Chapter 3

EXERCISE 3.2. Here is an account. Young adolescents (as everyone) should be prevented from using heroine. Hence the law, in most countries, qualifies possession of heroine as a crime, and punishes it by imprisonment. This involves coercion and violates the requirements of libertarian paternalism. But this is not always bad, and often it cannot be avoided. The message of the paper, therefore, cannot be that libertarian paternalism would always be a good thing. □

Chapter 4

EXERCISE 4.2.1.

a) EU does not hold because EU(0.9:4; 0.1:0) = 0.9*2 + 0.1*0 = 1.8 whereas EU(1) = 1. Hence, by EU, (0.9:4; 0.1:0) $\succ$ 1, violating the indifference in (*).

b) The indifference (*) implies

WU(0.9:4; 0.1:0) = WU(1). Because WU(1) = $\frac{1*f(1)U(1)}{1*f(1)}$ = U(1) = 1, we get

$$\frac{0.9*f(4)U(4) + 0.1*f(0)*0}{0.9*f(4)+0.1*f(0)} = U(1)$$

$$\frac{1.8*f(4)}{0.9*f(4)+0.1} = 1$$

1.8*f(4) = 0.9*f(4) + 0.1

0.9*f(4) = 0.1

f(4) = 1/9.

$\frac{1}{\lambda 4+1}$ =1/9. λ4 = 8. λ = 2.

WU(0.9:1; 0.1:0) =

$$\frac{0.9*f(1)U(1) + 0.1*f(0)*0}{0.9*f(1)+0.1*f(0)} = (\lambda = 2,\ f(1) = 1/3)$$

$\frac{0.9*(1/3)*1}{0.9*(1/3)+0.1}$ = 0.75. WU(1/4) = 0.5. Hence, (0.9:1; 0.1:0) is strictly preferred:

(0.9:1; 0.1:0) $\succ$ 1/4.

c) $\frac{0.9*\mathbf{f'(4)}U(4) + 0.1*f'(0)*0}{0.9*\mathbf{f'(4)}+0.1*f'(0)} < U(1) = 1$

$0.9*\mathbf{f'(4)}U(4) < 0.9*\mathbf{f'(4)} + 0.1$

$0.9*\mathbf{f'(4)} < 0.1$

$\mathbf{f'(4)} < 1/9 = f(4)$

We have $\mathbf{f'(4)} < f(4)$.

The only thing changed for the calculations here for Paul relative to John is that f(4) was replaced by the smaller bold **f´(4)**, leading to a lower weight

$$\frac{0.9*\mathbf{f'(4)}}{0.9*\mathbf{f'(4)}+0.1*f'(0)}$$

for the highest utility U(4) in the convex combination of the utilities U(4) and U(0) = 0 when evaluating the lottery. It shows that decreasing this decreases the value of the lottery, so, it enhances risk aversion. You can guess this without seeing the algebra if you know that decreasingness of f (f(4) versus f(0)) enhances risk aversion. □

EXERCISE 4.2.2. The WU value now is 5.25914, and the CE is $U^{-1}(5.25914) = 54.68^{1/0.2075} - 3000 = -19.52$. The risk premium now is 9.52, which is considerably more than under EU in Exercise 1.4.4. □

EXERCISE 4.2.3. I give two different solutions.

SOLUTION 1. We use the following fact: If

$$\frac{a}{c} = \frac{b}{d}, \text{ then these two are equal to } \frac{\lambda a + (1-\lambda)b}{\lambda c + (1-\lambda)d}\,. \qquad (*)^1$$

Assume that $x = (p_1{:}x_1,\ldots, p_n{:}x_n)$ and $y = (q_1{:}y_1,\ldots, q_m{:}y_m)$ have the same certainty equivalent CE (and, hence, the same WU value).

[1] If $\frac{a}{c} = r$ then $rc = a$, $rd = b$, implying $r(\lambda c + (1-\lambda)d) = \lambda a + (1-\lambda)b$. Divide by the nonzero $(\lambda c + (1-\lambda)d)$.

$$WU(x) = \frac{\sum_{i=1}^{n} p_i f(x_i)U(x_i)}{\sum_{i=1}^{n} p_i f(x_i)} \;=\; WU(y) = \frac{\sum_{i=1}^{m} q_i f(y_i)U(y_i)}{\sum_{i=1}^{m} q_i f(y_i)}$$

Apply (*) with:

a is the numerator of WU(x), which is $\sum_{i=1}^{n} p_i f(x_i)U(x_i)$;

c is the denominator of WU(x), which is $\sum_{i=1}^{n} p_i f(x_i)$.

b is the numerator of WU(y), which is $\sum_{i=1}^{m} q_i f(y_i)U(y_i)$;

d is the denominator of WU(y), which is $\sum_{i=1}^{m} q_i f(y_i)$.

(*) implies that the following number is equal to WU(x) and WU(y):

$$\frac{\lambda\sum_{i=1}^{n} p_i f(x_i)U(x_i) + (1-\lambda)\sum_{i=1}^{m} q_i f(y_i)U(y_i)}{\lambda\sum_{i=1}^{n} p_i f(x_i) + (1-\lambda)\sum_{i=1}^{m} q_i f(y_i)}.$$

This number is the WU value of $\lambda x + (1-\lambda)y$. Hence it has the same WU value as x and y. Weak betweenness holds.

SOLUTION 2. The weak betweenness axiom shows that indifference classes are closed under linear mixing, so that they are linear subspaces. It suggests, and indeed so it is, that every indifference class is an indifference class of an EU functional. EU does not hold because different indifference classes are indifference classes of different EU functionals. Recognizing this linearity in the functionals shows a way through the algebra. Here it follows.

Assume $x = (p_1{:}x_1,\ldots, p_n{:}x_n)$ has a certainty equivalent CE. For any constant θ, specified later, we define $V(\alpha) = U(\alpha) - U(\theta)$. We have

$$WU(x) = \frac{\sum_{i=1}^{k} p_i f(x_i)U(x_i)}{\sum_{i=1}^{k} p_i f(x_i)} = \frac{\sum_{i=1}^{k} p_i f(x_i)U(\theta)}{\sum_{i=1}^{k} p_i f(x_i)} + \frac{\sum_{i=1}^{k} p_i f(x_i)V(x_i)}{\sum_{i=1}^{k} p_i f(x_i)} =$$

$$U(\theta) + \frac{\sum_{i=1}^{k} p_i f(x_i)V(x_i)}{\sum_{i=1}^{k} p_i f(x_i)}$$

$\theta = CE$ holds if and only if the fraction is 0, meaning the numerator is 0. This means exactly that x has EU value 0 if we take $\alpha \mapsto f(\alpha)\big(U(\alpha) - U(CE)\big)$ as utility function in the EU criterion. Note that this EU function is different for different equivalence classes due to its dependency on CE. If $y \sim x$ then y has the same CE, and also has EU value 0 under the utility function specified. Because EU is linear in probability

$(EU(\lambda x + (1-\lambda)y) = \lambda EU(x) + (1-\lambda)EU(y))$, then $\lambda x + (1-\lambda)y$ also has EU value 0 under the utility function specified. This implies that it has the same CE, and is indifferent to x and y. □

EXERCISE 4.3.1.

a) The indifference (*) implies

DU(0.9:4; 0.1:0) = DU(1). Because DU(1) = U(1) = 1, we get

$$\frac{0.9*U(4) + (1+\beta)*0.1*0}{0.9+(1+\beta)*0.1} = U(1)$$

$$\frac{1.8}{0.9+(1+\beta)*0.1} = 1$$

$\beta = 8$.

DU(0.9:1; 0.1:0) =

$\dfrac{0.9*U(1) + (1+\beta)0.1*0}{0.9+(1+\beta)0.1}$ = 0.5. DU(1/4) = 0.5. Hence, we have indifference:

(0.9:1; 0.1:0) ~ ¼.

b) Now

$$\frac{0.9*U(4) + (1+\boldsymbol{\beta}')*0.1*0}{0.9+(1+\boldsymbol{\beta}')*0.1} < U(1)$$

$$\frac{1.8}{0.9+(1+\boldsymbol{\beta}')*0.1} < 1$$

$\boldsymbol{\beta}' > \beta$.

The only thing changed for the calculations here for Ringo relative to George is that β was replaced by the bold $\boldsymbol{\beta}'$, leading to a bigger weight

$\dfrac{(1+\boldsymbol{\beta}')*0.1}{0.9+(1+\boldsymbol{\beta}')*0.1}$ for the lowest utility $U(0) = 0$ in the convex combination of the utilities U(4) and U(0) when evaluationg the lottery. You can guess this without seeing the algebra if you know that β enhances risk aversion. □

EXERCISE 4.3.2. The DA value is 5.25606, and the CE is $U^{-1}(5.25606)$ = $5.25606^{1/0.2075} - 3000 = -27.92$. The risk premium now is 17.92. □

EXERCISE 4.5.3. The biseparable utility is 5.25078, and the CE is $U^{-1}(5.25078) = 5.25078^{1/0.2075} - 3000 = -42.26$. The risk premium now is 32.26. □

EXERCISE 4.6.1. Prospect (0.99:4.01, 0.01:3.01) has NAU 14.627. The NAU of receiving €4 for sure is 16. Receiving €4 for sure is preferred.

99% of the economists today are wrong in thinking that, also if we do not have the expected utility model, then still convex utility means risk seeking. □

Chapter 5

EXERCISE 5.4.1.

(a) DU = 9.86 and PV = 97.16.

(b) (0.5:100): DU = 9.86; PV = 97.16

(0.5+1:105): DU = 9.90; PV = 98.09

The late large payment (0.5+1:105) is preferred, suggesting decreasing impatience.

(c) (1.7635: 105.15) has DU = 9.86; PV = 97.16. So the two are indifferent.

(d) DU = 9.32 and PV = 86.81.

(e) (0.5:100): DU = 9.32; PV = 86.81.

(0.5+1:122.14): DU = 9.78 PV = 95.60.

The late large payment (0.5+1:122.14) is preferred, suggesting decreasing impatience.

(f) (2.9142: 122.14) has DU = 9.32; PV = 86.81. So the two are indifferent.

(g) For $\delta = 0.8$ the extra time the person is willing to wait after the 0.5 delay has increased by 0.2635 (from 1.5 to 1.7635), but for $\delta = 0.5$ it has increased by 1.4142 (from 1.5 to 2.9142). This suggests that impatience decreased more for $\delta = 0.5$. □