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BOOK REVIEW

Luce's Paradigm for Decision under Uncertainty

R. Duncan Luce. **Utility of Gains and Losses: Measurement-Theoretical and Experimental Approaches.** London: Lawrence Erlbaum Publishers, 2000. xii + 331 pp. $59.95. ISBN 0-8058-3460-5.

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To explain the historical meaning of Luce's book, let me start this review with some comments on Savage (1954). This work is generally praised for its impressive justification of subjective expected utility. Savage's book makes another important contribution by laying down the paradigm of decision under uncertainty as commonly used today: A decision maker determines his preference relation over "acts." The result of an act is a *consequence*. This consequence depends on which element of a set of *states* (of nature) is the true one. Formally, *acts* are functions from the state space to the consequence space, with $f(s)$ the consequence under act f if s is the true state of nature. For example, $(x, A; y, B; z, C)$ denotes an act that assigns consequence x to all states s in *event* A $(A \subset S)$, consequence y to all states s in event B, and consequence z to all states s in event C, where $\{A, B, C\}$ partitions S. The decision maker is uncertain about which state of nature is the true one and

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hence about the consequence of a chosen act. Savage's theory is one-shot and static, assuming only one decision at one moment in time and a one-time resolution of the uncertainty.

In early works, e.g., Ramsey (1931), a clear modeling of the primitives such as in Savage's approach was not yet available. After Savage (1954) virtually everyone in the field, the reviewer included, has routinely followed Savage's paradigm and used his primitives. In many situations, however, other models can be considered. Sometimes it is more natural to let states of nature be maps from acts to consequences (a manager considering investments for the coming year with uncertain net profits at the end) or to let consequences be pairs of states and acts. Finally, in most situations we need dynamic models rather than one-shot models.

Luce has, throughout the years, adopted a different and more creative style of working than most researchers. He developed, from scratch, his own model and primitives for decision under uncertainty so as to best suit his experimental and theoretical interests. The basic aspects of his ideas could already be recognized in Chapter 8 of Krantz *et al.* (1971), a book which after Savage (1954) and de Finetti (1937) was most influential for the reviewer's academic thinking. The basic differences with Savage's model were already discussed there (Section 8.6.3).

In the 1970s and 1980s Luce mostly worked on other topics. He returned to decision under uncertainty at the end of the 1980s, catching up with the rank-dependent approach initiated by Quiggin (1981) and Schmeidler (1989, first version 1982). Simultaneously with Tversky and Kahneman (1992) and independently of Starmer and Sugden (1989, Appendix), Luce developed his sign-dependent extension of the rank-dependent model, extending ideas of Luce and Narens (1985) to multiple-consequence gambles.

Unfortunately, many of the current generation of researchers, in particular economists, do not know Luce's basic model of the early 1970s. Hence his many papers based on this paradigm have not been well-understood and have not received the attention they deserve. Repeating all basic assumptions in every research paper is not feasible, and there was a danger that Luce's rich ideas would be lost. His current book solves the problem. It describes all his ideas from scratch, thus providing the key to his work.

Let me take Savage's model as a point of departure and describe how Luce's model varies. The differences are summarized in Table 1, the details of which are discussed hereafter. Luce uses "experiments" to consider what can be interpreted as conditional acts. For instance, in one experiment I may have a car available to drive to work and in another I may instead have a bike at my disposal to cycle to work. Savage's model requires consideration of a state of nature such as the car needing gas were I to have the car available *and* the brakes of my bike not functioning were I to have the bike. Luce only conditions on the separate experiments and hence need not consider such unrealistic combinations of events.

Following Krantz *et al.* (1971, Chapter 8), Luce deliberately does not distinguish between acts and states as strictly as Savage does. Experiments can be conditioned on events as well as on the choice of acts. In Luce's approach, my going by bike or car can be my own decision and also an uncertain event to which I can assign probabilities. In this respect Luce's model is similar to that of Jeffrey (1965).

TABLE 1

Points of Comparison between Savage and Luce

Savage	Luce
One universal event S (state space)	Conditioned on various "experiments"
Strict separation of acts and states	No strict separation of acts and states
Acts are functions; hence: $(x, A; y, B; y, C) \equiv$ $(x, A; y, B \cup C) \equiv$ $(x, A; y, C; y, B)$	Acts are $2n$-tuples; hence: $(x, A; y, B; y, C) \neq$ $(x, A; y, B \cup C) \neq$ $(x, A; y, C; y, B)$
Compound gambles are either not assumed or immediately reduced	Compound gambles are not reduced
Events are one-shot	Events can be repeated independently

Next consider the acts $(A, x;\ B, y;\ C, y)$ and $(A, x;\ B \cup C, y)$, for a partition $\{A, B, C\}$ of S. Both acts assign consequence x to all states in A and y to all states in B and C. In Savage's model these acts are identical *by definition*, both designating the same mathematical function. In Luce's approach, "coalescing" can be violated; i.e., $(A, x;\ B, y;\ C, y)$ is formally a sextuple and hence can be treated differently than the quadruple $(A, x; B \cup C, y)$. Luce permitted such formal differences already in the early 1970s. Almost everyone will agree that these differences do not have any normative basis, and hence this aspect of Luce's model originally received little interest. During the last decades, however, it became better understood that the deviations from rational models found empirically are at very basic levels. For example, experiments demonstrated that subjects can treat the act differently when it is described as $(A, x; B, y; C, y)$ than when it is described as $(A, x; B \cup C, y)$ (Tversky & Kahneman, 1986, p. 178, problem 7). Moreover, this effect seems to be a major cause for experimental violations of expected utility (Starmer & Sugden, 1993; Birnbaum, 1999). Whereas it was originally believed that such irrationalities are of minor importance and should not be incorporated in models, their importance is well understood today. In Savage's approach these phenomena cannot even be defined. Researchers interested in a formal framework for analyzing the effect will find it readily available in Luce's book.

A further difference is that Luce considers *compound* gambles (the consequences of which can be gambles again), whereas Savage either does not assume them or identifies them with their reduced forms. Luce permits independent repetitions of the events considered. The "experiment" of my taking the bike or the car to work can be repeated as often as desired. Finally, Luce often invokes another empirical primitive, i.e., a joint receipt of gambles which designates a generalized addition operation. I do not consider Luce's or Savage's approach superior. Which is more natural or complex depends on the context and application, and both approaches deserve to be studied.

Chapter 1 of Luce's book opens up with a highly valuable introduction to the primitives of his model (Sections 1.1.1–1.1.6) which provides the key to all of his work on uncertainty, followed by discussions of the meaning of preference. Chapter 2 discusses elementary choice principles (transitivity, monotonicity, and more

advanced ones). Chapter 3 discusses the rank-dependent models for binary (two-consequence) gambles. "Event commutativity" and other conditions based on compound gambles are used to provide appealing new derivations of rank- and sign-dependent models. In the same way, an appealing modification of Prelec's (1998) important result on probability transformations is presented in Theorem 3.4.1.

Chapter 4 introduces joint receipt, where the joint receipt of \$2 and \$3 need not be the same as the receipt of \$5. Joint receipt can also be applied to nonquantitative consequences. A "riskless" (my term) function V representing joint receipt can then be compared to a "risky" utility function U. Again, appealing conditions are presented by means of compound gambles. Chapter 5 turns to multiple-consequence rank- and sign-dependent models. I like the separate treatment of binary and multiple-consequence gambles because the former are more general and less controversial than the latter. Coalescing is discussed in detail. Chapters 6 and 7 consider the interplay between, first, the mixing of gains and losses and, second, the joint receipt operation. Treating gains differently than losses leads to new and more substantial departures from the classical linear model, with nonbilinear representations and violations of monotonicity for large probabilities. These departures are backed up by empirical evidence. Finally, Chapter 8 concludes. Errata are given at Luce's homepage.

The formal analyses are at a high level. They are illustrated and motivated by reviews of empirical evidence throughout the book. Luce thus demonstrates an impressive depth and broadness. These points constitute, however, not only a strong point but also a weak point of the book. Not many readers will be able to understand all empirical subtleties or all details of the formal analyses, and even fewer will be able to understand both. Several parts, while interesting in their own right, do not contribute to the line of the book. Examples are the criticisms of Tversky (1969) and Starmer (1999) (Subsections 2.2.3 and 5.5.2.1). Whereas the basic framework of Luce's theory makes this book a classic with a long-term impact, I think that many of its reviews of experimental details will lose their interest over time.

It should be understood that consequence monotonicity is assumed throughout the book. This condition is logically implied by, and usually considered intuitively equivalent to, two dynamic decision principles mostly known as consequentialism and dynamic consistency (Machina, 1989). Together with the accounting indifferences (= reduction of compound lotteries extended to uncertain events), consequence monotonicity implies expected utility. Luce deviates from expected utility by abandoning the accounting indifferences, for which there indeed is empirical evidence. There is, however, also evidence that the two conditions underlying consequence monotonicity are problematic as well (Cubitt, Starmer, & Sugden 1998; Busemeyer, Weg, Barkan, & Ma, 2000). Luce mentions the restrictive nature of consequence monotonicity on p. 45 l. 5, l. 21, but presents it as almost self-evident in many other places (p. 45 l. 5, p. 94 l. 1). More complex considerations arise for the mixed-consequences and joint-receipt model of Chapter 7 (pp. 250, 259, 260). The derivation of the power weighting function (Eq. 3.21), the central place given to it in his book as locally rational, and the criticism of weighting function families (e.g., of Tversky & Kahneman 1992, see Subsection 3.4.3.1) for not including the power function as a special case, all presuppose consequence monotonicity.

Luce's direct and outspoken way of criticizing other works, without wasting time and paper on inefficient diplomacy, is refreshing and stimulating. He often gives balanced descriptions of arguments counter to his own viewpoints.

Sometimes, however, I disagree with Luce's criticisms. Subsections 3.4.2.1 and 3.4.2.2 attempt to cast doubt on the empirical claims by Kahneman and Tversky (1979) and Tversky and Kahneman (1992). These empirical claims will be confirmed in discussions of other evidence, e.g., Subsections 3.4.2.3–3.4.2.5 on probability weighting. Apparently Luce wants to suggest that Kahneman and Tversky, while making the correct empirical claims, did not justify them properly. First I think that even if Luce's suggestion were correct, it would not be worth taking the reader's time for this criticism. Although Luce's first and third reasons mentioned on p. 94 may have a point, I disagree with his other suggestions. Kahneman and Tversky's justification, that the parametric families and parameters chosen fit the data well, is sound, but that point is completely ignored by Luce. In particular, Kahneman and Tversky give a justification for their power utility. On p. 94, Luce's "strong reason" for exponential utility is no more than it being a possible alternative,[1] and his suggestion that fitting exponential utility might give other qualitative conclusions is unfounded. In addition, some typos should be noted in Subsections 3.4.2.1 and 3.4.2.2: superadditivity should be subcertainty, subadditivity should be supercertainty, Eq. 3.26 is to be used for $p+q<1$ with x and y unconstrained as well as for $x>0>y$, and the reference to Subsection 4.4.6 on p. 94 should also include Section 4.5 (Luce, personal communication).

I also disagree with Luce's claim on p. 29 that axioms should be judged individually for scientific goals ("trying to understand what is going on") and his claim on p. 109 that restricted solvability does not impose restrictions on behavior as such. Section 9.1 of Krantz *et al.* (1971) gives an example where restricted solvability in isolation does not impose restrictions on behavior, but in combination with other axioms it does.

This book for the first time permits a study, from scratch, of Luce's fascinating paradigm for decision under uncertainty. Its central topic, the rank- and sign-dependent model, looks (to the reviewer) like the most promising one presently available. Many appealing axiomatizations are given, in particular when they are based on natural special cases of accounting indifferences. All results are based on up-to-date mathematical developments and are related to the latest empirical findings. This book has delivered mathematical psychology at its best. It will be a continuing source of inspiration for the next generation of researchers in this area.

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[1] Luce writes on p. 276: "power functions and exponential ones seem to do rather well."

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