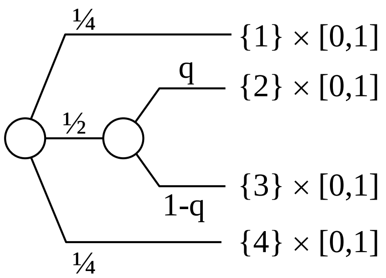
In the following example, E and E^c are unambiguous events, and there is "nonuniform" ambiguity conditional on E^c. This ambiguity is influenced by outcomes conditional on E through nonseparable interactions typical of nonexpected utility. Then E is ambiguous by Epstein & Zhang's (2001) definition, which is undesirable.

Consider the following scenario, in the spirit of Machina's (1982) fanning out. If the outcome at E is very high so that the acts are in favorable indifference classes, the decision maker is ambiguity seeking. Then he prefers to bet on ambiguous subevents of E^c over unambiguous subevents. If the outcome at E is very low so that the acts are in unfavorable indifference classes, the decision maker changes to ambiguity aversion. Then his preferences reverse in the sense that he prefers to bet on unambiguous subevents of E^c over ambiguous subevents. Epstein & Zhang (2001) then qualify E as ambiguous. The example below will be of this kind. Similar examples with ambiguity aversion everywhere, but increased ambguity aversion in lower indifference classes, can be constructed.

EXAMPLE [Betweenness-type ambiguity attitude]. Assume that the outcome set X is (0,1). $S = \{1,2,3,4\} \times [0,1]$. The following figure merely serves to illustrate the example. I emphasize that the model is static and not dynamic or multi-stage.



$P(\{1\} \times A) = P(\{4\} \times A) = \lambda(A)/4$ for all $A \subset [0,1]$. Here λ denotes the usual Lebesgue measure, assigning to each interval its length. Conditional upon $\{2\} \times [0,1]$, $(\{2\} \times A)$ has probability $\lambda(A)$, and so does $(\{3\} \times A)$ conditional upon $\{3\} \times [0,1]$. The probability q can be anything from [0,1] and is unknown, bringing ambiguity.

For given probabilities, the DM maximizes EV. For an act f, write x_j for the conditional expectation of f given {j}. The preference value of an act will be a function of (x_1,x_2,x_3,x_4). It will be

$$x_1/4 + \mu\gamma/2 + (1-\mu)\beta/2 + x_4/4,$$

where γ ("good") = $\max\{x_2,x_3\}$, β ("bad") = $\min\{x_2,x_3\}$, and μ is a parameter that reflects ambiguity aversion ($\mu \leq 0.5$) or ambiguity seeking ($\mu \geq 0.5$). It is a rank-dependent-type, biseparable-type, form of ambiguity attitude conditional on $\{2,3\} \times [0,1]$. We will allow μ to depend on $(x_1,\ldots,x_4)$. More precisely, we assume $\mu = c$ with c the certainty equivalent of the act ($0 < c < 1$). Thus, the better the act is, the more ambiguity seeking the decision maker is.

The dependence on the certainty equivalent of the act is in the spirit of betweenness models, although there also is a kind of rank-dependence. It suggests an implicit definition of μ and c, as is common in betweenness models. In our case, an explicit definition can be obtained; see below. The example is reminiscent of Machina's (1982) fanning out, where a DM becomes more risk seeking as he is better off. In our case, not risk attitude but ambiguity attitude is affected by well-being.

We have

$c = x_1/4 + c\gamma/2 + (1-c)\beta/2 + x_4/4.$

Writing $x = x_1/2 + x_4/2$ we get

$c = x/2 + c\gamma/2 + (1-c)\beta/2.$

$2c = x + c\gamma + (1-c)\beta.$

$2c - c\gamma + c\beta = x + \beta.$

$c(2 - \gamma + \beta) = x + \beta.$

$$c = \frac{x + \beta}{2 - \gamma + \beta}.$$

Thus, we have an explicit representation of c. The function is clearly strictly increasing in x and, hence, in x_1 and x_2, and in γ. Because $2- \gamma > 1 > x$, the function is also strictly increasing in β. Increasingness in β and γ implies increasingness in x_2 and x_3 (if increasing x_2 changes it from β, below x_3, to γ, above x_3, then first consider the increase up to x_3, and then beyond; increases in x_3 work similarly). Hence, monotonicity is satisfied.

We have, using the (x_1,x_2,x_3,x_4) notation now for acts constant on each conditional [0,1],

(0.9; **0.9**; 0.5; **0.1**) $\succ$ (0.9; **0.1**; 0.5; **0.9**)

but

(0.1; **0.9**; 0.5; **0.1**) $\prec$ (0.1; **0.1**; 0.5; **0.9**).

In the upper preference, both acts are favorable in the sense of having a certainty equivalent exceeding 0.5. Hence, $\mu > 0.5$ and there is ambiguity seeking. The bet on the ambiguous $\{2\} \times [0,1]$ is preferred to the bet on the unambiguous $\{4\} \times [0,1]$.
In the lower preference, both acts are unfavorable in the sense of having a certainty equivalent below 0.5. Hence, $\mu < 0.5$ and there is ambiguity aversion. The bet on the ambiguous $\{2\} \times [0,1]$ is preferred less than the bet on the unambiguous $\{4\} \times [0,1]$.

By Epstein & Zhang's definition of ambiguity, the interaction between $\{1\} \times [0,1]$ and its complement is interpreted as ambiguity of T. However, the natural interpretation is that T itself is unambiguous, but influences the ambiguity attitude off T. Such violations of separability are typical of nonexpected utility, where the risk attitude conditional on T^c and its subevents can depend on the constant outcome under T.

In the rest of this text I connect with the notation of Epstein & Zhang. To this effect, take T = {1} × [0,1], A = {2} × [0,1], B = {4} × [0,1], and R = {3} × [0,1]. x* = 0.9 = z, x = 0.1 = z´, and the act takes value 0.5 elsewhere (h(s) = 0 in Epstein & Zhang's notation) Then

(A:x*;B:x; R:0.5; T:z) $\succ$ (A:x;B:x*; R:0.5; T:z)

but

(A:x*;B:x; R:0.5; T:z´) $\prec$ (A:x;B:x*; R:0.5; T:z´)

In the upper choice, both acts have a certainty equivalent exceeding 0.5, and the DM is optimistic, preferring to gamble on the ambiguous event A. In the lower choice, both acts have a certainty equivalent below 0.5, and the DM is pessimistic, preferring to gamble on the unambiguous B. □

REFERENCES

Epstein, Larry G. & Jiangkang Zhang (2001), "Subjective Probabilities on Subjectively Unambiguous Events," *Econometrica* 69, 265–306.

Machina, Mark J. (1982), " 'Expected Utility' Analysis without the Independence Axiom," *Econometrica* 50, 277–323.