Testing for international business cycles: a multilevel factor model with stochastic factor selection

Tino Berger[1], Gerdie Everaert[2], and Lorenzo Pozzi*[3]

[1] *University of Goettingen*

[2] *Ghent University*

[3] *Erasmus University Rotterdam & Tinbergen Institute*

December 2020

Abstract

The empirical literature on common international business cycles has largely ignored model misspecification in estimated factor models as the various cycles are typically imposed but not tested for. This paper proposes a Bayesian stochastic factor selection approach for multilevel factor models. The procedure is applied to a three-level dynamic factor model with a global factor, six regional factors and three development level factors. We estimate the factor model using real GDP growth data for a panel of 60 countries over the period 1961 − 2017. We find robust evidence for the presence of a global business cycle, four regional cycles (Europe, North America, Latin America and Asia) and two development level cycles (industrial countries and emerging market economies). This suggests that both geographical proximity and the development level of countries are important dimensions of international business cycle synchronization that should be considered simultaneously, a point not previously made in the existing synchronization literature.

JEL Classification: F44, C52, C32

Keywords: Global business cycle, regional cycle, multilevel dynamic factor model, Bayesian, model selection

*Corresponding author: Department of Economics, P.O. Box 1738, 3000 DR Rotterdam, the Netherlands. Email: pozzi@ese.eur.nl. Website: http://people.few.eur.nl/pozzi.

1 Introduction

The empirical macroeconomic literature has devoted considerable attention to the occurrence and the extent of business cycle synchronization across countries. A branch of this literature has investigated business cycle synchronization using multilevel or hierarchical factor models (see e.g., Kose et al., 2003; Mumtaz et al., 2011; Kose et al., 2012; Hirata et al., 2013; Francis et al., 2017). An important limitation of most of this literature is that model specification is typically ignored, i.e., the precise structure of international interdependencies is imposed rather than tested, resulting in a potentially misspecified model. The type of factors (i.e., global, regional or other) that are included in the model are selected ex ante, without testing which factors are actually relevant in practice. Francis et al. (2017) show that misspecification leads to factor estimates that deviate substantially from the true ones and to severe reductions in model fit.

This paper contributes to the factor literature by suggesting a Bayesian stochastic model specification search for multilevel factor models. This allows to determine which factors are relevant model attributes and which are not. The approach is based on the Bayesian variable selection procedure of George and McCulloch (1993) for regression models with observed variables and its extension by Frühwirth-Schnatter and Wagner (2010) to model selection in state space models with unobserved components. The method differs from the literature that deals with determining the optimal number of common factors in general factor models (see e.g., Hallin and Liska, 2007; Amengual and Watson, 2007; Bai and Ng, 2007) as it specifically tests for the existence of predetermined types of factors in multilevel factor models. The most distinctive feature of this procedure is that it considers standardized factors multiplied by their standard deviation instead of the original scaled factors. These standard deviations then take the form of regression coefficients such that we can apply Bayesian variable selection. The selection procedure amounts to assigning binary indicators to each of the factors in the model. These binary indicators are sampled together with the unknown parameters using Markov Chain Monte Carlo (MCMC) methods. From the sampled binary indicators, we then calculate posterior factor inclusion probabilities. A small-scaled Monte Carlo simulation, tailored to the properties of the data that we will use, confirms that this procedure is adequate in selecting the relevant factors.

We apply our stochastic model specification search to investigate the presence of international business cycles in real GDP growth for a sample of 60 countries over the period 1961 − 2017. In particular, we estimate a three-level dynamic factor model with a global factor, six regional factors (i.e., Europe, North America, Oceania, Latin America, Africa, Asia) and three development group factors (i.e., industrialized countries, emerging market economies and developing countries). The literature that estimates dynamic factor models in the context of business cycle synchronization has until now focused mainly on specifications that include either global and regional factors (see e.g., Kose et al., 2003; Mumtaz et al., 2011; Hirata et al., 2013) or global and development factors (see e.g., Kose et al., 2012). One exception is Francis et al. (2017), who consider a two-level factor model including a global factor and a set of endogenously clustered factors. In particular, rather than imposing a predefined (geographical or other) structure, the data are used to endogenously assign each cross-sectional unit to a cluster. Their estimation results show that similarities in institutions (e.g., legal systems, language diversity) may be just as important as physical proximity for determining country clusters. However, an important restriction is that countries can only belong to a single cluster, which precludes clusters along different dimensions. Our testing procedure, which selects the relevant regional and/or development factors, allows us to conclude whether the drivers of synchronization are either geographical proximity, the level of development or both.

Our method selects a global business cycle, four regional cycles (Europe, North America, Latin America, Asia) and two development group cycles (industrial countries, emerging market economies). These results obtained from our factor selection procedure are new and suggest that it is important to estimate factor models with factors that capture *both* geographical proximity and the level of development of countries, a point that has not yet been made in the existing synchronization literature. Using variance decompositions, we compare the model chosen by our selection procedure to two benchmark models used in the literature - i.e., a model with a global factor and only regional factors (see e.g., Kose et al., 2003) and a model with a global factor and only development factors (see e.g., Kose et al., 2012) - and argue that the quantitative relevance of the various factors may be judged incorrectly when working with a misspecified model that incorrectly omits either regional or development group factors. We further investigate whether the importance of the various factors has changed over time by estimating our model over the pre-globalization period ($1961-1984$) and over the globalization period ($1985-2017$). In line with the existing literature, the results point towards increased regionalization in the globalization period, in particular for Asia, Latin America and for the emerging market economies subgroup.

The remainder of the paper is organized as follows. Section 2 outlines the multilevel factor model, introduces the testing procedure and provides details on the MCMC estimation method. Section 3 provides information on the performance of our procedure through the use of a Monte Carlo simulation. Section 4 presents the data, the baseline results, a model comparison using variance decompositions, and a number of robustness checks. Section 5 concludes.

2 Stochastic factor selection in a multilevel factor model

In this section, we present our multilevel factor model to characterize business cycle co-movements at the global, regional and development group level. We further outline the normalizations that we impose for identification, the stochastic model specification search that we use to determine which factors are relevant and a brief overview of the MCMC method to estimate the model.

2.1 A multilevel dynamic factor model

We consider a multilevel factor model in which real GDP growth y_{it} in country $i = 1, \ldots, N$ at time $t = 1, \ldots, T$ is driven by a global factor G_t, one of the J regional factors R_{jt} (for $j = 1, \ldots, J$) and one of the K development level factors D_{kt} (for $k = 1, \ldots, K$):

$$y_{it} = \gamma_i G_t + \theta_i R_{jt} + \phi_i D_{kt} + \mu_{it}, \tag{1}$$

where γ_i, θ_i and ϕ_i are factor loadings and μ_{it} is an idiosyncratic component.

Letting $y_t = (y_{1t}, \ldots, y_{Nt})'$ denote the $N \times 1$ vector stacking y_{it} over all countries i, the factor model in eq.(1) can be rewritten as,

$$\begin{aligned} y_t &= \Gamma G_t + \Theta R_t + \Phi D_t + \mu_t, \\ &= \Lambda F_t + \mu_t, \end{aligned} \tag{2}$$

with $\mu_t = (\mu_{1t}, \ldots, \mu_{Nt})'$, $F_t = (G_t, R_{1t}, \ldots, R_{Jt}, D_{1t}, \ldots, D_{Kt})'$ an $M = 1 + J + K$ column vector stacking the unobserved factors and $\Lambda = (\Gamma, \Theta, \Phi)$ an $N \times M$ loading matrix, with $\Gamma = (\gamma_1, \ldots, \gamma_N)'$, Θ an $N \times J$ matrix with typical entry $\Theta_{ij} = \theta_i$ if country i is in region j and zero otherwise, and Φ an $N \times K$ matrix with typical entry $\Phi_{ik} = \phi_i$ if country i is in development group k and zero otherwise.

The factors F_t and the idiosyncratic components μ_t are assumed to follow zero-mean autoregressive processes,

$$\mathrm{P}(L) F_t = \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}\left(0, \Sigma_\varepsilon^2\right), \tag{3}$$

$$\Pi(L) \mu_t = \nu_t, \qquad \nu_t \sim \mathcal{N}\left(0, \Sigma_\nu^2\right), \tag{4}$$

where $\mathrm{P}(L) = 1 - \mathrm{P}_1 L - \ldots - \mathrm{P}_p L^p$ and $\Pi(L) = 1 - \Pi_1 L - \ldots - \Pi_q L^q$ are lag polynomial of orders p and q respectively, with $\mathrm{P}_l = \mathrm{diag}(\rho_{1l}, \ldots, \rho_{Ml})$ for $l = 1, \ldots, p$ and $\Pi_l = \mathrm{diag}(\pi_{1l}, \ldots, \pi_{Nl})$ for $l = 1, \ldots, q$. We further assume that $\Sigma_\nu^2 = \mathrm{diag}(\sigma_\nu^2)$, with $\sigma_\nu^2 = (\sigma_{\nu_1}^2, \ldots, \sigma_{\nu_N}^2)'$, such that the innovations to the idiosyncratic components ν_{it} are mutually independent, which implies that all co-movements in the data are captured by the factors F_t. We will put further restrictions on Σ^2_ε to identify the model, as outlined in the next subsection.

2.2 Identification

Because of rotational indeterminacy (see e.g., Bai and Ng, 2013), the loadings Λ and the factors F_t in eq.(2) are not identified without additional restrictions. For instance, multiplying γ_i and G_t in eq.(1) by a constant leaves their product unaffected; or an observationally equivalent model can be obtained as $y_{it} = \gamma_i^* G_t + \theta_i R^*_{jt} + \phi_i D_{kt} + \mu_{it}$, with $\gamma_i^* = \gamma_i - \theta_i c$, $R^*_{jt} = R_{jt} + cG_t$ and c an arbitrary scalar. In a similar multilevel factor model, Kose et al. (2003) achieve (i) scale identification by normalizing the variance-covariance matrix of the factor innovations ε_t to be the identity matrix $\Sigma^2_\varepsilon = I_M$, and (ii) sign identification for each factor F_m, with $m = 1, \ldots, M$, by restricting one of the loadings to be strictly positive. Bai and Wang (2015) formally show that this identification scheme is sufficient to uniquely identify the multilevel factor model (see their proposition 3).

The main goal of this paper is to test which of the M included factors are relevant drivers of y_{it} and which are not. When using the Kose et al. (2003) identification scheme outlined above, finding irrelevant factors is not directly feasible for the following two reasons. First, testing whether factor innovation variances on the diagonal of Σ^2_ε are zero is not possible as these are all normalized to one for (scale) identification. Second, testing whether the loadings on factor F_m are simultaneously zero is not informative since at least one of these loadings is normalized to be strictly positive for (sign) identification.

To allow for stochastic factor selection, as outlined in the next subsection, we will therefore use the alternative identification scheme put forward by Bai and Wang (2015). This still assumes that $\Sigma^2_\varepsilon = \text{diag}(\sigma^2_\varepsilon)$, with $\sigma^2_\varepsilon = (\sigma^2_{\varepsilon_1}, \ldots, \sigma^2_{\varepsilon_M})'$ is a diagonal matrix (implying that the factors F_m are not correlated), but leaves the innovation variances σ^2_ε unrestricted and instead imposes M additional restrictions on the factor loadings. In particular, for each factor F_m one of corresponding loadings is restricted to be one to obtain both scale and sign identification. Alternatively, the average of the relevant (non-zero) factor loadings for each factor F_m can be normalized to be one:

$$\overline{\gamma} = \frac{1}{N}\sum_{i=1}^{N} \gamma_i = 1, \tag{5}$$

$$\overline{\theta}_j = \frac{1}{N_j}\sum_{i=1}^{N_j} \theta_i = 1, \qquad \text{for country } i \text{ in region } j, \text{ with } j = 1, \ldots, J, \tag{6}$$

$$\overline{\phi}_k = \frac{1}{N_k}\sum_{i=1}^{N_k} \phi_i = 1, \qquad \text{for country } i \text{ in development group } k, \text{ with } k = 1, \ldots, K, \tag{7}$$

where N_j and N_k are the number of countries in region j and development group k respectively. This alternative normalization also provides the required M restrictions and has the additional advantage that, unlike setting a single loading to be one, we don't have to select a specific country for normalization. The latter may be problematic as selecting a country that does not load on the corresponding factor would make this factor unidentified. By restricting the average loading to be one, we actually require that at least one country (in the relevant group) loads on the corresponding factor. When none of the countries load on a factor, it will not be retained by the stochastic model selection procedure outlined below.

2.3 Stochastic factor selection

The advantage of normalizing the factor loadings instead of the factors is that determining whether a factor F_{mt} is relevant or not can be done from an estimate of the corresponding factor innovation variance $\sigma^2_{\varepsilon_m}$. This is more straightforward than determining whether the factor loadings are simultaneous zero or not. In a Bayesian setting, $\sigma^2_{\varepsilon_m}$ is typically estimated using an inverse Gamma prior. However, Frühwirth-Schnatter and Wagner (2010) show that this approach tends to push the posterior distribution of $\sigma^2_{\varepsilon_m}$ away from zero as the inverse Gamma distribution has no probability mass at zero. As such, the importance of common factors could be overestimated in this approach. As an alternative, we therefore follow the stochastic model selection approach suggested by Frühwirth-Schnatter and Wagner (2010).

The first step is to rewrite the factor model in eq.(2) in terms of the standardized factors f_t,

$$y_t = \Lambda \Sigma_\varepsilon f_t + \mu_t, \tag{8}$$

where $\Sigma_\varepsilon = \text{diag}(\sigma_\varepsilon)$, with $\sigma_\varepsilon = (\sigma_{\varepsilon_1}, \ldots, \sigma_{\varepsilon_M})'$ and where the elements $f_{mt} = F_{mt}/\sigma_{\varepsilon_m}$ in $f_t = (f_{1t}, \ldots, f_{Mt})'$ are rescaled such that they have innovations with unit variances. This reparametrization of the factor model provides a natural framework to test for the inclusion or exclusion of common factors. As the standard deviations σ_ε are treated as regression coefficients, they can take on any value such that an irrelevant factor f_{mt} will automatically drop from the model as σ_{ε_m} will be zero in this case. Moreover, the signs of a standardized factor f_{mt} and its corresponding standard deviation σ_{ε_m} are *not* separately identified as it is possible to multiply both by -1 without changing their product.[1] As a result of this non-identification, the likelihood function is symmetric around zero along the σ_{ε_m} dimension. When F_{mt} exists, ($\sigma^2_{\varepsilon_m} > 0$), the likelihood function is bimodal with modes σ_{ε_m} and $-\sigma_{\varepsilon_m}$. When F_{mt} does not exist ($\sigma^2_{\varepsilon_m} = 0$), the likelihood function is unimodal around zero. This non-identification of the sign of σ_{ε_m} is convenient as it provides a first indication on whether a common factor F_{mt} is relevant or not.

[1] Note that this is different from the standard non-identification of the signs of Λ and F_t in equation (2), which is avoided by normalizing the average of the factor loadings Λ as outlined in Section 2.2.

To allow for formal model selection, the second step in the Frühwirth-Schnatter and Wagner (2010) approach entails multiplying each factor f_{mt} by a binary indicator δ_m that is equal to one if f_{mt} is to be included in the model (with σ_{ε_m} an unconstrained unknown parameter that is estimated from the data) and that is equal to zero if f_{mt} is to be excluded from the model (with σ_{ε_m} set equal to zero). That is, we rewrite eq.(8) to

$$y_t = \Lambda \Sigma_\varepsilon \Delta f_t + \mu_t, \tag{9}$$

where $\Delta = \text{diag}(\delta)$, with $\delta = (\delta_1, \ldots, \delta_M)'$. This allows to more formally test for factor relevance as sampling the binary indicators δ, together with the unknown parameters in the model, will provide posterior inclusion probabilities for each common factor. By combining the posterior factor probabilities, we can then calculate posterior model probabilities, i.e., the posterior probability of a particular *combination* of common factors.

2.4 Bayesian estimation

We estimate the multilevel factor model in eq.(9) using Bayesian methods. In this subsection, we first discuss our choices for the priors and next present the general outline of our MCMC algorithm.

2.4.1 Priors

As σ_ε is a vector of regression coefficients in eq.(9), an important advantage of our factor model expressed in terms of standardized factors f_t (rather than scaled factors F_t) is that it allows us to use a Gaussian prior centered at zero for σ_ε. Centering the prior distribution at zero makes sense as the distribution of σ_{ε_m} is symmetric around zero, for both $\sigma^2_{\varepsilon_m} = 0$ and $\sigma^2_{\varepsilon_m} > 0$. Frühwirth-Schnatter and Wagner (2010) show that - in contrast to a posterior density for $\sigma^2_{\varepsilon_m}$ obtained when imposing a standard inverse Gamma prior on the variance parameter $\sigma^2_{\varepsilon_m}$ - the posterior density of σ_{ε_m} is not very sensitive to the hyperparameters of the Gaussian distribution and is not pushed away from zero when $\sigma^2_{\varepsilon_m} = 0$. This is important as we are primarily interested in testing whether $\sigma^2_{\varepsilon_m} = 0$.[2] In particular, for each of the standard deviations σ_{ε_m} we use a Gaussian prior distribution centered at zero $\mathcal{N}(0, V_0)$ with an uninformative prior variance $V_0 = 10$.[3]

For the binary indicators δ that determine which factors in f_t should be included in the model, we

[2] Note that the using a Gaussian prior centered at zero for σ_{ε_m} instead of an inverse Gamma prior for $\sigma^2_{\varepsilon_m}$ is important for our factor selection approach but is not an issue in the standard dynamic factor model literature where the factor innovation variances are typically normalized to one for identification purposes.

[3] In principle, the inverse Gamma prior for the innovation variance $\sigma^2_{\varepsilon_m}$ can also be replaced by an alternative distribution that puts (some) weight on zero, like the exponential distribution used by Ferroni et al. (2019). However, as the standard deviation σ_{ε_m} enters as regression coefficient in the reparameterized factor model of eqs.(8) and (9), it is natural to use a conjugate normal prior distribution.

choose a Bernoulli prior distribution where each indicator has a prior probability p_0 of being included in the model, i.e., $p(\delta_m = 1) = p_0$. In our baseline scenario we set $p_0 = 0.5$, but as a robustness check we also report results for setting $p_0 = 0.25$ and $p_0 = 0.75$. The reason for this is that, as noted by Scott and Berger (2010), the prior choice $p_0 = 0.5$ does not provide multiplicity control for Bayesian variable - in our case, factor - selection. When the number of possible factors is large and each of the binary indicators has a prior probability $p_0 = 0.5$ of being equal to one, the fraction of selected factors will very likely be around 0.5.

We use a Gaussian prior for each of the factor loadings $\alpha = (\gamma_1, \dots, \gamma_N, \theta_1, \dots, \theta_N, \phi_1, \dots, \phi_N)$ in Λ and for each of the AR parameters in $\rho = (\rho_{11}, \dots, \rho_{Mp})$ and $\pi = (\pi_{11}, \dots, \pi_{Nq})$. In particular, the prior mean for the loadings is set to 1 (in line with the normalization) and to 0.5 for the AR parameters (to align with the moderate persistence in GDP growth rates). The prior variance for both the factor loadings and the AR parameters is set equal to 0.15^2 such that the prior distributions have support over the relevant range of parameter values. For the variances of the innovations to the idiosyncratic components in σ_ν^2, we use inverse Gamma distributions $IG(s_0T, s_0b_0T)$ with shape s_0T and scale s_0b_0T where b_0 is the prior belief concerning the value of the variance $\sigma_{\nu_i}^2$ and s_0 is the strength given to this belief expressed as a fraction of the sample size T (see e.g., Bauwens et al., 2000). We set $b_0 = 1$, which is a magnitude that is in accordance with the actual variance of the data and $s_0 = 0.1$ which corresponds to a relatively loose prior.

2.4.2 Outline MCMC algorithm

An MCMC approach is implemented to jointly sample the binary indicators δ, the other parameters $\psi = (\alpha, \rho, \pi, \sigma_\varepsilon, \sigma_\nu^2)$ and the factors $f = (f_1', \dots, f_T')$ conditional on the data $y = (y_1', \dots, y_T')$. The posterior density of interest is then given by $\mathscr{F}(\delta, \psi, f|y)$. Following the stochastic model specification procedure suggested by Frühwirth-Schnatter and Wagner (2010), our MCMC scheme is as follows:

1. Sample the binary indicators δ from $\mathscr{F}(\delta|\psi, f, y)$ while marginalizing over the parameters σ_ε for which the factor selection is carried out.

2. Sample the parameters ψ from the conditional distribution $\mathscr{F}(\psi|\delta, f, y)$.

   (a) Sample the standard deviations of the innovations to the factors σ_ε from $\mathscr{F}(\sigma_\varepsilon|\delta, \alpha, \rho, \pi, \sigma_\nu^2, f, y)$ using eq.(9) for those factors f_m for which $\delta_m = 1$. For the factors f_m for which $\delta_m = 0$, set $\sigma_{\varepsilon_m} = 0$.

   (b) Sample the factor loadings α and the variances σ_ν^2 jointly from $\mathscr{F}(\alpha, \sigma_\nu^2|\delta, \rho, \pi, \sigma_\varepsilon, f, y)$ using eq.(9). Impose the normalizations outlined in eqs.(5)-(7).

(c) Sample the AR parameters ρ from $\mathscr{F}(\rho|\delta, \alpha, \pi, \sigma_\varepsilon, \sigma_\nu^2, f, y)$ and π from $\mathscr{F}(\pi|\delta, \alpha, \rho, \sigma_\varepsilon, \sigma_\nu^2, f, y)$ using eqs.(3) and (4) respectively. The approach follows the method of Chib and Greenberg (1994) to deal with AR terms in Bayesian regression models.

3. Sample the factors f from the conditional distribution $\mathscr{F}(f|\delta, \psi, y)$.

   (a) The factors that are included in the model (i.e., those for which δ_m=1) can be sampled from $\mathscr{F}(f|\delta, \psi, y)$ using eqs.(9) and (3). We use a state space approach with multi-move sampling (see e.g., Carter and Kohn, 1994; Kim and Nelson, 1999). The factors that are excluded from the model (i.e., those for which δ_m=0) are sampled from their prior distribution $f_{mt} = 0.5 f_{m,t-1} + \mathscr{N}(0, 1)$.

   (b) Perform a random sign switch on σ_{ε_m} and f_m (for $m = 1, ..., M$) to exploit the non-identification of their signs, i.e., σ_{ε_m} and f_m are left unchanged with probability 0.5 while with the same probability they are replaced by $-\sigma_{\varepsilon_m}$ and $-f_m$.

4. Calculate additional quantities such as the scaled common factors $F_m \equiv \sigma_{\varepsilon_m} f_m$ and variance shares λ obtained from a variance decomposition applied to the estimated factor model (see Section 2.5).

Initial values are taken from the relevant prior distributions. Sampling from the above steps is then iterated W times and, after a sufficiently large number of burn-in draws B, the sequence of draws $(B + 1, ..., W)$ approximates a sample from the posterior distribution $\mathscr{F}(\delta, \psi, f|y)$. The results reported below are based on $W = 60\ 000$ iterations with the first $B = 10\ 000$ draws discarded as a burn-in sequence. The convergence diagnostics reported in Appendix B show that the convergence of the MCMC algorithm with the retained number of draws is satisfactory.

2.5 Variance decompositions

We also calculate variance shares from our selected factor model. Average variance shares for the global, regional and development factors as well as for the idiosyncratic component are calculated in two steps. First, using a variance decomposition applied to eq.(1), we calculate factor variance shares for each country i as the fraction of the variance in y_{it} that is explained by the global factor G_t,

$$\lambda_i^G = \frac{V(\gamma_i G_t)}{V(y_{it})}, \qquad \text{with} \quad G_t = \delta_1 \sigma_{\varepsilon_1} f_{1t}, \tag{10}$$

the regional factor R_{jt}

$$\lambda_i^R = \frac{V(\theta_i R_{jt})}{V(y_{it})}, \qquad \text{with} \quad R_{jt} = \delta_{1+j} \sigma_{\varepsilon_{1+j}} f_{1+j,t}, \tag{11}$$

the development group factor D_{kt}

$$\lambda_i^D = \frac{V(\phi_i D_{kt})}{V(y_{it})}, \qquad \text{with} \quad D_{kt} = \delta_{1+J+k}\sigma_{\varepsilon_{1+J+k}} f_{1+J+k,t}, \tag{12}$$

and the idiosyncratic component μ_{it}

$$\lambda_i^\mu = \frac{V(\mu_{it})}{V(y_{it})}. \tag{13}$$

Second, the variance shares are averaged across (groups of) countries to obtain the average variance share of the global, regional and development factors and of the idiosyncratic component over all countries over which the average is taken.

3 Monte Carlo simulation

This section presents Monte Carlo simulation results that illustrate the performance of the proposed factor selection procedure. We tailor the sample size and the data generating process to the empirical specification put forward in Section 2.1 and to the properties of the data that will be used in Section 4 below. In particular, we generate artificial data from eqs.(2)-(4) for two sample sizes that are in line with the dimensions of the available data when using the full sample ($N = 60$; $T = 60$) and subsamples ($N = 60$; $T = 30$). As in the real data, we consider factors at three levels, i.e., one global factor, three development group factors (industrialized countries, emerging market economies and developing countries) and six regional factors (Europe, North America, Oceania, Latin America, Africa and Asia). The assignment of countries to the development groups and regions is as in the real data (see Section 4.1 and Appendix A for details). The parameters are chosen to roughly match with those estimated using the real data, i.e.

- the standard deviation σ_{ν_i} of the innovations to the idiosyncratic component μ_{it} (for $i = 1, \ldots, N$) is set equal to one;
- the length of the polynomials $\mathrm{P}(L)$ and $\Pi(L)$ is set to one ($p = q = 1$) with each of the AR parameters equal to 0.5 ($\rho_m = \pi_i = 0.5$ for $m = 1, \ldots, M$ and $i = 1, \ldots, N$);
- the factor loadings $(\gamma_i, \theta_i, \phi_i)$ (for $i = 1, \ldots, N$) are independently drawn from a normal distribution with expected value one and standard deviation 0.25;
- the standard deviation σ_{ε_m} of the innovations to factor F_{mt} (for $m = 1, \ldots, M$) is set equal to one when factor m is relevant and equal to zero when irrelevant.

We consider the following five different cases: (c1) all factors are relevant; (c2) all factors are irrelevant; (c3) the world factor is irrelevant, all other factors are relevant; (c4) the development group factors are irrelevant, all other factors are relevant; (c5) the regional factors are irrelevant, all other factors are relevant. We also considered cases where only a specific selection of development group and regional factors are relevant (in line with the estimation results presented in Section 4), but, as these cases did not yield additional insights, we do not report these results here.

Table 1: Monte Carlo simulation results: factor selection frequencies

Factor (and number of countries loading on this factor)		World (60)	IND (23)	EME (18)	DEV (19)	Europe (18)	No.Am. (3)	Oceania (2)	Lat.Am. (18)	Africa (7)	Asia (12)
Case 1	Factor relevant	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	$T = 30$	100.0	99.4	100.0	100.0	95.6	85.5	65.0	99.8	99.8	100.0
	$T = 60$	100.0	100.0	100.0	100.0	99.9	96.7	86.8	100.0	100.0	100.0
Case 2	Factor relevant	No	No	No	No	No	No	No	No	No	No
	$T = 30$	0.2	0.0	0.2	0.3	0.0	0.0	0.0	0.0	0.1	0.1
	$T = 60$	0.1	0.1	0.1	0.1	0.0	0.0	0.0	0.2	0.0	0.1
Case 3	Factor relevant	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	$T = 30$	9.8	99.8	100.0	100.0	95.2	87.7	65.1	100.0	99.8	100.0
	$T = 60$	3.5	100.0	100.0	100.0	100.0	96.8	86.6	100.0	100.0	100.0
Case 4	Factor relevant	Yes	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes
	$T = 30$	100.0	21.2	0.8	2.0	100.0	87.9	60.9	100.0	99.8	100.0
	$T = 60$	100.0	7.8	0.3	0.6	100.0	97.7	87.5	100.0	100.0	100.0
Case 5	Factor relevant	Yes	Yes	Yes	Yes	No	No	No	No	No	No
	$T = 30$	100.0	100.0	100.0	100.0	1.3	0.0	0.1	0.6	0.0	0.1
	$T = 60$	100.0	100.0	100.0	100.0	2.4	0.0	0.0	0.5	0.2	0.4

Notes: The reported frequencies are calculated as the fraction of the Monte Carlo draws in which the considered factor is selected by the Bayesian model specification search, as indicated by a posterior inclusion probability larger than 0.5. Based on 1 000 Monte Carlo draws and 50 000 iterations of the Gibbs sampler. IND denotes industrial countries, EME denotes emerging market economies and DEV denotes developing economies.

Table 1 reports factor selection frequencies calculated as the fraction of Monte Carlo draws for which the posterior inclusion probability (i.e., the average of the binary indicator over the Gibbs iterations) of the considered factor is larger than 0.5. The results show that, overall, our Bayesian model specification search is quite effective in selecting the appropriate factors. For relevant factors, the selection frequency is mostly equal, or very close, to 100%. The only exceptions are the smaller regions North America (consisting of three countries) and Oceania (consisting of two countries), for which the selection frequencies of a relevant regional factor are around 85% and 65%, respectively, in the smallest sample $T = 30$. When $T = 60$, these frequencies are above 96% for North America and above 86% for Oceania. This shows that it is more difficult to pick up relevant factors when both T and the available number of cross-sectional units in a group are small. For irrelevant factors, the selection frequency is mostly lower than 10%. The only exception is the industrial group factor in case 4 for $T = 30$, which is selected in 21.2% of the cases while irrelevant. The reason for this is the strong overlap with the European region (18 out of the 23 industrial countries are located in Europe), which implies that the industrial countries group factor may be picking up part of the European factor, especially when T is small.

From the conducted analysis, we conclude that our factor selection procedure performs adequately. As such, we now report the results obtained when applying our factor selection procedure to the estimation of our multilevel factor model using real world data.

4 Estimation results

In this section, we report results from estimating the multilevel factor model discussed above using data on real GDP growth for $N = 60$ countries over the period $1961 - 2017$. Section 4.1 provides details on the data. Section 4.2 presents preliminary evidence on which factors are relevant. Section 4.3 reports the more formal model selection results. Section 4.4 plots and discusses the estimated common factors. Section 4.5 shows variance decompositions and compares the variance shares obtained from our selected factor model to those calculated for two factor models commonly estimated in the literature. Finally, Section 4.6 reports a number of robustness checks.

4.1 Data

Our data are taken from the Penn World Table (PWT) version 9.1 (see Feenstra et al., 2015). The sample consists of the $N = 60$ countries considered by Kose et al. (2003) and more recently also by Francis et al. (2017). In addition to a global factor, we focus on geographical regions (see e.g., Kose et al., 2003) and development groups (see e.g., Kose et al., 2012) as country sub-groups. The six regional factors are Europe, North America, Oceania, Latin America, Africa and Asia. The three development group factors are the industrialized countries (IND), emerging market economies (EME), and developing countries (DEV). As such, we have $M = 10$ common factors. The countries considered and the subgroups to which they belong are reported in Appendix A. The dataset consists of annual data over the period $1960-2017$. The data are log first-differenced and demeaned (see e.g., Kose et al. (2003) and many subsequent papers that estimate business cycles using dynamic factor models).[4] The effective sample period is therefore $1961 - 2017$ ($T = 57$). We also conduct our estimations using two separate subsamples, i.e., one for the subperiod $1961 - 1984$ ($T = 24$) and one for the subperiod $1985 - 2017$ ($T = 33$). This follows, among others, Kose et al. (2008, 2012) and Hirata et al. (2013), who assume that there is a demarcation point in the mid 1980s that effectively separates the pre-globalization period from the globalization period.

[4]Instead of calculating growth rates, an alternative approach to detrend the data would be to use an explicitly filtered measure obtained for instance from a Hodrick-Prescott or bandpass filter. As noted by Canova (1998) however, business cycle facts are not robust to the use of such filtering methods.

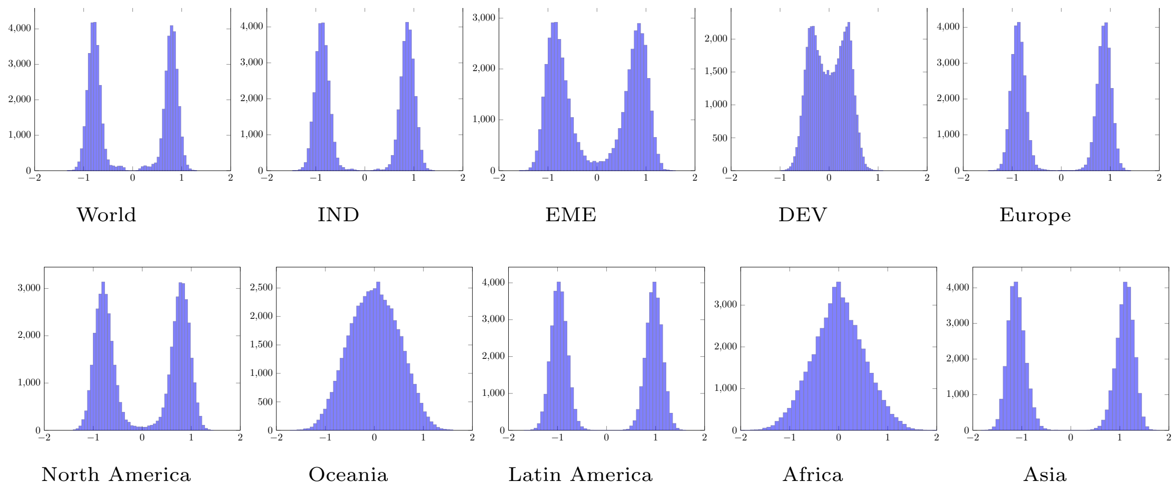
In the robustness checks discussed in Section 4.6, we make a further distinction between developed and developing Asian countries as in Kose et al. (2003). Appendix A further reports to which of these distinct groups each of our Asian countries belongs.

4.2 A first look at factor relevance

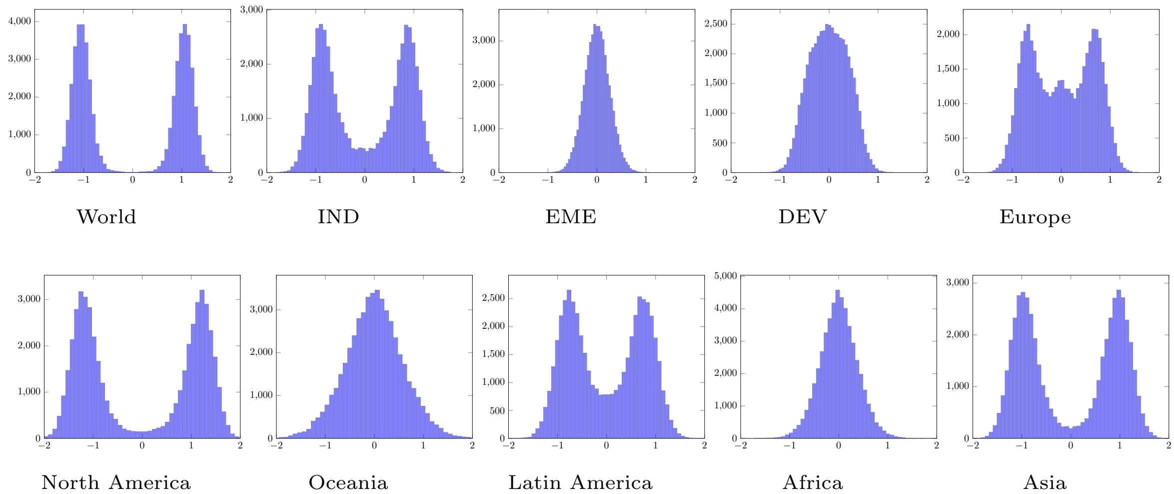
To obtain preliminary evidence on factor relevance, we first estimate the model with all binary indicators δ in eq.(9) set equal to one. We note that all baseline estimations are conducted using $AR(1)$ processes for all factors f_{mt} and for all idiosyncratic components μ_{it}, i.e., we set $p = q = 1$ in eqs.(3) and (4). In the robustness checks reported below, we show that our conclusions do not change when assuming higher-order AR processes. Figure 1 plots the posterior distributions of the standard deviations σ_{ε_m} of the innovations to the $M = 10$ common factors that we consider. As noted in Section 2.3, when the posterior distribution of a particular common factor is bimodal with low or no probability mass at zero, this can be considered as evidence that this factor belongs in the model, i.e., it suggests that $\sigma^2_{\varepsilon_m} > 0$. When, on the other hand, the posterior distribution of a particular common factor is unimodal with most of its probability mass around zero this can be considered as evidence that this factor does not belong in the model, i.e., it suggests that $\sigma^2_{\varepsilon_m} = 0$. We present the posterior distributions of σ_{ε_m} for the full sample period $1961 - 2017$ as well as for the subperiods $1961 - 1984$ and $1985 - 2017$.

Figure 1: Posterior distributions of the standard deviations σ_{ε_m} of the innovations to the common factors

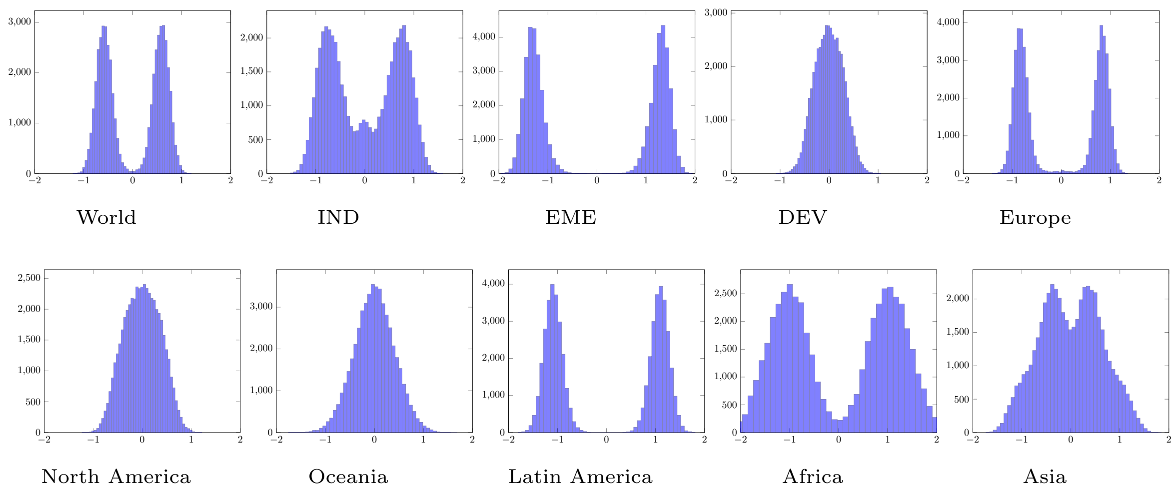
1. Sample period 1961-2017



2. Sample period 1961-1984



3. Sample period 1985-2017



Notes: The posterior distributions are obtained with binary indicators δ_m set to one (for $m = 1, ..., M$) in eq.(9). IND denotes industrial countries, EME denotes emerging market economies and DEV denotes developing economies.

The following conclusions can be drawn. First, for every sample period considered, the reported posterior distribution of the standard deviation of the innovations to the world factor is bimodal with no probability mass at zero. This shows that there is indeed a global business cycle.

Second, for the regional factors over the full period, we observe clear bimodality in the posterior distributions of σ_{ε_m} for Europe, North America, Latin America and Asia but not for Africa and Oceania. Looking at the subsamples, the distribution plots suggest that the regional cycles are less pronounced in the pre-globalization period $1961 - 1984$. Although we observe bimodality for Europe, North America, Latin America and Asia, especially for Europe and Latin America there is also considerable probability mass at zero. For the globalization period $1985 - 2017$, the distributions are clearly bimodal for Europe, Latin America and Africa, with almost no probability mass at zero. This suggests that these economies have become more integrated and synchronized during the globalization era. Asia and North America seem to have become less integrated. For North America, we should stress that this region only consists of three countries and that the Monte Carlo simulations have made clear that it may be difficult to detect common factors when both the cross-sectional and the time series dimension are small. For Asia, we note that this region is rather heterogeneous, so that different countries in this region may well have experienced a different evolution. In the robustness checks reported in Section 4.6 below, we argue that it is informative to make a distinction between more developed Asian economies and less developed Asian economies. We argue there that, while the former have become more integrated, this is not the case for the latter.

Third, for the development level factors over the full period, the posterior distributions of σ_{ε_m} show clear bimodality for the industrialized countries and emerging market economies subgroups. While there is some bimodality for the developing countries, there is also considerable probability mass at zero which suggests that a business cycle factor is probably less relevant for this subgroup. Looking at the subsamples, there is evidence that the industrial economies already shared a common cycle during the pre-globalization period. This is not the case for emerging economies for which there is clear unimodality during the period $1961 - 1984$ and clear bimodality during the period $1985 - 2017$. A development group factor does not show up in any of the two subsamples.

4.3 Stochastic factor selection

To more formally test for factor relevance, we sample the stochastic binary indicators δ in eq.(9) jointly with the other parameters in the model. Table 2 presents the posterior inclusion probabilities for each of the $M = 10$ considered factors. These probabilities are calculated as the average of the sampled binary indicators δ_m over the iterations of the Gibbs sampler. They are reported for the full sample period

(1961 − 2017), the pre-globalization sample period (1961 − 1984) and the globalization sample period (1985 − 2017) using three different prior factor inclusion probabilities, i.e., for the baseline case $p_0 = 0.5$ but also for $p_0 = 0.75$ and for $p_0 = 0.25$.

Table 2: Posterior inclusion probabilities of the common factors over different priors and sample periods

Period	Prior	Posterior factor inclusion probabilities									
		World	IND	EME	DEV	Europe	No. Am.	Oceania	Lat. Am.	Africa	Asia
1961-2017	$p_0 = 0.50$	1.00	1.00	0.81	0.60	0.97	0.99	0.18	1.00	0.25	1.00
	$p_0 = 0.25$	1.00	1.00	0.66	0.43	0.99	0.97	0.08	1.00	0.12	1.00
	$p_0 = 0.75$	1.00	1.00	0.92	0.72	1.00	0.99	0.38	1.00	0.44	1.00
1961-1984	$p_0 = 0.50$	1.00	0.87	0.10	0.28	0.45	0.94	0.21	0.49	0.13	0.59
	$p_0 = 0.25$	1.00	0.86	0.04	0.14	0.25	0.87	0.09	0.29	0.05	0.31
	$p_0 = 0.75$	1.00	0.87	0.23	0.45	0.64	0.98	0.42	0.75	0.31	0.83
1985-2017	$p_0 = 0.50$	1.00	0.96	1.00	0.47	0.87	0.34	0.19	1.00	0.84	0.50
	$p_0 = 0.25$	1.00	0.97	1.00	0.42	0.65	0.17	0.09	0.99	0.64	0.32
	$p_0 = 0.75$	1.00	0.96	1.00	0.58	0.92	0.54	0.39	1.00	0.94	0.68

Notes: The reported probabilities are calculated as the average of the binary indicators δ_m (for $m = 1, ..., M$) over the 50 000 iterations of the Gibbs sampler. IND denotes industrial countries, EME denotes emerging market economies and DEV denotes developing economies.

The results in Table 2 generally confirm the preliminary findings reported in Section 4.2. Looking at the full sample results in the top panel of the table, the following conclusions stand out. First, the posterior inclusion probability of the global factor always equals unity. As such, our formal testing procedure confirms that a global business cycle exists. Second, the regions Europe, North America, Latin America and Asia have distinct common cycles as the posterior inclusion probabilities of their factors are close to or equal to one, irrespective of the prior inclusion probabilities considered. We do not find distinct common business cycles for Africa nor for Oceania as the posterior factor inclusion probabilities of these regions are always well below their prior factor inclusion probabilities. Third, for the country groups based on the level of development, we find posterior factor inclusion probabilities close to one for industrial countries. For emerging market economies, we find posterior probabilities lower than one although still substantially higher than the prior factor inclusion probabilities. The evidence in favor of a distinct cycle for the developing countries is not very strong, with a posterior inclusion probability of only 60% when the prior inclusion probability equals 50%. Turning to the subsamples, the world as a whole and the industrial countries clearly have a common cycle in both periods. For Europe, Latin America, Africa and the emerging market economies, a group-specific business cycle seems to have emerged only during the globalization period. In general, our findings support the findings of Kose et al. (2012) who argue that subgroup specific factors have become more important in the globalization period.

As a final note with respect to the results reported in Table 2, we sometimes observe posterior probabilities that are lower in both subsamples as compared to those estimated over the full sample period. As highlighted by our Monte Carlo simulation results in Section 3, this could be caused by the reduction in the number of time series observations that are available when we estimate the model over subperiods. As this is more pronounced for factors of groups that consist of only a few countries, the subsample results for North America, for example, should be interpreted with caution. Note that we also observe lower probabilities in the subperiods for Asia. This may further be related to a different evolution of the integration of less developed versus more developed Asian economies, an issue that we discuss in Section 4.6 below.

Besides the posterior inclusion probabilities of the individual common factors, the model selection approach also allows to compute overall model probabilities, i.e., probabilities for combinations of common factors. As there are 10 binary indicators, there are 2^{10} possible models. In Table 3 we report the four most preferred models for the full sample period and for both subsamples, with the ranking based on results obtained under the prior factor inclusion probability $p_0 = 0.5$ as well as $p_0 = 0.25$ and $p_0 = 0.75$.

The results show that the specification that includes a global factor, four regional factors (Europe, North America, Latin America, Asia) and three development factors (industrial countries, emerging markets, developing countries) is the preferred model over the full sample period when the prior inclusion probabilities are set to either $p_0 = 0.5$ or $p_0 = 0.75$, while the same model without the developing countries group is the preferred model when the prior inclusion probabilities equal $p_0 = 0.25$. As such, contrary to the literature which has until now focussed mostly on the estimation of factor models that either include global and regional factors only (see e.g., Kose et al., 2003; Hirata et al., 2013) or that include global and development factors only (see e.g., Kose et al., 2012), this result suggests that *both* the geographical proximity and the level of development of countries is important. The subsamples once again highlight the emergence of group-specific factors over time. Specifically, for the pre-globalization subsample $(1961 - 1984)$, the preferred model for $p_0 = 0.5$ includes a global factor but only two regional factors and the industrialized countries subgroup. The preferred model for the globalization period $(1985 - 2014)$ when $p_0 = 0.5$ additionally includes the emerging market economies, Europe, Latin America and Africa but excludes North America. With respect to this exclusion of North America, we reiterate that the subsample results for groups consisting of only a few countries should be interpreted with caution.

Table 3: Posterior model probabilities of the four preferred models over different priors and sample periods

Period	Model										Posterior model probability		
	World	IND	EME	DEV	Europe	No. Am.	Oceania	Lat. Am.	Africa	Asia	$p_0 = 0.50$	$p_0 = 0.25$	$p_0 = 0.75$
1961-2017	1	1	1	1	1	1	0	1	0	1	0.29	0.22	0.23
	1	1	1	0	1	1	0	1	0	1	0.19	0.28	0.08
	1	1	1	1	1	1	0	1	1	1	0.09	0.03	0.17
	1	1	1	0	1	1	0	1	1	1	0.08	0.04	0.08
1961-1984	1	1	0	0	0	1	0	0	0	1	0.07	0.10	0.01
	1	1	0	0	0	1	0	1	0	1	0.06	0.03	0.03
	1	1	0	0	0	1	0	0	0	0	0.05	0.23	0.00
	1	1	0	0	1	1	0	1	0	1	0.05	0.03	0.06
1985-2017	1	1	1	0	1	0	0	1	1	1	0.12	0.08	0.07
	1	1	1	0	1	0	0	1	1	0	0.09	0.13	0.03
	1	1	1	1	1	0	0	1	1	0	0.09	0.08	0.05
	1	1	1	0	1	0	0	1	1	1	0.07	0.08	0.07

Notes: Per sample period, the models are ordered from most preferred to least preferred. The ordering is on the basis of posterior model probabilities obtained under prior $p_0 = 0.5$. An entry "1" denotes inclusion of the denoted common factor while an entry "0" denotes exclusion of the denoted common factor. IND denotes industrial countries, EME denotes emerging market economies and DEV denotes developing economies.

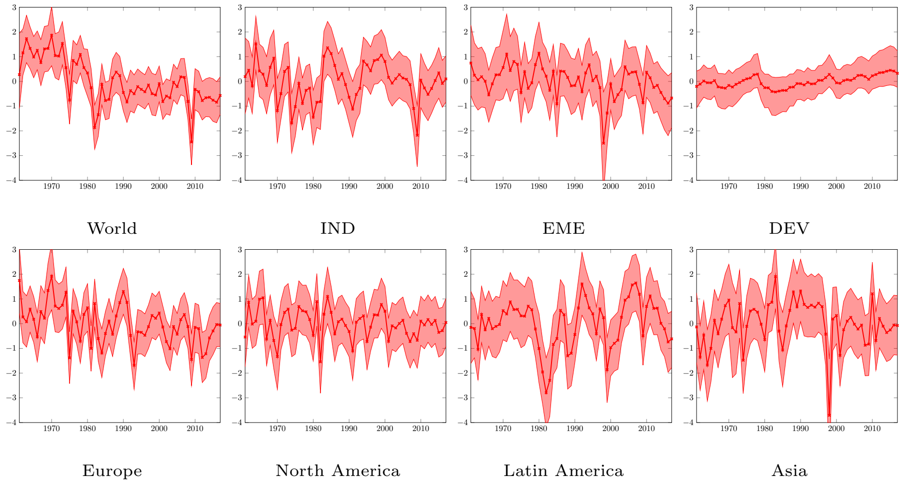
4.4 Estimated common factors

We present the scaled common factors $F_{mt} = \sigma_{\varepsilon_m} f_{mt}$ in Figure 2. The top panel shows the common factors estimated from our preferred factor model for the full sample period $1961-2017$. The bottom panel shows the estimated factors obtained from the preferred factor models for the subsamples $1961-1984$ and $1985-2017$ where a vertical bar separates both subperiods. The preferred factor models are reported in Table 3 for every period and are based on the orderings obtained with the posterior factor inclusion probabilities using prior $p_0 = 0.5$. The figure shows the means of the posterior distributions of the scaled common factors and the 90% highest posterior density (HPD) intervals, i.e., the 5th and 95th percentiles of these distributions.

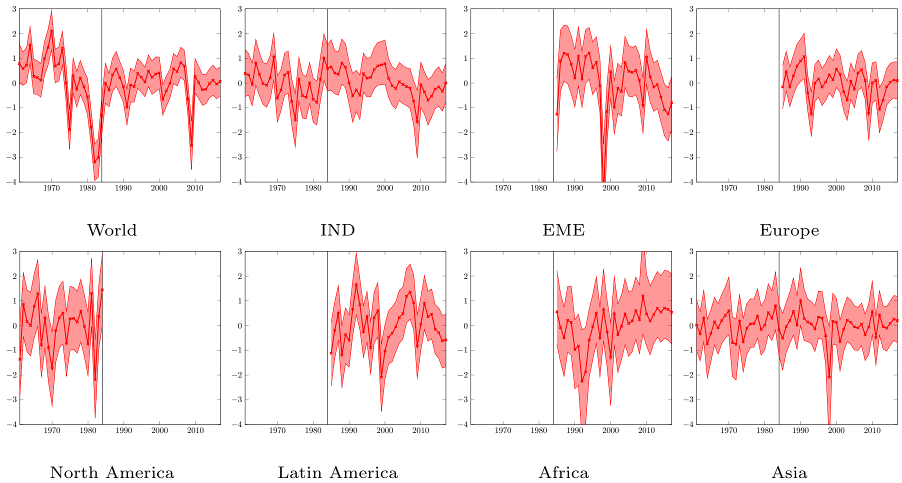
The global business cycle component picks up the recessions of the mid $1970s$ and early 1980s caused by sharp oil price increases as well as the Great Recession of $2007-09$. These events affected the majority of countries in our sample and hence are reflected in the evolution of the global factor. The industrial countries group factor tends to be positive in the 1960s and in the mid to late 1990s. The former period was characterized by high productivity growth in the aftermath of World War II, while during the 1990s many industrial countries experienced low levels of unemployment, relatively high levels of economic growth and a stable inflation rate. During the 1970s, the industrial economies factor remains below zero, suggesting that the recession caused by the oil price shock induced group-specific dynamics as well. Similarly, the decline of the industrial economies factor during the Great Recession points to the existence of group-specific dynamics during this period which are not captured by the global factor. The emerging market economies factor displays a decline around the time of the Asian crisis in 1997 which points out business cycle commonalities between Asian countries and other emerging markets. In accordance with its low inclusion probability as reported in the previous section, the business cycle factor of the developing countries group is close to zero throughout the whole sample period. The included regional business cycles also display periods of booms and bursts that accord well with established facts. The European factor, for instance, shows another downturn after the recovery from the Great Recession around 2012. This double dip recession was caused by the European debt crisis and was therefore specific to European countries but did not affect other industrialized countries. The Latin American factor clearly shows the severe recession associated with the Latin American debt crisis in the early 1980s. Finally, on the graph for the Asian factor the largest downturn that can be observed is due to the Asian financial crisis that started in 1997.

Figure 2: Estimated factors in the models preferred by the stochastic factor selection procedure

1. Full sample period 1961-2017



2. Subsamples 1961-1984 and 1985-2017 (separated by a vertical bar)



Notes: Reported are the mean, the 5th and the 95th percentile of the posterior distribution of the scaled common factors $F_{mt} = \sigma_{\varepsilon_m} f_{mt}$ estimated using the factor models that are preferred by the stochastic factor selection procedure in the full sample and in the two subperiods (as reported in Table 3 using the prior inclusion probability $p_0 = 0.5$). IND denotes industrial countries, EME denotes emerging market economies and DEV denotes developing economies.

4.5 Variance decompositions and model comparison

This section reports and discusses variance decompositions for the factor model that is selected by our stochastic factor selection procedure, i.e., the preferred model identified and discussed in Section 4.3 above. We refer to this model as sWRD (selection of world, regional and development group factors). To analyze the potential impact of misspecification, we also report variance decompositions for two benchmark models that have frequently been estimated in the literature, i.e., a model with a global factor and only regional factors (see e.g., Kose et al., 2003; Mumtaz et al., 2011; Hirata et al., 2013) and a model with a global factor and only development factors (see e.g., Kose et al., 2012). We refer to these models as WR and WD, respectively. Our estimation results suggest that both of these models are misspecified as the set of factors selected by our procedure in Section 4.3 is a combination of world, regional and development group factors.

Table 4 reports the contribution to the variance in real GDP growth of the world factor, the development level factor, the regional factor and the country-specific idiosyncratic component in the full sample period $1961-2017$, the pre-globalization subperiod $1961-1984$ and the globalization subperiod $1985-2017$ across the following groups of countries: all countries, industrial countries, emerging economies, developing countries, Europe, North America, Oceania, Latin America, Africa and Asia. We refer to Section 2.5 for details on the calculation of these variance shares and to Appendix A for the allocation of countries to the different groups.

Looking at the results, we first compare the variance shares obtained from the sWRD models selected by our procedure to those obtained from the WR and WD factor models frequently used in the literature. We notice that our selected models indicate that *both* the development level factors and the regional factors matter as they often drive a substantial share of the variance in real GDP growth. For example, over the full sample period $1961-2017$, the combined average variance share of the development and regional factors across all countries (ALL) is larger than the share of the global factor and equals 28% with each factor being responsible for exactly half this share (i.e., 14%). In many instances, we observe that the magnitude of the variance share of a given factor differs considerably depending on the model considered. Often, these differences occur because included factors in the restricted WD and WR models pick up the share of wrongfully omitted factors. For example, for the subperiod $1985-2017$, the average variance share across all countries (ALL) for the development factor and the regional factor in the sWRD model equals 14%, respectively 16%. In the restricted model WD (which excludes regional factors), the variance share of the development factor equals 27% whereas in the WR model (which excludes development level factors) the share of the regional factor equals 29%. Hence, the quantitative relevance of a factor can be severely misjudged when looking at models such as the WD and WR models that incorrectly omit relevant factors. Our factor selection procedure helps to avoid such misspecification by showing that it is important to include factors that capture both geographical proximity and the level of development of countries rather than factor models that only capture one of these two dimensions.

For the factor sWRD models that are selected by our procedure, we further observe the following from Table 4. First, in line with the high posterior inclusion probabilities found for the global factor, the variance shares of the world factor are consistently found to be quite substantial (i.e., between 10% and 43% across all country groups and all subperiods). The global factor is most important for industrial economies, followed by the emerging economies and then the developing economies. In line with this, it is most important for the regions North America and Europe, followed by Latin America, Oceania, Asia and Africa. Interestingly, when comparing the pre-globalization and globalization periods, we notice a substantial decrease in the variance shares of the global factor for all country groups during the globalization period. Second, the development level factor is less important than the global factor for almost all country groups. When comparing both subperiods, we notice an increase in the importance of this factor for most country groups during the globalization period. Together with the declining importance of the global factor, this result supports the findings reported by Kose et al. (2012) who argue that factors reflecting the level of development of countries have become more important during this period. This increased importance is most noticeable for the EME group where we observe an increase in the variance share from 0% in the period $1961 - 1984$ to 26% in the period $1985 - 2017$, such that it overtakes both the global and the regional factor in these economies during this period. When looking at the regions, we notice that the increase in the importance of the development factor is most noticeable in Asia where it rises from 1% to 26%. Third, the contribution of the regional factor to the variance of real output growth is also generally lower than that of the global factor. But, upon comparing the subperiods, there is evidence of increased regionalization, i.e., an increased variance share for the regional factor in the globalization period compared to the pre-globalization period. This phenomenon has been documented in the literature (see e.g., Hirata et al., 2013). Regionalization occurs globally as the increased variance share of the regional factor can be observed across all countries. Increases in the variance shares of the regional factor can further be observed for industrial economies, emerging markets and, especially, developing economies. The increase in regionalization is also pronounced in Europe, Latin America and Africa. Finally, we note that the idiosyncratic or country-specific components of our factor model are still responsible for at least 30% - and often substantially more - of the variance of output growth across all considered periods and country groups suggesting that there is still room for further synchronization of business cycles across countries.

Table 4: Variance decompositions

		Contribution of factors and idiosyncratic components to the variance in real GDP growth											
		1961-2017				1961-1984				1985-2017			
Country Group	Model	World	DEV	REG	Idio	World	DEV	REG	Idio	World	DEV	REG	Idio
ALL	sWRD	0.25	0.14	0.14	0.48	0.36	0.05	0.03	0.55	0.15	0.14	0.16	0.55
	WD	0.28	0.20	0	0.52	0.34	0.10	0	0.56	0.16	0.27	0	0.58
	WR	0.25	0	0.23	0.52	0.32	0	0.16	0.52	0.20	0	0.29	0.51
IND	sWRD	0.29	0.21	0.12	0.39	0.41	0.14	0.03	0.42	0.19	0.17	0.13	0.51
	WD	0.31	0.28	0	0.41	0.37	0.22	0	0.42	0.17	0.37	0	0.46
	WR	0.33	0	0.21	0.46	0.40	0	0.15	0.45	0.27	0	0.24	0.49
EME	sWRD	0.24	0.11	0.18	0.46	0.35	0	0.06	0.59	0.10	0.26	0.15	0.49
	WD	0.29	0.18	0	0.53	0.35	0.01	0	0.64	0.12	0.33	0	0.55
	WR	0.22	0	0.29	0.48	0.29	0	0.16	0.55	0.15	0	0.37	0.49
DEV	sWRD	0.21	0.08	0.12	0.59	0.31	0	0.02	0.67	0.14	0	0.21	0.65
	WD	0.25	0.11	0	0.64	0.30	0.05	0	0.65	0.18	0.07	0	0.75
	WR	0.19	0	0.19	0.62	0.24	0	0.18	0.58	0.16	0	0.28	0.56
Europe	sWRD	0.30	0.20	0.12	0.38	0.43	0.14	0	0.43	0.18	0.17	0.15	0.49
	WD	0.32	0.29	0	0.39	0.38	0.21	0	0.41	0.17	0.40	0	0.43
	WR	0.32	0	0.23	0.45	0.40	0	0.14	0.46	0.25	0	0.28	0.47
No.Am.	sWRD	0.34	0.18	0.17	0.31	0.34	0.08	0.25	0.35	0.25	0.19	0	0.54
	WD	0.38	0.19	0	0.43	0.37	0.17	0	0.46	0.20	0.27	0	0.53
	WR	0.42	0	0.20	0.39	0.36	0	0.29	0.35	0.38	0	0.05	0.58
Oceania	sWRD	0.21	0.23	0	0.56	0.31	0.16	0	0.53	0.16	0.14	0	0.71
	WD	0.18	0.21	0	0.61	0.26	0.25	0	0.49	0.13	0.17	0	0.69
	WR	0.31	0	0.09	0.60	0.38	0	0.08	0.54	0.25	0	0.06	0.70
Lat.Am.	sWRD	0.24	0.09	0.18	0.49	0.40	0	0	0.60	0.13	0.08	0.22	0.58
	WD	0.32	0.12	0	0.56	0.39	0.04	0	0.57	0.19	0.13	0	0.69
	WR	0.21	0	0.28	0.51	0.29	0	0.21	0.50	0.16	0	0.30	0.54
Africa	sWRD	0.19	0.09	0	0.72	0.26	0	0	0.74	0.11	0.05	0.21	0.64
	WD	0.18	0.09	0	0.72	0.24	0.04	0	0.72	0.11	0.08	0	0.81
	WR	0.19	0	0.07	0.74	0.24	0	0.04	0.73	0.14	0	0.28	0.57
Asia	sWRD	0.20	0.12	0.19	0.48	0.26	0.01	0.11	0.62	0.12	0.26	0.12	0.50
	WD	0.22	0.24	0	0.54	0.26	0.04	0	0.70	0.12	0.39	0	0.49
	WR	0.19	0	0.29	0.52	0.24	0	0.19	0.57	0.14	0	0.39	0.47

Notes: Reported are the means of the posterior distributions of the variance shares. For the calculation of the variance shares, we refer to Section 2.5. $sWRD$ denotes the factor model selected using our stochastic factor selection procedure under prior inclusion probability $p_0 = 0.5$ (see Table 3 for the models selected in each period). WD denotes the two-level factor model with a global factor G_t and a development level factor D_{kt} (industrial, emerging or developing economies factor). WR denotes the two-level factor model with a global factor G_t and a regional factor R_{jt} (European, North American, Oceanian, Latin American, African or Asian factor). The global factor G_t is denoted by 'World', the level of development factor D_{kt} by 'Dev' and the regional factor R_{jt} by 'Reg' and the country-specific or idiosyncratic component μ_{it} by 'Idio'. ALL denotes all countries in the sample, IND denotes industrial countries, EME denotes emerging market economies and DEV denotes developing economies. We refer to Appendix A for the list of countries included in the different groups.

4.6 Robustness checks

In this section, we check the robustness of the results reported in Section 4.3 – in particular the results reported in Table 2 – regarding the inclusion or exclusion of common factors over the full sample period and over the pre-globalization and globalization periods. The three robustness checks reported in Table 5 generally confirm our previous findings while providing some additional insights. First, we distinguish between developed and developing Asian economies as in Kose et al. (2003). The results are very similar to those reported in Table 2 for $p_0 = 0.5$. The subsample results indicate, however, that the distinction between developed and developing Asian economies is meaningful as only the former group seems to share a distinct business cycle over the globalization period $1985 - 2017$. This increase in integration for the developed Asian economies cannot be observed from the results reported in Table 2 where all Asian countries are lumped together in one region. Second, we assume AR(2), instead of AR(1), processes for all common factors f_{mt} and for all country-specific idiosyncratic components μ_{it} in eqs.(3) and (4) of Section 2.1 above. When comparing the results to those reported in Table 2 with $p_0 = 0.5$, we note that the estimation results obtained with AR(2) processes are quite similar to those obtained with AR(1) processes.

Table 5: Posterior inclusion probabilities of the common factors: robustness checks

Check	Period	Posterior factor inclusion probabilities									
		World	IND	EME	DEV	Europe	No. Am.	Oceania	Lat. Am.	Africa	Asia
Asia split	1961-2017	1.00	1.00	0.83	0.59	0.98	0.97	0.16	1.00	0.19	0.88^a, 1.00^b
	1961-1984	1.00	0.94	0.10	0.27	0.36	0.9	0.20	0.37	0.13	0.53^a, 0.28^b
	1985-2017	1.00	1.00	1.00	0.53	0.50	0.37	0.18	1.00	0.56	0.17^a, 1.00^b
AR(2)	1961-2017	1.00	1.00	0.85	0.76	1.00	0.99	0.21	1.00	0.20	1.00
	1961-1984	1.00	0.70	0.07	0.45	0.44	0.95	0.24	0.63	0.14	0.62
	1985-2017	1.00	0.90	1.00	0.62	0.87	0.40	0.21	1.00	0.84	0.61
106 countries	1961-2017	1.00	1.00	0.96	1.00	1.00	0.98	0.17	1.00	0.87	0.99
	1961-1984	1.00	0.82	0.08	0.22	0.64	0.94	0.21	0.81	0.18	0.45
	1985-2017	1.00	1.00	1.00	0.93	0.46	0.39	0.23	1.00	0.91	0.22

Notes: All reported results are based on prior $p_0 = 0.5$ but they are robust to the use of alternative priors $p_0 = 0.25$ or $p_0 = 0.75$."Asia split" is a robustness check in which a distinction is made between developed Asian economies and developing Asian economies resulting in $M = 11$ instead of $M = 10$ common factors with ^{a} denoting the results for the group of developing Asian countries and ^{b} denoting the results for group of developed Asian countries. In Appendix A, we report which countries belong to either group. "AR(2)" is a robustness check in which the dynamic factor model is estimated assuming AR(2) instead of AR(1) processes for all common factors f_{mt} and all idiosyncratic components μ_{it}, i.e., for $p = q = 2$. "106 countries" is a robustness check where the sample consists of $N = 106$ countries instead of the 60 countries reported in Appendix A where these countries can belong to either one of the three levels of development that we consider and to either one of the six regions that we consider. We refer to Kose et al. (2012) for the list of countries included in this extended sample.

Third, while we conduct our analysis with the more balanced, more reliable and more frequently used 60 countries dataset, we also consider a larger dataset consisting of 106 countries. We refer to Kose et al. (2012) for the countries included in this extended dataset. Most of the results reported in Table 2 are confirmed. One important difference, however, is that there is now more evidence that supports the presence of a factor for the developing countries (DEV) and for Africa, in particular during the globalization period $1985 - 2017$. Hence, the inclusion of a large number of additional countries in the DEV and Africa country groups in the larger 106 countries dataset provides indications that these groups may also command distinct business cycles.

5 Conclusions

The business cycle synchronization literature has so far largely ignored model misspecification in estimated dynamic factor models as the common factors in these models are typically imposed but not tested for. This can lead to misleading results. In this paper, we therefore propose a Bayesian stochastic model specification search to select the relevant factors in a hierarchical dynamic factor model with predetermined factors. Our factor model is estimated using real GDP growth data for a panel of 60 countries over the period $1961 - 2017$. Whereas until now the synchronization literature has focused mainly on factor models that either include global factors and regional factors or global factors and factors reflecting the level of development of countries, we instead estimate a three-level factor model with factors that capture *both* the geographical proximity of countries and their level of development and use our factor selection procedure to determine which of these factors are relevant.

For the full sample period $1961 - 2017$, our factor selection procedure strongly supports the presence of a global business cycle, four regional cycles (Europe, North America, Latin America, Asia) and two development group cycles (industrial countries, emerging market economies). These results are new and show that to investigate business cycle synchronization, it is in fact important to estimate a factor model that includes global, regional and development level factors simultaneously. We further analyze whether the importance of the various factors has changed over time by estimating our model over the pre-globalization period $(1961 - 1984)$ and over the globalization period $(1985 - 2017)$. In line with the existing literature, we find that the importance of the global factor has decreased over time, while the impact of the regional and development factors has become more pronounced during the globalization period.

References

Amengual, D. and Watson, M. W. (2007). Consistent estimation of the number of dynamic factors in a large N and T panel. *Journal of Business & Economic Statistics*, 25:91–96.

Bai, J. and Ng, S. (2007). Determining the number of primitive shocks in factor models. *Journal of Business & Economic Statistics*, 25:52–60.

Bai, J. and Ng, S. (2013). Principal components estimation and identification of the factors. *Journal of Econometrics*, 176:18–29.

Bai, J. and Wang, P. (2015). Identification and Bayesian estimation of dynamic factor models. *Journal of Business and Economic Statistics*, 33:221–240.

Bauwens, L., Lubrano, M., and Richard, J.-F. (2000). *Bayesian inference in dynamic econometric models.* Oxford University Press.

Canova, F. (1998). Detrending and business cycle facts. *Journal of Monetary Economics*, 41(3):475–512.

Carter, C. and Kohn, R. (1994). On Gibbs sampling for state space models. *Biometrika*, 81:541–53.

Chib, S. and Greenberg, E. (1994). Bayes inference in regression models with ARMA(p,q) errors. *Journal of Econometrics*, 64:183–206.

Feenstra, R., Inklaar, R., and Timmer, M. (2015). The next generation of the Penn World Table. *American Economic Review*, 105(10):3150–3182.

Ferroni, F., Grassi, S., and Leon-Ledesma, M. (2019). Selecting structural innovations in DSGE models. *Journal of Applied Econometrics*, 34(2):205–220.

Francis, N., Owyang, M., and Savascin, O. (2017). An endogenously clustered factor approach to international business cycles. *Journal of Applied Econometrics*, 32(7):1262–1276.

Frühwirth-Schnatter, S. and Wagner, H. (2010). Stochastic model specification search for Gaussian and partial non-Gaussian state space models. *Journal of Econometrics*, 154(1):85–100.

George, E. and McCulloch, R. (1993). Variable selection via Gibbs sampling. *Journal of the American Statistical Association*, 88:881–889.

Geweke, J. (1992). Evaluating the accuracy of sampling-based approaches to the calculation of posterior moments. In *IN BAYESIAN STATISTICS*, pages 169–193. University Press.

Hallin, M. and Liska, R. (2007). Determining the number of factors in the general dynamic factor model. *Journal of the American Statistical Association*, 102:603–617.

Hirata, H., Kose, M. A., and Otrok, C. (2013). Regionalization vs. globalization. *IMF Working Paper*, 13/19.

Kim, C.-J. and Nelson, C. (1999). *State-space models with regime switching. Classical and Gibbs-sampling approaches with applications.* MIT Press, Cambridge.

Kose, M. A., Otrok, C., and Prasad, E. (2012). Global business cycles: convergence or decoupling? *International Economic Review*, 53(2):511–538.

Kose, M. A., Otrok, C., and Whiteman, C. H. (2003). International business cycles: world, region, and country-specific factors. *American Economic Review*, 93(4):1216–1239.

Kose, M. A., Otrok, C., and Whiteman, C. H. (2008). Understanding the evolution of world business cycles. *Journal of International Economics*, 75(1):110–130.

Mumtaz, H., Simonelli, S., and Surico, P. (2011). International comovements, business cycle and inflation: a historical perspective. *Review of Economic Dynamics*, 14(1):176 – 198.

Scott, J. G. and Berger, J. O. (2010). Bayes and empirical Bayes multiplicity adjustment in the variable-selection problem. *Annals of Statistics*, 38(5):2587–2619.

Appendix A List of countries

The sample consists of 60 countries which, below, are grouped according to geographical region. Each country belongs to one of six regions: Europe, North America, Oceania, Latin America, Africa and Asia. This classification follows Kose et al. (2003). Each country also belongs to one of three development level groups: industrial economies (IND), emerging market economies (EME) and developing economies (DEV). This classification follows Kose et al. (2012). Between brackets we denote the development level of a country according to this classification. Between squared brackets the ISO classification of the country is added. In the robustness checks of Section 4.6, we further separate developed and developing Asian economies as in Kose et al. (2003). Below, the developed Asian economies are denoted with an asterisk.

Europe (18 countries):

Austria [AUT](IND), Belgium [BEL](IND), Denmark [DNK](IND), Finland [FIN](IND), France [FRA](IND), Germany [DEU](IND), Greece [GRC](IND), Iceland [ISL](IND), Ireland [IRL](IND) Italy [ITA](IND), Luxembourg [LUX](IND), Netherlands [NLD](IND), Norway [NOR](IND), Portugal [PRT](IND), Spain [ESP](IND), Sweden [SWE](IND), Switzerland [CHE](IND), United Kingdom [GBR](IND)

North America (3 countries):

Canada [CAN](IND), Mexico [MEX](EME), U.S. [USA](IND)

Oceania (2 countries):

Australia [AUS](IND), New Zealand [NZL](IND)

Latin America (18 countries):

Costa Rica [CRC](DEV), Dominican Republic [DOM](DEV), El Salvador [SLV](DEV), Guatemala [GTM](DEV), Honduras [HND](DEV), Jamaica [JAM](DEV), Panama [PAN](DEV), Trinidad [TTO](DEV), Argentina

[ARG](EME), Bolivia [BOL](DEV), Brazil [BRA](EME), Chile [CHL](EME), Columbia [COL](EME), Ecuador [ECU](DEV), Paraguay [PRY](DEV), Peru [PER](EME), Uruguay [URY](DEV), Venezuela [VEN](EME),

Africa (7 countries):

Cameroon [CMR](DEV), Ivory Coast [CIV](DEV), Kenya [KEN](DEV), Morocco [MAR](EME), Senegal [SEN](DEV), South Africa [ZAF](EME), Zimbabwe [ZWE](DEV),

Asia (12 countries):

Bangladesh [BGD](DEV), India [IND](EME), Indonesia [IDN](EME), Pakistan [PAK](EME), Philippines [PHL](EME), Sri Lanka [LKA](DEV), Hong Kong* [HKG](EME), Japan* [JAP](IND), Malaysia* [MYS](EME), Singapore* [SGP](EME), South Korea* [KOR](EME), Thailand* [THA](EME)

Appendix B Convergence diagnostics MCMC sampler

To check convergence of the MCMC sampler, we calculate the integrated autocorrelation time (IAT) as $\tau = 1 + 2\sum_{t=1}^{d} \tilde{c}(t)$ with $\tilde{c}(t)$ the estimated autocorrelation function at lag t and d the lag for which $\tilde{c}(t) < 0.01$. It can be interpreted as the factor by which the squared Monte Carlo standard error increases due to the dependence in the Markov chain. Based on the IAT, the number of independent samples is given by W/τ, where W denotes the size of the Markov chain. Table B-1 reports the IAT for the three most important models estimated over the full period $(1961 - 2017)$, i.e. the model with all binary indicators δ_m set equal to 1 that is behind Figure 1; the model with the binary indicators sampled that is behind Table 2; the preferred sWRD model that is used to calculate variance shares in Table 4. We find only a moderate degree of autocorrelation in the Markov chain, varying between 2.08 and 12.30 across the different models and parameters, implying that our choice of $W = 50,000$ yields a sufficiently high number of effective draws. Similar results are obtained for the two subperiods $(1961-1984; 1985-2017)$ and other models used in this paper. Trace plots and the Geweke (1992) test statistics (that compare the mean and variance of segments of the chain) also show proper mixing of the Markov chain. These results are available on request.

Table B-1: Integrated autocorrelation time for selected models over the full period 1961 − 2017

Model	Parameters					Factors									
	α	ρ	π	σ_ϵ	σ_ν^2	World	IND	EME	DEV	Europe	No. Am.	Oceania	Lat. Am.	Africa	Asia
$\delta_m = 1$	3.96	3.35	2.75	6.21	2.74	3.46	6.67	4.34	3.23	7.98	6.41	2.82	6.69	3.72	5.19
δ_m sampled	4.50	7.20	2.58	4.47	2.08	8.64	9.22	8.13	6.50	9.99	4.32	2.38	6.38	2.49	4.87
sWRD	5.30	6.71	2.85	5.49	2.37	7.54	12.30	7.75	4.26	9.08	5.00	–	5.10	–	5.69

Notes: The table shows the average integrated autocorrelation time. The first model denoted by $\delta_m = 1$ sets all binary indicators equal to and was used to produce Figure 1. The second model samples the binary indicators to calculate the factor inclusion probabilities reported in Table 2. The last is the *sWRD* model used to compute the variance shares of our preferred model in Table 4. It restricts δ_m to either zero or one, depending on the factor inclusion probability.