,y_n) \preccurlyeq (p_1,x_1;\ldots;p_n,x_n)\}$ are closed subsets of $\mathbb{R}^n$. Note that this continuity condition concerns only variation in outcomes. Continuity with respect to variation in probabilities is not required by our continuity condition.

Theorem 3. For the preference relation $\succcurlyeq$ on the set of lotteries over $\mathbb{R}$, the following two statements are equivalent:

(i) CPT *holds with a positive power utility for gains and a, possibly different, positive power utility for losses.*

(ii) *The preference relation $\succcurlyeq$ satisfies the following conditions*:
  (1) *weak ordering*;
  (2) *stochastic dominance*;
  (3) *simple continuity*;
  (4) *tail independence*;
  (5) *constant proportional risk aversion.* [5]

If (i) *holds then the probability transformations are uniquely determined and utility is a ratio scale, i.e. it is unique up to a positive scale factor. The powers of utility for gains and losses in* (i) *are identical if and only if strong constant proportional risk aversion holds.*

The theorem shows that not only constant proportional risk aversion, but also strong constant proportional risk aversion, leads to CPT forms and not necessarily to RDU forms. We next discuss the uniqueness of utility in more detail and define loss aversion. Suppose that the utility function for gains is σ^+x^α and that the utility function for losses is $-\sigma^-|x|^\beta$, for positive σ^+,σ^-. Utility can be freely multiplied by a positive factor, which however must be the same for gains and losses. Hence, we can choose any utility $\mu\sigma^+x^\alpha$ for

[5] In the weak version, for gains and losses separately.

gains and $-\mu\sigma^{-}|x|^{\beta}$ for losses. Taking $\mu = 1/\sigma^{+}$ and defining $\lambda = \sigma^{-}/\sigma^{+}$, we get the following result.

$U(x) = x^{\alpha}$ for gains;

$U(0) = 0$;

$U(x) = -\lambda|x|^{\beta}$ for losses.

The scaling of utility just described has been generally adopted in parametric studies of CPT. It means setting $U(1) = 1$ and scaling loss aversion with $\lambda = -U(-1)/U(1)$. This scaling was used by Fishburn (1977) and Holthausen (1981) within the expected utility framework and by Luce and Fishburn (1991, Eq. (4)) and Tversky and Kahneman (1992) in their studies of CPT. Empirical estimations have usually found high values of λ (loss aversion), typically exceeding 2 (Holthausen, 1981; Tversky and Kahneman, 1992). These degrees of loss aversion have been used in new explanations of the equity premium puzzle.

Most papers on decision under risk invoke weak continuity, requiring not only continuity in outcomes (implied by our simple continuity) but also continuity in probabilities. There is, however, empirical and theoretical interest in discontinuities of the probability transformation at 0 and 1 (Bell, 1985; Cohen and Jaffray, 1988; Hadar and Seo, 1995; Prelec, 1998). Hence, we have presented the above more general result. Continuity in probability is characterized in Observation A.7 in the appendix.

The next corollary characterizes rank-dependent utility by restricting the preceding analysis to nonnegative outcomes. It generalizes existing results (Theorems 3 and 8 of Ebert (1988), Corollary 1 of Miyamoto and Wakker (1996)) because it does not impose continuity restrictions on the probability transformation function.

Corollary 4. For the preference relation $\succcurlyeq$ *on the set of lotteries over* $\mathbb{R}_{+}$, *the following two statements are equivalent*:

(i) RDU *holds with a positive power utility.*

(ii) $\succcurlyeq$ *satisfies the following conditions*:

 (1) *weak ordering*;

 (2) *stochastic dominance*;

 (3) *simple continuity*;

 (4) *tail independence*;

 (5) *constant proportional risk aversion.*

The probability transformation in (i) in the corollary is uniquely determined. The utility function is a ratio scale if the convention $U(0) = 0$ is maintained. If we permit adding an arbitrary constant, as is possible and common under RDU, then U is cardinal, i.e. it is unique up to scale and location.

A characterization of constant proportional risk aversion under expected utility is obtained if tail independence in (4) of Theorem 3 is replaced by full-force von Neumann–Morgenstern independence. The theorem then shows two ways of extending constant proportional risk aversion to losses, either with weak constant proportional risk aversion and different powers for gains and losses or with strong constant proportional risk aversion and identical powers. Corollary 1.1 in Blackorby and Donaldson (1982) describes a similar result with identical powers for gains and losses, for a subjective expected utility functional in the context of social choice.

The extension of the above results to nonsimple lotteries follows from Wakker (1993a, Corollary 4.5). The extension to lotteries with bounded support only requires one additional condition, i.e. that there exists a certainty equivalent for each nonsimple lottery. The extension to all nonsimple lotteries with well defined RDU/CPT values and unbounded support follows by adding Wakker's truncation-continuity condition, a condition also used by Nakamura (1995).

4. Conclusion

Our result constitutes the simplest preference foundation of CPT that is presently available for the special case of power utility, the most popular utility specification in empirical studies. Only elementary conditions are used. CPT necessarily follows as soon as tail independence, a natural weakening of von Neumann–Morgenstern independence that lies at the heart of rank dependence, and constant proportional risk aversion are assumed. We hope that this result will increase the tractability and economic interest of CPT.

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Appendix A. Proofs and continuity in probability

The main line of the proof of Theorem 3 is simple. First tail independence and some other conditions are used to obtain an overal rank-dependent additively decomposable representation as in Green and Jullien (1988). Next Ebert's (1988) technique is applied to the positive and negative parts separately to establish that these are rank-dependent forms. Eq. (4) then yields an overall CPT form.

There remain, however, many technical complications due to differences in structural assumptions between our model and others existing in the literature, such as the absence of continuity in the probability dimension in our model. The complications are exacerbated because of rank dependence and sign dependence. Rank-dependent proofs are notorious for their complications which have led to several misunderstandings in the literature (Wakker, 1993b). Therefore, a lengthy proof cannot be avoided. We will use Miyamoto and Wakker's (1996) variation of Ebert's result. The latter authors were unaware of Ebert's precedence.

An auxiliary notation is as follows. For $n \in \mathbb{N}$ and a probability tuple $(p_1,\ldots,p_n)$, $L(p_1,\ldots,p_n)$ denotes all lotteries of the form $(p_1,x_1;\ldots;p_n,x_n)$. Let us repeat that it is implicitly understood in this notation that $x_1 \geqslant \cdots \geqslant x_n$ and that the probabilities p_j are nonnegative and sum to one. The following lemma, trivial in the absence of rank dependence, is adapted here to rank dependence.

Lemma A.1. For each finite number of sets $L(q_1^1,\ldots,q_{n_1}^1),\ldots,L(q_1^m,\ldots,q_{n_m}^m)$, there exists a set $L(p_1,\ldots,p_n)$ that contains them all. In particular, for each finite set of lotteries, there is a set $L(p_1,\ldots,p_n)$ that contains them all.

Proof. As an illustration, $L(1/2,1/2)$ and $L(1/3,2/3)$ are contained in $L(1/3,1/6,1/2)$. For the general case, define the set of cumulative probabilities, $q_i^{j\prime} = q_1^j + \cdots + q_i^j$ for all i,j, take the set of all such $q_i^{j\prime}$, including 0 and 1, and rank-order them from lowest to highest. The differentials between these successive levels of cumulative probabilities are the p_j's. Verification is left to the reader. □

Proof of Theorem 3. We first assume Statement (i) and derive Statement (ii). Weak ordering is immediate and stochastic dominance follows because utility and the probability transformations are strictly increasing. Simple continuity follows from continuity of utility. Tail independence follows from Lemma 2. Constant proportional risk aversion holds because of homogeneity of the CPT functional for gains and for losses. (ii) has been established.

In the following part of the proof, Statement (ii) is assumed and Statement (i), as well as the uniqueness results, are derived. In the first lemmas, up to Corollary A.5, a CPT representation is derived on a fixed set $L(p_1,\ldots,p_n)$. This set being a subset of the lottery domain, the representation is, formally speaking, a restriction of a CPT representation. Similarly, the probability transformation functions, being defined only on a finite number of probabilities within the unit interval, are restrictions of probability transformations. At the end of the proof, the utility function, probability transformations, and CPT representation are extended to the entire domain.

Lemma A.2. Whenever $n \geqslant 3$ and $p_1,\dots,p_n$ are positive, there exist continuous strictly increasing functions $V_1,\dots,V_n$ such that $V_j(0)=0$ for all j, $\sum_{j=1}^{n} V_j(1)=1$, and

$$(p_1,x_1;\dots;p_n,x_n) \mapsto \sum_{j=1}^{n} V_j(x_j) \tag{A.1}$$

represents $\succcurlyeq$ on $L(p_1,\dots,p_n)$. The V_j's are uniquely determined.

Proof. Define $\mathbb{R}^n_{\downarrow} = \{x \in \mathbb{R}^n\colon x_1 \geqslant \cdots \geqslant x_n\}$. This set is identified with $L(p_1,\dots,p_n)$ in the obvious manner. Therefore, the preference relation $\succcurlyeq$ on $L(p_1,\dots,p_n)$ induces a preference relation, denoted by $\succcurlyeq'$, on $\mathbb{R}^n_{\downarrow}$. $\succcurlyeq'$ is a continuous weak order that is *monotonic*, i.e. $x \succ' y$ whenever $x_j \geqslant y_j$ for all j and $x_j > y_j$ for at least one j. Monotonicity follows from positiveness of the probabilities and stochastic dominance.

Tail independence of $\succcurlyeq$ implies *tail independence* of $\succcurlyeq'$, that is, if two elements of $\mathbb{R}^n_{\downarrow}$ have the first i or the last j coordinates in common, then the preference between them is independent of these common coordinates. To establish this claim, we compare the preference between

$$(p_1,x_1;\dots;p_m,x_m;\,p_{m+1},c_{m+1};\dots;p_n,c_n)$$

and

$$(p_1,y_1;\dots;p_m,y_m;\,p_{m+1},c_{m+1};\dots;p_n,c_n)$$

with the preference between

$$(p_1,x_1;\dots;p_m,x_m;\,p_{m+1},c'_{m+1};\dots;p_n,c'_n)$$

and

$$(p_1,y_1;\dots;p_m,y_m;\,p_{m+1},c'_{m+1};\dots;p_n,c'_n).$$

Define $\sum_{j=1}^{m} p_j=\gamma$, $P=(p_1/\gamma,x_1;\dots;p_m/\gamma,x_m)$, $Q=(p_1/\gamma,y_1;\dots;p_m/\gamma,y_m)$ (note that $\sum_{j=m+1}^{n} p_j=1-\gamma$), $C=(p_{m+1}/(1-\gamma),c_{m+1};\dots;p_n/(1-\gamma),c_n)$, and $C'=(p_{m+1}/(1-\gamma),c'_{m+1};\dots;p_n/(1-\gamma),c'_n)$. Tail independence of $\succcurlyeq$ implies that the two preferences considered here are the same. We conclude that tail independence holds for $\succcurlyeq'$.

The existence of the V_j functions now follows from a generalization of Theorem 3.2 of Wakker (1993b) described by Chateauneuf and Wakker (1993). Unfortunately, the details of the proof have to combine several results from different papers. They are described briefly. For $n \geqslant 3$, the condition CI of Wakker (1993b) (complete independence of preferences from common coordinates, also called (con)joint independence or strong separability or the sure-thing principle in the literature) has been weakened to tail independence here. Additive representability, i.e. existence of functions V_j as in the lemma, then still holds on the (rank-ordered) set $\mathbb{R}^n_\downarrow$. This claim follows from Corollary C.5 of Chateauneuf and Wakker (1993) and Gorman (1968): In Gorman's terminology, separability of $\{1,\ldots,i\}$ and $\{i,\ldots,n\}$ implies, within each box within $\mathbb{R}^n_\downarrow$, separability of $\{1,\ldots,i-1,i+1,\ldots,n\}$. This separability implies that every preference is independent of any single common coordinate, i.e. Wakker's (1993b) CI holds within each box. By Gorman (1968) or Debreu (1960), additive representability follows on each box within $\mathbb{R}^n_\downarrow$, i.e. local additive representability follows on the interior of our rank-ordered domain. Global additive representability follows from this result, Chateauneuf and Wakker (1993, Lemma C.5), and the absence of maximal and minimal elements.

The V_j's derived in the literature are usually unique up to a location and a common scale. Our choices of locations and scale uniquely determine these functions. QED

Lemma A.3. For the lotteries in $L(p_1,\ldots,p_n)$ with nonnegative outcomes, the representation in Lemma A.2 is the restriction of a CPT *form, with U from the positive power family and $U(0)=0$, $U(1)=1$. U is unique and so is the (restriction of a) probability transformation function w^+ on its domain which consists of all values $\sum_{j=1}^{i} p_j$, $i=1,\ldots,n$. On this domain, w^+ is strictly increasing.*

Proof. Let $V_1,\ldots,V_n$ be as in Lemma A.2. Define $\succcurlyeq'$ on $\mathbb{R}^n_\downarrow$ as in the proof of Lemma A.2 and restrict attention to lotteries with nonnegative outcomes. If the representation in Lemma A.2 is a CPT form, then necessarily $U(x)=\sum_{j=1}^{n} V_j(x)$ for all nonnegative x. We therefore have to define U in that manner and show that CPT holds for this uniquely determined utility function. Note that $U(0)=0$ and $U(1)=1$.

Fix some $1 \leqslant m < n$. We restrict attention to n-tuples of nonnegative outcomes for which the first m coordinates are identical and so are the last $n-m$ ones. These n-tuples are written as $(x_{1,m}, x_{m+1,n})$ for $x_{1,m} \geqslant x_{m+1,n} \geqslant 0$. $\succcurlyeq'$ is represented on this two-dimensional subset of n-tuples by $V_{1,m}(x_{1,m}) + V_{m+1,n}(x_{m+1,n})$ where $V_{1,m}(x_{1,m}) = \sum_{j=1}^{m} V_j(x_{1,m})$ and $V_{m+1,n}(x_{m+1,n}) = \sum_{j=m+1}^{n} V_j(x_{m+1,n})$. Due to constant proportional risk aversion, $\succcurlyeq'$ is invariant under positive scalar-multiplication of outcomes on this subset of lotteries. Miyamoto and Wakker (1996, Theorem 2) consider only gains (>0) and show that $V_{1,m}$ and $V_{m+1,n}$ are proportional when restricted to gains. Obviously, they are also proportional if the zero outcome is included in the domain because all functions are zero at zero. $V_{1,m}$ and $V_{m+1,n}$ must be proportional to their sum $V_{1,m} + V_{m+1,n} = U$ and, hence, they are of the form

$$V_{1,m} = \pi_{1,m} U \quad \text{and} \quad V_{m+1,n} = \pi_{m+1,n} U \tag{A.2}$$

for positive uniquely determined $\pi_{1,m}$ and $\pi_{m+1,n}$ that sum to one. Theorem 2 of Miyamoto and Wakker (1996) implies also that, for gains, U is either from the positive power family, or logarithmic, or from the *negative power family*; the latter is defined as in Eq. (5) but with α and σ^+ negative. The logarithmic family and the negative power family are excluded in our case because strict increasingness (or continuity) at zero would imply that $V_{1,m}(0) = -\infty$ which is excluded, $V_{1,m}(0)$ being zero. We conclude that

$$U \text{ is a positive power function for nonnegative outcomes.} \tag{A.3}$$

Define $\pi_1 = \pi_{1,1}$, $\pi_j = \pi_{1,j} - \pi_{1,j-1}$ for $j = 2,\ldots,n-1$, and $\pi_n = \pi_{n,n}$ ($= \pi_{1,n} - \pi_{1,n-1}$ if we set $\pi_{1,n} = 1$). By definition, $V_{1,1} = V_1 = \pi_1 U$. Eq. (A.2) implies that $V_j = V_{1,j} - V_{1,j-1} = \pi_{1,j}U - \pi_{1,j-1}U = \pi_j U$ for $j = 2,\ldots,n-1$, and then also $V_j = \pi_j U$ for $j = n$. These equalities and monotonicity imply that all π_js are positive. From their definition it follows that the π_j's sum to one (this also follows from the equality $V_j = \pi_j U$ and the definition of U). A (restriction of a) probability transformation function w^+ is next defined so as to properly transform cumulative probabilities into cumulative decision weights. That is, $w^+(\sum_{i=1}^{j} p_i) = \sum_{i=1}^{j} \pi_i$. Because all π_i are positive, w^+ is strictly increasing on its domain.

It follows that the representation of Lemma A.2 is the restriction of a CPT form with respect to w^+ and U on the set of lotteries in $L(p_1,\ldots,p_n)$ with nonnegative outcomes. Uniqueness of U was established when it was defined, $\pi_j = V_j(1)/U(1)$ uniquely defines each π_j. QED

Lemma A.4. For the lotteries in $L(p_1,\ldots,p_n)$ with nonpositive outcomes, the representation in Lemma A.2 is the restriction of a CPT *form, with U from the positive power family and $U(0) = 0$. U is unique and so is the (restriction of a) probability transformation function w^- on its domain which consists of the values $\sum_{j=i}^{n} p_j$, $i = 1,\ldots,n$. On this domain, w^- is strictly increasing.*

Proof. Only lotteries with nonpositive outcomes are considered. It can be demonstrated by a reasoning similar to the proof of Lemma A.3 that the representation of Lemma A.2 is again a restriction of a CPT form. Because of the scaling convention $\sum_{j=1}^{n} V_j(1) = 1 = U(1)$, there is no more liberty for choosing the scale of utility for losses. It is uniquely determined by $U(-1) = \sum_{j=1}^{n} V_j(-1)$ (which was defined as the loss aversion parameter λ in the main text). Other than this, the reasoning for nonnegative outcomes can entirely be repeated. U is also a positive power function for losses, but may obviously have a different exponent than for gains. w^- is related to the decision weights for losses through the formulas $w^-(p_n)$, $w^-(p_n + p_{n-1}) - w^-(p_n)$, etc. QED

Corollary A.5. The representation in Lemma A.2 is (the restriction of) a CPT *representation with U from the positive power family for gains and also for losses, and $U(0) = 0$ and $U(1) = 1$. U is unique and so are the* (*restrictions of*) *the probability transformation function* w^+ *at the values* $\sum_{j=1}^{i} p_j$, $i = 1,\ldots,n$, *and* w^- *at the values* $\sum_{j=i}^{n} p_j$, $i = 1,\ldots,n$.

Proof. Lemmas A.3 and A.4 have demonstrated that the representation of Lemma A.2 is a CPT form for nonnegative outcomes, and also for nonpositive outcomes. Next consider a lottery P with both gains and losses. The additive representation of Lemma A.2 is a sum of its value at P^+ and at P^-. Hence, the value of the lottery P is $\text{CPT}(P^+) + \text{CPT}(P^-)$. By Eq. (4), CPT follows. QED

The following analysis extends the CPT representation from fixed sets $L(p_1,\ldots,p_n)$ to the set of all simple lotteries. Lemma A.6 prepares.

Lemma A.6. The CPT *representations on two different sets of lotteries* $L(p_1,\ldots,p_n)$ *and* $L(q_1,\ldots,q_m)$ *with* $m \geqslant 3$, $n \geqslant 3$, *and all probabilities positive*, *coincide in the sense that the utility function* U *is the same for both sets and* w^+ *and* w^- *agree on the intersection of their domains.*

Proof. By Lemma A.1, there exists a set $L(r_1,\ldots,r_i)$ that contains both sets of lotteries in the lemma. We may assume that all probabilities r_j are positive by dropping the zero probabilities. Obviously, $i \geqslant 3$. By the uniqueness result of Lemma A.4, the CPT representation on $L(r_1,\ldots,r_i)$ agrees with the one of $L(p_1,\ldots,p_n)$ and the one of $L(q_1,\ldots,q_m)$ on their respective domains. On their common domain both of the latter two CPT representations agree with the former and, hence, with each other. QED

Proof of Theorem 3 (*Conclusion*)**.** By considering all sets $L(p_1,\ldots,p_n)$, w^+ and w^- are determined on the entire unit interval $[0,1]$. They are strictly increasing by Lemmas A.3 and A.4. Because every lottery is contained in a set $L(p_1,\ldots,p_n)$ satisfying the requirements of Lemma A.2, a CPT value is determined for every lottery. By Lemma A.6, the CPT value is independent of the particular set $L(p_1,\ldots,p_n)$. Every pair of lotteries is contained in some set $L(p_1,\ldots,p_n)$, on this set CPT represents preference, therefore CPT represents preference between every pair of lotteries.

The uniqueness of the representation is derived next. For any other CPT representation with U^*, w^{+*}, w^{-*}, reconsideration of the preceding analysis shows that $U^*(.)/U^*(1)$ must agree with $U(\cdot)$, and next that the two probability transformations w^{+*} and w^{-*} must agree with w^+ and w^-, respectively. Conversely, for any positive τ, U can be replaced by τU.

Finally, we consider strong constant proportional risk aversion. It was explained in the main text that this condition holds under CPT with positive power utility functions with the same powers $\alpha = \beta$ for gains and losses. We next show that the condition does not hold if $\alpha \neq \beta$. Take any positive x, y

such that $(0.5,x;0.5,-y) \sim 0$. By stochastic dominance and continuity, such x and y can always be found. Then, because $\alpha \neq \beta$, $(0.5,2x;0.5,-2y)$ is not equivalent to 0, violating strong proportional risk aversion. Therefore, strong proportional risk aversion holds if and only if $\alpha = \beta$. The proof of Theorem 3 is now complete. □

Proof of Corollary 4. This proof follows from restricting the proof of Theorem 3 to nonnegative outcomes. Logarithmic and negative power utility are still excluded because the zero outcome is contained in the domain. If outcomes were restricted to $\mathbb{R}_{++}$, utility could also have been logarithmic or a negative power. □

We finally characterize continuity of the probability transformations. $\succcurlyeq$ satisfies *continuity in probabilities for gains* on $]0,1[$ if, for all probabilities p and outcomes $X > x > 0$:

$$\text{If } (p,X;1-p,0) \succ (1,x) \text{ then there exists } q < p \text{ such that still } (q,X;1-q,0) \succ (1,x), \tag{A.4}$$

and the same holds with the two preferences and the inequality reversed, i.e.

$$\text{If } (p,X;1-p,0) \prec (1,x) \text{ then there exists } q > p \text{ such that still } (q,X;1-q,0) \prec (1,x) \tag{A.5}$$

$\succcurlyeq$ satisfies *continuity in probabilities for gains at* 1 if Eq. (A.4) holds for $p = 1$. $\succcurlyeq$ satisfies *continuity in probabilities for gains at* 0 if Eq. (A.5) holds for $p=0$. Similar conditions can be defined for losses instead of gains by assuming that $X < x < 0$ in Eq. (A.4) and Eq. (A.5) (and reordering the outcomes under the notational convention of rank-ordered outcomes) and reversing the strict preferences. The next result is similar to Wakker's (1994) Theorem 12 for RDU.

Observation A.7. Assume that Statement (i) *in Theorem 3 holds. Then continuity conditions of the probability transformations agree with the related continuity conditions of $\succcurlyeq$ in probability, with w^+ related to gains preference conditions and w^- to loss preference conditions.*

Proof. The range of utility is an interval with 0 in its interior. Hence, the following facts are implied by substitution and strict increasingness of the probability transformation functions: Eq. (A.4) implies that w^+ cannot jump down to the left of p and, hence, is left continuous at p. Eq. (A.5) implies that w^+ cannot jump up to the right of p, and therefore is right continuous at p. Similar facts hold for w^- and losses. □

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