(correction 2nd part made by MFH)

Extreme Value Theory: An Introduction

by Laurens de Haan and Ana Ferreira

With this webpage the authors intend to inform the readers of errors or mistakes found in the book after publication. We also give extensions for some material in the book. We acknowledge the contribution of many readers.

Chapter 1: Limit Distributions and Domains of Attraction

page	line	error/unclear/missing	correction
8	-0	(complement)	**Corollary 1.1.3A** For $x>0$ $$\lim_{t\to\infty}\frac{a(tx)}{a(t)}=x^{\gamma}$$
12	14	$b_n' = (2\log n)^{1/2} - \dfrac{\log\log n+\log(4\pi)}{(2\log n)^{1/2}}$	$b_n' = (2\log n)^{1/2} - \dfrac{\log\log n+\log(4\pi)}{2(2\log n)^{1/2}}$
20	-3	… estimator of γ (Section 3.5). Next we show….	… estimator of γ (Section 3.5). The necessary and sufficient condition for a distribution function to belong to the domain of attraction of an extreme value distribution is sometimes called “the extreme value condition”. Next we show….

Chapter 2: Extreme and Intermediate Order Statistics

page	line	error/unclear/missing	correction
40	-2	…, as we shall see. The following result …	…, as we shall see. The asymptotic behavior of intermediate order statistics is important for statistics of extreme values since any meaningful estimator is based on (extreme and) intermediate order statistics (see Chapter 3). The following result …
42	10	$\sqrt{k}\left(\dfrac{n}{kU_{k+1,n}}-1\right)$	$\sqrt{k}\left(\dfrac{nU_{k+1,n}}{k}-1\right)$

Chapter 3: Estimation of the Extreme Value Index and Testing

page	line	error/unclear/missing	correction
77	6	$\frac{\lambda}{\rho c_2}$	$\left\vert\frac{\lambda}{\rho c_2}\right\vert$
104	1	$\Psi_{\gamma_-,\rho}(x)\vert \le \varepsilon x^{\gamma_- + \rho}$	$\Psi_{\gamma_-,\rho'}(x)\vert \le \varepsilon x^{\gamma_- + \rho'}$
111	7 10	Then (3.6.5) becomes and (3.6.6) becomes	Then (3.6.5) (multiplied by *f(t)*) becomes and (3.6.6) (multiplied by *f(t)*) becomes
113	1	The “negative Hill estimator” was proposed by Falk (1995).	The “negative Hill estimator” was proposed by Smith and Weissman (1985).

Chapter 4: Extreme Quantile and Tail Estimation

page	line	error/unclear/missing	correction
128	label vertical axis in Fig.4.2(b)	log (1+ γx) / x	log (1+ γx) / γ
130	-7	... the moment estimator of γ. We define…	... the moment estimator of γ. In order to find an estimator for the scale a (*n/k*) we use relation (3.5.3) for *j* = 1 and define…
134	-12	**Theorem 4.3.1**	**Theorem 4.3.1 (de Haan and Rootzén (1993))**
135	-5	$\frac{U(tx)-U(t)}{a(t)}$	$\frac{U(tx)-U(t)}{a_0(t)}$
138	10	**Theorem 4.3.8**	**Theorem 4.3.8 (Dijk and de Haan (1992))**
140	8	$1+4\gamma+5\gamma^2+2\gamma^3+2\gamma^4$	$1+4\gamma+5\gamma^2+2\gamma^3=(1+\gamma)^2(1+2\gamma)$

Chapter 6: Basic Theory in higher dimensional space

page	line	error/unclear/missing	correction
209	-8	$U_1([n])$ etc.	$U_1(n)$ etc.
213	-12 -9	**Remark 6.1.8** Relations (6.1.15) does not hold for all Borel sets $A_{x,y}$. … > 0 and $\nu(\partial A) = 0$, and any …	Relations (6.1.15) can be written $$P\left(A^c_{x,y}\right) = \exp\left\{-\nu\left(A_{x,y}\right)\right\} \qquad (6.1.15^*)$$ Where P is the probability measure with distribution function G_0. Relation (6.1.15*) does not necessarily hold for Borel sets A not of the form (6.1.16) … > 0, and any …
216	1 to 9	(6.1.24) … … $\int_0^{\pi/2}\left(\frac{\cos\theta}{x}\vee\frac{\sin\theta}{y}\right)\Psi(d\theta).$	(6.1.24) $= \iint_{r>\frac{x}{\cos\theta}\wedge\frac{y}{\sin\theta}} \frac{dr}{r^2}\Psi(d\theta)$ $= \int_0^{\pi/2}\left(\frac{\cos\theta}{x}\vee\frac{\sin\theta}{y}\right)\Psi(d\theta).$
217	-11	**Definition 6.1.13** We call …	**Definition 6.1.13** A distribution function G is called a **max-stable distribution** if there are constants A_n, C_n >0, B_n and D_n such that for all x,y and n= 1,2,… $$G^n(A_n x + B_n, C_n y + D_n) = G(x,y) .$$ It is easy to see that any distibution function G satisfying (6,1,25) is max-stable and also $G(\alpha x+\beta, \gamma y+\delta)$ where α, γ are arbitrary positive constants and β, δ arbitrary real constants. Since any max-stable distribution is in the class of limit distributions for (6.1.1), we get all the max-stable distributions this way. The class of…
224	-8	Clearly $G_L^{(n)}$ and $G_U^{(n)}$ …	Clearly $G_L^{(n)}$ and $G_U^{(n)}$ are max-stable distributions with marginal distributions $\exp(-c/x)$ where c is a generic constant and there …

231	6 8	converges to 2^{-1} … $= -\log G_0(x,y)$ with G_0 from Theorem 6.1.1.	converges to $g(x,y) := 2^{-1}$ … $= \iint_{\{s>x\}\cup\{t>y\}} g(s,t)\,ds\,dt$ for $x,y \in \mathbb{R}$.
232	5 15 16 18 -9	$r^3 q(\theta r, r(1-\theta))$. Let $\{r_{i,j}\}^d_{i,j=1}$ be a matrix the random vector $(\vee^d_{j=1} r_{1,j} V_j, \ldots, \vee^d_{j=1} r_{d,j} V_j)$ two-dimensional simple distribution function can be $\vee$ twice	$r^3 q(\theta r, r(1-\theta)) = q(\theta, (1-\theta))$ (cf. Coles and Tawn (1991)). Let $\{r_{i,j}\}_{i=1,2;\, j=1,2,\ldots,d}$ be a matrix the random vector $(\vee^d_{j=1} r_{1,j} V_j, \vee^d_{j=1} r_{2,j} V_j)$ two-dimensional distribution function with Fréchet marginals can be $\wedge$ twice
233	6	complement	**6.14.** Let (X,Y) have a standard spherically symmetric Cauchy distribution. Show that the probability distribution of $(\|X\|,\|Y\|)$ is in the domain of attraction of an extreme value distribution with uniform spectral measure Ψ. Show that the probability distribution of (X,Y) is also in a domain of attraction. Find the limit distribution.

Chapter 7: Estimation of the Dependence Structure

page	line	error/unclear/missing	correction
252	15	... a max-stable distribution function.	... a max-stable distribution function. The marginal distribution are $\hat{G}_0(x,\infty) = \exp-\left(a_1/x\right)$ and $\hat{G}_0(\infty,y) = \exp-\left(a_2/y\right)$ for some positive a_1 and a_2 not necessarily one. Hence $\hat{G}_0$ is not necessarily simple max-stable (cf.

			Definition 6.1.13).
261	1,2		Not all inequalities have to hold, but at least one of them
262	13 14	$Q(x,\infty)$, $Q(\infty,y)$.	$Q(x,\infty)=0$, $Q(\infty,y)=0$.
263	-12	=	$\rightarrow$
265	18 -10	… = $x + y - L(x,y)$. … does not imply asymptotic independence.	… = $x + y - L(x,y) = R(x,y)$. … does not imply asymptotic dependence.
268	-2	$EW(x_1, \ldots, x_d)\, W(x_1, \ldots, x_d) = \mu(R(x_1, \ldots, x_d) \cap R(x_1, \ldots, x_d))$	$EW(x_1, \ldots, x_d)\, W(y_1, \ldots, y_d) = \mu(R(x_1, \ldots, x_d) \cap R(y_1, \ldots, y_d))$
269	4	and N is a standard normal random variable.	and N indicates a normal probability distribution.

Chapter 8: Estimation of the Probability of a Failure Set

page	line	error/unclear/missing	correction
273	-0	complement	i.e. $Q_n = c_n S$ (assumption), where …
275	2	more detail below; cf. Theorems 8.2.1 and 8.3.1	more detail in section 8.2.; cf. (8.2.7), (8,2,8) and (8,2,15))
283	-7	$\frac{\log^2(c_n x)}{2}$	$\frac{\log^2(c_n x)}{2\sqrt{k}}$

Chapter 9: Basic Theory in C[0,1]

page	line	error/unclear/missing	correction
301	-11	>0 *and* $\upsilon(\partial A)=0$, *and any* $a>0$,	>0 *and any* $a>0$,
304	-1, -2, -3	$\zeta_1,\zeta_2,\zeta_3,\ldots$ be a realization of the point process. Define $\eta := \bigvee_{i=1}^{\infty} \zeta_1$.	$(Z_1,\pi_1),(Z_2,\pi_2),(Z_3,\pi_3),\ldots$ be a realization of the point process where $Z_1 \in (0,\infty]$ and $\pi_1 \in \bar{C}_1^+[0,1]$. Define $\eta := \bigvee_{i=1}^{\infty} Z_i\pi_i$.
306	7 to19		These lines should be indented (belong to (2)).
	11	First we note that this implies	First we note that this assumption implies
307	-12 -6 -4	$\eta(s) \stackrel{d}{=} \bigvee_{i=1}^{k} \zeta_i(s)$ **Corollary 9.4.5** *All simple max-stable processes in* $C^+[0,1]$ *can be generated in the following way.* *stochastic processes* $V_1, V_2,\ldots$ *in* $C^+[0,1]$	$\eta(s) = \bigvee_{i=1}^{k} \zeta_i(s)$ **Corollary 9.4.5** (cf. M. Schlather (2002)) *All simple max-stable processes* η *in* $C^+[0,1]$ *can be generated in the following way.* *stochastic processes* $V_1, V_2,\ldots$ *in* $\bar{C}^+[0,1] := \{f \in C[0,1] : f \geq 0\}$
308	6	… Resnick (1977).	… Resnick (1977). Let W^* be two-sided Brownian motion: $W^*(s) := \begin{cases} W^+(s) & \text{for } s \geq 0 \\ W^-(-s) & \text{for } s<0 \end{cases}$ where W^+ and W^- are independent Brownian motions. In the rest of the example change W into W^*.
311	-5	**Theorem 9.5.1**	**Theorem 9.5.1 (de Haan and Lin (2001))**

315	5	**Theorem 9.6.1 (Resnick and Roy (1991))**	**Theorem 9.6.1 (Resnick and Roy (1991) and de Haan(1984))**
320	11	… of the theorem is easy. Next we turn …	… of the theorem is easy. ■ **Corollary 9.6.8A** *Let* $\{(Z_i,T_i)\}_{i=1}^{\infty}$ *be a realization of a Poisson point process on* $(0,\infty]\times\mathbb{R}$ *with mean measure* $(dr/r^2)\times d\lambda$ (λ Lebesgue measure). *If* η *is a simple max-stable process in* $C^+(\mathbb{R})$, *then there exists a family of functions* $f_s(t)\,(s,t\in\mathbb{R})$ *with* 1. *for each* $t\in\mathbb{R}$ *we have a non-negative continuous function* $f_s(t):\mathbb{R}\to[0,\infty)$, 2. *for each* $s\in\mathbb{R}$ $$\int_{-\infty}^{\infty} f_s(t)\,dt=1, \qquad (9.6.7A)$$ 3. *for each compact interval* $I\in\mathbb{R}$ $$\int_{-\infty}^{\infty}\sup_{s\in I} f_s(t)\,dt<\infty,$$ *such that* $$\{\eta(s)\}_{s\in\mathbb{R}} \overset{d}{=} \left\{\bigvee_{i=1}^{\infty} Z_i\, f_s(T_i)\right\}_{s\in\mathbb{R}}. \qquad (9.6.8A)$$ *Conversely every process of the form exhibited at the right-hand side of (9.6.8A) with the stated conditions, is a simple max-stable process in* $C^+(\mathbb{R})$. *Proof*. Let H be a probability distribution function with a density H` that is positive for all real *x*. With the functions f_s from Theorem 9.6.7 define the functions $\tilde f_s(t):=f_s(H(t))H'(t)$. Since for any $s_1,s_2,...,s_d\in\mathbb{R}$ and $x_1,x_2,...,x_d$ positive $$\int_{-\infty}^{\infty}\max_{1\le i\le d}\frac{\tilde f_{s_i}(t)}{x_i}\,dt=\int_0^1\max_{1\le i\le d}\frac{f_{s_i}(t)}{x_i}\,dt\ ,$$

			the representation of the corollary follows easily from that of Theorem 9.6.8. ■ Next we turn …
	14,16,20,24	[0,1]	$\mathbb{R}$
	18, 22, 25	$\int_0^1$	$\int_{-\infty}^{\infty}$
321	16, 18, 21, 22, 23, 24	$\int_0^1$	$\int_{-\infty}^{\infty}$
	26	distributions.	distributions. ■
323	-1 (2#), -6, -8	W	W^*
	-6	… independent Brownian motions.	… independent two-sided Brownian motions (cf. correction to Example 9.4.6) .
324	5, 6	$e^{-x}\Phi\left(\frac{\sqrt{u}}{2}-\frac{y-x}{\sqrt{u}}\right)$	$e^{-x}\Phi\left(\frac{\sqrt{u}}{2}+\frac{y-x}{\sqrt{u}}\right)$
	1 (3#) 2 (2#) 3 (3#) 11 (2#) 12 , 13	W	W^*
	-5	x	y
325	10 (2#) 11 (2#) 12 (2#)	W	W^*

	-6	Hence for $s_1 < 0 < s_2$	Hence for $s_1 < 0 < s_2$ and in fact for all real s_1, s_2
326	2 -14	Let W be Brownian motion independent of Y. Consider the process… a>0	Let W^* be two-sided Brownian motion: $W^*(s) := \begin{cases} W^+(s) & \text{for } s \geq 0 \\ W^-(-s) & \text{for } s < 0 \end{cases}$ where W^+ and W^- are independent Brownian motions. Let Y and W^* be independent. Consider the process… In the rest of the example change W into W^*. a>1
326-328	Example 9.8.2	Remove the text of the example	**Example 9.8.2 (extension of Brown and Resnick (1977))** Let $\{X(s)\}_{s\in\mathbb{R}}$ be an Ornstein- Uhlenbeck process, i.e., $X(s) = 1_{\{s\geq 0\}} e^{-s/2}\left(N + \int_0^s e^{u/2} dW^+(u)\right) + 1_{\{s<0\}} e^{s/2}\left(N + \int_0^{-s} e^{u/2} dW^-(u)\right)$ with N, W^+ and W^- independent , N a standard normal random variable and W^+ and W^- standard Brownian motions. Since for $s \neq t$ the random vector ($X(s)$, $X(t)$) is multivariate normal with correlation coefficient less than 1, Example 6.2.6 tells us that relation (9.5.1) can not hold for any max-stable process in $C[0,1]$: since Y has continuous sample paths, $Y(s)$ and $Y(t)$ can not be independent. Hence we compress space in order to create more dependence, i.e., we consider the convergence of $\left\{\bigvee_{i=1}^{n} b_n\left(X_i\left(\frac{s}{b_n^2}\right) - b_n\right)\right\}_{s\in\mathbb{R}}$ (9.8.4) in $C[-s_0, s_0]$ for arbitrary $s_0 > 0$, where X_1, X_2, … are independent and identically distributed copies of X and the b_n are the appropriate normalizing constants for the standard one-dimensional normal distribution, e.g., (cf. Example 1.1.7) $b_n = (2\log n - \log\log n - \log(4\pi))^{1/2}$. Then

$$b_n\left(X\left(\frac{s}{b_n^2}\right)-b_n\right)$$

$$=e^{-|s|/\left(2b_n^2\right)}\left(b_n\left(N-b_n\right)+b_n\int_0^{|s|/b_n^2}e^{u/2}dW^{\pm}(u)+\left(1-e^{|s|/\left(2b_n^2\right)}\right)b_n^2\right)$$

where $W^{\pm}(s)$ is $W^{+}(s)$ for $s\geq 0$ and $W^{-}(s)$ for $s<0$. Note that uniformly for $|s|\leq s_0$

$$e^{-\left|s/\left(2b^2\right)\right|}=1+O\left(\frac{1}{b_n^2}\right).$$

Further, since $e^{u/2}=1+O\left(1/b_n^2\right)$ for $|u|<s_0/b_n^2$,

$$b_n\int_0^{|s|/b_n^2}e^{u/2}dW^{\pm}(u)=\left(1+O\left(\frac{1}{b_n^2}\right)\right)b_nW^{\pm}\left(\frac{|s|}{b_n^2}\right).$$

Finally, for $|s|\leq s_0$,

$$\left(1-e^{|s|/\left(2b_n^2\right)}\right)b_n^2=-\frac{|s|}{2}+O\left(\frac{1}{b_n^2}\right).$$

It follows that

$$b_n\left(X\left(\frac{s}{b_n^2}\right)-b_n\right)$$

$$=\left(1+O\left(\frac{1}{b_n^2}\right)\right)\left(b_n\left(N-b_n\right)+b_nW^{\pm}\left(\frac{|s|}{b_n^2}\right)-\frac{|s|}{2}\right)+O\left(\frac{1}{b_n^2}\right).$$

We write $\widetilde{W}(|s|):=b_nW^{\pm}\left(|s|/b_n^2\right)$. Then $\widetilde{W}$ is also Brownian motion. We have

$$\left\{\bigvee_{i=1}^{n}b_n\left(X_i\left(\frac{s}{b_n^2}\right)-b_n\right)\right\}_{s\in\mathbb{R}}$$

$$=\left(1+O\left(\frac{1}{b_n^2}\right)\right)\left\{\bigvee_{i=1}^{n}\left(b_n\left(N_i-b_n\right)+\widetilde{W}_i(|s|)-\frac{|s|}{2}\right)\right\}+O\left(\frac{1}{b_n^2}\right).$$

			Hence the limit of (9.8.4.) is the same as that of $$\left\{\bigvee_{i=1}^{n}\left(b_n\left(N_i-b_n\right)+\widetilde{W}_i\left(\lvert s\rvert\right)\right)-\frac{\lvert s\rvert}{2}\right\}_{s\in\mathbb{R}}. \qquad (9.8.5)$$ The rest of the proof runs as in the previous example. One finds that the sequence of processes (9.8.5) converges weakly in $C\left[-s_0,s_0\right]$ hence in $C(\mathbb{R})$, to $$\left\{\bigvee_{i=1}^{\infty}\left(\log Z_i+\widetilde{W}_i\left(\lvert s\rvert\right)\right)-\frac{\lvert s\rvert}{2}\right\}_{s\in\mathbb{R}}$$ with $\{Z_i\}$ the point process from (9.8.1). Note that the point process $\{Z_i\}$ and the random processes $\widetilde{W}_i$ are independent.
328	-7	independent of *V*	independent of *Y*
329	1	u>0	x>0

Chapter 10: Estimation in C[0,1]

page	line	error/missing	correction
332	-2	**Theorem 10.2.1**	**Theorem 10.2.1 (de Haan and Lin (2003))**
336	3	$1-\widehat{F}_{n,s}(x):=\frac{1}{n}\sum_{j=1}^{n}1_{\{X_i(s)>x\}}.$	$1-\widehat{F}_{n,s}(x):=\frac{1}{n}\sum_{j=1}^{n}1_{\{X_j(s)>x\}}.$
339	-3	**Theorem 10.4.1**	**Theorem 10.4.1 (de Haan and Lin (2003))**
341	-3	$\zeta_{n-k+1,n}$	$\zeta_{n-k,n}$
352	6	υ_n (S)	υ (S)

Appendix B: Regular Variation and Extensions

page	line	error	correction
365	-1 and -3	(B.1.6)	(B.1.16)
366	5	exp (integral}	exp(integral)
370	-10	$f(t) = \exp[\log t]$	$f(t) = \exp\{-[\log t]\}$
375	-10 -9	(B.1.23) (B.1.24) (B.1.24)	(B.2.12) (B.2.13) (B.2.13)
376	-9	*Hence f(t) is bounded for* $t \ge t_0$.	*Hence f(t) is locally bounded for* $t \ge t_0$.
379	3	$f(\infty) - f(t) =$	$f(\infty) - f(t) \sim$
380	9	$\frac{1-x^{\delta_1}}{\delta_1}$ (left side)	$\frac{1-x^{-\delta_1}}{-\delta_1}$
381	-8	From Remark B.2.14(2) it follows	From part 3 of the present proposition it follows

Further and Updated References

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Data files

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