APPENDIX E. ALGEBRAIC ELABORATIONS

of

"A Truth-Serum for Non-Bayesians: Correcting Proper Scoring Rules for Risk Attitudes,"

by

Theo Offerman, Joep Sonnemans, Gijs van de Kuilen, & Peter P. Wakker

DERIVATION OF FOOTNOTE 6, for r > 0.5:

Eq. 10 gives

$$B(E) = w^{-1}\left(\frac{r}{r + (1-r)\frac{U'(1-(1-r)^2)}{U'(1-r^2)}}\right) = \text{[because w is the identity]} =$$

$$\frac{r}{r + (1-r)\frac{U'(1-(1-r)^2)}{U'(1-r^2)}} = \text{[because } U'(p) = 0.26p^{-1.26}\text{]} =$$

$$\frac{r}{r + (1-r)\frac{0.26(1-(1-r)^2)^{-1.26}}{0.26(1-r^2)^{-1.26}}}$$

DERIVATION OF THE INVERSE OF EQUATION (18).

$exp(-\beta(-ln(p))^{\alpha}) = w(p) = y$

$-\beta(-ln(p))^{\alpha} = ln(y)$

$(-ln(p))^{\alpha} = -\frac{ln(y)}{\beta}$

$-ln(p) = (-\frac{ln(y)}{\beta})^{1/\alpha}$

$ln(p) = -(-\frac{ln(y)}{\beta})^{1/\alpha}$

$$p = w^{-1}(y) = exp(-(-\frac{ln(y)}{\beta})^{1/\alpha}) \quad \text{(E.1)}$$

DERIVATION OF THE EQUATION DISPLAYED ABOVE EQUATION (19)

We apply equation (10), and equation (E.1) as just derived.

We analyze the argument (input)

$$\frac{r}{r + (1-r)\frac{U'(1-(1-r)^2)}{U'(1-r^2)}}$$

of w^{-1} in equation (10), which was denoted y in equation (E.1), and show that it is

$$\frac{r(2r-r^2)^{1-\rho}}{(1-r)(1-r^2)^{1-\rho} + r(2r-r^2)^{1-\rho}}.$$

Substituting this in equation (E.1) then gives the desired result.

Here we go:

$$\frac{r}{r + (1-r)\frac{U'(1-(1-r)^2)}{U'(1-r^2)}} = [\text{substituting } U'(x) = \rho x^{\rho-1}] =$$

$$\frac{r}{r + (1-r)\frac{\rho(1-(1-r)^2)^{\rho-1}}{\rho(1-r^2)^{\rho-1}}} = [\text{dropping two } \rho\text{'s}]$$

$$\frac{r}{r + (1-r)\frac{(1-(1-r)^2)^{\rho-1}}{(1-r^2)^{\rho-1}}} = [1-(1-r)^2 = 2r - r^2]$$

$$\frac{r}{r + (1-r)\frac{(2r-r^2)^{\rho-1}}{(1-r^2)^{\rho-1}}} = [\text{multiplying nominator and denominator by } (2r-r^2)^{1-\rho}]$$

$$\frac{r(2r-r^2)^{1-\rho}}{r(2r-r^2)^{1-\rho} + (1-r)\frac{1}{(1-r^2)^{\rho-1}}} =$$

$$\frac{r(2r-r^2)^{1-\rho}}{r(2r-r^2)^{1-\rho} + (1-r)(1-r^2)^{1-\rho}} =$$

$$\frac{r(2r-r^2)^{1-\rho}}{(1-r)(1-r^2)^{1-\rho} + r(2r-r^2)^{1-\rho}}.$$

Substituting this for y in equation (E.1) indeed gives the equation displayed above equation (19).