Unknown Unknowns:

Uncertainty About Risk and Stock Returns

Online Supplement

Friday 1st May, 2015

Contents of the Online Supplement

This online supplement presents additional details on the procedures used to construct our data:

- **Annex A** describes the calculation of implied volatility (IV) from US OptionMetrics data (referred to in Section 2).
- **Annex B** describes in more detail how we build our final sample from CRSP and Compustat data (referred to in Section 2).
- **Annex C** describes how we constuct the vol-of-vol measure from European OptionMetrics data (referred to in Section 3.5).
- **Annex D** provides references to key papers for each of the control variables.
- **Annex E (Table A.1)** presents Fama-MacBeth regressions that demonstrate the vol-of-vol effect after controlling for the five sets of control variables (referred to in Section 3.3).
- **Annex F (Table A.2)** presents double-sorts using equal-weighted portfolios.
- **Annex G (Table A.3 and Figure A.1)** motivates, presents, and discusses several robustness checks (referred to in Section 3.4).
- **Annex H (Table Table A.4)** presents and discusses the distribution of (monthly) stock returns associated with varying levels of vol-of-vol (referred to in Section 6.4).

Annex A: Using OptionMetrics data for the U.S.

First, implied volatility can be calculated in several ways. For instance, IVs are often obtained through OptionMetrics' Volatility Surface file, which contains interpolated IVs for constant levels of maturity and moneyness. However, we find that these IVs may also vary because of arbitrary changes in the options used to calculate the Surface. For example, the OptionMetrics 30-day at-the-money put IV is interpolated from four put options, with strike prices straddling the stock price and maturities straddling 30 days. As the included options approach expiration, one or more of the four options will be replaced by other options, often causing a spurious change in the estimated implied volatility.

Other studies use so-called model-free implied volatilities that are computed from option prices without the use of any particular option-pricing model. While a model-free approach is robust to misspecification, it requires a large number of option prices over a whole grid for various moneynesses and maturities to construct the model-free IV measure. Especially for deep-OTM options on many individual names this tends to be a problem. For this reason, model-free approaches are generally used for IV estimates at the market level for very liquid markets such as the S&P 500 option market, but seem less suitable for options on a wide cross-section of individual stocks. In unreported analyses, we find that our results after applying model-free approaches are indeed more noisy but otherwise similar.

We use model-based IVs calculated as the average IV of the at-the-money (ATM) call option and ATM put option. Such ATM IV quotes approximate the risk-neutral expected volatility of a stock (Carr and Wu, 2005). Our ATM IVs are generally based on the most liquid, actual option prices, and hence do not require interpolation which potentially leads to distortions. While this could lead to fluctuations in the moneyness and maturity of selected options over time that result in undesired cross-sectional differences in vol-of-vol, we find such differences in vol-of-vol to be rather limited in our sample. OptionMetrics calculates these IVs from Cox et al. (1979)'s binomial tree-based algorithm that incorporates discrete dividend payments and early exercise.

Using these 'plain' IVs we apply the following screening criteria on all options to ensure that we select well-traded and well-priced options that contain reliable information:

- We exclude 'special' options that do not expire on the third Friday of a month. This filters out non-standard option series that are only partially available in the sample and generally have lower liquidity.
- We retain only those options that have positive open interest, positive best bid price, and at least 12 non-missing implied volatility values between 3 percent and 200 percent.
- We eliminate all options that have bid-ask spreads exceeding 25 percent of the average between the bid and ask price.

- Since most activity for options is concentrated at the short end, we require a time to maturity between 10 and 52 trading days for our main option measures, thereby selecting options with a remaining time to maturity (TTM) of approximately one month. We have verified that results are robust to changing the options' TTM to three, six or twelve months.

- We separate call and put options into at-the-money (ATM), out-of-the-money (OTM), and in-the-money (ITM) options following Ofek et al. (2004) and Xing et al. (2010). An option is defined as ATM when the ratio of the strike price to the stock price (strike-to-spot) is between 0.95 and 1.05. Similarly, an option is defined as OTM when the ratio is lower than 0.95 (but higher than 0.80), and ITM when the ratio is higher than 1.05 (but lower than 1.20). Options with ratios below 0.80 and above 1.20 are dropped from the sample. When multiple options fall into the same group, we select the option with moneyness closest to 1.00 (ATM), 0.95 (OTM) or 1.05 (ITM). We have verified that the results are qualitatively similar when weighing all ATM options by their volume or open interest.

Annex B: Other data choices

- From the CRSP data, we exclude "penny stocks" with prices below $5 (Amihud, 2002; Zhang, 2006), and "micro caps" by requiring a market capitalization of at least $225mln (e.g., D'Avolio, 2002) at the end of 2009 (discounted at the risk-free rate to ensure that the $225 threshold does not remove relatively more stocks in earlier (later) years, when market capitalization generally is smaller (larger)). This threshold roughly corresponds to the smallest 10% of NYSE stocks and effectively eliminates stocks with difficult-to-measure prices and fundamentals, and illiquid stocks with potential market microstructure problems. D'Avolio (2002) shows that about one-third of these excluded stocks are difficult to short since institutional lenders generally do not have a position in them, and have high shorting costs. Hence, these criteria imply that we select stocks with relatively low short-sale constraints.

- We adjust the CRSP returns for delisting biases (obtained from the CRSP delisting file) as suggested by Shumway (1997) and Shumway and Warther (1999), assuming a delisting return of -30 percent (for NYSE and AMEX stocks) or -55 percent (for NASDAQ stocks) if the corresponding delisting code is performance-related.

- Following Fama and French (1992), we match Compustat accounting data to CRSP after six months following fiscal year end. Accounting data are required to have a 3-year history to prevent survivorship bias. Realized earnings are obtained from Compustat's quarterly item 8 (income before extraordinary items) and matched to CRSP after the earnings announcement date.

Annex C: Using OptionMetrics data for Europe

For our examination of the vol-of-vol effect in European option markets in the paper, we make the following adjustments to account for the specifics of the European database. We select options on all common shares (issuetype 1) with a European region code. We restrict our sample to European companies and remove options on, e.g., ADRs on non-European stocks. We remove all stocks that are not included in the S&P Global Broad Market Index to prevent our results to be driven by micro caps. Because options and underlying stocks are traded across different countries, and some countries have several exchanges, issues can arise due to mismatches in trade time or currency. As Swiss stocks are particularly sensitive to this issue, we exclude this country from the sample.[1]

To calculate the vol-of-vol measure, we use IVs from options with positive open interest, positive best bid price, and IV values in the 5-200 percent range. Optionmetrics already accounts for a number of erroneous cases, e.g., it does not calculate implied volatilities for options with a price below intrinsic value. We further remove options for which the call and put IVs differ by more than 25%, or have large daily jumps in open interest (maximum factor of 10) or implied volatility (maximum factor of 2). These restrictions do not materially affect the results. While we aim for a remaining maturity of one month as in the US sample, European option exchanges differ considerably in the times to maturity that are common. We therefore allow time to maturity to vary across exchanges and select those options that have the maturity most frequently observed for a specific exchange, but within the 14-120 days range, based on the number of outstanding contracts. We select the option contract with moneyness closest to 1, as long as the moneyness is between 0.95 and 1.05. The IV used to calculate vol-of-vol is the average of the call IV and put IV.

As in Eq. (3), we calculate vol-of-vol each month as the standard deviation of daily IVs, divided by average IV over the month. We require a minimum number of ten observations, and account for a one day implementation lag by skipping the last day of the month. Subsequently, we sort stocks in ascending order into quintile portfolios on the basis of vol-of-vol. To account for differences across all European countries, we rank stocks and create quintile portfolios within each country, and calculate equal-weighted and value-weighted excess returns and alphas. Since within-country value-weighting can lead to large country exposures (e.g., when a country has many stocks that are large compared to other European stocks, the top quintile will have a disproportionate exposure to this country) we value-weigh each country within each quintile based on the country's aggregated market value.

[1]We thank Barclays Capital for guidance on this.

Annex D: References with respect to control variables

- Beta: Black et al. (1972); Fama and MacBeth (1973)
- Book-to-market: Polk et al. (2006)
- Size: Banz (1981)
- Momentum: Jegadeesh and Titman (1993)
- Short-term reversal: Jegadeesh (1990); Lehmann (1990)
- Return distribution characteristicsIdiosyncratic volatility: Ang et al. (2006); Bali et al. (2011)
- Maximum return: Bali et al. (2011)
- Skewness: Xu (2007)
- Amihud illiquidity: Amihud (2002)
- Turnover: Datar et al. (1998)
- At-the-money skew: Bali and Hovakimian (2009); Cremers and Weinbaum (2010)
- Out-of-the-money skew: Xing et al. (2010)
- Implied volatility - realized volatility spread: Bali and Hovakimian (2009); Goyal and Saretto (2009)
- Change in the ATM call IV: Ang et al. (2010)
- Change in ATM put IV: Ang et al. (2010)
- Age: Pastor and Veronesi (2003); Zhang (2006)

- Analyst coverage: Zhang (2006)
- Forecast dispersion: Diether et al. (2002); Zhang (2006)
- Volatility: Zhang (2006)
- Private information: Durnev et al. (2003)
- Leverage: Bhandari (1988)
- Stock price delay: Hou and Moskowitz (2005)
- Short-sale constraints: Nagel (2005)
- Changes in VIX: Ang et al. (2006)
- Change in the slope of the implied volatility skew: Cremers et al. (2011); Yan (2011)

Annex E: Fama-MacBeth regressions

We estimate Fama and MacBeth (1973) regressions to simultaneously control for a range of control variables, to avoid the specification of breakpoints, and to further take advantage of the cross-sectional variation in vol-of-vol and the control variables. Each month, we conduct cross-sectional regressions of stock returns on vol-of-vol and one or more control variables, each of which is winsorized at the 1st and 99th percentile to limit the effect of outliers. The regressions are estimated using OLS and take the following form:

$$r_{i,t+1} - r^f_{t+1} = \alpha + \beta X_{i,t} + \varepsilon_{i,t+1}$$

where $r_{i,t+1}$ is the realized return on stock i in month $t+1$, r^f_{t+1} is the risk free rate over month $t+1$, $X_{i,t}$ is a collection of predictor variables at time t for stock i, and $\varepsilon_{i,t+1}$ is the prediction error which is assumed to be normally distributed with mean zero.

Next, we conduct tests on the time-series averages of the slope coefficients from the regressions. To account for potential autocorrelation and heteroskedasticity in the coefficients, we compute Newey and West (1987)-adjusted t-statistics based on the time-series of the coefficient estimates. Table A.1 shows the results, classified in the same categories as in the main body of the paper. The coefficient on vol-of-vol remains highly significant with t-statistics generally well above 3, indicating that the vol-of-vol effect is not simply a combination of the cross-sectional return predictors above.

Table A.1: Fama-MacBeth regression results

This table presents coefficient estimates from monthly Fama-MacBeth (1973) regressions over our sample period from January 1996 to October 2014, of stock returns on vol-of-vol and one or more control variables, each of which is winsorized at the 1st and 99th percentile to limit the effect of outliers. The regressions take the following form:

$$r_{i,t+1} - r^f_{t+1} = \alpha + \beta X_{i,t} + \varepsilon_{i,t+1}$$

where $r_{i,t+1}$ is the realized return on stock i in month $t+1$, r^f_{t+1} is the risk free rate over month $t+1$, $X_{i,t}$ is a collection of predictor variables (classified in the same categories as in the main body of the paper) at time t for stock i, and $\varepsilon_{i,t+1}$ is the prediction error which is assumed to be normally distributed with mean zero. We regress excess stock returns over month $t+1$ against a constant, vol-of-vol, and a series of stock characteristics, all measured at the end of month t using a one-day implementation lag. The variable definitions are described in the paper (Appendix). Vol-of-vol is past month's volatility of option-implied volatility (IV), standardized by average IV (as in the main paper). IV is calculated from at-the-money call and put options with maturity closest to 30 days. We report t-statistics in parentheses that are Newey-West corrected. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

(a) Canonical characteristics

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	0.011***	0.012***	0.009**	0.008	0.009**	0.011***	0.012**
	(2.73)	(5.21)	(2.22)	(0.92)	(2.36)	(2.86)	(2.03)
Vol-of-vol	-0.034***	-0.029***	-0.034***	-0.03***	-0.034***	-0.032***	-0.023***
	(-3.91)	(-4.15)	(-4.03)	(-3.30)	(-4.08)	(-3.76)	(-3.56)
Beta		-0.001					-0.002
		(-0.54)					(-0.85)
Book-to-market			0.003				0.001
			(1.43)				(0.68)
ln(Size)				<0.001			>-0.001
				(0.44)			(-0.21)
Momentum					-0.002		-0.006**
					(-0.76)		(-2.30)
Short-term reversal						-0.002	-0.006**
						(-0.53)	(-2.03)
Adjusted R^2	0.002	0.046	0.011	0.018	0.015	0.012	0.073
Observations	244,538	244,538	244,538	244,538	244,538	244,538	244,538

Table A.1: Fama-MacBeth regression results (Continued)

(b) Returns distribution characteristics

	(1)	(2)	(3)	(4)	(5)
Constant	0.026***	0.016***	0.013**	0.013**	0.027***
	(4.42)	(2.88)	(2.13)	(2.08)	(4.60)
Vol-of-vol	-0.023***	-0.021***	-0.023***	-0.024***	-0.025***
	(-3.56)	(-3.16)	(-3.54)	(-3.66)	(-4.08)
Beta	>-0.001	-0.001	-0.002	-0.002	<0.001
	(-0.19)	(-0.63)	(-0.86)	(-0.86)	(0.05)
Book-to-market	>-0.001	0.001	0.001	0.001	>-0.001
	(-0.11)	(0.46)	(0.71)	(0.68)	(-0.08)
ln(Size)	-0.001**	>-0.001	>-0.001	>-0.001	-0.051**
	(-2.22)	(-0.76)	(-0.32)	(-0.27)	(-2.38)
Momentum	-0.005**	-0.005**	-0.006**	-0.006**	-0.006**
	(-2.13)	(-2.20)	(-2.22)	(-2.24)	(-2.12)
Short-term reversal	-0.006**	-0.005	-0.006**	-0.006**	-0.006*
	(-1.98)	(-1.53)	(-2.07)	(-2.04)	(-1.89)
Idiosync. volatility	-0.283***				-0.315***
	(-2.65)				(-2.77)
Maximum return		-0.034*			-0.006
		(-1.89)			(-0.44)
Skewness			-0.001		>-0.001
			(-1.36)		(-0.87)
Kurtosis				<0.001	<0.001*
				(0.34)	(1.77)
Adjusted R^2	0.082	0.077	0.075	0.074	0.087
Observations	243,445	243,445	243,445	243,445	243,445

Table A.1: Fama-MacBeth regression results (Continued)

(c) Liquidity characteristics

	(1)	(2)	(3)
Constant	0.013**	0.013**	0.014**
	(2.26)	(2.18)	(2.41)
Vol-of-vol	-0.023***	-0.024***	-0.024***
	(-3.51)	(-3.46)	(-3.42)
Beta	-0.002	-0.002	-0.002
	(-0.86)	(-0.88)	(-0.89)
Book-to-market	0.001	0.001	0.001
	(0.74)	(0.85)	(0.91)
ln(Size)	>-0.001	>-0.001	>-0.001
	(-0.44)	(-0.24)	(-0.46)
Momentum	-0.005**	-0.006**	-0.005**
	(-2.22)	(-2.25)	(-2.18)
Short-term reversal	-0.006**	-0.006*	-0.006**
	(-2.06)	(-1.93)	(-1.96)
Amihud illiquidity	-0.052		-0.068
	(-0.49)		(-0.65)
Turnover		>-0.001	>-0.001
		(-0.40)	(-0.39)
Adjusted R^2	0.075	0.078	0.081
Observations	244,529	244,529	244,529

Table A.1: Fama-MacBeth regression results (Continued)

(d) Options-based characteristics

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	0.011	0.011	0.011	0.010	0.010	0.011	0.012*
	(1.56)	(1.64)	(1.60)	(1.44)	(1.52)	(1.61)	(1.73)
Vol-of-vol	-0.033***	-0.035***	-0.033***	-0.034***	-0.038***	-0.038***	-0.031***
	(-2.87)	(-3.04)	(-3.02)	(-2.88)	(-3.11)	(-3.19)	(-2.53)
Beta	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.001
	(-0.87)	(-0.80)	(-0.86)	(-0.61)	(-0.81)	(-0.83)	(-0.56)
Book-to-market	0.002	0.002	0.002	0.002	0.002	0.002	0.002
	(0.72)	(0.75)	(0.77)	(0.76)	(0.84)	(0.87)	(0.72)
ln(Size)	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	>-0.001
	(0.07)	(0.12)	(0.18)	(0.32)	(0.22)	(0.16)	(-0.23)
Momentum	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002
	(-0.58)	(-0.62)	(-0.71)	(-0.71)	(-0.72)	(-0.73)	(-0.62)
Short-term reversal	-0.001	-0.001	-0.002	-0.002	-0.002	-0.002	-0.001
	(-0.37)	(-0.43)	(-0.63)	(-0.51)	(-0.42)	(-0.58)	(-0.39)
Option bid-ask spread	-0.001						-0.003
	(-0.19)						(-0.39)
ATM skew		0.057***					0.067***
		(3.61)					(3.04)
OTM skew			-0.010				0.006
			(-0.86)				(0.55)
IV-RV spread				-0.002			-0.002
				(-0.32)			(-0.33)
Change in call IV					-0.008		-0.015
					(-0.88)		(-0.91)
Change in put IV						-0.018**	0.002
						(-2.31)	(0.13)
Adjusted R^2	0.087	0.085	0.085	0.087	0.087	0.086	0.099
Observations	120,463	120,463	120,463	120,463	120,463	120,463	120,463

Table A.1: Fama-MacBeth regression results (Continued)

(e) Uncertainty characteristics and other characteristics

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Constant	0.013**	0.017**	0.016***	0.021***	0.014**	0.027***	0.018***	0.014**	0.028***
	(2.08)	(2.58)	(2.66)	(3.65)	(2.25)	(3.87)	(3.24)	(2.33)	(4.15)
Vol-of-vol	-0.023***	-0.023***	-0.021***	-0.021***	-0.021***	-0.023***	-0.023***	-0.023***	-0.020***
	(-3.50)	(-3.48)	(-3.33)	(-3.23)	(-3.37)	(-3.56)	(-3.56)	(-3.46)	(-3.37)
Beta	-0.002	-0.002	-0.001	-0.001	-0.002	-0.003	-0.003	-0.002	-0.001
	(-0.75)	(-0.89)	(-0.37)	(-0.28)	(-0.82)	(-1.15)	(-1.04)	(-0.81)	(-0.57)
Book-to-market	0.001	0.001	0.003	0.001	0.001	<0.001	0.001	0.001	0.002
	(0.63)	(0.66)	(1.61)	(0.36)	(0.57)	(0.17)	(0.49)	(0.67)	(1.22)
ln(Size)	>-0.001	-0.001	-0.001	-0.001	>-0.001	-0.101*	-0.001	>-0.001	-0.012***
	(-0.84)	(-1.28)	(-1.06)	(-1.50)	(-0.69)	(-1.76)	(-0.93)	(-0.60)	(-3.05)
Momentum	-0.006**	-0.005**	-0.006**	-0.005**	-0.006**	-0.006**	-0.005**	-0.006**	-0.006**
	(-2.24)	(-2.19)	(-2.22)	(-2.15)	(-2.40)	(-2.19)	(-2.15)	(-2.24)	(-2.40)
Short-term reversal	-0.006*	-0.006**	-0.007**	-0.006*	-0.007**	-0.006*	-0.006**	-0.006**	-0.007**
	(-2.05)	(-1.97)	(-2.09)	(-1.93)	(-2.13)	(-1.95)	(-1.96)	(-2.03)	(-2.19)
ln(Age)	-0.001								-0.001
	(-1.21)								(-0.99)
Analyst coverage		<0.001*							<0.001**
		(1.80)							(2.11)
Forecast dispersion			-0.489***						-0.543***
			(-4.65)						(-5.43)
Volatility				-0.169*					-0.043
				(-1.86)					(-0.49)
Private information					0.001				0.002
					(0.34)				(0.74)
Leverage						-0.059**			-0.004
						(-2.11)			(-1.23)
Stock price delay							-0.004		-0.001
							(-1.40)		(-0.26)
Short sale constraints								<0.001***	<0.001***
								(2.74)	(2.76)
Adjusted R^2	0.077	0.077	0.079	0.083	0.080	0.078	0.077	0.075	0.099
Observations	233,062	233,062	233,062	233,062	233,062	233,062	233,062	233,062	233,062

Annex F: Equal-weighted double sorts

Table A.2: Equal-weighted returns of portfolios sorted by stock characteristics and vol-of-vol

This table reports average monthly *equal-weighted* returns of portfolios sorted on stock characteristics and vol-of-vol over our sample period from January 1996 to October 2014. Vol-of-vol is past month's volatility of option-implied volatility (IV), standardized by average IV (see Section 2). IV is calculated from at-the-money call and put options with maturity closest to 30 days. Each month we sort stocks in ascending order into quintile portfolios on the basis of one of the characteristics described in Section 3. Each characteristic is defined in the Appendix. Within each characteristic quintile, we sort stocks into five additional portfolios ("Low", "2", "3", "4", and "High") based on vol-of-vol, and then average each of the vol-of-vol portfolios across the five quintiles that resulted from the first sort. We compute the returns on the resulting portfolios over the subsequent month, as well as the difference between portfolio High and portfolio Low ("High-Low"). We use a one trading day implementation lag and equal-weigh stocks in each portfolio. The column labeled "High-Low (4F alpha)" presents the difference in 4-factor alphas between portfolio High and portfolio Low. For liquidity characteristics, the table presents returns of the most liquid portfolios. For the remaining characteristics, the table presents the return of each vol-of-vol quintile, averaged over the five characteristic-sorted portfolios. We report t-statistics in parentheses that are Newey-West corrected. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

(a) Canonical characteristics

	Low	2	3	4	High	High-Low	High-Low (4F alpha)
Beta	0.73*	0.68*	0.63	0.53	0.42	-0.32***	-0.29***
	(1.89)	(1.76)	(1.64)	(1.33)	(1.06)	(-3.26)	(-2.90)
Book-to-market	0.82**	0.75*	0.62	0.57	0.36	-0.46***	-0.40***
	(2.03)	(1.84)	(1.55)	(1.40)	(0.89)	(-3.91)	(-3.24)
Size	0.69	0.73*	0.53	0.48	0.29	-0.40***	-0.34**
	(1.57)	(1.68)	(1.22)	(1.07)	(0.66)	(-2.88)	(-2.54)
Momentum	0.80**	0.73*	0.57	0.47	0.29	-0.51***	-0.47***
	(1.97)	(1.79)	(1.40)	(1.15)	(0.71)	(-4.26)	(-3.83)
Short-term reversal	0.71*	0.74*	0.52	0.44	0.29	-0.42***	-0.36***
	(1.73)	(1.78)	(1.26)	(1.07)	(0.70)	(-3.52)	(-2.86)

(b) Returns distribution characteristics

	Low	2	3	4	High	High-Low	High-Low (4F alpha)
Idiosync. volatility	0.70*	0.71	0.51	0.46	0.28	-0.42***	-0.38***
	(1.67)	(1.63)	(1.19)	(1.05)	(0.65)	(-3.83)	(-3.51)
Maximum return	0.72*	0.64	0.56	0.52	0.24	-0.48***	-0.45***
	(1.69)	(1.51)	(1.33)	(1.23)	(0.58)	(-4.33)	(-4.24)
Skewness	0.71*	0.72*	0.53	0.47	0.24	-0.47***	-0.40***
	(1.70)	(1.71)	(1.26)	(1.12)	(0.56)	(-3.81)	(-3.24)
Kurtosis	0.74*	0.74*	0.51	0.47	0.28	-0.46***	-0.40***
	(1.78)	(1.76)	(1.22)	(1.12)	(0.66)	(-3.65)	(-3.11)

Table A.2: Equal-weighted returns of portfolios sorted by stock characteristics and vol-of-vol (Continued)

(c) Liquidity characteristics (largest and most liquid stocks)

	Low	2	3	4	High	High-Low	High-Low (4F alpha)
Largest stocks	0.72**	0.82**	0.79**	0.52	0.21	-0.51**	-0.50**
(top size quintile)	(2.39)	(2.52)	(2.37)	(1.46)	(0.54)	(-2.35)	(-2.45)
Largest stocks	0.75**	0.66**	0.66**	0.44	0.1	-0.65***	-0.54**
(NYSE stocks only)	(2.50)	(2.26)	(2.24)	(1.38)	(0.28)	(-2.9)	(-2.41)
Most liquid stocks	0.84**	0.73**	0.74*	0.51	0.22	-0.62***	-0.54***
(Amihud)	(2.41)	(2.01)	(1.95)	(1.29)	(0.53)	(-2.97)	(-2.68)
Most liquid stocks	0.72	0.57	0.45	0.26	-0.08	-0.80***	-0.69***
(turnover)	(1.30)	(1.00)	(0.80)	(0.44)	(-0.15)	(-3.58)	(-3.04)

(d) Option-based characteristics

	Low	2	3	4	High	High-Low	High-Low (4F alpha)
Option bid-ask spread	0.73*	0.70*	0.56	0.40	0.29	-0.44***	-0.38***
	(1.75)	(1.69)	(1.37)	(0.95)	(0.68)	(-3.55)	(-3.01)
ATM skew	0.73*	0.73*	0.59	0.41	0.16	-0.57***	-0.52***
	(1.74)	(1.72)	(1.38)	(0.94)	(0.35)	(-4.01)	(-3.87)
OTM skew	0.71*	0.70*	0.58	0.44	0.17	-0.54***	-0.46***
	(1.67)	(1.66)	(1.38)	(1.00)	(0.38)	(-3.88)	(-3.18)
IV-RV spread	0.66	0.65	0.55	0.43	0.19	-0.47***	-0.42***
	(1.59)	(1.54)	(1.32)	(1.00)	(0.46)	(-3.82)	(-3.29)
Change in call IV	0.71	0.75*	0.63	0.41	0.32	-0.38**	-0.28**
	(1.59)	(1.81)	(1.57)	(1.04)	(0.75)	(-2.28)	(-1.96)
Change in put IV	0.69	0.63	0.61	0.45	0.31	-0.38**	-0.29**
	(1.52)	(1.47)	(1.48)	(1.13)	(0.78)	(-2.29)	(-1.99)

Table A.2: Equal-weighted returns of portfolios sorted by stock characteristics and vol-of-vol (Continued)

(e) Uncertainty-related characteristics

	Low	2	3	4	High	High-Low	High-Low (4F alpha)
Age	0.74*	0.63	0.56	0.39	0.19	-0.55***	-0.48***
	(1.75)	(1.46)	(1.30)	(0.90)	(0.45)	(-4.29)	(-3.83)
Analyst coverage	0.60	0.59	0.48	0.41	0.16	-0.45***	-0.38**
	(1.31)	(1.29)	(1.04)	(0.90)	(0.36)	(-2.85)	(-2.36)
Forecast dispersion	0.80*	0.81*	0.51	0.43	0.28	-0.52***	-0.46***
	(1.88)	(1.94)	(1.19)	(0.99)	(0.66)	(-4.02)	(-3.52)
Volatility	0.72*	0.70	0.49	0.53	0.29	-0.43***	-0.41***
	(1.69)	(1.62)	(1.16)	(1.21)	(0.70)	(-4.02)	(-3.8)
Private information	0.74*	0.72*	0.58	0.44	0.38	-0.36***	-0.30**
	(1.82)	(1.77)	(1.46)	(1.07)	(0.93)	(-2.97)	(-2.3)

(f) Other characteristics

	Low	2	3	4	High	High-Low	High-Low (4F alpha)
Leverage	0.76*	0.70*	0.57	0.48	0.31	-0.46***	-0.38***
	(1.88)	(1.74)	(1.41)	(1.17)	(0.74)	(-3.88)	(-3.22)
Stock price delay	0.78*	0.72*	0.55	0.50	0.34	-0.44***	-0.37***
	(1.91)	(1.80)	(1.37)	(1.23)	(0.85)	(-3.64)	(-2.95)
Short sale constraints	0.76*	0.73*	0.60	0.43	0.25	-0.51***	-0.44***
	(1.84)	(1.75)	(1.45)	(1.01)	(0.59)	(-3.84)	(-3.33)

Annex G: Robustness checks

In Table A.3 below we examine the sensitivity of our results to a series of robustness checks. First, by forming vol-of-vol portfolios that follow procedures that are standard in the literature, we ignore possible industry clustering within the portfolios. One could argue that high and low vol-of-vol stocks cluster in certain industries at various points in time, so that the vol-of-vol effect is partly driven by industry effects. Therefore, we create industry-neutral vol-of-vol portfolios by constructing value-weighted vol-of-vol quintile portfolios within each two-digit SIC code industry, and averaging each vol-of-vol portfolio over the industries. The rows labeled "Industry neutral vol-of-vol portfolios" present the results. The vol-of-vol effect is roughly of similar magnitude as before. However, removing between-industry variation within each of the vol-of-vol portfolios increases the t-statistic to -3.67 (-3.32). Hence, the vol-of-vol effect is robust to controlling for industry effects.

Second, we scale vol-of-vol with the average IV over the past month to break the correlation between the two measures (average cross-sectional rank correlations between IV and vol-of-vol are indeed zero). To confirm that such scaling is not driving the negative vol-of-vol effect, we also create portfolios based on unscaled vol-of-vol. The rows labeled "Vol-of-vol: No IV scaling" show that the High-Low difference for unscaled vol-of-vol is economically larger and continues to be statistically significant, if not more significant. Next, even though we include both volatility and the RV-IV spread in previous analysis, we further address concerns about the role of IV by first sorting on IV before sorting on the IV-scaled vol-of-vol measure. The rows labeled "IV neutral, IV-scaled vol-of-vol portfolios" indicate that the High-Low difference remains economically and statistically strong after double-sorting on IV. Hence, the vol-of-vol effect seems to capture something distinctly different from IV.

Third, we approximate the expected volatility of a stock by IV. However, IV contains not only the expected volatility of a stock, but also a volatility risk premium. To verify that the former rather than the latter drives our results, we replace IV in the vol-of-vol definition by the 'objective' expected volatility of a stock. Objective expected volatility is constructed from the fitted values of a daily regression of one month ahead realized volatility on the current values of IV and realized volatility. We use one-year rolling windows requiring at least 12 degrees of freedom and approximate the realized volatility over the coming month by the square root of summed squared daily returns over this period. The rows labeled "Vol-of-vol: Volatility of 'objective' volatility" show that the vol-of-vol effect based on objective expected volatility remains economically and statistically strong. Hence, the vol-of-vol effect also seems robust to using alternative measures of expected volatility.

Fourth, the negative vol-of-vol effect could be driven by the way we have defined vol-of-vol. For instance, since volatility tends to spike up from time to time and only gradually moves down, computing vol-of-vol based on the level of IV gives relatively higher importance to increases in volatility. Also, computing the volatility of IV levels implicitly assumes that IV realizations are merely noise around a constant mean, an assumption that ignores the empirical evidence on persistence in volatility. We therefore also present results for alternative vol-of-vol definitions such as the volatility of log-transformed IV, the volatility of IV increments (which implicitly assumes a random walk), and the volatility of log-transformed IV increments. The rows labeled “Vol-of-vol: volatility of ln(IV)”, “Vol-of-vol: volatility of increments in IV”, and ”Vol-of-vol: volatility of increments in ln(IV)” show that the vol-of-vol effect remains economically strong and statistically significant for each of these alternative vol-of-vol definitions. Hence, the vol-of-vol effect seems not to depend on a particular definition of vol-of-vol.

Finally, we examine the robustness of the vol-of-vol effect when we vary the historical window to compute vol-of-vol between three, six or twelve months. The rows labeled “Vol-of-vol: past$n$ month" present the results. The vol-of-vol effect is negative for all the different historical windows, but generally of smaller statistical significance. Please see the main text for further discussion of this.

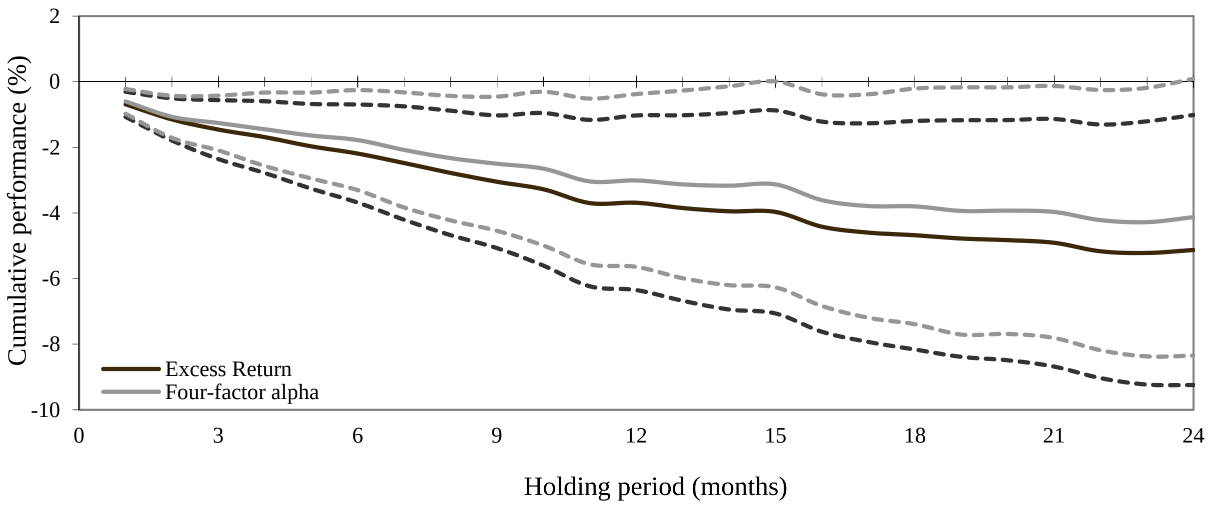
Table A.3: Robustness checks

This table reports robustness results on the vol-of-vol effect. Vol-of-vol is past month's volatility of option-implied volatility (IV), standardized by average IV unless stated otherwise. IV is calculated from at-the-money call and put options with maturity closest to 30 days. We use a one trading day implementation lag and value-weigh the stocks in each portfolio. The Table presents the results for a selection of robustness checks. The top rows restate the results from Table 3. The rows labeled "Industry neutral vol-of-vol portfolios" present the vol-of-vol effect after constructing industry-neutral vol-of-vol portfolios using the double-sorting procedure described in Section 3.3. The rows labeled "Vol-of-vol: No IV scaling" present the vol-of-vol effect when vol-of-vol is *not* scaled by average implied volatility. The rows labeled "IV neutral, IV-scaled vol-of-vol portfolios" present the vol-of-vol effect after constructing IV-neutral vol-of-vol portfolios using the double-sorting procedure described in Section 3.3, in addition to scaling vol-of-vol by average implied volatility. The rows labeled "Vol-of-vol: Volatility of ln(IV)", "Vol-of-vol: Volatility of increments in IV", "Vol-of-vol: Volatility of increments in ln(IV)", and "Vol-of-vol: Volatility of 'objective' volatility" present the vol-of-vol effect for a selection of transformations of IV in Eq. 1. 'Objective' volatility is expected volatility calculated as the fitted values from a regression of one-month ahead realized volatility on current realized volatility and implied volatility. The rows labeled "Vol-of-vol: past$n$ month" calculate vol-of-vol using implied volatilities over the past n months. We report t-statistics in parentheses that are Newey-West corrected. *, ** and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Low	High	High-Low	High-Low (4F alpha)
Benchmark vol-of-vol effect (Table 3)	0.81	0.12	-0.69***	-0.60***
	(2.61)	(0.32)	(-2.96)	(-2.62)
Industry neutral vol-of-vol portfolios	0.85	0.41	-0.44***	-0.42***
	(2.73)	(1.22)	(-3.67)	(-3.32)
Vol-of-vol: No IV scaling	0.68	-0.06	-0.74*	-0.94***
	(2.78)	(-0.11)	(-1.89)	(-3.33)
IV neutral, IV-scaled vol-of-vol portfolios	0.76	0.08	-0.68***	-0.59**
	(1.72)	(0.16)	(-3.19)	(-2.58)
Vol-of-vol: Volatility of ln(IV)	0.74	0.19	-0.55**	-0.49**
	(2.47)	(0.48)	(-2.27)	(-2.17)
Vol-of-vol: Volatility of increments in IV	0.73	0.20	-0.53**	-0.46**
	(2.32)	(0.50)	(-2.33)	(-2.14)
Vol-of-vol: Volatility of increments in ln(IV)	0.75	0.27	-0.47**	-0.45**
	(2.42)	(0.71)	(-2.10)	(-2.05)
Vol-of-vol: Volatility of 'objective' volatility	0.79	0.27	-0.52**	-0.48**
	(2.78)	(0.68)	(-2.33)	(-2.16)
Vol-of-vol: past 3-month	0.66	0.44	-0.22	-0.12
	(2.12)	(1.11)	(-0.90)	(-0.47)
Vol-of-vol: past 6-month	0.74	0.34	-0.41*	-0.31
	(2.34)	(0.86)	(-1.72)	(-1.32)
Vol-of-vol: past 12-month	0.84	0.30	-0.53**	-0.51**
	(2.82)	(0.72)	(-2.20)	(-2.36)

Figure A.1: Performance persistence of the High-Low vol-of-vol portfolio

This figure shows the average cumulative performance of the High-Low vol-of-vol portfolio that buys the top quintile portfolio and sells the bottom quintile portfolio. The sample period runs from January 1996 to October 2014. Vol-of-vol is past month's volatility of option-implied volatility (IV), standardized by average IV (see Section 2). IV is calculated from at-the-money call and put options with maturity closest to 30 days. Each month we sort stocks in ascending order into quintile portfolios ("Low", "2", "3", "4", and "High") on the basis of vol-of-vol. We use a one trading day implementation lag and value-weigh stocks in each portfolio. We then buy the High portfolio and sell the Low portfolio, and hold this position for the next 1 to 24 months. The graph plots the average excess returns (black line) and four-factor alphas (grey line) of this strategy, with dotted lines delineating the 95% confidence interval.



Annex H: Distribution of monthly stock returns for varying levels of vol-of-vol

Table A.3 contains the results of we examining the distribution of future monthly returns in more detail for each of the five quintile portfolios.. In panels (a) and (b), the first data column ("Avg") re-states the average excess returns on the five vol-of-vol portfolios. Panel (a) contains the time-series averaged statistics of the cross-sectional, value-weighted future returns distribution of the individual stocks included within each quintile portfolio, as well as the difference in statistics between the High vol-of-vol portfolio and the Low vol-of-vol portfolio ("High-Low"). Panel (b) presents statistics that describe the time-series returns distribution of each of the quintile portfolios statistics themselves, the difference in statistics between the High vol-of-vol portfolio and the Low vol-of-vol portfolio ("High-Low"), and the returns distribution of the High-Low quintile portfolio described in previous Sections ("Hedge").

Panel (a) reveals that the returns distribution of the "High" portfolio displays less upside potential, a comparable reduction in downside risk, but a fatter extreme left tail than the "Low" portfolio. Panel (b) shows that "High" vol-of-vol portfolios have lower returns over time up to the 75th percentile (columns "Min" to "P75") t the "Low" portfolio, which is hard to align with a risk-based explanation. Below-median return differences ("High-Low") are generally larger in absolute value than above-median differences, indicating downside risk that exceeds upside potential. By contrast, for the 90th percentile and up, the higher vol-of-vol portfolios generally achieve higher returns, but to a limited extend. Furthermore, when we look at the future returns distribution of the High vol-of-vol minus Low vol-of-vol portfolio ("Hedge"), negative returns again tend to be larger (in absolute terms) than positive returns. Hence, the future stock or portfolio returns distributions of various vol-of-vol portfolios provide no evidence for an asymmetric risk-based explanation.

Table A.4: Distribution of monthly stock returns associated with varying levels of vol-of-vol

This table presents distribution characteristics for portfolios sorted on vol-of-vol over our sample period from January 1996 to October 2014. Vol-of-vol is past month's volatility of option-implied volatility (IV), standardized by average IV (see Section 2). IV is calculated from at-the-money call and put options with maturity closest to 30 days. Each month we sort stocks in ascending order into quintile portfolios (Low, 2, 3, 4, and High) on the basis of vol-of-vol. We use a one trading day implementation lag and value-weigh stocks in each portfolio. Panel (a) presents the time-series average of the value-weighted cross-sectional mean (Avg), standard deviation (Std), minimum (Min), percentiles (P1-P99), and maximum (Max) of individual stock returns within each vol-of-vol portfolio, as well as the difference in returns between portfolio High and portfolio Low (High-Low). Panel (b) presents the mean (Avg), standard deviation (Std), minimum (Min), percentiles (P1-P99), and maximum (Max) of the time-series of returns on the vol-of-vol quintile portfolios, as well as the difference in returns between portfolio High and portfolio Low (High-Low). The rows labeled "Hedge" describe the returns distribution of the High-Low vol-of-vol portfolio (that buys the top quintile and sells the bottom quintile).

Panel (a): Cross-sectional distribution of individual stock returns

	Avg	Std	Min	P1	P5	P10	P25	P50	P75	P90	P95	P99	Max
Low	0.81	6.37	-29.36	-15.16	-9.31	-6.51	-2.61	1.46	5.43	9.30	12.51	19.90	41.87
2	0.61	6.77	-30.54	-13.64	-8.32	-5.96	-2.41	1.13	4.66	8.21	10.79	17.98	42.70
3	0.68	6.80	-32.23	-12.06	-7.20	-5.31	-1.93	1.30	4.56	8.04	10.44	16.88	38.38
4	0.40	6.74	-30.79	-12.56	-7.79	-5.37	-1.85	1.15	4.44	7.73	9.84	15.92	38.53
High	0.12	5.97	-34.95	-12.80	-7.51	-5.04	-2.04	1.17	4.47	7.62	10.80	16.94	39.55
High-Low	-0.69	-0.40	-5.59	2.36	1.80	1.47	0.57	-0.29	-0.96	-1.68	-1.71	-2.96	-2.32

Panel (b): Time-series distribution of portfolio returns

	Avg	Std	Min	P1	P5	P10	P25	P50	P75	P90	P95	P99	Max
Low	0.81	4.51	-18.66	-9.77	-6.45	-4.30	-1.80	1.37	4.44	6.10	7.67	10.27	10.96
2	0.61	4.46	-16.21	-9.13	-7.24	-4.54	-2.04	1.50	3.60	6.17	7.53	9.81	11.91
3	0.68	4.73	-17.96	-13.48	-7.38	-5.34	-1.88	1.23	3.93	6.45	7.91	10.37	11.58
4	0.40	4.88	-16.40	-12.95	-8.35	-5.86	-2.08	1.07	3.72	6.73	7.61	10.31	10.59
High	0.12	5.54	-22.00	-17.47	-10.28	-6.10	-2.42	0.87	3.83	6.10	8.10	12.17	17.47
High-Low	-0.69	1.03	-3.35	-7.70	-3.84	-1.80	-0.61	-0.50	-0.61	0.00	0.43	1.90	6.50
Hedge	-0.69	3.52	-19.50	-14.44	-5.52	-4.00	-2.08	-0.41	1.02	2.67	3.26	9.04	14.85

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