Relating Risky to Riskless Preferences, and Their Joint Irrationality: A Comment on Oprea (2024)

PETER P. WAKKER
Erasmus School of Economics, Erasmus University Rotterdam
Wakker@ese.eur.nl

April, 2025

ABSTRACT. This comment shows that Oprea's (2024) findings do not falsify, but corroborate, probability weighting and loss aversion, contrary to his claims. Complexity does not replace them, but is an important factor supporting and explaining them. Oprea's contribution lies in his ingenious stimuli. They reveal irrationalities in risky preferences, general perceptual principles underlying them, and the importance of biases for economics, more convincingly than done before.
(JEL C91, D81, D91)

Oprea (2024) uses ingenious riskless "mirror" stimuli and finds clearly irrational preference patterns between them. He argues that these preferences must be unrelated to preferences that (1) are rational and (2) concern risky stimuli. However, they are found to be closely related to probability weighting and loss aversion for risky preferences, called the classical pattern by Oprea, abbreviated CP henceforth. Oprea then concludes that CP is rejected, and that complexity can *replace* CP. No paper cited him differently as yet.

Banki et al. (2025), which preceded this comment, criticized Oprea's experiment and results. This comment is complementary. It accepts Oprea's basic empirical finding, that riskless stimuli can reveal irrational CPs similar to risky stimuli, a finding that has in fact been established for many decades. The novelty of this note is to criticize Oprea's aforementioned inferences: they are logically flawed. Contrary to his suggestions and the general citations of his paper, his findings do not falsify, but instead corroborate CP for risk, as will be explained. I will specify four confusing aspects of Oprea (2024) that have unavoidably led to the misunderstandings. Then implications are discussed. Finally, I specify what Oprea's valuable contribution is, and why it is important.

To prepare, two citations. Contrary to Oprea's suggestions, CP was not intended to be rational. Thus, Kahneman and Tversky (1979) wrote:

> that values are attached to changes rather than to final states, and that decision weights do not coincide with stated probabilities. These departures from expected utility must lead to *normatively unacceptable* consequences …In these circumstances the *anomalies* implied by prospect theory are expected to occur. [italics added] (p. 277; Observation 2)

OBSERVATION 1. Kahneman and Tversky (1979) did not intend CP to be rational[1], but qualified it as irrational.

Second, contrary to Oprea's suggestions, CP was not intended to be confined to risky preferences. Thus, Kahneman and Tversky (1979) wrote:

> carriers of value are changes in wealth or welfare … This assumption is compatible with *basic principles of perception and judgment*. Our perceptual

[1] I use this common term as equivalent to Orea's "welfare relevant" for proper policy decisions. Kahneman and Tversky did not consider CP to be welfare relevant in Oprea's sense. They also considered bounded rationality as a possible explanation (sometimes even rationalization) of CP.

> apparatus is attuned to the evaluation of changes …. When we respond to attributes such as brightness, loudness, or temperature, … defines an adaptation level, or reference point … The same principle applies to non-sensory attributes such as health, prestige, and wealth. … Many sensory and perceptual dimensions share the property that the psychological response is a concave function of the magnitude of physical change. … room temperature … this principle applies in particular to the evaluation of monetary changes … the value function … is normally concave above the reference point … and often convex below. … Some support for this hypothesis has been reported by Galanter and Pliner [17], who scaled the perceived magnitude of monetary and non-monetary gains and losses. The above hypothesis regarding the shape of the value function was based on responses to gains and losses in a *riskless context*. We propose that the value function which is derived from *risky choices* shares the same characteristics. (pp. 277-278) [italics added]

OBSERVATION 2. Based on their psychological expertise, Kahneman and Tversky derived CP from general cognitive and perceptual principles (insensitivity and reference dependence) that occur in many riskless contexts as they do under risk.

Online Appendixes C-F give many further citations, also from Tversky & Kahneman (1981, 1986, 1992) and Wakker's (2010) textbook on prospect theory, that confirm Observations 1 and 2. In particular, for riskless time preferences it has long been understood that they exhibit phenomena very similar to risk (Ebert & Prelec 2007).

Oprea's empirical findings are in full agreement with Observations 1 and 2. The simple conclusion at this stage is that Oprea did not provide any falsification of CP, or reason to replace it. More involved is the question how Oprea and his readers could have come to the opposite conclusion. Four confusing aspects of Oprea's paper have led to this misunderstanding, explained next.

First, Oprea does not explain that Kahneman and Tversky endorsed Observations 1 and 2, crediting neither them nor many other predecessors. This absence of crediting even implicitly and incorrectly suggests that CP would not have been based on these views. For instance, Oprea claims, contrary to Observation 2:

> a key prediction of standard risk preference-based interpretations of the classical pattern (e.g., prospect theory) is that the pattern **should only arise in the presence of risk** … that subjects …will be …complexity *insensitive*. (p. 3801) [italics from original] [bold added]

Obviously, no standard interpretation of prospect theory has ever made the purported extreme predictions.

Second, Oprea uses the term "preference" in an unconventional manner. His Footnote 1 explains that he lets it refer only to rational choices, whereas in economics it commonly refers to revealed preferences that may be irrational. Oprea writes:

> much of the behavior motivating our most important behavioral theories of risk derive from complexity-driven mistakes *rather than* true risk *preferences*. (p. 3789, abstract) (italics added)

With Oprea's terminology understood, this claim and similar claims are correct, but they are merely restatements of Observation 1 and they are not very informative. However, readers generally mistook Oprea's term "preference" in its common meaning (revealed), and then the claims entail empirical falsifications of CP. The more so as such claims are repeated throughout Oprea's paper (Online Appendix B cites 14 such claims by Oprea). Those claims would then be very informative, were it not that they then are incorrect. CP has been derived from well-observed "true risk preferences" if the latter term refers to observed revealed preferences that are allowed to be irrational.[2] An underlying logical flaw here: if risk preferences share properties with other preferences, it does not imply that they are not risk preferences.

Third, Oprea's Sections I-II overstate the similarities between lottery and mirror preferences, calling them "virtually identical" and of the same strength (Result 5). Those sections throughout mention complexity as the only factor explaining everything. Banki et al. (2025) criticized these suggestions. Only Oprea's Sections III-IV properly acknowledge differences between lotteries and mirrors, and explanatory factors typical of risk and beyond complexity, confirming Banki et al. (2025). Whereas risky preferences indeed share properties and underlying mechanisms with other preferences, they have them in different degrees and in part based on different underlying mechanisms. They also have unique properties not found in other domains, such as the rationality of separability/independence due to the mutual exclusiveness of disjoint events, a point recognized throughout history (Alchian 1953 p. 37; Moscati 2016 p. 225; Tversky & Kahneman 1986 p. S252; von Neumannn & Morgenstern 1947 §3.3.2). Such observations, and Oprea's Sections III-IV, go against his first two sections and take away most of his claims.

---

[2] If there are debates about this widely accepted claim, then Oprea's evidence only supports it via Observations 1 and 2.

Fourth, continuing on the preceding misunderstandings, Oprea often suggests that complexity should *replace* CP, as in the text cited above and the 14 similar texts cited in Online Appendix B. In reality, complexity is an important factor explaining and supporting CP, as has often been observed, rather than replacing it. Recent analyses from that perspective include Armantier & Treich (2016), Spiliopoulos & Hertwig (2023), and Zilker, Hertwig, & Pachur (2020).

The four confusing aspects have unavoidably led to the general misunderstanding in the field that Oprea (2024) would have falsified CP. All 35 citations of Oprea's paper listed by Google Scholar on 2 April 2025 cited him affirmatively[3], and, understandably, none showed awareness of the above problems. In particular, none of these modern references showed awareness of the precedence by Kahneman and Tversky, half a century ago, on Observations 1 and 2.

The important novelty of Oprea (2024) lies in his ingenious stimuli. They show how to maximize the similarity between risky and riskless stimuli, and how to demonstrate irrationalities more convincingly than ever before. That is, they show Observations 1 and 2 more clearly than ever before. Banki et al. (2025) revealed problems in Oprea's experiment, showing that more careful experiments are needed. I believe that with the right levels of incentives, cognitive ability of subjects, clarity of instructions, and, importantly, complexity of stimuli, CP can be found convincingly for mirror stimuli, confirming Oprea's basic empirical claims. A first investigation (Wu 2025) did not find such levels and it remains an important topic for future research. Initiated, indeed, by Oprea (2024).

Even if actual empirical demonstrations of the role of complexity are more involved than suggested by Oprea, the general validity of Observation 2 is beyond doubt. Oprea's (2024) stimuli show the relevance of biases and heuristics for economics, and the need for further studies, better than ever before. His overly provocative presentation, criticized in this comment, does have the useful impact of stirring the interests in the field. However, as explained in this comment, the implications are different than suggested by Oprea and commonly taken in the literature as yet.

[3] Banki et al. (2025) criticized the experiment but not the logic of Oprea (2024).

REFERENCES

Alchian, Armen A. (1953) "The Meaning of Utility Measurement," *American Economic Review* 43, 26–50.

Armantier, Olivier & Nicolas Treich (2016) "The Rich Domain of Risk," *Management Science* 62, 1954–1969.

Banki, Daniel, Uri Simonsohn, Robert Walatka, & George Wu (2025) "Decisions under Risk Are Decisions under Complexity: Comment," working paper.

Ebert, Jane E.J. & Drazen Prelec (2007) "The Fragility of Time: Time-Insensitivity and Valuation of the Near and Far Future," *Management Science* 53, 1423–1438.

Moscati, Ivan (2016) "How Economists Came to Accept Expected Utility Theory: The Case of Samuelson and Savage," *Journal of Economic Perspectives* 30, 219–236.

Kahneman, Daniel & Amos Tversky (1979) "Prospect Theory: An Analysis of Decision under Risk," *Econometrica* 47, 263–291.

Oprea, Ryan (2024) "Decisions under Risk are Decisions under Complexity," *American Economic Review* 114, 3789–3811.

Spiliopoulos, Leonidas & Ralph Hertwig (2023) "Variance, Skewness and Multiple Outcomes in Described and Experienced Prospects: Can One Descriptive Model Capture It All?," *Journal of Experimental Psychology: General* 152, 1188–1222.

Tversky, Amos & Daniel Kahneman (1981) "The Framing of Decisions and the Psychology of Choice," *Science* 211, 453–458.

Tversky, Amos & Daniel Kahneman (1986) "Rational Choice and the Framing of Decisions," *Journal of Business* 59, S251–S278.

Tversky, Amos & Daniel Kahneman (1992) "Advances in Prospect Theory: Cumulative Representation of Uncertainty," *Journal of Risk and Uncertainty* 5, 297–323.

von Neumann, John & Oskar Morgenstern (1944, 1947, 1953) "*Theory of Games and Economic Behavior*." Princeton University Press, Princeton NJ.

Wakker, Peter P. (2010) "*Prospect Theory: For Risk and Ambiguity*." Cambridge University Press, Cambridge, UK.

Wu, George (2025) "There is No Smoke with Mirrors when Instructions are Clear," working paper.

Zilker, Veronika, Ralph Hertwig, & Thorsten Pachur (2020) “Age Differences in Risk Attitude Are Shaped by Option Complexity,” *Journal of Experimental Psychology: General*, 149, 1644–1683.

For Online Publication: Online Appendix of

"Relating Risky to Riskless Preferences, and Their Joint Irrationality: A Comment on Oprea (2024)"

Peter P. Wakker

April 2025

Online Appendix A. Introduction

Texts are cited from a number of papers that illustrate points in the main text. The particular points supported are indicated by keywords. The following two keywords are used throughout.

CP = irrational
This keyword indicates that the text cited gives a restatement of Observation 1 of the main text: CP's deviations from expected utility are irrational.

Risky ≈ riskless:
This keyword indicates that the text cited gives a restatement of Observation 2 of the main text: CP is based on general principles also occurring in riskless preferences.

Online Appendix B. Citations from Oprea (2024)

This appendix presents citations from Oprea (2024) with keywords added. KT abbreviates Kahneman & Tversky (1979), Tversky & Kahneman (1981, 1986, 1992), Wakker (2010), and other references that stated Observations 1 and 2. For the keyword "*CP = irrational"*, Opres never acknowledges the precedence of KT.

Two more keywords are used.

Risky ≠ riskless [4]:

This keyword indicates that the text cited contradicts Observation 2 of the main text. The cited text erroneously suggests that revealed phenomena for risk (which can comprise irrationalities), cannot be the same as riskless phenomena, e.g. driven by complexity. These texts are used to suggest, again incorrectly, that CP, and theories based on it (e.g., prospect theory) would be falsified as soon as similar phenomena occur in riskless contexts. To illustrate the logical flaw, even if humans share 97% of their genes and properties with ("other") apes, we still call them human. Similarly, if risk preferences share properties with riskless preferences, such as being driven by complexity, then risk preferences are still risk preferences.

bold rational ⇒ trivial; bold irrational ⇒ incorrect

Citations with this keyword will be numbered 1-14. The main text refers to them for erroneously suggesting that complexity should *replace* CP. The keyword indicates that the cited text centers around ambiguity of the part part of the text where I added boldface. If bold parts such as "preference"[5] are taken rational, as they should according to Oprea's Footnote 1, then the text is merely a restatement of Observation 1, put forward long ago by KT, and the text is not informative. If the bold parts such as "preference" are taken in their usual sense ("revealed"), where they can be irrational, then the text would amount to a refutation of CP and prospect theory, were

[4] This keyword is comprised by the next keyword: *bold rational ⇒ trivial; bold irrational ⇒ incorrect*

[5] Oprea sometimes adds terms such as "true", "reliably", "taste" that however do not reduce the misunderstandings.

it not that the text then is incorrect, as with the preceding keyword "*Risky ≠ riskless"*, which is comprised by this keyword.

Now follow citations from Oprea (2024).

Oprea (2024) p. 3789 (abstract) (also cited in main text):
> much of the behavior motivating our most important behavioral theories of risk derive from complexity-driven mistakes rather than **true risk preferences**.

bold rational ⇒ trivial; bold irrational ⇒ incorrect (1)

Oprea (2024) p. 3789:
> "many important anomalies occur because lotteries are complex (costly or difficult to properly evaluate) *rather than* because they are risky." [italics added]

Risky ≠ riskless

Oprea (2024) p. 3789:
> "many anomalies that are commonly interpreted as expressions of **risk preferences** should *instead* be interpreted as systematic mistakes that are only *indirectly* related to risk." [italics added]

bold rational ⇒ trivial; bold irrational ⇒ incorrect (2)
This text is more nuanced by allowing a vague "indirectly".

Oprea (2024) p. 3789 (in his Footnote 1):
> "Throughout the paper, we will use the word "preferences" to refer to a decision-maker's welfare-relevant rank ordering of lotteries. This is a narrower way of using the term than some treatments in which "preference" refers simply to the decision-maker's observed choice (i.e., the revealed preference of an agent)."

Oprea's terminology, only explained in a footnote, where "preferences" refer to rational rather than actual preferences, deviates from common conventions. It contributes to the misunderstandings that his paper has created.

Oprea (2024) p. 3791:
> "(iii) … the severity of each of these anomalies in lotteries is strongly predicted by their severity in deterministic mirrors, suggesting that the behaviors in the two settings are strongly linked, deriving from a common behavioral mechanism (which, clearly, cannot be grounded in risk or **risk preferences**)."

bold rational ⇒ trivial; bold irrational ⇒ incorrect (3)

Oprea (2024) p. 3791:
> "lottery anomalies like probability weighting and loss aversion are not primarily rational expressions of nonstandard risk preferences as is often believed (e.g., in some interpretations of prospect theory)"

CP = irrational
The interpretations of prospect theory mentioned are not KT's.

Oprea (2024) p. 3791:
> "such anomalies *tell us little* about ***tastes*** for risk or loss and therefore should not be accommodated in welfare analysis or policy design." [italics added]

bold rational ⇒ trivial; bold irrational ⇒ incorrect (4)

The cited text does not recognize the usefulness of CP for debiasing and preference-purification techniques. For instance, Bleichrodt, Pinto, & Wakker (2001) qualified CP as irrational, in full agreement with Kahneman, Tversky, and Oprea. They then showed how CP can be removed from (irrational) revealed preferences to distill "true tastes" that are better suited for welfare analysis and policy design.This way, prospect theory does not attack the cornerstones of classic rational economics, but rather supports them by widening their relevance.

> Oprea (2024) p. 3791:
> "errors they describe likely distort choice in a far broader range of contexts than has so far been appreciated."

There has been wide appreciation of this insight (Observation 2).

> Oprea (2024) p. 3794:
> "probability weighting: a *putative* tendency" [italics added]

Oprea seeks to question CP whereas his findings should be taken to support it.

> Oprea (2024) p. 3795:
> "prospect theory (Kahneman and Tversky 1979; Tversky and Kahneman 1992; Wakker 2010). … To the degree the classical pattern is indeed driven by **risk preferences** (i.e., **tastes** for risk that cause valuations to deviate from expected value), it *should* disappear when we remove risk from lotteries in our Mirror treatment" [italics added]

bold rational $\Rightarrow$ *trivial; bold irrational* $\Rightarrow$ *incorrect* (5)

The word "Indeed" suggests that the works cited before adopt the rational "taste" interpretation of Oprea's Footnote 1, which is incorrect (Observation 1) and misleading.

> Oprea (2024) p. 3795:
> "we have evidence for an **alternative** interpretation of the classical pattern: that it is a pattern of systematic *mistakes*, arising **not** because lotteries are risky, per se, but **rather** because they are complex (costly or difficult to properly value)." [italics from original] [bold added]

CP = irrational; *Risky* $\neq$ *riskless*

This text, not included in the numbered texts, nevertheless also contributes to the misunderstanding that complexity would replace, rather than support, CP.

> Oprea (2024) p. 3800:
> "to the extent that evidence of anomalies is strongly *correlated* in lotteries and mirrors, we have evidence that they are likely both driven by the complexity of evaluation (the property lotteries and mirrors share) rather than by risk or **risk preferences**" [italics from original] [bold added]

bold rational $\Rightarrow$ *trivial; bold irrational* $\Rightarrow$ *incorrect* (6)

> Oprea (2024) p. 3800:
> "suggesting the two tendencies likely derive from *a related behavioral mechanism*." [italics added]

Risky $\approx$ *riskless*

> Oprea (2024) p. 3801:
> "driven in each case by the *same behavioral mechanism*" [italics added]

Risky $\approx$ *riskless*

Oprea (2024) p. 3801 (also cited in main text):
"a key prediction of standard risk preference-based interpretations of the classical pattern (e.g., prospect theory) is that the pattern *should only arise in the presence of risk* … that subjects …will be …complexity *insensitive*." [italics added]
Risky ≠ riskless

Oprea (2024) p. 3801:
"Most subjects therefore deviate from the most basic prediction of risk preference-based theories. By contrast, the vast majority of subjects (82 percent) can be classified as complexity sensitive"
Risky ≠ riskless; Again, although not included in the numbered texts, this text also takes complexity as replacing, rather than explaining, CP.

Oprea (2024) p. 3802:
"valuations of objects like lotteries and mirrors do *not* transparently reveal **preferences** but instead derive from subjects' use of relatively shallow heuristics that are highly sensitive to superficial details of the choice environment." [italics added]
bold rational ⇒ trivial; bold irrational ⇒ incorrect (7)

Oprea (2024) p. 3804:
"a **response not to risk** but rather to the complexity of valuation." [italics added]
|*bold rational ⇒ trivial; bold irrational ⇒ incorrect* (8)

Oprea (2024) p. 3804:
"preferences for even the simplest-seeming lotteries are not transparent to decision-makers … lottery valuations therefore *do not reliably reveal* subjects' **risk preferences**." [italics added]
bold rational ⇒ trivial; bold irrational ⇒ incorrect (9)

Oprea (2024) p. 3804:
"lottery valuations therefore often reveal the consequences of systematic heuristic mistakes *instead of* **true preferences for risk**." [italics added]
bold rational ⇒ trivial; bold irrational ⇒ incorrect (10)

Oprea (2024) p. 3804:
"behaviors in lotteries … typically used to measure *putative* components of **preferences** like probability weighting, reference dependence, and loss aversion in lotteries are likely to a great extent driven by heuristic mistakes" [italics added]
bold rational ⇒ trivial; bold irrational ⇒ incorrect (11)

Oprea (2024) p. 3804:
"We do not claim … that **risk preferences** … do not exist but only that they are unlikely to be reliably revealed in lottery valuations." [italics added]
bold rational ⇒ trivial; bold irrational ⇒ incorrect (12)

Oprea (2024) p. 3806:
"these styles of explanations …, *unlike* theories of behavioral risk preferences, … can explain why these patterns occur both with and without risk." [italics added]
Risky ≠ riskless

Oprea (2024) p. 3807:

> "anomalous phenomena often attributed to **preferences** … calling into question the idea that these phenomena measure welfare-relevant preferences at all."

bold rational ⇒ trivial; bold irrational ⇒ incorrect (13)

The phenomena can become welfare-relevant if debiased (Bleichrodt, Pinto, & Wakker 2001).

Oprea (2024) p. 3808:

> "Prospect theory describes the classical pattern as growing out of risk preferences"

Risky ≠ riskless

Oprea (2024) p. 3808:

> "whether prospect theory describes decision-makers' welfare-relevant tastes for risk and loss or whether it instead describes judgment errors. We view our results as strong support for the latter interpretation."

CP = irrational

Note that, contrary to what is suggested, KT also endorse the "latter interpretation" (Observation 1).

Oprea (2024) p. 3808:

> "to be insensitive to features of decision problems that matter for optimal choice. The classical pattern can be interpreted, in large part, as an outgrowth of just this sort of insensitivity, an observation that goes back at least to *Tversky and Kahneman (1992)*."
> [italics added]

The only place where Kahneman and Tversky are, possibly, credited for part of the findings in this paper, but then in an unclear and insufficient manner, and only for Observation 2. The unclarity is enhanced because the text is positioned within a modern context on complexity, with the vague term "outgrowth". Thus, no paper has as yet shown awareness of KT's precedence. Also, such a fundamental precedence should have been cited in the introduction. Introductions should clarify priority.

Oprea (2024) p. 3808:

> "These kinds of results underscore and expand upon our interpretation of our results by suggesting that the patterns of insensitivity that describe the classical pattern may be generic to the evaluation of complex things, a possibility that may unify a great number of anomalies in behavioral economics."

Risky ≈ riskless

Oprea (2024) p. 3808:

> "We provide evidence that some of the central lottery anomalies …are not special phenomena of risk and therefore are unlikely to reflect **decision-makers' risk preferences**." *Instead,* [italics added]

bold rational ⇒ trivial; bold irrational ⇒ incorrect (14)

Oprea (2024) p. 3808:

> "theories of risk preferences designed to explain these anomalies (e.g., prospect theory) are unlikely to contain much normative content and therefore should not be accommodated in the inference of welfare or the design of policy.

CP = irrational

They can become welfare-relevant if debiased (Bleichrodt, Pinto, & Wakker 2001).

Oprea (2024) p. 3809:

> "many of the phenomena that have animated the rich behavioral literature on decision-making under risk likely have a much broader scope of application *than has been so far appreciated*." [italics added]

Risky ≈ *riskless*

Online Appendix C. Citations from Kahneman & Tversky (1979)

This appendix presents citations from Kaheman & Tversky (1979) with, again, keywords added to interpret them.

> Kahneman & Tversky (1979) p. 277 (also cited in main text):
> "that values are attached to changes rather than to final states, and that decision weights do not coincide with stated probabilities. These departures from expected utility must lead to normatively unacceptable consequences … the anomalies implied by prospect theory are expected to occur."

CP = irrational

> Kahneman & Tversky (1979) p. 277 (also cited in main text):
> "carriers of value are changes in wealth or welfare … This assumption is compatible with basic principles of perception and judgment. Our perceptual apparatus is attuned to the evaluation of changes …. When we respond to attributes such as brightness, loudness, or temperature, … defines an adaptation level, or reference point … The same principle applies to non-sensory attributes such as health, prestige, and wealth."

Risky ≈ riskless

> Kahneman & Tversky (1979) p. 278 (also cited in main text):
> "Many sensory and perceptual dimensions share the property that the psychological response is a concave function of the magnitude of physical change. … room temperature … this principle applies in particular to the evaluation of monetary changes … the value function … is normally concave above the reference point … and often convex below. … Galanter and Pliner [17], … perceived magnitude of monetary and non-monetary gains and losses. The above hypothesis regarding the shape of the value function was based on responses to gains and losses in a riskless context. We propose that the value function which is derived from risky choices shares the same characteristics."

Risky ≈ riskless

> Kahneman & Tversky (1979) p. 288:
> "the proposed value function for money should apply to other attributes as well."

Risky ≈ riskless

Online Appendix D. Citations from Tversky & Kahneman (1981)

This appendix presents citations from Tversky & Kaheman (1981), with keywords added. The central topic of the paper is how general imperfections in perception and cognitive constraints make people behave differently in identical, but differently formulated problems, implying irrationalities. The problems are absolutely not restricted to risky choices.

Tversky & Kahneman (1981) p. 453 (abstract):
"The effects of frames on preferences are compared to the effects of perspectives on *perceptual* appearance." [italics added]
Risky ≈ riskless

Tversky & Kahneman (1981) p. 453 opening para:
"there is general agreement that rational choices should satisfy some elementary requirements of consistency and coherence. In this article we describe decision problems in which people systematically violate the requirements of consistency and coherence"
CP = irrational

Tversky & Kahneman (1981) p. 453 1st-2nd column:
"perceived relative height of two neighboring mountains … Because of *imperfections* of human perception … reverse … the relative desirability of options." [italics added]
CP = irrational; *Risky ≈ riskless*

Tversky & Kahneman (1981) p. 453 last sentence:
"When faced with a choice, a *rational* decision-maker will prefer the prospect that offers the *highest expected utility*." [italics added]
CP = irrational

Tversky & Kahneman (1981) p. 454 ll. 4-7:
"We have presented elsewhere (3) a *descriptive* model, called prospect theory" [italics added]
CP = irrational

Tversky & Kahneman (1981) p. 455 2nd and 3rd para:
"violations of dominance ... The respondents in problem 3 failed to combine options, although the integration was relatively simple and was encouraged by instructions (13). The *complexity* of practical problems of concurrent decisions, such as portfolio selection, would prevent people from integrating options without computational aids, even if they were inclined to do so." [italics added]
Remarkably, Kahneman and Tversky here obtained a direct violation of monotonicity leading to a sure loss, as Oprea did in his mirror choices. They also mention complexity.

Tversky & Kahneman (1981) p. 456, 1st para:

"The certainty effect reveals attitudes toward risk that are *inconsistent* with the axioms of *rational* choice" [italics added]

CP = irrational

Tversky & Kahneman (1981) p. 457 3rd column 2nd para:

"while traveling in a mountain range … Similarly … attractiveness of options …The susceptibility to perspective effects is of special concern in the domain of decision-making …The metaphor of changing perspective can be applied to other phenomena of choice"

Risky ≈ riskless

Online Appendix E. Citations from Tversky & Kahneman (1986)

This appendix presents citations from Tversky & Kaheman (1986), with keywords added. The central topic of the paper is expressed in the last sentence of the abstract: "no theory of choice can be both normatively adequate and descriptively accurate." (*CP = irrational*)

Tversky & Kahneman (1986) p. S251 (abstract):
"the psychophysical principles of evaluation embodied in prospect theory"
Risky ≈ riskless

Tversky & Kahneman (1986) p. S252:
"A descriptive model of choice is presented, which accounts for preferences that are anomalous in the normative theory."
CP = irrational

Tversky & Kahneman (1986) p. S272:
"Prospect theory differs from the other models in being unabashedly descriptive and in making no normative claims."
CP = irrational

Tversky & Kahneman (1986) p. S272:
"Perhaps the major finding of the present article is that the axioms of rational choice are generally satisfied in transparent situations and often violated in nontransparent ones"
"Nontransparent" is close to complex, so that the authors here are close to acknowledging the important role of complexity.

Tversky & Kahneman (1986) p. S273:
"the role of transparency and … consistent with the conception of bounded rationality … by Herbert Simon"
Again, the authors are close to recognizing the role of complexity.

Tversky & Kahneman (1986) p. S273:
"prospect theory is an attempt to articulate some of the principles of perception and judgment that limit the rationality of choice."
CP = irrational; *Risky ≈ riskless*

Tversky & Kahneman (1986) p. S274:
"Incentives … prevent errors that arise from insufficient attention and effort than errors that arise from misperception or faulty intuition. The example of visual illusion is instructive ...
Risky ≈ riskless.
The distinction between the two kinds of errors is useful. The visual illusion is due to misperception and faulty intuition having nothing to do with complexity, and so do many biases in risky perception.

Online Appendix F. Citations from Tversky & Kahneman (1992)

This appendix presents citations from Tversky & Kaheman (1992), with keywords added.

Tversky & Kahneman (1992) p. 297 (abstract):

> "Two principles, diminishing sensitivity and loss aversion, are invoked to explain the characteristic curvature of the value function and the weighting functions."

Risky ≈ riskless

Tversky & Kahneman (1992) p. 303:

> "v is concave above the reference point … and convex below the reference point … v is steeper for losses than for gains. The first two conditions reflect the principle of diminishing sensitivity … The principle of diminishing sensitivity applies to the weighting functions as well. …In the evaluation of uncertainty, there are two natural boundaries … certainty and impossibility … Diminishing sensitivity, therefore, gives rise to a weighting function that is concave near 0 and convex near 1."

Risky ≈ riskless

Tversky & Kahneman (1992) p. 316:

> "We have proposed an alternative descriptive theory"

CP = irrational

Tversky & Kahneman (1992) p. 317 last para of main text:

> "Prospect theory departs from the tradition that assumes the rationality of economic agents; it is proposed as a descriptive, not a normative, theory."

CP = irrational

Online Appendix G. Citations from Wakker (2010)

This appendix presents citations from Wakker (2010), a textbook on prospect theory, with keywords added. I collaborated with Tversky from 1990 until he passed away, and have one joint paper with Kahneman.

Wakker (2010) p. 2:
"The normative expected utility model … Kahneman & Tversky's (1979) prospect theory provided a major breakaway. It was the first descriptive theory that explicitly incorporated irrational behavior in an empirically realistic manner (Kahneman 2003 p. 1456), while at the same time being systematic and tractable. It was the first rational theory of irrational behavior, so to say."
CP = irrational

Wakker (2010) p. 3:
"In recent years, economics has been opening up to introspective and neuro-imaging data. It is to be expected that the concepts of prospect theory, in view of their sound psychological basis, will be well suited for such future developments and for connections with such domains of research."
Risky ≈ riskless

Wakker (2010) p. 64:
Independence, and its weakenings presented here, may seem to be completely self-evident at first sight, and I also think they *are* normatively compelling.
CP = irrational

Wakker (2010) p. 83:
"The preceding empirical findings on utility fit well with the ratio–difference principle from psychology (Stevens & Davis 1938; Baron 1997)."
Risky ≈ riskless

Wakker (2010) p. 143
"In the same way as Bernoulli's (1738) expected utility entailed a departure from objectivity, prospect theory entailed a departure from rationality."
CP = irrational

Wakker (2010) p. 147:
"Lopes (1987) wrote: "Risk attitude is more than the psychophysics of money" Psychophysics is the field of psychology that examines sensations generated by physical stimuli. Utility can be taken as the scale that describes the sensations generated by receiving money."
Risky ≈ riskless

Wakker (2010) p. 203:
"Fig. 2a shows another psychological phenomenon. It reflects "diminishing sensitivity" for probabilities, which we will call likelihood insensitivity."
Risky ≈ riskless

Wakker (2010) p. 204:
"The regressive shape in Fig. 2a, with weights correlating imperfectly with probabilities, and with as much overweighting of good as of bad outcomes, suggests

that *perceptual and cognitive limitations*, prior to any consideration of value, underlie this effect." [italics added]

CP = irrational; Risky ≈ riskless

Wakker (2010) p. 204:

"Kunreuther, Novemsky, & Kahneman (2001) and Reyna & Brainerd (1995) provided evidence supporting the cognitive interpretation of the inverse-S phenomenon."

Risky ≈ riskless

Wakker (2010) p. 222:

"Any interpretation of w as reflecting belief or probability (mis)perception is to be left to speculations beyond revealed preference. Such speculations are important because proper future relations with concepts from other domains such as artificial intelligence, cognitive psychology, or neuroscience are important for the future of decision theory."

CP = irrational; Risky ≈ riskless

Wakker (2010) p. 227:

"Likelihood insensitivity reflects diminishing sensitivity for a scale bounded from two sides."

Risky ≈ riskless

Wakker (2010) p. 228:

"According to many, including the author, probability weighting is irrational"

CP = irrational

Wakker (2010) p. 234:

"reference dependence … entails, I think, a bigger deviation from rationality than probability weighting."

CP = irrational

Wakker (2010) p. 238:

"Bipolar perceptions … with a neutrality point chosen … have been observed in many psychological domains (Nowlis & Nowlis 1956; Peeters & Czapinski 1990; Russell & Carroll 1999; Schimmack 2001). If we experience warmth, then … In color vision … As in other domains, the corresponding physical level is adapted to circumstances … ("neutrality adaptation"; Hurvich & Jameson 1951; see also Hevell & Kingdom 2008 and Grabisch & Labreuche 2008 §3). Based on a large-scale study, Nichol & Epstein (2008) argued for a separate treatment of gains and losses for health outcomes. Tom et al. (2007) provided neural data, and Hardisty & Weber (2009) provided intertemporal data."

Risky ≈ riskless

Wakker (2010) p. 240:

"The stimuli that generate a neutral experience may be different in different contexts. For the perception of warmth … In decision under risk"

Risky ≈ riskless

Wakker (2010) p. 241:

"modeling of outcomes as changes with respect to a reference point really deviates from the modeling of outcomes as (referring to) final wealth. Such deviations entail major *irrationalities*" [talics added]

CP = irrational

Wakker (2010) p. 245:
"Whereas traditional EU is, in my opinion, the hallmark of rationality"
CP = irrational

Wakker (2010) p. 245:
"any deviation from final wealth due to reference dependence is utterly irrational."
CP = irrational

Wakker (2010) p. 264:
"Descriptive utility is driven not only by the perception of purchasing power (diminishing marginal utility) but also by the perception of numbers and diminishing sensitivity (Köbberling, Schwieren, & Wakker 2007)."
Risky ≈ riskless

Wakker (2010) p. 265:
"The violations of asset integration entailed by reference dependence (the basis of loss aversion) are more irrational than for instance probability weighting."
CP = irrational

Wakker (2010) p. 276:
"The great contribution of OPT [1979 prospect theory] was that it was the first theory that combined indispensable psychological concepts for risk theories, being sensitivity to probability and loss aversion, into an accessible theory with empirical realism."
Risky ≈ riskless

Wakker (2010) p. 326:
like the author of this book [Wakker] a strong advocate of expected utility for normative purposes
CP = irrational

Wakker (2010) p. 382:
I do not consider nonexpected utility to be normative
CP = irrational

References for Online Appendix

Bleichrodt, Han, José Luis Pinto, & Peter P. Wakker (2001) "Making Descriptive Use of Prospect Theory to Improve the Prescriptive Use of Expected Utility," *Management Science* 47, 1498–1514. https://doi.org/10.1287/mnsc.47.11.1498.10248

Kahneman, Daniel & Amos Tversky (1979) "Prospect Theory: An Analysis of Decision under Risk," *Econometrica* 47, 263–291. https://doi.org/10.2307/1914185

Oprea, Ryan (2024) "Decisions under Risk are Decisions under Complexity," *American Economic Review* 114, 3789–3811. https://doi.org/10.1257/aer.20221227

Tversky, Amos & Daniel Kahneman (1981) "The Framing of Decisions and the Psychology of Choice," *Science* 211, 453–458. https://doi.org/10.1126/science.7455683

Tversky, Amos & Daniel Kahneman (1986) "Rational Choice and the Framing of Decisions," *Journal of Business* 59, S251–S278.

Tversky, Amos & Daniel Kahneman (1992) "Advances in Prospect Theory: Cumulative Representation of Uncertainty," *Journal of Risk and Uncertainty* 5, 297–323. https://doi.org/10.1007/BF00122574

Wakker, Peter P. (2010) "*Prospect Theory: For Risk and Ambiguity*." Cambridge University Press, Cambridge, UK.