Suggestions for Presenting Tradeoff-Consistency Preference Conditions

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1 Introduction

This note explains how tradeoff consistency preference conditions can be explained transparently. Section 2 explains the condition in full, involving four indifferences, which may be complex. Section 3 gives a simpler presentation by focusing on two central indifferences, specifying the central intuition, and then giving a verbal formulation of the axiom. Section 4 gives a version with preferences instead of indifferences. Versions with indifferences are easier to understand and give more general theorems (Köbberling & Wakker 2003), but versions with preferences may work better for normative discussions and some empirical applications. Then follow yet an alternative version, background, and a conclusion.

2 The condition in full and its intuition

2.1 Notation

We assume a preference relation $\succcurlyeq$ over options of an agent satisfying common conditions such as weak ordering and monotonicity. This section discusses the following condition:

$\boldsymbol{\alpha}_i g \sim \boldsymbol{\beta}_i G$ & $\boldsymbol{\alpha}_{i'} g' \sim \boldsymbol{\beta}_{i'} G'$ &
$\boldsymbol{\gamma}_i g \sim \boldsymbol{\delta}_i G$ imply

$$\boldsymbol{\gamma}_{i'} g' \sim \boldsymbol{\delta}_{i'} G' \quad (1)$$

Here, $\alpha, \beta, \gamma, \delta$ denote outcomes, monetary or otherwise. i denotes a coordinate/attribute if choice options are, for instance, n-tuples, say temporal outcome streams over n timepoints or allocations over n persons. i may denote a state or (multi-state) event in Savage's (1954) uncertainty framework. G, g, G', g' may also denote outcomes, or $n-1$ tuples, or (non-constant) restrictrions of Savagean acts to the event complementary to event i, as the case may be. Thus, $\boldsymbol{\alpha}_i g$ is a choice option yielding outcome α for attribute/event i and assigning to all other attributes or the event complementary to i what g gives there. It is g but with α substituted at i.

2.2 Intuition

Together with common assumptions, tradeoff consistency characterizes weighted utility representations such as subjective expected utility

$$(x_1, \ldots, x_n) \mapsto \sum_{j=1}^{n} p_j U(x_j) \quad (2)$$

We assume that i is nonnull (i.e., it sometimes impacts preferences) to avoid triviality, and, for simplicity, $\alpha \succ \beta$ and $\gamma \succ \delta$. The first (left upper) indifference in Eq. 1 shows that the improvement α instead of β exacty offsets getting g instead of G. In the left lower indifference in Eq. 1, we replaced α by γ and β by δ. The improvement γ instead of δ, again, exactly offsets getting g instead of G. That is:
The improvement
[α instead of β]
does exactly the same as the improvement
[γ instead of δ].
The interpretation is that the strength of preference of α over β is as strong as that of γ over δ. As Prelec formulated briefly: α is to β what γ is to δ. The outcomes g, G served as gauges here and we, accordingly, call them *gauge outcomes* or *gauges*.

After presenting the above concept, I recommend quickly giving a lemma that under a particular theory, such as Eq. 2, we obtain (assuming i nonnull)

$$U(\alpha) - U(\beta) = U(\gamma) - U(\delta). \quad (3)$$

Thus, replacements such as [α instead of β] capture utility differences. One can also say that [α instead of β] is an equally good improvement as [γ instead of δ].

As a route to preference axiomatizations, one can then point out that as a necessary condition for the theory considered, there must be consistency between the above elicitations, because otherwise contradictory utility-difference equalities would result. Then can come a theorem saying that the consistency condition is not only necessary but also sufficient, i.e., gives an axiomatization of the theory concerned.

To have a general term that does not commit to any preference between the outcomes, and that also can readily be used for gauges, I often use the term tradeoff instead of improvement. Tradeoff consistency requires consistency in the above inferences about tradeoffs. Thus, in a context with different i, g, G (indicated by primes in Eq. 1), we should find consistent inferences. The condition entails that tradeoffs have an independent meaning, independent of what happens at other coordinates/events (separability), and independent of the underlying coordinate/event i.[1]

2.3 Transparent visual display

I recommend displays much like Eq. 1. γ is exactly below α, and δ is exactly below β; similar with the rest of the left indifferences. $\alpha, \beta, \gamma, \delta$ are made salient by enlarging and/or bold face. In the two left indifferences, readers immediately see what has changed and what remained the same. For this visual reason, the two left indifferences are below each other rather than on the same line.

Similar observations apply to the two right indifferences in Eq. 1. Although primes are heavy notation, they so well clarify in a unified manner how the right indifferences are changed relative to the left indifferences, that I still used them here.

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[1] If invariance over different i, i' is dropped by imposing Eq. 1 only for $i = i'$, then the condition reduces to what is called the Reidemeister condition and, with preferences instead of indifferences, triple cancellation (Krantz et al. 1971). That condition is often equivalent to additively decomposable representability, or state-dependent expected utility, and to separability.

3 A simpler partial presentation

The condition as displayed in Eq. 1 is complex, involving many symbols. The essence of the intuition is to recognize the strength of preference inference coming from the left two indifferences. It often works better to single out that part, and introduce a notation for it. Thus, I write the following $\sim^t$ indifference between so-called tradeoffs

$$\alpha \ominus \beta \sim^t \gamma \ominus \delta \tag{4}$$

whenever there exist G, g and nonnull i such that

$$\begin{aligned} &\boldsymbol{\alpha}_i g \sim \boldsymbol{\beta}_i G \ \ \& \\ &\boldsymbol{\gamma}_i g \sim \boldsymbol{\delta}_i G \end{aligned} \tag{5}$$

Tradeoff consistency can then be formulated as: strictly improving any outcome in an $\sim^t$ indifference breaks that indifference.[2]

4 Preference versions

A preference version of tradeoff consistency is as follows:

$$\begin{aligned} &\boldsymbol{\alpha}_i g \preccurlyeq \boldsymbol{\beta}_i G \quad \& \qquad \boldsymbol{\alpha}_{i'} G' \succcurlyeq \boldsymbol{\beta}_{i'} g' \quad \& \\ &\boldsymbol{\gamma}_i g \succcurlyeq \boldsymbol{\delta}_i G \quad \text{imply} \\ &\qquad\qquad\qquad\qquad \boldsymbol{\gamma}_{i'} g' \succcurlyeq \boldsymbol{\delta}_{i'} G' \end{aligned} \tag{6}$$

Intuitively, the left two preferences show that the tradeoff γ instead δ ($\gamma \ominus \delta$) is better than the tradeoff α instead of β ($\alpha \ominus \beta$). And so on. The preference condition is stronger, more restrictive, than the indifference version (proof left to readers). The preference condition works better for some empirical purposes because preferences are easier to measure than indifferences and stronger preference conditions are easier to falsify.

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[2] Bob Nau suggested this sentence. Duncan Luce suggested using the symbol $\ominus$.

One can define corresponding orderings of tradeoffs:

$\alpha \ominus \beta \preccurlyeq^t \gamma \ominus \delta$
whenever there exist nonnull i and G, g such that
$\boldsymbol{\alpha}_i g \preccurlyeq \boldsymbol{\beta}_i G$ &
$\boldsymbol{\gamma}_i g \succcurlyeq \boldsymbol{\delta}_i G$ (7)

and define

$\alpha \ominus \beta \succ^t \gamma \ominus \delta$
whenever there exist nonnull i' and G', g' such that
$\boldsymbol{\alpha}_{i'} G' \succcurlyeq \boldsymbol{\beta}_{i'} g'$ &
$\boldsymbol{\gamma}_{i'} g' \prec \boldsymbol{\delta}_{i'} G'$ (8)

Tradeoff consistency precludes the inonsistent [$\alpha \ominus \beta \succcurlyeq^t \gamma \ominus \delta$ and $\alpha \ominus \beta \prec^t \gamma \ominus \delta$]. A difficulty of the preference conditions concerns remembering the required directions of preference.

That for most preference conditions, versions with indifferences suffice because, whereas they by themselves are weaker, they still imply the preference conditions in the presence of usual conditions, can be seen in Wakker (1989), Theorem III.6.6 (p. 70), Statement (ii), together with Remark III.7.3. The only nonindifference condition needed is weak separability, which for monetary outcomes is implied by monotonicity.

Wakker (1988) used tradeoff consistency with indifferences but without monotonicity to generalize subjective expected utility by allowing for negative probabilities.

5 A yet simpler interpretation and why it is not used

The following strength of preference iterpretation is yet easier to understand. Consider again
$\boldsymbol{\alpha}_i g \sim \boldsymbol{\beta}_i G$ &
$\boldsymbol{\gamma}_i g \sim \boldsymbol{\delta}_i G$ (9)

Let us assume $\gamma \succ \alpha$ and $\delta \succ \beta$. We now compare each upper option with the one below, saying that the left option has improved as much as the right option, and we write

$$\gamma \ominus \alpha \sim^t \delta \ominus \beta \tag{10}$$

Whereas this interpretation is easier to understand than Eq. 4, I still do not use it. Consider the tradeoffs in Eq. 4, such as $\alpha \ominus \beta$. It plays a role in the indifference $\boldsymbol{\alpha}_i g \sim \boldsymbol{\beta}_i G$, where it should offset getting g instead of G. The following section will explain that these tradeoffs capture the "influence" of the agent. The tradeoffs $\gamma \ominus \alpha$ and $\delta \ominus \beta$ in Eq. 10 do not play a role in any single decision situation. I, therefore, expect that they will be less useful concepts and I do not use them.

6 Philosophical background

For simplicity of terminology, I focus on binary choices here. Multiple options can be taken as combinations of binary choices. During the marginal revolution in economics around 1870 (Jevons 1871, Menger 1871, Walras 1874), it became understood that utility differences, rather than utilities in an absolute sense, are often central in decision making. A deciding entity ("agent") specifies, for an option realized, a counterfactual option and takes it as its decision that the realized option rather than the counterfactual one occurs. Pairs of options rather than options themselves are the most basic entities of decision making. Replacing one option by another is what a decision is, the "influence" of the agent. Tradeoffs as analyzed above are such influences, conditioned upon i (nature's choice of true state in Savage's framework). Hence, I expect that analyses that put such tradeoffs central will give results that are intuitive and that are theoretically and mathematically strong.

An illustration of the above claim concerns ambiguity theory today. The Anscombe-Aumann (1963) framework is popular today because it conveniently gives cardinal utility. However, it uses a monotonicity assumption that does not fit well with ambiguous events (amounting to weak separability of those ambiguous events), leading for instance to historical accidents discussed in Wakker (2010 §11.6 last para). For the ambiguity models that I like to work on, I used the tradeoff technique to axiomatize them and measure them empirically. This approach naturally and directly gives cardinal utility, avoiding the drawbacks of the Anscombe-Aumann framework.

7 Conclusion

Getting to know the tradeoff technique takes a prior investment, essentially learning to understand Eq. 4. But once understood, it provides a powerful and intuitve tool for many decision theories.

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