ERIK KOLE

On Crises, Crashes and Comovements

On crises, crashes and comovements

Erik Kole

On Crises, Crashes and Comovements

Over crises, crashes en afhankelijkheid in koersverloop

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Kalm en netjes

J.C. Boldoot

Vertel, Muze, vertel van de wrok van Achilles. Daar kwam voor de Grieken grote ellende uit voort.

... antwoorde de legeraanvoerder Agamemnon, "(...) Zoals Apollo mij Chryseïs ontneemt, die ik op mijn eigen schip met mijn eigen mannen wegzend, zo zal ik gaan naar uw tent, Achilles, en mij laten halen uw prijs, het meisje Briseïs..."

Ilias – Homeros

Voorwoord

Dit proefschrift richt zich op crises en crashes in financiële markten. Crises zijn zo oud als de wereld, zoals het bovenstaande citaat illustreert. Bovendien – zo voel ik me aan mijn vrouwelijke collegae verplicht op te merken – is het begrip crisis net als zondeval te herleiden op een vrouw. Crises en crashes vormen een boeiend onderwerp van onderzoek. Crises en crashes spreken tot de verbeelding, waardoor het makkelijker is uit te leggen waar je nu precies onderzoek naar doet. Crises en crashes zijn ook een relevant onderwerp. Ze kunnen immers ernstige consequenties hebben voor beleggers. Daarnaast houden crises en crashes een uitdaging in, voor een onderzoeker in het algemeen omdat nog lang niet alles bekend is, en voor de econometrist omdat het bestuderen van crises en crashes een creatieve toepassing van econometrische technieken vergt.

De onderzoeken naar crises en crashes in dit proefschrift zijn tot stand gekomen in een periode van ruim vier jaar. In die tijd heb ik als Assistent-in-Opleiding (AiO) veel geleerd. Ik weet inmiddels een stuk meer van financiering en op econometrisch gebied is mijn ervaring gegroeid. Vooral heb ik echter geleerd hoe je onderzoek vorm geeft. Het belangrijkste daarbij is het organiseren van een klankbord. Zonder een groep mensen rondom je die kritisch zijn, je uitdagen en de juiste vragen stellen wordt onderzoeken al snel een moeizaam gebed zonder einde. Ik prijs me dan ook gelukkig met de vakgroep Financial Management, waar geen gebrek was aan belangstellende, kritische collegae.

Onderzoek *leren* doen is geen sinecure. Het AiO-programma van de onderzoeksschool ERIM vormt een goede start, maar het meest leer je door simpelweg onderzoek te doen. Door terug te kijken op de tot stand koming van een artikel kun je nagaan wat de grootste problemen waren die je ondervonden hebt en bedenken hoe deze voortaan te voorkomen. Als econometrist kom je er dan achter dat nieuwe of geavanceerdere methoden weliswaar een voor de hand liggende motivatie van nieuw onderzoek zijn, maar dat het toch beter is onderzoek vanuit een probleem te motiveren. Evaluaties van het onderzoeksproces komen in mijn optiek nog te weinig voor.

Ik kijk met veel plezier terug op mijn AiO-tijd. Je hebt bijzonder veel vrijheid, kunt naar eigen goeddunken je onderzoek bepalen en tijd indelen. Een keerzijde hiervan is dat je je afvraagt voor wie je nu eigenlijk dat onderzoek doet op momenten dat je onderzoek niet wil vlotten. Een jaar en drie maanden wachten op een reactie van een tijdschrift helpt dan ook niet mee. Ik was dan ook erg opgetogen dat ik het artikel *Portfolio implications of systemic crises* in 2005 op verschillende conferenties kon presenteren. Helemaal fantastisch was de acceptatie van dit artikel voor publicatie in het *Journal of Banking & Finance* op de een na laatste dag van mijn AiO-contract.

Voor hun bijdragen aan dit proefschrift wil ik een aantal mensen bedanken. Grote dank ben ik verschuldigd aan mijn promotoren. Beste Kees, je hebt me op belangrijke punten gemotiveerd, me uitgedaagd kort en bondig de bijdragen van een onderzoek uit te leggen, en me geleerd dat in ieder onderzoek een moment komt waarop je besluit dat het goed is en stopt met het invullen van details. Beste Marno, jij hebt me het vertrouwen gegeven dat ik de onderzoeken tot een goed einde kon brengen, je hebt me geleerd kritisch te zijn door het geduld waarmee je (tussen)resultaten en voorlopige versies bestudeerde, en moedigde me aan nieuwe ideeën te formuleren. Ook tijdens conferenties was je een grote inspirator... Samen vormden jullie een goed team door de verschillende hoeken vanwaaruit jullie je bijdragen leverden.

Ook wil ik op deze plek mijn collegae bij Financial Management bedanken. Ik heb altijd veel plezier ontleent aan de gezamenlijke lunches (in de *grote* mensa uiteraard), en zie de *Fun, Friendship & Finance Ski Experiences* nog steeds als het hoogtepunt van collegialiteit. Een aantal collegae wil ik met naam noemen. In de eerste plaats gaat mijn dank uit naar Nadja. We hebben bijzonder prettig samengewerkt aan het onderzoek dat geresulteerd heeft in Hoofdstuk 2 en ik verwacht veel van onze verdere samenwerking op dit gebied. Daarnaast heb je herhaaldelijk stukken tekst van mijn hand van commentaar voorzien. Ook Ben Jacobsen wil ik bedanken voor de samenwerking rondom Hoofdstuk 2. Mathijs bedank ik voor zijn belangstelling voor en raadgevingen aangaande mijn onderzoek. In Joop vond ik een kamergenoot met wie ik prima alle gebeurtenissen op de universiteit en daarbuiten kon kritiseren. Guillermo, you taught me how to dance Argentinian tango, a perfect way to relax. Aan de activiteiten tijdens conferenties en buiten het werk met Gerard, Ben, Reggy, Willem, Mathijs, Nadja, Joop en Marieke en andere collegae bewaar ik warme herinneringen.

Natuurlijk moet ik hier een plaatsje reserveren voor mijn vrienden annex belastingbetalers. “Een AiO is en blijft natuurlijk toch een soort klaploper die op andermans kosten loopt te freewheelen op de universiteit”, zo verzekerden jullie mij van tijd tot tijd. En meestal had ik dan weer een conferentie in het vooruitzicht die vanuit jullie loonstrookjes gefinancierd zou worden. Een speciaal woord van dank geldt mijn (oud-)huisgenoten Robbie, Ester, Bram en Eduard die geduldig luisterden naar mijn verhalen over het werk. Bij de dames uit Valkenswaard kon ik lekker zeuren over onderzoek doen, en jullie zeurden dan even hard terug. Ik ben ook erg gelukkig met mijn vrienden die tijdens de verdediging naast me staan en meehelpen bij de organisatie van de dag eromheen, de paramilitairen Joost B. a.k.a. the Machine en Dennis van P. alias der Schnitzer, en ceremonieMeesters Janneke.

Het laatste dankwoord is bestemd voor mijn familie. Jullie hebben je vanaf het begin vol belangstelling verdiept in mijn bezigheden, al was het soms moeilijk te doorgronden wat ik nu precies deed. Lieve Marly en Koen, jullie leefden altijd met me mee. Lieve ouders, van jullie heb ik belangrijke eigenschappen voor een onderzoeker meegekregen, zoals doorzettingsvermogen en precisie, maar ook mijn beruchte eigenwijsheid. Voor alle steun, bedankt!

Erik Kole
Valkenswaard, 31 December 2005

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When you know a thing, to hold that you know it; and when you do not know a thing, to allow that you do not know it – this is knowledge.
The Confucian Analects – CONFUCIUS

Chapter 1

Introduction

Crises and crashes in financial markets are what investors fear most. Investors associate them directly with large price decreases of financial assets and substantial increases in the risk and the uncertainty of holding and trading these assets. This deterioration in the trade-off between risk and return causes investors serious harm and turns financial markets into a less attractive place. Consequently, investors still regard past crises and crashes as the specter of markets turning mad and investments going awry, examples of which include the historic stock market crashes of October 1929 and October 1987, and – more recently – the emerging markets crises of the 1990s and the strong, prolonged decreases in 2001 and 2002 after the internet hype.

In this dissertation we study crises and crashes from the perspective of an investor in financial markets. The two key concerns that an investor has regarding crises and crashes are their influence on his risk exposure and their effect on his asset allocation decisions. We analyze these questions in several ways. In the analyses we pay specific attention to the comovements in asset returns.

Dependent on the aggregation level, we define a crash as a large price decrease of a single asset, a single sector or a single market. We consider a crisis as a period with high uncertainty that affects many assets in an industry, a single market or several markets worldwide. So while a crash refers to a specific event in one asset, industry or market on its own, a crisis refers to a period of turmoil in several assets, industries or markets at the same time. Consequently, crises have a more pronounced and prolonged character than crashes. A crash can be pinpointed to one or several specific days, while a crisis can last several months. The crash of October 1987 took place on October 19. The Asian crisis started with the devaluation of the Thai Baht on July 2, 1997, spread to other Asian countries in the following months and died down slowly in 1998 (see Kaminsky and Schmukler, 1999). Moreover, the effects of a crash are mostly limited to the industry or market in which it takes place, while a crisis can disrupt the functioning of a country's economy in every aspect. Also in this sense, we call the plunge of October 1987 a crash, and the turmoil in Asia in 1997 and 1998 a crisis. A crash can be the onset of a crisis, as in 1929 when the collapse of the stock market marked the start of the Great Depression.

Comovements constitute the third important element of this dissertation. Asset prices do not move independently of one another, nor do markets function on a stand-alone basis. Moreover, investors generally hold a portfolio of assets. Should a crash remain limited to a single asset, its effects can be mitigated by diversification over different assets and markets. However, the crashes of 1929 and 1987 reverberated across equity markets worldwide, and the Asian crisis spread from one economy to another until its effects were felt by all financial markets around the globe. Consequently, we should not only study a crisis or crash on its own, but also take their interaction with other assets, industries or markets into account. Comovements are therefore an important ingredient of an investigation of crises and crashes.

1.1 Motivation

In the classical view asset prices fluctuate because news reaches financial markets. Small news items lead to small movements; large news items produce large price changes. From this perspective, crashes occur when ominous news reaches the markets. News is hardly purely idiosyncratic and is likely to influence more assets. Therefore, assets do not move independently. If the news is fundamental to the economy as a whole, all assets are affected, producing a crisis in case the news is really shattering. Consequently, we should expect a crash to occur from time to time and a crisis to break out occasionally though less frequently than a crash.

Unfortunately, this classical view fails to sufficiently explain the crises and crashes observed in the past. For several reasons, crises and crashes cannot be treated as an integral part of the normal way in which financial markets function. First, crashes and crisis are only to a small degree related to news. Shiller (1981) and Roll (1988b) conclude that generally only part of the changes in asset prices can be explained by news reaching markets. Cutler *et al.* (1989) and Shiller (2000, Ch. 4) find that this also holds for big price changes, including crisis and crashes.

Second, empirical evidence indicates a pronounced difference between the behavior of asset returns during tranquil periods and their behavior during crisis periods.

Central in the analysis of returns are their means and volatilities. The normal distribution would then be the obvious candidate to use in a study of returns. However, already in the 1960s Mandelbrot (1963) and Fama (1965) pointed out that a normal distribution leads to predictions on the frequency of extreme negative (or extreme positive) returns that are too low compared to observed frequencies. As Fama (1965, p. 50) put it "... under the Gaussian hypothesis for any given stock an observation more than five standard deviations from the mean should be observed about once every 7,000 years. In fact such observations seem to occur about once every three to four years." A normal distribution would result if asset returns follow a diffusion process. Empirical evidence showed subsequently that jumps occur in the evolution of asset prices from time to time (see Oldfield Jr. *et al.*, 1977; Jarrow and Rosenfeld, 1984; Ball and Torous, 1985), though Jorion (1988) finds limited impact for monthly or yearly stock returns. However, more recent evidence from option markets by Andersen *et al.* (2002) and Bates (2000) establishes that jumps are a necessary complement to diffusions to explain observed option prices. Concluding, asset returns contain a diffusion component that is dominating during tranquil times and a (mostly negative) jump component that manifests itself during crisis periods. As a consequence, asset returns exhibit fat tails.

Third, crises and crashes have an increasing and persistent effect on volatility. One of the central findings in empirical finance is the tendency of large price changes to cluster. This phenomenon has led to the development of the class of GARCH models, pioneered by Engle (1982), who was awarded a Nobel prize for it, and Bollerslev (1986). Moreover, the phenomenon is asymmetric in the sense that large price decreases lead to stronger future fluctuations than large price increases (see Black, 1976; Christie, 1982). Combining this effect with the presence of both a diffusion and a jump component in asset returns indicates that the two components influence each other. Bates (2000) and Eraker *et al.* (2003) provide evidence that jumps show up both in returns themselves but also in the associated volatility. Consequently, if a crash hits an asset or a market, the volatility afterwards remains on a higher level for a prolonged period of time.

Fourth, the comovements of assets become stronger when markets are under stress. Most of the evidence supporting a strengthening of comovements is based on correlations that increase for extreme positive or negative returns and for bear markets. Ang and Chen (2002) document such an increase for US stocks, whereas Ramchand and Susmel (1998) and Campbell *et al.* (2002) provide international evidence. In a statistically more robust approach than the correlation approach Longin and Solnik (2001) reject the hypothesis that correlation between international stock markets remains constant over tranquil and crisis periods. By means of a slightly different approach, Hartmann *et al.* (2004) reject this hypothesis for both equity and bond markets. Bae *et al.* (2003) find that this result also holds for emerging markets. Crisis and crashes are more contagious than can be justified by comovements during normal periods.

While these four reasons all relate to the behavior of asset prices, the fifth reason to study crises and crashes in relation to comovements stems from investor behavior. It is by now widely accepted that investors do not behave as the traditionally assumed expected utility maximizing agent. Kahneman and Tversky (1979) and Tversky and Kahneman (1991) argue that investors are particularly averse to losses. Benartzi and Thaler (1995) and Barberis *et al.* (2001) show that investors' loss aversion helps explaining the equity premium. Crises and crashes being anathemas to a loss averse investor is another reason to call for a detailed investigation of crises and crashes.

These five reasons indicate that crises and crashes cannot be considered as simply the mirror images of good times in financial markets. They do not adhere to the same rules or laws that events in financial markets normally do. They happen too often, and not because bad news shows up more often than it should. Moreover, crashes tend to occur together more frequently than expected based on normal periods, and crises spread more vehemently. Into the bargain, investors do not treat crises and crashes in the same way as other events in financial markets, but exhibit a particular aversion to crises and crashes. To make things even worse, crises and crashes do not simply pass by, but their effects can be felt in markets for a prolonged period of time. Reasons enough to put crises, crashes and comovements under further scrutiny.

1.2 Effects of crises, crashes and comovements

Conceptually, finance from an investor perspective studies the risks and the returns that investing in financial markets entails. In this respect, crises and crashes are an important part of the downside of financial markets. In this section we indicate in more detail how the different characteristics of crises and crashes affect investors and financial markets. We discuss current consensus and issues for further research.

The first and most obvious aspect to be influenced is risk management. We concluded that asset returns behave differently during tranquil times than during normal times. Consequently, specific techniques have been developed to study extreme returns. During the last two decades Extreme Value Theory (EVT) has been put forward as a robust and flexible way to deal with the fat tails of asset returns.[1] Longin (1996) shows that the extremes of asset returns are best approximated by a

[1] For a rigorous treatment of Extreme Value Theory we refer to Embrechts *et al.* (1997) and Reiss and Thomas (1997).

Fréchet distribution, while a normal distribution would imply a Gumbel distribution for extreme values. Longin (2000) applies EVT to calculate the risk of a portfolio in aggregate. Campbell (2001) applies EVT to determine the exposure to various sources of financial risk.

Extreme Value Theory has been shown to be very successful in a univariate setting. Deheuvels (1978) discusses EVT in a general multivariate setting. While multivariate EVT seems promising in areas dealing with natural disasters (see Coles and Tawn, 1991; Embrechts *et al.*, 2000), applications to financial risk management suffer from two problems. First, incorporating more than three dimensions increases the complexity of calculations considerably. The second problem is more subtle. Multivariate EVT provides a robust framework to study joint extreme events, such as two markets crashing at the same time, or one market crashing given that another market crashes (see Longin and Solnik, 2001; Hartmann *et al.*, 2004; Poon *et al.*, 2004). However, to asses such risks in a portfolio setting assumptions have to be made on the comovements of assets during both tranquil and stressed periods, which turn out to be crucial for the outcomes. Correlations are the traditional tool for describing comovements, but Embrechts *et al.* (2002) point out the limitations of correlation and recommend the more flexible class of copulas. A copula is a function that calculates the joint cumulative probability of a set of events from the marginal cumulative probability of each event. However, it is not a priori clear which copula to choose for which application from the many available copulas.

In Chapter 3 we propose goodness-of-fit tests for copulas and apply them to examine copulas for stress tests of a portfolio of stocks, bonds and real estate. These tests have the advantage over existing tests that they directly compare the dependence that a specific copula implies with the dependence that is present in the data. As a consequence, they can be widely applied to any copula of any dimension. For daily returns on stocks, bonds and real estate we confirm the failure of the Gaussian copula, implied by the correlation approach. The tests also reject the Gumbel copula, which is based on multivariate EVT. On the contrary, the Student's t copula provides a good fit in both the center and the tails of the distribution, as it is not rejected. To clarify the importance of choosing an accurate copula, we show that the Gaussian copula significantly underestimates the risk of joint downside returns, while the Gumbel copula overestimates it. In accordance with the test results, the Student's t copula does not lead to an assessment of the risk of downside returns that differs significantly from the observed probabilities.

Crises and crashes also influence portfolio formation. The central aim of asset allocation is constructing a portfolio with an optimal trade-off between the risk and the expected return. Four of the effects that we discussed play an important role here and are reinforcing each other. In the first place, if crashes are accurately taken into account, each asset on its own will be regarded more risky. Second, if the dependence between assets becomes stronger for extreme negative returns, diversification opportunities decline. Third, persistence causes a prolonged influence of the first two effects. Finally, because of their aversion to large losses, investors are particularly sensitive to this kind of price behavior. Based on these arguments, we hypothesize that investors invest more prudently, if they accurately take crises, crashes and comovements into account. Moreover, investors should show a preference for assets that are less prone to crises and crashes.

Various authors have considered asset allocation from this perspective. Das and Uppal (2004) study international asset allocation in the presence of synchronous crashes in all countries, but find that the effects are small. However, their model design precludes persistence.[2] Liu *et al.* (2003) show that jumps have a large impact on asset allocation, if the synchronous and persistent rise in volatility as shown by Bates (2000) and Eraker *et al.* (2003) is incorporated. Unfortunately, their univariate setting limits the study of effects on diversification. Ang and Chen (2002) find that it is costly to ignore the reduction of diversification opportunities due to the rise in correlations during times of stress. Ang and Bekaert (2002) report that diversification effects do not disappear during bear markets. Since the last two studies focus on bear markets in general and not explicitly on crises and crashes, it is not yet clear how systemic crises with persistent effects on volatilities and correlations affect diversification opportunities and asset allocations.

In Chapter 4 we investigate the consequences of systemic crises on portfolio choice. We use a regime switching model to capture persistence in the rise of volatilities and correlations. This approach extends the methods of Ang and Bekaert (2002), since we introduce a separate crisis regime on top of more general bear market regimes. Moreover, we show how the predictions of regime switching models can be included in a continuous-time formulation of an investor's asset allocation problem. For a representative set of developed and emerging markets, we report pronounced changes in the optimal portfolios after inclusion of the crisis regime. The compensation that a log-utility investor requires for incorrectly ignoring the crisis regime is substantial and can easily exceed 3% per month. Diversification possibilities erode rapidly. Investors that face short sales restrictions completely withdraw from financial markets. These findings stress the importance of persistence in the effects of crises.

[2] Das and Uppal (2003) allow for persistence, albeit at a relatively low level. For stronger persistence, the importance of these jumps increase, which indicates that higher levels persistence may lead to different conclusions.

A third important aspect of crises and crashes is their influence on asset pricing. In the discussion so far, we concluded that crises and crashes are expected to lead to less risky asset allocations and a preference for less crash-prone assets. Nevertheless, the strengthening of comovements during times of stress indicates that crises and crashes are difficult to evade. The persistent effect of crises and crashes turns financial markets in aggregate into a riskier place. Since this persistence and more severe comovements lead to an increase of systematic risk, investors should be rewarded for bearing this risk, according to standard financial theory. Hence, the equity premium should contain a component that reflects crash risk. If investors are particularly averse to large losses, this component will be quite large.

A premium for crash risk has been advocated by Rietz (1988) as a solution for the equity premium puzzle put forward by Mehra and Prescott (1985), but it is debated whether such a premium could indeed solve the puzzle completely (see Mehra and Prescott, 1988, 2003). Bates (1991, 1996, 2000) and Andersen *et al.* (2002) provide evidence of a risk premium for large negative jumps in the market as a whole. Moreover, they conclude that jumps are necessary to explain the return distributions that option prices imply. Consequently, an individual asset should contain a premium for market crash risk that corresponds with the asset's sensitivity to a market crash. However, Bakshi *et al.* (2003) show that the option-implied return distributions for individual stocks deviate from the implied return distribution for the market. Therefore, it is not clear whether a crash risk premium shows up as a distinguishable component in individual asset returns.

We examine this hypothesis in Chapter 5. We extend the traditional CAPM of Sharpe (1964) and Lintner (1965) with a crash risk factor and formulate three measures to determine a stock's sensitivity to market crashes. By sorting stocks into portfolios based on these measures we test whether a crash risk premium can be identified. Stocks with a high sensitivity to market crashes pay on average a significant extra return of 2.3% to 4.0% per year, on top of the regular return due to traditional market risk. This extra return cannot be explained by other risk factors, including coskewness and cokurtosis. For stocks with a low sensitivity we do not find a significant extra average return. We find mild evidence that a crash factor helps explaining the cross section of stock returns. For momentum portfolios the traditional CAPM is rejected, while a crash-CAPM is not. For other portfolios differences are smaller. Our work extends Harvey and Siddique (2000), who introduced coskewness as a risk factor, and Dittmar (2002), who introduced cokurtosis. While coskewness and cokurtosis can proxy for an asset's sensitivity to market crashes, we find a significant premium on crash risk portfolios, but not on coskewness nor on cokurtosis portfolios. Consequently, our approach may be better suitable to capture crash risk.

The last aspect that we want to discuss is the influence of crises and crashes on our understanding of financial markets. Since news can only partially explain the occurrence of crises and crashes, other explanations have been put forward. Brunnermeier (2001) lists four categories. The first category consists of liquidity shortage models, implying that crashes only have a temporary effect, which vanishes after liquidity has been restored. As a second category he discusses models with sunspots or multiple equilibria, in which a crash occurs when the economy switches from one equilibrium to another or when a sunspot takes place. The third category comprises bubble models in which a crash is a correction for a preceding run up in prices. The fourth category contains models in which restrictions on investors or traders, or trading costs cause information to come out in lumps.

The models of the third and fourth category are interesting from an investor perspective. If an investor can recognize the symptoms of a bubble or information hold-up in the market, he can improve his predictions on the likelihood of a crisis or a crash. Shiller (2000) relates the large crashes of 1929 and 1987. Kindleberger (2000) also argues that many crashes can be seen as a correction for price run ups. However, their analyses do not consider prediction but take place after the fact. Temin and Voth (2004) and Brunnermeier and Nagel (2004) show that informed investors were able to ride the bubble and get out of the market in time during the South Sea Bubble of 1720 and the technology bubble of the last decade, respectively.

In Chapter 2 we follow a more systematic approach. We take the perspective of an investor who wants to use the presence of a bubble to predict the likelihood of a crash. The investor perceives a bubble if the average monthly abnormal return over the last one to five years exceeds a given threshold. The presence of a bubble makes investing in financial markets more risky. Its presence leads to a significant doubling of the likelihood of a crash during the next period. To make things worse, the likelihood of a more severe crashes increases even more. To increase the number of available bubbles and crashes we base these findings on US industries. We conclude that this approach works well, as our results also apply to the market as a whole as well. Moreover, we provide evidence in favor of the model of Abreu and Brunnermeier (2003), as we do not reject hypotheses based on it. The strength of a bubble is positively related to crash likelihood, while the length of a bubble is unrelated to it.

To summarize, crises and crashes have been put under scrutiny in several respects. In particular with regard to univariate models for extreme returns, academia largely agrees on the superiority of Extreme Value Theory. However, some issues are still unresolved. In this dissertation we fill in some of theses gaps. Chapter 2 relates crashes to prediction based on bubbles in past returns. Chapter 3 links crashes and comovements to risk management. Chapter 4 discusses the implications of a persistent systemic crisis for asset allocation. Chapter 5 investigates the consequences of crashes for asset pricing. In Chapter 6 we discuss our findings from a more general perspective. We indicate how our research can improve the understanding of crises, crashes and comovements and their consequences for finance.

But how do we know when irrational exuberance has unduly escalated asset values, which then become subject to unexpected and prolonged contractions as they have in Japan over the past decade?
ALAN GREENSPAN

Chapter 2

Bubbles and Crashes in Industries*

2.1 Introduction

The crashes of 1929 and 1987 stand out as the archetypical stock market crashes. Because these crashes cannot be explained by dramatic news reaching financial markets, they are commonly explained as corrections to the run up in prices of the preceding years (see Shiller, 2000, Ch. 4). The run up in prices and the subsequent crash are then presented as evidence of bubbles showing up in stock markets. Because of the challenge that bubbles pose to rational models of financial markets, they have often been studied from a theoretical perspective (see Brunnermeier, 2001; LeRoy, 2004). Shiller (2000, Ch. 6) provides empirical evidence of bubble-crash patterns in different equity markets throughout the world. However, a thorough analysis of these patterns from an investor perspective is still missing. Our analysis fills this gap and provides investors with an estimate of the possibility of a crash, based on the presence of a bubble. As this analysis leads to an improved understanding of the risk of severe losses, its added value for risk management and investment decisions is obvious.

*This chapter is based on the article by Kole, Guenster, and Jacobsen (2005a).

Although several papers analyze specific bubbles and crashes[1], empirical research on the relation between bubbles and crashes in aggregate is scarce for at least two reasons. First, theoretical research explaining how stock market bubbles relate to crashes is limited. By the argument of backward induction, classical finance theory precludes divergence of asset prices from their fundamental values. Starting with De Long *et al.* (1990) theoretical models have been developed showing that assets can diverge – positively or negatively – from their fundamental values. De Long *et al.* explain these deviations by feedback trading. Although their model explains both bubbles and crashes separately, it does not show how a bubble leads to a crash. More recently, Abreu and Brunnermeier (2003) provide an explanation for the link between bubbles and crashes. They challenge proponents of the efficient market hypothesis (e.g. Fama, 1965) and prove mathematically that even in the presence of rational arbitrageurs bubbles can exist and lead to crashes. The second reason is that stock market crashes are extreme, infrequent events, thereby limiting the number of observations available for empirical analysis.

In this chapter we conduct an empirical investigation of the relation between bubbles and crashes from an investor perspective. The investor wants to predict the probability of a crash occurring next month, using currently available information. To expand the number of observations, we focus on bubbles and crashes in industries. The investor perceives a bubble in an industry, if the series of abnormal returns over the last one to five years exhibits an average above a specified threshold. We use the CAPM to find the fundamental value of an industry and to construct abnormal returns. We define a crash as next month's abnormal return below a specified threshold. Unlike related literature (e.g. Chen *et al.*, 2001) we refrain from using skewness as a measure of crash likelihood since investors cannot incorporate findings based on skewness directly into their risk assessment. Though several authors (e.g. Harvey and Siddique, 2000; Kraus and Litzenberger, 1976) have shown that skewness can be included in optimal portfolio selection by a Taylor series expansion of the utility function, we regard direct knowledge of crash likelihood as more informative.

Our main finding is that investors can use the presence of a bubble to predict crashes. In the basic setting, we choose the crash threshold such that 5% of all observations qualify as a crash. We consider 48 US industries from 1926 to 2004. Conditional on perceiving a bubble, the likelihood of a crash in the next month almost doubles to 7.7%, compared to 4.2% if a bubble is not observed. The investor

[1] Kindleberger (2000) provides a general overview of bubbles and crashes. Temin and Voth (2004) and Dale *et al.* (2005) analyze the South Sea Bubble, Rappoport and White (1994) study the crash of 1929, Bates (1991) examines the 1987-crash, and Ofek and Richardson (2003) and Brunnermeier and Nagel (2004) investigate the recent internet bubble.

perspective with conditional information is crucial in our analysis to establish this predictive effect of a bubble, and extends the unconditional analysis of Shiller (2000, Ch. 6). Moreover, we show that bubbles and crashes occur in all industries, and not only in new industries.

A further inspection of our results shows that the effect of a bubble on crash likelihood strengthens, if we restrict the analysis to more severe crashes. For the 20% severest crashes, the presence of a bubble triples the crash probability. A bubble that grew up to the last available observation has a larger impact on crash likelihood than a bubble that stopped growing in the last two to six months. Based on the model of Abreu and Brunnermeier (2003), we develop empirically testable hypotheses relating the characteristics of a bubble to crash likelihood. We find that the strength of the bubble positively affects the probability of a crash. The length of the bubble has no significant effect. We also show that these results are robust to changes in our research setting.

While the results on industries are interesting for investment strategies based on sector rotation, the big question is whether they carry over to the market as whole. From a similar investor perspective we investigate the presence of bubbles and crashes in the market. As expected, the analysis lacks statistical power, but in a qualitative sense the results for the market are largely similar to those for sectors. Consequently, we conclude that investors encounter a considerable increase of the risk of a market crash in the next month, if a bubble occurs in the last six months.

This chapter is structured as follows. In Section 2, we review the literature from a practical perspective and derive empirically testable hypotheses. Section 3 presents our data and the investor perspective towards bubbles and crashes. In Section 4, we analyze the impact of a bubble and its characteristics on crash likelihood. Section 5 compares our findings for industries to the market. Because our approach requires some arbitrary choices, we discuss several robustness checks in Section 6. Section 7 concludes.

2.2 Literature review

Most theoretical literature on bubbles and crashes focuses on establishing conditions under which a bubble can occur. Irrespective of whether a theoretical model can accommodate the presence of a bubble, the empirical evidence on bubbles and crashes so far in Kindleberger (2000) and Shiller (2000) compels further research. As the main purpose of this chapter we empirically investigate the hypothesis that the presence of a bubble increases the probability of a crash in the next period. We refrain from the theoretical debate on bubbles. However, the findings of the theoretical research can help us steering our examination and set up the investor perspective that we take towards bubbles and crashes.

A bubble is commonly defined as a period during which the price of an asset exceeds its fundamental value (see e.g. Brunnermeier, 2001; LeRoy, 2004). Asymmetric information is crucial for the occurrence of a bubble. Santos and Woodford (1997) show that a bubble can only exist under very strict conditions in a case with symmetric information. Abreu and Brunnermeier (2003) and Conlon (2004) show that bubbles can exist in a setting where some investors know about the bubble while others do not, and investors do not know who is informed and who is not.

The start of a bubble is commonly related to displacement in a Minsky model (see Kindleberger, 2000, p. 14) or new-economy thinking (see Shiller, 2000). Improvements in the fundamentals of an industry increase its outlook, and consequently asset prices in that industry grow at a faster rate than before. However, uninformed market participants extrapolate this faster growth rate and expect it to hold in perpetuity.[2] De Long *et al.* (1990) argue that the behavior of noise traders, who base their trades on such extrapolation (called feedback trading), can lead to the continuation of a bubble. Instead of trading against the bubble, the informed investors will try to ride the bubble at the expense of the noise traders. Abreu and Brunnermeier (2003) prove the optimality of this trading strategy for a setting where informed traders are unaware of the proportion of informed traders. Temin and Voth (2004) and Brunnermeier and Nagel (2004) provide empirical evidence of this behavior.

Most bubble models either explicitly state that a bubble ends with a crash (see for instance Blanchard and Watson, 1982; Abreu and Brunnermeier, 2003), or imply that a bubble ends with a crash, because the bubble becomes common knowledge. Though likely, a bubble does not necessarily have to end with a crash. It can also deflate without a crash. In theoretical models like Abreu and Brunnermeier (2003) the noise traders are assumed to be fully unaware of a bubble taking place, contrary to the informed traders who are fully aware of it. In reality, investors cannot be characterized as fully aware or fully unaware, but will show varying degrees of awareness. As a consequence, feedback trading may vary over time, and gradually decreasing feedback trading can lead to a soft landing of the bubble.

In our research we investigate whether bubble characteristics like its size and length help in determining the probability of a crash. We base two hypotheses mainly on the model of Abreu and Brunnermeier (2003). The investor perspective they use in their model makes it easy to relate their model to our empirical setting. In the model of Abreu and Brunnermeier a bubble has a maximum size. All traders start being

[2] This behavior is examined empirically by Frankel and Froot (1988) and experimentally by Andreassen and Kraus (1990).

uninformed, but per unit of time a fixed proportion of traders becomes fully aware of the bubble. Though they know its maximum, they do not know when it started, and consequently they do not know how long it will last before it bursts. Because traders do not know whom of the other traders are informed, a coordination problem arises. Informed traders will ride the bubble and try to sell out before it bursts. A bubble always ends with a crash, when it reaches its maximum size. This maximum can be exogenously given or endogenously arise as the point where selling pressure of the informed traders exceeds the buying capacity of the noise traders. We hypothesize that a bubble with a stronger growth rate will burst sooner, because it reaches its maximum size sooner.[3] Our hypothesis is in line with the hypothesis of Youssefmir *et al.* (1998) that larger bubbles are more susceptible to shocks, which they base on simulations. A crucial assumption in the model of Abreu and Brunnermeier (2003) is that the investors do not know the exact start date of the bubble, which makes it difficult to determine its length. Therefore, we test whether the length of the bubble that the investor infers does not help in predicting the probability of a crash.

The assumption that a crash happening after a bubble is related to it is implicit in our approach. While crashes may occur because of news reaching the market, crashes in the presence of a bubble are mostly too large to be explained by that news (see Shiller, 2000, Ch. 4). Abreu and Brunnermeier (2003) argue that news can act as a synchronizing event, leading to massive sell out by the informed investors.[4] If selling pressure exceeds the buying capacity of the noise traders, the bubble will burst and the asset price will fall. However, if this coordinated attack fails, a temporary strengthening of the bubble will set in, followed by new crashes. We call these crashes aftershocks.

In the next section we investigate whether these theoretical aspects of bubbles and crashes are present in industries. We test the hypotheses, but we do not test whether a specific model describes bubbles and crashes accurately. However, since the investor perspective of Abreu and Brunnermeier (2003) can be easily related to our approach, our findings can be interpreted as support in favor or against their model.

[3] In the case that the maximum size of a bubble arises endogenously as selling pressure exceeding buying capacity, the effect of a larger growth rate in the model of Abreu and Brunnermeier (2003) is twofold. On the one hand, traders have a stronger incentive to ride the bubble. Consequently, the bubble will be larger when selling pressure bursts it. On the other hand, it will reach this size sooner.

[4] In broader sense, this argument can apply to ‘real’ news that has a serious impact on the outlook of a sector, but also to sunspots or lumpy information that has been held up by restricted investors as in Hong and Stein (2003) and Cao *et al.* (2002).

2.3 Data and Concepts

In this section we formulate the concepts of a bubble and a crash from the perspective of an investor. In our setup, the investor does not receive a signal telling him with certainty that a bubble occurs as in the model of Abreu and Brunnermeier (2003), but he has to make inferences from the data. The investor wants to know whether a bubble has been inflating until recently, and tries to use that information to improve his prediction of the distribution of next period's return. This means that he is less interested in pinpointing exactly during which periods bubbles were taking place. How an investor perceives a bubble is discussed in the next subsection, followed by the definition of a crash. First, we briefly introduce the data set.

We use the industry indexes as in Fama and French (1997), which are available on French's website[5]. The data set consists of monthly value-weighted returns for 48 industries from July 1926 to December 2004. Eight industries (Soda, Health, Rubber, Fabricated Products, Guns, Gold, Personal Services and Paper) have a shorter time-series of returns available. Therefore, these industries are marked with a dagger in our tables. We also use the risk-free rate and the market index in our analysis. The risk-free rate is the one-month Treasury bill rate from Ibbotson Associates. We proxy the market index by the CRSP all share index. The market index and the risk free rate are obtained from French's website. We transform all discrete returns to log returns.

2.3.1 Bubbles

We take the viewpoint of an investor that wants to predict the probability that a crash occurs in the next month $t+1$, based on information available up to the current month t. We investigate whether knowledge about a bubble helps in this respect. We focus on bubbles that have been inflating up to time t, or that have stopped growing recently. In line with the theoretical research, the investor perceives a bubble in an industry, if it has shown larger price increases than can be justified based on its fundamentals. We assume throughout our analysis that the fundamental evolution of an industry index is given by the CAPM. Our analysis can be easily adjusted for other models for the fundamental evolution such as the Arbitrage Pricing Theory of Ross (1976). As a robustness check we consider the three-factor model of Fama and French (1993).

The investor examines past price patterns by means of a regression. To capture the conditional nature of the investor's examination we use a rolling regression frame-

[5]The data can be downloaded from `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`. We have used the data set constructed with the new specifications.

work, in which the investor uses 120 months of history. If the industry index evolves according to its fundamental, i.e. its returns have a linear relation with the market returns, a regression will produce residuals that do not systematically deviate from zero. If the industry index exhibits a bubble, the residuals will be systematically larger than zero during the bubble period. However, the investor does not a priori know the length of the bubble, and can look for bubbles of different lengths. Therefore, we distinguish between two windows in the regression framework: an estimation window which is constant and equal to 120 months, and a candidate window of variable length for the presence of a bubble, within the estimation window. We assume that the investor estimates the following regression over the estimation window from month $t-119$ to month t

$$r_{i\tau} - r_{f\tau} = \alpha_{i0t}(1 - D_{i\tau}) + \alpha_{i1t}D_{i\tau} - \beta_{it}(r_{m\tau} - r_{f\tau}) + \epsilon_{i\tau}, \qquad (2.1)$$

where $r_{i\tau}$ is the time τ log return on industry i, $r_{m\tau}$ represents the return on the market index, $r_{f\tau}$ is the risk-free rate, $D_{i\tau}$ is a dummy variable that equals one during the candidate period and zero otherwise, and α_{i0t}, α_{i1t}, β_{it} are coefficients. We interpret α_{i1t} as the growth rate of the bubble.[6]

The question is whether a bubble was present in the candidate window. The investor perceives a bubble during the candidate window, if the t-ratio of the α_{i1t}-coefficient exceeds a specified threshold. The candidate window is not fixed, and instead the investor estimates this regression for each admissible candidate window. We set the minimum and maximum of the candidate window equal to 12 and 60 months respectively. Moreover, the candidate window should end at most 6 months before the current month t. If several candidate periods qualify as a bubble, the period that ends closest to the end of the estimation period is selected as the perceived bubble. If two candidate periods qualifying as a bubble end equally close to the end of the estimation period, the longest period is selected.[7]

For each industry, Eq. (2.1) is estimated as a rolling regression with a window of 120 months over our complete sample period 1926-2004. Since the information set changes if we move from month t (with a prediction for month $t+1$) to month

[6] We do not follow the cointegration approach of Sinha and Sun (2004), since the long series needed for cointegration tests would not fit in with the rolling regression framework.

[7] This setup for selecting a candidate window as a bubble looks like conducting a series of t-tests on different α_{i1t} estimates. We commonly use a threshold equal to 2.358, which corresponds with a cumulative probability of 0.99 for a Student's t distribution with 120 degrees of freedom. However, because of the selection rules, the coefficient of α_{i1t} is not tested for statistical significance. The test on the α_{i1t}-coefficient is conditional on the outcome of the prior tests, implying that the test-statistic does not have a standard Student's t distribution. We thank Marno Verbeek for pointing out this issue.

$t+1$ (with a prediction for month $t+2$), the perception of a bubble changes as well. The investor updates his perception of a bubble based on the information that he obtains during month $t+1$. Because the inferred growth rate can change from time to time, the level of feedback trading can vary as well. Consequently, a bubble can be inferred to have stopped growing. Later, the investor can infer that the bubble has resumed its growth, that it burst with a crash, or that it was gradually deflating. In all cases, knowledge about a bubble can influence the investor's predictions, which is why we allow candidate periods ending up to 6 months before the current month. With hindsight, we can try to make precise statements on the actual period that the bubble was present. However, we do not follow this approach to avoid look-ahead bias.

Our setup can incorporate the asymmetric information that is crucial for the existence of a bubble. Investors that put different restrictions on the candidate window, use other fundamentals or a different threshold for the t-ratio of α_{i1t} will come to other conclusions regarding the presence of a bubble. Moreover, investors will generally not know how other investors conduct their analyses.

Table 2.1 presents the number of months for which we find evidence of a bubble in industries. We count each month that shows up at least once in a candidate window that is selected as a bubble. For each industry, we find on average 217 months during which a bubble occurs. These 217 months represent about 23.9% of our sample. The industry experiencing the fewest bubbles is Fabricated Products. However, for this industry we only have a limited time-series of observations. The industry experiencing the fewest bubbles months for which we have a complete time-series of observations is Laboratory Equipment. The Beer industry experiences most bubble months. Table 2.1 also shows the raw returns as well as the risk-adjusted returns during the bubble months. Generally, we find that both returns are on average positive. The only exception is the Fabricated Product industry which has a negative average raw return. We find the largest positive return, raw as well as risk-adjusted, for the Gold industry and the smallest risk-adjusted return for the Household Industry.

2.3.2 Crashes

Extraordinarily large negative returns are the distinguishing characteristic of a crash. We investigate crashes as a correction for the run up in prices during the bubble. Since a bubble is taken as a period of exceedance of fundamental values (i.e. according to the CAPM), we define a crash also with respect to fundamentals. In the CAPM

Table 2.1: Bubble months

industry	# months	$\bar{r}$	$\bar{\eta}$	industry	# months	$\bar{r}$	$\bar{\eta}$
Agric	74	30.2%	37.4%	Guns†	101	21.3%	20.7%
Food	395	18.4%	9.7%	Gold†	161	39.8%	38.1%
Soda†	240	32.0%	17.0%	Ships	245	24.7%	17.9%
Beer	485	21.9%	11.5%	Mines	182	24.1%	15.6%
Smoke	371	20.8%	15.4%	Coal	187	21.3%	14.7%
Toys	200	26.2%	18.6%	Oil	245	23.1%	13.4%
Fun	283	31.1%	17.2%	Util	232	18.6%	9.0%
Books	279	27.5%	13.0%	Telcm	308	20.7%	8.9%
Hshld	223	16.4%	7.6%	PerSv†	294	26.5%	17.8%
Clths	207	25.1%	15.9%	BusSv	255	25.6%	13.4%
Health†	159	36.2%	21.9%	Comps	162	32.1%	16.4%
MedEq	323	21.6%	14.6%	Chips	270	32.2%	14.3%
Drugs	413	20.4%	9.8%	LabEq	64	33.1%	20.5%
Chems	208	17.5%	10.0%	Paper†	96	3.9%	12.7%
Rubbr†	122	15.5%	12.5%	Boxes	280	20.6%	11.2%
Txtls	288	27.5%	13.0%	Trans	149	10.3%	10.9%
BldMt	190	13.2%	10.1%	Whshl	194	20.8%	12.1%
Cnstr	215	19.5%	14.9%	Rtail	265	18.1%	9.8%
Steel	134	18.7%	12.1%	Meals	334	22.0%	13.6%
FabPr†	16	-27.3%	38.6%	Banks	316	19.2%	10.9%
Mach	131	19.1%	9.2%	Insur	127	19.6%	12.0%
ElcEq	66	26.2%	12.0%	RlEst	186	18.8%	14.9%
Autos	145	27.2%	13.4%	Fin	258	19.6%	9.8%
Aero	191	23.2%	15.3%	Other	139	31.7%	14.5%
Pooled	10408	22.8%	13.8%				

For each industry we report the number of months that show up at least once in a bubble period in the regression in Eq. (2.1). For these months we calculate the average returns and the average abnormal returns. The abnormal return for sector i in month t is constructed as $r_{it} - r_{ft} - \hat{\beta}(r_{mt} - r_{ft})$, where $\hat{\beta}$ results from the regression over months $t-1$ to $t-120$. For each industry we report the average return $\bar{r}$ and average abnormal return $\bar{\eta}$ on an annual basis. Finally, we report the number of bubble months, the average return and average abnormal return for the pooled series. A dagger after an industry name indicates that less observations are available.

setting this means that we construct an abnormal return η_{it+1} as

$$\eta_{it+1} = r_{it+1} - r_{ft+1} - \hat{\beta}_{it}(r_{mt+1} - r_{ft+1}), \tag{2.2}$$

where the variables are defined as in Eq. (2.1) and $\hat{\beta}_{it}$ is the estimate for the CAPM-β based on the regression

$$r_{i\tau} - r_{f\tau} = \alpha_{i0t} + \beta_{it}(r_{m\tau} - r_{f\tau}) + \epsilon_{i\tau}, \quad E[\epsilon_{i\tau}] = 0,\ E[\epsilon^2_{i\tau}] = \sigma^2_{it}, \tag{2.3}$$

estimated over the previous 120 months of returns. This approach is a logical consequence of the conditional setup with the rolling regressions we proposed in the previous section. The investor tries to make a prediction of the next periods abnormal return η_{it+1} based on information up to time t. He uses the return series from $t - 119$ up to t to estimate the relation with the market, and to determine whether a bubble is present.[8] Moreover, this approach prohibits including one market crash 48 times.

Based on the abnormal return η_{it+1} we determine whether industry i is experiencing a crash during month $t + 1$. To accommodate time-varying volatilities and different volatilities across industries, we define a crash with respect to the volatility of abnormal returns.[9] Our five threshold levels are $-1.65\sigma_{it}$, $-2\sigma_{it}$, $-2.25\sigma_{it}$, $-2.5\sigma_{it}$ and $-3\sigma_{it}$. We consider an industry as experiencing a crash, if we observe a negative abnormal return at least $1.65\sigma_{it}$ away from zero. As we show below, this threshold level qualifies roughly 5% of the observations as a crash, which is reasonable. Throughout the analysis we will refer to this type as a category 0 crash. If we find a negative return below $2\sigma_{it}$ we call it a category 1 crash, and so on. The five different crash thresholds allow us to investigate whether crashes of different magnitudes have different characteristics and prior return patterns. Further, they ensure the robustness of our findings. A disadvantage of this approach might be that crashes are defined conditionally on past returns via the volatility estimate. We do not think this argument really bites, as a reference window of 10 years combined with slowly adaptation to changes may quite well capture the approach that investors have towards financial markets.

Crashes happening closely after each other are probably related. Therefore we count two consecutive months with large negative returns as one crash. Neither

[8]Since we do not want the abnormal return to depend on the selection of a bubble in Eq. (2.1), we do not include a candidate window in Eq. (2.3).

[9]For this reason we deviate from Longin and Solnik (2001), who use absolute thresholds in their analysis of extreme returns in different countries. We also deviate from Bae *et al.* (2003), who define extreme returns as one that lies below 5% of the return distribution, because this would determine crash probabilities a priori, while we want to estimate them at a later stage.

do we consider a pattern of a large negative return, a small positive return and again a large negative return as two separate crashes. Instead, borrowing from the earthquake terminology, we name the second large negative return an “aftershock”. More formally, all crashes that happen after another crash are called aftershocks, given that the industry has not fully recovered. We consider an industry as fully recovered when the sum of abnormal returns after the previous crash is positive. For all following aftershocks the same principle applies. A third crash is named an aftershock if the industry has not recovered from the first aftershock. If the industry has recovered, specifically the sum of abnormal returns since the first aftershock is larger than zero, we consider the third crash as a new crash. Aftershocks can happen up to twelve months after a preceding crash.

Table 2.2 provides summary statistics of the abnormal returns and the numbers of crashes for the 48 industries. In order to account for different volatilities across industries, we divide the abnormal returns by their respective standard deviation. Because of the rolling regression framework, the abnormal returns are constructed out-of-sample. Therefore we can use them to investigate the accuracy of the CAPM as the fundamental model. The abnormal returns center around zero. For six of the 48 sectors we find average abnormal returns that deviate significantly from zero. The pooled average abnormal return, which is an equal-weighted average of the industry averages, does not deviate significantly from zero. We conclude that the CAPM does not lead to systematic mispricing. The volatilities of the standardized abnormal returns are close to one. However, from a statistical perspective 29 out of 48 deviate significantly from one. It seems that the one-factor model in Eq. (2.3) underestimates the true volatilities. Since the deviations are economically small, we decide to continue the analysis with these estimates. Of the 48 industries, 22 are negatively skewed and 26 are positively skewed. The pooled series shows hardly any skewness. The kurtosis coefficients show that all industry return series exhibit fat tails. However, as our crash definition does not make assumptions regarding fat tails, this causes no problems for our analysis.

Table 2.2 also shows the number of crashes per industry for category 0. For the interested reader, Table 2.10 in the appendix provides information on the number of crashes per industry for the different crash categories. On average an industry encounters 45 crashes during the 70 years we consider. The Steel industry experiences with 58 crashes the most crashes, which can be related to its cyclical nature. The industry experiencing the fewest crashes is the Fabricated Products industry, which has a limited return series starting in July 1961. The industry experiencing the smallest number of crashes for which we have a complete return series is Beer. This finding is intuitively appealing because the beer industry is operating in a relatively

Table 2.2: Summary statistics of abnormal returns per industry

industry	# obs	mean	vol.	skew	kurt	min	max	# first crashes	# after-shocks
Agric	822	−0.02	1.03	0.02	4.93	−5.08	4.41	22	19
Food	822	0.09*	1.12*	0.32*	5.14	−3.96	6.10	28	12
Soda†	702	0.03	1.10*	0.31*	4.10	−3.95	4.76	23	18
Beer	822	0.09*	1.06*	−0.20*	6.66	−6.08	6.04	18	13
Smoke	822	0.09*	1.11*	−0.28*	4.13	−5.32	3.41	31	11
Toys	822	−0.04	1.01	−0.54*	8.12	−6.80	3.84	14	19
Fun	822	−0.01	1.01	0.30*	5.12	−4.88	5.16	26	18
Books	822	−0.02	1.00	−0.18*	3.93	−4.58	3.08	29	16
Hshld	822	0.05	1.05*	−0.36*	5.98	−6.40	4.72	25	15
Clths	822	−0.05	1.07*	−0.21*	3.97	−4.67	3.53	20	31
Health†	402	−0.05	1.07*	−0.60*	5.49	−5.45	3.13	15	13
MedEq	822	0.01	0.98	−0.05	3.84	−4.58	3.89	24	20
Drugs	822	0.07	1.06*	−0.22*	5.43	−5.56	4.29	20	18
Chems	822	−0.04	1.07*	0.06	5.67	−4.69	6.35	27	24
Rubbr†	774	−0.01	1.04	−0.28*	5.11	−5.16	4.53	23	20
Txtls	822	−0.05	1.07*	−0.24*	4.33	−4.50	4.78	24	33
BldMt	822	−0.06	1.07*	−0.33*	4.79	−5.17	3.66	22	28
Cnstr	822	−0.06	1.01	0.02	3.99	−4.01	4.29	22	25
Steel	822	−0.08*	1.07*	0.44*	5.25	−3.21	6.77	25	33
FabPr†	402	−0.16*	1.09*	−0.24	5.07	−4.77	4.78	11	16
Mach	822	−0.06	1.05*	0.15	4.16	−4.19	5.17	30	21
ElcEq	822	−0.04	1.03	−0.05	4.74	−4.77	4.98	26	19
Autos	822	−0.03	1.05*	0.07	6.02	−5.59	5.86	26	18
Aero	822	−0.03	1.03	−0.17*	5.38	−6.22	3.69	24	27
Guns†	522	−0.01	1.07*	−0.89*	9.67	−8.00	4.46	15	14
Gold†	522	0.02	1.15*	0.32*	4.36	−3.55	5.92	24	12
Ships	822	−0.04	1.08*	0.04	6.18	−5.58	5.10	25	18
Mines	822	0.00	1.05*	0.38*	4.25	−3.71	4.62	26	15
Coal	822	0.02	1.10*	0.51*	6.90	−4.86	6.63	21	18
Oil	822	0.04	1.04*	0.12	3.84	−3.78	3.69	20	22
Util	822	0.03	1.06*	0.02	3.83	−3.81	4.03	27	30
Telcm	822	0.07	1.10*	0.69*	7.85	−3.92	8.45	24	20
PerSv†	810	−0.02	1.03	−0.85*	10.01	−8.74	3.92	21	20
BusSv	822	0.03	0.98	0.34*	5.43	−4.76	4.46	26	13
Comps	822	−0.02	1.07*	−0.03	4.05	−4.49	4.07	27	27
Chips	822	0.00	1.03	0.07	4.04	−3.89	4.65	20	21
LabEq	822	−0.05	1.06*	0.20*	5.25	−4.27	6.12	29	14
Paper†	786	−0.04	0.99	0.24*	5.94	−4.71	5.98	27	13
Boxes	822	0.00	1.08*	0.05	6.17	−4.39	6.82	23	26
Trans	822	−0.05	1.03	0.37*	4.45	−3.65	4.72	24	19
Whshl	822	−0.06	1.02	0.06	7.42	−6.60	5.83	22	18
Rtail	822	0.02	1.06*	−0.26*	4.03	−3.83	4.13	26	30
Meals	822	0.03	1.04	0.05	4.57	−5.07	3.95	22	16
Banks	822	0.03	1.00	−0.40*	5.39	−5.08	4.03	17	18
Insur	822	0.01	1.03	−0.19*	5.46	−6.05	4.85	27	18
RlEst	822	−0.09*	1.07*	0.11	6.39	−6.76	4.68	22	26
Fin	822	−0.01	1.04*	−0.37*	7.58	−7.91	4.25	21	24
Other	822	0.01	1.02	0.24*	4.05	−3.13	4.83	25	14
Pooled	37800	−0.01	1.05*	−0.01	5.38	−8.74	8.45	1116	953

This table report summary statistics of the abnormal returns constructed with the factor model in Eq. (2.3) with a 120-month estimation window. Each abnormal return η_{it+1} is divided by the corresponding volatility estimate σ_{it} to correct for time-varying volatility. We report the number of observations, mean, standard deviation, skewness, kurtosis, minimum and maximum per industry and for the pooled set of adjusted abnormal returns. We also include the number of crashes based on a threshold of $-1.65\sigma_{it}$, split up in first crashes and aftershocks. A dagger after an industry name indicates that less observations are available. An asterisk denotes a significant difference from zero in case of means and skewness coefficients, and a significant difference from one in case of volatility, all at a 5% confidence level. Standard errors of the skewness coefficient are calculated as $\sqrt{6/T}$ (see Tabachnick and Fidell, 2001).

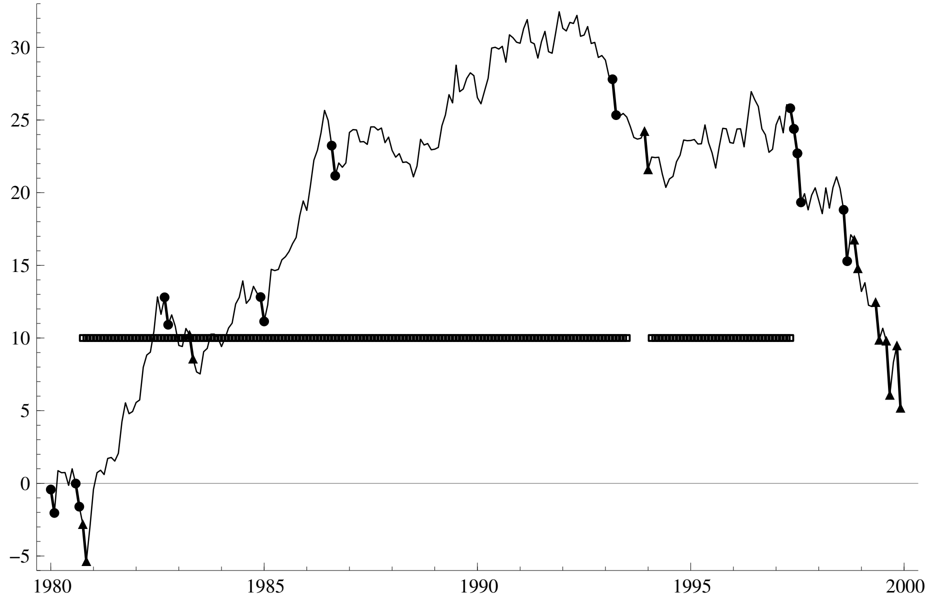
stable business environment. However, in the previous section we found that the Beer industry had most bubble months, which suggests that the presence of a bubble diminishes the crash probability. Looking at the beer industry in more detail in Figure 2.1, we see that the bubble months can be attributed to only four long lasting bubble periods, which are interrupted by crashes. However, these crashes are not large enough to end the bubbles and therefore the bubbles continue beyond the crashes. This pattern also illustrates the temporary strengthening after a coordinated attack, as put forward by Abreu and Brunnermeier (2003).

Figure 2.2(a) shows the distribution of crashes over the course of the years. We find that October is the most dangerous month. For our broadest definition of crashes, category 0, we find 145 crashes in October compared to an overall average of 93 crashes per month. For other threshold values, October remains the most dangerous month, with the exception of the worst category. In that case September shows 24 crashes and October 22. It is less obvious during which months investors are safest. Overall the fewest crashes (category 0) happen in February. However, this result seems to be driven by the few extra crashes, moving from category 2 to categories 1 and 0. For instance, we see the smallest number of the most severe crashes (category 4) in May.

Figure 2.2(b) shows that the number of crashes varies considerably over the years. We count most crashes in 1980 (34) and 1950 (33). During the Second World War few crashes occurred which may be related to the adjustments in the economy caused by it. Also 1977 and 2003 stand out as relatively safe years. Overall, it seems that the number of crashes per year increases slightly until 1965, after which the average number of crashes remains stable. Until 1965, the average number of crashes is 12.7. The average over 1966-2004 is 18.8. The variation in the number of crashes over the years is in line with the variation in dispersion as reported by Solnik and Roulet (2000).

We define industry crashes in a similar way as Longin and Solnik (2001) and Bae *et al.* (2003). Our approach can also be understood in terms of the exchange

Figure 2.1: Cumulative abnormal returns, bubbles and crashes in the Beer sector over 1980 - 2000

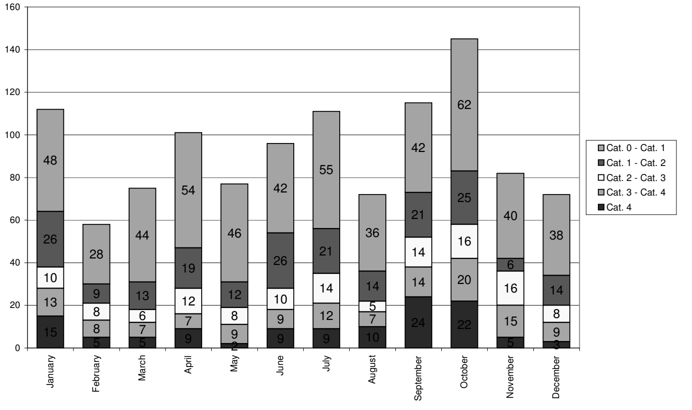


This figures shows the evolution of the cumulative abnormal returns in the Beer sector over the period 1980-2000 (y-axis in %). The bubble months during this period are indicated by the horizontal band in the graph. Crashes are indicated by the thick line segments in the cumulative abnormal returns. Circles mark the beginning and end of first crashes. Triangles indicate the beginning and end of aftershocks.

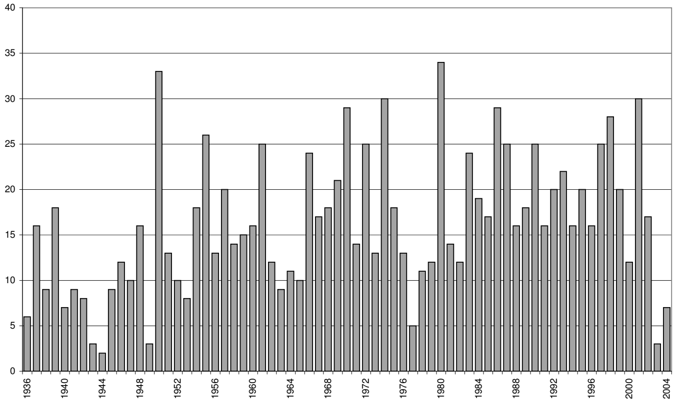
market pressure indexes that are used to define currency crises (see Eichengreen *et al.*, 1996; Kaminsky *et al.*, 1998).[10] We refrain from using skewness as a measure for the likelihood of a crash (see e.g. Chen *et al.*, 2001; Bates, 2000; Bakshi and Madan, 1999). First, skewness is an imperfect measure of crash likelihood as it does not focus exclusively on a pre-specified part of the return distribution that is of interest for crashes. In Table 2.2 we observe that the Beer sector, which has the fewest crashes, is left-skewed, while the Steel industry, encountering most crashes, is actually right-skewed. In the appendix we show that the skewness coefficient is not strongly related to the number of crashes in an industry. Second, investors cannot use findings based on a skewness measure in their assessment of risk. For instance, it is not clear how to interpret changes in skewness: does a change in skewness from

[10]The exchange market pressure indexes weigh different sources of pressure, with weights given by the standard deviation of the variables representing the sources. As a consequence, the exchange market pressure is unit free and comparable across currencies. We realize the same for the industries.

Figure 2.2: Crash distribution over the course of the year



(a) per month



(b) per year

We plot the number of first crashes per month in panel (a) and the number of first crashes of category zero per year in panel (b). In panel (a) we split up the set of category zero crashes to indicate the effect of the different thresholds. Starting with the most severe category, we show how many crashes are added if the threshold is increased with one step.

-0.10 to -0.20 imply a doubling of the crash likelihood? In contrast to previous studies we make direct inferences of crash probabilities depending on past returns. Only in this way, investors can use our findings in their risk management decisions.

2.4 Bubbles and Crashes

In this section we bring together our concepts of bubbles and crashes to see how they are related. In the first subsection, we use a simple technique to show that the presence of a bubble increases the likelihood of a crash. In the second subsection, we use a logit model to test the hypotheses based on the model of Abreu and Brunnermeier (2003).

2.4.1 Crash probability

To analyze the relation between bubbles and crashes we first look at the percentage of crashes that are preceded by bubbles. Second, we test the hypothesis that a bubble increases the likelihood of a crash. Table 2.3 shows that about 25.8% of all crashes are preceded by bubbles. The percentage of crashes preceded by a bubble increases monotonically with the strength of the crash up to 35.6% for category 4 crashes. We differentiate between a bubble that grew until the month before the crash (BUBBLE) and a bubble that stopped growing two to six months before the crash (LBUBBLE). Our evidence shows that crashes usually happen if the bubble is still growing (19.5%), compared to only 6.3% of crashes happening in the following two to six months after the bubble stopped growing. The last columns of Table 2.3 show that aftershocks are less frequently preceded by bubbles. Only 2.4% of all aftershocks are preceded by bubbles. This result is not surprising since aftershocks are preceded by definition by crashes. However, it also indicates that, as pointed out by Abreu and Brunnermeier (2003), not all bubbles end with a single one-time crash. If the first crash, or in the words of Abreu and Brunnermeier "coordinated attack", is not strong enough, the bubble can continue and even get stronger. This pattern repeats itself until there is a final "coordinated attack" that results in a crash strong enough to end the bubble. As in the case of first crashes, the percentage of crashes preceded by a bubble increases the more severe the aftershock is.

In Table 2.4, panel (a), we compare the unconditional probability of a crash in the next period to the conditional probabilities given that either a bubble has been perceived or not in the current period. The unconditional probability of a crash in category 0 is estimated at 4.7%. So, independent of the occurrence of a bubble, a crash happens on average every 21 months. Conditional on the absence of a bubble

Table 2.3: Percentage of crashes preceded by a bubble

	first crashes				aftershocks			
	all	BUBBLE	LBUBBLE	total	all	BUBBLE	LBUBBLE	total
cat. 0	1116	19.5%	6.3%	25.8%	953	2.4%	5.6%	8.0%
cat. 1	581	23.9%	6.7%	30.6%	561	3.4%	6.5%	10.0%
cat. 2	375	26.9%	5.9%	32.8%	416	4.0%	7.7%	11.7%
cat. 3	248	29.0%	6.0%	35.1%	305	4.4%	7.7%	12.1%
cat. 4	118	28.8%	6.8%	35.6%	188	6.8%	11.0%	17.8%

We distinguish a bubble that inflated up to the last observation (column labeled BUBBLE) and a bubble that stopped growing two to six months earlier (column labeled LBUBBLE). We report the percentages for the different crash categories.

over the last six months, the probability that a crash occurs decreases to 4.2%. Given that there was a bubble during the last month, the probability of a crash rises significantly to 7.7% (a relative increase of 80%). If the bubble stopped growing two to six months ago, the probability of a crash increases significantly from 4.2% to 5.6%.

In line with the results presented in Table 2.3, we find that the presence of a bubble has a stronger effect for more severe crashes. For category 4 crashes, the unconditional probability of a crash is 0.5%. Independent of the presence of a bubble, such a severe crash happens approximately once per 17 years. Conditional on the absence of a bubble the probability of a category 4 crash diminishes marginally to 0.4% (once every 21 years). However, given that there was a bubble during the last month the probability of a category 4 crash triples to once every 7 years. Our results demonstrate that the occurrence of a bubble has an economically large and statistically significant impact on the likelihood of a crash. The effect is more pronounced for bubbles that are still inflating than for bubbles that have stopped growing.

We also analyze the relation between bubbles and aftershocks, since a first crash does not necessarily need to end the bubble. Table 2.4 panel (b) shows that the probability of an aftershock independent of the presence of a bubble is large (8.7%). However, it cannot be compared directly with the unconditional probability of a first crash. The probability of an aftershock is not a completely unconditional probability, since aftershocks can only happen after a first crash. Our estimates demonstrate that in line with Abreu and Brunnermeier (2003) a crash is not a one-time event, but that several "coordinated attacks" can follow a bubble. However, independent of the strength of the aftershocks, we find no relation between aftershocks and prior

Table 2.4: Unconditional and conditional probabilities of a crash.

(a) first crashes

	cat. 0	cat. 1	cat. 2	cat. 3	cat. 4
unconditional on a bubble					
p	4.7**	2.4**	1.6**	1.0**	0.5**
	0.1	0.1	0.1	0.1	0.0
conditional on a bubble					
p_{NO}	4.2**	2.0**	1.3**	0.8**	0.4**
	0.1	0.1	0.1	0.1	0.0
$\Delta p_{\mathsf{BUBBLE}}$	3.5**	2.9**	2.3**	1.7**	0.8**
	0.5	0.4	0.4	0.3	0.2
	[84%]	[141%]	[180%]	[213%]	[213%]
p_{BUBBLE}	7.7	4.9	3.6	2.5	1.2
$\Delta p_{\mathsf{LBUBBLE}}$	1.4*	1.1*	0.5	0.4	0.3
	0.7	0.5	0.4	0.3	0.2
	[34%]	[54%]	[39%]	[48%]	[67%]
p_{LBUBBLE}	5.6	3.1	1.8	1.2	0.6

(b) Aftershocks

	cat. 0	cat. 1	cat. 2	cat. 3	cat. 4
unconditional on a bubble					
p	8.6**	5.0**	3.7**	2.7**	1.7**
	(0.27)	(0.21)	(0.18)	(0.15)	(0.12)
conditional on a bubble					
p_{NO}	8.7**	5.1**	3.7**	2.8**	1.7**
	(0.28)	(0.22)	(0.19)	(0.16)	(0.13)
$\Delta p_{\mathsf{BUBBLE}}$	-1.6	0.2	0.2	0.1	0.4
	(1.34)	(1.16)	(1.01)	(0.87)	(0.74)
	[-19%]	[4%]	[5%]	[4%]	[25%]
p_{BUBBLE}	7.1	5.2	3.9	2.9	2.1
$\Delta p_{\mathsf{LBUBBLE}}$	-0.8	-0.2	-0.0	-0.3	-0.0
	(1.00)	(0.80)	(0.70)	(0.57)	(0.47)
	[-9%]	[-4%]	[-1%]	[-12%]	[-1%]
p_{LBUBBLE}	7.9	4.8	3.7	2.4	1.7

We calculate probabilities for the occurrence of a crash in the next period for first crashes (panel a) and aftershocks (panel b). p gives the estimate, unconditional on the presence of a bubble. p_{NO} gives the probability of a crash conditional on the absence of a bubble. $\Delta p_{\mathsf{BUBBLE}}$ is the estimated effect of the presence of bubble that has been inflating up to the last observation, and p_{BUBBLE} gives the resulting probability of a crash, given that a bubble is present. The same applies to $\Delta p_{\mathsf{LBUBBLE}}$ for bubbles that stopped growing in the last two to six months. As a dependent variable we consider crashes of the different categories. For each model we report the estimates (in %), the standard errors in parentheses and the relative size of $\Delta p_{\mathsf{BUBBLE}}$ and $\Delta p_{\mathsf{LBUBBLE}}$ to p_{NO} in brackets. A single (double) asterisk indicates significance at the 5% (1%) level.

bubbles. This results may be due to the fact that relatively few aftershocks are preceded by prior bubbles as shown in Table 2.3.

2.4.2 Bubble characteristics and crash likelihood

In Section 2.2 we make several predictions regarding the characteristics of a bubble and the likelihood of a subsequent crash, based on the theoretical model of Abreu and Brunnermeier (2003). First, we hypothesize that a stronger growth rate of the bubble increases the likelihood of a crash. Second, if a bubble is difficult to date, the length of the bubble should be unrelated to the probability of a crash. In this section we analyze these hypotheses empirically.

We use two measures for the strength of the bubble. The first measure, labeled STRENGTH1, is simply the t-ratio of α_{i1t} in Eq. (2.1) for the candidate window that is selected as a bubble. It indicates the value with which the bubble exceeds the threshold. However, since our bubble search procedure is based on maximizing the length of the bubble period and not the strength, we also slightly modify our search procedure to find the strongest bubble during each estimation period. Instead of choosing the longest bubble during each estimation period, we select the bubble with the maximum t-ratio given that it fulfills the minimum and maximum length requirements. This procedure leads to our second measure of the strength of a bubble called STRENGTH2. For each of the two measures of bubble strength we also compute the corresponding length of the bubble, called LENGTH1 and LENGTH2, respectively.

The average value for STRENGTH1 is 2.57. As expected, the average t-statistic of STRENGTH2, 3.10, is larger than STRENGTH1. LENGTH1 equals on average 39 months, while the average of LENGTH2 is shorter with 29 months. The respective standard deviations are 0.34, 0.69, 17 and 15. We use these at a later stage to determine the economic significance of the variables with respect to crash likelihood.

We analyze how the different bubble characteristics affect the probability of a crash in a standard logit model:

$$\begin{aligned} Pr[\eta_{it+1} \leq -1.65\sigma_{it}|\mathbf{x}_{it}] &= F(a + \mathbf{b}'\mathbf{x}_{it}) \\ F(y) &= \frac{\mathrm{e}^y}{1+\mathrm{e}^y}, \end{aligned} \tag{2.4}$$

where η_{it+1} is the abnormal return of sector i at time $t+1$, $-1.65\sigma_{it}$ reflects the category zero threshold and $\mathbf{x}_{it}$ is a vector of explanatory variables. As explanatory variables we include a dummy for the presence of a bubble during the previous six months (BUBBLE$'$), a dummy if there was a bubble during the last two to six months (LBUBBLE), the number of months between the crash and the last bubble (LAGS) as well as the bubble characteristics that we discussed earlier.

The results of this analysis are presented in Table 2.5. In panel (a) we focus on the bubble characteristics STRENGTH1 and LENGTH1. For crash category 2 to 5 our results confirm that the strength of the bubble affects the likelihood of a crash. We calculate that an increase in STRENGTH1 by one standard deviation (from 2.57 to 2.91) causes a relative increase in the likelihood of a crash by 28%.[11] For more severe crashes the effect of strength becomes larger. For category 5 crashes, we find that for every increase of one standard deviation in the t-statistic the probability of a crash increases by 63%. If the volatility remains at a similar level of say 5% per month, and the bubble has an average length of 39 months, this means that the bubble yields an extra return of $0.34 \cdot \sqrt{39} \cdot 5\% = 10\%$ over its life span. In none of the regressions do we find a significant relation between the length of the bubble and the probability of a crash.

In Table 2.5, panel b, we investigate the relation between crash likelihood and STRENGTH2 and LENGTH2. We find again that the strength of a bubble is significantly related to the likelihood of a crash, whereas the length is not. For the other explanatory variables such as BUBBLE′ and LAGS, the results are also similar to our prior findings. In panel c, we report the estimates for the model with STRENGTH1 and STRENGTH2 included. The results confirm the findings above, and indicate that STRENGTH2 has more explanatory power than STRENGTH1.

The logit model enables us to investigate whether the influence of a bubble becomes less and less if it stopped growing longer ago, or that it only matters whether a bubble is still inflating or not. If the influence becomes gradually less, the coefficient for the LAGS variables should be significant. If there is mainly a difference between still inflating and stopped growing, the coefficient on LBUBBLE should be significant. Since the coefficient on LBUBBLE is not significant in any of the settings, while the coefficient on LAGS is in most, we interpret this as evidence supporting a gradually decreasing influence of a bubble.

Our results in this section support the hypothesis that the strength of the bubble increases the likelihood of a crash. We also find evidence that a bubble is difficult to date. Finally we find that the effect of a bubble diminishes gradually after it stopped growing. These conclusions are robust to a different construction of the variables.

[11] We base this calculation on the first order approximation of the logistic model in Eq. (2.4). For STRENGTH1 and category one crashes we find an increase of 0.56%, which is a relative increase of 28% compared to the situation when a bubble is absent (which has a probability 2.0% according to Table 2.4).

Table 2.5: Logit models for the probability that a first crash occurs

	Cat. 0		Cat. 1		Cat. 2		Cat. 3		Cat. 4	
model a										
constant	-3.13^{**}	(0.03)	-3.88^{**}	(0.04)	-4.35^{**}	(0.05)	-4.81^{**}	(0.06)	-5.56^{**}	(0.09)
BUBBLE'	0.62^{**}	(0.07)	0.84^{**}	(0.09)	0.99^{**}	(0.11)	1.09^{**}	(0.13)	1.01^{**}	(0.20)
LBUBBLE	−0.33	(0.21)	0.06	(0.26)	0.03	(0.33)	−0.05	(0.40)	0.73	(0.56)
LAGS	0.02	(0.06)	-0.18^{*}	(0.09)	-0.27^{*}	(0.12)	−0.26	(0.14)	-0.63^{*}	(0.26)
STRENGTH1	0.13	(0.17)	0.42^{*}	(0.17)	0.52^{**}	(0.18)	0.47^{*}	(0.21)	0.77^{**}	(0.26)
LENGTH1	0.001	(0.004)	−0.002	(0.005)	−0.002	(0.006)	0.000	(0.007)	−0.017	(0.010)
$\log L$	−4474.8		−2689.7		−1891.6		−1347.6		−723.73	
model b										
constant	-3.13^{**}	(0.03)	-3.88^{**}	(0.04)	-4.35^{**}	(0.05)	-4.81^{**}	(0.06)	-5.56^{**}	(0.09)
BUBBLE'	0.59^{**}	(0.07)	0.82^{**}	(0.09)	0.96^{**}	(0.11)	1.03^{**}	(0.14)	1.00^{**}	(0.20)
LBUBBLE	−0.31	(0.20)	0.05	(0.26)	0	(0.33)	−0.05	(0.40)	0.71	(0.56)
LAGS	0.04	(0.06)	−0.16	(0.09)	-0.24^{*}	(0.12)	−0.23	(0.14)	-0.60^{*}	(0.26)
STRENGTH2	0.22^{**}	(0.08)	0.29^{**}	(0.09)	0.35^{**}	(0.11)	0.41^{**}	(0.12)	0.39^{*}	(0.17)
LENGTH1	−0.007	(0.004)	−0.005	(0.005)	−0.007	(0.007)	−0.005	(0.008)	−0.010	(0.011)
$\log L$	−4470.4		−2687.6		−1889.6		−1344.4		−724.84	
model c										
constant	-3.13^{**}	(0.03)	-3.88^{**}	(0.04)	-4.35^{**}	(0.05)	-4.81^{**}	(0.06)	-5.56^{**}	(0.09)
BUBBLE'	0.58^{**}	(0.07)	0.85^{**}	(0.09)	0.97^{**}	(0.11)	1.04^{**}	(0.13)	1.08^{**}	(0.18)
LAGS	−0.06	(0.03)	-0.15^{**}	(0.05)	-0.24^{**}	(0.07)	-0.24^{**}	(0.08)	-0.29^{*}	(0.12)
STRENGTH1	−0.14	(0.19)	0.10	(0.21)	0.17	(0.23)	−0.01	(0.26)	0.24	(0.36)
STRENGTH2	0.29^{**}	(0.10)	0.27^{*}	(0.12)	0.30^{*}	(0.14)	0.44^{**}	(0.15)	0.30	(0.23)
$\log L$	−4474.7		−2690.9		−1890.3		−1345.3		−726.92	

We estimate the probability that a first crash occurs using standard logit models with different bubble characteristics as explanatory variables. We include a constant and a dummy for a bubble that may end at most six months before the last observation (BUBBLE′). LBUBBLE equals one if the bubble does not include the last observation. LAGS counts the number of months since the bubble has ended. Further, we consider the t-statistic of the bubble (STRENGTH1), its length (LENGTH1), the maximum t-statistic over a subperiod within the bubble (STRENGTH2) and its length (LENGTH2). All characteristics have been demeaned. As a dependent variable we consider the number of first crashes in the five categories. We report the estimates with standard errors in parentheses, and the value of the log likelihood function. A single (double) asterisk after an estimate indicates significance at the 5% (1%) confidence level.

Table 2.6: Distribution of market crashes and crashes for the average industry

	market crashes						industry crashes					
	all crashes		first crashes		aftershocks		all crashes		first crashes		aftershocks	
cat. 0	41		22		19		45		24		21	
cat. 1	24	59%	8	36%	16	84%	25	55%	13	52%	12	59%
cat. 2	20	49%	7	32%	13	68%	17	38%	8	34%	9	44%
cat. 3	18	44%	7	32%	11	58%	12	27%	5	22%	7	32%
cat. 4	10	24%	4	18%	6	32%	7	15%	3	11%	4	20%

Market crashes are defined in similar way as for sectors. Abnormal returns are constructed by subtracting the long run average market return from the observed returns. The average industry crashes are constructed by dividing the pooled crashes (see Tables 2.2 and 2.10) by the total number of observations (37,800) and multiplying it by the number of market observations (822). We consider all crashes, first crashes and aftershocks.

2.5 Bubbles and crashes in the market

We investigate whether our results for industries are generalizable to the market as a whole. In this case, we obviously cannot define crashes and bubbles relative to the market. Instead, we use the long run equity premium as a benchmark, and compute the abnormal returns as deviations from it. For simplicity, we assume that this premium is constant over time. We estimate it as the long run average excess return of the CRSP all share index, which equals 5.98% per annum over the period July 1926 - December 2004.

Table 2.6 compares the number and distribution of market crashes to the average industry crashes. The market experiences 41 crashes compared to 45 crashes per industry. In both cases about half of the crashes are first crashes. The distribution of crashes over the categories also shows similarities. Market crashes tend to be more extreme, since category three and four contain more market crashes than average industry crashes. In particular market aftershocks seem to be more severe. Overall, the descriptive statistics for market and industry crashes look very similar.

More interesting is the question whether we also find similar results regarding the bubble-crash patterns. Table 2.7 shows that 18% of market crashes (category 0) are preceded by a bubble over the last six months. About half of these bubbles are inflating until the month just before crash and the other half stops growing between two to six month before the crash. These results differ only slightly from our findings for industry crashes (see Table 2.3). For industry crashes, we find that almost 20% are preceded by an inflating bubble. For 6.3% of the industry observations, we find evidence of a bubble that stopped growing. Looking at the different crash categories,

Table 2.7: Percentage of market crashes preceded by a bubble

	first crashes				aftershocks			
	all	BUBBLE	LBUBBLE	total	all	BUBBLE	LBUBBLE	total
cat. 0	22	9%	9%	18%	19	0%	5%	5%
cat. 1	8	13%	13%	25%	16	0%	13%	13%
cat. 2	7	14%	14%	29%	13	0%	14%	14%
cat. 3	7	14%	14%	29%	11	0%	0%	0%
cat. 4	4	0%	25%	25%	6	0%	0%	0%

This table is similar to Table 2.3. We distinguish a bubble that inflated up to the last observation (column labeled BUBBLE) and a bubble that stopped growing two to six months earlier (column labeled LBUBBLE). We report the percentages for the different crash categories.

the picture is similar. It seems that industry crashes are only slightly more frequently preceded by bubbles than market crashes.

In line with our analysis for the industries, we also investigate the relation between market crashes and prior bubbles in a conditional probability framework. Unfortunately, the statistical power of our analysis is very limited due to the small number of market crash and bubble observations. In Table 2.8, we show that a preceding inflating bubble increases the likelihood of a crash from 3.7% to 6.9% (a relative increase of 87%). This increase makes the effect of the presence of a market bubble comparable to the effect of an industry bubble. If the bubble stopped growing two to six month ago, the probability increases further to 8.3%. In that sense a market bubble has an even stronger effect than an industry bubble. In line with our results for industries, it seems that the impact of a bubble on crash likelihood increases for more severe crashes. However, although the changes in crash likelihood are economically large, they are statistically insignificant due to a lack of sufficient observations. For crash category 4, we can actually not even estimate the conditionally probability given that there was an inflating bubble, since we have no observation at all available. Due to a lack of observations, we are also not able to estimate the logit model in Eq. (2.4) in order to analyze the relation between the bubble characteristics and the probability of a market crash.

2.6 Robustness checks

In Section 2.3 we propose a flexible approach towards bubbles and crashes. Consequently, we have made some arbitrary choices in the application of this approach. We have selected the CAPM as the fundamental model for an industry. We have fixed the length of the estimation window at 120 months, and put the maximum of

Table 2.8: Unconditional and conditional probabilities of a market crash

Panel (a) First crashes

	cat. 0	cat. 1	cat. 2	cat. 3	cat. 4
unconditional on a bubble					
p	4.1**	1.5**	1.3**	1.3**	0.7*
	(0.85)	(0.52)	(0.49)	(0.49)	(0.37)
conditional on a bubble					
p_{NO}	3.7**	1.2*	1.0*	1.0*	0.6
	(0.85)	(0.50)	(0.46)	(0.46)	(0.35)
$\Delta p_{\mathsf{BUBBLE}}$	3.2	2.2	2.4	2.4	NA
	(4.78)	(3.42)	(3.42)	(3.42)	
	[87%]	[180%]	[237%]	[237%]	
p_{BUBBLE}	6.9	3.4	3.4	3.4	
$\Delta p_{\mathsf{LBUBBLE}}$	4.6	2.9	3.1	3.1	3.6
	(5.71)	(4.11)	(4.10)	(4.10)	(4.09)
	[126%]	[239%]	[307%]	[307%]	[578%]
p_{LBUBBLE}	8.3	4.2	4.2	4.2	4.2

(b) Aftershocks

	cat. 0	cat. 1	cat. 2	cat. 3	cat. 4
unconditional on a bubble					
p	8.6**	7.2**	5.9**	5.0**	2.7*
	(1.88)	(1.74)	(1.58)	(1.46)	(1.09)
conditional on a bubble					
p_{NO}	8.5**	7.1**	5.7**	5.2**	2.8*
	(1.91)	(1.76)	(1.59)	(1.52)	(1.14)
$\Delta p_{\mathsf{BUBBLE}}$	NA	NA	NA	NA	NA
p_{BUBBLE}					
$\Delta p_{\mathsf{LBUBBLE}}$	1.5	2.9	4.3	NA	NA
	(9.68)	(9.65)	(9.62)		
	[18%]	[41%]	[77%]		
p_{LBUBBLE}	10.0	10.0	10.0		

This table is similar to Table 2.4. We calculate probabilities for the occurrence of a crash in the next period for first crashes (panel a) and aftershocks (panel b). p gives the estimate, unconditional on the presence of a bubble. p_{NO} gives the probability of a crash conditional on the absence of a bubble. $\Delta p_{\mathsf{BUBBLE}}$ is the estimated effect of the presence of bubble that has been inflating up to the last observation, and p_{BUBBLE} gives the resulting probability of a crash, given that a bubble is present. The same applies to $\Delta p_{\mathsf{LBUBBLE}}$ for bubbles that stopped growing in the last two to six months. As a dependent variable we consider crashes of the different categories. For each model we report the estimates (in %), the standard errors in parentheses and the relative size of $\Delta p_{\mathsf{BUBBLE}}$ and $\Delta p_{\mathsf{LBUBBLE}}$ to p_{NO} in brackets. A single (double) asterisk indicates significance at the 5% (1%) level.

the candidate window for a bubble at 60 months. To qualify as a bubble the t-ratio of the α_{i1t}-coefficient of a candidate window should exceed a threshold of 2.358. An aftershock can happen up to 12 months after a previous crash. In this section we briefly review the robustness checks that we conducted on these choices. The main results are presented in Table 2.9. In Appendix 2.B we provide more details.

In the first robustness check we consider the three-factor model of Fama and French (1993) as an alternative for the CAPM. This means that the expected return of an industry becomes a linear function of the market return, the return on the size hedge portfolio (SMB) and the value hedge portfolio (HML).[12] We add these portfolios to the regressions in Eqs. (2.1) and (2.3) and the construction of the abnormal returns in Eq. (2.2).

Using the Fama and French (1993) model does not lead to large differences. While it can be argued that the industry bubbles that we find under the CAPM approach are related to size or value effects, a correction for them does not affect the number of bubble months much. Table 2.9(a) shows that we find 9,312 bubble months using the Fama and French (1993) model, compared to 10,408 under the CAPM. In panels (b) and (c) we see a slight increase in the number of crashes from 2069 to 2228 (category 0) and from 306 to 342 (category 4). As a consequence, the estimated crash probabilities change only marginally. The effect of observing a bubble on crash likelihood is still large. For category zero crashes it increases from 4.7% to 7.9% (an inflating bubble) or 6.5% (a bubbled that stopped growing). In case of the CAPM we found an increase from 4.2% to 7.7% or 5.6%. For category four crashes (panel c) we also see large similarities. In Appendix 2.B.1 we show that also in case of the Fama and French (1993)-model, the strength of a bubble positively affects the probability of a crash. We conclude that our results are not substantially affected by using the Fama and French (1993)-model.

As a second robustness check, we replicate the analysis with a shorter estimation window of 60 months (instead of 120 months) and a maximum length for the candidate window for a bubble of 36 months (instead of 60 months).[13] The minimum length of a bubble remains 12 months. A shorter window may lead to estimates that quicker adapt to changes in the economic environment. However, in Appendix 2.B.2 we show that shorter windows do not lead to improved estimation results.

[12]For information on how these factor are constructed please refer to French's website (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html`) and Fama and French (1993).

[13]In this setting we ignore the first 60 observations in the data set, to keep it comparable with the data set of the basic model.

Table 2.9: Summary statistics of robustness checks

	basic	FF	short	threshold 1.98	aftershocks no	aftershocks 6 months	aftershocks 18 months
(a) Bubbles							
# bubble months	10,408	9,312	9,306	15,229	10,408	10,408	10,408
$\bar{r}$	22.8%	22.8%	26.3%	20.5%	22.8%	22.8%	22.8%
$\bar{\eta}$	13.8%	12.5%	12.6%	8.1%	13.8%	13.8%	13.8%
(b) Category 0 crashes							
# crashes	2069	2228	2171	2069	2069	2069	2069
# first crashes	1116	1169	1199	1116	2069	1584	904
p	4.7^{**}	5.1^{**}	5.2^{**}	4.7^{**}	5.9^{**}	5.3^{**}	4.5^{**}
p_{NO}	4.2^{**}	4.7^{**}	4.8^{**}	4.1^{**}	5.7^{**}	5.0^{**}	3.9^{**}
p_{BUBBLE}	7.7^{**}	7.9^{**}	8.0^{**}	6.7^{**}	7.6^{**}	7.7^{**}	7.8^{**}
p_{LBUBBLE}	5.6^{*}	6.5^{*}	6.6^{*}	5.6^{**}	6.5	5.4	5.7^{**}
# afterhocks	953	1059	972	953	0	485	1165
p	8.6^{**}	8.9^{**}	8.2^{**}	8.6^{**}	-	9.3^{**}	8.9^{**}
p_{NO}	8.7^{**}	8.9^{**}	8.3^{**}	8.8^{**}	-	9.5^{**}	8.9^{**}
p_{BUBBLE}	7.1	8.1	6.2	7.8	-	5.7^{*}	6.8
p_{LBUBBLE}	7.9	8.6	7.3	6.8	-	9.5	7.5
(c) Category 4 crashes							
# crashes	306	342	326	306	306	306	306
# first crashes	118	139	134	118	306	204	91
p	0.5^{**}	0.6^{**}	0.6^{**}	0.5^{**}	0.9^{**}	0.7^{**}	0.5^{**}
p_{NO}	0.4^{**}	0.5^{**}	0.5^{**}	0.4^{**}	0.8^{**}	0.6^{**}	0.3^{**}
p_{BUBBLE}	1.2^{**}	1.3^{**}	1.2^{**}	1.0^{**}	1.3^{**}	1.3^{**}	1.2^{**}
p_{LBUBBLE}	0.6	1.1	0.9	0.7	1.0	0.7	0.6
# afterhocks	188	203	192	188	0	102	215
p	1.7^{**}	1.7^{**}	1.6^{**}	1.7^{**}	-	2.0^{**}	1.4^{**}
p_{NO}	1.7^{**}	1.7^{**}	1.7^{**}	1.7^{**}	-	2.0^{**}	1.4^{**}
p_{BUBBLE}	2.1	0.8	0.6	1.9	-	1.4	2.0
p_{LBUBBLE}	1.7	1.5	0.9^{*}	1.5	-	1.8	1.6

This table presents summary statistics for the results of the different robustness checks. Each column corresponds with a certain model setting. We report the basic model, the Fama and French (1993) model for the fundamental values (FF), the CAPM model estimated over 60 months and a maximum length of 36 months for a bubble (short), a bubble threshold of 1.98, no partition in first crashes and aftershocks (no aftershocks), an aftershock window of 6 months and an aftershock window of 18 months. In panel (a) we report the number of bubble months, and the average return ($\bar{r}$) and average abnormal return ($\bar{\eta}$) of these bubble months. In panel (b) we report the unconditional crash probability (p), the crash probability conditional on the absence of a bubble p_{NO}, the crash probability conditional on the presence of a bubble inflating until the last observation p_{BUBBLE} and the crash probability conditional on the presence of a bubble that stopped growing in the last two to six months p_{LBUBBLE}, both for first crashes and aftershocks of category 0. In panel (c) we report the values of the same parameters for category 4 crashes. A single (double) asterisk behind p and p_{NO} indicates significance at 5% (1%) level. A single (double) asterisk behind p_{BUBBLE} and p_{LBUBBLE} indicates a significant difference from p_{NO} at 5% (1%) level.

Shorter windows for estimation and candidate windows have little influence on the results. In column 3 of Table 2.9 we observe fewer bubble months, which could be expected since we have used a stricter definition of a bubble. We find 102 crashes extra in category zero and 20 extra in category four, which are mainly first crashes. These differences lead to slightly higher parameter estimates. The probability of a crash, if a bubble has been inflating is estimated at 8.0% (was 7.7% in the basic model). If the bubble has stopped growing two to six months ago, the probability of a crash in the next month has increased to 6.6%. Since we count fewer aftershocks, aftershock probabilities decrease. For category four crashes we report a similar pattern. We conclude that the estimation window and maximum bubble length are not crucial for the conclusions drawn from the basic model.

In the third robustness check we lower the threshold for the t-ratio of the α_{i1t}-coefficient from 2.358 to 1.98. Consequently, it is easier for a candidate window to qualify as a bubble. In column 4 of Table 2.9 we see that this leads to a large increase in the number of bubble months, adding up to 15,229. As expected, the average return and in particular the average abnormal return decrease. However, the average abnormal return of 8.1% per annum is still considerable.

The effect of a bubble on the probability of a first crash in the next period has diminished. Since bubbles have generally decreased in strength in this setting, this is line with our earlier evidence on the strength of a bubble. However, the increases in crash likelihood remain significant. For aftershocks we do not see a clear pattern in the changes. Estimating the probability of a first crash as a function of the presence of a bubble with the 2.358 threshold as well as the presence of a bubble with the 1.98 threshold, indicates that effect of the presence of a bubble steams mainly from the stronger bubble. A significant effect for bubbles with t-stats on α_{i1t} between 1.98 and 2.358 is rejected.[14]

In the fourth robustness check we investigate the sensitivity of our results with regard to aftershocks. Throughout the analysis we distinguish between first crashes and aftershocks, where aftershocks can happen up to 12 months after a preceding crash. Given the differences that we observed between first crashes and aftershocks and their relation with bubbles, we investigate the sensitivity to this period of 12 months. We consider three settings, in which we (1) do not distinguish between first crashes and aftershocks, (2) impose a maximum of 6 months between the previous crash and an aftershock and (3) a maximum of 18 months in between. We keep the requirement that a sector should not be fully recovered.

Columns 5 to 7 in Table 2.9 indicate that the numbers of first crashes and aftershocks vary widely with the settings for aftershocks. The total number of 2069

[14]Results are available upon request.

crashes in category zero is of course unaffected, but we see a considerable increase in the number of aftershocks, moving from no aftershocks via a 6-month period, 12-month period to a 18-month period in which an aftershock can happen. The effects on the crash probabilities confirm our earlier conclusion that bubbles do not affect the probability of aftershocks. The estimated probability of a crash after perceiving a bubble remains stable between 7.6% and 7.8%. The probability of a crash if a bubble has not been observed varies between 3.9% (if aftershocks can happen up to 18 months after a previous crash) and 5.7% (no distinction between first crashes and aftershocks). This pattern is present in the aftershock probabilities as well. The probability of an aftershocks within 6 months is larger than an aftershock within 12 or 18 months. The patterns for category 4 crashes lead to similar results. We conclude that the probability of a crash can be predicted both by the presence of bubbles in the last 6 months and by crashes in the last 18 months. While both the presence of bubbles and crashes increase the likelihood of a crash in the next period, they do not reinforce each other. This indicates that most crashes are large enough to burst the bubble.

From the results on these robustness checks, we conclude that our flexible approach leads to reliable outcomes. In all settings we see that observing a bubble leads to an increase in the probability of a crash during the next period. While the magnitude varies, the effect is always significant and large enough to be taken into account. The effect is driven by bubbles that have a large t-ratio (exceeding 2.358) and disappears for less strong bubbles. These checks cover the main assumptions that we make.

2.7 Conclusion

Bubbles and crashes are among the most intriguing events in financial markets that have puzzled both academics and practitioners for decades. Much of the research on bubbles and crashes either aims at establishing conditions under which bubbles can exist in theory, or focuses on a specific bubble with the subsequent, mostly dramatic crash. In this chapter we have provided more general empirical evidence on the presence of bubble and crash patterns in US industries. Moreover, we have shown that the results based on industries carry over to the markets as a whole.

We have taken the viewpoint of an investor who tries to make a forecast on the probability of a crash in next month's return. He can use the series of prior returns to detect a bubble, which he perceives as long period of outperformance. A crash consists of one or more extraordinarily large negative returns. We distinguish several crash categories, based on the size of the crash. We have found that 25% of the broadest crashes and up to 38% of the severest crashes are preceded by a bubble. Knowing that a bubble inflated until the previous month significantly increases the estimates of the likelihood of a crash. Moreover, the economic impact is pronounced, as the probability of a crash of the broadest crash category almost doubles from 4.2% to 7.7%, conditional on the presence of a bubble. The likelihood of a more extreme crash increases even more; for the most severe crash category the presence of a bubble triples crash likelihood. The presence of a bubble that stopped growing two to six months ago produces a significant increase to 5.6%. The strength of a bubble increases the probability of a crash. We find no evidence that the length of the bubble has an impact on crash likelihood.

We conclude from this research that investors who try to ride a bubble face a severe risk of encountering a crash. The probability of such a crash is at least twice as large as normally. Moreover, bubbles that are easier to detect because they show stronger outperformance are more susceptible to crashes. We found widespread evidence for bubbles and crashes, as we found them in many industries. More importantly, we observe a similar pattern in the market, though we lack statistical power to formally test for significance. Given these similarities, we conclude that pooling industry data helps studying infrequent events.

We have chosen to work with a flexible investor perspective, necessitating some arbitrary choices, which may affects our findings. Therefore, we conduct extensive robustness checks. Instead of using the CAPM to compute abnormal returns, we replicate our analysis using the Fama and French (1993)-model. Further, we vary the length of the estimation window, aftershock window and the maximum length of the bubble. None of these changes has a substantial effect on our findings.

2.A Crashes, booms and skewness

Other studies (e.g. Chen *et al.*, 2001; Bates, 2000; Bakshi and Madan, 1999) have used skewness as a measure for crash likelihood. In order to compare our approach to the use of the skewness coefficient we introduce the concept of a boom. We consider booms to be the exact opposite of crashes. The general idea of our comparison is that a negative skewness coefficient should indicate that a crash is more likely than a boom and a more negative skewness coefficient should coincide with more crashes. If the skewness coefficient is not informative, we expect that its sign coincides in about 50% of the cases with the difference between booms and crashes. Table 2.10 compares the number of booms and crashes per industry to their skewness coefficients. For 25 out of the 48 industries, that is only 52% of the cases, the skewness coefficient correctly indicates whether crashes exceed booms or vice versa. Under the hypothesis that the skewness coefficient is not informative, an outcome of at least 25 correct predictions has a probability of 67%, based on a binomial distribution. We conduct a similar analysis for the other categories. Only for the fourth category we reject the hypothesis that skewness is uninformative (for 33 industries we find accordance in signs; under the null hypothesis of skewness being uninformative the p-value equals 0.002).

We also analyze the relation between the number of booms versus crashes and the skewness coefficient by the regression:

$$\mathrm{BMC}_i = a + b \cdot \mathrm{SKEW}_i + u_i, \tag{2.5}$$

where BMC_i is the difference between the number of booms and crashes for industry i, and SKEW_i is industry i's skewness coefficient. The first 3 columns of Table 2.11 show the results. For category 0, all crashes and booms, we find that the skewness coefficient has no explanatory power. The p-value of the skewness coefficient is 0.45 and the R^2 is about 1%. Since the weight that an observation receives in the skewness measure increases if it is more extreme, the explanatory power of skewness increases if we consider more extreme crashes and booms. However, for the most severe category, the R^2 is still low at 38%. Moreover the most severe category considers only 15% of the number of crashes that we think are important to investors, and we show in Section 2.4 that such a crash happens on average once every 17 years.

We also examine whether a more negative skewness coefficient coincides with more crashes:

$$\mathrm{CR}_i = a + b \cdot \mathrm{SKEW}_i + u_i, \tag{2.6}$$

where CR_i is the number of crashes divided by the number of observations. Table 2.11, columns 4-6 present the results. We find that the skewness coefficient

Table 2.10: Number of crashes and booms per category

category	crashes 0	1	2	3	4	booms 0	1	2	3	4	skewness
Agric	41	27	16	12	7	40	23	20	15	8	0.02
Food	40	21	16	13	8	53	36	26	23	15	0.32*
Soda†	41	23	14	9	6	47	30	18	16	11	0.31*
Beer	31	18	14	10	7	56	39	24	15	6	-0.20
Smoke	42	28	21	15	7	63	33	26	17	7	-0.28
Toys	33	22	16	13	8	35	25	16	10	6	-0.54
Fun	44	20	9	5	3	46	22	14	10	7	0.30*
Books	45	25	16	11	5	47	21	15	11	1	-0.18
Hshld	40	21	17	11	4	53	25	16	12	7	-0.36
Clths	51	31	22	16	10	46	24	15	11	5	-0.21*
Health†	28	15	10	7	6	17	12	8	4	2	-0.60*
MedEq	44	21	9	4	2	42	23	13	6	2	-0.05*
Drugs	38	26	19	12	8	50	29	23	18	7	-0.22
Chems	51	31	21	16	7	41	23	16	11	6	0.06
Rubbr†	43	28	21	15	8	30	22	15	7	4	-0.28*
Txtls	57	35	27	23	13	45	24	13	7	4	-0.24*
BldMt	50	29	20	15	9	36	19	15	10	5	-0.33*
Cnstr	47	27	21	15	6	38	23	12	11	6	0.02
Steel	58	29	24	16	6	41	28	18	15	7	0.44
FabPr†	27	20	15	10	8	18	8	6	3	1	-0.24*
Mach	51	27	18	12	6	44	26	17	13	5	0.15
ElcEq	45	25	16	14	8	48	18	13	11	3	-0.05
Autos	44	24	17	12	8	41	20	16	12	6	0.07
Aero	51	21	14	7	4	47	27	19	12	5	-0.17*
Guns†	29	13	6	4	4	26	9	7	4	2	-0.89*
Gold†	36	18	12	7	6	35	26	13	8	5	0.32
Ships	43	21	16	13	8	42	26	17	11	7	0.04
Mines	41	19	16	9	6	53	29	17	13	8	0.38*
Coal	39	26	19	11	6	48	25	22	20	14	0.51*
Oil	42	23	15	7	5	49	27	20	15	7	0.12*
Util	57	27	20	11	6	49	28	21	16	9	0.02
Telcm	44	24	15	7	4	49	30	19	16	10	0.69*
PerSv†	41	24	18	13	7	39	18	14	10	4	-0.85*
BusSv	39	15	8	7	3	41	25	20	15	8	0.34*
Comps	54	28	17	12	7	48	27	18	15	7	-0.03*
Chips	41	23	17	10	6	49	21	14	10	6	0.07*
LabEq	43	27	17	13	8	44	17	11	6	4	0.20*
Paper†	40	21	14	11	5	33	14	11	7	6	0.24
Boxes	49	30	22	16	9	43	28	16	13	5	0.05
Trans	43	20	10	7	4	42	24	17	15	8	0.37
Whshl	40	25	19	11	4	39	28	20	14	6	0.06
Rtail	56	28	26	21	10	50	26	16	12	3	-0.26*
Meals	38	22	15	10	6	43	26	19	16	9	0.05*
Banks	35	19	17	13	8	41	21	13	10	5	-0.40
Insur	45	21	14	12	9	46	21	18	13	5	-0.19
RlEst	48	29	18	10	4	43	25	17	12	9	0.11
Fin	45	22	13	10	5	44	27	21	20	5	-0.37*
Other	39	23	14	6	2	41	22	16	13	5	0.24*
Pooled	2069	1142	791	544	306	2061	1150	791	584	293	-0.01*

This table reports the number of crashes and the number of booms for the different categories for each industry. The thresholds for the different categories are given by $-1.65\sigma_{it}$ (Cat. 0), $-2\sigma_{it}$ (Cat. 1), $-2.25\sigma_{it}$ (Cat. 2), $-2.5\sigma_{it}$ (Cat. 3), $-3\sigma_{it}$ (Cat. 4). For booms the categories are defined by the positive equivalents of these thresholds. The last column presents the skewness coefficient as reported in Table 2.2. An asterisk behind a skewness coefficient indicates that the number of booms in category zero minus the number of crashes in category zero on the one hand and the skewness coefficient on the other hand carry the same sign. We also report pooled results. A dagger after an industry name indicates that less observations are available.

cannot really explain the number of crashes. The estimated coefficient is only significant in the regression for the most severe crashes. Further, the R^2's are very low. We estimate similar regressions for booms, where we use the number of booms divided by the number of observations as dependent variable. The results, presented in columns 7-9, look very different. The skewness coefficient is statistically significantly related to the number of booms for all boom categories, except category 0.

We conclude from the analyses that skewness is a reasonably good proxy for the occurrence of booms but a limited proxy for crashes. Of course, it can be that the relation between skewness and the number of booms and the number of crashes is non-linear, implying that we should interpret the regression results with care. However, this limitation does not apply to the sign-test result which also indicates that skewness is not a good measure for crash likelihood.

2.B Robustness checks

In this section, we provide more information on the robustness checks discussed in Section 2.6. We concentrate on the abnormal returns that are constructed based on

Table 2.11: Relation between the number of crashes and booms, and skewness

	booms minus crashes			crash proportion			boom proportion		
	b	p-val	R^2	b	p-val	R^2	b	p-val	R^2
cat. 0	2.93	0.45	0.012	-0.0003	0.93	0.000	0.0057	0.10	0.059
cat. 1	5.59	0.06	0.077	-0.0020	0.44	0.013	0.0068	0.02	0.110
cat. 2	4.50	0.07	0.072	-0.0026	0.29	0.024	0.0036	0.07	0.071
cat. 3	5.46	0.01	0.145	-0.0022	0.26	0.028	0.0059	0.00	0.169
cat. 4	7.09	0.00	0.381	-0.0037	0.01	0.138	0.0061	0.00	0.350

This table reports the estimated coefficient on skewness (b) in a standard OLS regression, together with the p-value of the hypothesis $b = 0$ and the R^2 of the regression. As dependent variable we consider the number of booms minus the number of crashes (columns 1-3), the proportion of crashes (columns 4-6) and the proportion of booms (columns 7-9). The dependent variables are constructed from the numbers reported in Tables 2.2 and 2.10.

the fundament model, and the logit regressions. Because the robustness check on aftershocks does not entail a different fundamental model, we do not provide more details on the aftershocks.

2.B.1 The Fama and French (1993)-model

The abnormal returns that result from the Fama and French (1993)-model are shown in Table 2.12. For 17 sectors we report significant average abnormal returns, eleven more than in the CAPM case. Also, the pooled returns series exhibits a significantly positive average abnormal return. Moreover, the volatility estimates are also worse, as we find that 39 sectors have a higher volatility than predicted by the Fama and French (1993)-model (for the CAPM this applied to 29 industries). The skewness and kurtosis estimates are for both models rather similar. Based on the Fama and French-model we find in total 1169 first crashes, compared to only 1116 when our estimation is based on the one-factor market model. We find also slightly more aftershocks for the Fama and French-model than for the market model. Overall, our statistics on the abnormal returns and crashes show no evidence at all that the Fama and French-model is superior, and neither did the number of bubble months (see Table 2.9).

We also replicate our analysis on bubble characteristics and crash likelihood. Table 2.13 presents the results. As before, we find that the strength of a bubble is significantly positively related to the probability that a crash of category 2 to 5 occurs. If we include both measures simultaneously, it turns out that STRENGTH2 accounts for most of the effect. In line with our previous results, there is no evidence that the length of a bubble affects the likelihood of a crash. We see that the effect of a bubble does not diminish if it stops growing.

This table report summary statistics of the abnormal returns constructed with the Fama and French (1993)-model with a 120-month estimation window. It is similar to Table 2.2. Each abnormal return η_{it+1} is divided by the corresponding volatility estimate σ_{it} to correct for time-varying volatility. We report the number of observations, mean, standard deviation, skewness, kurtosis, minimum and maximum per industry and for the pooled set of adjusted abnormal returns. We also include the number of crashes based on a threshold of $-1.65\sigma_{it}$, split up in first crashes and aftershocks. A dagger after an industry name indicates that less observations are available. An asterisk denotes a significant difference from zero in case of means and skewness coefficients, and a significant difference from one in case of volatility, all at a 5% confidence level. Standard errors of the skewness coefficient are calculated as $\sqrt{6/T}$ (see Tabachnick and Fidell, 2001).

Table 2.12: Summary statistics of abnormal returns per industry based on the Fama and French (1993)-model

								number of	
industry	# obs	mean	vol	skew	kurt	min	max	crashes	shocks
Agric	822	−0.03	1.04*	0.09	5.11	−4.98	4.90	24	17
Food	822	0.09*	1.13*	0.40*	4.97	−3.53	6.02	29	12
Soda†	702	0.02	1.12*	0.22*	4.11	−3.73	5.02	30	25
Beer	822	0.11*	1.10*	−0.10	6.07	−5.43	6.00	21	12
Smoke	822	0.09*	1.14*	−0.36*	4.22	−5.35	3.46	32	20
Toys	822	−0.06	1.03	−0.71*	11.41	−8.89	4.21	18	18
Fun	822	−0.02	1.03	0.27*	5.79	−5.94	5.53	19	31
Books	822	−0.02	1.02	−0.21*	4.10	−5.09	3.29	31	15
Hshld	822	0.09*	1.10*	−1.16*	15.06	−10.69	4.54	23	15
Clths	822	−0.10*	1.12*	−0.28*	5.45	−5.80	4.52	32	25
Health†	402	−0.04	1.14*	−0.96*	8.44	−6.77	3.95	13	14
MedEq	822	0.04	1.02	−0.07	4.11	−4.69	3.97	20	20
Drugs	822	0.13*	1.10*	−0.15	4.73	−5.23	4.20	24	20
Chems	822	−0.07	1.08*	0.07	5.12	−4.35	6.00	24	29
Rubbr†	774	−0.03	1.04*	−0.28*	4.70	−5.26	4.44	22	28
Txtls	822	−0.10*	1.08*	−0.05	4.48	−3.92	5.92	31	29
BldMt	822	−0.09*	1.09*	−0.15	4.00	−5.02	3.71	24	35
Cnstr	822	−0.07*	1.04*	0.01	3.75	−3.51	4.22	26	30
Steel	822	−0.14*	1.10*	0.32*	4.94	−4.10	6.49	29	37
FabPr†	402	−0.27*	1.12*	0.04	4.52	−3.94	4.91	9	31
Mach	822	−0.08*	1.06*	0.15	3.87	−3.66	4.72	27	29
ElcEq	822	−0.06	1.04*	−0.05	4.50	−4.81	4.79	25	29
Autos	822	−0.08*	1.07*	0.12	5.67	−4.54	6.06	27	24
Aero	822	−0.05	1.04*	−0.29*	5.44	−6.73	3.48	24	29
Guns†	522	−0.05	1.09*	−0.65*	8.96	−7.83	4.90	15	15
Gold†	522	0.00	1.15*	0.35*	4.71	−3.75	6.24	20	17
Ships	822	−0.06	1.11*	0.07	6.52	−6.74	5.43	24	21
Mines	822	−0.03	1.06*	0.40*	4.60	−3.72	5.19	28	20
Coal	822	0.01	1.12*	0.44*	6.67	−4.74	6.49	25	18
Oil	822	0.01	1.06*	−0.02	3.81	−3.76	3.78	25	22
Util	822	−0.01	1.07*	0.02	4.08	−4.02	3.84	27	32
Telcm	822	0.06	1.12*	0.72*	7.92	−3.86	8.59	24	17
PerSv†	810	−0.01	1.04*	−0.95*	11.62	−9.40	3.79	27	11
BusSv	822	0.09*	0.98	0.26*	5.28	−4.71	4.68	27	8
Comps	822	0.03	1.08*	0.14	3.73	−4.12	3.63	24	20
Chips	822	0.03	1.04*	0.03	3.46	−3.60	3.28	19	22
LabEq	822	−0.03	1.07*	0.04	4.63	−4.74	4.95	26	18
Paper†	786	−0.06	0.99	0.16	5.43	−4.36	5.66	27	14
Boxes	822	−0.03	1.10*	0.17*	7.11	−4.71	7.55	24	25
Trans	822	−0.12*	1.06*	0.17	4.38	−3.97	5.24	26	29
Whshl	822	−0.10*	1.05*	0.05	7.12	−5.87	6.01	21	22
Rtail	822	0.04	1.09*	−0.22*	4.21	−4.26	4.38	25	29
Meals	822	0.04	1.06*	−0.02	4.59	−5.31	4.26	29	12
Banks	822	0.02	1.00	0.07	4.06	−4.04	3.93	22	19
Insur	822	0.01	1.02	0.04	3.75	−3.51	3.23	26	20
RlEst	822	−0.15*	1.10*	−0.05	7.82	−7.87	5.31	22	33
Fin	822	−0.03	1.07*	−0.28*	6.63	−7.79	3.76	26	22
Other	822	0.03	1.03	0.28*	4.33	−3.41	5.01	26	19
Pooled	37800	−0.02*	1.07*	−0.03*	5.64	−10.69	8.59	1169	1059

Table 2.13: Logit models for the probability that a first crash occurs using the Fama and French (1993)-model as fundamental model.

	Cat. 0		Cat. 1		Cat. 2		Cat. 3		Cat. 4	
model a										
constant	-3.02^{**}	(0.03)	-3.75^{**}	(0.04)	-4.17^{**}	(0.04)	-4.57^{**}	(0.05)	-5.31^{**}	(0.08)
BUBBLE'	0.50^{**}	(0.08)	0.76^{**}	(0.10)	0.87^{**}	(0.11)	0.96^{**}	(0.13)	0.89^{**}	(0.19)
LBUBBLE	−0.21	(0.20)	−0.35	(0.26)	−0.35	(0.30)	−0.45	(0.36)	−0.90	(0.54)
LAGS	0.04	(0.06)	0.10	(0.07)	0.08	(0.09)	0.09	(0.10)	0.31^{*}	(0.14)
STRENGTH1	0.34	(0.18)	0.44^{*}	(0.20)	0.56^{**}	(0.21)	0.51^{*}	(0.25)	0.76^{**}	(0.28)
LENGTH1	−0.001	(0.004)	−0.001	(0.005)	−0.002	(0.006)	−0.007	(0.006)	−0.006	(0.009)
$\log L$	−4589.2		−2792.5		−2056.3		−1524.2		−832.92	
model b										
constant	-3.02^{**}	(0.03)	-3.75^{**}	(0.04)	-4.17^{**}	(0.04)	-4.57^{**}	(0.05)	-5.31^{**}	(0.08)
BUBBLE'	0.48^{**}	(0.08)	0.73^{**}	(0.10)	0.82^{**}	(0.12)	0.88^{**}	(0.14)	0.82^{**}	(0.20)
LBUBBLE	−0.24	(0.20)	−0.39	(0.25)	0	(0.30)	−0.47	(0.36)	−0.99	(0.55)
LAGS	0.07	(0.06)	0.13	(0.07)	0.13	(0.08)	0.13	(0.10)	0.36^{**}	(0.14)
STRENGTH2	0.21^{*}	(0.09)	0.30^{**}	(0.10)	0.41^{**}	(0.11)	0.40^{**}	(0.12)	0.45^{**}	(0.17)
LENGTH1	−0.006	(0.004)	−0.007	(0.005)	−0.007	(0.006)	−0.014	(0.007)	−0.017	(0.011)
$\log L$	−4587.1		−2789.7		−2052.2		−1519.6		−831.07	
model c										
constant	-3.02^{**}	(0.03)	-3.75^{**}	(0.04)	-4.17^{**}	(0.04)	-4.57^{**}	(0.05)	-5.31^{**}	(0.08)
BUBBLE'	0.48^{**}	(0.06)	0.73^{**}	(0.08)	0.81^{**}	(0.09)	0.87^{**}	(0.11)	0.87^{**}	(0.15)
STRENGTH1	0.15	(0.21)	0.14	(0.24)	0.10	(0.26)	−0.09	(0.30)	0.31	(0.40)
STRENGTH2	0.20	(0.11)	0.28^{*}	(0.13)	0.39^{**}	(0.14)	0.48^{**}	(0.16)	0.32	(0.24)
$\log L$	−4592.1		−2792.8		−2053.8		−1522.5		−833.95	

We estimate the probability that a first crash occurs using standard logit models with different bubble characteristics as explanatory variables. Bubbles and crashes are defined with respect to the Fama and French (1993)-model. This table is similar to Table 2.5. We include a constant and a dummy for a bubble that may end at most six months before the last observation (BUBBLE′). LBUBBLE equals one if the bubble does not include the last observation. LAGS counts the number of months since the bubble has ended. Further, We consider the t-statistic of the bubble (STRENGTH1), its length (LENGTH1), the maximum t-statistic over a subperiod within the bubble (STRENGTH2) and its length (LENGTH2). All characteristics have been demeaned. As a dependent variable we consider the number of first crashes in the five categories. We report the estimates with standard errors in parentheses, and the value of the log likelihood function. One (two) asterisk(s) after an estimate indicates significance at the 5% (1%) confidence level.

2.B.2 Shorter windows

To investigate the sensitivity of our results to the horizons of the estimation window and the candidate window, we replicate our analysis with an estimation window of 60 months and a candidate window of 36 months. Table 2.14 shows that a shorter estimation window does not lead to improved estimation of the model parameters. For 15 series we find a significant average abnormal return, and for only one series do we not reject the hypothesis that the volatility is correctly predicted. It can be that the presence of bubbles combined with the shorter estimation window, leads to estimates that extrapolate bubble behavior when present. If this is the case, this approach would characterize a noise trader, and the significant average abnormal returns combined with the underestimation of the true volatility are evidence of the mistakes he can make.

Shortening the maximum length of the bubble perception leads to a different relation between bubble characteristics and crash likelihood. From Tables 2.5 and 2.13 we concluded that the strength of a bubble was positively related to the probability of a crash, while length did not have predictive power. In Table 2.15, panel a, we see that instead of STRENGTH1, LENGTH1 is now significant, though STRENGTH2 still shows up significantly in panel b. In panel c, it is not clear which effect dominates. In case that the maximum length is too short to capture a bubble accurately, censoring may influence the calculation of the values for the characteristics.

This table report summary statistics of the abnormal returns constructed with the factor model in Eq. (2.3) with a 60-month estimation window. It is similar to Table 2.2. Each abnormal return η_{it+1} is divided by the corresponding volatility estimate σ_{it} to correct for time-varying volatility. We report the number of observations, mean, standard deviation, skewness, kurtosis, minimum and maximum per industry and for the pooled set of adjusted abnormal returns. We also include the number of crashes based on a threshold of $-1.65\sigma_{it}$, split up in first crashes and aftershocks. A dagger after an industry name indicates that less observations are available. An asterisk denotes a significant difference from zero in case of means and skewness coefficients, and a significant difference from one in case of volatility, all at a 5% confidence level. Standard errors of the skewness coefficient are calculated as $\sqrt{6/T}$ (see Tabachnick and Fidell, 2001).

Table 2.14: Summary statistics of abnormal returns per industry based on the CAPM with a 60 month estimation window

industry	# obs	mean	vol	skew	kurt	min	max	number of first crashes	number of after-shocks
Agric	822	−0.02	1.08*	0.18*	4.91	−4.98	5.46	23	20
Food	822	0.10*	1.14*	0.22*	4.47	−4.36	5.12	25	16
Soda†	702	0.02	1.11*	0.21*	4.05	−3.67	4.88	20	15
Beer	822	0.12*	1.13*	−0.01	5.41	−5.20	6.17	19	16
Smoke	822	0.08*	1.14*	−0.37*	4.32	−5.19	3.71	33	13
Toys	822	−0.04	1.14*	−0.35*	8.34	−8.14	4.47	21	23
Fun	822	−0.02	1.11*	−0.47*	10.67	−9.76	4.86	24	21
Books	822	−0.03	1.09*	−0.18*	3.86	−4.79	3.47	28	30
Hshld	822	0.09*	1.17*	−1.17*	14.68	−11.17	5.73	28	16
Clths	822	−0.08*	1.13*	−0.14	4.21	−4.82	3.63	23	32
Health†	402	0.00	1.13*	−0.60*	6.51	−6.05	4.10	16	14
MedEq	822	0.03	1.05*	0.04	3.76	−3.79	3.93	28	19
Drugs	822	0.13*	1.11*	−0.03	4.66	−4.80	4.62	26	17
Chems	822	−0.06	1.11*	0.13	4.98	−4.99	4.99	29	25
Rubbr†	774	−0.02	1.10*	−0.18*	4.88	−5.42	5.19	24	24
Txtls	822	−0.09*	1.12*	−0.11	4.05	−4.63	5.08	26	27
BldMt	822	−0.09*	1.09*	−0.12	3.63	−4.35	4.05	24	28
Cnstr	822	−0.05	1.07*	0.05	3.86	−4.21	3.75	25	22
Steel	822	−0.15*	1.11*	0.34*	4.94	−4.04	6.54	26	38
FabPr†	402	−0.26*	1.14*	0.14	4.24	−3.62	4.85	13	19
Mach	822	−0.08*	1.09*	0.19*	3.82	−3.88	3.96	28	20
ElcEq	822	−0.05	1.08*	−0.02	4.30	−4.89	4.21	25	20
Autos	822	−0.09*	1.12*	0.11	5.54	−4.99	5.95	23	21
Aero	822	−0.06	1.09*	−0.12	4.49	−5.47	3.99	26	30
Guns†	522	−0.06	1.13*	−0.80*	8.72	−7.73	5.20	16	13
Gold†	522	0.00	1.11*	0.39*	4.96	−4.04	6.13	21	11
Ships	822	−0.06	1.14*	0.23*	6.70	−6.37	6.89	20	24
Mines	822	−0.04	1.10*	0.54*	4.95	−3.46	6.24	22	19
Coal	822	0.01	1.13*	0.52*	7.67	−5.32	7.81	25	16
Oil	822	0.01	1.09*	0.07	3.92	−3.68	4.20	27	18
Util	822	0.02	1.08*	0.05	4.29	−4.19	5.03	31	17
Telcm	822	0.05	1.13*	0.43*	7.03	−4.50	8.20	27	22
PerSv†	810	0.01	1.09*	−0.59*	8.26	−8.62	3.67	25	12
BusSv	822	0.07	1.04	0.11	4.76	−4.43	5.05	28	11
Comps	822	0.00	1.13*	0.17	3.61	−3.63	3.56	30	22
Chips	822	0.03	1.09*	0.14	3.35	−3.13	4.22	23	21
LabEq	822	−0.03	1.07*	0.06	4.08	−4.18	4.46	30	17
Paper†	786	−0.05	1.04	0.09	4.35	−3.84	5.10	30	14
Boxes	822	−0.04	1.13*	0.22*	6.33	−4.16	7.40	25	25
Trans	822	−0.13*	1.12*	0.01	4.27	−5.04	4.58	27	30
Whshl	822	−0.09*	1.12*	−0.02	5.81	−5.06	5.21	25	22
Rtail	822	0.06	1.10*	−0.22*	3.82	−3.98	4.37	27	27
Meals	822	0.04	1.12*	−0.18*	5.79	−6.76	4.41	24	15
Banks	822	0.01	1.09*	−0.07	4.76	−4.84	4.36	19	21
Insur	822	0.02	1.07*	0.14	3.58	−3.39	3.48	32	20
RlEst	822	−0.14*	1.12*	0.07	6.08	−6.55	5.06	28	18
Fin	822	−0.03	1.10*	−0.20*	6.93	−8.21	4.42	26	18
Other	822	0.03	1.09*	0.21*	4.18	−4.27	5.19	28	13
Pooled	37800	−0.02*	1.11*	−0.02	5.42	−11.17	8.20	1199	972

Table 2.15: Logit models for the probability that a first crash occurs, using 60 months to estimate the CAPM.

	Cat. 0		Cat. 1		Cat. 2		Cat. 3		Cat. 4	
model a										
constant	−2.99**	(0.03)	−3.75**	(0.04)	−4.19**	(0.04)	−4.54**	(0.05)	−5.32**	(0.08)
BUBBLE′	0.52**	(0.08)	0.75**	(0.10)	0.69**	(0.13)	0.55**	(0.16)	0.74**	(0.22)
LBUBBLE	0.00	(0.20)	−0.16	(0.28)	0.08	(0.32)	0.14	(0.40)	−0.12	(0.54)
LAGS	−0.07	(0.06)	−0.08	(0.09)	−0.11	(0.10)	−0.09	(0.12)	0.02	(0.16)
STRENGTH1	−0.08	(0.20)	−0.05	(0.24)	−0.11	(0.31)	0.22	(0.33)	0.50	(0.34)
LENGTH1	0.019*	(0.009)	0.032**	(0.011)	0.039**	(0.014)	0.034	(0.018)	0.041	(0.024)
$\log L$	−4690.6		−2774.4		−1978.6		−1464.4		−812.18	
model b										
constant	−2.99**	(0.03)	−3.75**	(0.04)	−4.19**	(0.04)	−4.54**	(0.05)	−5.32**	(0.08)
BUBBLE′	0.50**	(0.08)	0.73**	(0.10)	0.66**	(0.13)	0.53**	(0.16)	0.75**	(0.22)
LBUBBLE	0.05	(0.20)	−0.10	(0.27)	0	(0.32)	0.17	(0.39)	−0.13	(0.54)
LAGS	−0.05	(0.06)	−0.07	(0.08)	−0.09	(0.10)	−0.07	(0.12)	0.04	(0.16)
STRENGTH2	0.23*	(0.09)	0.26*	(0.12)	0.31*	(0.14)	0.38*	(0.17)	0.43*	(0.21)
LENGTH2	0.002	(0.008)	0.016	(0.011)	0.017	(0.013)	0.016	(0.016)	0.033	(0.021)
$\log L$	−4690.4		−2775.4		−1979.8		−1464.6		−812.81	
model c										
constant	−2.99**	(0.03)	−3.75**	(0.04)	−4.19**	(0.04)	−4.54**	(0.05)	−5.32**	(0.08)
BUBBLE′	0.47**	(0.06)	0.67**	(0.08)	0.64**	(0.10)	0.51**	(0.13)	0.73**	(0.17)
STRENGTH2	0.20*	(0.10)	0.20	(0.12)	0.22	(0.14)	0.26	(0.17)	0.29	(0.21)
LENGTH1	0.010	(0.01)	0.023*	(0.01)	0.027	(0.01)	0.028	(0.02)	0.037	(0.02)
$\log L$	−4691		−2778.6		−1979.6		−1463.9		−812.85	

We estimate the probability that a first crash occurs using standard logit models with different bubble characteristics as explanatory variables. Bubbles and crashes are defined with respect to the CAPM estimated over a horizon of 60 months. This table is similar to Table 2.5. We include a constant and a dummy for a bubble that may end at most six months before the last observation (BUBBLE′). LBUBBLE equals one if the bubble does not include the last observation. LAGS counts the number of months since the bubble has ended. Further, We consider the t-statistic of the bubble (STRENGTH1), its length (LENGTH1), the maximum t-statistic over a subperiod within the bubble (STRENGTH2) and its length (LENGTH2). All characteristics have been demeaned. As a dependent variable we consider the number of first crashes in the five categories. We report the estimates with standard errors in parentheses, and the value of the log likelihood function. One (two) asterisk(s) after an estimate indicates significance at the 5% (1%) confidence level.

Alas, in the partners' [of Long Term Capital Management] *lingo, "the correlations [among the trades] had gone to one." Every bet was losing simultaneously.*
When Genius Failed – Roger Lowenstein

Chapter 3

Testing copulas to model financial dependence*

3.1 Introduction

Modelling dependence is of key importance to all economic fields in which uncertainty plays a large role. It is a crucial element of decision making under uncertainty and risk analysis. Consequently, an inappropriate model for dependence can lead to suboptimal decisions and inaccurate assessments of risk exposures. Traditionally, correlation is used to describe dependence between random variables, but recent studies have ascertained the superiority of copulas to model dependence, as they offer much more flexibility than the correlation approach. Clemen and Reilly (1999) discuss the application of copulas in decision making. Frees and Valdez (1998) show the use of copulas in actuarial risk analyses. Embrechts *et al.* (2002) advocate using copulas in finance. An important reason to consider other copulas than the correlation-implied Gaussian copula is the failure of the correlation approach to capture dependence between extreme events, as shown by Longin and Solnik (2001), Bae *et al.* (2003) and Hartmann *et al.* (2004). However, up to now no consensus has been reached on which copula to use in specific applications or on how to test the accuracy of a specific copula.

In this chapter we propose a new approach to evaluate copulas. Generally, theory offers little guidance in choosing a copula, making the selection an empirical

*This chapter is based on the article by Kole, Koedijk, and Verbeek (2005b).

issue. Since a copula is equivalent to a distribution function, we consider traditional tests designed for the fit of a distribution on a sample. We show how modifications of the Kolmogorov-Smirnov test and the Anderson-Darling test can be applied. These goodness-of-fit tests are based on a direct comparison of the dependence implied by the copula with the dependence observed in the data. If dependence over the complete distribution is important, as in the case of investment decisions, the Kolmogorov-Smirnov tests can be chosen because of their focus on the fit in the distribution's center. If dependence of extreme values is of interest, as in the case of risk management, the Anderson-Darling tests are preferable, because they pay more attention to the tails. Using these direct tests of the fit of a copula has several advantages over alternative approaches proposed in the literature. First of all, they are applicable to any copula, not only to the Student's t and Gaussian copula. Second, it can be used for copulas of any dimension, not only for bivariate copulas. Third, they indicate whether a copula captures the observed dependence accurately, and not only whether it can be rejected against another specific copula. Finally, if the tests that we propose for selecting a copula are used, the decision is based on the complete dependence pattern, contrary to selection procedures that consider only part of the dependence pattern (i.e. dependence of extreme observations).

We apply the goodness-of-fit tests to select a copula for the risk management of an asset portfolio consisting of stocks, bonds and real estate, which are among the main investment opportunities available to investors. As investors are generally averse to downside risk, it is important to capture the risk of joint downside movements of asset prices, without failing to exploit the diversification possibilities that assets offer. Therefore, we consider the Gaussian, the Student's t and the Gumbel copula to model the dependence. We approximate the returns on the different investment categories by indexes: the Standard & Poor's 500 Composite stock index, the JP Morgan US Government bond index and the NAREIT All index (real estate). The Gaussian copula is the traditional candidate for modelling dependence. The Student's t copula is a natural second candidate, because it can capture dependence in the tails without giving up flexibility to model dependence in the center. We include the Gumbel copula, because it is directly related to multivariate extensions of extreme value theory, which has gained popularity in risk management over the last decade (see e.g. Longin, 1996). Our tests provide clear evidence against the Gaussian and Gumbel copulas, but do not reject the Student's t copula. As a comparison, we apply the approach of Poon *et al.* (2004), which is based on bivariate dependence in the tails and show that it does not facilitate a decision. A detailed comparison of the tail behavior present in the data with the tail behavior of the copulas shows the importance of choosing the right copula for risk management. While the Gaussian copula leads to a serious underestimation of the risk of joint downside movements and the Gumbel copula overestimates it, the Student's t copula captures this risk accurately. Moreover, these differences are significant.

Our study adds to the ongoing debate on modelling dependence in finance, particularly regarding asset returns. This debate concentrates not only on which copula to use, but also on the methods used in selection. Different authors have proposed different methods, based on either dependence in the tails[1], likelihood ratio tests[2], correlations conditional on the size of returns[3] or regime switching models[4]. However, contrary to our approach, none of these methods directly tests the fit of a dependence model. Moreover, the recent methods based on dependence in the tails can only handle bivariate dependence and take only part of the dependence pattern into account. Likelihood ratio tests can only handle nested copulas, while the methods based on size-conditional correlations are sensitive to (corrections for) biases (see Forbes and Rigobon, 2002; Corsetti *et al.*, 2005). Finally, the evidence supplied by the different methods on relatively similar data sets is mixed. We try to overcome the drawbacks of these existing methods by proposing tests that relate directly to the fit of a dependence model. We extend the work of Malevergne and Sornette (2003), who only consider the Gaussian copula.

The remainder of this chapter is structured as follows. Section 2 discusses the tests and their application to the Gaussian, the Student's t and the Gumbel copulas. In Section 3 we show how to use the tests to select a copula to model dependence in a risk management application. We demonstrate the advantages of our approach by comparing it with the test procedure in Poon *et al.* (2004). A detailed analysis of tail behavior shows the importance of choosing the right copula. Section 4 concludes.

3.2 Goodness-of-fit tests for copulas

In this section we explain how the Kolmogorov-Smirnov and Anderson-Darling tests can be implemented for copulas. We start with a short introduction on copulas.[5] In the second subsection we present the tests. The third subsection discusses the implementation.

[1]See Hartmann *et al.* (2004), Poon *et al.* (2004) and Longin and Solnik (2001).

[2]See Mashal *et al.* (2003) and Mashal and Zeevi (2002).

[3]See Ang and Chen (2002), Campbell *et al.* (2003), Campbell *et al.* (2002), Forbes and Rigobon (2002) and Loretan and English (2000).

[4]See Ang and Bekaert (2002), Edwards and Susmel (2001) and Ramchand and Susmel (1998).

[5]A more rigorous treatment of copulas can be found in Joe (1997) and Nelsen (1999). For a discussion applied to finance we refer to Cherubini *et al.* (2004) and Bouyé *et al.* (2000).

3.2.1 Copulas

Dependence between random variables can be modelled by copulas. A copula returns the joint probability of events as a function of the marginal probabilities of each event. This makes copulas attractive, as the univariate marginal behavior of random variables can be modelled separately from their dependence. For a random vector $\boldsymbol{X}$ of size n with marginal cumulative density functions (cdf) F_i, the copula with cdf $C(\cdot)$ gives the cumulative probability for the event $\boldsymbol{x}$:

$$P(\boldsymbol{X} \leq \boldsymbol{x}) = C(F_1(x_1), \ldots, F_n(x_n)). \tag{3.1}$$

The applicability of copulas is wide, as Sklar (1959) proves that each multivariate distribution with continuous marginals has a unique copula representation. Moreover, any function $C : [0,1]^n \rightarrow [0,1]$ satisfying some regularity restrictions implies a copula.[6]

Tail dependence is an important property of copulas. It describes the behavior of copulas when the value of the marginal cdf F_i reaches its bounds of zero (lower tail dependence) or one (upper tail dependence) and is defined as the limiting probability that a subset of the variables in $\boldsymbol{X}$ has extreme values, given that the complement has extreme values.[7] If the limiting probability equals zero, a copula exhibits tail independence; if the probability exceeds zero, it exhibits tail dependence.

The traditional use of correlation to model dependence implies using the Gaussian copula[8] which has cdf:

$$C_n^{\Phi}(\boldsymbol{u}; \boldsymbol{\Omega}^{\Phi}) = \Phi_n(\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_n); \boldsymbol{\Omega}^{\Phi}), \tag{3.2}$$

where $\boldsymbol{u}$ is a vector of marginal probabilities, Φ_n denotes the cdf for the n-variate standard normal distribution with correlation matrix $\boldsymbol{\Omega}^{\Phi}$, and Φ^{-1} is the inverse of the cdf for the univariate standard normal distribution. For imperfectly correlated variables, the Gaussian copula implies tail independence (see Embrechts *et al.*, 2002).

Closely related to the Gaussian copula is the Student's t copula, with cdf:

$$C_n^{\Psi}(\boldsymbol{u}; \boldsymbol{\Omega}^{\Psi}, \nu^{\Psi}) = \Psi_n(\Psi^{-1}(u_1; \nu^{\Psi}), \ldots, \Psi^{-1}(u_n; \nu^{\Psi}); \boldsymbol{\Omega}^{\Psi}, \nu^{\Psi}), \tag{3.3}$$

where Ψ_n denotes the cdf of an n-variate Student's t distribution with correlation matrix $\boldsymbol{\Omega}^{\Psi}$ and degrees of freedom parameter $\nu^{\Psi} > 2$, and Ψ^{-1} is the inverse of the

[6] See Definition 1 in Embrechts *et al.* (2002).

[7] Joe (1997) Sec. 2.1.10 gives a definition for the bivariate case, which is generalized by Schmidt and Stadmüller (2003) to $n > 2$ dimensions.

[8] Correlation can always be used as a dependence *measure*. However, if correlation is used as a *model*, i.e. a complete characterization, of dependence it implies the Gaussian copula

cdf for the univariate Student's t distribution with mean zero, dispersion parameter equal to one and degrees of freedom ν^{Ψ}. The Gaussian and Student's t copula belong to the class of elliptic copulas. A higher value for ν decreases the probability of tail events. As the Student's t copula converges to the Gaussian copula for $\nu \to \infty$, the Student's t copula assigns more probability to tail events than the Gaussian copula. Moreover, the Student's t copula exhibits tail dependence (even if correlation coefficients equal zero). For large ν^{Ψ}, differences between the Student's t and Gaussian copula are negligible.

The third copula we consider in this chapter is the Gumbel copula, which belongs to the class of Archimedean copulas. The Gumbel copula is an extreme value copula.[9] Its standard cdf is given by

$$C_n^{\mathrm{G}}(\boldsymbol{u}; a) = \exp\left(-\left(\sum_{i=1}^{n}(-\log u_i)^a\right)^{1/a}\right), \tag{3.4}$$

with $a \geq 1$, where $a = 1$ implies independence. Because the standard Gumbel copula implies the same dependence between all combinations of marginal variables u_i, we use the extension proposed by Bouyé (2002). He uses a recursive definition, in which the dependence of the marginal probability u_{i+1} with the preceding marginal probabilities $u_1, \ldots, u_i$ is characterized by a specific parameter a_i:

$$C_n^{\mathrm{B}}(u_1, \ldots, u_n; a_1, \ldots, a_{n-1}) = \begin{cases} C_2^{\mathrm{G}}(u_1, u_2; a_1) & \text{if } n = 2 \\ C_2^{\mathrm{G}}(C_{n-1}^{\mathrm{B}}(u_1, \ldots, u_{n-1}; a_1, \ldots, a_{n-2}), u_n; a_{n-1}) & \text{if } n > 2, \end{cases} \tag{3.5}$$

with $a_1 \geq a_2 \geq \ldots \geq a_{n-1} \geq 1$. $C_2^{\mathrm{G}}()$ denotes the standard bivariate Gumbel copula as defined in Eq. (3.4). The restrictions on the a's impose a descending dependence order: the dependence between u_1 and u_2, governed by a_1 is at least as strong as the dependence between u_1 and u_2 on the one hand and u_3 on the other, governed by a_2. The ordering of the variables is therefore important. The Gumbel copula exhibits upper tail dependence but lower tail independence, which can be reversed by using the survival copula.[10]

[9] Joe (1997) provides a detailed, general discussion of extreme value theory in relation to copulas, while Bouyé (2002) discusses it from a risk management perspective.

[10] The cumulative joint probability of events $\boldsymbol{u}$ is calculated by the survival copula: $P(\boldsymbol{U} \leq \boldsymbol{u}) = \bar{C}(\boldsymbol{\imath}_n - \boldsymbol{u})$, where $\bar{C}$ denotes the joint survival function. For a random vector $\boldsymbol{X}$ with (multivariate) density function $F(\boldsymbol{x})$ (not necessarily a copula) the joint survival function is defined as $\bar{F}(\boldsymbol{x}) = P(\boldsymbol{X} \geq \boldsymbol{x})$. Joe (1997) (p. 10, item 39) gives the general formula that relates $\bar{F}$ to F (e.g. for the two dimensional case $\bar{F}(x_1, x_2) = 1 - F_1(x_1) - F_2(x_2) + F(x_1, x_2)$, where F_i denotes a marginal distribution).

3.2.2 Test statistics for the fit of copulas

The tests we propose belong to the large class of goodness-of-fit tests for distributions. Suppose that we want to test whether a specific distribution for a random variable accurately fits the corresponding observations. Under the hypothesis that this is the case, the empirical cumulative distribution of the observations F_{E} will converge to the hypothesized cumulative distribution F_{H} almost surely, as stated by the Glivenko-Cantelli theorem (see Mittelhammer, 1996, p. 313). Therefore, we can use the deviations of the empirical distribution from the hypothesized distribution to test the fit. Let $\boldsymbol{x}_t$ be a realization of the random variable $\boldsymbol{X}$ out of sample of T realizations. We propose the following four statistics:

$$D_{\mathrm{KS}} = \max_t |F_{\mathrm{E}}(\boldsymbol{x}_t) - F_{\mathrm{H}}(\boldsymbol{x}_t)|; \tag{3.6}$$

$$D_{\overline{\mathrm{KS}}} = \int_{\boldsymbol{x}} |F_{\mathrm{E}}(\boldsymbol{x}) - F_{\mathrm{H}}(\boldsymbol{x})| \,\mathrm{d}F_{\mathrm{H}}(\boldsymbol{x}); \tag{3.7}$$

$$D_{\mathrm{AD}} = \max_t \frac{|F_{\mathrm{E}}(\boldsymbol{x}_t) - F_{\mathrm{H}}(\boldsymbol{x}_t)|}{\sqrt{F_{\mathrm{H}}(\boldsymbol{x}_t)(1 - F_{\mathrm{H}}(\boldsymbol{x}_t))}}; \tag{3.8}$$

$$D_{\overline{\mathrm{AD}}} = \int_{\boldsymbol{x}} \frac{|F_{\mathrm{E}}(\boldsymbol{x}) - F_{\mathrm{H}}(\boldsymbol{x})|}{\sqrt{F_{\mathrm{H}}(\boldsymbol{x})(1 - F_{\mathrm{H}}(\boldsymbol{x}))}} \,\mathrm{d}F_{\mathrm{H}}(\boldsymbol{x}). \tag{3.9}$$

The first distance measure is commonly referred to as the Kolmogorov-Smirnov distance, of which the second is an average. The third distance measure is known as the Anderson-Darling distance after Anderson and Darling (1952), and the fourth is again an average of it. The Kolmogorov-Smirnov distances are more sensitive to deviations in the center of the distribution, whereas the Anderson-Darling distances give more weight to deviations in the tails. Originally, the measures focus on the largest deviation in a sample but to get more complete information on the goodness-of-fit the average can be used as well. To reduce the influence of outliers in the Anderson-Darling distances, we follow Malevergne and Sornette (2003) by replacing the original $(F_{\mathrm{E}}(\boldsymbol{x}_t) - F_{\mathrm{H}}(\boldsymbol{x}_t))^2$ term by $|F_{\mathrm{E}}(\boldsymbol{x}_t) - F_{\mathrm{H}}(\boldsymbol{x}_t)|$. The distributions of the statistics under the null hypothesis are non-standard. Moreover, the parameters for the hypothesized distribution are often estimated on the same data. Therefore, simulations are necessary to evaluate the test statistics.

One way to test the fit of a specific copula is to derive the test statistics directly, by transforming each observation to the corresponding marginal probabilities, based on which the distance measures are then calculated. The hypothesized and empirical copulas take the place of F_{H} and F_{E}, respectively. The empirical copula C_{E} based on a sample $\mathcal{X}$ gives the joint probability for a vector of marginal probabilities u as follows:

$$C_{\mathrm{E}}(\boldsymbol{u};\mathcal{X}) = \frac{1}{T}\sum_{t}^{T} I(x_{1,t} \leq x_1^{\lfloor u_1 \cdot T \rfloor}) \cdot \ldots \cdot I(x_{n,t} \leq x_n^{\lfloor u_n \cdot T \rfloor}), \tag{3.10}$$

where $I(\cdot)$ is the indicator function, which equals 1 if the statement in parentheses is true and zero otherwise, and $x_j^{\lfloor u_j \cdot T \rfloor}$ is the k^{th} (ascending) order statistic, k being the largest integer not exceeding $u_j \cdot T$.

Inspired by Malevergne and Sornette (2003) we propose a slightly different approach for elliptic copulas. As the cumulative distribution functions of elliptic distributions are generally not available in closed form, calculation of the hypothesized probabilities will be computationally demanding if the number of dimensions increases. We use a faster procedure and evaluate the fit of elliptic copulas in terms of the fit of a univariate random variable. This approach i based on the property that the density functions of elliptic distributions are constant on ellipsoids. Each elliptically distributed random variable implies a univariate random variable with a specific distribution that corresponds with the radii of the ellipsoids of constant density (see Fang *et al.*, 1990, for a formal treatment). Instead of considering the observation itself we consider the squared radius of the ellipsoid of constant density that it implies. We compare the empirical distribution of the squared radii with their theoretical distribution, which are standard distributions in case of the Gaussian and Student's t copula.

For a random vector $\boldsymbol{U} = (U_1, \ldots, U_n)'$ with marginal uniform distributions on $[0,1]$ and dependence given by the Gaussian copula with correlation matrix $\boldsymbol{\Omega}^{\Phi}$, we construct the squared radius as:

$$Z_{\Phi} = \tilde{\boldsymbol{U}}'(\boldsymbol{\Omega}^{\Phi})^{-1}\tilde{\boldsymbol{U}}, \tag{3.11}$$

where $\tilde{\boldsymbol{U}} = (\Phi^{-1}(U_1), \ldots, \Phi^{-1}(U_n))'$ and $\Phi^{-1}(\cdot)$ is the inverse of the standard normal cdf. The random variable Z_{Φ} has a χ^2_n-distribution. This follows easily upon realizing that $\tilde{\boldsymbol{U}}$ has a normal distribution with correlation matrix $\boldsymbol{\Omega}^{\Phi}$, which makes Z_{Φ} the sum of n squared random variables that are independently, standard normally distributed. So, starting with a sample having uniform marginal distributions, we transform each observation $\boldsymbol{u}$ to $z = \tilde{\boldsymbol{u}}'\boldsymbol{\Omega}^{-1}\tilde{\boldsymbol{u}}$ and calculate its associated cumulative probability by the cdf of the χ^2_n-distribution.

For the Student's t copula we use a similar transformation. Suppose that $\boldsymbol{V} = (V_1, \ldots, V_n)'$ is a random vector with each V_i being uniformly distributed on $[0,1]$ and whose dependence is given by the Student's t copula with correlation matrix $\boldsymbol{\Omega}^{\Psi}$ and degrees of freedom ν^{Ψ}. Now we construct the squared radius as

$$Z_{\Psi} = \tilde{\boldsymbol{V}}'(\boldsymbol{\Omega}^{\Psi})^{-1}\tilde{\boldsymbol{V}}/n, \tag{3.12}$$

where $\tilde{\boldsymbol{V}} = (\Psi^{-1}(V_1;\nu^\Psi),\ldots,\Psi^{-1}(V_n;\nu^\Psi))'$ and $\Psi^{-1}(V_j;\nu^\Psi)$ is the inverse function of the standard Student's t distribution with degrees of freedom parameter ν^Ψ. The variable Z_Ψ is distributed according to an F-distribution with degrees of freedom parameters n and ν^Ψ. Note that the variable $\tilde{\boldsymbol{V}}$ has a Student's t distribution and can therefore be written as $\boldsymbol{W}/\sqrt{S/n}$, with $\boldsymbol{W}$ being an n-dimensional normally distributed random variable with correlation matrix $\boldsymbol{\Omega}^\Psi$ and S being a univariate random variable with a $\chi^2_{\nu^\Psi}$-distribution. Consequently, we can write

$$Z_\Psi = \frac{\boldsymbol{W}'(\boldsymbol{\Omega}^\Psi)^{-1}\boldsymbol{W}/n}{S/\nu^\Psi},$$

which makes Z_Ψ the ratio of two χ^2-distributed variables divided by their respective degrees of freedom. Therefore, it has a Snedecor's F_{n,ν^Ψ} distribution. So when we test the Student's t copula, we start with a sample having uniform marginal distributions, transform each observation $\boldsymbol{v}$ to $\tilde{\boldsymbol{v}}'(\boldsymbol{\Omega}^\Psi)^{-1}\tilde{\boldsymbol{v}}/n$ and calculate the cumulative probability with the cdf of the F_{n,ν^Ψ} distribution.

3.2.3 The procedure

Suppose that we want to use a specific copula with cdf C and parameter vector $\boldsymbol{\theta}$ to model the dependence of a random variable X for which we have a sample available of size T. The procedure that we propose to evaluate the fit of this copula consist of four steps:

Estimation step We estimate the parameters $\boldsymbol{\theta}$. In general two approaches can be used for estimating copula parameters. For our test procedure we advocate the inference functions for margins method (IFM) (see Joe, 1997, Ch. 10). In this two-step approach the parameters for the marginal models are estimated first. In the second step, the copula parameters are estimated with the marginal distribution parameters treated as given.[11] It is also possible to apply maximum likelihood to jointly estimate the parameters for the marginal models and the copula. The IFM is less efficient than one-step maximum likelihood, but it is computationally more attractive and allows larger flexibility in the estimation techniques for the marginal models.

Evaluation step We evaluate the fit of the copula with the estimated parameters by calculating the four distance measures of the previous subsection. If the copulas belong to the elliptical family, we base the calculation on a transformation of

[11] The resulting estimators $\hat{\boldsymbol{\theta}}$ belong to the general class of sequential estimators (see Newey, 1984).

the uniform marginals. We use $\hat{d}_{\mathrm{KS}}$, $\hat{d}_{\overline{\mathrm{KS}}}$, $\hat{d}_{\mathrm{AD}}$ and $\hat{d}_{\overline{\mathrm{AD}}}$ to refer to the distance measures for the original sample.

Simulation step To test whether the distance measures provide evidence against the fit of the copula, we need to construct the distribution of the distance measures under the null hypothesis of accurate fit. Given the form of the distance measures and the fact that the parameters of the copula are not known but estimated, simulations have to be used. For each simulation, we generate a random sample of size T from the copula with parameters $\hat{\boldsymbol{\theta}}$.[12] We apply the estimation and evaluation step on this simulated sample (and find a new estimate for $\boldsymbol{\theta}$). Each simulation yields new values for the distance measures. Combined, the simulations result in a distribution of random variables corresponding to $\hat{d}_{\mathrm{KS}}$, $\hat{d}_{\overline{\mathrm{KS}}}$, $\hat{d}_{\mathrm{AD}}$ and $\hat{d}_{\overline{\mathrm{AD}}}$.

Test step Finally, we use the distribution that results from the simulation step to judge the values $\hat{d}_{\mathrm{KS}}$, $\hat{d}_{\overline{\mathrm{KS}}}$, $\hat{d}_{\mathrm{AD}}$ and $\hat{d}_{\overline{\mathrm{AD}}}$, by determining their p-value. p-values below the commonly used thresholds of 10%, 5% or 1% lead to rejection of the fit of the copula on that sample.

This procedure can be implemented straightforwardly. Note that the estimation step within the simulation step should be applied to the marginal parameters as well. If, for example, the empirical distributions are used to model the marginal distributions of the original sample, they should be used for the simulated sample, too.

3.3 A risk management application

In this section we consider three copulas to model the dependence between asset returns. The asset returns we consider are the returns on a stock, a bond and a real estate index. When investors determine their asset allocations, stocks, bonds and real estate are among the main investment opportunities available to them. The investor's objective is to construct a portfolio that has an optimal risk-return trade-off. The risk that a specific portfolio entails is directly related to the dependence between the portfolio's constituents. Consequently, the model used for dependence is of key importance for the construction of an optimal asset allocation.

[12]Simulation techniques for copulas can be found in Bouyé *et al.* (2000). General simulation techniques are discussed in Devroye (1986). Aas (2004) discusses a specific simulation technique for Gumbel copulas.

Overwhelming evidence has been established that investors are sensitive to downside risk, implying that investors pay specific attention to extreme negative returns.[13] This makes it important to capture the risk entailed by the joint tail behavior of returns, without failing to exploit the diversification possibilities represented by the center of the return distribution. Therefore, we consider the Gaussian, the Student's t and the Gumbel copula to model dependence. The Gaussian copula, the traditional method to model dependence, mostly reflects dependence in the center of the distribution and implies tail independence. The Gumbel copula mostly reflects tail dependence. Being an extreme value copula, it extends the successful univariate extreme value theory techniques in risk management, as shown by Longin (1996) and Jansen *et al.* (2000). In their study of dependence of extreme returns Longin and Solnik (2001) and Poon *et al.* (2004) also use the Gumbel copula. The Student's t copula can capture both dependence in the center and the tails of the distribution, and has been proposed as an alternative to the Gaussian copula by several authors including Glasserman *et al.* (2002), Campbell *et al.* (2003), Mashal *et al.* (2003), Valdez and Chernih (2003) and Meneguzzo and Vecchiato (2004). Though the evidence for tail dependence is actually mixed, as Longin and Solnik (2001) and Hartmann *et al.* (2004) find positive evidence for it in international asset returns while Poon *et al.* (2004) reject it by applying a different test, its importance for downside risk averse investors is large enough not to exclude it a priori.

Our tests can be used to determine which copula to apply. If the Gaussian copula fits the data well, the center is the dominating factor and the correlation matrix suffices to describe dependence. If the Student's t copula fits the data well and the Gaussian copula does not, the first captures dependence in the tails accurately while the latter fails to do so. The Student's t copula converges to the Gaussian copula, if the degrees of freedom parameter increases. Consequently, a good fit of the Gaussian copula will necessarily imply a good fit of the Student's t copula for a sufficiently high value for the degrees of freedom parameter. In that case the Gaussian copula should be preferred, as it is more parsimonious. Finally, if the Gumbel copula fits the data well, dependence in the tails is the dominating factor.

In the next subsection we introduce the data. We briefly discuss how the marginal distributions for each return can be modelled. In the second subsection we test the fit of the Gaussian, the Student's t and the Gumbel copulas. We compare the outcome of the selection with the outcome of the method proposed by Poon *et al.* (2004). In

[13] See Kahneman and Tversky (1979) and Tversky and Kahneman (1991) for a general discussion. Benartzi and Thaler (1995) and Berkelaar *et al.* (2004) discuss the implications of downside risk aversion from a finance perspective.

Table 3.1: Summary statistics and tail indices.

(a) Summary statistics

	stocks	bonds	real estate
mean	0.012	0.022	0.065
volatility	1.26	0.34	0.81
skewness	0.18	−0.34	−0.33
kurtosis	4.60	3.83	7.34
minimum	−5.83	−1.38	−5.19
maximum	5.73	1.08	4.68

(b) Tail indices

α_l	5.31	4.62	2.60
α_r	4.44	4.96	4.23
$\alpha_l = \alpha_r$	4.51	5.89	3.27

Panel (a) reports summary statistics for the three index return series (in %) in our sample: S&P 500 Composite Index (stocks), JP Morgan Government Bond Index (bonds) and NAREIT All Index (real estate). The series consist of 1499 returns from January 1, 1999 to December 17, 2004. Panel (b) reports estimates for the left and the right tail indices (α_l and α_r respectively) and the estimates for the tail indices under the restriction that the left and right tail indices are equal ($\alpha_l = \alpha_r$). The tail indices are estimated by Huisman *et al.* (2001)'s modified Hill-estimator, with the maximum number of observations used (κ) equal to 149.

the last subsection we analyze the dependence in the tails in more detail and discuss its implications for risk management.

3.3.1 Data and marginal models

We use indexes to proxy for the returns on stocks, bonds and real estate: Standard & Poor's 500 Composite Index (stocks), JP Morgan's US Government Bond Index (bonds) and the NAREIT All Index (real estate). Because it is important to pay attention to dependence in the tails of the distribution, a reasonable number of tail observations should be included. Therefore, we calculate daily total returns for all indexes. We collect data from DataStream over the period January 1, 1999 to December 17, 2004. Excluding non-trading days the sample consist of 1499 returns. Panel (a) in Table 3.1 presents summary statistics on the returns in the sample. Our data exhibit the well-known stylized facts: asymmetry as indicated by non-zero skewness and fat tails as indicated by excess kurtosis.

To model the marginal distributions, we use the semi-parametric method of Daníelsson and de Vries (2000). This method uses the empirical distribution for the center of the distribution and rely on univariate extreme value theory to model the tails. It enables us to combine the good approximation to the center of the actual distribution offered by the empirical distribution, and the statistical rigor from extreme value theory to model the tails of the distribution. Central in univariate extreme value theory is the tail index α, which characterizes the limiting behavior of a density. A distribution is fat tailed if the hypothesis $1/\alpha = 0$ is rejected in favor of the alternative $1/\alpha > 0$. In that case, the tail of the distribution can be modelled by the Pareto distribution. Tail index estimation is commonly based on the Hill-estimator (Hill, 1975). We use the modified Hill-estimator developed by Huisman *et al.* (2001) because of its unbiasedness.[14]

Table 3.1(b) reports estimates for the left tail index (α_l) and for the right tail index (α_r), and estimates for the tail indices under the assumption that the left and right tail indexes are equal. The hypothesis $1/\alpha = 0$ is rejected in all cases. The hypothesis of equal left and right tails cannot be rejected for stocks and bonds. For real estate, this hypothesis is marginally rejected with a p-value of 0.088. We model the tails separately, the left (right) tail applying to cumulative probabilities below 0.01 (above 0.99).

3.3.2 Selecting a copula

We use the procedure outlined in Section 3.2.3 to select from the Gaussian, Student's t and Gumbel copulas. The copula parameters are estimated by IFM method of Joe (1997), with the marginal distributions based on the semi-parametric method of Daníelsson and de Vries (2000) and maximum likelihood estimation in the second step. We calculate the distance measures and evaluate them by constructing their distributions under the null hypothesis of an accurate fit by simulating 10,000 samples based on the parameters that resulted from the estimation step.

The outcomes of this analysis are reported in Table 3.2. The parameter estimates in panel (a) for the Gaussian copula reveal that the correlations are negative though close to zero for stocks and bonds, and bonds and real estate, and moderate and positive for stocks and real estate. For an investor these estimates offer an attractive perspective, as they indicate large diversification possibilities. However, this conclusion is premature, since the test statistics in panel (d) indicate that the Gaussian copula does not capture the actual dependence well. For 3 out of 4 statistics, p-values are below 5%, rejecting the hypothesis of an accurate fit.

The Student's t copula performs better as the p-values for the distance measures reported in Table 3.2(d) exceed the 5% critical values by far. Its estimates for the

[14]Other methods for tail index estimation can be found in Daníelsson *et al.* (2001) and Drees and Kaufmann (1998). Brooks *et al.* (2005) conclude that the modified Hill-estimator by Huisman *et al.* (2001) outperforms other methods for tail index estimation when applied in Value-at-Risk calculations.

Table 3.2: Estimation and test results

(a) Gaussian copula

$\rho_{\mathrm{s,b}}$	-0.200	(0.024)
$\rho_{\mathrm{s,r}}$	0.471	(0.018)
$\rho_{\mathrm{b,r}}$	-0.073	(0.026)
$\log L$	218.44	

(b) Student's t copula

$\rho_{\mathrm{s,b}}$	-0.195	(0.026)
$\rho_{\mathrm{s,r}}$	0.471	(0.020)
$\rho_{\mathrm{b,r}}$	-0.074	(0.027)
ν	12.1	(2.76)
$\log L$	230.47	

(c) Gumbel copula

$a_{\mathrm{s,r}}$	1.42	(0.028)
$a_{\mathrm{(s,r),b}}$	1.000	(0.038)*
$\log L$	183.55	

(d) Test results

	Gaussian		Student's t		Gumbel	
$\hat{d}_{\mathrm{KS}}$	0.026	[0.013]	0.0095	[0.98]	0.035	[$< 0.5 \cdot 10^{-4}$]
$\hat{d}_{\overline{\mathrm{KS}}}$	0.012	[0.0006]	0.0024	[0.9980]	0.0082	[0.0003]
$\hat{d}_{\mathrm{AD}}$	0.058	[0.34]	0.044	[0.69]	0.28	[0.33]
$\hat{d}_{\overline{\mathrm{AD}}}$	0.030	[0.00073]	0.0065	[0.9993]	0.026	[0.0016]

Estimation and test results for the Gaussian, Student's t and Gumbel copula. Panels (a) to (c) report the parameter estimates and log likelihood values. The copulas are estimated on daily returns from the S&P 500 Composite Index, the JP Morgan Government Index and NAREIT All Index from January 1, 1999 to December 17, 2004 using the IFM method (Joe, 1997). The marginal distributions are constructed by the semi-parametric method of Daníelsson and de Vries (2000), with cut-off probabilities 0.01 and 0.99 for the left and right tail respectively, and tail indices estimated by the modified Hill-estimator of Huisman *et al.* (2001) (see Table 3.1). For both the Gaussian and the Student's t copula we report the correlation coefficients for stocks and bonds ($\rho_{\mathrm{s,b}}$), stocks and real estate ($\rho_{\mathrm{s,r}}$), and bonds and real estate ($\rho_{\mathrm{b,r}}$). For the Student's t copula we include the degrees of freedom parameter ν. The parameters for the Gumbel copula refer to Bouyé (2002)'s extension of the standard Gumbel copula, applied to the survival copula. $a_{\mathrm{s,r}}$ refers to the dependence between stocks and real estate; $a_{\mathrm{(s,r),b}}$ to the dependence between stocks and real estate on the one hand, and bonds on the other. Standard errors are reported in parentheses. In the estimation $a_{\mathrm{(s,r),b}} = 1+x^2$ is used; the standard error marked with an asterisk corresponds with x. Panel(d) reports the distance measures resulting from the tests. The values for the distance measures result from the evaluation step, applying the transformation in Eq. (3.11) for the Gaussian copula and in Eq. (3.12) for the Student's t copula. The p-values, based on 10,000 simulations as described in the simulation step, are reported in brackets.

correlation coefficients in panel (b) are largely equal to the estimated correlation coefficients for the Gaussian copula, but the degrees of freedom parameter is relatively low. This indicates that extreme events have a stronger tendency to occur jointly than captured by the Gaussian copula. Consequently, stocks, bonds and real estate still offer ample diversification opportunities, but dependence in the tails hampers the diversification of downside risk. This effect will be stronger for investors with a strong aversion to downside risk.

The results for the Gumbel copula show that basing a dependence model on tail dependence does not lead to good results. We use Bouyé (2002)'s extension of the standard Gumbel copula, which makes the ordering of the variables important. By definition, the dependence between the first two random variables is stronger than the dependence between the first two and the third random variables, i.e. $a_{1,2} \leq a_{(1,2),3} \leq 1$.

To determine that order, we estimate a bivariate copula for the three possible combinations. We put the two variables with the highest a estimate first (being stocks and real estate) and the other variable last (bonds).[15] Since we focus on downside risk, we use the survival copula to allow for lower tail dependence (see footnote 10 on page 55). The estimate for $a_{\mathrm{s,r}}$ shows dependence and lower tail dependence between stocks and real estate, but the $a_{(\mathrm{s,r}),\mathrm{b}}$ estimate being equal to one indicates independence of bond returns from the returns on stocks and real estate. Three out of four tests (including the average Kolomogorov-Smirnov and average Anderson-Darling statistic) provide evidence against the Gumbel copula.

We conclude that the followed procedure provides a clear positive advise for selecting the Student's t copula. None of the four distance measures indicates rejection, while both for the Gaussian and the Gumbel copula three out of four distance measures lead to a negative advice. Both the parameter estimates and the test results indicate that dependence in the tails is not accurately captured by the Gaussian copula. However, the Gumbel copula fails to capture the dependence in the center. The likelihood ratio test proposed by Mashal *et al.* (2003) and Mashal and Zeevi (2002) confirms the preference for the Student's t copula over the Gaussian copula, but the outcome of their test alone does not indicate that the Student's t copula fits the data

[15] To stress the ordering, we deviate from the notation in section 2, and attach it as a subscript to a. So, $a_{(s,r),b}$ is the coefficient for the dependence between stocks and real estate on the one hand and bonds on the other hand.

well.[16] A selection based on AIC or BIC also leads to a preference of the Student's t copula, but again, this does not imply by itself a good fit.[17]

3.3.3 Using tail dependence to select a copula

In the literature, alternative procedures have been proposed to select copulas. Because of the problems with the correction for biases in the size-conditional correlation approach pointed out by Corsetti *et al.* (2005), selection based on tail dependence seems the most promising. If tail dependence is found, all copulas exhibiting tail independence can be eliminated, and vice versa. In a two-dimensional setting this approach is appealing, but for realistic problems with more than tweo dimensions, basing a selection on tail dependence becomes problematic, since different forms of tail dependence can be defined (see Schmidt and Stadmüller, 2003). One approach would be to base the decision on pairwise comparisons. Another drawback is that only choices between a copula with and a copula without tail dependence can be made.

To compare the outcome of a selection based on tail dependence with our results, we apply the method proposed by Poon *et al.* (2004). In a bivariate setting, measures for tail dependence are derived from the limiting behavior of one random variable, conditional on the other being more and more extreme. Coles *et al.* (1999) construct two tail dependence parameters, one describing the behavior of two asymptotically dependent random variables, and the other for two asymptotically independent random variables. Both dependence parameters can be directly estimated and used to test for dependence or independence, but Ledford and Tawn (1996) show that tests based on the parameter estimate that describes the behavior of asymptotically dependent random variables are biased towards rejecting independence, which is why Poon *et al.* (2004) use the second measure.

[16] Because the Student's t copula converges to the normal copula for $\nu \to \infty$, a likelihood ratio test for the restriction that the degrees of freedom are high (say $\nu = 10,000$) can be used to test the Gaussian copula versus the Student's t copula. We estimate a log likelihood value of 218.47 for a Student's t copula with $\nu = 10,000$ degrees of freedom, which results in an adjusted likelihood ratio statistic of $-2 \cdot (218.47 - 230.47)/2 = 12$. The original statistic is halved to take the estimation of the parameters of the marginal models into account when comparing with the usual critical value. The p-value of 0.00053 leads to rejection of the restriction and hence the Gaussian copula.

[17] Using the value for the log likelihood function in Table 3.2, we find for the AIC values of 430.88 (Gaussian), 452.94 (Student's t) and 363.10 (Gumbel); and for BIC of 196.50 (Gaussian), 201.22 (Student's t) and 168.92 (Gumbel).

Poon *et al.* (2004) define the tail dependence measure $\bar{\chi}$, describing the behavior of asymptotically independent random variables X_1 and X_2 as:

$$\bar{\chi} = \lim_{s\to\infty} \frac{2\log \Pr(S_2 > s)}{\log \Pr(S_1 > s, S_2 > s)} - 1, \tag{3.13}$$

with $S_i = -1/\log F_i(X_i)$, F_i being the marginal cdf for X_i. By construction, $-1 \leq \bar{\chi} \leq 1$, while $\bar{\chi} = 1$ indicates that the two variables are asymptotically dependent. Rejection of the hypothesis $\bar{\chi} = 1$ leads to copulas exhibiting tail independence, and failure to reject it leads to copulas exhibiting tail dependence. Poon *et al.* construct their estimate for $\bar{\chi}$ based on the estimated (right) tail index α for the variable $S_{\min} = \min(S_1, S_2)$ by $\bar{\chi} = 2/\alpha - 1$.[18]

One way to extend their approach to a setting with more than two dimensions, is to apply it to each possible bivariate combination of the random variables. This results in three analyses. For consistency we construct the marginal distributions in the same way as for the estimation of the copula parameters. We measure tail dependence both for the left tails and for the right tails of the distribution. The results in Table 3.3 are mixed, however: for three combinations of stock, bond and real estate returns, tail dependence is clearly rejected, for two combinations, the tests clearly fail to reject, and for one combination (right tail-independence for bond and real estate returns) the hypothesis is rejected at the 5% level but not at the 2.5% level. Based on these results, we cannot decide which copula to use.

The tests that we propose directly consider the fit of the copulas on the observed data, instead of being based on pairwise analyses. Moreover, they yielded a clear preference for the Student's t copula. As a trivariate Student's t copula implies bivariate Student's t copulas for each combination of two out of the three variables, the results of Poon *et al.*'s approach seem inconsistent with ours. However, as their approach considers variables two by two, it is less efficient, which can influence the outcomes, in particular for dependence models. Furthermore, Coles *et al.* (1999) remark that the estimation of $\bar{\chi}$ can also be subject to biases. To get more insight in the actual tail behavior we investigate the tails in more detail in the next subsection.

3.3.4 Tail behavior

Figure 3.1 presents the tail behavior of the return series themselves and the tail behavior that the Gaussian, Student's t and Gumbel copulas imply given the parameter estimates. As a starting point we take returns with a marginal cumulative probability of 0.10 and calculate the joint probability of returns below these values.

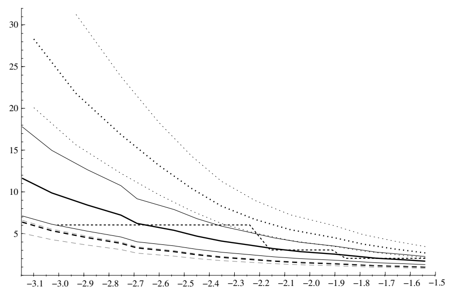
[18] This approach is based on Ledford and Tawn (1996, 1998).

Table 3.3: Estimates for the asymptotic independence parameter $\bar{\chi}$

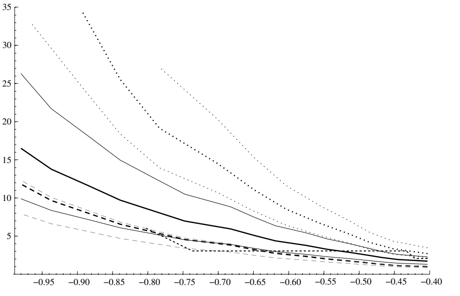
	stocks and bonds	stocks and real estate	bonds and real estate
$\bar{\chi}_{\mathrm{l}}$	0.0049 (0.19)	0.69 (0.32)	0.36 (0.26)
	[$< 10^{-3}$]	[0.17]	[0.007]
$\bar{\chi}_{\mathrm{r}}$	0.019 (0.20)	0.619 (0.31)	0.48 (0.29)
	[$< 10^{-3}$]	[0.11]	[0.035]

We report the estimated asymptotic independence parameters for both the left tails and the right tails of the different combinations of the returns on stock, bond and real estate indexes. The estimates are based on the tail index estimate for the right tail resulting from Huisman *et al.* (2001)'s modified Hill-estimator, which is applied to the series that results from taking the minimum of each couple of transformed observations. The observations are the returns on the S&P 500 Composite Index, the JP Morgan Government Index and NAREIT All Index from January 1, 1999 to December 17, 2004. All observations x_i are transformed to $-1/\log F_i(x_i)$ for the left tail-independence parameter and $-1/\log(1 - F_i(x_i))$ for the right tail-independence parameter. The marginal distributions are constructed by the semi-parametric method by Daníelsson and de Vries (2000) (see Table 3.1). Standard errors are reported in parentheses. In brackets the p-values for the hypothesis $\bar{\chi} = 1$ are reported.

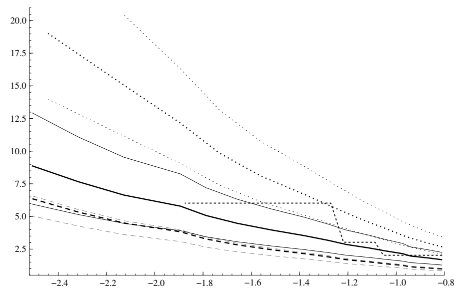
Figure 3.1: Expected waiting time of extreme events



(a) stocks



(b) bonds



(c) real estate

This figure presents the expected waiting time (in years) for the joint occurrence of returns below thresholds. The expected waiting time is calculated as the inverse of the joint probability. The basic thresholds are selected as those returns with a marginal cumulative probability of 0.10, which gives -1.54% for stocks, -0.40% for bonds and -0.81% for real estate. For each of the three categories, the corresponding subfigure shows the expected waiting time if the corresponding threshold is reduced, while the others remain at their basic level. We plot waiting times for the Gaussian (dotted), Student's t (solid), Gumbel (long dashed) and empirical (dashed, piecewise linear) copulas. The thick lines show the point estimates, the thin lines show the 95% confidence intervals. The parameter estimates are reported in Tables 3.1 and 3.2. The confidence intervals are based on 200 parameters drawings based on the estimated Hessian matrix.

By calculating the joint probability after reducing one of the returns we show the influence of the copula on the probability of more and more extreme events. We also graph 95%-confidence intervals for the calculated probabilities, based on the estimated variance of the parameter estimates, to show whether the copulas entail significantly different probabilities.

The choice between the copulas has a large impact on the joint probabilities. Under the assumption of independence, the joint probability of three returns below the threshold returns that have a marginal cumulative probability of 0.10 simply equals $0.10^3 = 0.001$ or one day per 48 months. Using the Gaussian copula, this probability becomes 0.0015 (one day per 30 months), for the Student's t copula it increases to 0.0024 (one day per 20 months), while it equals 0.0042 (one day per 11 months) for the Gumbel copula. If the events get more extreme (i.e. the returns are reduced), the probabilities implied by the Gaussian copula decrease much faster than those implied by the Student's t or Gumbel copula, and consequently, the average waiting periods implied by the Gaussian copula increase much faster.

A second conclusion from these graphs, is that the differences between the different probabilities are significant, as indicated by the non-overlapping confidence intervals. Because the probabilities implied by the empirical copula fall largely in the confidence interval for the Student's t probabilities, this shows once again that the Gaussian and Gumbel copulas differ significantly from the empirical dependence patterns, while the Student's t copula does not, exactly what our tests indicated. The Student's t copula provides an accurate estimate of the risk of joint downside movements. On the contrary, the Gaussian copula significantly underestimates this risk, while the Gumbel copula overestimates it.

This analysis can be linked directly to stress tests for risk management. In a stress test, a risk manager analyzes a portfolio of assets for an extreme event taking place (see Longin, 2000; Kupiec, 1998). Berkowitz (2000) argues that the probability for stress tests should be included in such an analysis to retain consistency with other elements of the risk management system. A stress test for a portfolio for which the prices of stocks, bonds and real estate are the risk factors would hence consist of specifying extreme events, i.e. returns below a threshold, calculating the probability of the event and analyzing the impact on the portfolio. Our analysis shows that the weight given to a stress test is largely influenced by the chosen copula, as the stress test has a considerably different probability of occurrence depending on which copula is used.

3.4 Conclusions

In this chapter we have considered copula selection. Because accurately modelling dependence is crucial to many fields in finance, a dependence model should be selected prudently. Both recent theoretical and empirical evidence have cast doubt on the accuracy of the Gaussian copula that is implied by using correlations. We discuss how traditional tests for distributional assumptions, being the Kolmogorov-Smirnov and Anderson-Darling tests, can be implemented to determine the accuracy of the Gaussian and alternative copulas, such as the Student's t and Gumbel copula. These tests are preferable to existing tests in the literature, as they directly compare the fit of the copula on observed dependence, while the existing tests only use indirect comparisons. Moreover, they can be applied more generally, while several existing tests can only be used in bivariate cases or for elliptical copulas. Finally, while the choice of test leaves some flexibility – the Kolmogorov-Smirnov-based tests are more sensitive to fit in the center and the Anderson-Darling-based tests more to fit in the tails – the complete dependence pattern is taken into account, contrary to approaches that focus exclusively on dependence of extreme returns.

We apply the tests to choose between the Gaussian, Student's t and Gumbel copula to model the dependence between three broad indexes for stocks, bonds and real estate. Since investors are typically averse to downside risk, the dependence model that they use should not only capture dependence in the center, but also the dependence in the tails, to accurately incorporate the risk of joint downside movements. The Gaussian copula, which focuses on dependence in the center and exhibits tail independence, and the Gumbel copula, which focuses mostly on dependence in the tails, are clearly rejected, while the Student's t copula, which can capture both central and tail dependence, is not. In contrast, we show that the selection procedure proposed by Poon *et al.* (2004) does not lead to unambiguous results. As this procedure is based on bivariate tail dependence, this comparison demonstrates the disadvantages of using a procedure based on an analysis of pairwise dependence. In a detailed inspection of the tails we find that the Student's t copula captures the empirical tail behavior accurately, while the Gaussian copula underestimates the risk of joint downward movements and the Gumbel copula overestimates it. While this result has a direct impact on stress tests in a risk management system, it can also influence investor's optimal allocation, in particular when downside risk aversion is taken into account.

In this chapter we have taken an unconditional approach. While it can be argued that conditional aspects should be taken into account (e.g. ARCH-effects), it is debated whether models for extreme returns benefit from a conditional approach (see also the discussion in Daníelsson and de Vries, 2000). In particular for stress tests, a conditional approach may be undesirable.

At the heart of the concept [systemic risk] *is the notion of "contagion", a particular strong propagation of failures from one institution, market or system to another.*
Systemic Risk: a Survey – Olivier de Bandt & Philipp Hartmann

Chapter 4

Portfolio implication of systemic crises*

4.1 Introduction

In this chapter we focus on the consequences of crises and crashes on asset allocation. De Bandt and Hartmann (2000) and Dow (2000) provide excellent surveys on the characteristics and causes of systemic crises for the different financial markets, including banking, currency, credit and equity markets. We concentrate on the consequences of systemic crises for investors in international equity markets. International investors suffer from the deterioration of the risk and return characteristics, as a systemic crisis exhibits a sharp drop in returns, an upswing in volatilities and a rise of the correlations between financial markets on a global scale. Evidence of this behavior has been based on the October 1987 stock market crash, and the crises that originated from the emerging markets in the 1990s (e.g. the Mexican crisis of 1994, the Asian crisis of 1997 and the Russian crisis of 1998).[1] Due to their irregular

*This chapter is forthcoming as Kole, Koedijk, and Verbeek (2006) in the *Journal of Banking & Finance*.

[1]For research directly aimed at the October 1987 crash we refer to Roll (1988a, 1989), Bertero and Mayer (1990) and King and Wadhwani (1990). Calvo and Reinhart (1996) discusses the Mexican crisis, Kaminsky and Schmukler (1999) and Baig and Goldfajn (1999) investigate the Asian crisis, while Kaminsky and Reinhart (2002) cover the Asian and Russian crises. A more general overview is given in De Bandt and Hartmann (2000).

and rare occurrence, standard models that investors use to support their asset allocation decisions typically fail to account for systemic crises, resulting in suboptimal international asset allocations.

The implications of systemic crises for equity portfolios have been studied by Das and Uppal (2004), who conclude that they are limited. However, their approach assumes that a systemic crises is a short-lived event that is hardly persistent. On the contrary, recent crises and their aftermaths have lasted several months, indicating persistence. If the risk-return trade-off deteriorates for a longer period, the impact of systemic crises for investors will be more severe. In order to include possible persistence, we propose to investigate this issue by means of a regime switching model in the style of Ang and Bekaert (2002), which we combine with optimal portfolio construction as set out by Merton (1969, 1971). This approach allows us to model the behavior of asset returns on a regime by regime basis, making it both simple and flexible. Formulating and solving the asset allocation problem in continuous time ensures analytical tractability.

We distinguish between two strategies that a utility-maximizing investor can adopt to solve his asset allocation problem: a crisis conscious and a crisis ignorant strategy. The crisis conscious strategy includes a systemic crisis as a distinct regime in which all markets encounter a shock, while the crisis ignorant strategy does not. For both strategies, we construct optimal portfolios. By comparing the portfolios we assess the implications and importance of a systemic crisis. For a US-based global investor, who can invest in stock markets in the US, Europe, Japan, Hong Kong, Thailand, Korea and Brazil, and a riskless asset, we find that the crisis conscious strategy leads to a reduction of the investments in risky assets and a shift to countries less prone to a crisis. A small probability of a crisis (of say 5%) already causes these adjustments, and they quickly become more pronounced if the probability increases. Ignoring a crisis is costly, as the investor requires a certainty equivalent return of 1.13% per year as a compensation if he has no information on the ex-ante probability of a crisis. If the investor knows with almost certainty that a crisis occurs, this compensation can easily exceed 3% per month.

We make several contributions to the literature investigating the influence of extreme returns and regime switches on asset allocation. We extend the analysis of Das and Uppal (2004) in three important aspects. First, our model is better able to capture the persistence of a crisis, because we include a systemic crisis as a distinct regime in a regime-switching model, while they incorporate it by adding a perfectly correlated jump to a geometric Brownian motion.[2] Second, we analyze the impact of systemic crises in a dynamic setting, which can adapt to changes in the behavior of asset prices. Third, our model without a crisis is more realistic, as the model proposed by Das and Uppal implies a normal distribution with a constant mean and variance. We also extend the work of Liu *et al.* (2003) by showing the effects of systemic crises on diversification, while their model is limited to a univariate setting with one risky asset only. Our finding that persistence is an important aspect of systemic crisis is consistent with their results. Our approach is complementary to Ang and Bekaert (2002, 2004), who consider international asset allocations in a regime-switching framework, as we use a similar framework to concentrate on the effects of systemic crises. Because of the severity of the crisis regime, we find larger effects of regime switches on diversification. As another extension to their work we show how the resulting allocation problem can be solved in continuous time.

In a broader sense our study can be seen as an investigation of the hypothesis that diversification advantages fail to be realized due to increasing correlations during market downturns, such as systemic crises. This claim has been put forward by various authors[3], but it is not clear how strong this effect is. Ang and Chen (2002) conclude that the costs of ignoring increasing correlations during bear markets are substantial, but Ang and Bekaert (2002) find that diversification advantages remain present. In our approach, an increase in correlations is inherent in a crisis[4]. If the probability with which a crisis hits increases, diversification possibilities erode rapidly and cause large divestments. If the investor faces short sales constraints, he completely withdraws from equity markets.

The outline of the chapter is as follows. In Section 2 we discuss how the crisis conscious and crisis ignorant strategies produce optimal portfolios and how the portfolios can be compared. Section 3 presents the actual design of the study, including the data. We discuss the estimation results in section 4, and derive and and compare the allocations produced by the different strategies in section 5. Section 6 concludes.

[2] In a related paper, Das and Uppal (2003) also consider a regime switching model to allow for stronger persistence and conclude that it does not change their main conclusions. However, the degree of persistence they consider is fairly low compared to our analysis. For higher levels of persistence systemic crises are likely to have more severe consequences, which is also indicated by the result in Das and Uppal (2003) that the effects of systemic crises are increasing and convex for increasing levels of persistence.

[3] See, for instance, Boyer *et al.* (1999); Loretan and English (2000); Longin and Solnik (2001); Campbell *et al.* (2002); Ang and Chen (2002); Ang and Bekaert (2002); Campbell *et al.* (2003).

[4] It is widely discussed whether the tendency of markets to move downward together is a form of contagion or can be explained by joint shocks (see Forbes and Rigobon, 2002; Corsetti *et al.*, 2005).

4.2 A crisis conscious and a crisis ignorant strategy

The investor can adopt two strategies to construct an optimal portfolio: a crisis conscious and a crisis ignorant strategy. Both strategies contain a model for the return process and a formulation of the asset allocation problem as their main ingredients. In both cases, a Markov regime switching model describes the return process because of its flexibility to capture heteroskedasticity (see Hamilton and Susmel, 1994) and fat tails (see Timmermann, 2000). The difference between the strategies is the presence of distinct crisis regimes in the model employed by the crisis conscious strategy. The model in the crisis ignorant strategy can be interpreted as a restricted version of that in the crisis conscious strategy.

We assume that the investor formulates and solves his asset allocation problem in an expected utility, continuous time framework. Because of the continuous time approach, the problem has a closed-form solution, contrary to the numerical approach of Ang and Bekaert (2002). The different models for the return process will lead to different allocations for the two strategies. Because the investor constructs his asset allocation under expected utility, we can determine the economic importance of those differences by calculating the certainty equivalent return needed to compensate the investor for incorrectly using the crisis ignorant strategy.

In the first subsection, we discuss the models for the return process. In the next one, we formulate and solve the investor's asset allocation problem. Subsection 3 shows how the continuous time return process in the model of the asset allocation problem should be constructed to make it consistent with the predictions resulting from the Markov regime switching models. In subsection 4 we derive the compensation (as certainty equivalent return) that the investor requires if he incorrectly adopts the crisis ignorant strategy. We conclude by showing how the differences between the allocations of the crisis conscious and crisis ignorant strategies can be explained.

4.2.1 Regime switching models for the return process

We start with the more general model that is used in the crisis conscious strategy, and consider the restricted version in the crisis ignorant strategy subsequently. The general model for the return process consist of several regimes. The behavior of the return process in a regime corresponding to a normal period is made up of basic components only, while its behavior in a regime corresponding with a systemic crisis contains both basic and crisis components. By choosing this setup, a systemic crisis can be clearly interpreted as a simultaneous shock to all assets, which comes on top of the normal behavior of the asset. We assume that the investor can invest in n assets.

First, consider the set of states that apply to the return process. We assume that each asset i's basic return component can be in a regime Q_i from a set of K regimes. For the crisis component two regimes Q_c are available: presence ($Q_\mathrm{c} = 1$) and absence ($Q_\mathrm{c} = 0$), and since the crisis is systemic the crisis regime applies to all assets. Consequently, the state that applies to the joint returns, $\tilde{\boldsymbol{Q}}$, is completely defined by the combination of each asset's basic regime and the crisis regime[5]:

$$\tilde{\boldsymbol{Q}} \equiv (Q_1, Q_2, \dots, Q_n, Q_\mathrm{c}). \tag{4.1}$$

We use $\boldsymbol{Q}$ to denote the combination of basic regimes, $\boldsymbol{Q} \equiv (Q_1, Q_2, \dots, Q_n)$. The sets $\mathbb{Q}$ and $\tilde{\mathbb{Q}}$ collect all possible state vectors $\boldsymbol{Q}$ and $\tilde{\boldsymbol{Q}}$, respectively. The actually prevailing state will never be known with certainty. Instead, each state prevails with a certain probability, inferred from the data.

The return vector can be split into a basic component and a crisis component. We assume that for each state $\boldsymbol{Q} \in \mathbb{Q}$ the basic component $\mathbf{x}$ is a normally distributed random vector, with a state-specific mean $\boldsymbol{\mu}_{\boldsymbol{Q}}$ and variance matrix $\boldsymbol{\Omega}_{\boldsymbol{Q}}$. The marginal distribution of asset i's basic component depends only on the regime Q_i that applies. The crisis component consists of a shock, represented by a univariate random variable x_c, to which each asset has a specific sensitivity δ_i. Following Das and Uppal (2004) and Liu *et al.* (2003) we assume that the shock has a normal distribution, with mean μ_c and variance ω_c. Combining the two components gives the return vector:

$$\boldsymbol{r} = \mathbf{x} + Q_\mathrm{c} x_\mathrm{c} \boldsymbol{\delta}, \quad Q_\mathrm{c} \in \{0, 1\}, \tag{4.2}$$

where $\boldsymbol{\delta}$ is the vector of sensitivities. Conditional on the state $\tilde{\boldsymbol{Q}} \in \tilde{\mathbb{Q}}$, the return is normally distributed, being the sum of two (conditionally) normally distributed variables. Under the assumption that the shock and the basic component are independent, the mean vector $\boldsymbol{\mu}_{\tilde{\boldsymbol{Q}}}$ and variance matrix $\boldsymbol{\Omega}_{\tilde{\boldsymbol{Q}}}$ of the return can be written as:

$$\boldsymbol{\mu}_{\tilde{\boldsymbol{Q}}} = \boldsymbol{\mu}_{\boldsymbol{Q}} + Q_\mathrm{c} \mu_\mathrm{c} \boldsymbol{\delta}, \tag{4.3}$$

$$\boldsymbol{\Omega}_{\tilde{\boldsymbol{Q}}} = \boldsymbol{\Omega}_{\boldsymbol{Q}} + Q_\mathrm{c} \omega_\mathrm{c} \boldsymbol{\delta}\boldsymbol{\delta}' \tag{4.4}$$

Because we want the shock to have the same direction for each asset, we require $\delta_i \geq 0$ for each i. Consequently, each variance and covariance term will increase if $Q_\mathrm{c} = 1$.[6]

[5] We use the expression state to refer to a combination of the basic regimes and the crisis regimes.

[6] The correlation will rise if the relative increase in covariance exceeds the product of relative increase in volatilities. This condition will generally be satisfied for correlations not too close to 1. In our model this change in correlation is completely due to the occurrence of a crisis. Consequently, using the terminology of Forbes and Rigobon (2002), our model implies interdependence of assets but not contagion.

The transaction probabilities are constant over time, and we impose a specific structure to capture volatility spill-over effects, being the tendency of high volatility in one asset's return to spread to other assets. These effects are widespread and important.[7] Let π^{ab} denote the conditional probability of a switch to state $\tilde{\boldsymbol{Q}}^a$, given that the current state is $\tilde{\boldsymbol{Q}}^b$. First, we impose a structure based on the crisis regime.

- If a crisis occurs neither in the current state ($Q_\mathrm{c}^b = 0$) nor in the destination state ($Q_\mathrm{c}^a = 0$), we model π^{ab} as the product of the marginal conditional probabilities $\pi_i(Q_i^a|\tilde{\boldsymbol{Q}}^b)$ and the conditional probability that a crisis does not occur, given the current state $\tilde{\boldsymbol{Q}}^b$. Here, π_i^{ab} gives the probability that asset i switches to regime Q_i^a, given that the current state is $\tilde{\boldsymbol{Q}}^b$. The dependence on the state $\tilde{\boldsymbol{Q}}^b$ instead of the asset-specific regime Q_i^b introduces dependence across assets and enables the incorporation of volatility spill-over effects. We use a multinomial logistic model to model this dependence, which we discuss in Appendix 4.A.1.[8] The alternative of free parameters would lead to an exploding number of free parameters equal to $2K^n \cdot (2K^n - 1)$.

- If the assets enter a crisis regime (so $Q_\mathrm{c}^a = 1$ and $Q_\mathrm{c}^b = 0$, the regime processes switch to the regime with the highest volatility. This restriction imposes that global stress triggers local stress.

- For the case that a crisis remains, the same restriction as in the previous case applies. It prohibits illogical switches from high volatility regimes to lower volatility regimes, while a crisis remains present.

- If the assets leave the crisis regime ($Q_\mathrm{c}^a = 0$ and $Q_\mathrm{c}^b = 1$), the basic regime processes remain in the highest volatility regimes in the next period. After that, they can switch to other regimes. This restriction captures a gradual cooling down of assets after a crisis.

The crisis transition probabilities are independent of the basic regimes. This leads to two parameters for the crisis transition probabilities: π_c^{10}, the probability that a crisis occurs, and π_c^{11}, the probability that a crisis remains. If we assume without loss of generality that the regimes are ordered in ascending order of volatility, we can represent our model as follows:

[7] Several authors have established volatility spill-over effects for different markets: Hamao *et al.* (1990) for the US, UK and Japan; Bekaert and Harvey (1997) and Ng (2000) for the US, Japan, and other Pacific-Basin markets; Edwards and Susmel (2001) for emerging markets in Asia and Latin America; Lee *et al.* (2004) for the US and Asian markets; and Baele (2005) for European countries.

[8] Bae *et al.* (2003) use a related model to investigate contagion.

$$\pi^{ab} = \begin{cases} \pi_1^{ab} \cdot \pi_2^{ab} \cdots \pi_n^{ab} \cdot (1 - \pi_{\mathrm{c}}^{10}) & \text{if } Q_{\mathrm{c}}^a = 0, Q_{\mathrm{c}}^b = 0 \\ \pi_{\mathrm{c}}^{10} & \text{if } Q_{\mathrm{c}}^a = 1, Q_{\mathrm{c}}^b = 0, \forall i\ Q_i^a = K \\ \pi_{\mathrm{c}}^{11} & \text{if } Q_{\mathrm{c}}^a = 1, Q_{\mathrm{c}}^b = 1, \forall i\ Q_i^a = K \\ 1 - \pi_{\mathrm{c}}^{11} & \text{if } Q_{\mathrm{c}}^a = 0, Q_{\mathrm{c}}^b = 1, \forall i\ Q_i^a = K \\ 0 & \text{otherwise} \end{cases} \tag{4.5}$$

The crisis ignorant strategy imposes the restriction that transitions to $Q_{\mathrm{c}}^a = 1$ have zero probability, i.e. $\pi_{\mathrm{c}}^{10} = 0$. Consequently, the crisis ignorant strategy leads to different inferences and forecasts about the prevailing regime. Also, if the investor incorrectly follows an ignorant strategy and estimates the parameters of the process, the parameter estimates are likely to be biased.

4.2.2 The asset allocation problem

The investor is risk averse and maximizes his utility over terminal wealth W_T. We assume he has a power utility function:

$$U(W_T) = \frac{W_T^{1-\gamma}}{1-\gamma}, \quad \gamma > 0, \gamma \neq 1, \tag{4.6}$$

where γ is the investor's coefficient of relative risk aversion[9]. To focus on the effect of a systemic crisis on asset allocation, we do not allow intermediate consumption. As such, our analysis is comparable to other studies such as Ang and Bekaert (2002), Liu *et al.* (2003) and Das and Uppal (2004). The investor can trade in continuous time. At each point in time t he will choose to invest proportions of his wealth in the n risky assets, collected in the vector $\boldsymbol{\phi}_t$ and the remaining part $1 - \boldsymbol{\phi}_t' \boldsymbol{\imath}_n$ in the riskless asset ($\boldsymbol{\imath}_n$ being a vector of size n with ones) in order to maximize expected utility.

We assume that the investor has an initial endowment W_0. This assumption and a process for the asset prices enables us to derive the investor's self-financing constraint, which describes the dynamics of the wealth process. The returns follow an Itô process

$$\mathrm{d}\boldsymbol{r} = \boldsymbol{\mu}(\boldsymbol{r}, t)\,\mathrm{d}t + \boldsymbol{\Lambda}(\boldsymbol{r}, t)\,\mathrm{d}\mathbf{Z}, \tag{4.7}$$

where $\boldsymbol{\mu}(\boldsymbol{r}, t)$ is the vector of instantaneous drift rates, which can depend on the return up to time t and time itself, $\boldsymbol{\Lambda}(\boldsymbol{r}, t)$ is a lower triangular $n \times n$ matrix that can also be a function of $\boldsymbol{r}$ and t, and $\mathrm{d}\mathbf{Z}$ is a vector of n independent Wiener

[9]For $\gamma = 1$ the utility function is defined as log utility $U(W_T) = \ln W_T$.

processes. Consequently, the instantaneous variance rate $\boldsymbol{\Omega}(\boldsymbol{r},t)$ is given by $\boldsymbol{\Omega}(\boldsymbol{r},t) = \boldsymbol{\Lambda}(\boldsymbol{r},t)\boldsymbol{\Lambda}(\boldsymbol{r},t)'$. Below we describe a specific function for the drift and variance rate that makes them consistent with the predictions of the regime switching models of the previous section. For notational convenience we drop the time and return dependence of $\boldsymbol{\mu}$, $\boldsymbol{\Lambda}$ and $\boldsymbol{\Omega}$. After applying Itô's lemma to find the price processes we end up with the self-financing condition

$$\frac{\mathrm{d}W}{W} = r_\mathrm{f}\,\mathrm{d}t + \boldsymbol{\phi}'\boldsymbol{\alpha}\,\mathrm{d}t + \boldsymbol{\phi}'\boldsymbol{\Lambda}\,\mathrm{d}\mathbf{Z}, \tag{4.8}$$

where $\boldsymbol{\alpha} \equiv \boldsymbol{\mu} + \frac{1}{2}\mathrm{diag}(\boldsymbol{\Omega}) - r_\mathrm{f}\boldsymbol{\imath}_n$[10], r_f is the risk-free rate, and $\mathrm{diag}(\boldsymbol{\Omega})$ denotes a vector containing the diagonal elements of $\boldsymbol{\Omega}$.

The asset allocation problem consists of maximizing the investors expected utility subject to the self financing condition in Eq. 4.8. It can be solved using standard stochastic control techniques.[11] We show in Appendix 4.A.2 that the optimal portfolio is given by

$$\boldsymbol{\phi}^* = \gamma^{-1}\boldsymbol{\Omega}^{-1}\boldsymbol{\alpha} = \gamma^{-1}\boldsymbol{\Omega}^{-1}\left(\boldsymbol{\mu} + \tfrac{1}{2}\mathrm{diag}(\boldsymbol{\Omega}) - r_\mathrm{f}\boldsymbol{\imath}_n\right). \tag{4.9}$$

Though this expression has the same structure as the solution to a standard mean-variance optimization problem, both $\boldsymbol{\mu}$ and $\boldsymbol{\Omega}$ will be shown to depend on time and the observed returns, making the weights depending on them as well. This expression also applies to the log-utility investor.

4.2.3 The Itô process for returns

Brigo (2002) describes a way to derive continuous time processes whose corresponding density at a certain point in time is a mixture of densities from the same family.[12] This approach has two advantages. First, it facilitates the use of the powerful techniques developed for continuous time finance. Second, the continuous time processes that result from his approach include the different regimes directly in their parameters without introducing extra state variables for the different regimes. Consequently, the regimes are implicitly present in the portfolio optimization (i.e. in the parameters of the Itô process), and do not lead to regime-specific Itô processes.

Since the distribution of $\boldsymbol{r}_{\tau+1}$ conditional on its filtration is a mixture of normal distributions (see Hamilton, 1994, Ch. 22), we can apply a multivariate extension of Theorem 2 in Brigo (2002). Consequently, the Itô process in Eq. (4.7) starting at

[10] The expression can be interpreted as an excess, arithmetic mean return.

[11] See for example Léonard and Van Long (1992).

[12] Applications of this technique can be found in Alexander and Narayanan (2001), Alexander and Scourse (2004) and Brigo and Mercurio (2002).

$t_0 = \tau$ with $\boldsymbol{r}_{t_0} = \mathbf{0}$ has a mixture density at time $\tau + 1$ which corresponds with the mixture model implied by the regime switching model, if the instantaneous drift rate $\boldsymbol{\mu}(r,t)$ and instantaneous variance rate $\boldsymbol{\Omega}(r,t)$ are given by:

$$\boldsymbol{\mu}(r,t) = \sum_{\tilde{\boldsymbol{Q}} \in \tilde{\mathbb{Q}}} \pi(\tilde{\boldsymbol{Q}}, \boldsymbol{r}, t) \mu_{\tilde{\boldsymbol{Q}}} \tag{4.10}$$

$$\boldsymbol{\Omega}(r,t) = \sum_{\tilde{\boldsymbol{Q}} \in \tilde{\mathbb{Q}}} \pi(\tilde{\boldsymbol{Q}}, \boldsymbol{r}, t) \boldsymbol{\Omega}_{\tilde{\boldsymbol{Q}}} \tag{4.11}$$

with

$$\pi(\tilde{\boldsymbol{Q}}, \boldsymbol{r}, t) = \frac{\xi_{\tau+1|\tau}(\tilde{\boldsymbol{Q}}) \cdot f\left(\boldsymbol{r}_t; \boldsymbol{\mu}_{\tilde{\boldsymbol{Q}}}(t-\tau), \boldsymbol{\Omega}_{\tilde{\boldsymbol{Q}}}(t-\tau)\right)}{\sum_{\hat{\boldsymbol{Q}} \in \tilde{\mathbb{Q}}} \xi_{\tau+1|\tau}(\hat{\boldsymbol{Q}}) \cdot f\left(\boldsymbol{r}_t; \boldsymbol{\mu}_{\hat{\boldsymbol{Q}}}(t-\tau), \boldsymbol{\Omega}_{\hat{\boldsymbol{Q}}}(t-\tau)\right)}, \tag{4.12}$$

where $\tau < t \leq \tau{+}1$, and $\xi_{\tau+1|\tau}(\tilde{\boldsymbol{Q}}) = \Pr(\tilde{\boldsymbol{Q}}_{\tau+1}|\mathcal{F}_\tau)$ gives the forecast probability that state $\tilde{\boldsymbol{Q}}$ is prevailing at time $\tau{+}1$.[13] $\boldsymbol{\Lambda}(\boldsymbol{r}, t)$ can then be found by applying a Cholesky decomposition to $\boldsymbol{\Omega}(\boldsymbol{r}, t)$. For $t = \tau$, Eq. (4.12) reduces to $\pi(\tilde{\boldsymbol{Q}}, 0, \tau) = \xi_{\tau+1|\tau}(\tilde{\boldsymbol{Q}})$.

The drift and variance rates constructed by Eqs. (4.10) to (4.12) have an appealing interpretation, as they are probability weighted averages of the mean and variance parameters for the different states. These probabilities have a clear interpretation as inference probabilities, which are commonly used in regime switching models (see Hamilton, 1994, Eq. 22.4.5). Furthermore, Eq. (4.12) follows a Bayesian update rule, with $\xi_{\tau+1|\tau}(\tilde{\boldsymbol{Q}}) = \Pr(\boldsymbol{Q}_{\tau+1}|\mathcal{F}_\tau)$ as prior probability for the prevailing regime and $\pi(\tilde{\boldsymbol{Q}}, \boldsymbol{r}, t) = \Pr(\boldsymbol{Q}_{\tau+1}|\boldsymbol{r}_t, \mathcal{F}_\tau)$ as its posterior probability.

4.2.4 Comparing portfolios

Though the expression for the optimal portfolio Eq. (4.9) is the same for both the crisis conscious and the crisis ignorant strategy, the resulting portfolios ($\boldsymbol{\phi}^{\mathrm{c}}$ and $\boldsymbol{\phi}^{\mathrm{i}}$ respectively) will differ because of differences in $\boldsymbol{\mu}_{\tilde{\boldsymbol{Q}}}$, $\boldsymbol{\Omega}_{\tilde{\boldsymbol{Q}}}$ and $\pi(\tilde{\boldsymbol{Q}}, \boldsymbol{r}, t)$. To assess the economic impact of the differences in portfolios we calculate the certainty equivalent return needed to compensate the investor for using the crisis ignorant strategy, when he should have used the crisis conscious one. Since the first does not take a crisis into account, the resulting portfolio will be suboptimal and yield lower utility. The certainty equivalent return shows by how much the initial wealth of the investor should be raised to compensate him for this utility loss and hence the cost of ignoring a crisis. We derive in Appendix 4.A.2 that the certainty equivalent return $\bar{r}$ needed for compensation equals

$$\bar{r} = [h(\boldsymbol{\phi}^{\mathrm{c}}) - h(\boldsymbol{\phi}^{\mathrm{i}})](T - t), \tag{4.13}$$

[13] We follow the notation in Hamilton (1994).

where $\boldsymbol{\phi}^{\mathrm{c}}$ denotes the crisis conscious portfolio, $\boldsymbol{\phi}^{\mathrm{i}}$ denotes the crisis ignorant portfolio and $h(\boldsymbol{\phi}) = \boldsymbol{\phi}'\boldsymbol{\alpha} - \frac{1}{2}\gamma\boldsymbol{\phi}'\boldsymbol{\Omega}\boldsymbol{\phi}$. This expression depends only on the coefficient of risk aversion γ via the function h and the portfolio $\boldsymbol{\phi}$. It is easy to show that $h(\gamma^{-1}\boldsymbol{\phi}) = \gamma^{-1}h(\boldsymbol{\phi})$. Consequently, the certainty equivalent return needed to compensate a power utility investor can be derived from the certainty equivalent return for the log utility investor. Moreover, the certainty equivalent return is a linear function of the investor's horizon T.

The portfolio differences can stem from differences in the estimates for the basic regimes, the estimation effect, and the absence of crisis regimes, the crisis effect. An analysis of these differences provides insights into the importance of both sources. Suppose that the differences in parameter estimates explain just a small part of the changes in the optimal allocations. In that case, the crisis regime is the main driver of the portfolio adjustments. Alternatively, if the differences in parameter estimates explain most of the changes in optimal portfolios, the influence of the crisis itself is limited. The observations that belong most likely to the crisis regime cause outlier problems in the crisis ignorant case.

In order to disentangle the differences between the optimal allocations produced by the crisis conscious and ignorant strategies we introduce a myopic strategy. This strategy uses the same estimates as the crisis conscious strategy, but excludes a crisis regime in the forecasts it makes. Instead of forecasts for state vectors in the complete state space $\xi_{\tau+1|\tau}(\tilde{\boldsymbol{Q}})$, only forecasts for the basic states are constructed $\xi^{\mathrm{m}}_{\tau+1|\tau}(\boldsymbol{Q})$:

$$\xi^{\mathrm{m}}_{\tau+1|\tau}(\boldsymbol{Q}) = \xi_{\tau+1|\tau}(\boldsymbol{Q}, Q_{\mathrm{c}} = 0) + \xi_{\tau+1|\tau}(\boldsymbol{Q}, Q_{\mathrm{c}} = 1), \quad \boldsymbol{Q} \in \mathbb{Q}. \tag{4.14}$$

The myopic strategy produces an allocation $\boldsymbol{\phi}^{\mathrm{m}}$. We interpret the differences between the myopic and the crisis ignorant strategy $\boldsymbol{\phi}^{\mathrm{e}} \equiv \boldsymbol{\phi}^{\mathrm{m}} - \boldsymbol{\phi}^{\mathrm{i}}$ as the estimation effect, and the differences between the crisis conscious and myopic strategy as the crisis effect, $\boldsymbol{\phi}^{\mathrm{s}} \equiv \boldsymbol{\phi}^{\mathrm{c}} - \boldsymbol{\phi}^{\mathrm{m}}$. We also calculate $h(\boldsymbol{\phi}^{\mathrm{m}})$, and derive the economic importance of the estimation effect as $\bar{r}^{\mathrm{e}} = [h(\boldsymbol{\phi}^{\mathrm{m}}) - h(\boldsymbol{\phi}^{\mathrm{i}})](T-t)$ and the importance of the crisis effect as $\bar{r}^{\mathrm{s}} = [h(\boldsymbol{\phi}^{\mathrm{c}}) - h(\boldsymbol{\phi}^{\mathrm{m}})](T-t)$.

4.3 Design of the analysis

Central in our analysis of the impact of systemic crises is a US investor who wants to diversify his portfolio internationally. He can invest worldwide and does not only consider the developed markets US, Europe, Japan and Hong Kong, but also the emerging markets Thailand, Korea and Brazil, which can extend diversification opportunities.[14] We represent each market by an index. For each index we construct a return series, on which the models for the crisis conscious and the crisis ignorant strategies are estimated. We assume that for each country two regimes can be distinguished. The estimation results are used to construct allocations. Based on the differences between the allocations resulting from the crisis conscious strategy and the crisis ignorant strategy we determine the impact of systemic crises.

We base the analysis on monthly returns, mainly because monthly data are available with the longest history. A systemic crises is a rare event, necessitating a relatively long history to get an accurate estimate of the probability of a systemic crisis. A longer horizon also improves the estimate for the mean returns in the different regimes. Each developed market is approximated by its corresponding gross return index from Morgan Stanley Capital International (MSCI). For the emerging markets we use the gross return indexes provided by Standard & Poors / International Finance Corporation (IFC), both provided by DataStream. We use the start of the IFC indexes, December 31, 1975 as a starting point for our analysis and collect the index values in dollars till December 31, 2004, resulting in 348 returns. We construct excess returns by subtracting the 1-month T-bill return from Ibbotson Associates, Inc.[15]

The summary statistics in Table 4.1 show the familiar picture of small, positive means, non-zero skewness and fat tails. Generally, the minimum exceeds the maximum in absolute value. The correlation matrix shows low levels of correlation, particularly for the emerging markets, implying the presence of diversification opportunities. However, Hong Kong, Thailand, Korea and Brazil may be less attractive due to their relatively high levels of volatility.

The regime switching models we propose in Section 4.2 belong to the standard regime switching models as discussed in Hamilton (1994). We use the expectation maximization algorithm (see Dempster *et al.*, 1977; Hamilton, 1990) to estimate the parameters in the models. Central in this algorithm is a filtering technique to determine the state probabilities at each point in time. If the improvement in the likelihood function falls below a specified limit, the algorithm stops. To ensure that the estimate covariance matrix is positive definite, we assume that the correlation matrix is independent of the basic state vectors.

[14]Early studies (see e.g. Harvey, 1995) find significant diversification opportunities, but more recent studies show these may be less when transaction costs and investment constraints are taken into account (see Bekaert and Harvey, 2003, for a discussion).

[15]We use the series available on the website of Kenneth French, http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Table 4.1: Descriptive statistics

(a) univariate statistics

	US	Europe	Japan	Hong Kong	Thailand	Korea	Brazil
mean	0.50	0.53	0.31	0.69	0.31	0.41	0.32
volatility	4.38	4.75	6.48	9.31	10.22	10.54	15.42
skewness	−0.76	−0.72	0.07	−1.08	−0.44	0.36	−0.49
kurtosis	5.98	4.78	3.48	8.43	6.01	5.81	6.01
minimum	−24.45	−21.65	−22.18	−57.58	−41.88	−41.37	−84.79
maximum	12.05	12.69	21.04	28.37	38.14	53.17	44.84

(b) correlation matrix

	US	Europe	Japan	Hong Kong	Thailand	Korea	Brazil
US	1	0.64	0.30	0.42	0.32	0.28	0.21
Europe	0.64	1	0.48	0.50	0.31	0.25	0.25
Japan	0.30	0.48	1	0.30	0.25	0.37	0.15
Hong Kong	0.42	0.50	0.30	1	0.39	0.21	0.21
Thailand	0.32	0.31	0.25	0.39	1	0.39	0.13
Korea	0.28	0.25	0.37	0.21	0.39	1	0.12
Brazil	0.21	0.25	0.15	0.21	0.13	0.12	1

The data set consists of the monthly excess gross returns (in %) for the MSCI US, MSCI Europe, MSCI Japan, MSCI Hong Kong, IFC Thailand, IFC Korea and IFC Brazil indexes, running from January 1976 to December 2004. Panel (a) presents univariate statistics, panel (b) shows the correlation matrix.

The expression for the optimal portfolio in Eq. (4.9) defines an asset allocation strategy in continuous time. This means that we can derive the evolution of portfolios over time for a given price path. We construct paths of portfolios based on daily prices of the mentioned gross return indexes in dollar terms, also gathered from DataStream. To keep the daily and monthly data sets consistent, we use the 1-month T-bill rate that was prevailing at the beginning of the month to compute the daily excess returns.

The portfolio at a given day of the month reflects two sources of information: a prior probability based on the information at the beginning of the month, and the information present in the returns observed until that day. The second source of information is used to update the prior probability to a posterior probability as in Eq. (4.12). We interpret the prior probability as the outcome of an investor's thorough analysis of the likelihood of a state. Because of its thoroughness such an analysis is conducted at a limited frequency, i.e. once per month. We represent the outcomes of the analysis by a Markov chain. The prior probability can be regarded as an informative prior, because it is based on all available information at the moment it is determined. We will also consider the allocation that results if the investor does not use the observed stock price path to determine the prior probabilities. Instead, the investor uses unconditional probabilities for each regime, which are only based on the transition matrices. The results from this analysis can serve as a benchmark and can be compared to the results in Das and Uppal (2004).

4.4 Estimation results

In this section we present and discuss the results from estimating the models for the returns in the two strategies. We split the discussion in two: first we consider the estimates for the distributions under the different regimes; next we turn to the parameter estimates for the transition probabilities of the Markov chains.

Table 4.2 presents the estimates for the parameters of the marginal normal distributions. The main difference between the crisis ignorant and crisis conscious strategy is the presence of the crisis regime in the latter one. The crisis regime contains a shock with an estimated mean of -0.63% and a volatility of 1.54%. The shock has been normalized such that the US has a sensitivity of 1. The other countries (except Europe) are more sensitive to the shock. In particular the emerging countries Korea and Thailand are more than 10 times as sensitive as the US. The considerable difference in log likelihood values[16] of 19.0, provides evidence in favor of the addition of a crisis regime. However standard statistical tests cannot be used, because many parameters are not identified under the null hypothesis.[17] In both models the two basic regimes for each asset can be distinguished by their volatility levels, as reported by Ramchand and Susmel (1998), Ang and Bekaert (2002) and Graflund and Nilsson (2003). In the remainder we will therefore use the terms low volatility regime and high volatility regime to distinguish between the regimes.

Combining the estimates for the basic high volatility regimes with the estimates for the crisis shock yields the crisis regime, which exhibits a sharp drop in expected returns and an increase in volatilities. Because of their large sensitivity to a crisis, these effects are most pronounced for emerging markets, which may also offer an explanation for the fat tails reported by Susmel (2001). Table 4.3 shows that the correlations in the crisis regime are also higher, as has been reported before by Ang and Chen (2002) and Forbes and Rigobon (2002). It is obvious that the risk-

[16]The log likelihood values for the models without and with a crisis equal -8029.5 and -8010.5, respectively.

[17]Hansen (1992) proposes a method to formally test whether the addition of a regime is a significant improvement. In this method, the likelihood function is maximized over different combinations of fixed values for the restricted and nuisance parameters. The number of combinations grows exponentially in the number of parameters, which makes the method less attractive to test the significance of the crisis regime.

Table 4.2: Univariate parameter estimates

		crisis ignorant		crisis conscious		
		low	high	low	high	crisis
US	μ	0.90	0.35	0.95	0.47	−0.16
	$\sqrt{\omega}$	2.47	4.99	2.21	4.73	4.97
						1.00
EU	μ	1.10	0.23	1.17	0.26	−0.30
	$\sqrt{\omega}$	3.09	5.46	3.07	5.35	5.52
						0.88
Japan	μ	0.44	0.03	0.20	0.74	−1.46
	$\sqrt{\omega}$	5.14	8.29	4.92	8.26	9.83
						3.47
Hong Kong	μ	0.93	0.57	0.92	0.74	−2.15
	$\sqrt{\omega}$	6.00	13.34	5.77	12.98	14.75
						4.55
Thailand	μ	0.80	−0.53	−0.22	1.51	−7.03
	$\sqrt{\omega}$	6.29	15.28	5.24	11.44	23.62
						13.46
Korea	μ	0.34	1.28	0.19	1.94	−6.55
	$\sqrt{\omega}$	7.26	15.69	6.56	12.44	24.04
						13.40
Brazil	μ	0.96	−0.73	0.86	−0.01	−2.43
	$\sqrt{\omega}$	9.83	20.91	8.49	19.49	20.35
						3.81
	μ_c					−0.63
	$\sqrt{\omega_c}$					1.54

This tables reports the estimates for the mean parameters (μ) and volatility parameters ($\sqrt{\omega}$) of the marginal distributions of the excess monthly equity return (in %) for the US, Europe Japan, Hong Kong, Thailand Korea and Brazil under the low and high volatility regimes. The first two columns present the estimates for the crisis ignorant strategy; the second two for the crisis conscious strategy. The parameters for the crisis conscious strategy also contain estimates for the shock: a mean (μ_c) and variance ($\sqrt{\omega_c}$). The last column contains the estimates for the sensitivity to a systemic crisis (δ) and the resulting mean and volatility. The sensitivity of the US market has been normalized to 1.

return trade-off for each asset deteriorates, while the correlation matrix shows that diversification possibilities also become less. Consequently, risky assets become less attractive on a global scale. However, the correlations between the emerging markets on the one hand and the US and Europe on the other hand remain low. The exact consequences become clear in the next section.

The addition of a crisis regime has important effects on the other estimates as well. Decreasing volatility estimates indicate that the risk within each regime is actually less, particularly those in the high volatility regime. Moreover, the means in the high volatility regimes increase considerably, indicating that the few crisis observations differ substantially from normal high volatility periods.

Table 4.4 presents the estimates for the parameters of the logistic functions that we use to construct the regime transition probabilities (see Appendix 4.A.1 for more details). In total, we have $2 \cdot 7^2 = 98$ parameters from which we construct the 128×128 $(= 2^7)$ basic transition matrix. The diagonal elements give the estimates that correspond with no regime switch for a country, given that the other countries are currently in their low volatility regimes. The off-diagonal elements give the volatility spill-over estimates, which are restricted to be negative (positive) for switches from the low (high) volatility regimes. Because of these restrictions, volatility spill-over effects increase the probability that countries are in their high volatility regimes. As an example, consider the Hong Kong market. The probability that the Hong Kong market remains in its low volatility regime, given that the other markets are also in their low volatility regimes equals $\mathrm{e}^{3.75}/(1+\mathrm{e}^{3.75}) = 0.98$. However, we find volatility spill-over from the US to Hong Kong: if the US is in its high volatility remain, the probability that Hong Kong switches from low to high volatility is $\mathrm{e}^{3.75-0.48}/(1+\mathrm{e}^{3.75-0.48}) = 0.96$.

We draw several conclusions from the estimates in Table 4.4. First of all, each regimes on itself is strongly persistent, as indicated by high values for the positive diagonal elements (exceptions are the high volatility regimes for Japan and Korea). Second, volatility spill-over effects mainly affect the probability of a switch from the low volatility to the high volatility regimes; the probability that a country remains in its high volatility regime, given that other countries are also in their high volatility regimes, is less affected. Third, volatility spill-over effects are mainly present among the developed markets, and from emerging markets to developed markets. Other studies finds volatility spill-over effects from the US and Japan to other Asian markets, such as Hong Kong, Thailand and Korea (see Bekaert and Harvey, 1997; Ng, 2000; Lee *et al.*, 2004), but this may be due to differences in the applied methods (GARCH-models versus regime switching models; daily vs. monthly data).

Table 4.3: Correlation parameter estimates

(a) crisis ignorant

	US	EU	JP	HK	TH	KO	BR
US	1	0.65	0.29	0.46	0.28	0.27	0.23
EU	0.65	1	0.48	0.53	0.30	0.27	0.27
JP	0.29	0.48	1	0.32	0.18	0.38	0.15
HK	0.46	0.53	0.32	1	0.35	0.22	0.25
TH	0.28	0.30	0.18	0.35	1	0.24	0.11
KO	0.27	0.27	0.38	0.22	0.24	1	0.13
BR	0.23	0.27	0.15	0.25	0.11	0.13	1

(b) crisis conscious

	US	EU	JP	HK	TH	KO	BR
US	1	0.64	0.30	0.47	0.33	0.26	0.24
EU	0.64	1	0.49	0.54	0.32	0.26	0.27
JP	0.30	0.49	1	0.32	0.15	0.35	0.16
HK	0.47	0.54	0.32	1	0.37	0.21	0.26
TH	0.33	0.32	0.15	0.37	1	0.21	0.10
KO	0.26	0.26	0.35	0.21	0.21	1	0.14
BR	0.24	0.27	0.16	0.26	0.10	0.14	1

(c) crisis regime

	US	EU	JP	HK	TH	KO	BR
US	1	0.67	0.41	0.54	0.42	0.39	0.31
EU	0.67	1	0.53	0.57	0.37	0.34	0.32
JP	0.41	0.53	1	0.49	0.53	0.62	0.28
HK	0.54	0.57	0.49	1	0.57	0.50	0.36
TH	0.42	0.37	0.53	0.57	1	0.80	0.30
KO	0.39	0.34	0.62	0.50	0.80	1	0.32
BR	0.31	0.32	0.28	0.36	0.30	0.32	1

Estimates for the correlations between the different countries for the crisis ignorant (panel a) and crisis conscious strategy (panel b) and the resulting correlations for the crisis regime (panel c). The correlations are assumed to be independent of the basic regimes.

Table 4.4: Parameter estimates for the multinomial model for the regime transition probabilities

(a) crisis ignorant: low volatility regimes

	US	EU	JP	HK	TH	KO	BR
US	39.90	−87.42	−37.81	−0.48	0.00	0.00	−15.53
EU	−37.40	106.71	0.00	0.00	0.00	−16.33	0.00
JP	0.00	0.00	120.41	0.00	0.00	0.00	−0.16
HK	0.00	−36.40	−56.48	3.75	0.00	0.00	−1.15
TH	−38.54	0.00	−81.05	0.00	3.33	−0.08	0.00
KO	−609.63	−18.79	−0.18	0.00	0.00	18.74	0.00
BR	−37.85	−36.55	−47.41	−0.05	0.00	0.00	19.43

(b) crisis ignorant: high volatility regimes

US	EU	JP	HK	TH	KO	BR	
US	1.53	0.00	16.52	0.00	0.06	0.00	0.00
EU	0.00	1.67	28.34	0.00	0.00	12.40	0.00
JP	0.00	0.00	−46.09	0.00	0.00	0.00	0.00
HK	0.00	0.00	0.00	2.22	0.00	19.54	0.00
TH	14.49	0.00	0.00	0.00	1.42	20.98	0.00
KO	0.00	0.00	0.00	1.34	13.64	−19.11	0.00
BR	0.00	0.00	31.20	0.00	0.00	0.00	3.45

(c) crisis conscious: low volatility regimes

	US	EU	JP	HK	TH	KO	BR
US	3.97	−95.63	0.00	0.00	0.00	0.00	0.00
EU	0.00	137.55	0.00	0.00	0.00	−14.32	0.00
JP	0.00	0.00	32.26	0.00	0.00	0.00	0.00
HK	0.00	−27.92	−27.99	3.74	0.00	0.00	−2.60
TH	0.00	−26.31	−30.46	0.00	2.66	−2.56	−0.37
KO	0.00	−14.86	0.00	0.00	0.00	17.86	0.00
BR	0.00	−56.18	−2.18	−0.64	0.00	0.00	4.62

(d) crisis conscious: high volatility regimes

US	EU	JP	HK	TH	KO	BR	
US	4.80	2.04	22.83	1.73	0.00	14.46	1.65
EU	0.00	−0.10	0.00	0.00	0.00	0.00	0.00
JP	0.00	0.00	−59.11	0.00	0.00	0.00	0.00
HK	0.00	0.00	11.75	1.14	0.00	12.33	0.00
TH	0.00	0.00	35.48	0.00	1.69	11.25	0.83
KO	0.00	0.00	0.00	0.00	13.72	−23.39	0.00
BR	0.00	0.00	25.46	0.00	0.00	0.00	1.51

This table presents the parameter estimates for the crisis ignorant (panels a and b) and the crisis conscious strategy (panels c and d). The multinomial model is specified in appendix 4.A.1. Each column in the panels (a) and (c) (panels (b) and (d)) corresponds with remaining in the low (high) volatility regime. The diagonal elements correspond with the case that all countries are in their low volatility regimes. The off-diagonal elements in a column give the volatility spill-over effects from the row-country to the column-country. The off-diagonal elements in the panels (a) and (c) (panels (b) and (d)) are restricted to be negative (positive).

Finally, we consider the probability estimates for a systemic crisis. A crisis has a probability of 0.0031 to occur, given that currently no crisis occurs. However, if a crisis occurs, it is highly persistent, as indicated by the probability of 0.93 of remaining in the crisis regime. Unconditionally, if no prior information on the prevailing regimes is available, the crisis regime occurs with a probability of 0.045. Das and Uppal (2004) estimate a probability on a systemic jump of 0.0501 for the developed markets and of 0.0138 for the US with emerging markets, which is comparable to the unconditional probability that we find. However, in their model, a systemic jump at a certain instant does not affect the probability of a jump in the next instant, which remains relatively low as a consequence. This illustrates the main difference between their model and our model.

4.5 Portfolio construction

Based on the estimates of the previous section we construct optimal portfolios for the crisis conscious and crisis ignorant strategies. The portfolios vary over time and depend on the filtration of the return processes, limiting the relevance of an analysis of static portfolios. Instead, we concentrate on two situations. First, we consider the influence of a crisis when the investor uses uninformative forecast probabilities for the likelihood of the different regimes, or in other words has no prior knowledge on the state of the economy. In the second situation we analyze the effects of a crisis when the investor uses informative forecasts in a period in which the probability that a crisis was actually prevailing was high, being the second half of 1997, when the Asian crisis took place. We concentrate on October 1997, the month in which the Hong Kong market crashed. The second situation is more interesting, since we can observe how both strategies perform in a real-life situation. The first situation will be useful as a benchmark and enables a comparison with the results of Das and Uppal (2004) and Liu *et al.* (2003).

For both cases we conduct an analysis consisting of the same steps. We present and motivate the steps here, together with their main outcomes. We start by deriving and comparing the optimal allocations for the log utility investor. Though the assumption of log utility is unrealistic due to its low degree of risk aversion, the log utility portfolio is popular in asset allocation studies. The optimal portfolio for a power utility investor is the log utility portfolio scaled by the inverse of his coefficient of relative risk aversion and an investment in the riskless asset to meet the budget constraint. The considerable differences between the crisis conscious and crisis ignorant strategies that we find indicate that the crisis conscious strategy invests less in risky assets. An inspection of only the risky part shows that the crisis conscious strategy shifts investment to countries less prone to a crisis.

Next, we determine the economic importance of the differences between the two strategies. We calculate the certainty equivalent return that the investor requires as a compensation for adopting the crisis ignorant strategy, when the crisis conscious strategy is appropriate. For the uninformative case, the costs of ignoring the possibility of a systemic crisis are limited, but large enough not to neglect it, particularly for longer horizons. In the second situation, when a crisis takes place with almost certainty, the certainty equivalent return rises substantially, also for more risk averse investors.

We conclude the analysis by investigating what can explain the differences between the crisis conscious and crisis ignorant portfolios: the differences in the parameters estimates for the basic component, or the hedging demand due to the possibility of a crisis. To accomplish this we use the myopic strategy introduced in Section 4.2.4. This strategy uses the same estimates for the basic part as the crisis conscious model, but excludes the crisis regime from the forecasts it makes. Consequently, the differences between the portfolios produced by the crisis ignorant and the myopic strategies are due to different parameter estimates for the basic component. On the other hand, the differences between the myopic strategy and the crisis conscious strategy stem solely from the crisis regime. These latter differences have the clear interpretation of a hedging demand. We find that the investor hedges against a crisis by taking a long position in the US, Europe, and the riskless asset and a short position in the stock markets of the other countries.

In the next two subsections we report the actual analysis in more detail. In the last subsection we discuss some robustness checks.

4.5.1 Static analysis

In Table 4.5 we present the portfolios that a log utility investor will construct, if he has no information on the price path of the assets so far. Most importantly, the crisis conscious strategy results in a less aggressive allocation than the crisis ignorant strategy. Overall, the position is less leveraged: the investor lends 3.79 times his initial wealth opposed to 4.71 under the crisis ignorant strategy. Das and Uppal (2004) report similar, though less pronounced results. Leverage is large because we report the log utility portfolio. An investor with coefficient of relative risk aversion equal to 5 would invest 4% in the risk free asset, adopting the crisis conscious strategy, or lend 14% of his wealth, if he adopts the crisis ignorant strategy. The risky asset portfolio itself does not change much.

Table 4.5: Static optimal portfolio weights

	log utility portfolio				risky assets portfolio			
	crisis ign.	crisis cons.	ϕ^{e}	ϕ^{s}	crisis ign.	crisis cons.	ϕ^{e}	ϕ^{s}
US	1.61	1.15	−0.68	0.22	0.28	0.24	−0.11	0.06
Europe	3.59	2.98	−1.06	0.44	0.63	0.62	−0.15	0.14
Japan	−0.70	−0.39	0.45	−0.13	−0.12	−0.08	0.07	−0.03
Hong Kong	0.26	0.27	0.08	−0.07	0.05	0.06	0.02	−0.01
Thailand	0.02	0.07	0.55	−0.50	0.00	0.01	0.10	−0.09
Korea	0.60	0.44	0.30	−0.46	0.11	0.09	0.06	−0.08
Brazil	0.33	0.27	−0.03	−0.03	0.06	0.06	0.00	0.00
risk free	−4.71	−3.79	0.40	0.52	0	0	0	0

Optimal portfolios for the crisis ignorant and crisis conscious strategies for different situations: log utility and an investment in risky assets only. The portfolios are the initial portfolios ($t = 0$) based on the unconditional inference probabilities. The portfolio weights for the different countries and the risk-free asset are reported in the first two columns. The differences between the allocations are decomposed in an estimation effect ϕ^{e} and a crisis effect ϕ^{s}.

It is costly to ignore the possibility of a crisis. A log-utility investor who incorrectly adopts the crisis ignorant strategy requires a certainty equivalent return of 0.09% per month (or 1.13% per year) as compensation. For more risk averse investors, the required compensation becomes less, as the certainty equivalent return should be divided by their coefficient of relative risk aversion. A comparison of this result with findings of Das and Uppal (2004) highlights the importance of persistence. Das and Uppal (2004) report that an investor with coefficient of relative risk aversion of 3 and a horizon of 1 year requires a return of 0.1% as compensation for incorrectly ignoring the systemic jumps in their model. A similar investor in our approach would require a return of 0.38% for incorrectly following the crisis ignorant strategy. Since both systemic events have a comparable probability of occurrence, we conclude that persistence increases the importance of systemic crises. Das and Uppal also show that the certainty equivalent return is an increasing and convex function of the probability of a systemic crisis (see Das and Uppal, 2003, Section 5.2.5, Appendix A.2 and Figure 4), but the degree of persistence they consider is fairly low.

Overall, the decomposition of the differences shows that the estimation effect causes a more prudent allocation: leverage is decreased by 0.40. However, within the risky asset part of the portfolio, investments shift from the US and Europe to Asia. A comparison of the estimates for the high volatility regimes in the crisis conscious model with those in the crisis ignorant model, shows that particularly the high volatility regimes for the Asian markets become more attractive. Of course, this implies that the crisis regimes entails substantial risk for investments in Asian equity. The crisis effect causes large divestments in Asia, which are partly directed towards the US and European markets and partly to the riskless asset. In the uninformative case, the estimation effect and crisis effect cancel out more or less for the Asian markets, but for the developed market the estimation effect dominates. Also in an economic sense the crisis effect is the more important. A log-utility investor requires a compensation of 0.31% per month for ignoring the crisis effect. If he also ignores the estimation effect, the required compensation reduces to 0.1% per month.

We conclude that the implications of the possibility of systemic crises are already visible in an uninformative, static setting. Including a crisis regime in the model for asset returns leads to improved estimates for the basic regime, and boosts investments in emerging markets, but the probability of large negative returns curbs it. Though the estimation effect dominates, the combined effects shift investments to the riskless asset, indicating that incorporating a crisis leads to prudence. Ignoring this prudence can cost up to 1.13% per year.

4.5.2 October 1997: the Asian crisis

To study an informative setting, we investigate the implications of the strategies for asset allocation in October 1997, the month during which the Hong Kong market crashed. We take the estimates presented in the previous sections as given. The inference probabilities that the investor uses are constructed by applying the filtering technique described in Hamilton (1994) on the returns up to September 1997. This setup enables us to observe how the dive of the Hong Kong market influenced the inference probabilities and consequently the asset allocation. The calculation of certainty equivalent returns and decompositions can help us to understand the changes in optimal asset allocations over time, caused by the continuous updating of the inference probabilities.

The Asian crisis hit financial markets during the second half of 1997.[18] In August 1997 the Thai market crashed. Only after the crash of the Hong Kong market, the shocks in Asia were considered as a global crisis. An inspection of the inference probabilities of our model produces a similar picture. By the end of August and September the inferred probability for the crisis regime did not exceed 0.05, but by the end of October it had risen to almost 1. The smoothed inference probabilities resulting from our model confirm the view that the sharp drop in the Thai market could be seen as an overture to the Asian crisis. After August and September, the

[18]See Kamin (1999) for a broad discussion of the symptoms of the Asian crisis. Kaminsky and Schmukler (1999) investigate the causes of daily market fluctuations during the Asian crisis.

smoothed inference probability of the crisis regime was above 0.80, and from October onwards it remained close to 1.

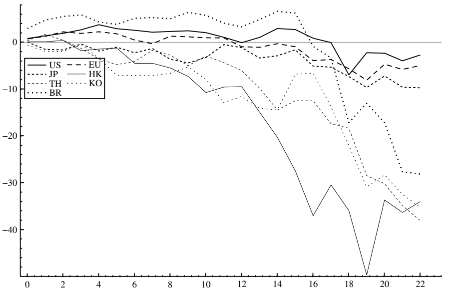
Figure 4.1(a) plots the cumulative excess returns for the different countries. The cumulative returns in the Asian markets are already negative during the beginning of the month, but the US, Europe and Brazil realize small, positive returns. However, after October 17, 1997 (the 13th trading day of that month) the Hong Kong market starts to dive: from -9.5% to -50% in 7 days. The Thai and Korean market move in lock step, but the other markets also suffer large price drops, in particular the Brazilian market. At the end of October 20, a Monday, the inference probability for the crisis regime in the crisis conscious model climbs to 0.71 and remains high for the rest of the month. So the conscious strategy deems the crisis regime most likely one the second half of the month.

To see what the investor would have inferred had he not taken a crisis into account, we plot the inferences for the crisis ignorant model in Figure 4.1(c). In the first half of the month, it is inferred that the US and the Thai market, probably accompanied by the Hong Kong market, are in their high volatility regimes. After the sharp decline in the Hong Kong market it is inferred that in all markets the high volatility regimes are prevailing, though some doubt exists regarding Japan and Brazil.

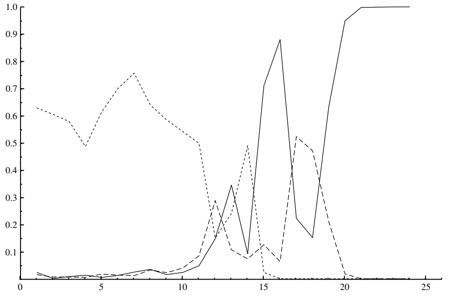
The allocations to which the crisis conscious and crisis ignorant strategies lead during the month are plotted in Figure 4.2. Adopting the crisis conscious strategy leads to less risky allocations. First of all, when applying the crisis conscious strategy, an investor uses less leverage as can be concluded from Figure 4.2(h). Both strategies start with approximately the same degree of leverage, but as October goes by and the probability of a crisis increases, leverage decreases faster in the crisis conscious strategy. By the end of the month the crisis conscious strategy has a long position in the risk free asset, while the crisis ignorant portfolio is still leveraged. It is particularly interesting to see that following the crisis conscious strategy leads to a sharp decrease in leverage already before the dive of the Hong Kong market.

In the crisis conscious strategy, foreign markets quickly become less attractive as the probability of a crisis rises. For four of them (Japan, Hong-Kong, Thailand and Korea), the conscious strategy even advises short positions. When applying the crisis ignorant strategy, the investor decreases his exposure to some of these markets (Japan, Hong Kong), but he increases his exposure to Hong Kong and Korea by the end of the month and maintains long positions in these markets throughout most of the month. Again it is reassuring to see that the investor adopting the crisis conscious strategy withdraws quickly from the Hong Kong market, when the probability of a crisis rises. When adopting the crisis ignorant strategy, the Hong Kong market remains much longer attractive.

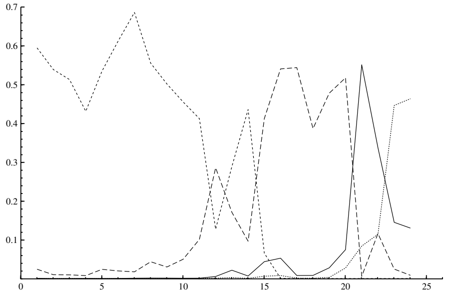
Figure 4.1: Returns and inference probabilities during October 1997



(a) return path



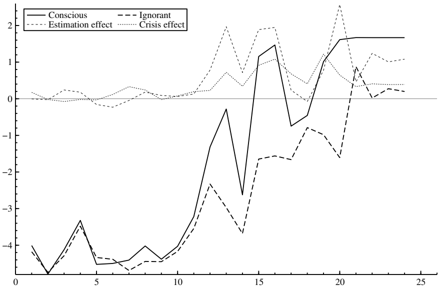
(b) conscious



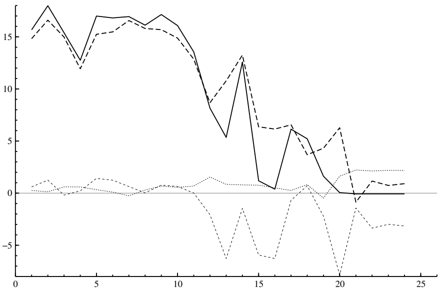
(c) ignorant

The cumulative return path (excess returns, in %) of the indexes for the US, Europe (EU), Japan (JP), Hong Kong (HK), Thailand (TH), Korea (KO) and Brazil (BR) (panel a) and the resulting inferences for the crisis conscious strategy (panel b) and the crisis ignorant strategy (panel c) for each trading day in October 1997 (numbered consecutively). The inference probabilities are constructed by updating the forecast probabilities based on the returns to September 1997 in a Bayesian fashion as given in Eq. (4.12). We only plot the inferences for a state vector, if the inferences have exceeded 0.4 at least once: for the crisis conscious strategy that is US and Thailand high volatility, others low (dashed line), US, Hong Kong and Thailand high volatility, others low (long dashed line), and the crisis state (solid line); for the crisis ignorant strategy that is US and Thailand high volatility, others low (dashed line), US, Hong Kong and Thailand high volatility, others low (long dashed line), US, Europe, Hong Kong, Thailand and Korea high volatility, others low (solid line) and the state in which all countries have high volatility (dotted line).

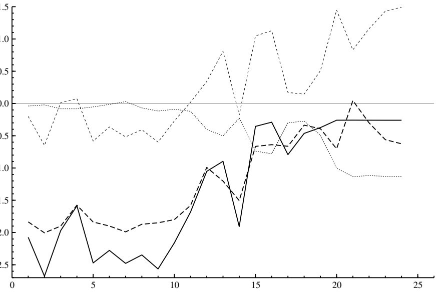
Figure 4.2: Optimal portfolio weights during October 1997



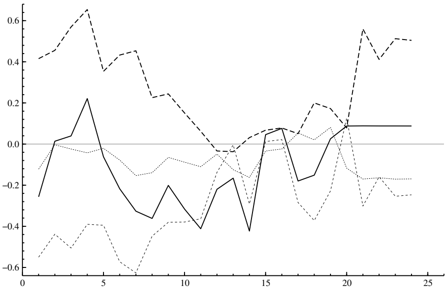
(a) US



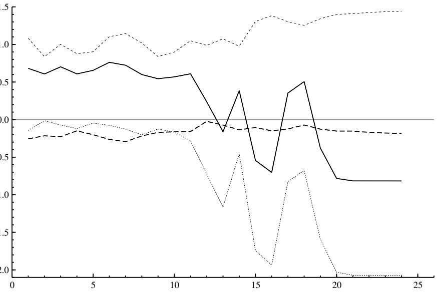
(b) EU



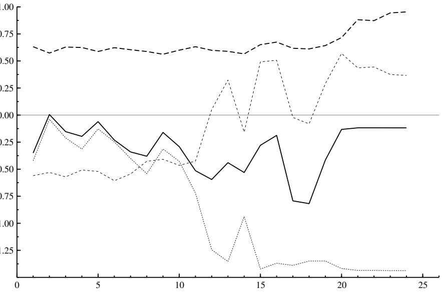
(c) Japan



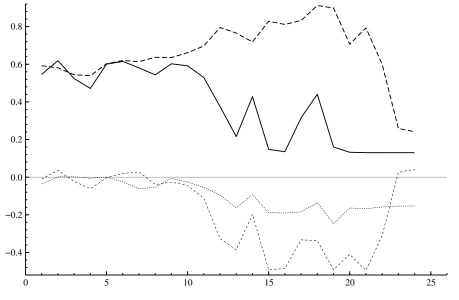
(d) Hong Kong



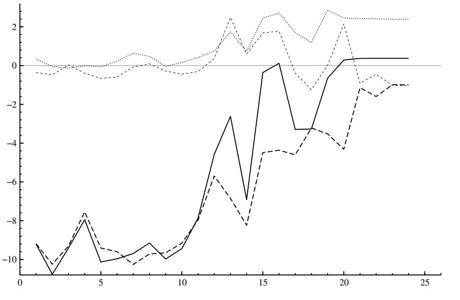
(e) Thailand



(f) Korea



(g) Brazil



(h) risk free asset

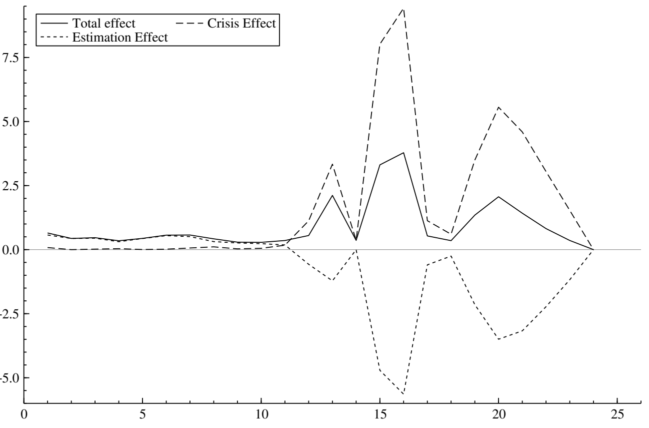
The proportion of wealth invested in the different countries for the crisis conscious and crisis ignorant strategy and a decomposition of the differences for each trading day in October 1997 (numbered consecutively). We assume the investor has a log utility function. The portfolios are based on the estimates in Tables 4.2 and 4.3 and the inference probabilities that are constructed by updating the forecast probabilities based on the returns to September 1997 in a Bayesian fashion as given in Eq. (4.12). The portfolio differences between the two portfolios are decomposed in an estimation and a crisis effect.

The differences between the allocations in Figure 4.2 are not only pronounced but also economically important. Both before and after the dive of the Hong Kong market, the certainty equivalent in Figure 4.3 is higher than 0.3% per month, clearly exceeding the 0.09% per month of the uninformative case. Moreover, it rises dramatically (to at most 4.0%) after the crash of the Hong Kong market. Of course, these returns are lower for more risk averse investors, but we stress that they correspond with a 1-month horizon.

The results also indicate that diversification opportunities deteriorate rapidly if the inference probability of a crisis increases. Already before the crisis hits, leverage decreases, investments are reduced and in some countries short position are taken. These findings can be seen as an addition to the results in Ang and Bekaert (2002), who conclude that diversification opportunities do not disappear when the bearish regime in their model is prevailing. Since the crisis regime in our model consists of the bear regime of each country and a shock (with a negative mean) on top of that, conditions are much worse in the crisis regime, and consequently we do observe such a deterioration.

We conclude the analysis by considering the estimation and crisis effects over October 1997, which are also plotted in Figure 4.2. The crisis effect indicates that the US and European market and the riskless asset can be used to hedge against a crisis. We observe a positive demand for the US and European asset, and a strong and increasing positive demand for the riskless asset. Positions in other markets are more

Figure 4.3: Certainty equivalent returns



This figure plots the certainty equivalent return (in %) needed to compensate the investor for adopting the suboptimal ignorant strategy and a decomposition in an estimation and crisis effect for each trading day in October 1997 (numbered consecutively). We assume that the investor has a log utility function and a horizon of 1 month. The portfolios during October 1997 and the corresponding decompositions are given in Figure 4.2.

and more reduced, particularly in crisis prone Korea and Thailand. The estimation effect presents a less clear picture. In the uninformative case, the estimation effect causes investments to shift from the riskless asset to the risky assets. Now we observe a preference for Japan, Korea and Thailand. Within the risky asset part we observe a tendency to more aggressive allocations, but leverage is not increased much. The certainty equivalent returns associated with missing the crisis effect are also considerable, being always positive and rising rapidly. Of course, missing the crisis effect becomes extremely expensive after a crisis has occurred (with a maximum certainty equivalent return at 10%). The estimation effect does not always harm the investor's utility. Finally, the estimation effect influences utility less (in absolute sense) than the crisis effect.

The portfolio differences and the corresponding certainty equivalent returns lead to several important conclusions. First, the differences and their importance become rapidly larger when the inference probability of a crisis increases. Second, an inference probability of a crisis of around 0.10 already leads to portfolio differences that are much larger than in the uninformative case (in that case the probability of a crisis equals 0.05) and much more costly to ignore. Third, an investor that adopts the crisis conscious strategy takes precautions well before the inference probability reaches high levels (exceeding 0.5). Finally, the occurrence of a crisis rapidly diminishes diversification opportunities.

4.5.3 Robustness checks

In order to gauge the strength of our results, we subject them to several robustness checks. We start by repeating our analysis of the allocations during October 1997 for four different months during the Asian crisis: August, September, November and December. We continue by investigating the effects of short selling constraints on the different allocations. Next, we determine the portfolio turnover for the different strategies and use it to discuss the possible impact of transaction costs. In the following subsections we briefly discuss each robustness check.[19]

Other months during the Asian crisis

The Asian crisis was perceived as a global crisis after the collapse of the Hong Kong market and the subsequent fall of other markets in October 1997. The crisis actually built up during August and September, but it was not perceived as a systemic crisis then. After October, the crisis was perceived to take place with almost certainty. This makes August, September, November and December interesting months to study as well.

In August the Thai market crashed, but other markets did not follow it as much as the Hong Kong crash in October. In September the Thai market showed sharp fluctuations and the Korean market decreased considerably (at most -14%). In November all Asian markets declined steadily. December showed again large declines for the emerging markets (in particular Korea and Thailand), though the other markets seemed to stabilize. Inference probabilities for a systemic crisis are low in August and September and high in November and December.

We perform the analysis discussed in section 4.5.2 for these months and find large similarities, which supports our conclusions based on October. In all months the crisis conscious strategy advises a smaller exposure to emerging markets. In August and September, it advises a higher exposure to the developed markets while in November and December, leverage is decreased. Because the inference probability for a crisis is large throughout November and December, the certainty equivalent return to compensate for the crisis ignorant strategy is also large in those months (well above 5.0% for a log-utility investor with a 1 month horizon). In August and September, the inference probability for a crisis is low, but we still observe certainty equivalent returns that exceed the 0.09% that resulted from the uninformative case.

[19] The complete results are available from the authors.

Short sales constraints

Large short positions as in Figure 4.2 may not be feasible. Therefore, we construct optimal allocations for both strategies for an investor facing short sales constraints. In Appendix 4.A.3 we derive the optimal portfolio under the restriction that the portfolio weights for the risky assets are nonnegative.[20] We use that result to determine the optimal allocation for both the uninformative case and the informative case of October 1997.

In the uninformative case, the consequences of short sales constraints are limited. Both strategies do not invest in Japan. The investments in the other countries are slightly reduced, but we mainly observe an increase in leverage of both portfolios. Because the portfolio adjustments are similar for both strategies, the decomposition in the estimation and the crisis effect, and the certainty equivalent return needed to compensate for the crisis ignorant strategy remain largely unaffected.

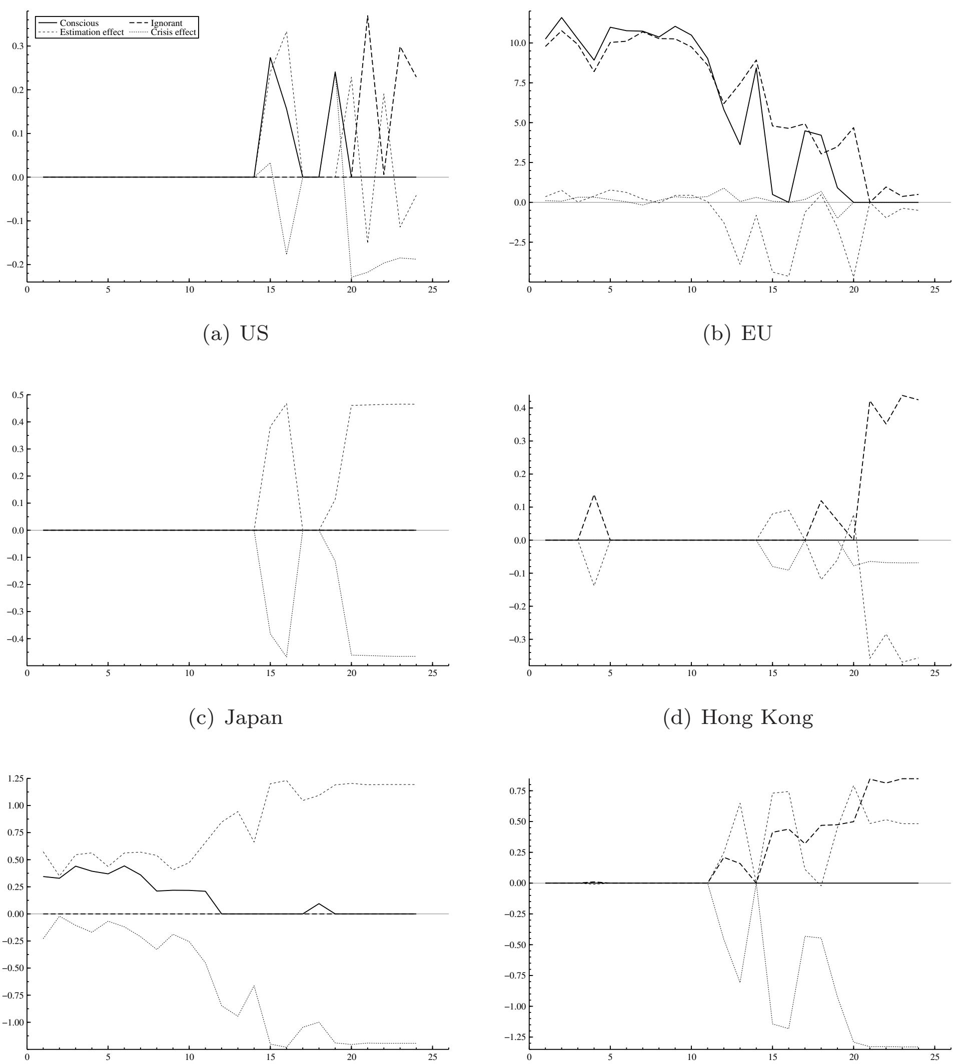
The analysis of October 1997 yields more interesting insights. First of all, the investor who uses the crisis conscious strategy leaves all markets, if a crisis occurs. In Appendix 4.A.3, we show that the investor does not invest in assets for which $\alpha_i \equiv \mu_i + \frac{1}{2}\omega_i - r_{\mathrm{f}}$ is negative and the covariance with other assets is positive. Based on the parameters reported in Tables 4.2 and 4.3 we conclude that these conditions are satisfied. During October 1997, a crisis does not occur with certainty, but its inferred probability is large enough.

Figure 4.4 shows the allocations for the trading days in October 1997. Imposing short sales constraints results in less volatile and less aggressive allocations for both strategies. The conscious strategy stays out of the Japanese, Hong Kong and Korean market during the complete month; the allocations to the Thai and Brazilian market are quickly reduced. During the last days of the month, the crisis conscious strategy advises to leave all markets, while the crisis ignorant strategy advises to leave only Japan and Thailand. Despite their high volatility, the other markets remain attractive. Because the differences between the allocations become smaller in absolute sense, the certainty equivalent return decreases slightly. The other months (August till December) confirm these findings. In August and September, the crisis conscious strategy advises to invest neither in the Korean market.

We conclude that the impact of short sales constraints on allocations can be large, particularly for high probabilities of a systemic crises. However, even if short sales constraints are imposed it remains economically important to take the possibility of a crisis into account.

[20]For a more general treatment of optimal portfolio selection for investors with CRRA utility facing short sales constraints, see Teplá (2000).

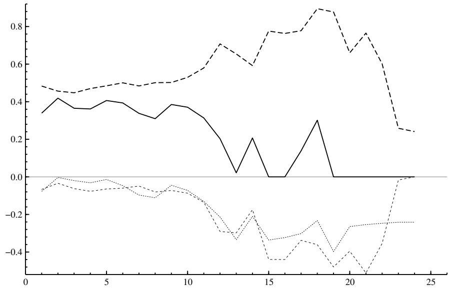
Figure 4.4: Optimal portfolio weights during October 1997 under short sales constraints



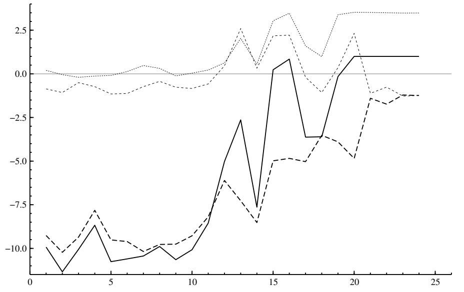
(a) US (b) EU

(c) Japan (d) Hong Kong

(e) Thailand (f) Korea



(g) Brazil



(h) risk free asset

The proportion of wealth invested in the different countries for the crisis conscious and crisis ignorant strategy and a decomposition of the differences for each trading day (numbered consecutively) in October 1997 assuming that short sales are not allowed. We assume the investor has a log utility function. The portfolios are based on the estimates in Tables 4.2 and 4.3 and the inferences presented in Figure 4.1. The portfolio differences between the two portfolios are decomposed in an estimation and a crisis effect.

Portfolio turnover

In this subsection we shortly discuss the portfolio turnover that results from the different strategies. We use this information to gauge the possible impact of transaction costs on our results. Though transaction costs are present in the real world and can seriously impact dynamic trading strategies (see e.g. Liu and Loewenstein, 2002; Balduzzi and Lynch, 1999), including transaction costs in the asset allocation problem would impede our analysis and is not our chief interest. Instead, we turn attention to the portfolio turnover of the different strategies as a measure of their variability. Higher variability typically leads to higher transaction costs. If the crisis conscious strategy leads to a higher portfolio turnover than the crisis ignorant strategy, introducing transaction costs will affect the crisis conscious allocations most.

We calculate the portfolio turnover by summing the absolute changes in the portfolio weights for the risky assets. Transactions costs will be a fraction of this measure, if transaction costs are proportional to the value of assets bought or sold. For the log utility investor, we find that the turnover in October 1997 equals 123.5 for the crisis conscious strategy versus 97.6 for the crisis ignorant strategy, 1.25 times as much, indicating that the crisis ignorant strategy is more stable. The turnover is large, but this is partly due to the assumption of log utility. In order to move to power utility, the log utility turnover should be divided by the coefficient of relative risk aversion. For the other months we calculate lower numbers, as the large drops during October lead to large adjustments of the portfolio weights. We interpret the relatively small differences in turnover as evidence that our results are robust to transaction costs.

4.6 Conclusions

A systemic crisis in international equity markets can put investors in dire straits, because of the simultaneous decrease in expected returns and increase in volatilities and correlations. However, standard models supporting asset allocation decisions typically fail to fully capture systemic crises due to their irregular and relatively rare occurrences. In this chapter, we have proposed a framework to determine the impact of systemic crises on asset allocations, which combines regime switching models with optimal portfolio construction in continuous time. In this framework, an investor can adopt a crisis conscious strategy that includes systemic crises, and, as an alternative, a crisis ignorant strategy which excludes it. We have studied the allocations of a US-based, global investor, who maximizes his expected utility by investing in equity markets in the US, Europe, Japan, Hong Kong, Thailand, Korea and Brazil, and a riskless asset. We have considered the case in which he has no information on the likelihood of the different states in the regime switching model, and the case in which he has information up to October 1997, which belongs to the Asian crisis. If the investor adopts the crisis conscious strategy, he decreases leverage and shifts investments to countries that are less prone to a crisis. The certainty equivalent returns indicate that these differences are economically important. The investor requires a substantial compensation for incorrectly adopting the crisis ignorant strategy in the uninformative case, and this compensation rises quickly during October 1997.

The pronounced portfolio differences and their economic impact indicate that persistence is an important characteristic of systemic crises. We have estimated that the probability of remaining in the crisis regime for another month equals 0.93. Because of this persistence, we find stronger evidence advocating the incorporation of systemic crises in asset allocation decisions than reported by Das and Uppal (2004, 2003), while the corresponding unconditional probabilities on a crisis are similar (0.045 versus 0.0501). Crisis persistence also explains the large differences between the crisis conscious and ignorant strategies and the large required compensation if a crisis occurs with almost certainty. This would also explain the well-known fact that investors stay away from financial markets after a systemic crises for a relatively long period of time.

Systemic crises seriously diminish diversification possibilities. If a crisis has a small probability of occurrence, this effect is present but limited. As the probability increases, the impact becomes larger and leads to short positions in several markets. When the investors faces short sales constraints, he withdraws from equity markets completely. These findings are complementary to Ang and Bekaert (2002) who conclude that the presence of a bear regime in a regime switching model does not extinguish diversification possibilities. However, because of its severity the crisis regime in our model causes a much stronger deterioration in the risk-return trade-off than the bear regime in their model.

This chapter can motivate further research in several ways. Some parts of our model are kept at a basic level for clarity. It would be interesting, however, to see the influence of a crisis when other economic variables are used to predict the likelihood of a crisis or the corresponding means and variances. A more normative model for a crisis can also be interesting. Our finding that crisis conscious strategies shift allocations from countries that are relatively prone to a crisis to countries that are less prone to it can add to the research on the home bias puzzle (see Lewis, 1999, for an overview), particularly in relation to emerging markets. At a more fundamental level, an equilibrium analysis of systemic crises can shed more light on their consequences for asset pricing.

4.A Model details

4.A.1 A multinomial model for regime transition probabilities

In this appendix we discuss the specification of the multinomial logistic model that we propose for the regime transition probabilities in Section 4.2.1 in more detail. π_i^{ab} gives the marginal probability that asset i will switch to regime Q_i^a, given that the current regime is $\boldsymbol{Q}^b$, and that a crisis is absent in both the current and the destination regime. Its functional form reads

$$\pi_i^{ab} = \frac{\exp f(Q_i^a, \boldsymbol{Q}^b)}{1 + \sum_{k=1}^{K-1} \exp f(Q_i^a = k, \boldsymbol{Q}^b)} \tag{4.15}$$

The summation in the denominator excludes the K^{th} basic regime to ensure that the probabilities add up to 1. We specify the function $f(Q_i^a, \boldsymbol{Q}^b)$ as follows:

$$f(Q_i^a = k, \boldsymbol{Q}^b) = \psi_{i,k,Q_i^b} + \sum_{j=1,\ j\neq i}^{n} \sum_{k'=2}^{K} \psi_{i,j,k,k'} I(Q_j^b = k'), \tag{4.16}$$

where ψ_{i,k,Q_i^b} and $\psi_{i,j,k,k'}$ are constant, and $I()$ denotes the indicator function. If all assets other than asset i are in their low volatility regimes ($Q_j^b = 1$), the function returns the constant ψ_{i,k,Q_j^b}. For the assets that are in a higher volatility level k', constants $\psi_{i,j,k'}$ are added to it. We require $\psi_{i,j,k,k'} < 0$ for $k < k'$, $\psi_{i,j,k,k'} > 0$ for $k \geq k'$, and $\psi_{i,j,k,k'} < \psi_{i,j,k,k''}$ for $k' < k''$ to ensure that the volatility spill-over effects increase the probability of higher volatility regimes.

4.A.2 Solving the asset allocation problem

In this appendix we derive the optimal portfolio and the certainty equivalent return to compensate for suboptimal portfolios. The optimal portfolio solves

$$\begin{aligned} &\max_{\boldsymbol{\phi}_s, t\leq s\leq T} \mathrm{E}_t\left[U(W_T)\right] \\ &\text{subject to} \quad \frac{\mathrm{d}W}{W} = r_\mathrm{f}\,\mathrm{d}t + \boldsymbol{\phi}'\boldsymbol{\alpha}\,\mathrm{d}t + \boldsymbol{\phi}'\boldsymbol{\Lambda}\,\mathrm{d}\boldsymbol{Z}. \end{aligned} \tag{4.17}$$

Via the indirect utility function

$$V(W,t) \equiv \max_{\boldsymbol{\phi}_s, t\leq s\leq T} \mathrm{E}_t\left[U(W_T)\right], \tag{4.18}$$

we derive the Hamilton-Jacobi-Bellman equation:

$$\max_{\boldsymbol{\phi}} \left(\frac{\partial V}{\partial t} + (r_\mathrm{f} + \boldsymbol{\alpha}\boldsymbol{\phi}')W\frac{\partial V}{\partial W} + \frac{1}{2}\boldsymbol{\phi}'\boldsymbol{\Omega}\boldsymbol{\phi}W^2\frac{\partial^2 V}{\partial W^2} \right) = 0. \tag{4.19}$$

We conjecture (and verify) that the indirect utility function is of the form

$$V(W,t) = C(t)\frac{W^{1-\gamma}}{(1-\gamma)}, \tag{4.20}$$

derive expressions for the derivatives in Eq. (4.19) and substitute them.[21] Differentiating Eq. (4.19) with respect to $\boldsymbol{\phi}$, and solving the first-order condition yieldd the optimal portfolio in Eq. (4.9).

The certainty equivalent return $\bar{r}$ to compensate the investor for selecting the inappropriate crisis ignorant portfolio $\boldsymbol{\phi}^{\mathrm{i}}$ instead of the optimal crisis conscious portfolio $\boldsymbol{\phi}^{\mathrm{c}}$ solves:

$$V\left(\mathrm{e}^{\bar{r}}W_t, t; \boldsymbol{\phi}^{\mathrm{i}}\right) = V\left(W_t, t; \boldsymbol{\phi}^{\mathrm{c}}\right). \tag{4.21}$$

Using the functional form of the value function in Eq. (4.20), we find after some rearrangements that

$$\bar{r} = (\ln C(t;\boldsymbol{\phi}^{\mathrm{c}}) - \ln C(t;\boldsymbol{\phi}^{\mathrm{i}}))/(1-\gamma),$$

which is independent of wealth. To identify $C(t)$, consider the Hamiltonian Eq. (4.19) at the presumed optimal solution $\boldsymbol{\phi}^*$. This equation implies an ordinary differential equation for $C(t)$:

$$\mathrm{d}C = -(1-\gamma)\left[r_{\mathrm{f}} + \boldsymbol{\phi}^{*\prime}\boldsymbol{\alpha} - \tfrac{1}{2}\gamma\boldsymbol{\phi}^{*\prime}\boldsymbol{\Omega}\boldsymbol{\phi}^*\right]C(t),$$

that can be solved straightforwardly, yielding:

$$C(t;\boldsymbol{\phi}^*) = \exp[(1-\gamma)(r_{\mathrm{f}} + h(\boldsymbol{\phi}^*))(T-t)]. \tag{4.22}$$

with $h(\boldsymbol{\phi}^*) = \boldsymbol{\phi}^{*\prime}\boldsymbol{\alpha} - \frac{1}{2}\gamma\boldsymbol{\phi}^{*\prime}\boldsymbol{\Omega}\boldsymbol{\phi}^*$. We use the boundary condition $V(W,T;\boldsymbol{\phi}^*) = U(W_T)$, to solve for the integration constant. Substituting this into the expression for $\bar{r}$ yields the expression in Eq. (4.13).

4.A.3 Short sales constraints

Short sales constraints can be included in the asset allocation problem straightforwardly. The basic optimization problem, constituted by Eq. (4.17) is now extended with short sales constraints $\phi_{i,t} \geq 0,\ \forall t,\ i = 1,\ldots,n$. The new first order conditions for optimality include Kuhn-Tucker conditions for the new restrictions. To derive these we start with the Hamilton-Jacobi-Bellman equation (13), combined with the guess for the indirect utility function (14):

$$\max_{\boldsymbol{\phi}}\left(\frac{\partial C(t)}{\partial t}\frac{W^{1-\gamma}}{1-\gamma} + (r_{\mathrm{f}} + \boldsymbol{\phi}'\boldsymbol{\alpha})\,W^{1-\gamma} - \tfrac{1}{2}\gamma\boldsymbol{\phi}'\boldsymbol{\Omega}\boldsymbol{\phi}W^{1-\gamma}\right) = 0. \tag{4.23}$$

[21] For $\gamma = 1$ we use $V = \ln[C(t)W]$.

We introduce Lagrange multipliers $\lambda_{i,t} \geq 0 \; \forall t$ for the short sales constraint on asset i and construct a Lagrangian function by adding the term $\boldsymbol{\phi}'\boldsymbol{\lambda}$ to the maximand in the HJB-equation. Differentiation yields the following first order conditions:

$$\boldsymbol{\alpha} - \gamma \boldsymbol{\Omega} \boldsymbol{\phi} = -\boldsymbol{\lambda} W^{-(1-\gamma)} \tag{4.24}$$

$$\phi_i \lambda_i = 0 \tag{4.25}$$

$$\phi_i, \; \lambda_i \geq 0, \tag{4.26}$$

where the last two restrictions are the complementary slackness conditions.

This system of (in)equalities resembles the system of (in)equalities resulting from a mean-variance portfolio optimization problem with short sales constraints (see Teplá, 2000; de Roon *et al.*, 2001). Consequently, the optimal portfolio can also be characterized similarly. Let I^{p} be the set of indices for which the short selling constraints are not binding, and use the superscript $^{\mathrm{p}}$ to denote the subvectors and submatrices with respect to that set. The optimal positive weights are given by:

$$\phi^{\mathrm{p}} = \gamma^{-1} (\boldsymbol{\Omega}^{\mathrm{p}})^{-1} \boldsymbol{\alpha}^{\mathrm{p}}. \tag{4.27}$$

The Lagrange multipliers for the assets in the complement I° of I^{p} can be found as:

$$\boldsymbol{\lambda}^{\circ} = (-\boldsymbol{\alpha}^{\circ} + \gamma \boldsymbol{\Omega}^{\circ \mathrm{p}} \boldsymbol{\phi}^{\mathrm{p}}) W^{1-\gamma} = (-\boldsymbol{\alpha}^{\circ} + \boldsymbol{\Omega}^{\circ \mathrm{p}} (\boldsymbol{\Omega}^{\mathrm{p}})^{-1} \boldsymbol{\alpha}^{\mathrm{p}}) W^{1-\gamma} \geq 0 \tag{4.28}$$

where $\boldsymbol{\Omega}^{\circ \mathrm{p}}$ denotes the $n^{\circ} \times n^{\mathrm{p}}$ covariance matrix of the returns of the assets in I° with those in I^{p}. Using positivity of wealth, a partition of the assets into subsets I^{p} and I° is valid if and only if $-\boldsymbol{\alpha}^{\circ} + \boldsymbol{\Omega}^{\circ \mathrm{p}} (\boldsymbol{\Omega}^{\mathrm{p}})^{-1} \boldsymbol{\alpha}^{\mathrm{p}} \geq 0$. In general, the validity of a partition will have to be checked using this condition. However, it easy is to verify that assets i for which $\alpha_i \leq 0$ and $\Omega_{ij} \geq 0 \; \forall j$ will always be in I°.

Der Streik, die Krise nähern euch dem Ziele, durch den großen Kladderadatsch werdet ihr ins Paradies eingeführt.
KARL MARX

Chapter 5

Crash risk in the cross section of stock returns*

5.1 Introduction

Crashes have dramatic consequences for investors. Because comovements of assets become stronger when crashes occur, these consequences are hard to diversify and difficult to evade. For these reasons, financial theory predicts that investors receive a premium for the crash risk that they bear. If asset returns exhibit different characteristics during a crash than during quiet periods, or if investors are particularly averse to crashes, the crash risk premium should be distinguishable from the risk premium for quiet times. Knowledge about this premium and its structure can then increase our understanding of investor preferences and the expected returns of assets both over time and in the cross section.

In this chapter, we examine whether we can identify a risk premium for crash risk in the cross section of stock returns. Up to now, evidence of a premium for crash risk is largely based on options on stock market indexes (see Andersen *et al.*, 2002; Bates, 1991, 2000). However, Bakshi *et al.* (2003) show that the distributions of stock returns implied by individual stock options differ from the implied distribution for market index returns, as they are less skewed. Chen *et al.* (2001) report wide variation in the skewness of stock return distributions. This evidence can indicate that individual stocks vary in their sensitivity to market crashes, and differ from the

*This chapter is based on the article by Kole and Verbeek (2005).

market in aggregate. As a consequence, the expected returns of stocks can differ according to their exposure to market crash risk.

To investigate systematic crash risk in asset prices, we derive an extension of the Capital Asset Pricing Model (CAPM) that includes a premium for crash risk. Based on Bates (2001), we model a crash as a Poisson-jump in the dividend process underlying the stock price. To capture investors' aversion to large losses we include a specific crash aversion part in the utility function. Individual stocks can vary in their tendency to crash given that the market crashes. The expected return for a stock contains a reward for crash risk proportional to its sensitivity for it. We derive that a stock's sensitivity to crash risk consists of two parts: the probability that it crashes, given that the market crashes, and a ratio of the crash magnitude of the asset to that of the market. The premium for crash risk is a function of general risk aversion and crash aversion.

We use this model to set up an empirical analysis of crash risk in the cross section of stock returns. We derive three measures for the conditional crash likelihood that shows up in an asset's sensitivity to crash risk. The intuition behind these three measures is that crash risk can explain the difference between the actual dependence of an asset and the market, and the correlation-implied dependence (see Hartmann *et al.*, 2004; Longin and Solnik, 2001). So, large deviations reflect a large exposure to crash risk. For each stock we construct estimates for the values of the three measures and use them to sort the stocks into value-weighted portfolios.

Based on the CRSP-database from June 1964 to November 2003, we provide evidence that crash risk shows up as a separate premium in the cross section of stock returns. After correcting for market risk as in the traditional CAPM, portfolios with stocks that score in the top 33% of crash risk exposure yield on average an extra significant return of 2.3% to 4% per annum, depending on which of the three measures is used for sorting the stocks. These extra returns cannot be explained by established factors, such as size, value or momentum. We do find that these portfolios are related to the coskewness effect of Harvey and Siddique (2000), and the cokurtosis effect of Dittmar (2002), but the extra returns remain significant after correcting for these effects. The portfolios with stocks that exhibit little or no exposure to crash risk do not pay an extra significant average return.

We find mild evidence supporting the inclusion of a crash risk factor in asset pricing tests. For portfolios constructed on momentum a crash factor clearly helps explaining cross-sectional variation in returns. The traditional CAPM produces pricing errors that are jointly significant, but this significance vanishes after including a crash risk factor. For portfolios sorted on industry, size or value versus growth we find small improvements, but the traditional CAPM suffices to explain these cross sections of portfolio returns. An analysis of the entire cross section of stock returns also indicates that pricing errors and their standard errors are lower after addition of a crash portfolio. The risk premium estimated at 8.4% per year indicates that the premium for crash risk has the same magnitude as the premium on market diffusion risk.

This chapter adds to the ongoing debate on asset pricing. We investigate the presence of a factor that can be related directly to risk, which distinguishes our research from the more data based approaches underlying the size and value factors of Fama and French (1993, 1995) and the momentum factor of Jegadeesh and Titman (1993) and Carhart (1997). Our research is in line with the literature that extends the traditional CAPM with higher order moments, such as coskewness in Harvey and Siddique (2000) and Barone Adesi *et al.* (2004), or cokurtosis in Dittmar (2002) and Christie-David and Chaudhry (2001). Coskewness and cokurtosis can be interpreted as proxies for conditional crash likelihood. Indeed, we find a relation between the crash risk portfolios and the coskewness and cokurtosis portfolios. However, for the period we consider, the returns on the coskewness and cokurtosis hedge portfolios are insignificant. This can indicate that coskewness and cokurtosis are imperfect proxies for crash risk.

This chapter is structured as follows. In Section 2 we derive the CAPM extended with crash risk. In Section 3 we derive the measure for conditional crash likelihood. Section 4 discusses the portfolio formation based on these measure and reports their relation with other risk factors. Section 5 considers the cross section of stock returns in relation to crash risk, while section 6 concludes.

5.2 Extending the CAPM with crash risk

The traditional Capital Asset Pricing Model (CAPM) as put forward by Sharpe (1964) and Lintner (1965) posits that the following equilibrium relation between the excess return $R^{\mathrm{e}}_{i,t+1}$ on asset i and the excess return on the wealth portfolio $R^{\mathrm{e}}_{\mathrm{w},t+1}$ holds at time t:

$$\mathrm{E}_t\left[R^{\mathrm{e}}_{i,t+1}\right] = \beta_{i,t}\,\mathrm{E}_t\left[R^{\mathrm{e}}_{\mathrm{w},t+1}\right], \tag{5.1}$$

where $\mathrm{E}_t[\cdot]$ denotes the conditional expectation given information available at time t. In this expression, $\beta_{i,t}$ measures the sensitivity of the asset's excess return with respect to the return on the wealth portfolio. In empirical work, the wealth portfolio is commonly approximated by the market portfolio.

The CAPM can be derived straightforwardly by assuming an endowment economy in which the representative agent has power utility and the systematic uncertainty stems from diffusion processes.[1] While such an approach provides a nice starting point for studying asset pricing, it is too restrictive to study the influence of crash risk on asset prices. Following Bates (2001), we propose modifications of the standard approach to address important drawbacks. We use the insights of the extended CAPM to steer the empirical research in the remainder of this chapter.

We derive the extended CAPM from the standard CAPM setting in which we assume an endowment economy with a finite-lived representative agent who consumes at the final date T. In this economy $n+1$ assets are available. The first asset is a contract that provides a certain pay-off at the end date. The other n assets entitle the owner to an uncertain final pay-off, denoted by X_{iT}, $i = 1, \ldots, n$. We assume that the riskless asset is in zero net supply, while each of the risky assets have a net supply of unity. The market then consists of the sum of the uncertain pay-offs, which we call the market claim and denote as $X_{\mathrm{m}t}$. The CAPM enables us to derive the price dynamics of these assets over time, based on the dynamics of the underlying processes. We use S_{it} to denote the time t price of the asset i, and $S_{\mathrm{m}t}$ for the price of the market claim.

As a first extension we add jumps as a source of systematic risk to the diffusions present in the standard CAPM. We assume that the growth rate of the final market pay-off follows

$$\mathrm{d}\log X_{\mathrm{m}} = \mu_{\mathrm{m}}\,\mathrm{d}t + \sigma_{\mathrm{m}}\,\mathrm{d}Z_{\mathrm{m}} + \kappa_{\mathrm{m}}\,\mathrm{d}N_{\mathrm{m}}, \tag{5.2}$$

where $\mu_{\mathrm{m}}\,\mathrm{d}t$ is the drift rate, Z_{m} is a Wiener process, N_{m} is a Poisson process with arrival rate λ_{m}, κ_{m} is the jump size, which we assume to be negative, and $\sigma_{\mathrm{m}}^2\,\mathrm{d}t$ is the variance rate of the process, conditional on no jumps occurring over $\mathrm{d}t$. We assume that the Wiener process and the Poisson process evolve independently.

Poisson processes are essential to capture the sudden price decreases during crashes. Empirical evidence indicates that stock prices are not only driven by diffusion processes but exhibit negative jumps as well (see Das and Uppal, 2004, and references therein). Moreover, evidence from option markets in Andersen *et al.* (2002) and Bates (2000, 1996, 1991) indicates that investors expect downward jumps to occur and require a premium for this risk. Stock price processes that are constructed with diffusions only (including stochastic volatility) cannot generate the distributions that are implied by option prices. This implies that jumps have a systematic component, which should show up in the pricing kernel.

[1] See Cochrane (2001, Ch. 9.1) for other sets of assumptions that lead to the CAPM.

Second, we add a crash discount to the power utility function of the representative agent, formulated indirectly in terms of wealth:

$$U(W_t, N_{\mathrm{m}t}) = E_t\left[\frac{\mathrm{e}^{\delta N_{\mathrm{m}T}} W_T^{1-\gamma} - 1}{1-\gamma}\right], \quad \gamma > 1, \quad \delta \geq 0, \tag{5.3}$$

where W_t denotes the investor's wealth at time t, $N_{\mathrm{m}t}$ denotes the number of market crashes up to time t, γ is the investor's coefficient of relative risk aversion and δ reflects his crash aversion.

Standard utility functions are ill-suited to capture investors' aversion to downside risk.[2] In our setup an extra crash multiplies the value of the utility function with a factor $\mathrm{e}^{\delta} > 0$, reducing the value of the utility function (for $\gamma > 1$ we have $W_t^{1-\gamma}/(1-\gamma) < 0$). This setting implies loss aversion in a limited sense, since it only applies to crash losses and not to losses in general. However, this is not a substantial limitation, as we are specifically interested in the effect of crash risk. Furthermore, in the absence of market crashes $N_{\mathrm{m}} = 0$, our model design leads to the traditional CAPM, providing clear insights what the changes imply.

The third extension of the traditional CAPM posits a structure for the stochastic processes that underlie the specific pay-off X_{iT} that is similar to the structure for the process for the market pay-off in Eq. (5.2):

$$d\log X_i = \mu_i\,\mathrm{d}t + \sigma_i\,\mathrm{d}Z_i + \kappa_i\,\mathrm{d}N_i, \tag{5.4}$$

where Z_i is a Wiener process, and N_i is a Poisson process with arrival rate λ_i, independent from Z_i. The Wiener processes Z_{m} and Z_i are related via $E[\mathrm{d}Z_i\,\mathrm{d}Z_{\mathrm{m}}] = \rho_{i\mathrm{m}}\,\mathrm{d}t$. The Poisson process N_i is different from Nm, but not independent from it. We assume that the joint process $\boldsymbol{N} = \begin{pmatrix} N_{\mathrm{m}} & N_i \end{pmatrix}'$ evolves evolves according to:

$$\mathrm{d}\boldsymbol{N} = \begin{cases} (1\ 1)' & \text{with probability } \lambda_{i\mathrm{m}}\,\mathrm{d}t \\ (1\ 0)' & \text{with probability } (\lambda\mathrm{m} - \lambda_{i\mathrm{m}})\,\mathrm{d}t \\ (0\ 1)' & \text{with probability } (\lambda_i - \lambda_{i\mathrm{m}})\,\mathrm{d}t \\ (0\ 0)' & \text{with probability } (1 - \lambda_i - \lambda_{\mathrm{m}} + \lambda_{i\mathrm{m}})\,\mathrm{d}t. \end{cases} \tag{5.5}$$

To guarantee that each arrival rate falls in the $[0,1]$ interval, we impose the restriction $\lambda_{i\mathrm{m}} \leq \min\{\lambda_i, \lambda_{\mathrm{m}}\}$. The processes N_{m} and N_i can be interpreted as the marginal processes of the process $\mathbf{N}$ (with marginal arrival rates equal to λ_{m} and λ_i, respectively). N_{m} and N_i are independent iff $\lambda_{i\mathrm{m}} = \lambda_i\lambda_{\mathrm{m}}$.[3]

[2]For a general discussion of loss aversion we refer to Kahneman and Tversky (1979) and Tversky and Kahneman (1991). Benartzi and Thaler (1995), Barberis *et al.* (2001) and Berkelaar *et al.* (2004) discuss loss aversion in a finance context.

[3]Independence is equivalent with $\Pr[dN_i = 1|dN_m = 1] = \lambda_{i\mathrm{m}}/\lambda_{\mathrm{m}}\,\mathrm{d}t = \lambda_i\,\mathrm{d}t = \Pr[dN_i = 1]$.

The structure of this bivariate Poisson process captures our basic idea that individual assets need not all behave in the same way when the market encounters a crash. A value of $\lambda_{i\mathrm{m}}/\lambda_{\mathrm{m}}$ close to one implies that an individual asset has a high likelihood to crash if the market crashes, whereas a value for $\lambda_{i\mathrm{m}}/\lambda_{\mathrm{m}}$ close to zero implies that this probability is negligible. As we show later, this conditional probability is crucial for the crash risk premium present in an individual asset. This structure distinguishes our design from Ho *et al.* (1996), where one jump process is present in all processes underlying the assets, and from Merton (1971), who studies asset pricing in the presence of idiosyncratic jumps.

Based on these assumptions we derive the pricing kernel and the price processes of the assets in the economy. We provide the derivation in Appendix 5.A and discuss the resulting equilibrium equations for the expected returns here. The instantaneous expected excess return on the market asset equals

$$\mathrm{E}_t\left[R_{\mathrm{m}}\right] \equiv \mathrm{E}_t\left[\frac{\mathrm{d}S_{\mathrm{m}}}{S_{\mathrm{m}}}\right] = \gamma\sigma_{\mathrm{m}}^2\,\mathrm{d}t + \lambda_{\mathrm{m}}\left(\mathrm{e}^{\delta-\gamma\kappa_{\mathrm{m}}}-1\right)\left(1-\mathrm{e}^{\kappa_{\mathrm{m}}}\right)\mathrm{d}t. \tag{5.6}$$

The first term is the risk premium that the agent requires in the traditional CAPM setting. In our setting it is the risk premium associated with diffusion risk. The price of a unit of diffusion risk σ_{m}^2 is γ. The second term reflects crash risk. Its expression is more complicated than the expression for diffusion risk, but it has a similar structure. The effect of a crash consists of its probability λ_{m} times its impact on wealth, being a decrease with a factor $1-\mathrm{e}^{\kappa_{\mathrm{m}}}$ (the exponent arises because the crash takes place in the growth rate of $X_{\mathrm{m}T}$). To find the premium associated with crash risk, this expression is multiplied by $\mathrm{e}^{\delta-\gamma\kappa_{\mathrm{m}}}-1$, which can be interpreted as the price of one unit of crash risk. It is a function of general risk aversion γ and an extra term δ capturing crash aversion.

The expected excess return on asset i as underlying equals

$$\mathrm{E}_t\left[R_i\right] \equiv \mathrm{E}_t\left[\frac{\mathrm{d}S_i}{S_i}\right] = \frac{\sigma_i}{\sigma_{\mathrm{m}}}\rho_{i\mathrm{m}}\zeta_{\mathrm{d}}\,\mathrm{d}t + \frac{1-\mathrm{e}^{\kappa_i}}{1-\mathrm{e}^{\kappa_{\mathrm{m}}}}\frac{\lambda_{i\mathrm{m}}}{\lambda_m}\zeta_{\mathrm{c}}\,\mathrm{d}t, \tag{5.7}$$

where we use $\zeta_{\mathrm{d}} \equiv \gamma\sigma_{\mathrm{m}}^2$ and $\zeta_{\mathrm{c}} \equiv \lambda_{\mathrm{m}}\left(\mathrm{e}^{\delta-\gamma\kappa_{\mathrm{m}}}-1\right)\left(1-\mathrm{e}^{\kappa_{\mathrm{m}}}\right)$. The first term is again a reward for the diffusion risk that the contract on X_{iT} entails. It has the same structure as in the traditional CAPM. The expression $\sigma_i/\sigma_{\mathrm{m}}\cdot\rho_{i\mathrm{m}}$ gives the asset's β with respect to the market as in Eq. (5.1). Note that this β gives a sensitivity, conditional on no crashes occurring, which is why we call it a diffusion-β. The second term covers the premium for crash risk. Similar as the diffusion risk premium, it consists of the risk premium for market crashes multiplied with a sensitivity factor. This sensitivity factor can be easily compared to the diffusion-β. The ratio $(1-\mathrm{e}^{\kappa_i})/(1-\mathrm{e}^{\kappa_{\mathrm{m}}})$ reflects the relative magnitude of a crash in X_{iT} in the same way as the ratio $\sigma_i/\sigma_{\mathrm{m}}$

give the relative magnitude of the diffusion in X_{iT}. The conditional probability $\lambda_{i\mathrm{m}}/\lambda_{\mathrm{m}}$ reflects the dependence between crashes in X_{iT} and market crashes and can be compared to the parameter $\rho_{i\mathrm{m}}$, which corresponds with the dependence of the two diffusion processes. Consequently, we interpret the product of the magnitude ratio and the conditional probability as the asset's crash-β.

Equations (5.6) and (5.7) offer the main insights of the crash-CAPM. Under the crash-CAPM, the expected return on a stock can be split in a part related to its sensitivity to the market during normal periods and a part related to its sensitivity to the market in times of a market crash. The premium on crash risk reflects the "normal" aversion to the risk that crashes entail and the extra aversion that agents have to crash losses. As a consequence, the crash risk premium can be quite pronounced.

5.3 Conditional crash likelihood

The insights of the crash-CAPM are useful in guiding the empirical research in this chapter. We want to shed more light on two issues. First, we investigate whether we can identify a crash risk premium in the cross section of stock returns. Second, we examine whether crash risk helps explaining the cross-sectional variation of stock returns. To answer the first question we use the common technique of sorting stocks into portfolios, in this case based on their sensitivity to market crashes (Harvey and Siddique, 2000, follow the same approach for coskewness). To answer the second question we use those portfolios again. We do not base the answers on direct estimation of the Crash-CAPM, but use the portfolio approach instead, because it needs less assumptions.

In this section we propose three measures to determine a stock's market crash sensitivity. According to Eq. (5.7) the sensitivity consists of two components: a ratio of crash magnitude and a conditional probability of a crash, given that the market crashes. Our measures concentrate on that conditional probability, and are based on the difference between the actual dependence and the dependence implied by the diffusion processes.[4] The conditional crash probability affects the dependence between an asset and the market. If it equals zero, this dependence originates solely from the dependence in the diffusion processes, which is measured by the correlation coefficient $\rho_{i,m}$. If the conditional probability is larger than zero, the actual dependence between an asset and the market reflects both the general dependence from the diffusion processes and the dependence due to crashes. Indeed, empirical evidence finds that the dependence between stocks and markets increases for more

[4]We expect that the cross-sectional variation in stocks' crash β stems mainly from this component, in particular if we take into account that crash magnitudes will actually be stochastic.

extreme observations.[5] In other words, if the proportion of observations in a sample that qualify as crashes increases, the observed dependence becomes stronger.

The three measures are derived from copulas, because they constitute a flexible and powerful tool to model dependence.[6] A copula is a function that models the dependence between random variables. It returns the joint cumulative probability of a set of events as a function of the cumulative marginal probabilities of each event.[7] Since we are interested in dependence between an asset and the market, we will use bivariate copulas in this paper. This means that we model the joint distribution of the return on asset i and the market return with a copula function $C()$ as

$$\Pr(R_i \leq r_i, R_m \leq r_m) = C(\Pr(R_i \leq r_i), \Pr(R_m \leq r_m)). \tag{5.8}$$

The dependence due to the diffusion processes implies a Gaussian copula, which is related to the normal distribution. If the conditional crash probability is high, the actual dependence will deviate from the diffusion-implied Gaussian copula, and this deviation will be larger for a larger conditional crash probability. We exploit this idea in the three measures that we construct. The first measure is based on the difference between the empirical copula and the Gaussian copula. The second and third measure use the Student's t copula. We discuss the construction of the measures in more detail in the next subsection. Since copulas only model the dependence between random variables, we first discuss how the marginal return distribution can be modelled.

5.3.1 Marginal models

The marginal distributions that Eqs. (5.22) and (5.25) in Appendix 5.A imply for the market return and the individual asset return, respectively, would be normal distributions in the absence of crash risk. Its presence will lead to distributions that deviate from normal distributions and have negative skewness and fat tails. As a consequence, we model the marginal distributions by a skewed Student's t distribution. It can be constructed from the regular Student's t distribution by the method proposed by Fernández and Steel (1998) as shown by Lambert and Laurent (2001) and de Jong and Huisman (2000). Its density function is given by:

$$\psi_{sk}(x; \mu, \sigma, \nu, \xi) = \begin{cases} c \cdot \psi\left(\xi \dfrac{x-\mu}{\sigma}; \nu\right) & \text{if } x - \mu \leq 0 \\ c \cdot \psi\left(\dfrac{1}{\xi}\dfrac{x-\mu}{\sigma}; \nu\right) & \text{if } x - \mu > 0, \end{cases} \tag{5.9}$$

[5] See among others Hartmann *et al.* (2004), Bae *et al.* (2003), Ang and Chen (2002) and Longin and Solnik (2001).

[6] See also Chapter 3 and the references therein.

[7] Copula theory is discussed in general terms in Joe (1997) and Nelsen (1999), and in a financial setting by Cherubini *et al.* (2004) and Bouyé *et al.* (2000).

where $\psi(z;\nu)$ is the standard Student's t distribution with degrees of freedom parameter $\nu > 2$, μ, $\sigma > 0$ and $\xi > 0$ are the location, dispersion and skewness parameters, respectively, and $c = 2\xi/(\sigma(\xi^2+1))$ is a constant. Because of the transformation, the parameters cannot be interpreted directly as moments. For $\xi < 1$, the distribution is left-skewed, for $\xi = 1$ it is symmetric, and for $\xi > 1$ it is right-skewed.

The main reason for choosing the skewed Student's t distribution lies in its flexibility. Direct estimation of the parameters in Eqs. (5.23) and (5.25) would be an attractive alternative under the assumption that the model part for crashes is correctly specified.[8] Using the skewed Student's t distribution does not require an assumption on the specific structure of the crash model.

5.3.2 Measures for conditional crash likelihood

The first dependence measure we use is based on the Gaussian copula and the empirical copula. The Gaussian copula is related to the normal distribution and describes the dependence implied by Wiener processes. Its bivariate functional form is given by

$$C_2^{\Phi}(u,v;\rho^{\Phi}) = \Phi_2\left(\Phi^{-1}(u), \Phi^{-1}(v), \rho^{\Phi}\right), \quad u,v \in [0,1], \tag{5.10}$$

where $\Phi_2()$ denotes the standard normal cumulative distribution function with correlation coefficient ρ^{Φ}, and Φ^{-1} denotes the inverse of the univariate standard normal cumulative distribution function.

The empirical copula is the copula version of the empirical distribution function. For a given set of T observations $\boldsymbol{r}_i$ and $\boldsymbol{r}_{\mathrm{m}}$, its functional form can be written as

$$C_{\mathrm{E}}(u,v;\boldsymbol{r}_{\mathrm{m}},\boldsymbol{r}_i) = \frac{1}{T}\sum_{t}^{T} I\left(r_{it} \leq r_i^{\lfloor uT \rfloor}\right) \cdot I\left(r_{\mathrm{m}t} \leq r_{\mathrm{m}}^{\lfloor vT \rfloor}\right), \quad u,v \in [0,1], \tag{5.11}$$

where $r^{\lfloor uT \rfloor}$ is the k^{th} (ascending) order statistic, k being the largest integer not exceeding uT, and $I()$ is the indicator function.

The first measure for conditional crash likelihood, which we call the empirical measure, is constructed as

$$\lambda_{i|\mathrm{m}}^{\mathrm{emp}} = \frac{C_{\mathrm{E}}(u,v;\boldsymbol{r}_i,\boldsymbol{r}_{\mathrm{m}})}{v} - \frac{C_2^{\Phi}(u,v,\rho_{i\mathrm{m}}^{\Phi})}{v}. \tag{5.12}$$

The first term gives the empirical probability that the individual asset return falls below the quantile associated with probability u, given that the market return lies below the v-quantile. The second term gives the same conditional probability using the Gaussian copula. If the dependence between the individual asset return and the

[8] Das and Uppal (2004) follow this approach by using GMM.

market return are driven largely by diffusions, the empirical copula and the Gaussian copula will yield approximately the same result. However, if crash dependence is present, the empirical copula will give a higher joint probability than the Gaussian copula. Consequently, we interpret a high value for $\lambda_{i|\mathrm{m}}^{\mathrm{emp}}$ as a conditional crash probability. Cappiello *et al.* (2005) use a similar measure to investigate comovements to identify contagion. Throughout the chapter we will use $u = v$.

The other two measures are based on the Student's t copula. The functional form of the bivariate Student's t copula reads

$$\begin{gathered} C_2^{\Psi}(u, v; \nu^{\Psi}, \rho^{\Psi}) = \Psi_2\left(\Psi^{-1}(u, \nu^{\Psi}), \Psi^{-1}(v, \nu^{\Psi}); \nu^{\Psi}, \rho^{\Psi}\right), \\ u, v \in [0, 1],\ \nu^{\Psi} > 2, \end{gathered} \tag{5.13}$$

where $\Psi_2()$ denotes the cumulative distribution function of the bivariate Student's t distribution with correlation coefficient ρ^{Ψ} and degrees of freedom parameter ν^{Ψ}, and $\Psi^{-1}()$ denotes the inverse of the univariate standard Student's t cumulative distribution with degrees of freedom parameter ν^{Ψ}.

The main difference between the Student's t copula and the Gaussian copula is the tail dependence that the Student's t copula entails. Tail dependence χ is the limit of the conditional probability of an extreme realization of a random variable U, given that the realization of a random variable V is extreme

$$\chi \equiv \lim_{u \downarrow 0} \Pr(U \leq u | V \leq u) = \lim_{u \downarrow 0} \frac{\Pr(U \leq u, V \leq u)}{\Pr(V \leq u)} = \lim_{u \downarrow 0} \frac{C_2(u, u)}{u}, \tag{5.14}$$

where U and V are assumed to have a marginal uniform distribution (See Joe, 1997, Ch. 2.1.10). If $\chi = 0$ the two variables do not exhibit tail dependence, and if $\chi > 0$ they do. Embrechts *et al.* (2002) shows that a Gaussian copula with $\rho^{\Phi} \neq 1$ implies tail independence. On the other hand, the Student's t copula implies tail dependence, even for $\rho^{\Psi} = 0$. The Student's t copula and the Gaussian copula belong to the class of elliptic copulas. The Student's t copula converges to the Gaussian copula for $\nu^{\Psi} \to \infty$.

Because tail dependence can be interpreted as the limit of the conditional crash probability in Eq. (5.7) for crashes getting more and more severe, we use it as an asymptotic measure for conditional crash probability. We base it on the Student's t copula, because the Student's t copula can also capture the elliptic dependence implied by the diffusion processes. Embrechts *et al.* (2002) derive a closed form expression for the tail dependence implied by the bivariate Student's t copula as

$$\lambda_{i|\mathrm{m}}^{\mathrm{a}} = 2 \cdot \Psi\left(-\sqrt{(\nu_{i\mathrm{m}}^{\Psi} + 1)\frac{1 - \rho_{i\mathrm{m}}^{\Psi}}{1 + \rho_{i\mathrm{m}}^{\Psi}}}; \nu_{i\mathrm{m}}^{\Psi} + 1\right). \tag{5.15}$$

We call it the asymptotic measure for conditional crash likelihood.[9]

The third measure we use for conditional crash likelihood is based on the degrees of freedom parameter ν^{Ψ} of the Student's t copula. If the conditional crash probability is low, it will not influence the actual dependence much, which will then come close to a Gaussian copula. That means that the degrees of freedom parameter estimate should be high. On the other hand, if the degrees of freedom parameter is low, the actual dependence deviates strongly from the Gaussian copula, indicating that the conditional crash likelihood is high. To give this measure the same domain as the other two measures, we define the degrees of freedom measure as

$$\lambda^{\nu}_{i|\mathrm{m}} = 1/(\nu^{\Psi} - 1), \tag{5.16}$$

which ensures $\lambda^{\nu}_{i|m} \in (0, 1]$. Of course, it does not have the interpretation of a probability.

We use these measures for conditional crash likelihood because they complement each other. The first measure is mainly based on crash observations. However, as the number of crashes in a sample is typically low, this measure may not be very precise (i.e. have a large standard error). The other two measures are parametric, which gives the advantage that they can be estimated more precisely. However, they are based on all observations, and as a consequence they reflect other parts of the distribution as well. Since the tail dependence implied by the Student's t copula is a function of ν^{Ψ}, the second and third measure are likely to be strongly related. However, the tail dependence is a function of ρ as well, which can lead to differences. In the empirical part we investigate the relation of the three measures in more detail.

5.4 Crash portfolios

5.4.1 Data and Methods

In this section we examine portfolios constructed with the three different measures for crash likelihood. In the portfolio construction we use all regular stocks (share code 10 and 11) in the stock database of the Center for Research in Security Prices (CRSP) at the University of Chicago during the period June 1964 - November 2003. The market return series also comes from the CRSP database. The risk-free rate is the one-month Treasury bill rate from Ibbotson Associates. Both series are available on the web site of French[10].

[9] In fact, we can write $\lambda^{\mathrm{a}}_{i|\mathrm{m}} = \lim_{u\downarrow 0} \{C^{\Psi}_2(u, u; \nu, \rho^{\Psi})/u - C^{\Phi}_2(u, u; \rho^{\Phi})/u\}$, since the limit of the second term equals zero. This shows that we can interpret $\lambda^{\mathrm{a}}_{i|\mathrm{m}}$ as an asymptotic version of $\lambda^{\mathrm{emp}}_{i|\mathrm{m}}$, where the Student's t copula replaces the empirical copula.

[10] `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`

The main step in the empirical analysis of this chapter is the construction of crash portfolios. To construct portfolios that have a low, intermediate or high probability of crashing if the market crashes, we use the following approach. In each month and for each stock we calculate the values of the three crash likelihood measures presented in Section 5.3.2. We base these calculations on the estimates for the copula parameters over a history of 120 months (implying that stocks with a shorter history are omitted). We use the inference functions for margins method (IFM) proposed by Joe (1997) for estimation. This two step procedure first estimates the parameters for the marginal distributions, i.e. the skewed Student's t distributions. The second step yields the estimates for the copulas, treating the marginal distribution parameters as given. In both steps we use maximum likelihood estimators. It is possible to apply maximum likelihood estimation to jointly estimate the parameters for the marginal distributions and the copulas. While IFM is less efficient than one-step maximum likelihood estimation, it is computationally more attractive. Moreover, it guarantees that the estimates for the marginal distribution of the market return do not vary depending on which individual asset's returns are included in the estimation.

Based on crash likelihood we sort the stocks into portfolios: portfolio L if the stock belongs to the bottom third, portfolio M if the stock falls in the middle third and portfolio H in the top third. The portfolios are value-weighted, with each stock weighted by its market value at the beginning of the month. Each portfolio contains the same number of stocks. At the end of the month the portfolio return is calculated. We also construct hedge portfolios, entailing a long position in the portfolio with the high conditional crash probability-stocks and a short position in the low conditional crash probability-stocks. We use the portfolio returns to investigate the presence of a crash risk premium. After correcting for diffusion risk, the portfolios from stocks with high conditional crash probabilities should offer a significantly positive abnormal return. On the contrary, portfolios of stocks with low conditional crash probabilities should offer insignificant abnormal returns. The average return on the hedge portfolio should be significantly positive.

Finally, we examine whether the returns on the crash portfolios can be explained by other hedge portfolio returns. We consider the size and value hedge portfolios from Fama and French (1993), a momentum portfolio as in Jegadeesh and Titman (1993), and portfolios based on coskewness as in Harvey and Siddique (2000) and cokurtosis as in Dittmar (2002). The returns on the hedge portfolios for the size, value and momentum effects are available from the web site of French (as SMB, HML and UMD, respectively). We construct hedge portfolios for coskewness and cokurtosis ourselves (see Appendix 5.B for details).

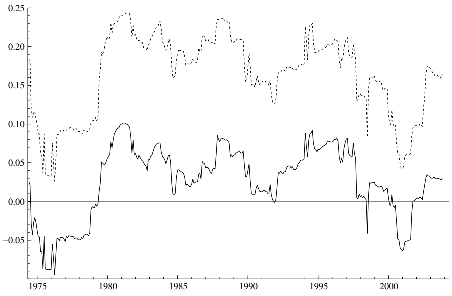
5.4.2 Constructing crash portfolios

The crash portfolios are constructed based on the three measures for conditional crash likelihood. Figure 5.1 shows the evolution of the different measures over time. It reports the one third quantiles (solid lines) and the two thirds quantiles (dashed lines) that are used in the portfolio construction. Figure 5.1(a) plots the quantiles for the empirical measure. It is based on the difference between the empirical copula and the Gaussian copula. For both copulas we calculate the probability that a stock crashes given that the market crashes. A crash is defined here as a return below the u-quantile, where we have set u equal to 5%. If the empirical and the Gaussian copulas do not differ significantly from each other, the empirical measure will be close to zero. We observe that the one third quantile is close to zero. Sometimes, it lies below zero, indicating that for some stocks the Gaussian copula implies a higher conditional probability of returns below the 5% quantile than empirically observed. For many stocks the empirical conditional probability of returns below the 5% quantile exceeds the Gaussian implied probability. For one third of the stocks this difference is easily 0.15 or larger. We will see later whether these stocks offer on average higher returns.

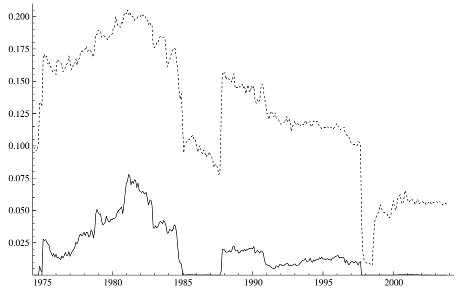
The asymptotic measure in Figure 5.1(b) and the degrees of freedom measure in Figure 5.1(c) are based on the Student's t copula. Because both measures are functions of the degrees of freedom parameter, the patterns of the quantiles show a clear resemblance. For the one third and the two thirds quantiles the correlation coefficients equal 0.69 and 0.82, respectively. We clearly see the effect of the crash of October 1987 as both measures show an immediate increase after it. However, the figures indicate that the crash of October 1987 ends a short period of low conditional crash probabilities, as it seems that the evolution of the measures over 1988-1997 is a continuation of 1975-1985. An inspection of the data shows that July, August and September of 1974 were notorious crash months with market returns as low as -7.79%, -9.37% and -11.78% (and a rebounce of 16.05% in October 1974). Because these months drop from the estimation horizon from July 1984 onwards, we see a decrease towards the beginning of 1985. At the end of 1997 we see a similar decrease in the conditional crash probability measures. By the end of the 1990s the measures revert partially. The time trend that seems to be present in the two thirds quantile of the asymptotic measure does not show up in the two thirds quantile of the degrees of freedom measure.

All measures exhibit considerable time variation, justifying our preference for the portfolio approach combined with a rolling estimation window. In each subfigure, the solid and dashed lines follow a similar pattern, indicating that changes in conditional crash likelihood are similar across stocks (the correlations between the one third

Figure 5.1: Evolution of crash likelihood measures over time



(a) empirical measure $\lambda^{\mathrm{emp}}_{i|\mathrm{m}}$



(b) asymptotic measure $\lambda^{\mathrm{a}}_{i|\mathrm{m}}$

(c) degrees of freedom measure $\lambda^{\nu}_{i|\mathrm{m}}$

This figure shows the one third and two thirds quantiles of the distribution of the three measures for the probability that a stock crashes given that the market crashes. The empirical measure (panel a) is calculated as the conditional probability of a stock return below the 5%-quantile, given that the market return falls below the 5%-quantile based on the empirical copula minus that conditional probability according to the Gaussian copula as in Eq. (5.12). The asymptotic measure (panel b) is calculated as the tail dependence coefficient that is implied by the Student's t copula, using Eq. (5.15). The degrees of freedom measure (panel c) is the transformation in Eq. (5.16) of the degrees of freedom parameters of the Student's t copula. We calculate the measure for each stock in each month, if the return history is long enough. The empirical copula and the parameter estimates for the Gaussian and Student's t copulas are based on the previous 120 monthly excess stock returns and excess market returns, using the IFM method of Joe (1997). Based on the cross section of stocks in each month, we calculate the values of the measures below which one third and two thirds of the stocks in that month lie. The solid (dashed) line corresponds with the one third (two thirds) quantile.

Table 5.1: Portfolio statistics for different sorts

	sorted on $\lambda^{\text{emp}}_{i\|\text{m}}$			sorted on $\lambda^{\text{a}}_{i\|\text{m}}$			sorted on $\lambda^{\nu}_{i\|\text{m}}$		
	L	M	H	L	M	H	L	M	H
$\overline{\lambda_{\text{L}}}$	−1	0.026	0.16	0	0.017	0.12	0	0.050	0.17
$\overline{\lambda_{\text{U}}}$	0.026	0.16	1	0.017	0.12	1	0.050	0.17	1
μ	5.92	7.98	8.66	5.80	8.17	7.30	5.80	8.13	7.43
σ	13.23	13.46	14.11	15.83	13.33	13.26	15.84	13.27	13.07
μ/σ	0.45	0.59	0.61	0.37	0.61	0.55	0.37	0.61	0.57

This table presents summary statistics of the different portfolios into which the stocks are sorted based on a measure for its conditional crash probability, given that the market crashes. We use the three different measures for this probability and calculate the value of these measures for each stock in each month as described in the caption of Figure 5.1. For each month we construct three value-weighted portfolios with an equal number of stocks, based on the values of the measures for the conditional crash probability, using the market values at the beginning of the month as weights. We label the portfolio L, M and H reflecting low, intermediate and high values of the conditional crash likelihood measure. For each portfolio and each measure we report the average lower and upper bound used to construct the portfolio ($\overline{\lambda_{\text{L}}}$, and $\overline{\lambda_{\text{U}}}$), the average excess return μ (in % per annum), the volatility σ (in % per annum) and the annual Sharpe ratio μ/σ.

quantiles and two thirds quantiles are 0.96 for the empirical measure, 0.85 for the asymptotic measure and 0.94 for the degrees of freedom measure). The empirical measure and the other two measures do not seem to be much related. Considering the one third quantiles we find correlation coefficients of 0.29 between the empirical measure and the asymptotic measure, and 0.44 between the empirical measure and the degrees of freedom measure. For the two thirds quantiles correlation coefficients are 0.24 and 0.37 respectively.

A first glance on the different portfolios that we construct based on the conditional crash likelihood measures is provided in Table 5.1. For each measure, we show the characteristics of the three portfolios, portfolio L with the stocks that have the lowest values for that measure (below the one third quantile), portfolio M with stock that fall in intermediate range (between the one third and two thirds quantile), and portfolio H with stocks in the top third quantile. The average values for the one third quantile are all close to zero (0.026 for the empirical measure, 0.017 for the asymptotic measure and 0.050 for the degrees of freedom measure). The two thirds quantiles equal on average 0.162, 0.124 and 0.165, respectively, and Figure 5.1 shows that they are larger than zero for each month. We interpret the relatively low values for the two third quantiles as an indication that idiosyncratic shocks account for a large proportion of the crashes in individual stocks. However, given the investor's general aversion to market-wide losses, the premium can still have a considerable impact on expected returns.

The average excess returns that we find for the different portfolios point in the direction of a reward for crash risk. The average excess returns on the L portfolios are considerably lower than the returns on the M and H portfolios. The difference varies from 1.63% to 2.74% on an annual basis. Moreover, in case of the asymptotic measure and the degrees of freedom measure, the volatility of the L portfolios exceeds the volatility of the M and H portfolios. Consequently, the Sharpe ratios for the L portfolios are considerably less attractive than for the M and H portfolios. Gathering all stocks into one value-weighted portfolio produces a Sharpe ratio of 0.54 over the period June 1974 - November 2003.[11] Of course, we have not taken differences in standard market risk exposure into account. In the next subsection, we check whether the M and H portfolios outperform the L portfolios after a correction for market risk. We also check whether the return series are related to other risk factors.

5.4.3 Crash portfolio analysis

In this subsection we put the different portfolios constructed in the previous subsection under further scrutiny. The first results in Table 5.1 indicate that the M and H portfolios perform better than the L portfolios. However, this difference may be caused by different exposures to market (diffusion) risk. In this subsection we correct for this exposure, and examine whether the pattern of Table 5.1 remains. By constructing a hedge portfolio we test whether a long position in stocks with high conditional crash probabilities and a short position in stocks with low conditional crash probabilities yields a positive average pay-off.

It is also possible that the better performance of the M and H portfolios can be explained by other trading strategies that yield significant outperformance. We consider the familiar strategies based on size and value versus growth, as proposed by Fama and French (1993, 1995) and momentum as put forward by Jegadeesh and Titman (1993). Harvey and Siddique (2000) show that a Taylor expansion of the pricing kernel leads to the inclusion of coskewness and Dittmar (2002) extends this approach to cokurtosis. Under regular assumptions on utility functions, investors have a preference for increasing coskewness and decreasing cokurtosis. We construct coskewness and cokurtosis hedge portfolios and examine whether the crash portfolios are related to these portfolios. In appendix 5.B we examine the trading strategies in more detail and discuss the construction of the coskewness and cokurtosis portfolios. We include two sets of coskewness and cokurtosis portfolios in our analysis. The first

[11] The Sharpe ratio for the complete market over this period equals 0.43, which is considerably lower. However, the portfolios we construct contains only stocks with a history of more than 10 years, which means that all stocks that have not been listed for 10 years are excluded.

set is constructed similarly as the crash portfolios, using a 120-months estimation window. This set can indicate whether our crash measures capture to a large degree the same information as coskewness and cokurtosis measures would. The second set uses an estimation window of 60 months, and more resembles the approach of Harvey and Siddique (2000).

Table 5.2 shows the regression results for the portfolios constructed with the empirical measure for conditional crash likelihood. Panels (a) to (c) consider the portfolios L, M and H and panel (d) reports the results on the hedge portfolio. We conduct simple OLS regression and calculate Newey-West standard errors. The coefficients on r_{m} indicate that the L, M and H portfolios all exhibit significant exposures to traditional market risk. After correcting for this exposure, the M and H portfolios still show significant α's of 0.26% (portfolio M) and 0.33% (portfolio H) per month. The α for the L portfolio is insignificant. We interpret this result as evidence that a portfolio with a higher crash exposure offers an extra return that cannot be explained by exposure to the market. The actual return due to exposure to market crashes may be even higher, because the estimated coefficient on r_{m} will capture crash exposure for a small part.

The portfolios have only limited exposure to other trading strategies. The coefficients on the size, value or momentum portfolios are significant in a few cases only, and the coefficients are generally small. The small negative coefficients on SMB may indicate that our selection procedure is slightly biased towards big firms. The relation with coskewness is stronger. The portfolios M and H, which have a relatively large exposure to crash risk, have significant coefficients on the coskewness hedge portfolios. The coefficient on the 120-months coskewness portfolio is larger than the coefficient on the 60-month coskewness portfolio. However, since the 120-month coskewness portfolio does not yield a significant positive return (see Table 5.8 in Appendix 5.B), the α's of the crash portfolios are not much affected. If we use the 60-month coskewness portfolio, which yields a significant return, the α's of the M and H portfolios decrease but remain significant at the 5% level.[12] The L and H portfolios show significantly negative exposures to the cokurtosis portfolios. Because the return on the kurtosis portfolio is insignificant, the α's of the crash portfolios do no change much. Depending on the risk corrections in the regressions, the portfolio with stocks with the largest exposure to crash risk yield an extra return of 2.5% to 4% on an annual basis.

[12] Including both the 120 months and the 60 months coskewness portfolios in one regression, shows that the explanatory effect of the 60 month portfolio is completely captured by the 120 month portfolio.

Table 5.2: Analysis of portfolios constructed based on the empirical conditional crash likelihood measure.

(a) portfolio L^{emp} (bottom third)

α	0.49^{b}	(0.22)	0.12	(0.10)	-0.01	(0.14)	0.11	(0.10)	0.11	(0.11)
r_m			0.64^{a}	(0.05)	0.68^{a}	(0.05)	0.62^{a}	(0.05)	0.64^{a}	(0.05)
SMB					-0.04	(0.05)				
HML					0.10	(0.06)				
UMD					0.07	(0.07)				
NMP							0.04	(0.07)		
LMP							-0.23^{a}	(0.07)		
NMP60									-0.01	(0.07)
LMP60									-0.18^{a}	(0.06)

(b) portfolio M^{emp} (middle third)

α	0.66^{a}	(0.22)	0.26^{a}	(0.08)	0.13	(0.10)	0.24^{a}	(0.08)	0.21^{b}	(0.09)
r_m			0.69^{a}	(0.05)	0.75^{a}	(0.04)	0.67^{a}	(0.05)	0.69^{a}	(0.05)
SMB					-0.08^{b}	(0.04)				
HML					0.12^{c}	(0.07)				
UMD					0.07	(0.05)				
NMP							0.21^{a}	(0.06)		
LMP							-0.15^{b}	(0.08)		
NMP60									0.15^{b}	(0.07)
LMP60									-0.07	(0.06)

(c) portfolio H^{emp} (top third)

α	0.72^{a}	(0.23)	0.33^{a}	(0.11)	0.22^{c}	(0.12)	0.31^{a}	(0.11)	0.26^{b}	(0.12)
r_m			0.67^{a}	(0.05)	0.72^{a}	(0.05)	0.62^{a}	(0.04)	0.66^{a}	(0.04)
SMB					-0.07^{c}	(0.04)				
HML					0.11	(0.08)				
UMD					0.06	(0.05)				
NMP							0.38^{a}	(0.07)		
LMP							-0.28^{a}	(0.08)		
NMP60									0.20^{b}	(0.08)
LMP60									-0.25^{a}	(0.07)

(d) hedge portfolio (portfolio H^{emp} - portfolio L^{emp})

α	0.23^{b}	(0.10)	0.21^{b}	(0.10)	0.23^{b}	(0.11)	0.20^{b}	(0.09)	0.15^{c}	(0.09)
r_m			0.03	(0.03)	0.04	(0.04)	0.00	(0.02)	0.02	(0.03)
SMB					-0.04	(0.04)				
HML					0.01	(0.05)				
UMD					-0.01	(0.03)				
NMP							0.34^{a}	(0.06)		
LMP							-0.05	(0.05)		
NMP60									0.21^{a}	(0.05)
LMP60									-0.07^{c}	(0.04)

This table present the regression results for different regressions of the monthly returns of the L, M and H portfolios on a constant, the excess market return and other factor portfolios. The L, M and H portfolios are value weighted portfolios with equal numbers of stocks, constructed based on the empirical measure for conditional crash likelihood (see Eq. 5.12) for the different stocks, estimated over the preceding 120 months. As regressors we consider the excess market return (r_m), the size factor (SMB), value factor (HML) and momentum factor (UMD) available on the website of French, and a coskewness factor (NMP) and cokurtosis factor (LMP) (both constructed as described in Appendix 5.B, based on an estimation window of 120 months and of 60 months (labeled NMP60 and LMP60)). Newey-West standard errors are in parentheses. Superscripts a, b or c indicates significance at the 1%, 5% or 10% level, respectively.

In panel (d), we report the results on the hedge portfolio, constructed by a long position in portfolio H and a short position in portfolio L. The average return on this portfolio equals 0.23% per month and is significant at the 5% level. This hedge portfolio does not have a significant exposure to the market return, nor to the size, value or momentum portfolios. Since portfolio H has an exposure to the coskewness factor, whereas portfolio L does not, the hedge portfolio shows a similar exposure to the coskewness factor as portfolio H. A correction for this exposure based on 120-month coskewness portfolio does not affect α much as it remains significant at 0.20% per month. This entails a yearly outperformance of 2.5%. If the 60-month coskewness portfolio is used, the α decreases but remains marginally significant.

In Table 5.3 we consider the portfolio constructed with the asymptotic measure. The L portfolio contains stocks with hardly any tail dependence with the market (the average upper bound for this portfolio is reported in Table 5.1 as 0.017). Zero tail dependence is consistent with dependence completely driven by the diffusion processes. If we correct for the correlation with the market return, the abnormal return of the L portfolio is positive but not significantly different from zero.[13] The M and H portfolios show dependence that cannot stem from diffusion processes, as discussed in the theoretical section. Panels b and c of Table 5.3 show that the α's of the M and H portfolios remain significantly positive after correcting for market risk. Strangely, the α of the M portfolio exceeds the α of the H portfolio, though not significantly.

Including the hedge portfolios SMB, HML and UMD in the regressions yields similar estimates as in Table 5.2. The coefficients are small and mostly insignificant. Each portfolio has a significant sensitivity to the coskewness factor of about the same size, which is a bit puzzling. Apparently, selecting stocks on their tail dependence with the market is different from selecting stocks on coskewness. Sensitivities to cokurtosis are different across portfolios. The L portfolio has a significant negative exposure to the cokurtosis hedge portfolio, indicating that it is more platykurtic. We find that H portfolio is more leptokurtic. Since tail dependence is an asymptotic concept, it should reflect the joint behavior in the extreme parts of the distribution. Consistent with this statement, the sensitivities of the portfolios in Table 5.3 compared with those in Table 5.2 show that cokurtosis gains importance while coskewness loses.

The results on the hedge portfolio in panel (d) of Table 5.3 indicate that shorting assets with hardly any tail dependence and investing in assets with relatively high

[13] In case of the empirical measure, both positive and negative deviations from the Gaussian copula are possible. This means that the L portfolio of the empirical measure cannot be related directly to no deviations from the Gaussian copula, contrary to the case of the asymptotic measure.

Table 5.3: Analysis of portfolios constructed based on the asymptotic conditional crash likelihood measure

(a) portfolio L^a (bottom third)

α	0.48^c	(0.27)	0.11	(0.15)	-0.05	(0.20)	0.09	(0.14)	0.08	(0.15)
r_m			0.63^a	(0.06)	0.66^a	(0.08)	0.57^a	(0.06)	0.62^a	(0.07)
SMB					0.04	(0.08)				
HML					0.09	(0.10)				
UMD					0.11	(0.10)				
NMP							0.22^b	(0.10)		
LMP							-0.63^a	(0.13)		
NMP60									0.00	(0.13)
LMP60									-0.50^a	(0.11)

(b) portfolio M^a (middle third)

α	0.68^a	(0.21)	0.30^a	(0.09)	0.12	(0.11)	0.28^a	(0.09)	0.25^a	(0.09)
r_m			0.65^a	(0.05)	0.71^a	(0.04)	0.62^a	(0.05)	0.64^a	(0.05)
SMB					-0.06	(0.05)				
HML					0.14^c	(0.07)				
UMD					0.12^b	(0.06)				
NMP							0.22^a	(0.07)		
LMP							-0.23^a	(0.09)		
NMP60									0.13	(0.08)
LMP60									-0.17^b	(0.08)

(c) portfolio H^a (top third)

α	0.61^a	(0.22)	0.19^b	(0.10)	0.11	(0.10)	0.19^c	(0.10)	0.16	(0.10)
r_m			0.70^a	(0.04)	0.76^a	(0.04)	0.69^a	(0.04)	0.70^a	(0.04)
SMB					-0.12^a	(0.03)				
HML					0.09	(0.06)				
UMD					0.05	(0.04)				
NMP							0.14^a	(0.05)		
LMP							-0.03	(0.05)		
NMP60									0.12^b	(0.06)
LMP60									-0.01	(0.05)

(d) hedge portfolio (portfolio H^a - portfolio L^a)

α	0.12	(0.14)	0.08	(0.15)	0.17	(0.17)	0.10	(0.13)	0.08	(0.12)
r_m			0.07^b	(0.03)	0.10^b	(0.04)	0.12^a	(0.03)	0.08^b	(0.03)
SMB					-0.16^b	(0.06)				
HML					0.00	(0.07)				
UMD					-0.06	(0.07)				
NMP							-0.08	(0.07)		
LMP							0.60^a	(0.12)		
NMP60									0.12	(0.10)
LMP60									0.49^a	(0.10)

This table present the regression results for different regressions of the monthly returns of the L, M and H portfolios on a constant, the excess market return and other factor portfolios. The L, M and H portfolios are value weighted portfolios with equal numbers of stocks, constructed based on the empirical measure for conditional crash likelihood (see Eq. 5.15) for the different stocks, estimated over the preceding 120 months. As regressors we consider the excess market return (r_m), the size factor (SMB), value factor (HML) and momentum factor (UMD) available on the website of French, and a coskewness factor (NMP) and cokurtosis factor (LMP) (both constructed as described in Appendix 5.B, based on an estimation window of 120 months and of 60 months (labeled NMP60 and LMP60)). Newey-West standard errors are in parentheses. Superscripts a, b or c indicates significance at the 1%, 5% or 10% level, respectively.

tail dependence does not yield a significant abnormal return, though it is on average positive. The hedge portfolio inherits the sensitivity to cokurtosis from the L portfolio.

Constructing portfolios with the degrees of freedom measure for conditional crash likelihood is almost the same as using the asymptotic measure. The results in Table 5.4 are virtually the same as in Table 5.3. The returns to the hedge portfolio are somewhat larger, but remain insignificant. The correlation coefficient of the portfolios constructed with the degrees of freedom measure with their respective companion portfolios from the asymptotic measure are all larger than 0.99. The hedge portfolios are also highly correlated.

Based on the empirical analysis in this section we conclude that a crash risk premium can be found in the cross section of stock returns. Buying stocks that score highly on the measures that we have constructed yields an extra return of 2% to 4% after correction for other risk exposures. However, only the empirical measure leads to a profitable hedge portfolio with a statistically significant extra return of 2.5%. This hedge portfolio is almost completely uncorrelated with the hedge portfolios of the asymptotic and the degrees of freedom measures, indicating that these measures are complements and not substitutes.

5.5 Explaining the cross section of stock returns

The question that remains to be answered is whether crash risk can contribute to explaining the cross section of stock returns. In the previous section we established that portfolios with a relatively high exposure to crash risk earn on average an extra pay-off after correcting for exposure to market risk and possible other risk factors. This indicates that a premium for crash risk is present. In this section we examine whether a combination of exposure to market diffusion risk and exposure to market crash risk leads to insignificant pricing errors. We start with an investigation of the explanatory power of the crash risk portfolio on a set of well-known portfolios. Then we consider the cross section of individual stock returns.

5.5.1 Portfolios tests

In its search for explanations for the cross section of stock returns, empirical research has established several groups of portfolios whose return differences could not be explained sufficiently by the traditional CAPM. From these portfolios the hedge portfolios are constructed that are used many times as a risk factor, i.e. the size, value and momentum factors. While these portfolios are specifically constructed for

Table 5.4: Analysis of portfolios constructed based on the degrees of freedom measure for conditional crash likelihood

(a) portfolio L^{ν} (bottom third)

α	0.48^{c}	(0.27)	0.10	(0.14)	-0.05	(0.20)	0.08	(0.14)	0.07	(0.15)
r_{m}			0.64^{a}	(0.06)	0.67^{a}	(0.08)	0.59^{a}	(0.06)	0.64^{a}	(0.07)
SMB					0.03	(0.08)				
HML					0.09	(0.10)				
UMD					0.10	(0.10)				
NMP							0.21^{b}	(0.10)		
LMP							-0.61^{a}	(0.13)		
NMP60									0.00	(0.12)
LMP60									-0.48^{a}	(0.11)

(b) portfolio M^{ν} (middle third)

α	0.68^{a}	(0.22)	0.26^{a}	(0.08)	0.15	(0.10)	0.25^{a}	(0.08)	0.22^{a}	(0.08)
r_{m}			0.70^{a}	(0.04)	0.74^{a}	(0.04)	0.67^{a}	(0.04)	0.69^{a}	(0.04)
SMB					-0.06^{c}	(0.04)				
HML					0.08	(0.05)				
UMD					0.08^{b}	(0.04)				
NMP							0.20^{a}	(0.05)		
LMP							-0.22^{a}	(0.06)		
NMP60									0.13^{b}	(0.06)
LMP60									-0.15^{a}	(0.06)

(c) portfolio H^{ν} (top third)

α	0.62^{a}	(0.21)	0.22^{b}	(0.10)	0.12	(0.09)	0.22^{b}	(0.10)	0.18^{c}	(0.10)
r_{m}			0.67^{a}	(0.05)	0.74^{a}	(0.04)	0.66^{a}	(0.05)	0.67^{a}	(0.05)
SMB					-0.14^{a}	(0.03)				
HML					0.12^{c}	(0.07)				
UMD					0.06	(0.04)				
NMP							0.14^{b}	(0.06)		
LMP							0.01	(0.06)		
NMP60									0.15^{b}	(0.06)
LMP60									0.02	(0.05)

(d) hedge portfolio (portfolio H^{ν} - portfolio L^{ν})

α	0.14	(0.14)	0.12	(0.15)	0.17	(0.16)	0.13	(0.13)	0.11	(0.12)
r_{m}			0.03	(0.04)	0.06	(0.04)	0.08^{a}	(0.03)	0.03	(0.03)
SMB					-0.17^{a}	(0.06)				
HML					0.03	(0.08)				
UMD					-0.04	(0.07)				
NMP							-0.07	(0.07)		
LMP							0.61^{a}	(0.11)		
NMP60									0.14	(0.10)
LMP60									0.50^{a}	(0.09)

This table present the regression results for different regressions of the monthly returns of the L, M and H portfolios on a constant, the excess market return and other factor portfolios. The L, M and H portfolios are value weighted portfolios with equal numbers of stocks, constructed based on the empirical measure for conditional crash likelihood (see Eq. 5.16) for the different stocks, estimated over the preceding 120 months. As regressors we consider the excess market return (r_{m}), the size factor (SMB), value factor (HML) and momentum factor (UMD) available on the website of French, and a coskewness factor (NMP) and cokurtosis factor (LMP) (both constructed as described in Appendix 5.B, based on an estimation window of 120 months and of 60 months (labeled NMP60 and LMP60)). Newey-West standard errors are in parentheses. Superscripts a, b or c indicates significance at the 1%, 5% or 10% level, respectively.

asset pricing tests, industry portfolios are also often used as they suffer less from data snooping.

In this subsection we consider portfolio sets based on industries, size, value and momentum. For each set we conduct a cross sectional test as described in Cochrane (2001, Ch. 12). We estimate the exposure to risk factors for each portfolio i in a time series regression

$$R^{\mathrm{e}}_{it} = a_i + \boldsymbol{\beta}_i' \boldsymbol{f}_t + e_{it}, \tag{5.17}$$

where R^{e}_{it} is the excess return on portfolio i, $\boldsymbol{f}_t$ is a vector with the factor values, i.e. excess returns, at time t, and $\boldsymbol{\beta}$ is a vector with sensitivities and e_{it} denotes the error term. We use these sensitivities to estimate the risk premia $\boldsymbol{\zeta}$ for the factors:

$$E[R^{\mathrm{e}}_{it}] = \alpha_i + \boldsymbol{\zeta}' \boldsymbol{\beta}_i. \tag{5.18}$$

The term α_i has the interpretation of a pricing error. If the risk factors $\boldsymbol{f}$ accurately explain the cross section of stocks returns, the pricing errors should be zero. This hypothesis can be tested formally by constructing the statistic $\hat{\boldsymbol{\alpha}}' \widehat{\mathrm{cov}[\boldsymbol{\alpha}]}^{-1} \hat{\boldsymbol{\alpha}}$, where $\hat{\boldsymbol{\alpha}}$ is a vector of empirical pricing errors. We estimate the model in Eqs. (5.17) and (5.18) in a GMM framework. In this way we automatically include the Shanken (1992) correction in the covariance matrix of the pricing errors for the fact that the sensitivities are estimated. Moreover, by using the weighting scheme from Newey and West (1987) for the spectral density matrix, we can incorporate autocorrelation and heteroskedasticity. Since Eq. (5.18) implies a number of moments equal to the number of portfolios and only one parameter (the risk premium) per risk factor, we weigh the moments by $\boldsymbol{\beta}$ to construct the GMM objective, as discussed in Cochrane (2001). In this setup we can use the TJ_T statistic to test for significant pricing errors. Under the null hypothesis of zero pricing errors, this statistic follows a χ^2-distribution with degrees of freedom equal to the number of portfolios minus the number of risk premia.

The data for the industry, size and value portfolios are based on the CRSP database and come from the website of French. The industry set consists of 10 portfolios based on SIC codes: Consumer Non-Durables (1), Consumer Durables (2), Manufacturing (3), Energy (4), High Tech (5), Telecom (6), Shops (7), Health (8), Utilities (9) and Others (10). The size portfolios are 10 portfolios with stocks sorted on market equity from small (1) to large (10). The value portfolios are 10 portfolios with stocks sorted on the book-to-market ratio, from low (growth, 1) to high (value, 10).[14] The set of momentum portfolios is based on the CRSP database and can be

[14] See the web site of Kenneth French, `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`, for detailed information on portfolio construction.

downloaded from the website of Van Vliet.[15] This set consists of 10 portfolios with stocks sorted on their performance over the one to twelve months prior to portfolio formation (See Post and van Vliet, 2004). The industry, size and value portfolios are available over the entire period for which we have constructed crash risk portfolios (June 1974 - November 2003). The momentum portfolios end at December 2002, implying a slightly shorter horizon for the test based on the momentum portfolios.

For each set of portfolios, we conduct three cross-sectional tests. First, we test the traditional CAPM. In the second test we include the hedge portfolio based on the empirical measure for conditional crash likelihood. In the third test we include the excess returns on the H portfolio based on the asymptotic measure for conditional crash likelihood. We do not include the hedge portfolio based on the asymptotic measure for two reasons. First, the exposure to crash risk should be concentrated completely in the H portfolio, when using the asymptotic measure. For the empirical measure this does not apply by definition. Second, the hedge portfolio did not yield a significant expected positive pay-off, whereas the H portfolio did. We do not consider the portfolios based on the degrees of freedom measure, because they are to a large extent identical to the portfolios based on the asymptotic measure.

An alternative approach would be to use the fact that all factors we consider are themselves returns and conduct a time series test (i.e. test whether the a_i's in Eq. (5.17) are insignificant). The factor risk premium is then taken equal to the average factor value. However, we have a specific reason not to use this approach. We want to establish two risk factors: a market diffusion factor and a market crash factor. However, as they are both present in the market return, we cannot estimate the market diffusion premium as the time series average of the market return. In a cross-sectional approach we do not have to make such an assumption.

Table 5.5 shows the results from the cross-sectional tests on the different portfolios. It reports the estimated risk premia and the pricing errors, together with their standard errors, and the TJ_T-statistic with a p-value. We draw several conclusions based on this table. First, the addition of the $\mathrm{H^a}$ portfolio improves the explanatory power of the model. Generally, we see a decrease of the TJ_T statistic and an increase of the p-value. This improvement is most notable for the momentum portfolios (see panel d). The traditional CAPM is rejected for momentum portfolios, but after the addition of the $\mathrm{H^a}$ portfolio, the pricing errors are not jointly significant anymore. In the appendix we show that the momentum effect is by far larger than the size and the value effect. In case of the traditional CAPM, six out of ten portfolios exhibit significant pricing errors. Not all pricing errors vanish, but they become smaller and less significant.

[15]See `http://www.few.eur.nl/few/people/wvanvliet/datacenter/index.htm`.

Table 5.5: Cross sectional tests with and without crash risk on single sorted portfolios

(a) Industry portfolios

ζ_{d}	0.67^{a}	(0.25)	0.66^{a}	(0.25)	0.63^{b}	(0.25)
$\zeta_{\mathrm{c}}^{\mathrm{emp}}$			0.57	(0.55)		
$\zeta_{\mathrm{c}}^{\mathrm{a}}$					0.76^{a}	(0.26)
1	0.27^{c}	(0.16)	0.27	(0.18)	0.15	(0.12)
2	−0.08	(0.18)	−0.15	(0.19)	−0.03	(0.19)
3	−0.13	(0.11)	−0.08	(0.13)	-0.19^{c}	(0.11)
4	0.18	(0.23)	0.23	(0.22)	0.07	(0.21)
5	−0.30	(0.25)	−0.20	(0.20)	−0.08	(0.16)
6	0.05	(0.20)	0.03	(0.20)	0.20	(0.18)
7	0.00	(0.13)	−0.06	(0.12)	−0.01	(0.13)
8	0.10	(0.18)	0.15	(0.20)	−0.09	(0.17)
9	0.23	(0.18)	0.06	(0.13)	0.09	(0.15)
10	0.03	(0.10)	−0.02	(0.09)	0.03	(0.10)
TJ_T	9.31	[0.41]	6.79	[0.56]	8.51	[0.39]

(b) Size portfolios

ζ_{d}	0.77^{a}	(0.26)	0.71^{a}	(0.25)	0.69^{a}	(0.24)
$\zeta_{\mathrm{c}}^{\mathrm{emp}}$			−1.10	(1.23)		
$\zeta_{\mathrm{c}}^{\mathrm{a}}$					−0.04	(0.44)
1	0.16	(0.16)	0.03	(0.08)	0.01	(0.06)
2	0.09	(0.09)	−0.02	(0.04)	−0.04	(0.05)
3	0.03	(0.06)	0.01	(0.06)	0.00	(0.05)
4	0.00	(0.05)	−0.07	(0.07)	−0.02	(0.05)
5	0.05	(0.04)	0.04	(0.06)	0.05	(0.04)
6	−0.05	(0.05)	0.01	(0.08)	−0.01	(0.05)
7	0.03	(0.05)	0.07	(0.05)	0.08^{c}	(0.05)
8	−0.06	(0.07)	0.02	(0.05)	−0.03	(0.05)
9	−0.06	(0.09)	0.03	(0.05)	0.03	(0.04)
10	−0.21	(0.16)	−0.14	(0.14)	−0.07	(0.08)
TJ_T	9.03	[0.43]	6.22	[0.62]	7.37	[0.50]

(c) Value portfolios

ζ_{d}	0.80^{a}	(0.25)	0.78^{a}	(0.24)	0.65^{b}	(0.25)
$\zeta_{\mathrm{c}}^{\mathrm{emp}}$			0.84	(0.80)		
$\zeta_{\mathrm{c}}^{\mathrm{a}}$					1.70^{b}	(0.70)
1	-0.51^{b}	(0.20)	-0.50^{b}	(0.20)	-0.41^{b}	(0.19)
2	−0.14	(0.09)	−0.14	(0.10)	−0.11	(0.09)

3	−0.10	(0.07)	−0.09	(0.09)	−0.04	(0.09)
4	0.00	(0.09)	0.04	(0.09)	−0.09	(0.09)
5	0.04	(0.09)	−0.04	(0.05)	−0.03	(0.11)
6	0.07	(0.07)	0.07	(0.09)	0.11	(0.10)
7	0.20^b	(0.10)	0.19^b	(0.09)	0.09	(0.09)
8	0.15	(0.10)	0.12	(0.09)	0.00	(0.09)
9	0.21^b	(0.10)	0.21^c	(0.12)	0.24^c	(0.13)
10	0.29^c	(0.17)	0.33^c	(0.18)	0.39^b	(0.19)
TJ_T	8.39	[0.50]	7.53	[0.48]	5.91	[0.66]

(d) Momentum portfolios

ζ_{d}	0.45^c	(0.25)	0.46^c	(0.26)	0.44^c	(0.25)
$\zeta_{\mathrm{c}}^{\mathrm{emp}}$			−1.72	(1.63)		
$\zeta_{\mathrm{c}}^{\mathrm{a}}$					1.02^a	(0.32)
1	-1.01^a	(0.25)	-1.05^b	(0.45)	-0.50^b	(0.21)
2	-0.35^c	(0.21)	−0.16	(0.23)	−0.31	(0.24)
3	−0.13	(0.13)	−0.23	(0.27)	−0.22	(0.17)
4	0.09	(0.12)	0.12	(0.15)	−0.09	(0.11)
5	−0.07	(0.10)	−0.03	(0.14)	-0.23^b	(0.10)
6	0.04	(0.09)	0.09	(0.14)	-0.19^b	(0.08)
7	0.25^b	(0.12)	0.26	(0.25)	0.00	(0.08)
8	0.38^a	(0.13)	0.53^c	(0.32)	0.19	(0.12)
9	0.43^b	(0.18)	0.34	(0.23)	0.44^b	(0.22)
10	0.69^a	(0.26)	0.51^b	(0.23)	0.90^b	(0.40)
TJ_T	21.28^b	[0.01]	11.38	[0.18]	9.86	[0.27]

This table reports the results of cross-sectional tests of three asset pricing models on four sets of portfolios: portfolios based on industry (panel a), on size (panel b), on book-to-market ratios (panel c) and on momentum (panel d). We consider the CAPM, the CAPM extended with the hedge portfolio based on the empirical measure for conditional crash likelihood and the CAPM extended with the H portfolios based on the asymptotic measure for conditional crash likelihood. We estimate the sensitivities and risk premia in a GMM framework. For the industry, size and value portfolios we use observations from June 1974 - November 2003. For the momentum portfolios we use observations from June 1974 - December 2002. We report premia on market diffusion risk (ζ_{d}), on the empirically based hedge portfolio ($\zeta_{\mathrm{c}}^{\mathrm{emp}}$), and on the asymptotically based H portfolio ($\zeta_{\mathrm{c}}^{\mathrm{a}}$). After the numbers 1 to 10 we report the pricing errors for the different portfolios. The industry portfolios are ordered as Consumer Non-Durables (1), Consumer Durables (2), Manufacturing (3), Energy (4), High Tech (5), Telecom (6), Shops (7), Health (8), Utilities (9) and Others (10). The size portfolios are ordered from small (1) to big (10). The value portfolios are ordered from low (1) to high (10). The momentum portfolios are ordered from loser (1) to winner (10). After each estimate the Newey and West (1987) consistent standard error is reported in parentheses. TJ_T reports the value of the TJ_T statistic, with the p-value for an insignificant TJ_T statistic based on a χ^2 distribution in brackets. Superscripts a, b or c indicates significance at the 1%, 5% or 10% level, respectively.

Second, the hedge portfolio based on the empirical measure for conditional crash likelihood performs worse than the H^{texta} portfolios. While we generally see a decrease of the TJ_T statistic and an increase of the p-value, we also observe large standard errors for the estimates of the pricing errors. Addition the H^{a} portfolio leads to more precise estimates of the pricing errors than the addition of the hedge portfolio based on the empirical measure.

Third, we observe that the estimates for the risk premium associated with the H^{a} portfolio are positive and significant, with exception of the case of the size portfolios. Moreover, in the other three cases the premium is considerable and exceeds the premium for market risk. The premium on the hedge portfolio based on the empirical measure is not significant. The size of this premium stresses the importance of crash risk.

Evidence in Post and van Vliet (2004) indicates that many anomalies are related to trading characteristics of small firms (e.g. transaction costs and liquidity). Therefore, we investigate a set of 25 portfolios, double sorted on size and value, which can also be downloaded from the website of French.[16] Table 5.6 shows cross-sectional tests for both the CAPM and the CAPM with the crash factors for this set of portfolios. The CAPM cannot explain the cross-sectional variation of these portfolio returns. Significant pricing errors seems to be concentrated in small firms and growth firms. However, addition of crash risk portfolios does not produce an improvement. While the TJ_T-statistic provides some evidence in favor of the empirically based hedge portfolio, its premium is not significant. The risk premium for the H portfolio based on the asymptotic measure is significant, but the TJ_T-statistic increases in this case. The average absolute pricing error is lowest for the last model, but this is driven by size portfolios 2, 3 and 4, and not by the portfolios of the small firms, nor by the portfolios of big firms. It can be that the way in which we calculate the crash likelihood measures and form the portfolios, creates a bias against the inclusion of small firms, as they tend to have a shorter history.

The evidence in Table 5.5 indicates that crash risk may contribute to the explanation of the cross section of stock returns, but the evidence is not overwhelming. However, for the industry, value and momentum portfolios, the asymptotically based H portfolio points at a priced risk factor. The premium for this factor is large and exceeds the premium for diffusion risk. Moreover, this risk factor can account for part of the momentum effect, which is the largest compared to the size and value effects (see Table 5.8). Also, we see for several portfolios that both the pricing errors and the corresponding standard errors decrease, which it also a clear improvement, though it does not necessarily lead to less significant pricing errors. The results on

[16] We refer to this website for details on construction.

Table 5.6: Cross-sectional tests with and without crash risk for double sorted portfolios on size and value.

ζ_{d}		0.83^a	(0.27)	0.85^a	(0.26)	0.89^a	(0.27)
$\zeta_{\mathrm{c}}^{\mathrm{emp}}$				0.62	(0.81)		
$\zeta_{\mathrm{c}}^{\mathrm{a}}$						1.11^a	(0.36)
Small	Growth	-0.81^a	(0.26)	-0.75^a	(0.19)	-0.47^a	(0.10)
	2	0.10	(0.14)	0.18^c	(0.10)	0.33^a	(0.09)
	3	0.31^b	(0.12)	0.39^a	(0.11)	0.41^a	(0.11)
	4	0.55^a	(0.14)	0.62^a	(0.15)	0.63^a	(0.14)
	Value	0.54^a	(0.16)	0.58^a	(0.18)	0.62^a	(0.17)
2	Growth	-0.58^a	(0.17)	-0.54^a	(0.14)	-0.43^a	(0.11)
	2	-0.07	(0.08)	-0.03	(0.08)	-0.05	(0.08)
	3	0.24^b	(0.09)	0.24^a	(0.09)	0.18^c	(0.09)
	4	0.40^a	(0.12)	0.40^a	(0.11)	0.36^a	(0.11)
	Value	0.32^b	(0.14)	0.36^b	(0.16)	0.31^b	(0.14)
3	Growth	-0.48^b	(0.19)	-0.42^b	(0.17)	-0.37^a	(0.13)
	2	0.03	(0.08)	-0.04	(0.12)	-0.01	(0.08)
	3	0.12	(0.10)	0.08	(0.08)	0.02	(0.07)
	4	0.26^b	(0.12)	0.19^b	(0.09)	0.13	(0.09)
	Value	0.45^a	(0.17)	0.42^a	(0.15)	0.37^a	(0.14)
4	Growth	-0.31^c	(0.18)	-0.31^c	(0.19)	-0.21	(0.17)
	2	-0.09	(0.10)	-0.09	(0.11)	-0.20^a	(0.08)
	3	0.10	(0.10)	0.05	(0.08)	-0.03	(0.07)
	4	0.21^c	(0.12)	0.17^b	(0.08)	0.10	(0.08)
	Value	0.22	(0.15)	0.09	(0.12)	0.08	(0.10)
Big	Growth	-0.37^c	(0.21)	-0.41^b	(0.20)	-0.49^b	(0.22)
	2	-0.10	(0.14)	-0.11	(0.14)	-0.24^c	(0.13)
	3	-0.04	(0.14)	-0.12	(0.10)	-0.16	(0.13)
	4	0.04	(0.15)	-0.01	(0.11)	-0.15^c	(0.08)
	Value	0.04	(0.18)	0.04	(0.21)	-0.07	(0.15)
		41.42^b	[0.01]	37.64^b	[0.03]	41.94^a	[0.01]

This table reports the results of cross-sectional tests of three asset pricing models on a sets of 25 portfolios constructed based on size and value. We estimate the CAPM, the CAPM extended with the hedge portfolio based on the empirical measure for conditional crash likelihood and the CAPM extended with the H portfolios based on the asymptotic measure for conditional crash likelihood. Estimation of the sensitivities and risk premia takes place in a GMM framework. We use observations from June 1974 - November 2003. We report the premium on market diffusion risk (ζ_{d}), the premium on the empirically based hedge portfolio ($\zeta_{\mathrm{c}}^{\mathrm{emp}}$), and the premium on the asymptotically based H portfolio ($\zeta_{\mathrm{c}}^{\mathrm{a}}$). The pricing errors for the portfolios are reported by (size, value)-quintile combination. After each estimate the Newey and West (1987) consistent standard error is reported in parentheses. TJ_T reports the value of the TJ_T statistic. In brackets we report the p-value for an insignificant TJ_T statistic, based on a χ^2 distribution. Superscripts a, b or c indicates significance at the 1%, 5% or 10% level, respectively.

the double sorted size/value portfolios indicate that the measures for crash likelihood do not work well for explaining the return on small firms.

5.5.2 Individual stocks

In this subsection we examine the added value of a crash risk factor on the entire cross section of stock returns instead of several portfolios as in the previous subsection. Crucial for a correct estimation of a risk premium (and hence the pricing errors) is enough variation in the sensitivities for the risk factors. Considering the entire cross section yields the largest possible variation. On the other hand, a direct test of the hypothesis that the pricing errors are jointly zero on each stock cannot be conducted, since too few observations are available to estimate the covariance matrix of the pricing errors. Therefore, we present the results in this subsection as complementary to the results in the previous subsection.

We conduct this analysis in a Fama and MacBeth (1973)-framework to allow changes in the sensitivities over time. We estimate the time series regression in Eq. (5.17) over 60 months. For month 61 we estimate the cross-sectional regression

$$R_i^e = \alpha_i + \boldsymbol{\zeta}'\boldsymbol{\beta}_i, \tag{5.19}$$

where R_i^e is the excess return for stock i for month 61, $\boldsymbol{\beta}_i$ is the vector with sensitivities estimated over the prior 60 months, α_i is the pricing error and $\boldsymbol{\zeta}$ is the vector of risk premia. Based on the outcomes we calculate the cross-sectional average pricing error. To be included in this regression for a certain month, a stock needs to have a complete return series over 61 months. We start with the month 1 to 60 for the time series regression and 61 for the cross-sectional regression, then we consider month 2 to 61 for the time series regression and 62 for the cross-sectional regression, and so on. After this procedure is ended, we have a time series for each risk premium and for the average pricing error. Based on these series we calculate the time series average of the risk premia and the time series average of the (average) pricing error, and the corresponding standard error.

Table 5.7 presents the results of this analysis. We report two estimates for standard errors: an estimate that is only based on the time series of premia and pricing errors, and another estimate that includes corrections due to Shanken (1992) for the fact that the sensitivities $\boldsymbol{\beta}_i$ are estimated as discussed in Cochrane (2001, Ch. 12). We find that the CAPM is rejected, as it leads to significant pricing errors of 0.51% per month. Adding the empirically based hedge portfolio is an improvement, since the average pricing error decreases. However, the premium for this factor is not significant. Adding the asymptotically based hedge portfolio leads to a further

Table 5.7: Cross-sectional test with an without crash risk for the entire cross section of stocks

	est	se	se+	est	se	se+	est	se	se+
ζ_d	0.85	0.26	0.37	0.78	0.24	0.36	0.70	0.24	0.35
ζ_c^{emp}				0.11	0.12	0.16			
ζ_c^{a}							0.71	0.22	0.32
$\bar{\alpha}$	0.51	0.13	0.14	0.43	0.11	0.11	0.26	0.07	0.08

This table reports the results for a cross-sectional test of three asset pricing models on the entire cross section of stocks. The test is conducted in a Fama and MacBeth (1973)-framework, with an estimation window of 60 months. The returns of month 61 are then used to estimate the risk premia and to construct pricing errors. We consider all stocks in the CRSP database from June 1974 to November 2003. To be included in the analysis a stock should have at least a complete series 61 observations. We report the time series averages of the risk premia. For each month, we calculate the cross-sectional average pricing error, and we report the time series average of this series ($\bar{\alpha}$). We report the premium on market diffusion risk (ζ_d), the premium on the empirically based hedge portfolio (ζ_c^{emp}), and the premium on the asymptotically based H portfolio (ζ_c^{a}). We also report estimates of the standard errors that are only based on the time series of the premia and pricing errors (column se) and standard error estimates that include a correction for the fact that the risk sensitivities are estimated based on Shanken (1992) (column se+). We calculate the additive correction as the variance of the risk factors divided by the number of observations, and the multiplicative correction as $\zeta'\Sigma_f^{-1}\zeta$, where ζ denotes the vector of estimated risk premia, and Σ_f is the factor variance matrix (see Cochrane, 2001, Ch. 12, for a discussion). All standard errors are based on a Newey and West (1987) correction for autocorrelation and heteroskedasticity.

reduction of the average pricing error to 0.26% per month. The estimated standard errors decrease as well, and consequently the average pricing error is still significant. The premium for this crash risk factor is 0.71% per months and is significant at the 1% level for the time series standard error and at the 5% level if Shanken (1992)-corrections are included.

Combining the results of this analysis with those of the previous subsection, we conclude that crash risk contributes to explaining the cross section of stock returns. For both the empirical measure and the asymptotic measure we find lower pricing errors and lower corresponding standard errors. Moreover, the H portfolio based on the asymptotic measure leads to a significant premium. The estimates for this premium vary based on the techniques and samples, but confidence intervals of the premium estimate all include the 0.71% that is estimated based on the broadest cross section. This would put crash risk on equal importance as market diffusion risk.

5.6 Conclusion

We have investigated whether crash risk is present in the cross section of stock returns. Based on Bates (2001) we show how the traditional CAPM can be extended to capture crash risk. We assume that the dividend process that underlies an asset price process contains a Brownian motion and a Poisson process. The market price of crash risk reflects both general risk aversion and specific aversion to crashes. Each asset pays a premium for crash risk that is the product of the market crash risk premium and the sensitivity that the asset exhibits with regard to market crashes. This sensitivity is the product of an asset's conditional crash likelihood and the ratio of the asset's crash magnitude with that of the market. If crash aversion is high or the crash magnitude is large, crash risk exposure can have a considerable impact on the cross section of stock returns.

We use the crash-CAPM to guide our further empirical research. We derive three measures to determine an asset's likelihood to crash, if the market crashes. These measures are based on the difference between the actual dependence of extreme negative returns and the dependence that is implied by the correlation from the Brownian motions. The empirical measure is calculated as the difference between the empirical copula and the Gaussian copula. The asymptotic measure is calculated as the tail dependence that is implied by a Student's t copula estimated for the dependence between the market return and an asset return. The last measure is a transformation of the degrees of freedom parameter of the Student's t copula.

For each stock in the CRSP database we calculate the values for these measures. To capture time-varying coefficients, we use a rolling regression framework of 120 months. We sort these stocks into three equally sized, value weighted portfolios, and use the next month's returns to calculate the portfolio return. For all three measures we find that the portfolio with stocks exhibiting the highest exposure to crash risk yield a significant, positive expected pay-off after correction for market risk. This pay-off varies from 2.3% per year for the asymptotic measure to 4% for the empirical measure. The portfolios with low exposure stocks do not produce a significant expected pay-off. The portfolios are only weakly related to size, value and momentum effects. We report a stronger relation with coskewness and cokurtosis portfolios, but we show that these portfolios do not lead to a significant positive expected pay-off. The hedge portfolio that we construct based on the empirical measure pays a significant expected return of 2.8% per year that cannot be explained by other risk factors.

We investigate whether these portfolios help in explaining the cross section of stock returns. We find that the CAPM suffices to explain the cross sectional variation in industry, size and value portfolios. Inclusion of the portfolio with stocks with a high exposure to market crashes based on the asymptotic measure, leads to small improvements. In the case of momentum portfolios, the CAPM leads to significant pricing errors. The inclusion of crash portfolios improves the fit of the model. The CAPM is also rejected based on 25 portfolios, double sorted on size and value. In this case, adding crash portfolios does not produce improvements. If we conduct a test on the entire cross section of stocks returns, we find that adding a crash portfolio lowers pricing errors considerably. We estimate a crash risk premium equal to 8.4% per year.

The evidence that we present in this chapter indicates that crashes possibly play an important role in the cross section of stock returns. In particular, the fact that crash risk portfolios can explain part of the momentum effect seems promising. The results based on the broad cross-section of stock returns indicate that crash risk may be equally important as market diffusion risk. Of course, the measures that we use are not perfect estimates of conditional crash likelihood. However, given that they all point in the same direction, we conclude that our findings are robust.

5.A Derivation of the crash-CAPM

In this appendix we derive the extended version of the CAPM that takes crash risk into account, based on Bates (2001). We show how the CAPM arises in an endowment economy with a representative agent who consumes at a final date, T. The assumption underlying this model are stated in Section 5.2. Before we start we state an extended version of the lemma in Bates (2001, p. 12) that we use in the derivation.

Lemma *Let Y_{1t} and Y_{2t} be two random variables that follow stochastic processes*

$$\mathrm{d}Y_{it} = \mu_i\,\mathrm{d}t + \sigma_i\,\mathrm{d}Z_i,\ i = 1, 2,$$

where $\mathrm{d}Z_1$ and $\mathrm{d}Z_2$ are correlated Wiener processes with $\mathrm{E}\left[\mathrm{d}Z_1\,\mathrm{d}Z_2\right] = \rho$. Let $\boldsymbol{N}_t = (N_{1t}\ N_{2t})'$ be a bivariate Poisson process that evolves according to

$$\mathrm{d}\boldsymbol{N} = \begin{cases} (1\ 1)' & \textit{with probability}\ \lambda_{11}\,\mathrm{d}t \\ (1\ 0)' & \textit{with probability}\ \lambda_{10}\,\mathrm{d}t \\ (0\ 1)' & \textit{with probability}\ \lambda_{01}\,\mathrm{d}t \\ (0\ 0)' & \textit{with probability}\ \lambda_{00}\,\mathrm{d}t, \end{cases}$$

where the arrival rates are larger than or equal to zero and sum to one. The Wiener processes and Poisson processes are independent.
The expectation of a function

$$F\left(Y_{1t}, Y_{2t}, N_{1t}, N_{2t}\right) = \exp\left(c_1 Y_{1t} + c_2 Y_{2t} + d_1 N_{1t} + d_2 N_{2t}\right),$$

with c_1, c_2, d_1 and d_2 deterministic constants, can then be found as

$$\begin{aligned} &\mathrm{E}_t\left[F\left(Y_{1T}, Y_{2T}, N_{1T}, N_{2T}\right)\right] = \\ &\quad F\left(Y_{1t}, Y_{2t}, N_{1t}, N_{2t}\right)\cdot\exp\Big\{\big(c_1\mu_1 + c_2\mu_2 + \tfrac{1}{2}c_1^2\sigma_1^2 + c_1\sigma_1\rho c_2\sigma_2 + \tfrac{1}{2}c_2^2\sigma_2^2 + \\ &\qquad\qquad \lambda_{11}(\mathrm{e}^{d_1+d_2} - 1) + \lambda_{10}(\mathrm{e}^{d_1} - 1) + \lambda_{01}(\mathrm{e}^{d_2} - 1)\big)\left(T - t\right)\Big\}. \end{aligned}$$

Proof Use Itô's lemma to derive

$$\begin{aligned} \mathrm{E}_t\left[\mathrm{d}F\right] =& F_t\big(c_1\mu_1 + c_2\mu_2 + \tfrac{1}{2}c_1^2\sigma_1^2 + c_1\sigma_1\rho c_2\sigma_2 + \tfrac{1}{2}c_2^2\sigma_2^2 + \\ &\lambda_{11}(\mathrm{e}^{d_1+d_2} - 1) + \lambda_{10}(\mathrm{e}^{d_1} - 1) + \lambda_{01}(\mathrm{e}^{d_2} - 1)\big)\,\mathrm{d}t, \end{aligned}$$

and use the standard solution for the partial differential equation. ■

First we derive the pricing kernel. In equilibrium the market clears and the representative agent holds the market claim. His marginal utility as time T can be found by differentiation Eq. (5.3)

$$\eta_T \equiv \partial_W U(W_T, N_T, T)|_{W_T = X_{\mathrm{m}T}} = \mathrm{e}^{\delta N_{\mathrm{m}T}} X_{\mathrm{m}T}^{-\gamma}. \tag{5.20}$$

Consequently, a pricing kernel that is valid at time t would be η_T/η_t.

We can use this pricing kernel to price assets. Since we are interested in excess returns, we use the riskless asset as numeraire. As a consequence, $\eta_t = \mathrm{E}_t[\eta_T]$. Using the lemma we find

$$\mathrm{E}_t[\eta_T] = \mathrm{e}^{\delta N_{\mathrm{m}t}} X_{\mathrm{m}t}^{-\gamma} \exp\left\{\left(-\gamma\mu_{\mathrm{m}} + \tfrac{1}{2}\gamma^2\sigma_{\mathrm{m}}^2 + \lambda_{\mathrm{m}}\left(\mathrm{e}^{\delta-\gamma\kappa_{\mathrm{m}}} - 1\right)\right)(T-t)\right\}. \tag{5.21}$$

The price $S_{\mathrm{m}t}$ of the market claim satisfies $\eta_t S_{\mathrm{m}t} = \mathrm{E}_t[\eta_T X_{\mathrm{m}T}]$ and based on the lemma we derive

$$S_{\mathrm{m}t} = X_{\mathrm{m}t} \exp\left\{\left(\mu_{\mathrm{m}} + \tfrac{1}{2}\sigma_{\mathrm{m}}^2 - \gamma\sigma_{\mathrm{m}}^2 + \lambda_{\mathrm{m}}\mathrm{e}^{\delta-\gamma\kappa_{\mathrm{m}}}\left(e^{\kappa_{\mathrm{m}}} - 1\right)\right)(T-t)\right\} \tag{5.22}$$

Applying Itô's lemma yields the process followed by the market asset

$$\frac{\mathrm{d}S_{\mathrm{m}}}{S_{\mathrm{m}}} = \left(\gamma\sigma_{\mathrm{m}}^2 - \lambda_{\mathrm{m}}\mathrm{e}^{\delta-\gamma\kappa_{\mathrm{m}}}(\mathrm{e}^{\kappa_{\mathrm{m}}} - 1)\right)\mathrm{d}t + \sigma\,\mathrm{d}Z_{\mathrm{m}} + (\mathrm{e}^{\kappa_{\mathrm{m}}} - 1)\,\mathrm{d}N_{\mathrm{m}} \tag{5.23}$$

Taking expectation produces the expected return on the market asset in (5.6).

The price of the each individual asset i satisfies the fundamental relation $\eta_t S_{it} = \mathrm{E}_t[\eta_T X_{iT}]$ as well. Apply the lemma to find

$$\begin{aligned} S_{it} =& X_{it} \exp\left\{\left(\mu_i + \tfrac{1}{2}\sigma_i^2 - \gamma\sigma_i\rho_{i\mathrm{m}}\sigma_{\mathrm{m}} + (\lambda_m - \lambda_{i\mathrm{m}})\left(\mathrm{e}^{\delta-\gamma\kappa_{\mathrm{m}}} - 1\right) + \right.\right. \\ & \qquad \lambda_{i\mathrm{m}}\left(\mathrm{e}^{\delta-\gamma\kappa_{\mathrm{m}}+\kappa_i} - 1\right) + (\lambda_i - \lambda_{i\mathrm{m}})\left(\mathrm{e}^{\kappa_i} - 1\right) - \\ & \qquad \left.\left.\lambda_{\mathrm{m}}(e^{\delta-\gamma\kappa_{\mathrm{m}}} - 1)\right)(T-t)\right\} \\ =& X_{it}\exp\left\{\left(\mu_i + \tfrac{1}{2}\sigma_i^2 - \gamma\sigma_i\rho_{i\mathrm{m}}\sigma_{\mathrm{m}} + \right.\right. \\ & \qquad \left.\left.\left(\lambda_i + \lambda_{i\mathrm{m}}\left(\mathrm{e}^{\delta-\gamma\kappa_{\mathrm{m}}} - 1\right)\right)\left(e^{\kappa_i} - 1\right)\right)(T-t)\right\}. \end{aligned} \tag{5.24}$$

The process for the asset can then be derived by applying Itô's lemma

$$\begin{aligned} \frac{\mathrm{d}S_i}{S_i} =& \left\{\gamma\sigma_i\rho_{i\mathrm{m}}\sigma_{\mathrm{m}} - \left(\lambda_i + \lambda_{i\mathrm{m}}\left(\mathrm{e}^{\delta-\gamma\kappa_{\mathrm{m}}} - 1\right)\right)\left(\mathrm{e}^{\kappa_i} - 1\right)\right\}\mathrm{d}t + \\ & \sigma_i\,\mathrm{d}Z_i + (\mathrm{e}^{\kappa_i} - 1)\,\mathrm{d}N_i, \end{aligned} \tag{5.25}$$

and taking expectations produces the expected return in Eq. (5.7).

5.B Trading strategies

In this appendix we discuss the hedge portfolios used in Section 5.4.3. Financial researchers have established several investment strategies that yield a positive significant abnormal return that cannot be explained by exposure to other risk factors. These strategies generally consist of constructing hedge portfolios: buy assets that score highly on a certain measure and sell assets that score lowly on it.

The two best-known strategies are due to Fama and French (1993, 1995). Their first strategy exploits the small firm effect by buying stocks of small firms and selling stocks of big firms. The resulting portfolio is commonly referred to as SMB (Small Minus Big). The second entails buying value stocks (firms having a high value for the ratio of book equity to market equity) and selling those of growth stocks (firms with a low book-to-market ratio). It is often denoted as HML (High Minus Low). French's web site provides returns on both hedge portfolio based on the CRSP database from 1926 onwards. Over the sample period that we consider in this chapter, the returns on the SMB and the HML portfolios add up to 3.36% and 5.13% per year, respectively.

Jegadeesh and Titman (1993) show that buying stocks that did relatively well in the recent past (based on a history of three up to twelve months) and selling stocks that performed relatively poorly in the recent past yields a profit. The UMD (Up Minus Down) portfolio, available on French's web site, is based on this strategy. Over the period June 1974 - November 2003, the average return on this portfolio equals 10.67% per year.

The other strategies we consider are based on higher order extensions of the CAPM. One of the possible explanations for the outperformance of the size, value and momentum portfolios argues that these portfolios capture non-linearities in the pricing kernel. Under the assumption that the market portfolio is an accurate proxy of the wealth portfolio, this means that the expected return on a specific stock is a non-linear function of the market return. Harvey and Siddique (2000) derive an extension of the CAPM in which the pricing kernel is a function of the market return and the squared market return. They show that their model implies that coskewness is priced. Coskewness measures to which degree an asset return increases when the squared market return increases. Because standard utility theory prescribes decreasing absolute risk aversion for increasing wealth (see Arditti, 1967), stocks with negative coskewness should pay a premium. Dittmar (2002) carries the extension of Harvey and Siddique (2000) one step further by adding the cubic market return to the pricing kernel equation. This introduces cokurtosis as a factor determining the expected return of an asset. Positive cokurtosis indicates that an asset moves in the same direction as the cubic market return. Under the assumption of decreasing absolute prudence (see Kimball, 1993), investors require a premium for stocks with positive cokurtosis.

As standard accepted portfolios are not available for coskewness and cokurtosis, we construct hedge portfolios capturing their effects. We follow a the same approach as we did for the crash portfolios (see Section 5.4.1), meaning that we start by estimating coskewness and cokurtosis over 120 months. We follow Harvey and Siddique (2000) and construct a coskewness measure $\beta_{i,t}^{\text{SKD}}$ as

$$\beta_{i,t}^{\text{SKD}} = \frac{\mathrm{E}_t\left[\epsilon_{i,t+1}\epsilon_{m,t+1}^2\right]}{\sqrt{\mathrm{E}_t\left[\epsilon_{i,t+1}\right]}\,\mathrm{E}_t\left[\epsilon_{m,t+1}^2\right]}, \tag{5.26}$$

where $\epsilon_{i,t+1}$ is the abnormal return on stock i at time t that results from the standard CAPM, and $\epsilon_{m,t+1}$ is the abnormal return on the market. We estimate this measure based on the residuals of the CAPM regression for asset i over the previous 120 months and the market returns minus their 120 months average. Because Dittmar (2002) does not define a measure for cokurtosis, we define a cokurtosis measure $\beta_{i,t}^{\text{KTD}}$ similar to the measure for coskewness

$$\beta_{i,t}^{\text{KTD}} = \frac{\mathrm{E}_t\left[\epsilon_{i,t+1}\epsilon_{m,t+1}^3\right]}{\sqrt{\mathrm{E}_t\left[\epsilon_{i,t+1}\right]}\left(\mathrm{E}_t\left[\epsilon_{m,t+1}^2\right]\right)^{3/2}}. \tag{5.27}$$

Based on these measures we construct three equally sized, value weighted portfolios, and use next month's stock returns to calculate portfolio returns. Finally, we construct a hedge portfolio for coskewness by a long position in the portfolio with stocks that have low values for the coskewness measure and a short position in portfolio with stocks that score highly. The average return on this NMP (Negative Minus Positive) portfolio equals 1.05% per annum. For cokurtosis we construct a hedge portfolio by a long position in the portfolio of assets with high cokurtosis and a short position in the low cokurtosis portfolio. For this LMP (Leptokurtic Minus Platykurtic) portfolio we find an average return of -0.72%.

The returns we find on the coskewness portfolio is low compared to the returns on the other hedge portfolios. Harvey and Siddique (2000) report a return on their coskewness hedge portfolio of 3.60% per annum. However, they estimate the coskewness measure in Eq. (5.26) over 60 months and calculate the average return over the period July 1963 - December 1993. If we select the same stocks as in the 120-month setup but use an estimation window of 60 months, we end up with an average return of 3.60% per annum.[17] The correlation between the 60-months and 120-months portfolio equals 0.66. Therefore, we will consider this hedge portfolio as well.

[17] If we include all stocks for which a 60 month history is available, we find an average return of 3.71%. This indicates that the estimation window is crucial to forming hedge portfolios based on coskewness.

The return on the cokurtosis hedge portfolio is also small and has the wrong sign. In this case, the consequences of changing the estimation window are small, as the average return based on a 60 month estimation window equals -0.93%. Dittmar (2002) remarks that the inclusion of human capital into the wealth portfolio is crucial for determining the risk premium on kurtosis. The absence of human capital in our wealth portfolio may explain the low value and wrong sign of the result on the kurtosis portfolio. Including human capital is beyond the scope of this research. Another explanation can be that the measure for cokurtosis in Eq. (5.27) is not accurate. However, if we use the regression coefficient of the stock return on the cubic market return (similarly, Harvey and Siddique, 2000, use the regression coefficient of the stock return on the squared market return as alternative measure for coskewness) or the measures proposed by Christie-David and Chaudhry (2001)[18], we find similar results.

In Table 5.8 we report the average market return and the average abnormal return (α) of the different hedge portfolios, after correcting for exposure to market risk and other hedge portfolios. We find that the average return on the market is 0.59% per month or 7.06% per annum. Of the two Fama and French-factors, only HML yields a significantly positive α over the period June 1974 - November 2003 of 7.45% per year. The momentum factor is not much affected by exposure to market risk or the size and the value effect, and remains high at a yearly abnormal return of 12.50%. Of the coskewness and cokurtosis hedge portfolios only the coskewness portfolio based on a 60 month estimation window shows a (marginally) significant α, which equals 3.28% on a yearly basis.

[18]They propose to measure cokurtosis as $\mathrm{E}_t\left[R_{it+1}\epsilon_{m,t+1}\right]/\,\mathrm{E}_t\left[\epsilon^4_{m,t+1}\right]$, where $\epsilon_{m,t+1}$ is the abnormal return on the market.

Table 5.8: Analysis of the market portfolio and hedge portfolios

	r_m	SMB	HML	UMD	NMP	NMP60	LMP	LMP60
constant	0.59^{b}	0.19	0.62^{a}	1.04^{a}	0.04	0.27^{c}	−0.03	−0.14
	(0.24)	(0.16)	(0.18)	(0.23)	(0.14)	(0.15)	(0.12)	(0.13)
r_m		0.16^{a}	-0.33^{a}	−0.13	0.07	0.05	-0.08^{b}	−0.03
		(0.04)	(0.05)	(0.09)	(0.06)	(0.05)	(0.03)	(0.04)
SMB				0.09				
				(0.16)				
HML				−0.24				
				(0.19)				
NMP							0.20	
							(0.13)	
NMP60								0.27^{c}
								(0.15)

This table presents the average market return and the regression results for the monthly return series (in %) of the excess market index and the different hedge portfolio over the period June 1974 - November 2003. We consider the market return r_m, and the hedge portfolios based on the size effect (SMB), the value effect (HML), the momentum effect (UMD), coskewness (NMP, NMP60) and cokurtosis (LMP and LMP60). The SMB, HML and UMD return series are available on French's web site. To construct hedge portfolios for coskewness and cokurtosis we first construct market value weighted L, M and H portfolios for the two measures. The measure for coskewness is given in Eq. (5.26), and is estimated over 120 months (NMP) or 60 months (NMP60). The measure for cokurtosis, given in Eq. (5.27), is also estimated over 120 months (PML) or 60 months (PML60). The coskewness hedge portfolios are constructed as a long position in L portfolio and a short position in the H portfolio. The cokurtosis hedge portfolios are constructed with a long position in the H portfolio and a short position in the L portfolio. We regress the hedge portfolios on a constant, the excess market return and other hedge portfolios. Each columns presents the results for a different hedge portfolio. Newey and West (1987) standard errors are in parentheses. Superscripts a, b or c indicates significance at the 1%, 5% or 10% level, respectively.

The heart of the discerning acquires
knowledge; the ears of the wise seek it out
The Bible

Chapter 6

Summary and conclusion

In this dissertation we have studied crises, crashes and comovements in financial markets. We have identified five reasons to put crises and crashes under scrutiny. First, the frequency of crises and crashes is too high to be explained by news. Second, returns on financial assets behave like a diffusion process during tranquil times but show more jump-like behavior during periods of turmoil. Third, crises and crashes have a persistent strengthening effect on the fluctuation of assets returns. Fourth, both individual assets and markets as a whole exhibit stronger comovements in times of stress. Finally, investors are particularly averse to the large losses that crashes and crises entail. These reasons reinforce each other and turn financial markets into a considerably less attractive place. Crises and crashes happen relatively often and directly harm investors by large losses, but the subsequent increase in volatility that they cause makes investing riskier as well. Moreover, crises and crashes are difficult to evade, as they tend to spread more vehemently than comovements during tranquil periods indicate.

We have discussed four aspects of investor behavior on financial markets that are affected by the different behavior of asset returns during periods of turmoil compared to more quiet periods. First, when managing the risk of an investment in financial markets, an investor should pay specific attention to crises, crashes and their consequences. Second, an investor should also take such risks into account when he constructs his portfolio. Third, investors can expect to receive a premium for their exposure to the risks of crises and crashes, to the extent that they cannot diversify these risks. Fourth, if crises or crashes do not occur because of news, but because of bubbles or congestion of information, a timely recognition of such situations can help investors in predicting the likelihood of a crisis or crash. We have studied these issues one by one in the four main chapters of this dissertation.

In Chapter 2 we have investigated whether the presence of a bubble in the current period leads to a higher crash likelihood for the next period. Patterns of bubbles and crashes have been the subject of many theoretical studies (see Brunnermeier, 2001, for an overview). Empirical research on bubbles concentrates mostly on a specific bubble with its subsequent crash, though Shiller (2000) and Kindleberger (2000) present a more general approach. We analyze the pattern of bubbles and crashes in industries from an investor perspective. This means that we do not identify bubbles with hindsight. Instead, the investor perceives a bubble if the average abnormal return over the last one to five years exceeds a specified threshold. The abnormal returns are constructed based on the CAPM. A crash takes place if the abnormal return of the next period falls below a threshold. We find that the presence of a bubble multiplies the probability of a crash by a factor two for the broadest category of crashes. If we restrict our analysis to more severe crashes the multiplication factor increases to three. If a bubble is stronger than average, we see a further increase in crash likelihood. Our evidence is based on US industries, because this allows us to investigate a large sample. An investigation of bubbles and crashes in the market shows that the number of observations is too low to make statistically sound inferences, but in a qualitative sense the results for the market are similar to those for industries. These results are robust to changes in the research design. Overall, the results imply that riding the bubble entails a serious risk of encountering a crash.

In Chapter 3 we have shown how standard econometric tests can be used to test the fit of a copula. As pointed out by Embrechts *et al.* (2002) copulas provide a flexible tool to model dependence. Correlations and the associated Gaussian copula only capture dependence completely, if it is linear. Other copulas can handle non-linear dependence. Evidence by Longin and Solnik (2001), Ang and Chen (2002), Bae *et al.* (2003) and Hartmann *et al.* (2004) clearly demonstrates the shortcomings of the Gaussian copula to capture stronger comovements of individual assets and of markets in aggregate during crisis periods. While other copulas can handle such behavior, it is not a priori clear which copula to use. We have proposed modifications of the familiar Kolmogorov-Smirnoff and Anderson-Darling tests, which are based on a comparison of the hypothesized distribution (i.e. copula) with the empirical distribution. These tests can be applied more widely than tests proposed by Mashal *et al.* (2003) and Poon *et al.* (2004). We have applied the modified tests to select a copula for stress tests in the risk management of a portfolio of stocks, bonds and real estate. The tests reject the Gaussian copula and the Gumbel copula, but support the Student's t copula. An inspection of the tails reveals that the Gaussian copula significantly underestimates the risk of joint extreme returns, while the Gumbel copula overestimates this risk.

In Chapter 4 the focus shifts to the relation between crises and investment decisions. We have investigated the portfolio implications of systemic crises, i.e. joint, synchronous shocks to all equity markets. We show how a regime switching model in the style of Ang and Bekaert (2002) can be used to capture the main characteristics of a systemic crisis, being decreases of expected returns, increases of volatilities and correlations, and persistence of these effects. For given predictions of the regime switching models we derive the solution of the asset allocation problem of an expected utility maximizing investor in continuous time. We use these theoretical results to empirically examine the consequences of systemic crises for a global investor based on the period 1975-2004. We conclude that it is costly to ignore the risk of such an event. These costs add up to a certainty equivalent return of 1.13% when the investor does not have any prior information on the likelihood of crisis, but rises sharply for relatively small increases in this likelihood. A crisis conscious strategy that incorporates the probability of a crisis advises less leverage and less exposure to crisis-prone assets than a strategy that ignores the risk of systemic crises. The difference between our findings and those of Das and Uppal (2004) indicates that persistence is an important element of systemic crises. Moreover, diversification opportunities erode rapidly. If the investor faces short sales constraints he completely withdraws from equity markets.

The results of the previous chapters show that crises and crashes are difficult to evade. Into the bargain, investor are particularly averse to their consequences. In Chapter 5 we have examined whether investors are rewarded for the crash risk they bear. While evidence exists of a crash risk premium in aggregate market returns (see Bates, 1991, 2000; Andersen *et al.*, 2002), such a premium has not been established for individual stock returns. In Chapter 5 we show how the traditional CAPM can be extended to capture crashes, modelled as negative jumps. The model implies that an asset pays in expectation a premium for crash risk that is proportional to its sensitivity to market crashes. An important factor of this sensitivity is the probability that an asset crashes conditional on a market crash. Based on copulas we have derived three measures to calculate this conditional crash likelihood. Sorting assets on these measures shows that assets belonging to the top third in terms of conditional crash likelihood pay an extra significant yearly return of 2.4% to 4.0% after correction for market risk. Assets that score low on conditional crash likelihood do not pay significantly extra. The crash risk portfolios are related to the coskewness portfolios of Harvey and Siddique (2000) and the cokurtosis portfolios of Dittmar (2002), but these portfolios do not pay a significantly positive average return over our sample period. Finally, we have established evidence that a crash risk factor helps in explaining the cross section of stock returns. For portfolios sorted on momentum the traditional CAPM is rejected, while the CAPM with a crash factor is not. The size of the crash risk premia indicates that crash risk can be equally important as diffusion risk.

As we have discussed in the introduction, we have treated the aspects of the relation of an investor with financial markets separately in this thesis. This approach enabled us to concentrate specifically on each aspect and to use different techniques that fitted best for a specific aspect. The advantages of this approach come with the disadvantage that it is more complicated to draw general conclusions. We cannot offer one big model that can be used to capture all characteristics of crises and crashes and to investigate its consequences in all aspects. Nevertheless, our research also increases current insights in crises and crashes and their consequences for investors in more general terms. Its contributions lie mainly in three directions.

First we show that stronger comovements and strong persistence are two crucial characteristics of crises and crashes. The probability of a joint crash of two assets is larger than correlation-based models predict, even if the univariate probability of a crash is correctly predicted. Consequently, the probability that one asset crashes given that the other asset crashes is also larger than regular models predict. This effect becomes stronger if more assets encounter a crash. Concluding, crashes spread more fiercely and can end in a crisis. Persistence exacerbates the effects of crises and crashes. The occurrence of a crash increases the probability of another crash to follow. Moreover, volatilities go up. Crashes and crises do not only spread to other assets and markets but also over time, as a crash of an asset during this month makes the asset riskier for the month thereafter.

Concluding, crises and crashes make financial markets a riskier place than an analysis based on a single asset, industry or market indicates. Because of the strengthening of comovements, crises and crashes are difficult to evade. Diversification effects disappear when they are needed most. Because of their persistent effects, it is more difficult to sit out crises and crashes. It takes more time for crises and crashes to die down. The fall of Long-Term Capital Management illustrates the dramatic consequences of failing to take these two aspects into account. All the hedge funds' investments went awry and lost money for many days in a row during August and September 1998.[1] Crises and crashes may lurk in the background of financial markets, but if they occur they are a center stage problem for investors.

[1]For a description of the collapse of Long-Term Capital Management see Edwards (1999), Jorion (2000) or the more popular book by Lowenstein (2000).

Second, we have also found some good news. Investors do not have to sit and wait for the next crash to strike, but can use the presence of bubbles to make inferences on the likelihood of a crash. Moreover, investors can expect a reward for that part of the risk of crises and crashes that cannot be evaded. The size of the crash risk premium indicates that the representative investor is aware of the risk that crises and crashes entail.

A third direction into which we extend empirical financial research is by highlighting how econometric techniques can be used in relation to crises and crashes. In Chapter 2 we have proposed a novel methodology to analyze bubbles and crashes. We have shown that skewness is probably not the best way to investigate crash likelihood. In Chapters 3 and 5 we have demonstrated how copulas can be helpful in research on crashes. In Chapter 4 we have made full use of the flexibility of regime switching models, and we have shown how discrete-time regime switching models can be combined with continuous-time optimization. While our outcomes are sensitive to the techniques we use and the data we choose, we have conducted robustness checks to establish that our main conclusions are unaffected. All these techniques extend the toolbox with which risk managers and financial researchers can analyze the consequence of crises, crashes and comovements.

The main challenge for future research that results from this dissertation lies in the combined effect that the strengthening of comovements and the persistence of crises and crashes have on their consequences for investors. Our results indicate that both effects exacerbate the consequences of crises and crashes. The model for a systemic crises that we use in Chapter 4 captures persistence and implies a rise in correlations, but conditional on the regimes, dependence is still linear. Combining the models that we have proposed throughout this dissertation can further extend knowledge on crises, crashes and comovements.

Nederlandse samenvatting (Summary in Dutch)

Inleiding

Crises en crashes in financiële markten vormen voor beleggers een reden tot grote vrees. De sterke koersdalingen tijdens een crisis of crash en de toename van risico erna berokkenen beleggers veel schade. Hierdoor gelden beruchte crises en crashes zoals die van 1929, 1987 en de crises in opkomende financiële markten van de jaren negentig nog steeds als schrikbeeld.

In dit proefschrift doen we onderzoek naar crises en crashes, waarbij we specifiek aandacht besteden aan de wederzijdse afhankelijkheid in het koersverloop van effecten. Onder een crash verstaan we een sterke daling van één aandeel, één sector of één markt. Een crisis is een periode van grote onzekerheid die meerdere effecten, sectoren of markten beïnvloedt. Een crash is vaak te herleiden op een of enkele dagen, terwijl een crisis meestal langer aanhoudt. Ook zijn de consequenties van een crisis verstrekkender dan die van een crash. De crash van 1987 vond plaats op 19 oktober. De Azië-crisis daarentegen, begon in juli 1997 en doofde pas uit in de eerste helft van 1998. Soms vormt een crash het begin van een crisis, zoals in 1929 toen de crash op de aandelenmarkten de crisis van de jaren '30 inluidde.

De samenhang tussen effecten neemt een belangrijke plaats in in dit proefschrift. De koersen van effecten bewegen niet onafhankelijk van elkaar, en voor markten geldt hetzelfde. Daarnaast houden beleggers in het algemeen meerdere effecten aan op verschillende markten. Als een crash zich zou beperken tot één effect of één markt, kunnen zijn gevolgen beperkt worden door beleggingen te spreiden. Helaas, crashes en crises zijn besmettelijk en verspreiden zich snel naar andere effecten en markten. Daarom is het belangrijk deze afhankelijkheid te betrekken in de analyse.

De klassieke financieringstheorie verklaart crises en crashes door nieuwe informatie die financiële markten bereikt. We zien dan een crash in het geval van slecht nieuws. Omdat nieuws vaak relevant is voor meerdere effecten of markten, zien we zijn effect meermalen weerspiegeld. Als het nieuws erg slecht is, en genoeg effecten en markten beïnvloedt treedt een crisis op.

Helaas schiet deze klassieke benadering tekort. We onderscheiden vijf redenen, waarom we crashes en crises niet kunnen beschouwen als een integraal deel van het normale functioneren van financiële markten. In de eerste plaats zijn koersfluctuaties, inclusief crises en crashes, maar gedeeltelijk te herleiden op nieuws (zie Roll, 1988b; Cutler *et al.*, 1989; Shiller, 1981, 2000).

In de tweede plaats wijken de statistische kenmerken van de rendementen op effecten tijdens crisisperioden fundamenteel af van die tijdens rustige perioden. In rustige perioden kan de normale verdeling gebruikt worden om rendementen te beschrijven, maar deze verdeling voorspelt een veel lagere frequentie voor extreme rendementen dan we waarnemen (zie o.a. Fama, 1965). Een normale verdeling voor rendementen komt voort uit een diffusieproces. Echter, empirisch bewijs toont aan dat koersen niet alleen geleidelijke veranderingen vertonen, zoals behorend bij een diffusieproces, maar ook grotere sprongen laten zien. Volgens Bates (2000) en Andersen *et al.* (2002) zijn deze sprongen zelfs noodzakelijk om de marktprijzen van opties te verklaren. We concluderen hieruit dat rendementen een diffusiecomponent bevatten die domineert tijdens rustige perioden en een sprongcomponent die zich manifesteert tijdens crisisperioden.

In de derde plaats hebben crises en crashes een langdurig versterkend effect op koersfluctuaties. Grote prijsveranderingen treden op in clusters. Bovendien is het effect van grote koersdalingen op deze clustervorming groter dan grote koersstijgingen. Hieruit volgt dat de sprong die zich voordoet ten tijde van een crisis of crash ervoor zorgt dat de fluctuaties naderhand toenemen en langdurig op dat niveau blijven.

In de vierde plaats neemt de afhankelijkheid tussen effecten en tussen markten toe ten tijde van onrust. Ang en Chen (2002) laten zien dat correlaties toenemen voor negatieve rendementen van individuele aandelen. Met een statistisch robuustere aanpak laten Longin en Solnik (2001) zien dat dit ook voor ontwikkelde aandelenmarkten als geheel geldt. Hartmann *et al.* (2004) tonen hetzelfde effect tussen aandelen- en obligatiemarkten aan. Bae *et al.* (2003) doen dit voor opkomende markten. De verspreiding van crises en crashes is dus groter dan normale afhankelijkheid kan verklaren.

De vijfde reden ligt in de aversie die beleggers vertonen jegens grote verliezen. Onderzoek van Kahneman en Tversky (1979) en Tversky en Kahneman (1991) verwerpt het traditionele model van een consument die zijn verwacht nut maximaliseert. Benartzi en Thaler (1995) en Barberis *et al.* (2001) laten zien dat de aversie jegens grote verliezen een bijdrage kan leveren aan de verklaring van de grootte van de risicopremie op aandelen. Dat crises en crashes bij uitstek de grootste verliezen omvatten vormt een belangrijke reden voor dit onderzoek.

We onderscheiden vier terreinen waarop deze eigenschappen die crises en crashes onderscheiden van rustige perioden beleggers en financiële markten beïnvloeden. Het eerste terrein is risico management. Voor het bestuderen van een crash op zichzelf biedt extreme waarde-theorie uitkomst. Longin (1996) toont aan dat de extreme rendementen van effecten benaderd kunnen worden met een Fréchet verdeling. Hierin wijken rendementen af van een normale verdeling, die een Gumbel verdeling impliceert voor de extremen. Longin (2000) laat zien hoe extreme waarde-theorie gebruikt kan worden voor een enkel aandeel of portefeuille. Het is echter niet duidelijk hoe multivariate extreme waarde-theorie toegepast kan worden in financieel risico management. Deze theorie is reeds gebruikt voor het bestuderen van gelijktijdige crashes in twee markten (zie Longin en Solnik, 2001; Poon *et al.*, 2004; Hartmann *et al.*, 2004), maar de terugkoppeling naar gevolgen op het niveau van portefeuilles is sterk afhankelijk van de onderliggende aannames aangaande afhankelijkheid. Embrechts *et al.* (2002) betogen dat correlaties, het traditionele model voor afhankelijkheid, te restrictief zijn en vervangen moeten worden door copulas. In Hoofdstuk 3 gaan we na hoe de geschiktheid van een copula bepaald kan worden.

Portefeuillekeuze is het tweede terrein waarop crises en crashes effect hebben. Een belegger probeert een portefeuille zodanig te construeren dat deze een optimale afweging tussen verwacht rendement en risico heeft. De genoemde kenmerken van crises en crashes versterken elkaar en veranderen deze afweging. In de eerste plaats wordt ieder effect op zichzelf riskanter, als de juiste kans op een crash wordt meegenomen. Daarnaast worden effecten gezamenlijk ook riskanter door hun sterkere afhankelijkheid in tijden van onrust. In de derde plaats houdt de invloed van een crisis of crash langer aan. Tenslotte zijn beleggers nu net bijzonder gevoelig voor deze invloeden. Hieruit volgt dan ook de hypothese dat beleggers voorzichtiger zullen beleggen, als ze deze kenmerken correct in ogenschouw nemen. Ook zullen ze een voorkeur vertonen voor effecten die minder gevoelig zijn voor een crisis of crash. In Hoofdstuk 4 gaan we na hoe sterk deze gevolgen zijn.

Als derde terrein noemen we de prijsvorming van effecten. Als crises en crashes moeilijk te ontlopen zijn in verband met een versterking van de afhankelijkheid tussen effecten, en als ze langere tijd aanhouden, behoren beleggers hiervoor een premie te ontvangen. Financiële theorie schrijft immers voor dat beleggers beloond worden voor het systematische risico dat zij lopen, en dat neemt door crises en crashes toe. Aan de hand van optieprijzen laten Bates (1991, 2000) en Andersen *et al.* (2002) zien dat rendementen op aandelen inderdaad een risicopremie voor marktcrashes bevatten. Deze bewijzen zijn echter gebaseerd op marktindices. Voor individuele aandelen is nog geen empirisch onderzoek gedaan. We leveren hieraan een bijdrage in Hoofdstuk 5.

Het laatste terrein betreft ons begrip van financiële markten. Omdat nieuws slechts ten dele het optreden van crises en crashes kan verklaren zijn andere verklaring naar voren gebracht (zie Brunnermeier, 2001, H. 6). Een van deze verklaringen is dat crashes een correctie zijn voor zeepbellen, sterke koersstijgingen van effecten. Mogelijk kan een belegger de aanwezigheid van een zeepbel gebruiken voor het bepalen van de kans op een crash. Hoewel Kindleberger (2000) en Shiller (2000) al enig bewijs leveren voor de relatie tussen zeepbellen en crashes, is nog niet duidelijk of een zeepbel ook een effect heeft op het voorspellen van crashes. We onderzoeken dit in Hoofdstuk 2.

Zeepbellen en crashes in sectoren (H. 2)

In Hoofdstuk 2 onderwerpen we de relatie tussen zeepbellen en crashes in het koersverloop van effecten aan een onderzoek. Een zeepbel is een periode van bovenmatige koersstijgingen die niet uit fundamentele verbeteringen van vooruitzichten kan worden verklaard. Zeepbellen zijn reeds lang het onderwerp van theoretisch onderzoek. Dit richt zich dan op de omstandigheden waaronder een zeepbel kan voorkomen (Brunnermeier, 2001, H. 2, geeft hiervan een overzicht). Asymmetrische informatie blijkt hiervoor een cruciale factor te zijn. Daarnaast is er empirisch onderzoek gedaan naar zeepbellen en crashes. Vaak beperkt zo'n onderzoek zich tot één zeepbel en de daaropvolgende crash, zoals Brunnermeier en Nagel (2004) en Ofek en Richardson (2003) naar de recente technologie zeepbel of Temin en Voth (2004) naar de "South Sea Bubble" van 1720. Shiller (2000) en Kindleberger (2000) presenteren een meer systematische analyse van zeepbellen en crashes, maar hun benadering vindt plaats in retrospectief.

Wij benaderen zeepbellen en crashes vanuit een beleggersperspectief. Een belegger neemt een zeepbel waar als het gemiddelde abnormale rendement over een periode tussen de laatste een tot vijf jaar boven een bepaalde grens ligt. In dit hoofdstuk kiezen we ervoor deze abnormale rendementen te baseren op het "Capital Asset Pricing Model" (CAPM). Een crash treedt op wanneer een abnormaal rendement onder een bepaalde grens valt. De belegger wil weten of de recente aanwezigheid van een zeepbel de kans op een crash gedurende de volgende maand verhoogt. Om een grotere steekproef te kunnen onderzoeken, richten we ons hierbij op de 48 verschillende sectoren van de economie van de VS in plaats van de markt als geheel.[2] Voor de

[2]Deze indeling wordt ook gebruikt in Fama en French (1997) en is beschikbaar op de website van French.

grootste groep crashes rapporteren we een verdubbeling van de kans op een crash, als zich een zeepbel voordoet. Als we ons beperken tot ernstigere crashes vinden we een verdriedubbeling. Op basis van het theoretische model voor zeepbellen en crashes van Abreu en Brunnermeier (2003) formuleren we twee hypothesen die de kenmerken van een zeepbel relateren aan de kans op een crash. Uit testen blijkt dat een zeepbel die sterker is dan gemiddeld de kans op een crash verder vergroot. De lengte van de zeepbel zoals de belegger die waarneemt, levert geen bijdrage aan het voorspellen van een crash.

Deze resultaten zijn belangrijk voor beleggers, omdat ze aantonen dat het profiteren van een zeepbel gepaard gaat met een aanzienlijke stijging van crash-risico. Ook tonen we aan dat historische koerspatronen van waarde zijn bij het voorspellen van een crash. De waarnemingen van zeepbellen en crashes in een markt als geheel zijn te beperkt om statistisch onderbouwde conclusies te trekken, maar de resultaten voor de VS markt komen in kwalitatief opzicht overeen met onze resultaten voor de 48 sectoren. We concluderen hieruit dat ook een zeepbel in een markt tot verhoogd crash-risico leidt.

Het testen van copulas voor het modelleren van financiële afhankelijkheid (H. 3)

Het correct modelleren van afhankelijkheid is belangrijk voor het nemen van beslissingen bij onzekerheid, als deze onzekerheid voortkomt uit verschillende bronnen. Correlaties vormen de traditionele maat voor het beschrijven van afhankelijkheid, maar recent is bezwaar gerezen tegen correlaties als model voor afhankelijkheid. In theoretische zin laten Embrechts *et al.* (2002) zien dat correlaties hun beperkingen hebben, omdat ze slechts lineaire afhankelijkheid modelleren. Copulas vormen een klasse van modellen voor afhankelijkheid, die meer mogelijkheden biedt dan de correlatiebenadering. Een copula geeft de gezamenlijke cumulatieve kans op een verzameling gebeurtenissen als functie van de marginale cumulatieve kansen van die gebeurtenissen. Hierdoor is het mogelijk de marginale verdelingen los van de gezamenlijke verdeling te modelleren. Is de afhankelijkheid tussen kansvariabelen lineair, dan en slechts dan is de Gaussische copula de juiste. Deze copula wordt geheel en al geparametriseerd door een correlatiematrix. Uit empirisch onderzoek volgt echter dat rendementen op effecten niet-lineair samenhangen. Ang en Chen (2002) rapporteren dat de correlaties tussen de rendementen van individuele aandelen stijgen bij neergaande markten. Longin en Solnik (2001), Bae *et al.* (2003) en Hartmann *et al.* (2004) laten zien dat de samenhang tussen extreme rendementen van markten als geheel sterker is dan correlaties impliceren. We concluderen hieruit dat correlaties te beperkt zijn voor het modelleren van de afhankelijkheid tussen rendementen. Een andere copula dan de Gaussische kan mogelijk uitkomst bieden.

In Hoofdstuk 3 stellen we testen voor die we kunnen gebruiken om te bepalen of een copula geschikt is voor het modelleren van de afhankelijkheid tussen stochasten. Zoals gezegd voldoet de Gaussische copula niet voor de rendementen op effecten. Financiële theorie schrijft echter niet bij voorbaat een alternatief voor, zodat het vinden van een goede copula een empirische aangelegenheid wordt. Een copula kan worden opgevat als een multivariate verdelingsfunctie. Daarom stellen we voor aangepaste versies van de standaard toetsen voor de juistheid van een univariate verdelingsaanname te gebruiken, zoals de Kolmogorov-Smirnoff toets en de Anderson-Darling toets. Deze aangepaste toetsen vergelijken de afhankelijkheid zoals deze volgt uit de gehypothetiseerde copula direct met de waargenomen afhankelijkheid. Daardoor zijn deze toetsen breed toepasbaar op alle copulas en op alle dimensies, en daarmee aantrekkelijker dan bestaande toetsen zoals die van Mashal *et al.* (2003) en Poon *et al.* (2004). De Kolmogorov-Smirnoff toets geeft vooral de juistheid voor het centrum van de verdeling weer, terwijl de Anderson-Darling toets meer de nadruk legt op de staarten van de verdeling. We beschrijven hoe de precieze test kan worden uitgevoerd door middel van een simulatieprocedure .

We passen deze testen vervolgens toe om een copula te selecteren voor het uitvoeren van een "stress test" in het risico management van een portefeuille die bestaat uit aandelen, obligaties en onroerend goed, alle in de VS. We beschouwen de traditionele Gaussische copula, de Student's t copula en de Gumbel copula. De Student's t copula behoort tot de elliptische familie, evenals de Gaussische copula, maar de Student's t copula impliceert een sterkere afhankelijkheid in de staarten dan de Gaussische copula. De Gumbel copula behoort tot de familie van archimedische copulas en is een extreme waarde-copula (zie Bouyé, 2002). We gaan aan de hand van dagdata van 1 januari 1999 tot 17 december 2004 na, of een van deze drie copulas voldoet voor het modelleren van de samenhang. De toetsen verwerpen de Gaussische copula en de Gumbel copula, maar de Student's t copula niet. Een nadere inspectie van de staarten van de verdeling laat zien dat de Student's t copula tot een juiste inschatting van het risiso op gezamenlijke negatieve rendementen leidt. Daarentegen onderschat de Gaussische copula dit risico significant, terwijl de Gumbel copula het overschat. De toets van Poon *et al.* (2004) biedt geen duidelijke selectie. Dit illustreert het belang van testen die direct gericht zijn op afhankelijkheid.

De implicaties van systeemcrises voor beleggingsportefeuilles (H. 4)

Hoofdstuk 4 richt zich op de gevolgen van een systeemcrisis op het samenstellen van een beleggingsportefeuille. Een systeemcrisis is een schok die voelbaar is in het gehele (mondiale) systeem van financiële markten (zie De Bandt en Hartmann, 2000). In dit hoofdstuk beperken we ons tot een schok die gelijktijdig alle aandelenmarkten wereldwijd treft. Zo'n schok verslechtert beleggingsmogelijkheden door de wereldwijde negatieve rendementen en daaropvolgende stijging van volatiliteiten en correlaties. Das en Uppal (2004) hebben de gevolgen voor beleggers van een dergelijke crisis onderzocht, en concluderen dat deze beperkt zijn. Hun aanpak, die gebaseerd is op de combinatie van een diffusie proces met een Poisson-proces, houdt echter geen rekening met persistentie in de effecten van een systeemcrisis. Liu *et al.* (2003) laten zien dat sprongen die ook doorwerken in de volatiliteit een geprononceerd effect hebben op de selectie van portefeuilles. Omdat hun benadering univariaat is, blijft het onduidelijk hoe een crisis diversificatiemogelijkheden beïnvloedt.

Wij stellen een nieuwe benadering van dit probleem voor. We introduceren een belegger die zijn verwacht nut maximaliseert en wiens nutsfunctie behoort tot de functies met constante relatieve risicoaversie. De belegger kan twee strategieën volgen, namelijk een strategie waarin hij bewust rekening houdt met een systeemcrisis, of een strategie waarin hij onwetend is wat de mogelijkheid van een systeemcrisis betreft. Beide strategieën zijn gebaseerd op een regime-switching model als in Ang en Bekaert (2002). Zo'n model biedt rijke en flexibele mogelijkheden om persistentie in volatiliteit en persistentie in een crisis te modelleren. Ieder land kan zich in een hoog of een laag volatiliteitsregime bevinden. In de crisisbewuste strategie is er bovendien een specifiek regime aanwezig voor het geval er een crisis optreedt. Aan de hand van de regime-switching modellen doen we een voorspelling voor de rendementen van de volgende periode. We gebruiken een techniek uit Brigo (2002) om een Itô-proces te construeren dat consistent is met deze voorspellingen, en lossen vervolgens het optimaliseringprobleem van de belegger op als in Merton (1969).

We analyseren de portefeuillekeuzes van een belegger in de VS die zijn beleggingen wil spreiden over de VS, Europa, Japan, Hong Kong, Thailand, Korea en Brazilië. Hij betrekt nadrukkelijk opkomende markten in zijn beleggingsbeslissing wegens de diversificatiemogelijkheden en aantrekkelijke rendementen die zij bieden. Onze analyse richt zich op twee situaties. In de eerste situatie heeft de belegger geen verdere informatie over de stand van de economie. In dat geval leidt de crisisbewuste strategie tot een minder riskante portefeuille, en een reductie van posities in opkomende markten in vergelijking met de onwetende strategie. Als de belegger deze laatste strategie gebruikt, hoewel de crisisbewuste strategie de juiste is, verlangt hij een zekere compensatie van 1.13% op jaarbasis. In de tweede situatie gebruikt de belegger informatie die voorhanden was in oktober 1997, een van de maanden van de Azië-crisis. Halverwege oktober maakte de markt van Hong Kong een duikvlucht, die gevolgd werd door alle andere markten. We zien dat het in deze situatie veel belangrijker is rekening te houden met een systeemcrisis. De belegger verlangt een compensatie van meer dan 3% per jaar in de eerste helft van de maand, dus *voordat* de markt van Hong Kong crashte. Na de crash loopt deze compensatie snel op richting de 3% *per maand*. De crisisbewuste strategie is opnieuw minder riskant dan de strategie die crises negeert. Daarenboven zien we dat de crisisbewuste strategie posities sneller afbouwt.

De resultaten die we rapporteren zijn sterker dan die van Das en Uppal (2004). We concluderen hieruit dat persistentie een belangrijk element is van een systeemcrisis. We borduren voort de uitkomst van Ang en Bekaert (2002) dat het belangrijk is rekening te houden met omschakelingen tussen regimes met een hoge en lage volatiliteit door te laten zien dat ook de omschakeling naar een crisis regime belangrijk is. Evenals Ang en Chen (2002) vinden we een sterke afname van de diversificatiemogelijkheden. Als short gaan niet is toegestaan verlaat de belegger zelfs alle markten wanneer een crisis toeslaat.

Crashrisico in de doorsnede van aandeelrendementen (H. 5)

In Hoofdstuk 5 gaan we na of we een premie voor crashrisico kunnen identificeren in de doorsnede van aandeelrendementen. Uit voorgaande hoofdstukken volgt dat crises en crashes grote, langdurige gevolgen kunnen hebben voor beleggers, en dat het tegelijkertijd moeilijk is deze gevolgen te ontlopen. Volgens de financiële theorie behoren beleggers dan een premie te ontvangen voor het systematische crash-risico dat zij lopen. Uit onderzoek naar marktprijzen voor opties op marktindices in Bates (1991, 2001) en Andersen *et al.* (2002) volgt dat neerwaartse sprongen in markten noodzakelijk zijn om deze prijzen te verklaren, en dat beleggers een premie eisen voor het risico op deze sprongen. Bakshi *et al.* (2003) laten echter zien dat de rendementsverdeling die opties op individuele aandelen impliceren afwijkt van de door marktindexopties geïmpliceerde verdeling. Het is daardoor niet op voorhand duidelijk of en in welke mate een premie voor crashrisico aanwezig is in de rendementen op individuele aandelen.

Om deze vraag te beantwoorden leiden we eerst een uitgebreidere versie van het CAPM af, waarin we expliciet rekening houden met crash-risico. We modelleren een crash als een grote, neerwaartse sprong. We veronderstellen de aanwezigheid van een representatieve belegger wiens nutsfunctie een aparte component voor crashes bevat om de aversie jegens verliezen van een crash te modelleren. In deze versie van het CAPM bevat het verwachte rendement op een aandeel een premie die proportioneel is met de gevoeligheid voor een markt-crash. Deze gevoeligheid bestaat uit de conditionele kans dat een aandeel crasht gegeven dat de markt crasht, en de grootte van de crash van het aandeel in verhouding tot de marktcrash.

Op basis van deze theoretische resultaten geven we ons empirisch onderzoek vorm. We construeren drie maatstaven voor de kans dat een aandeel crasht, gegeven dat de markt crasht. Voor ieder aandeel in de CRSP databank in de periode van juni 1964 tot november 2003 berekenen we voor elke maand de waarde van deze maatstaven. Vervolgens gebruiken we deze drie maatstaven om drie portefeuilles te bouwen met respectievelijk een hoge, middelmatige en lage conditionele crash-kans. Uit de analyse van de rendementen op deze portefeuilles volgt dat de portefeuilles met een hoge conditionele crashkans een significant positief rendement bieden van 2.4% tot 4.0% per jaar, na correctie voor normaal marktrisico. Dit extra rendement kan niet verklaard worden aan de hand van anomalieën zoals het grootte-effect en waarde-effect van Fama en French (1993, 1997) of het momentumeffect van Jegadeesh en Titman (1993) of aan de hand van risicofactoren zoals coskewness in Harvey en Siddique (2000) en cokurtosis in Dittmar (2002). Als we een factor voor crash-risico toevoegen aan het CAPM treden verbeteringen op. Het traditionele CAPM wordt verworpen op basis van de rendementen op momentumportefeuilles, maar na toevoeging van een factor voor crash-risico is dat niet langer het geval. Voor andere portefeuilles zien we minder duidelijk veranderingen, maar de premie voor crash-risico ligt in dezelfde orde van grootte als de premie voor diffusierisico. We leiden hieruit af dat crash-risico een belangrijke risicofactor is.

Onze onderzoeksresultaten leveren een bijdrage aan de voortgaande discussie over de doorsnede van aandelenrendementen. Onze onderzoeksopzet ligt in het verlengde van Harvey en Siddique (2000) en Dittmar (2002) die het CAPM uitbreiden met statistische momenten van een hogere orde dan covariantie. De gevoeligheid van een statistisch moment voor extreme waarden neemt toe met de orde van het moment. Onze maatstaven concentreren zich direct op de extreme waarden, namelijk crashes. Over onze steekproefperiode vinden we een positief significant extra gemiddeld rendement voor portefeuilles met aandelen met een hoge conditionele crashkans, maar aandelen met een grote negatieve coskewness of aandelen met een hoge positieve cokurtosis leveren gemiddeld geen positief extra rendement op. We vinden wel een relatie tussen de crashportefeuilles en de coskewness- en cokurtosisportefeuilles. We concluderen hieruit dat extreme waarden, dus crashes, belangrijk zijn voor de prijsvorming van effecten.

Conclusies

In zijn geheel beschouwd kunnen we een drietal conclusies trekken uit de resultaten van dit proefschrift. In de eerste plaats vormen zowel persistentie als versterking van afhankelijkheid in koersverloop een cruciaal kenmerk van crises en crashes. De kans op een crash in de koers van een bepaald effect wordt groter naarmate meer andere effecten een crash doormaken. Deze toename is groter dan reguliere modellen impliceren. Ook neemt de kans op een crisis hierdoor toe. Daarnaast verergert persistentie de consequenties van een crash. Tezamen zorgen deze twee kenmerken ervoor dat financiële markten riskanter zijn dan ze op het eerste gezicht lijken. Waartoe een onderschatting van deze kenmerken kan leiden wordt duidelijk geïllustreerd door de ineenstorting van Long Term Capital Management in 1998. Long Term Capital Management was een Amerikaans hedge fund ($\approx$ minder gereguleerd, besloten beleggingsfonds) dat wereldwijd grote speculatieve posities opbouwde. De toegenomen onrust in augustus en september van 1998 na de Roebelcrisis zorgde voor grote langdurige verliezen op alle posities, waarna het fonds gered moest worden van faillisement.

In de tweede plaats bevat dit proefschrift goed nieuws. We laten zien dat beleggers de kans op een crash kunnen voorspellen door naar zeepbellen te kijken. We vinden ook dat beleggers ter compensatie van het grotere risico dat crashes en crises inhouden een premie kunnen verwachten. De grootte van deze premie geeft aan dat crash risico een belangrijke component vormt van aandelenrendementen.

In de derde plaats demonstreren we hoe econometrische technieken gebruikt kunnen worden voor het bestuderen van crises en crashes. In Hoofdstuk 2 gebruiken we een nieuwe methode voor het analyseren van zeepbellen en crashes, die mogelijk beter is dan onderzoek gebaseerd op scheefheidscoëfficiënten. In Hoofdstukken 3 en 5 illustreren we het nut van copulas. In Hoofdstuk 4 maken we gebruik van de ruime mogelijkheden van regime-switching modellen. Deze methoden vergroten de technische mogelijkheden van onderzoekers op het gebied van empirische financiering.

De belangrijkste uitdaging voor nieuw onderzoek die volgt uit dit proefschrift ligt in het gecombineerde effect van persistentie en de versterking van afhankelijkheid in koersverloop op de consequenties van crises en crashes voor beleggers. Onze resultaten geven aan dat beide kenmerken op zichzelf de consequenties versterken, maar hoe ze elkaar versterken is nog onduidelijk. Een combinatie van de modellen en technieken de we gebruiken in dit proefschrift kan onze kennis van crises, crashes en de afhankelijkheid in koersverloop verder uitbreiden.

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