Can untargeted subsidies select the best firms?*

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Abstract

Untargeted subsidies are widely predicted to worsen firm selection and reduce aggregate productivity. We show this prediction fails in a broad class of environments where firms face kinked operating decisions, so that policy moves firms across discrete incentive regimes rather than along smooth margins. In such environments, untargeted subsidies can increase competition and, thus, raise equilibrium productivity cutoffs. We develop a general equilibrium model of monopolistic competition which nests canonical firm-dynamics frameworks and incorporates threshold-based operating incentives. Subsidies operate through two channels: a standard channel that lowers survival thresholds and a domain-switching channel that raises equilibrium cutoffs through increased competitive pressure. We test the mechanism using the COVID-19 policy response in the U.S. restaurant industry. We document asymmetric exit behavior around cost reference points and show subsidies large enough to move firms across domains reduce exit sharply while leaving entry largely unchanged. Calibrating the model to these patterns, we find the domain-switching channel accounts for roughly half of the observed decline in exit and generates productivity gains comparable in magnitude to those observed in the data.

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1 Introduction

A central implication of modern theories of firm dynamics is that untargeted subsidies worsen selection. In canonical models (Hopenhayn, 1992; Melitz, 2003), firms draw productivity upon entry and operate only if variable profits cover fixed costs. Untargeted subsidies lower survival thresholds and allow less productive firms to remain active, reducing average productivity among operating firms. This result is sharp and widely used to evaluate industrial policy and crisis-era firm support.

This paper shows the benchmark prediction is not robust once operating decisions involve threshold distortions rather than smooth adjustments. When small changes in fixed costs can move firms across distinct incentive regimes, subsidies affect selection through two channels: a mechanical channel that lowers survival thresholds and a regime-switching channel that increases competitive pressure in equilibrium. In such environments, untargeted subsidies can raise rather than lower productivity cutoffs.

The key mechanism is domain switching: policy changes that move firms across discrete incentive regimes rather than along smooth margins. When operating decisions involve such thresholds, small changes in fixed costs can produce discrete shifts in continuation behavior and competitive pressure. As a result, subsidies affect selection not only by mechanically lowering survival thresholds, but also by increasing competition and, thus, raising equilibrium productivity cutoffs.

We formalize this mechanism in a general equilibrium model of monopolistic competition that nests the standard Melitz (2003) framework. Firms differ in productivity and fixed costs, and operating decisions depend on benchmark-relative costs. As a tractable and empirically grounded micro-foundation for threshold-based incentives, we allow firms to evaluate costs relative to a reference point, generating distinct gain and loss domains. When costs are perceived as losses, they receive greater weight in operating decisions, creating a kink in effective fixed costs. While we focus on reference dependence, the logic extends to other environments with threshold-based operating incentives.

The model delivers three theoretical results. Under expected utility, fixed-cost subsidies unambiguously reduce average productivity, reproducing the standard benchmark. Once operating decisions exhibit domain switching, however, the effect of subsidies on productivity becomes ambiguous. Under reference-dependent preferences, there exists a threshold degree of gain-loss sensitivity above which the benchmark prediction reverses and subsidies raise average productivity. We characterize this threshold analytically and show how it depends on preference parameters, the distribution of fixed costs, and market competitiveness.

We test this mechanism in the context of the COVID-19 policy response in the U.S. restaurant industry, which provides a natural laboratory for domain switching. Despite large untargeted fixed-cost subsidies during this period, labor productivity in the sector rose by roughly 15 percent (Goolsbee, Syverson, Goldgof and Tatarka, 2025), a pattern difficult to reconcile with canonical selection models. The sector received large, untargeted fixed-cost support through the Paycheck Protection Program. COVID-related disruptions required many restaurants to adopt new production technologies, effectively confronting incumbents with new cost and productivity draws. These features make the episode particularly well suited to studying how subsidies interact with threshold-based operating decisions.

The restaurant sector is particularly informative because owner-operators are prevalent and individual preferences directly affect continuation decisions. Prior evidence shows small business owners often accept lower earnings in exchange for non-pecuniary benefits (Benz and Frey, 2004; Hamilton, 2000; Hurst and Pugsley, 2011). Direct measurements of loss aversion among restaurant owners document substantial heterogeneity in gain–loss sensitivity and strong associations with exit behavior (Emami Namini and Kapoor, 2025, 2026).

Before turning to calibration, we present descriptive evidence consistent with domain switching. Exit rates respond asymmetrically to changes in payroll: establishments are significantly more likely to exit when payroll rises relative to the prior year (the loss domain) than when it falls (the gain domain). This asymmetry is concentrated among small establishments, where owner decision-making is most direct. The pattern is consistent with benchmark-relative operating costs shaping continuation decisions. We document similar patterns in the Netherlands, where hospitality exits declined sharply in 2021 despite persistently negative payroll growth, consistent with generous wage and fixed-cost subsidies moving firms across domains.

We calibrate the model using U.S. county-level data, targeting the differential exit response of establishments that cross incentive domains following subsidy receipt. The estimated gain–loss sensitivity is consistent with independent survey evidence among small business owners. In the calibrated model, COVID-era subsidies reduce exit rates by approximately 4 percentage points, with the domain-switching channel accounting for roughly half of this effect. While the standard model predicts subsidies worsen selection and reduce productivity, incorporating domain switching reverses this implication: equilibrium productivity rises at empirically grounded parameter values, consistent in magnitude with the productivity gains observed in the data.

Our primary contribution is to the theory of firm dynamics and selection originating with Hopenhayn (1992) and Melitz (2003). In these canonical models, firms operate only if revenues cover fixed costs, implying that untargeted subsidies weaken selection by lowering productivity thresholds and reducing average productivity among active firms. This prediction plays a central role in policy evaluation, where firm support is typically expected to reduce aggregate productivity. Much of this reasoning builds on frameworks in which distortions affect firm behavior through wedges in input or output prices, generating smooth adjustments in production and survival decisions (e.g., Hsieh and Klenow, 2009; Restuccia and Rogerson, 1993). In contrast, we introduce reference-dependent operating incentives that produce non-smooth, kinked decision rules. Policies can therefore induce discrete changes in firms' behavior by shifting them across incentive regimes rather than merely altering marginal conditions. This mechanism generates selection effects that are absent in standard misallocation environments and can overturn the usual prediction that subsidies weaken selection. We characterize analytically when this reversal arises and provide a tractable general equilibrium framework that nests the canonical benchmark as a special case.

We provide a tractable microfoundation for threshold-based operating incentives by introducing reference-dependent continuation payoffs inspired by Kőszegi and Rabin (2006). Our specification captures a broader class of environments in which operating outcomes are evaluated relative to a benchmark, generating endogenous non-convexities in firms' participation decisions. While reference dependence is well documented in individual choice (e.g., Kahneman and Tversky, 1979), its implications for firm selection and market equilibrium remain largely unexplored (Angelis, 2024; Armstrong and Huck, 2010).[1] Embedding such incentives in a heterogeneous-firm general equilibrium framework allows us to characterize how deviations from standard profit maximization affect selection, equilibrium productivity, and the incidence of firm support policies.

Our analysis also contributes to the literature evaluating large-scale firm support policies, particularly the Paycheck Protection Program (PPP). Standard selection models predict that untargeted subsidies weaken selection by allowing low-productivity firms to remain active. However, recent evidence shows that sectors more exposed to PPP support experienced stable or rising productivity (e.g., Crane, Decker, Flaaen, Hamins-Puertolas and Kurz, 2022; Goolsbee, Syverson, Goldgof and Tatarka, 2025), a pattern difficult to reconcile with canonical frameworks. Our model provides a structural mechanism through which untargeted fixed-cost subsidies can increase equilibrium productivity cutoffs when firms face threshold-based

[1] Recent empirical evidence shows that loss-averse firm owners are significantly less likely to exit, consistent with reference-dependent incentives affecting continuation decisions (Emami Namini and Kapoor, 2025).

operating incentives. The framework also offers a quantitative interpretation of evidence that a substantial share of PPP funds accrued to business owners rather than workers (Autor, Cho, Crane, Goldar, Lutz, Montes, Peterman, Ratner, Villar and Yildirmaz, 2022), consistent with policies affecting continuation incentives rather than marginal production decisions.

More broadly, our results highlight that policy effects depend critically on the nature of operating incentives. Many economic decisions involve thresholds and non-convexities, including financing constraints, regulatory cutoffs, and organizational adjustment costs. In such environments, policies can have qualitatively different effects on selection and aggregate outcomes than predicted by standard frameworks based on smooth distortions.

The remainder of the paper proceeds as follows. Section 2 presents the model and formalizes the domain-switching mechanism. Section 3 derives the theoretical results on subsidies and selection. Section 4 introduces the data and documents empirical patterns consistent with threshold-based operating decisions. Section 5 quantifies the model and decomposes the effects of subsidies. Section 6 presents counterfactual exercises. Section 7 provides external validation using Dutch hospitality data to assess the qualitative plausibility of the mechanism in a setting with observable revenue shocks and subsidy offsets. Section 8 concludes.

2 Model

We develop a static general equilibrium model with monopolistically competitive firms and reference-dependent firm owners. The model builds on the stationary equilibrium structure of Melitz (2003), but departs from expected utility by allowing owners to evaluate operating costs relative to a reference point, following Kőszegi and Rabin (2006). This feature introduces distinct gain and loss domains with different effective operating costs and generates a domain-switching mechanism that lies at the core of our results.

The key economic force is simple. When fixed costs exceed the reference level, they are perceived as losses and receive greater weight in the operating decision, raising the effective fixed cost and the productivity threshold for survival. A subsidy which reduces fixed costs can move firms from the loss to the gain domain, generating a discrete reduction in perceived costs. Through general equilibrium effects due to shifts in competition, these domain switches affect equilibrium productivity cutoffs.

2.1 Environment

Consider an economy with a measure L of workers who supply labor inelastically. Workers consume a CES aggregate of differentiated varieties:

$$Q = \left[\int_{\phi \in \Omega} q(\phi)^{\frac{\sigma-1}{\sigma}} d\phi \right]^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1, \tag{1}$$

where Ω denotes the set of available varieties, each indexed by the productivity ϕ of the firm producing it. The parameter σ governs the elasticity of substitution across varieties.

Demand for variety ϕ is given by

$$q(\phi) = RP^{\sigma-1} p(\phi)^{-\sigma}, \tag{2}$$

where R is aggregate expenditure, P is the CES price index, and $p(\phi)$ is the price of variety ϕ.

2.2 Production Technology

Each firm produces a unique variety using labor. A firm with productivity ϕ and fixed labor requirement f uses

$$l = \frac{q}{\phi} + f \tag{3}$$

units of labor to produce q units of output. We normalize the wage to $w = 1$, so fixed costs equal $F = f$.

Under monopolistic competition, firms charge a constant markup over marginal cost:

$$p(\phi) = \frac{\sigma}{\sigma - 1} \cdot \frac{1}{\phi}. \tag{4}$$

Revenues and profits are

$$r(\phi) = RP^{\sigma-1} \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma-1} \phi^{\sigma-1}, \tag{5}$$

$$\pi(\phi, F) = \frac{r(\phi)}{\sigma} - F. \tag{6}$$

2.3 Reference-Dependent Firm Owners

The key departure from standard models is that firm owners evaluate operating costs relative to a reference point. We model preferences using a reference-dependent utility function following Kőszegi and Rabin (2006). Owner utility is given by

$$V(\phi, F) = (1-\eta)\,\pi(\phi, F) - \eta \cdot v(F \mid F^r), \tag{7}$$

where $\pi(\phi, F)$ denotes economic profit, $\eta \in [0, 1)$ is the weight placed on gain-loss utility, F^r is the reference level of fixed costs, and $v(\cdot)$ captures the evaluation of realized costs relative to this reference point. We specify the gain-loss function as

$$v(F \mid F^r) = \begin{cases} \omega_G \cdot (F - F^r) & \text{if } F \leq F^r: \quad \text{gain domain (“G”)} \\ \omega_L \cdot (F - F^r) & \text{if } F > F^r: \quad \text{loss domain (“L”)}, \end{cases} \tag{8}$$

where $\omega_L > \omega_G$ governs the asymmetry between losses and gains. When fixed costs exceed the reference level, owners experience a loss which is weighted more heavily than an equivalent gain.

Our empirical analysis assumes the reference point equals lagged fixed costs: $F^r = F_{t-1}$. This specification treats the status quo as the reference point, consistent with the experimental evidence of Baillon, Bleichrodt and Spinu (2020), who find that the most common reference points are the status quo and a security level, with little support for expectations-based reference points. The specification is also consistent with the Kőszegi and Rabin (2006) framework if costs follow a martingale, in which case the prior-period value is the rational expectation of the current value.

Our formulation nests expected utility as the special case $\eta = 0$ and delivers two regimes with distinct effective operating costs. While we refer to this asymmetry as loss aversion, the structure more generally captures reference dependence with asymmetric sensitivity to unfavorable realizations.

Remark. We model reference dependence over fixed costs rather than profits. Fixed costs, particularly payroll and rent, are salient, recurring, and commonly evaluated relative to recent experience by small business owners. This choice delivers tractable expressions for operating thresholds while preserving the essential economics of domain switching.

Scope of the preference formulation. The piecewise-linear gain-loss function in equation (8), together with the three parameters $(\eta, \omega_G, \omega_L)$, nests a range of reference-dependent theories as special cases. The key degree of freedom is what the reference point F^r represents and which parameter restrictions are imposed. Table 1 summarizes.

Table 1: Theories nested by our preference formulation

Theory	Reference point F^r	Restriction
Loss aversion (Kőszegi and Rabin, 2006)	Status quo / expectations	$\omega_G = 1$, $\omega_L > 1$
Disappointment aversion (Bell, 1985)[a]	Expected outcome	$\omega_G = 0$, $\omega_L > 0$
Envy (Fehr and Schmidt, 1999)	Peer outcome	$\omega_G = 0$, $\omega_L > 0$
Habit formation[b]	Weighted past consumption	$\omega_G = \omega_L$
Regret / elation (Loomes and Sugden, 1982)	Foregone alternative	$\omega_G < \omega_L$
Aspiration-based firm behavior (Cyert and March, 1963)	Industry/historical benchmark	$\omega_G < \omega_L$
Misallocation (Hsieh and Klenow, 2009)[c]	—	$\omega_G = \omega_L$; cost wedge only

Notes: Each row is obtained from the general formulation (equations 7–8) by redefining F^r and restricting parameters. All rows assume piecewise linearity. Expected utility is the special case $\eta = 0$.

[a] Following Roy (1952) and formalized axiomatically in Gul (1991).

[b] Obtained by setting the reference point to past consumption within the Kőszegi and Rabin (2006) framework; cf. Pollak (1970) for the canonical ($\eta = 1$) formulation.

[c] Unlike Hsieh and Klenow (2009), where firms face independent distortions on revenue and capital costs, our formulation generates cost wedges only.

Our formulation is *not* a generalization of Kőszegi and Rabin (2006) in full generality. The Kőszegi and Rabin (2006) model allows for two features we shut down: diminishing sensitivity (curvature in the gain-loss function) and distributional reference points (where the reference is the full probability distribution of rational expectations, requiring integration over all possible outcomes). Our preferences generalize the restricted Kőszegi and Rabin (2006) case where both of these channels are also shut down — that is, where the gain-loss function is linear and the reference point is a single value. The additional degree of freedom relative to that restricted case is the separation of ω_G and ω_L, which is what allows our formulation to nest the theories listed above.

We adopt linearity for two reasons specific to our setting. First, the mechanism of interest involves discrete regime changes: subsidies move firms across the reference point, switching them between gain and loss domains. From this perspective, what matters is the local behavior around the kink — the difference in slopes on either side of F^r — not the curvature far from it. Piecewise linearity isolates exactly this channel. Second, the standard justification for diminishing sensitivity is psychophysical: individuals become less sensitive to marginal changes as they move further from the reference point. For firms, the opposite case is plausible. A firm suffering extreme cost overruns may become *more* distressed as problems cascade — credit downgrades, supplier withdrawal, employee attrition — each amplifying the pain of additional losses. Similarly, a firm experiencing large cost reductions may become *more* enthusiastic as strategic opportunities compound. This suggests enhanced sensitivity rather than diminishing sensitivity away from the reference point. In the absence of clear empirical guidance on the direction of curvature for firms, linearity is the most neutral assumption.

2.4 Entry and Exit

Potential entrants pay a sunk entry cost F^e to enter the market. Upon entry, a firm draws productivity ϕ from distribution $G(\phi)$ with density $g(\phi)$ and fixed costs F from distribution $H(F)$ with density $h(F)$. We assume ϕ and F are independent.

Assumption 1. Productivity follows a Pareto distribution: $1 - G(\phi) = (\underline{\phi}/\phi)^k$ for $\phi \geq \underline{\phi}$, where $k > 2\,(\sigma - 1)$.

Note: This assumption strengthens the standard condition $k > \sigma - 1$ to ensure finite variance of firm revenues. Under monopolistic competition with CES demand, revenues are given by $r(\phi) = B\phi^{\sigma-1}$ for a constant B, so the variance of revenues is finite if and only if $k > 2(\sigma - 1)$. To see this, note that the n-th moment of a Pareto distribution with shape parameter k and lower bound $\underline{\phi}$ is:

$$\mathbb{E}[\phi^n] = \int_{\underline{\phi}}^{\infty} \phi^n \cdot \frac{k\underline{\phi}^k}{\phi^{k+1}} d\phi = k\underline{\phi}^k \int_{\underline{\phi}}^{\infty} \phi^{n-k-1} d\phi = \left[\frac{\phi^{n-k}}{n-k}\right]_{\underline{\phi}}^{\infty}, \tag{9}$$

which converges if and only if $n - k < 0$, i.e. $k > n$. Setting $n = 2(\sigma - 1)$, the second moment of revenues $\mathbb{E}[\phi^{2(\sigma-1)}]$ is finite if and only if $k > 2(\sigma - 1)$. Since the variance equals $\mathbb{E}[r(\phi)^2] - (\mathbb{E}[r(\phi)])^2 = B^2\left(\mathbb{E}[\phi^{2(\sigma-1)}] - (\mathbb{E}[\phi^{\sigma-1}])^2\right)$, finite variance requires $k > 2(\sigma - 1)$, which also implies $k > \sigma - 1$ since $\sigma > 1$. Finite revenue variance ensures that aggregate productivity calculations are not dominated by extreme firms and that all equilibrium values are well-defined. Our calibrated parameters $k = 8$ and $\sigma = 1.5$ satisfy this condition comfortably, with $2(\sigma - 1) = 1$.

Assumption 2. Fixed costs follow a Pareto distribution with density $h(F) = \kappa\underline{F}^\kappa / F^{\kappa+1}$ for $F \geq \underline{F}$ and shape parameter $\kappa > 0$ on $[\underline{F}, \infty)$.

After observing (ϕ, F), the firm decides whether to operate. Production occurs if $V(\phi, F) \geq 0$, i.e. firms with $\phi \geq \phi^*(F)$ operate. This defines a productivity cutoff $\phi^*(F)$ satisfying

$$(1 - \eta)\left[\frac{r(\phi^*(F))}{\sigma} - F\right] - \eta \cdot v(F \mid F^r) = 0. \tag{10}$$

Rearranging,

$$\frac{r(\phi^*(F))}{\sigma} = F + \frac{\eta}{1-\eta} \cdot v(F \mid F^r) \equiv \Phi(F), \tag{11}$$

where $\Phi(F)$ denotes the effective fixed cost perceived by the owner. Assuming a Pareto-distribution for ϕ with shape parameter k and defining an average productivity parameter as $\widetilde{\phi} = \left(\int_{\phi^*}^{\infty} \phi^{\sigma-1} \frac{g(\phi)}{1-G(\phi^*)} d\phi\right)^{\frac{1}{\sigma-1}}$ implies $\frac{\phi^*}{\widetilde{\phi}} = \left(\frac{k-\sigma+1}{k}\right)^{\frac{1}{\sigma-1}}$. Considering $r(\phi^*) = r(\widetilde{\phi}) \left(\frac{\phi^*}{\widetilde{\phi}}\right)^{\sigma-1}$ leads to:

$$\frac{r(\widetilde{\phi}(F))}{\sigma} = \frac{k}{k-\sigma+1}\Phi(F). \tag{12}$$

The implied effective fixed cost is

$$\Phi(F) = \begin{cases} \dfrac{F(1-\eta+\eta\omega_G) - \eta\omega_G F^r}{1-\eta} & \text{if } F \le F^r \\ \dfrac{F(1-\eta+\eta\omega_L) - \eta\omega_L F^r}{1-\eta} & \text{if } F > F^r. \end{cases} \tag{13}$$

This piecewise structure generates a kink at the reference point. Since $\omega_L > \omega_G$, firms in the loss domain face higher effective fixed costs and therefore higher productivity thresholds. Policies that reduce F can move firms across this boundary, producing discrete changes in operating incentives. Moving firms across this kink alone does not mechanically strengthen or weaken selection — the aggregate implications depend on equilibrium entry and competitive pressure.

The equilibrium share of active entrants μ is:

$$\mu = \int_{\underline{F}}^{F^r} [1 - G(\phi_G^*(F))]\, h(F)\, dF + \int_{F^r}^{\infty} [1 - G(\phi_L^*(F))]\, h(F)\, dF. \tag{14}$$

μ is determined endogenously in equilibrium through productivity cutoffs $\phi_G^*(F)$ and $\phi_L^*(F)$.

2.5 Free Entry

In equilibrium, expected utility from entry equals the entry cost:

$$\int_{\underline{F}}^{F^r} [1 - G(\phi_G^*(F))]\, V(\widetilde{\phi}_G(F), F)\, h(F)\, dF + \int_{F^r}^{\infty} [1 - G(\phi_L^*(F))]\, V(\widetilde{\phi}_L(F), F)\, h(F)\, dF = F^e, \quad (15)$$

where $\widetilde{\phi}_x(F)$, $x = G, L$, denotes average productivity among active firms conditional on fixed cost F. To solve the free entry condition for the mass of entrants M_e, we first consider the price index for the aggregate consumption good:

$$P = \left\{ M_e \left[\int_{\underline{F}}^{F^r} [1 - G(\phi_G^*(F))]\, p(\widetilde{\phi}_G(F))^{1-\sigma} h(F)\, dF + \int_{F^r}^{\infty} [1 - G(\phi_L^*(F))]\, p(\widetilde{\phi}_L(F))^{1-\sigma} h(F)\, dF \right] \right\}^{\frac{1}{1-\sigma}}. \quad (16)$$

Substituting the definition of V (equation 7) and the zero cutoff profit condition (equation 12) into the free entry condition (equation 15), the latter results as:

$$\int_{\underline{F}}^{F^r} [1 - G(\phi_G^*(F))]\, r(\widetilde{\phi}_G(F))\, h(F)\, dF + \int_{F^r}^{\infty} [1 - G(\phi_L^*(F))]\, r(\widetilde{\phi}_L(F))\, h(F)\, dF = \frac{\sigma k}{\sigma - 1} \cdot \frac{F^e}{1 - \eta}, \quad (17)$$

Using $r(\widetilde{\phi}_x(F)) = RP^{\sigma-1} p(\widetilde{\phi}_x(F))^{1-\sigma}$, $x = G, L$, together with P from equation (16), allows us to solve for the mass of entrants:

$$M_e = \frac{(\sigma - 1)R}{\sigma k} \cdot \frac{1 - \eta}{F^e}. \quad (18)$$

2.6 Equilibrium

Definition. *An equilibrium consists of:*

1. *A mass of entrants* M_e
2. *Productivity cutoffs* $\phi^*(F)$ *for each* F
3. *Average productivities* $\widetilde{\phi}(F)$ *for each* F

4. *Sector-wide average productivity* $\widetilde{\widetilde{\phi}}$

5. *A price index* P *and aggregate output* Q

such that: (i) firms optimize pricing and entry/exit decisions; (ii) free entry holds; and (iii) labor and goods markets clear.

We now characterize the key equilibrium objects. Define

$$\begin{aligned} \Theta &\equiv \left[\frac{(\sigma-1)(1-\eta)}{(k-\sigma+1)f_e} \underline{\phi}^k \left(\frac{k}{k-\sigma+1} \right)^{\frac{k}{\sigma-1}} \right]^{\frac{1}{\sigma-1}}, \\ \mathbb{E}(\Phi) &\equiv \int_{\underline{F}}^{F^r} \Phi_G(F)^{\frac{\sigma-1-k}{\sigma-1}} h(F)\, dF + \int_{F^r}^{\infty} \Phi_L(F)^{\frac{\sigma-1-k}{\sigma-1}} h(F)\, dF, \end{aligned}$$

where $\Phi_G(F)$ and $\Phi_L(F)$ are the effective fixed costs in the gain and loss domains defined in equation (13).

The average productivity among active firms with fixed cost F is:

$$\widetilde{\phi}_x(F) = \Theta^{\frac{\sigma-1}{k}} \mathbb{E}(\Phi)^{\frac{1}{k}} \Phi(F)^{\frac{1}{\sigma-1}}, \qquad x \in \{G, L\}. \tag{19}$$

The mass of entrants is determined by free entry:

$$M_e = \frac{(\sigma-1)R}{\sigma k} \cdot \frac{1-\eta}{F^e}. \tag{20}$$

The mass of active firms equals the mass of entrants times the equilibrium share of active entrants μ (equation (14)):

$$M = M_e \cdot \mu. \tag{21}$$

Expected profits of the average entrepreneur are:

$$\begin{aligned} \chi = \int_{\underline{F}}^{F^r} & [1 - G(\phi_G^*(F))]\, \pi\left(\widetilde{\phi}_G(F)\right) h(F)\, dF \\ & + \int_{F^r}^{\infty} [1 - G(\phi_L^*(F))]\, \pi\left(\widetilde{\phi}_L(F)\right) h(F)\, dF - F^e. \end{aligned} \tag{22}$$

Finally, sector-wide average productivity aggregates over all active firms:

$$\widetilde{\widetilde{\phi}} = \left(\frac{\mathbb{E}(\widetilde{\phi}^{\sigma-1})}{\mu} \right)^{\frac{1}{\sigma-1}}, \tag{23}$$

where $\mathbb{E}(\widetilde{\phi}^{\sigma-1}) \equiv \int_{\underline{F}}^{F^r} [1 - G(\phi_G^*(F))] \, \widetilde{\phi}_G(F)^{\sigma-1} h(F) dF + \int_{F^r}^{\infty} [1 - G(\phi_L^*(F))] \, \widetilde{\phi}_L(F)^{\sigma-1} h(F) dF$. The price index can equivalently be written as $P = \left[M \cdot p(\widetilde{\widetilde{\phi}})^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$, where M is the mass of active firms and $\widetilde{\widetilde{\phi}}$ is the sector-wide average productivity.

Loss aversion affects $\widetilde{\widetilde{\phi}}$ through two counteracting channels. First, higher ω_L increases perceived losses for firms with $F > F^r$, raising the productivity cutoff $\phi_L^*(F)$ at each fixed cost level in the loss domain and thereby selecting for more productive firms. This increases $\widetilde{\widetilde{\phi}}$. Second, the higher productivity cutoff $\phi_L^*(F)$ implies fewer firms are active in the loss domain, reducing μ and therefore the mass of active firms M overall in the market. With fewer active firms, competition decreases, which lowers the productivity cutoff $\phi_G^*(F)$ at each fixed cost level in the gain domain and therefore decreases $\widetilde{\widetilde{\phi}}$. Which effect dominates depends on the location of the reference point F^r and the distribution of fixed costs. In Appendix A.1.4, we show that under a Pareto distribution for F with empirically relevant shape parameter $\kappa = 0.3$, the selection effect dominates and $\widetilde{\widetilde{\phi}}$ increases with ω_L.

3 Subsidies and Firm Selection

This section studies how untargeted fixed-cost subsidies affect firm selection and aggregate productivity. We begin with the expected-utility benchmark, reproducing the canonical result that subsidies reduce average productivity by allowing less productive firms to remain active. We then show how reference-dependent preferences introduce a new selection channel operating through domain switching. This channel can overturn the standard prediction.

3.1 Benchmark: Expected Utility

We first establish the standard result in models of firm dynamics. Consider a proportional subsidy that reduces fixed costs by a factor $\Delta \in (0, 1)$, so that firms face fixed costs ΔF (a lower Δ corresponds to a larger subsidy). Under expected utility ($\eta = 0$), operating decisions depend only on productivity and realized fixed costs. The productivity cutoff satisfies:

$$\phi^*(\Delta F) = \left(\frac{\sigma \Delta F}{B} \right)^{\frac{1}{\sigma-1}}, \tag{24}$$

where B collects demand-side terms. A reduction in Δ lowers the cutoff, allowing less productive firms to operate.

Proposition 1 (Expected Utility Benchmark)**.** *Under expected utility ($\eta = 0$), a fixed-cost subsidy unambiguously reduces average productivity:*

$$\frac{\partial \tilde{\tilde{\phi}}}{\partial \Delta} > 0. \tag{25}$$

Proof. See Appendix A.1.5.4. □

This result is the canonical selection effect: untargeted subsidies lower survival thresholds and keep marginal low-productivity firms active.

3.2 Reference Dependence and Domain Structure

With reference-dependent preferences ($\eta > 0$), operating decisions depend on whether fixed costs lie above or below the reference point F^r. This creates two regimes:

- *Gain domain* ($F \leq F^r$): realized fixed costs fall short of the reference point. The shortfall $F^r - F \geq 0$ is perceived as a positive surprise, weighted by ω_G, reducing the perceived burden of fixed costs.
- *Loss domain* ($F > F^r$): realized fixed costs exceed the reference point. The excess $F - F^r > 0$ is perceived as a negative surprise, weighted more heavily by $\omega_L > \omega_G$, increasing the perceived burden of fixed costs.

As shown in Section 2, these preferences generate an effective fixed cost $\Phi(F)$ that is steeper in the loss domain than in the gain domain. Firms in the loss domain therefore face inflated effective costs and higher productivity cutoffs, even holding productivity fixed.

A subsidy affects average productivity through two channels:

1. It reduces fixed costs mechanically, lowering productivity thresholds and reducing average productivity (the standard channel).
2. It moves some firms across the reference point, shifting them from the loss domain to the gain domain, which increases the mass of active firms and, thus, competition. The increase in competition raises productivity thresholds and average productivity (the domain-switching channel).

The second channel is absent under expected utility.

3.3 Subsidies with Reference-Dependent Preferences

We now characterize how subsidies affect aggregate productivity when owners exhibit reference-dependent preferences.

Proposition 2 (Ambiguous Effect of Subsidies)**.** *Under reference-dependent preferences ($\eta > 0$), the effect of a fixed-cost subsidy on average productivity is ambiguous:*

$$\frac{\partial \widetilde{\widetilde{\phi}}}{\partial \Delta} \lesseqgtr 0. \tag{26}$$

Proof. See Appendix A.1.5.5. □

The ambiguity arises because the domain-switching channel can counteract, and potentially dominate, the standard selection effect.

3.4 Domain Switching and Selection

To understand the mechanism, consider firms with fixed costs just above the reference point. Without subsidies, these firms lie in the loss domain and face sharply elevated effective costs, requiring higher productivity to survive. A subsidy that reduces fixed costs can move such firms into the gain domain, discretely lowering their effective costs. This domain switching causes substantially more firms to become active than the direct subsidy effect alone, increasing competition and, thus, raising productivity thresholds across all firms.

Crucially, the increased competition from domain switching can raise average productivity even though the direct subsidy effect lowers productivity thresholds. When this competitive channel dominates, subsidies increase rather than decrease $\widetilde{\widetilde{\phi}}$, overturning the canonical prediction.

3.5 Reversal of the Standard Prediction

We now show that when the weight on gain–loss utility is sufficiently large, the domain-switching channel dominates.

Proposition 3 (Reversal Threshold)**.** *There exists a threshold $\eta^* \in (0, 1)$ such that:*

$$\frac{\partial \widetilde{\widetilde{\phi}}}{\partial \Delta} < 0 \quad \textit{for all } \eta > \eta^*. \tag{27}$$

Proof. See Appendix A.1.5.6. □

The threshold η^* depends on primitive features of the economy.

Corollary 1 (Comparative Statics of η^*)**.** *The threshold η^* is:*

1. *decreasing in the strength of loss weighting ω_L,*
2. *increasing in the elasticity of substitution σ.*

These comparative statics formalize when behavioral distortions in operating decisions are most likely to overturn the canonical selection result. When reference dependence is sufficiently strong, subsidies can improve selection by correcting distortions that disproportionately penalize firms in the loss domain.

3.6 Effects on Market Size

We now characterize how subsidies affect the mass of active firms. Let M denote the number of firms that choose to operate in equilibrium.

Proposition 4 (Effects on Market Size)**.**

1. *Under expected utility ($\eta = 0$), a fixed-cost subsidy unambiguously increases the mass of active firms:*

$$\frac{\partial M}{\partial \Delta} < 0. \tag{28}$$

2. *Under reference-dependent preferences ($\eta > 0$), the effect is amplified. There exists $\bar{\eta} \in (0, 1)$ such that for all $\eta > \bar{\eta}$:*

$$\left| \frac{\partial M}{\partial \Delta} \right|_{\eta > 0} > \left| \frac{\partial M}{\partial \Delta} \right|_{\eta = 0}. \tag{29}$$

Proof. See Appendix A.1.5.7. □

The first result is standard: subsidies allow less productive firms to cover their fixed costs, lowering productivity cutoffs and allowing more firms to survive. The second result reflects the domain-switching mechanism. Firms in the loss domain face inflated effective fixed costs due to $\omega_L > \omega_G$ and exit even when their underlying productivity would permit operation under expected utility. A subsidy that moves these firms into the gain domain discretely lowers their effective costs, producing a larger increase in survival than the standard channel alone.

Corollary 2 (Entry and Exit Rates)**.** *In a stationary equilibrium where entry equals exit, a fixed-cost subsidy:*

1. *Reduces exit rates by lowering the productivity threshold for survival.*
2. *Increases entry rates by raising expected profits conditional on entry.*

3. *Under loss aversion, these effects are concentrated among firms near the reference point, generating asymmetric responses in the gain and loss domains.*

The corollary provides a direct link to the empirical patterns documented in Section 4. The model predicts exit rates should decline more sharply for firms moving from the loss domain to the gain domain, while entry rates respond to the change in expected profitability across domains. This asymmetry is consistent with the evidence in Figures 1 and 2.

4 Data and Descriptives

We study the U.S. restaurant industry using county-level data from 1998-2019 for the pre-COVID period, and 2020-2021 for the COVID period. This section describes the data, defines our reference point measure, and documents empirical patterns consistent with reference-dependent firm behavior.

4.1 Data

Our primary data source is the County Business Patterns (CBP) from the U.S. Census Bureau, which provides annual data on the number of establishments, employment, and payroll by county and industry. We focus on NAICS code 7225 (Restaurants and Other Eating Places), which covers restaurants and other eating places.[2]

We supplement CBP with several additional data sources. Business Dynamics Statistics (BDS) from the U.S. Census Bureau provides entry and exit rates at the county-sector level. We also merge county-level GDP from the Bureau of Economic Analysis, population estimates and business applications data from the Census Bureau, house price indices from the Federal Housing Finance Agency, and Small Business Administration data on PPP and RRF disbursements.

Table 2 presents summary statistics for the pre-COVID period, including measures of market size, payroll, and firm entry and exit.

4.2 Reference Point

We set the reference point F^r equal to lagged average payroll per establishment. This reflects backward-looking expectations where owners evaluate current costs against recent

[2] Prior to 2012, this industry was classified under NAICS codes 7221-7223. For years before 1998, we use the equivalent SIC codes (5800, 5810, 5812).

experience. Formally, where m indexes counties and t indexes years:

$$F_{mt}^r = \frac{\text{Total payroll}_{m,t-1}}{\text{Number of establishments}_{m,t-1}}. \tag{30}$$

Our specification implies:

$$\text{Gain domain: } F_{mt} \leq F_{mt}^r \quad \Rightarrow \quad \text{payroll per establishment fell year-over-year} \tag{31}$$

$$\text{Loss domain: } F_{mt} > F_{mt}^r \quad \Rightarrow \quad \text{payroll per establishment rose year-over-year} \tag{32}$$

During COVID-19, payroll fell sharply for most restaurants in 2020, placing them in the gain domain. By 2021, as the economy recovered, payroll rose relative to the depressed 2020 levels, placing most restaurants in the loss domain. The specification of the reference point as lagged payroll is consistent with standard industry practice. Restaurant operators routinely benchmark current performance against prior-period outcomes through same-store comparisons of current revenue relative to revenue the same period last year.[3]

Since payroll data are only available at the county level, a county is classified as being in the loss (gain) domain if average payroll per establishment rose (fell) relative to the prior year. This county-level classification serves as an empirical approximation of the firm-level domain assignment in the model.

We do not claim $F^r = F_{t-1}$ is the uniquely correct reference point. The identification strategy only requires reference points to adjust sluggishly relative to realized cost shocks. Any specification where owners form expectations based on recent experience will generate qualitatively similar predictions. We verify this claim in Section 5.3 by showing our main results hold under alternative reference point specifications.

4.3 Entry and Exit Around the Reference Point

We begin by examining whether the data exhibit the asymmetric entry and exit patterns implied by reference-dependent preferences. The model predicts higher exit rates in the loss domain (where costs exceed the reference point). Entry rates, by contrast, should be less sensitive to domain: potential entrants evaluate expected profitability before observing their realized costs, so the asymmetry between gain and loss domains is muted.

Figure 1 plots entry and exit rates from the Business Dynamics Statistics against changes

[3] See, e.g., Incentivio, "Financial Planning and Budgeting for Restaurant's Long-Term Success," available at incentivio.com.

in average payroll per establishment. We residualize all variables by county and year fixed effects to remove permanent county characteristics and aggregate time shocks. The plotted values add back sample means to preserve interpretable levels. Entry rates are plotted against lagged payroll changes, reflecting that potential entrants observe the prior year's cost environment before deciding to enter. Exit rates are plotted against contemporaneous payroll changes, reflecting that incumbents respond to current cost realizations. Panel (b) shows the conversion rate—entries per 100 business applications—which captures the intensive margin of new business formation conditional on application.

To quantify asymmetry around the reference point, we compute slope ratios: the slope in the loss domain (right of zero) divided by the slope in the gain domain (left of zero). A ratio near 1.0 indicates symmetric responses across domains. Ratios substantially different from 1.0 indicate asymmetric behavior consistent with reference-dependent preferences.

The results support model predictions. Exit rates exhibit substantial asymmetry, with a steeper response to cost increases in the loss domain than to cost decreases in the gain domain. Entry rates and conversion rates, by contrast, are relatively flat across both domains. This pattern aligns with the theory: exit decisions involve incumbents evaluating realized costs against their reference point, whereas entry decisions involve potential entrants evaluating expected profitability before observing their actual cost draw. The asymmetry predicted by loss aversion should therefore be concentrated in exit rather than entry.

Figure 2 disaggregates these patterns by establishment size using County Business Patterns data.[4] For small establishments (fewer than 20 employees), entry and exit rates track each other closely in the gain domain, both hovering around 2–4%. In the loss domain, however, the two diverge: exit rates rise steeply—reaching approximately 12% at +30% payroll growth—while entry rates remain relatively flat. This divergence increases with the magnitude of the loss. A standard model with symmetric responses to cost changes could not generate this pattern, and the asymmetry is consistent with loss-averse exit decisions.

For large establishments (20 or more employees), the pattern is different—and counter-intuitive under the loss aversion interpretation. Exit rates are highest for the largest gains (around 10% at −30% payroll change) and decline steadily as payroll changes become more positive. This reflects a measurement issue: for large firms, payroll increases often reflect scale expansion—hiring to meet demand—rather than exogenous cost shocks. If payroll growth proxies for successful growth rather than cost pressure, then what we label the "loss

[4] Because the Business Dynamics Statistics does not report entry and exit rates separately by establishment size, we compute net rates from CBP establishment counts: positive year-over-year growth is classified as net entry, negative growth as net exit.

domain" actually contains thriving firms, explaining their low exit rates.

This scale interpretation is less applicable to small establishments, where payroll increases more plausibly reflect genuine cost pressure rather than expansion. We therefore focus the remainder of the empirical analysis on small establishments. This restriction is motivated both by theory—loss aversion should be most salient for owner-operators with concentrated exposure to business outcomes—and by measurement—payroll changes more cleanly proxy for cost shocks when scale expansion is less relevant.

4.4 Domain Switching During COVID-19

The policy response to COVID-19 provides a unique setting for studying the domain-switching mechanism. In 2020, restaurant payrolls fell sharply due to lockdowns and reduced demand, placing most counties in the gain domain. At the same time, the U.S. federal government provided substantial fixed-cost subsidies through the Paycheck Protection Program (PPP) and the Restaurant Revitalization Fund (RRF). PPP provided forgivable loans to small businesses to cover payroll and operating expenses, RRF provided grants specifically targeted to food service establishments. We compute county-level subsidy rates by aggregating disbursements to the restaurant sector and dividing by total payroll from County Business Patterns.

These comparisons are descriptive and intended to assess consistency with the model's qualitative predictions rather than to estimate causal effects of subsidies.

The COVID-19 disruption to restaurant operations has a natural interpretation in our framework. Lockdowns and health regulations required restaurants to adopt new production technologies rather than simply adjust margins of an existing model. In terms of our model, incumbents were effectively drawing new productivity and cost parameters and deciding whether to operate under the new regime. This interpretation bridges the gap between the model's selection margin—which formally applies to new entrants drawing productivity upon entry—and the observed survival decisions of incumbent restaurants during COVID.

A county in the loss domain switches to the gain domain if its subsidy rate is large enough to offset its payroll increase. Formally, switching occurs when $g \leq s/(1-s)$, where g is payroll growth and s is the subsidy rate—for example, a 13% subsidy can offset payroll growth up to approximately 15%. We define 'switching counties' as those satisfying this condition.

A county is classified as being in the loss domain if average payroll per establishment rose relative to the prior year, and switches to the gain domain if the subsidy rate is large enough to offset this average payroll increase.

Table 3 reports domain composition, subsidy characteristics, and exit rates for 2020 and 2021. In 2020, despite the overall contraction, 32% of counties experienced payroll increases (placing them in the loss domain). These counties tended to have less severe lockdowns or different industry composition which partially offset COVID-related declines. Among these loss-domain counties, 68% received subsidies large enough to switch them to the gain domain.

The 2021 pattern differs markedly. As the economy recovered, payroll rose relative to the depressed 2020 base in most counties—even where payroll remained below pre-COVID levels. This placed 85% of counties in the loss domain. However, only 40% of these counties received subsidies sufficient to switch domains, reflecting both smaller subsidy rates in 2021 and larger payroll increases to offset.

A central implication of the model is that switchers—firms that move from the loss to gain domain with subsidies—should exhibit lower exit rates than non-switchers. Figure 3 presents this comparison.

Switchers have substantially lower exit rates (6.47%) compared to non-switchers (12.98%), a difference of 6.51 percentage points that is highly statistically significant ($p < 0.001$). Entry rates, by contrast, are similar across the two groups (1.79% vs. 1.49%, $p = 0.54$). This asymmetry is consistent with our model: reference dependence affects incumbents evaluating realized costs, not potential entrants evaluating expected profitability. Subsidies that move firms out of the loss domain reduce the psychological weight on cost increases, lowering exit rates while leaving entry unaffected. Consistent with our model, this pattern suggests domain switching preserves establishments whose underlying productivity would permit operation absent distorted cost perceptions, rather than simply retaining the least productive firms.

4.5 Pre-COVID Selection Test

A potential concern is that the exit gap between switchers and non-switchers reflects stable firm heterogeneity rather than behavioral domain switching. If firms near the switching threshold are systematically different—for example, better managed or more resilient—they would exhibit lower exit rates regardless of subsidies. We test this by comparing pre-COVID exit rates (2018-2019) between counties that later switched domains during COVID-19 and those that did not.

Table 4 reports the results. Future switchers exhibit similar pre-COVID exit rates to future non-switchers (2.80% vs. 3.12%, $p = 0.17$). This gap of 0.32 percentage points is not statistically significant and is an order of magnitude smaller than the 6.51 pp gap observed during COVID.

The sharp widening of the exit gap—from an insignificant 0.32 pp before COVID to 6.51 pp during COVID, coincident with subsidy availability—is difficult to reconcile with stable differences in firm quality and instead aligns with domain switching triggered by subsidy availability.

5 Calibration and Model Fit

We now calibrate our model to match the empirical patterns documented in the previous section.

5.1 Parameter Calibration

Externally calibrated parameters. We set the following parameters based on existing literature and industry characteristics (Table 5, Panel A).

The elasticity of substitution $\sigma = 1.5$ is lower than the values of 4–8 commonly used in trade models calibrated to manufacturing. We use a lower value because restaurants are highly differentiated along dimensions including cuisine, ambiance, location, and service style. Consumers do not readily substitute between a neighborhood Thai restaurant and an Italian trattoria, nor even between two Italian restaurants with distinct atmospheres. High implied markups ($\sigma/(\sigma - 1) = 3$ in this case) are consistent with survey evidence showing restaurant owners perceive inelastic demand at current prices (Emami Namini and Kapoor, 2026).

Behavioral parameters. The key behavioral parameters are η (weight on gain-loss utility), ω_G (gain weight) and ω_L (loss weight). We normalize $\omega_G = 1$ and calibrate the remaining parameters to match the observed difference in exit rates between domain-switching and non-switching counties documented in Section 4.4. Specifically, we target two moments from the 2020 data:

1. The exit rate in domain-switching counties: 5.92%
2. The exit rate in non-switching counties: 12.27%

Our calibration procedure minimizes the sum of squared errors between model-predicted and observed exit rates.[5] The calibrated parameters are reported in Table 5, Panel B.

[5]The exit specification exit $= \alpha \cdot \tilde{\Phi}^{\beta}$ is a reduced-form assumption. The theory predicts exit is increasing in effective costs but yields no closed-form mapping. The values of η and ω_L are disciplined by the difference

The calibrated value of $\omega_L = 10$ is higher than the median estimate of approximately 2.0 from meta-analyses of individual loss aversion (Brown, Imai, Vieider and Camerer, 2024). However, direct survey evidence from restaurant owners supports loss aversion coefficients of this magnitude. Emami Namini and Kapoor (2026) elicit loss aversion using the Abdellaoui, Bleichrodt, L'Haridon and van Dolder (2016) method among 107 restaurant owners and managers in the Netherlands, finding a mean $\omega_L = 10.1$ and median $\omega_L = 1.6$. While 74% of owners exhibit loss aversion ($\omega_L > 1$), 30% have coefficients exceeding 3, indicating a substantial right tail of highly loss-averse owners. The same study analyzes stopping decisions in restaurant labor demand, estimating $\omega_L = 4.2$ overall and $\omega_L = 9.5$ on slow days when management teams are smaller and individual decisions carry more weight. The mean from the survey (10.1) and the slow-day stopping estimate (9.5)—conditions resembling COVID-era operations when owners made more decisions directly—are consistent with our calibrated value and suggest high loss aversion among restaurant owners is empirically grounded.

The calibrated value of $\eta = 0.20$ implies gain-loss utility receives a meaningful weight in owners' decisions—20% of the owner's value function reflects reference-dependent evaluation. To interpret these magnitudes, consider how effective costs differ across domains. For a switcher with 5% payroll growth and 9% subsidy, the normalized effective cost is approximately $\tilde{\Phi} = 0.95$. For a non-switcher with the same payroll growth but no subsidy (remaining in the loss domain), $\tilde{\Phi} \approx 1.17$—23% higher effective costs. This difference in effective costs, combined with the exit elasticity β, generates the observed 6.51 percentage point gap in exit rates between switchers and non-switchers.

5.2 Model Fit

The model is calibrated to match the observed exit rates for switchers (5.92%) and non-switchers (12.27%). Figure 4 shows the model fit across the distribution of payroll growth rates. The model captures the key qualitative pattern: exit rates are higher in the loss domain (positive payroll growth) than in the gain domain (negative payroll growth), with a kink at the reference point.

in $\tilde{\Phi}$ between switchers and non-switchers via equation (13); the parameters α and β simply scale this to match exit rate levels.

5.3 Reference Point Robustness

The baseline specification uses lagged payroll $F^r = F_{t-1}$ as the reference point. We verify our main findings are robust to alternative specifications that allow for more gradual reference point updating or trend adjustment.

Smoothed reference. Owners may anchor on a longer history of costs rather than the single most recent period. We consider $F^r = (F_{t-1} + F_{t-2})/2$, which averages the two most recent payroll realizations.

Trend-adjusted reference. Owners in growing markets may expect costs to rise, adjusting their reference point upward. We consider $F^r = F_{t-1}(1 + \bar{g}_m)$, where $\bar{g}_m$ is the county's average payroll growth rate in 2018–2019.

For each specification, we reclassify firms into gain and loss domains, recompute switcher status, and compare exit rates between switchers and non-switchers. Table 6 reports the results.

The exit rate gap between non-switchers and switchers ranges from 4.60 to 6.97 percentage points across specifications, and all estimates are highly significant. The robustness confirms identification does not depend on the precise reference point formulation—the domain-switching mechanism operates under any specification in which owners form reference points based on recent cost experience.

6 Counterfactual Exercises

We use the calibrated model to quantify how COVID-era subsidies affect firm exit and selection within the model.

6.1 Exit Rate Counterfactuals

Table 7 considers two scenarios:

1. **No Subsidy.** We simulate exit rates if COVID subsidies (PPP and RRF) had not been provided. Firms face their actual 2020 fixed costs without any offset.

2. **Actual (2020).** The observed equilibrium with actual subsidy disbursements.

Comparing the no-subsidy scenario (10.12%) to the actual outcome (7.75%), the model predicts COVID subsidies reduced the exit rate by 2.44 percentage points—a 24% reduction in exit probability.

6.2 Decomposing the Subsidy Effect

The total effect of subsidies on exit operates through two channels. The first is the standard channel: subsidies reduce fixed costs mechanically, lowering the productivity threshold for survival. This channel operates for all firms, regardless of domain. The second is the domain-switching channel: subsidies move firms across the reference point, discretely reducing their effective fixed costs. This channel operates only for firms near the boundary between domains.

To decompose the standard and domain-switching channel, we compare the model with expected utility ($\eta = 0$) to a counterfactual with reference-dependent preferences ($\eta = 0.20$, $\omega_L = 10$, $\omega_G = 1$). Under expected utility, the model predicts subsidies would reduce exit by approximately 1.1 percentage points through the standard channel alone. The reference-dependent model predicts a larger reduction of 2.4 percentage points. The additional 1.3 percentage points of reduction is attributable to the domain-switching channel. The standard channel accounts for approximately 47% of the total effect, while the domain-switching channel contributes about 53%.

6.3 Productivity Effects

We now examine how subsidies affect aggregate productivity. Under expected utility, Proposition 1 establishes that subsidies *reduce* average productivity: by lowering fixed costs, subsidies allow low-productivity firms, that would otherwise exit, to remain active, reducing average productivity. This is the standard prediction of canonical models.

With reference-dependent preferences, the domain-switching channel can reverse this prediction. Firms in the loss domain face inflated effective costs due to loss aversion, and subsidies that move these firms into the gain domain produce discrete cost reductions which go beyond the mechanical subsidy effect. These discrete cost reductions increase the mass of active firms, intensifying competition and raising productivity thresholds across all firms. When this channel is sufficiently strong, subsidies can *increase* rather than decrease average productivity.

Figure 5 illustrates how the productivity effect varies with the reference dependence parameter η for different levels of loss aversion ω_L (with $\omega_G = 1$ throughout). At $\eta = 0$ (expected utility), subsidies *reduce* productivity by approximately 1.3%, consistent with the standard adverse selection prediction. As η increases, the domain-switching channel becomes more powerful: firms in the loss domain face increasingly elevated effective costs, so moving them to the gain domain produces larger cost reductions. With higher loss aversion ω_L, this amplification is stronger, and the productivity effect crosses from negative to positive at lower values of η.

At our calibrated parameters ($\eta = 0.20$, $\omega_L = 10$, $\omega_G = 1$), the model predicts a productivity increase of approximately 5%—a reversal of the standard negative prediction. The dotted line at 15% marks the empirical estimate from Goolsbee et al. (2025). With higher loss aversion, the model can generate larger effects: at $\omega_L = 50$, the productivity gain rises to approximately 12%, and at $\omega_L = 100$, the model approaches the empirical estimate. The key insight is that for any $\omega_L > 1$ and $\omega_G = 1$, there exists a threshold η^* above which subsidies *increase* rather than decrease productivity. Even at empirically grounded parameter values, the domain-switching channel is strong enough to reverse the standard prediction.

The empirical estimate of roughly 15% lies above our baseline effect of 5%, but the model generates larger gains as loss aversion increases. The key insight is not to match a specific magnitude, but to show that once operating incentives exhibit threshold distortions, subsidies need not worsen selection and can plausibly improve it at empirically grounded parameter values.

7 External Validity: Evidence from the Netherlands

The Netherlands provides an independent setting in which to assess the domain-switching mechanism. Unlike the U.S. analysis, where financial distress must be inferred from exit behavior, the Dutch data allow direct observation of both the revenue shock and the associated subsidy response at the sector level.

The Dutch evidence is descriptive and intended to assess the qualitative plausibility of the mechanism in a setting with observable revenue shocks, rather than to provide structural identification.

During COVID-19, the Dutch government implemented two major support programs: NOW (Tijdelijke Noodmaatregel Overbrugging voor Werkgelegenheid), a wage subsidy analogous to PPP, and TVL (Tegemoetkoming Vaste Lasten), a fixed-cost subsidy analogous to RRF. Table 8 shows the hospitality sector (Horeca) received substantially higher support than other sectors: 59.9% of hospitality firms received NOW subsidies versus 22.9% for other sectors, and 85.1% received TVL subsidies versus 24.4% for other sectors.

Figure 6 presents time-series evidence for the Dutch hospitality sector. Panel (a) shows revenue per establishment indexed to 2019. The sequence of events is clear: COVID shocked revenue in this sector, causing it to collapse to 63% of its 2019 level in 2020—a 37% decline. This increased exit. The government then introduced subsidies which offset some of the revenue decline.

We construct a back-of-the-envelope estimate of effective revenue by combining observed revenue per establishment with imputed NOW and TVL transfers using published coverage and replacement rates. Combining NOW wage subsidies (covering approximately 85% of wages for 60% of firms) with TVL fixed-cost subsidies (covering approximately 70% of fixed costs for 85% of firms), we estimate that effective revenue—actual revenue plus subsidy transfers—reached 75 percent of its 2019 level (indexed to 100) in 2020 and 80 percent in 2021. Critically, subsidies recovered approximately 35% of the revenue loss, but *not* the full amount.

Panel (b) shows the exit rate (business deaths as a share of the establishment stock). Under a standard expected-utility model, incomplete compensation should lead to persistently elevated exit rates. In fact, they go down. Despite revenue remaining 20–35% below pre-COVID levels even with subsidies, the exit rate fell from 7.3% in 2020 to 5.1% in 2021—the lowest rate in the sample period. Exit rates even fell *below* pre-COVID levels. When subsidies ended in 2022, exit rates rebounded to 7.2% despite revenue fully recovering.

This pattern is difficult to reconcile with a smooth expected-utility model in which exit responds continuously to profitability. Because subsidies only partially offset the revenue loss (Figure 6), a standard framework would predict persistently elevated exit rates. Instead, exit rates fell below pre-COVID levels. This pattern is consistent with threshold-based operating incentives: transfers of the observed magnitude may have been sufficient to move firms across an incentive boundary, reducing exit even though revenues remained below their pre-COVID level.

The 2022 rebound addresses a potential selection concern—that elevated 2020 exit simply removed marginal firms, mechanically lowering subsequent exit rates. If selection explained the 2021 decline, exit rates should have remained low in 2022 among the "surviving" high-quality firms. Instead, exit rates returned to elevated levels precisely when subsidies ended, more consistent with a temporary incentive shift than with a pure selection story.

8 Conclusion

This paper shows that when firms' operating decisions exhibit threshold distortions, the standard prediction that untargeted subsidies reduce productivity can be overturned. In our framework, subsidies move firms across discrete incentive regimes, increasing competition and raising equilibrium productivity cutoffs.

Using reference-dependent preferences as a tractable micro-foundation for such threshold-based behavior, we calibrate the model to the U.S. restaurant sector during COVID-19. The model implies that subsidies reduced small-firm exit by approximately 2.4 percentage points, with the domain-switching channel accounting for roughly half of this effect.

The implications for productivity are substantive. Under expected utility, the model predicts a 1.3% decline in average productivity following subsidies, reflecting the standard selection effect. With reference-dependent preferences at empirically grounded parameter values ($\omega_L = 10$), the prediction reverses: average productivity increases by approximately 5%. The reversal arises entirely from the domain-switching channel, which amplifies competition by shifting firms out of the loss domain.

More broadly, our results highlight that the effects of untargeted firm support depend crucially on how operating incentives respond to financial shocks. When these incentives are smooth, subsidies weaken selection. When they exhibit threshold distortions, subsidies can instead improve it by correcting distortions that disproportionately penalize firms near critical operating margins.

Our analysis abstracts from several dimensions that could be incorporated in future work, including dynamic reference point formation and heterogeneity in behavioral parameters across firms, as well as richer labor market structures such as multi-factor production or labor market frictions. In our baseline environment with a single factor of production, proportional changes in wages do not affect selection, so the mechanism operates entirely through changes in operating incentives rather than factor price adjustments. While these extensions would refine the quantitative predictions, the central qualitative insight, however, is robust: once operating decisions exhibit threshold-based incentives, untargeted subsidies can strengthen rather than weaken selection.

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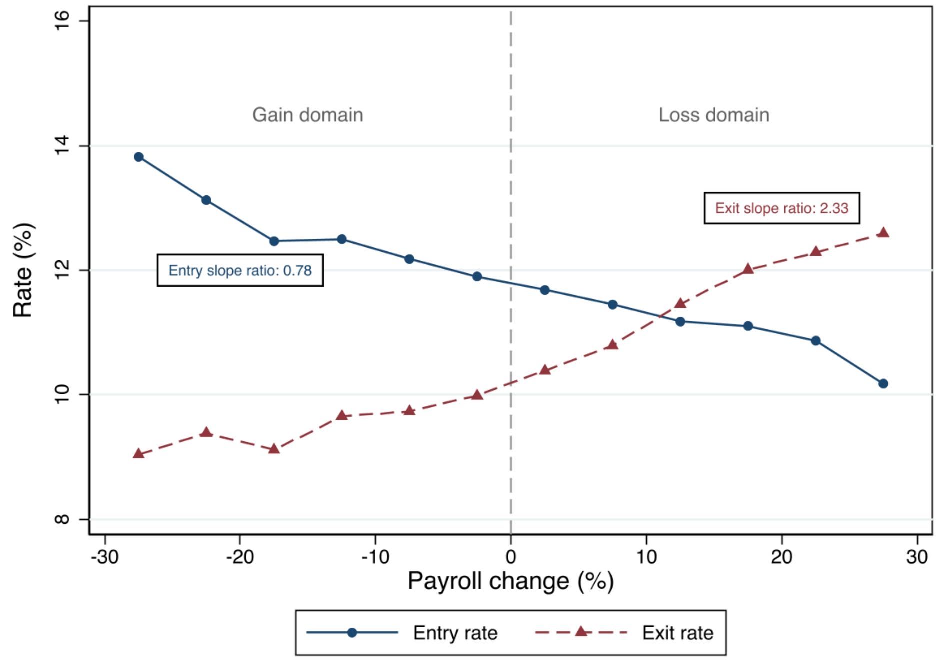
Figures and Tables

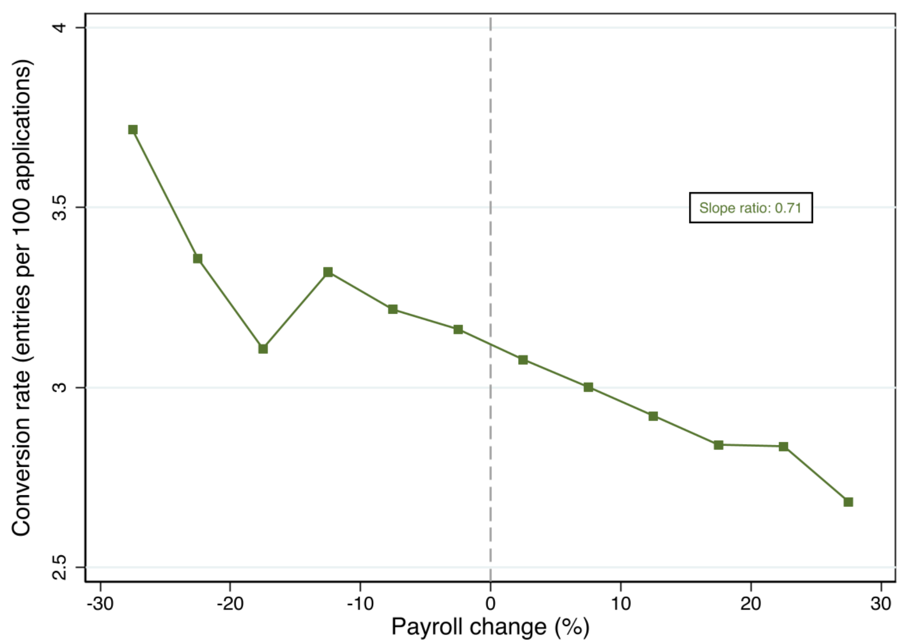
Table 2: **Summary Statistics for U.S. Restaurant Industry (1998–2019)**

	Mean	SD	Min	Max	N
Establishments (total number)	163.78	562.34	1.00	20,840.00	68279
GDP (thousands of 2021 dollars)	4,963,404.88	21,505,449.61	4,418.00	785361615.00	59131
Business applications	880.25	3,635.64	0.00	128,114.00	47138
Employees (total number)	2,922.73	10,190.92	0.00	372,966.00	68279
Housing price index (base year = 100)	251.12	161.69	54.56	2,079.17	59185
Population density (per square mile)	257.46	1,733.39	0.04	71,797.62	69548
Annual payroll per establishment (US$ 1000s)	154.69	94.02	0.00	754.80	68279
Annual payroll per establishment (change)	6.01	26.88	-681.05	754.80	68169
Above average payroll (frequency)	0.69	0.46	0.00	1.00	71541
Below average payroll (frequency)	0.31	0.46	0.00	1.00	71541
Above average payroll	11.37	19.01	0.00	754.80	68169
Below average payroll	-5.36	15.47	-681.05	0.00	68169
Entry rate (%)	4.80	11.58	0.00	325.00	68169
Exit rate (%)	5.88	12.71	0.00	87.50	68169

Notes: Statistics based on County Business Patterns and Business Dynamics Statistics data from the U.S. Census Bureau. Unit of observation is the county-year. Above average payroll refers to the change in payroll per establishment when the change is positive (payroll increased, i.e. loss domain). Below average payroll refers to the change when payroll decreased (gain domain). Entry and exit rates are from the Business Dynamics Statistics at the county-sector-year level. Sample restricted to counties with at least 10 restaurant establishments.



(a) Entry and Exit Rates



(b) Conversion Rate

Figure 1: Entry and Exit Rates by Payroll Changes

Notes: Panel (a) shows entry and exit rates from the Business Dynamics Statistics, plotted against changes in payroll per establishment. Entry rates are plotted against lagged payroll changes (year $t-1$), exit rates against contemporaneous changes (year t). Panel (b) shows the conversion rate (entries per 100 business applications), capturing the intensive margin of business formation. All variables are residualized by county and year fixed effects; sample means are added back. Slope ratios (right slope divided by left slope) quantify asymmetry around the reference point: values different from 1.0 indicate steeper responses in the loss domain (right of zero) versus the gain domain (left of zero). Exit rates show substantial asymmetry, while entry and conversion rates are relatively flat across domains. Sample includes county-years 1998-2019 with payroll changes within ± 30 percent. The vertical dashed line shows the reference point (zero change in payroll).

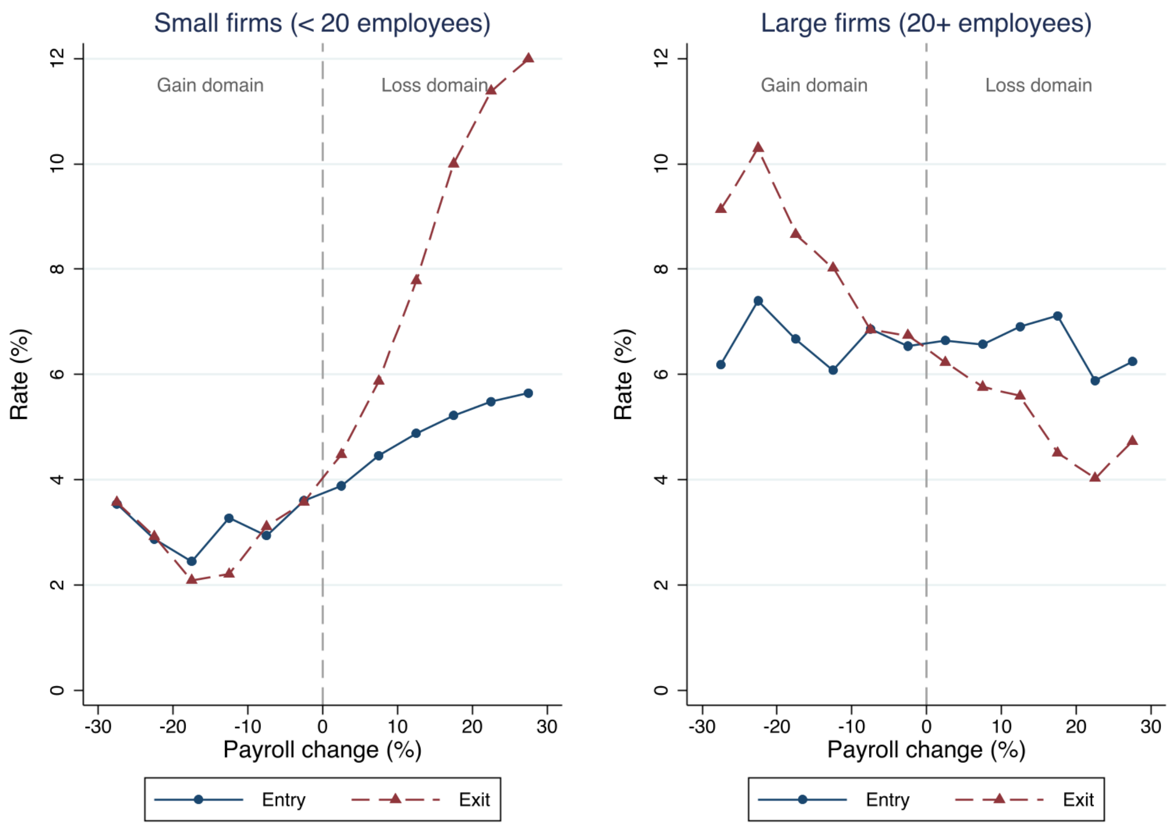


Figure 2: Net Entry and Exit Rates by Payroll Changes and Establishment Size

Notes: Net entry and exit rates by establishment size, plotted against changes in payroll per establishment. Because the Business Dynamics Statistics does not report entry and exit separately by firm size, we compute net rates from County Business Patterns: positive establishment growth is classified as net entry, negative growth as net exit. Small establishments are defined as those with fewer than 20 employees; large establishments have ≥ 20 employees. Entry rates are plotted against lagged payroll changes (year $t-1$), exit rates against contemporaneous changes (year t). All variables are residualized by county and year fixed effects; sample means are added back for interpretable levels. For small establishments, exit rates rise steeply in the loss domain while entry rates remain relatively flat—consistent with loss-averse exit decisions. For large establishments, the pattern reverses, likely reflecting that payroll growth proxies for expansion rather than cost pressure. Sample includes county-years from 1998–2019 with payroll changes within ±30 percent. The vertical dashed line shows the reference point (zero change in payroll).

Table 3: Summary of Key Empirical Findings (Small Establishments)

	2020	2021
Domain Composition		
Share in loss domain (%)	29	85
Share in gain domain (%)	71	15
Subsidy Characteristics		
Median subsidy rate (%)	13.0	9.1
Mean subsidy per establishment ($)	32,890	44,854
Domain Switching (among loss-domain)		
Switching rate (%)	56.1	39.6
Exit Rates (loss-domain firms)		
Non-switchers (%)	12.27	2.46
Switchers (%)	5.92	0.34
Difference (pp)	-6.35^{***}	-2.12^{***}

Notes: Small establishments defined as those with fewer than 20 employees. Loss domain = payroll increased relative to prior year; gain domain = payroll decreased. Switching = firm would move from loss to gain domain after accounting for subsidies. In 2020, most firms were in the gain domain (payroll fell during COVID), while in 2021 most were in the loss domain (payroll recovering). $^{***}p < 0.01$. Sample: 2,038 counties with valid data in 2020; 2,053 in 2021. 595 loss-domain counties in 2020 (261 non-switchers, 334 switchers). 1,746 loss-domain counties in 2021 (1,055 non-switchers, 691 switchers). Source: County Business Patterns, SBA.

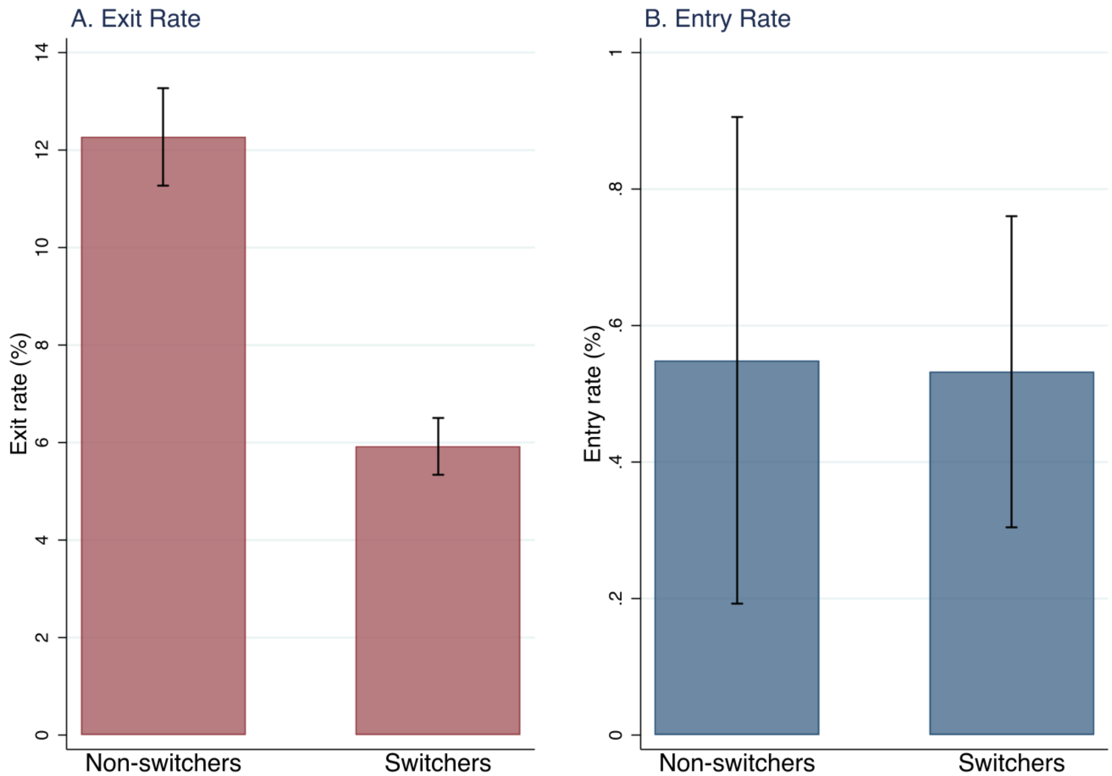


Figure 3: Exit and Entry Rates by Domain Switching Status (2020)

Notes: Average exit rates (Panel A) and entry rates (Panel B) for small establishments (fewer than 20 employees) in loss-domain counties in 2020, by switching status. Switchers ($N = 334$ counties) are firms that would move from the loss to gain domain after accounting for subsidies. Non-switchers ($N = 261$ counties) remain in the loss domain despite subsidies. The exit rate difference is 6.35 percentage points ($p < 0.0001$); the entry rate difference is 0.30 percentage points (not statistically significant). Error bars show 95% confidence intervals. Source: County Business Patterns, SBA.

Table 4: Pre-COVID Selection Test: Exit Rates by Future Switcher Status

Group	N (obs)	Exit Rate (%)	SE
Future non-switchers	824	3.12	0.186
Future switchers	1,202	2.80	0.139
Difference	—	0.32	0.232
p-value		0.170	

Notes: Pre-COVID (2018–2019) exit rates by future COVID-period switcher status. "Future switchers" are counties that switched from loss to gain domain at least once in 2020–2021. Sample: U.S. counties with non-missing exit rates in 2018–2019.

Table 5: Calibrated Parameters

Parameter	Symbol	Value	Source
Panel A: Externally Calibrated			
Elasticity of substitution	σ	1.5	Implied by restaurant markups
Pareto shape (productivity)	k	8	Farrokhi and Soderbery (2024), adjusted
Pareto shape (fixed costs)	κ	0.3	CBP establishment size distribution
Minimum productivity	$\underline{\phi}$	1	Normalization
Panel B: Internally Calibrated			
Reference dependence strength	η	0.20	Matched to exit rate gap
Gain weight	ω_G	1	Normalization
Loss weight	ω_L	10.00	Matched to exit rate gap
Exit scale	α	0.081	Matched to exit rate levels
Exit elasticity	β	0.96	Matched to exit rate levels

Notes: Panel A reports parameters set based on existing literature and industry characteristics. Panel B reports parameters calibrated to match exit rates by switching status in 2020. The parameter η controls the weight on gain-loss utility; $\omega_L > \omega_G$ implies loss aversion. Exit rates follow exit $= \alpha \cdot \tilde{\Phi}^{\beta}$, where $\tilde{\Phi}$ is the normalized effective cost. The calibration targets switcher exit rate (6.47%) and non-switcher exit rate (12.98%).

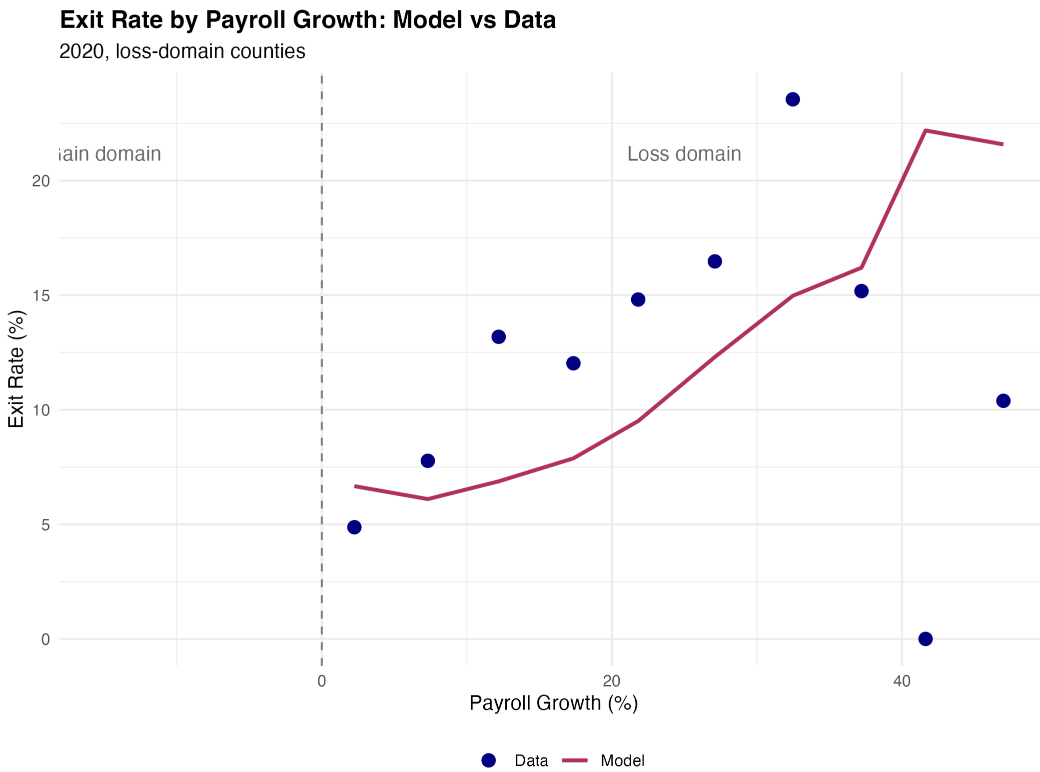


Figure 4: Exit Rate by Payroll Growth: Model vs. Data

Notes: Model-predicted exit rates (line) compared with observed exit rates (points) across bins of payroll growth for small establishments (fewer than 20 employees). The model incorporates reference-dependent preferences with calibrated parameters $\eta = 0.20$, $\omega_G = 1$, $\omega_L = 10$. The vertical dashed line separates the gain domain (left) from the loss domain (right). Sample: 532 loss-domain U.S. counties in 2020.

Table 6: Reference Point Robustness: Exit Rate Gap

Reference Point	N (Loss)	Switchers (%)	Exit: Non-sw.	Exit: Sw.	Gap (pp)
Baseline: F_{t-1}	525	81.5	13.44	6.47	6.97***
Smoothed: $(F_{t-1} + F_{t-2})/2$	614	81.6	11.95	5.88	6.07***
Trend-adjusted: $F_{t-1}(1 + \bar{g})$	459	69.7	11.28	6.68	4.60***

Notes: Exit rate comparison between non-switchers and switchers under alternative reference point specifications. All gaps significant at $p < 0.001$. Sample: Loss-domain U.S. counties in 2020 with positive subsidy rates ($N = 525$); seven counties with zero subsidy rates are excluded. *** $p < 0.01$.

Table 7: Counterfactual Analysis: Effect of Subsidies on Exit Rates

Scenario	Exit Rate (%)	Change from Actual (pp)
No subsidy	10.12	+2.44
Actual (2020)	7.75	—

Notes: Model-predicted exit rates for loss-domain firms in 2020. "No subsidy" simulates exit rates if COVID subsidies had not been provided. Calibrated parameters: $\eta = 0.20$, $\omega_G = 1$, $\omega_L = 10$. Sample: 532 loss-domain counties.

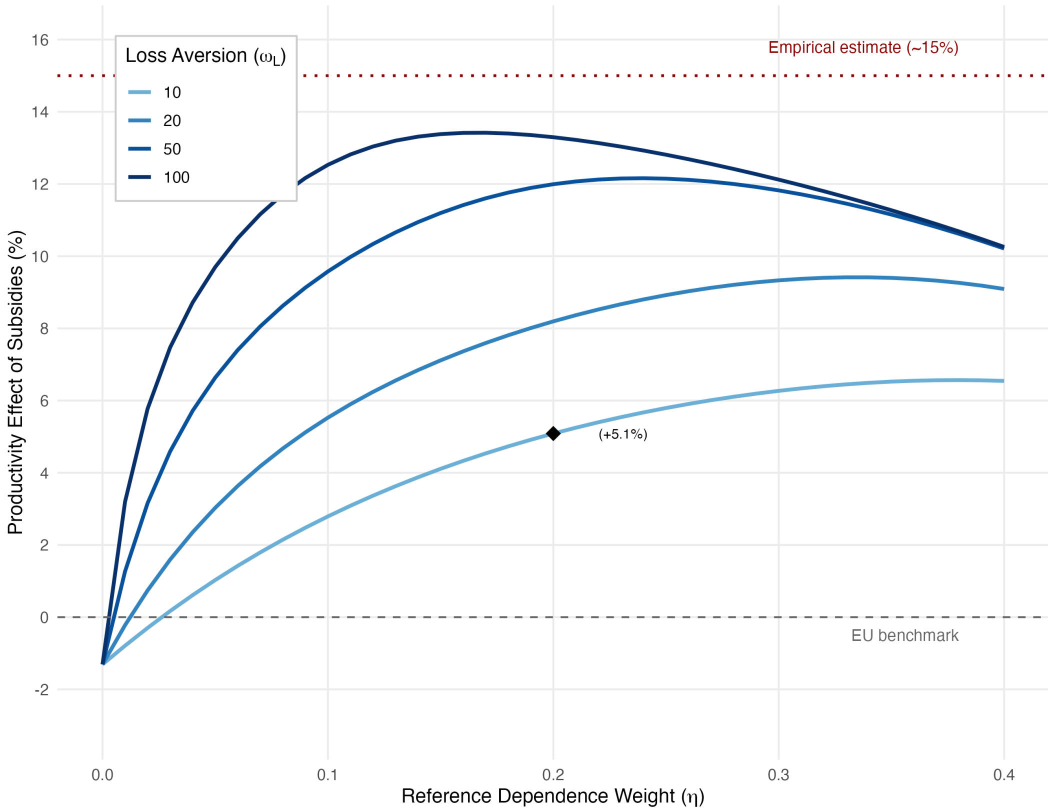


Figure 5: Productivity Effect of Subsidies

Notes: Productivity effect of a 10% subsidy as a function of the reference dependence parameter η, for different levels of loss aversion. The dashed line at zero marks the expected utility benchmark: at $\eta = 0$, subsidies reduce productivity by 1.3% through adverse selection. As η increases, the domain-switching channel dominates and the effect becomes positive. The dotted red line shows the empirical estimate of approximately 15% from Goolsbee et al. (2025). The black diamond marks the calibrated parameters ($\eta = 0.20$, $\omega_G = 1$, $\omega_L = 10$), where the model predicts a 5% productivity increase—a reversal of the standard prediction. With higher loss aversion, the model can generate larger effects approaching the empirical estimate. Computed using $\sigma = 1.5$, $k = 8$.

Table 8: COVID-19 Support by Industry Sector: Netherlands

Sector	NOW (%)	TVL (%)	Tax Deferral (%)
Hospitality (Horeca)	**59.9**	**85.1**	**44.5**
Other services	52.3	62.6	35.2
Transport and storage	37.7	42.7	37.9
Culture, sport and recreation	37.1	54.3	25.7
Trade	28.9	36.9	31.6
Manufacturing	26.9	22.7	28.0
Business services	22.7	18.3	22.6
Information and communication	21.9	21.0	22.9
Agriculture, forestry and fishing	19.9	17.2	27.7
Construction	17.8	14.9	27.7
Education	14.6	12.5	12.9
Real estate	14.3	17.9	22.3
Health and social work	12.3	7.0	12.9
Financial services	8.2	9.9	13.4
Mean (excl. Horeca)	22.9	24.4	23.4

Notes: Percentage of firms with 2 or more employees receiving each type of COVID-19 support through June 30, 2022. NOW = Tijdelijke Noodmaatregel Overbrugging voor Werkgelegenheid (wage subsidy, analogous to U.S. PPP). TVL = Tegemoetkoming Vaste Lasten (fixed cost subsidy, analogous to U.S. RRF). Hospitality (Horeca) includes restaurants, cafes, and hotels (SBI section I). Source: CBS "Twee jaar coronasteun" (2022).

Figure 6: Netherlands Hospitality Sector: Revenue, Subsidies, and Exit During COVID

Notes: Panel (a) shows revenue per establishment for the Dutch hospitality sector (Horeca), indexed to $2019 = 100$. The green line shows actual revenue; the blue line shows revenue plus estimated NOW and TVL subsidies. NOW subsidies are estimated as wage costs $\times$ coverage rate (59.9%) $\times$ replacement rate (85%). TVL subsidies are estimated as fixed costs (25% of revenue) $\times$ revenue loss $\times$ coverage rate (85.1%) $\times$ replacement rate (70%). Panel (b) shows the exit rate (business deaths / establishment stock). The shaded region indicates the COVID period with active NOW/TVL subsidies (2020–2021). Despite subsidies only partially offsetting revenue losses (effective revenue of 75–80 vs. baseline of 100), exit rates fell to 5.1% in 2021—the lowest in the sample—before rebounding to 7.2% when subsidies ended in 2022. Source: CBS StatLine.

Can untargeted subsidies select the best firms?

Online Appendix

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April 8, 2026

A.1 Equilibrium derivation and proofs

This appendix provides the full derivation of equilibrium conditions and formal proofs of all propositions. We begin with notation and firm-level decisions, continue with the general equilibrium characterization, and then analyze the effects of subsidies.

A.1.1 Notation and setup

We define the following notation used in the following:

$$\theta \equiv \frac{\sigma - 1}{k} \in (0, 1), \tag{33}$$

$$\alpha \equiv 1 - \frac{1}{\theta} = \frac{\sigma - 1 - k}{\sigma - 1} \in (-\infty, 0), \tag{34}$$

$$\beta \equiv \alpha - 1 = -\frac{k}{\sigma - 1} < -1, \tag{35}$$

$$B \equiv RP^{\sigma-1} \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma-1}, \tag{36}$$

where R is aggregate expenditure and P is the CES price index. Note that $\alpha < 0$ and $\beta < -1$ under assumption 1 ($k > \sigma - 1$).

For analyzing subsidies, we also define:

$$a(\Delta) \equiv \frac{F^r}{\Delta}, \tag{37}$$

$$c_G \equiv 1 - \eta + \eta\omega_G, \tag{38}$$

$$c_L \equiv 1 - \eta + \eta\omega_L. \tag{39}$$

With a proportional subsidy $\Delta \in (0, 1]$, a firm with pre–subsidy fixed cost F faces post–subsidy fixed cost ΔF. The firm is in the gain domain if $\Delta F \leq F^r$, i.e. if $F \leq a(\Delta)$.

A.1.2 Firm–level decisions

A.1.2.1 Firm optimization

A firm with productivity ϕ and fixed cost F chooses output q to maximize profits:

$$\max_q \left\{ p(\phi)q - \frac{q}{\phi} - F \right\} \tag{40}$$

subject to the demand function $q = RP^{\sigma-1}p^{-\sigma}$.

The first–order condition leads to the standard markup pricing:

$$p(\phi) = \frac{\sigma}{\sigma - 1} \cdot \frac{1}{\phi}. \tag{41}$$

Substituting back, revenues and profits are:

$$r(\phi) = B\phi^{\sigma-1}, \tag{42}$$

$$\pi(\phi, F) = \frac{r(\phi)}{\sigma} - F = \frac{B}{\sigma}\phi^{\sigma-1} - F. \tag{43}$$

A.1.2.2 Entry decision with reference dependent preferences

The owner's utility is:

$$V(\phi, F) = (1-\eta)\pi(\phi, F) - \eta \cdot v(F|F^r). \tag{44}$$

The firm operates if $V(\phi, F) \geq 0$, which requires:

$$(1-\eta)\left(\frac{B}{\sigma}\phi^{\sigma-1} - F\right) - \eta \cdot v(F|F^r) \geq 0. \tag{45}$$

Rearranging for the productivity cutoff:

$$\phi^*(F) = \left[\frac{\sigma}{B} \cdot \Phi(F)\right]^{\frac{1}{\sigma-1}}, \tag{46}$$

where the effective fixed cost $\Phi(F)$ depends on the domain (gain vs. loss domain), and it is derived in the following subsection.

A.1.2.3 Effective fixed cost derivation

Case 1: gain domain ($F \leq F^r$).

$$\begin{aligned} V(\phi, F) &= (1-\eta)\frac{B}{\sigma}\phi^{\sigma-1} - (1-\eta)F - \eta\omega_G(F - F^r) & (47)\\ &= (1-\eta)\frac{B}{\sigma}\phi^{\sigma-1} - Fc_G + \eta\omega_G F^r. & (48) \end{aligned}$$

Setting $V = 0$ and solving:

$$\frac{B}{\sigma}(\phi^*)^{\sigma-1} = \frac{Fc_G - \eta\omega_G F^r}{1-\eta} \equiv \Phi(F). \tag{49}$$

Case 2: loss domain ($F > F^r$).

$$\begin{aligned} V(\phi, F) &= (1-\eta)\frac{B}{\sigma}\phi^{\sigma-1} - (1-\eta)F - \eta\omega_L(F - F^r) & (50)\\ &= (1-\eta)\frac{B}{\sigma}\phi^{\sigma-1} - Fc_L + \eta\omega_L F^r. & (51) \end{aligned}$$

Setting $V = 0$ and solving:

$$\frac{B}{\sigma}(\phi^*)^{\sigma-1} = \frac{Fc_L - \eta\omega_L F^r}{1-\eta} \equiv \Phi(F). \tag{52}$$

Dis–/continuity at the reference point:

$$\Phi(F^r) = \frac{Fc_G - \eta\omega_G F^r}{1-\eta} = F^r, \tag{53}$$

$$\Phi(F^r) = \frac{Fc_L - \eta\omega_L F^r}{1-\eta} = F^r. \tag{54}$$

Thus, the effective fixed costs function is continuous at F^r, but its derivative is not:

$$\frac{\partial\Phi}{\partial F} = \frac{c_G}{1-\eta}, \tag{55}$$

$$\frac{\partial\Phi}{\partial F} = \frac{c_L}{1-\eta}. \tag{56}$$

Since $\omega_L > \omega_G$, we have $\partial\Phi_L/\partial F > \partial\Phi_G/\partial F$: effective fixed costs rise faster in the loss domain.

A.1.3 Equilibrium characterization

A.1.3.1 Average productivity conditional on F

This subsection derives the average productivity among active firms conditional on fixed cost F, properly accounting for general equilibrium. The key challenge is that the productivity cutoff $\phi^*(F)$ depends on B, which depends on the price index P, which itself depends on the productivity cutoffs $\phi^*_G(F)$ and $\phi^*_L(F)$ in the gain and loss domain, respectively.

Step 1: Zero cutoff profit condition.

The productivity cutoff $\phi^*(F)$ satisfies:

$$(1-\eta)\left(\frac{r(\phi^*)}{\sigma} - F\right) - \eta v(F|F^r) = 0. \tag{57}$$

Under Pareto-distributed productivity with shape parameter k, the relationship between the cutoff $\phi^*(F)$ and the average productivity $\widetilde{\phi}(F)$ among active firms is:

$$\widetilde{\phi}(F) = \left[\int_{\phi^*}^{\infty} \phi^{\sigma-1}\frac{g(\phi)}{1-G(\phi^*)}d\phi\right]^{\frac{1}{\sigma-1}} \Rightarrow \frac{\phi^*}{\widetilde{\phi}} = \left(\frac{k-\sigma+1}{k}\right)^{\frac{1}{\sigma-1}}. \tag{58}$$

Using $r(\phi^*) = r(\tilde{\phi}) \left(\frac{\phi^*}{\tilde{\phi}}\right)^{\sigma-1}$, we rewrite the zero cutoff profit condition as:

$$r(\widetilde{\phi}(F))\frac{k-\sigma+1}{\sigma k} = \Phi(F), \tag{59}$$

where $\Phi(F)$ is the effective fixed cost.

Step 2: Free entry condition.

In equilibrium, expected utility from entry equals the sunk market entry cost:

$$\int_{\underline{F}}^{F^r} [1 - G(\phi_G^*(F))]V(\widetilde{\phi}_G(F))h(F)dF + \int_{F^r}^{\bar{F}} [1 - G(\phi_L^*(F))]V(\widetilde{\phi}_L(F))h(F)dF = F_e. \tag{60}$$

Step 3: Price index.

$$P = \left(M_e\left[\int_{\underline{F}}^{F^r} [1 - G(\phi_G^*(F))]\, p(\widetilde{\phi}_G(F))^{1-\sigma}h(F)\, dF \right.\right.$$
$$\left.\left. + \int_{F^r}^{\infty} [1 - G(\phi_L^*(F))]\, p(\widetilde{\phi}_L(F))^{1-\sigma}h(F)\, dF\right]\right)^{\frac{1}{1-\sigma}}. \tag{61}$$

with M_e denoting the mass of entrants.

Step 4: Mass of Entrants.

Combining the free entry condition with the zero cutoff profit condition:

$$\int_{\underline{F}}^{F^r} [1 - G(\phi_G^*(F))]r(\widetilde{\phi}_G(F))h(F)dF + \int_{F^r}^{\infty} [1 - G(\phi_L^*(F))]r(\widetilde{\phi}_L(F))h(F)dF = \frac{\sigma k}{\sigma-1}\frac{F_e}{1-\eta}. \tag{62}$$

Using $r(\widetilde{\phi}_x(F)) = RP^{\sigma-1}p(\widetilde{\phi}_x(F))^{1-\sigma}$ and the price index expression (61), this yields:

$$M_e = \frac{(\sigma-1)R}{\sigma k}\frac{1-\eta}{F_e}. \tag{63}$$

Step 5: Intermediate expression for $\widetilde{\phi}(F)$.

Substituting the expressions for revenues, the price index, and the mass of entrants into equation (59), and using the Pareto distribution for ϕ, we obtain:

$$\widetilde{\phi}_x(F) = \Theta\left(\int_{\underline{F}}^{F^r} h(F)[\widetilde{\phi}_G(F)]^{\sigma-k-1}dF + \int_{F^r}^{\infty} h(F)[\widetilde{\phi}_L(F)]^{\sigma-k-1}dF\right)^{\frac{1}{\sigma-1}} \Phi(F)^{\frac{1}{\sigma-1}}, \tag{64}$$

where $x \in \{G, L\}$ indexes the gain and loss domains, and

$$\Theta \equiv \left[\frac{(\sigma-1)(1-\eta)}{(k-\sigma+1)F_e} \underline{\phi}^k \left(\frac{k}{k-\sigma+1} \right)^{\frac{k}{\sigma-1}} \right]^{\frac{1}{\sigma-1}}. \tag{65}$$

Step 6: Solving the fixed-point problem.

Equation (64) is a fixed-point problem: $\widetilde{\phi}_x(F)$ appears on both sides. To solve it, we exploit that the bracketed integral term is the same for all values of F. This means:

$$\widetilde{\phi}_x(F) = \widetilde{\phi}_G(\underline{F}) \left[\frac{\Phi(F)}{\Phi(\underline{F})} \right]^{\frac{1}{\sigma-1}}. \tag{66}$$

Define $b \equiv \frac{\sigma-1-k}{\sigma-1} < 0$. Substituting the proportionality relationship into the integral in equation (64) evaluated at $F = \underline{F}$:

$$\widetilde{\phi}_G(\underline{F}) = \Theta[\widetilde{\phi}_G(\underline{F})]^b \left(\int_{\underline{F}}^{F^r} \left[\frac{\Phi(F)}{\Phi(\underline{F})} \right]^b h(F)dF + \int_{F^r}^{\infty} \left[\frac{\Phi(F)}{\Phi(\underline{F})} \right]^b h(F)dF \right)^{\frac{1}{\sigma-1}} \times \Phi(\underline{F})^{\frac{1}{\sigma-1}}. \tag{67}$$

Rearranging:

$$[\widetilde{\phi}_G(\underline{F})]^{1-b} = \Theta \left(\int_{\underline{F}}^{F^r} \Phi(F)^b h(F)dF + \int_{F^r}^{\infty} \Phi(F)^b h(F)dF \right)^{\frac{1}{\sigma-1}} \Phi(\underline{F})^{\frac{1}{\sigma-1} - b\frac{1}{\sigma-1}}. \tag{68}$$

Since $1 - b = \frac{k}{\sigma-1}$:

$$\widetilde{\phi}_G(\underline{F}) = \Theta^{\frac{\sigma-1}{k}} \Phi(\underline{F})^{\frac{1}{\sigma-1}} \{E(\Phi)\}^{\frac{1}{k}}, \tag{69}$$

where we define:

$$E(\Phi) \equiv \int_{\underline{F}}^{F^r} \Phi(F)^b h(F)dF + \int_{F^r}^{\infty} \Phi(F)^b h(F)dF. \tag{70}$$

Step 7: General solution.

Using the proportionality relationship $\widetilde{\phi}_x(F) = \widetilde{\phi}_G(\underline{F}) \left[\frac{\Phi(F)}{\Phi(\underline{F})} \right]^{\frac{1}{\sigma-1}}$, we substitute the solution for $\widetilde{\phi}_G(\underline{F})$ to obtain the closed-form expression for average productivity conditional on *any* F:

$$\widetilde{\phi}_x(F) = \Theta^{\frac{\sigma-1}{k}} E(\Phi)^{\frac{1}{k}} \Phi(F)^{\frac{1}{\sigma-1}}, \quad x \in \{G, L\}. \tag{71}$$

This shows that $\widetilde{\phi}_x(F)$ depends on: (1) the effective fixed cost $\Phi(F)$ for firms with realized fixed cost F; (2) the term $E(\Phi)$, which captures general equilibrium effects through the price index; and (3) the constant Θ.

A.1.3.2 Sector-wide average productivity

The sector-wide average productivity aggregates $\widetilde{\phi}_x(F)$ over all active firms:

$$\widetilde{\widetilde{\phi}} = \left(\frac{\mathbb{E}(\widetilde{\phi}^{\sigma-1})}{\mu} \right)^{\frac{1}{\sigma-1}}, \tag{72}$$

where μ is the equilibrium share of active entrants (equation (14)) and:

$$\mathbb{E}(\widetilde{\phi}^{\sigma-1}) \equiv \int_{\underline{F}}^{F^r} [1 - G(\phi_G^*(F))] \, \widetilde{\phi}_G(F)^{\sigma-1} h(F) \, dF \\ + \int_{F^r}^{\infty} [1 - G(\phi_L^*(F))] \, \widetilde{\phi}_L(F)^{\sigma-1} h(F) \, dF. \tag{73}$$

Using the Pareto distribution and the closed-form solution for $\widetilde{\phi}_x(F)$ (equation (19)):

$$\widetilde{\widetilde{\phi}}^{\sigma-1} = \Theta^{\frac{(\sigma-1)^2}{k}} \cdot \frac{\left[\int_{\underline{F}}^{F^r} \Phi(F)^b \, h(F) \, dF + \int_{F^r}^{\infty} \Phi(F)^b \, h(F) \, dF \right]^{\frac{b-2}{b-1}}}{\int_{\underline{F}}^{F^r} \Phi(F)^{b-1} \, h(F) \, dF + \int_{F^r}^{\infty} \Phi(F)^{b-1} \, h(F) \, dF}. \tag{74}$$

The price index can equivalently be written as $P = [M \cdot p(\widetilde{\widetilde{\phi}})^{1-\sigma}]^{\frac{1}{1-\sigma}}$, confirming internal consistency.

A.1.3.3 Labor market clearing

Due to loss preferences, firm owners might realize monetary profits beyond their wage, or losses which they pay out of their wage income. As firm owners are part of L, they also receive wage w. These monetary profits or losses impact demand for goods and, thus, for labor.

To quantify the monetary profits or losses, we rewrite the free entry condition (equation 60):

$$\int_{\underline{F}}^{F^r} [1 - G(\phi_G^*(F))] \pi(\widetilde{\phi}_G(F)) h(F) dF + \int_{F^r}^{\infty} [1 - G(\phi_L^*(F))] \pi(\widetilde{\phi}_L(F)) h(F) dF - F_e \\ = \frac{\eta}{1-\eta} \Bigg(F_e + \omega_G \int_{\underline{F}}^{F^r} [1 - G(\phi_G^*(F))] h(F)(F - F^r) \, dF \\ + \omega_L \int_{F^r}^{\infty} [1 - G(\phi_L^*(F))] h(F)(F - F^r) \, dF \Bigg). \tag{75}$$

If $\eta = 0$, the average firm owner only receives wage w as part of L. If $\eta > 0$, the average firm owner realizes monetary profits or losses beyond w, denoted by χ, which equals the left-hand side of equation (75).

The sign of χ depends on whether firms expect to operate predominantly in the gain or loss domain. If the expected realization of fixed costs is substantially larger than the reference level F^r, or if the parameter of loss aversion ω_L is large, the positive term $\omega_L \int_{F^r}^{\infty} [1 - G(\phi_L^*(F))]$ $h(F)\,(F - F^r)\,dF$ dominates the right-hand side of equation (75), implying a lower mass of active firms in equilibrium and, thus, positive monetary profit $\chi > 0$ for the average active firm owner. Conversely, if the expected realization of fixed costs is substantially smaller than F^r, or if ω_G is large, the negative term $\omega_G \int_{\underline{F}}^{F^r} [1 - G(\phi_G^*(F))]\,h(F)\,(F - F^r)\,dF$ dominates, implying a higher mass of active firms and, thus, losses $\chi < 0$ for the average active firm owner. In both cases, χ represents additional demand (if positive) or reduced demand (if negative) for the average variety equal to $\frac{\chi}{\tilde{\tilde{\phi}}}$. The markup $\frac{\sigma}{\sigma-1}$ is left out from this demand term, as firm owners not only pay the markup, but also collect it. The corresponding change in labor demand equals $\tilde{\tilde{\phi}} \frac{\chi}{\tilde{\tilde{\phi}}}$ (which reduces to χ), and is balanced by the adjustment in the mass of active firms, maintaining the labor market clearing condition.

Thus, the labor market clearing condition is:

$$L = M_e \Bigg[F_e + \int_{\underline{F}}^{F^r} h(F)[1 - G(\phi_G^*(F))] \left(F + \frac{q(\widetilde{\phi}_G(F))}{\widetilde{\phi}_G(F)} \right) dF + \int_{F^r}^{\infty} h(F)[1 - G(\phi_L^*(F))] \left(F + \frac{q(\widetilde{\phi}_L(F))}{\widetilde{\phi}_L(F)} \right) dF + \chi \Bigg], \quad (76)$$

with χ being equal to the left hand side of equation (75).

Considering (i) $\frac{q(\phi)}{\phi} = r(\phi)\frac{\sigma-1}{\sigma}$, (ii) $r(\phi) = \frac{Rp(\phi)^{1-\sigma}}{P^{1-\sigma}}$ and (iii) the price index (equation 61), equation (76) can be rewritten:

$$L = M_e \left[F^e + \int_{\underline{F}}^{F^r} [1 - G(\phi_G^*(F))]\, F h(F) dF + \int_{F^r}^{\infty} [1 - G(\phi_L^*(F))]\, F h(F) dF + \frac{\sigma-1}{\sigma}\frac{R}{M_e} + \chi \right]. \quad (77)$$

Substituting $\frac{r(\widetilde{\phi}_x(F))}{\sigma}$ from the zero cutoff profit condition (equation 12) into the free entry condition (equation 15) and simplification leads to:

$$\int_{\underline{F}}^{F^r} h(F)\,[1 - G(\phi_G^*(F))]\, F dF + \int_{F^r}^{\infty} h(F)\,[1 - G(\phi_L^*(F))]\, F dF \;=\; F^e \frac{k - (\sigma-1)(1-\eta)}{(1-\eta)(\sigma-1)} - \chi. \quad (78)$$

Combining equations 77 and 78 leads to:

$$L \;=\; M_e \left[F^e + F^e \frac{k - (\sigma-1)(1-\eta)}{(\sigma-1)(1-\eta)} - \chi + \frac{\sigma-1}{\sigma}\frac{R}{M_e} + \chi \right]. \quad (79)$$

Finally, considering $M_e = \frac{(\sigma-1)R}{\sigma k}\frac{1-\eta}{F^e}$, equation (79) simplifies to: $1 = \frac{1}{\sigma k}\frac{\sigma k}{1}$.

A.1.4 Comparative Statics: Effect of Loss Weighting on Average Productivity

We study how the loss weighting parameter ω_L affects sector-wide average productivity. A symmetric analysis applies for ω_G, with inequalities reversed since higher ω_G reduces the gain-domain effective cost. From equation (74):

$$\tilde{\tilde{\phi}}^{\sigma-1} = \Theta^{\frac{(\sigma-1)^2}{k}} \frac{\left[\int_{\underline{F}}^{F^r} \Phi(F)^b h(F)dF + \int_{F^r}^{\infty} \Phi(F)^b h(F)dF\right]^{\frac{b-2}{b-1}}}{\int_{\underline{F}}^{F^r} \Phi(F)^{b-1} h(F)dF + \int_{F^r}^{\infty} \Phi(F)^{b-1} h(F)dF}. \tag{80}$$

We define

$$\alpha_G \equiv \frac{c_G}{1-\eta} = 1 + \frac{\eta\omega_G}{1-\eta}, \qquad \alpha_L \equiv \frac{c_L}{1-\eta} = 1 + \frac{\eta\omega_L}{1-\eta}, \tag{81}$$

so that $\partial\Phi_G/\partial F = \alpha_G$ and $\partial\Phi_L/\partial F = \alpha_L$, with $\alpha_L > \alpha_G > 0$ whenever $\omega_L > \omega_G$. Since $\sigma - 1 > 0$, the comparative statics of $\tilde{\tilde{\phi}}^{\sigma-1}$ with respect to ω_L are qualitatively identical to those of $\tilde{\tilde{\phi}}$. Linearizing $\Phi(F)^b$ and $\Phi(F)^{b-1}$ around a point F_0 via a first-order Taylor approximation yields:

$$\begin{aligned}
\Phi(F)^b\big|_{F<F^r} &\approx \left(F^0\alpha_G - (\alpha_G-1)F^r\right)^b + b\left(F^0\alpha_G - (\alpha_G-1)F^r\right)^{b-1}\alpha_G(F-F^0),\\
\Phi(F)^b\big|_{F\geq F^r} &\approx \left(F^0\alpha_L - (\alpha_L-1)F^r\right)^b + b\left(F^0\alpha_L - (\alpha_L-1)F^r\right)^{b-1}\alpha_L(F-F^0),\\
\Phi(F)^{b-1}\big|_{F<F^r} &\approx \left(F^0\alpha_G - (\alpha_G-1)F^r\right)^{b-1} + (b-1)\left(F^0\alpha_G - (\alpha_G-1)F^r\right)^{b-2}\alpha_G(F-F^0),\\
\Phi(F)^{b-1}\big|_{F\geq F^r} &\approx \left(F^0\alpha_L - (\alpha_L-1)F^r\right)^{b-1} + (b-1)\left(F^0\alpha_L - (\alpha_L-1)F^r\right)^{b-2}\alpha_L(F-F^0).
\end{aligned}$$

$\tilde{\tilde{\phi}}$ increases (decreases) with ω_L if the elasticity of the numerator with respect to ω_L exceeds (falls short of) the elasticity of the denominator:

$$\frac{b-2}{b-1}\frac{\frac{\partial}{\partial\omega_L}\int_{F^r}^{\bar{F}} \Phi(F)^b h(F)dF}{\int_{\underline{F}}^{F^r} \Phi(F)^b h(F)dF + \int_{F^r}^{\bar{F}} \Phi(F)^b h(F)dF} > (<) \frac{\frac{\partial}{\partial\omega_L}\int_{F^r}^{\bar{F}} \Phi(F)^{b-1} h(F)dF}{\int_{\underline{F}}^{F^r} \Phi(F)^{b-1} h(F)dF + \int_{F^r}^{\bar{F}} \Phi(F)^{b-1} h(F)dF}. \tag{82}$$

$\bar{F}$ denotes the upper bound of the support of F. For distributions with unbounded support (Case 2 below and the standard case in our paper), $\bar{F} = \infty$.

Case 1: Uniform distribution.

Lemma 1. *If F follows a uniform distribution on $[0,1]$, the sign of inequality (82) is ambiguous: both "$>$" and "$<$" are possible.*

Proof. Assume (i) a uniform distribution for F on $[0, 1]$, so the densities $h(F)$ cancel, and (ii) $F^0 = F^r$. Solving the integrals in inequality (82) and taking the partial derivatives with respect to λ yields:

$$\frac{\frac{b-2}{b-1} b \left[\left(F^r\right)^2 \frac{1}{2} - F^r + \frac{1}{2} \right]}{\left(F^r\right)^2 b \frac{\alpha_L - \alpha_G}{2} + F^r \left(1 - b\alpha_L\right) + \frac{b\alpha_L}{2}} \gtrless \frac{(b-1) \left[\left(F^r\right)^2 \frac{1}{2} - F^r + \frac{1}{2} \right]}{\left(F^r\right)^2 (b-1) \frac{\alpha_L - \alpha_G}{2} + F^r \left(1 - (b-1)\alpha_L\right) + \frac{(b-1)\alpha_L}{2}}.$$

Further simplification leads to:

$$\left(F^r\right)^2 \frac{\alpha_L - \alpha_G}{2} - F^r \left[\alpha_L + \frac{1}{b(1-b)} \right] + \frac{\alpha_L}{2} \gtrless 0. \tag{83}$$

Boundary case $F^r = 0$: Inequality (83) becomes $\frac{\alpha_L}{2} > 0$, which always holds.
Boundary case $F^r = 1$: Substituting and using the definitions of α_G and b, inequality (83) becomes:

$$-\frac{1}{2(1-\eta)} - \frac{(\sigma-1)^2}{(\sigma-1-k)k} \gtrless 0.$$

The left-hand side is negative if $\eta \to 1$, or if $\eta \to 0$ and, e.g., $\sigma = 2$ and $k > 2$.

Since ">" holds for $F^r = 0$ while "<" holds for $F^r = 1$ under appropriate parameter values, both directions of inequality (82) are possible. □

Case 2: Pareto distribution.

Lemma 2. *If F follows a Pareto distribution on $[\underline{F}, \infty)$ with density $h(F) = \frac{\kappa \underline{F}^\kappa}{F^{\kappa+1}}$ and $\kappa < 1$, then inequality (82) holds with ">". Consequently, $\tilde{\tilde{\phi}}$ unambiguously increases with ω_L.*

Proof. Inequality (82) leads to an ∞/∞ indeterminate form on both sides, since the terms involving $F^{1-\kappa}$ diverge as $\overline{F} \to \infty$. This becomes evident when evaluating the integrals:

$$\int_{F^r}^{\infty} \Phi(F)^b h(F)\, dF = \kappa \underline{F}^\kappa \frac{\left[\frac{F^0 \alpha_L (1-b) - (\alpha_L - 1) F^r}{-\kappa} F^{-\kappa} + \frac{b\alpha_L}{1-\kappa} F^{1-\kappa} \right]_{F^r}^{\infty}}{\left[F^0 \alpha_L - (\alpha_L - 1) F^r\right]^{1-b}}, \tag{84}$$

$$\int_{F^r}^{\infty} \Phi(F)^{b-1} h(F)\, dF = \kappa \underline{F}^\kappa \frac{\left[\frac{F^0 \alpha_L (2-b) - (\alpha_L - 1) F^r}{-\kappa} F^{-\kappa} + \frac{(b-1)\alpha_L}{1-\kappa} F^{1-\kappa} \right]_{F^r}^{\infty}}{\left[F^0 \alpha_L - (\alpha_L - 1) F^r\right]^{2-b}}. \tag{85}$$

Applying L'Hôpital's rule with respect to $\overline{F}^{1-\kappa}$ (notice: $\overline{F} \to \infty$) to both sides of inequality (82) and simplifying yields:

$$\frac{b-2}{b-1} \frac{\partial \alpha_L}{\partial \omega_L} b \frac{(F^0 - F^r) b\alpha_L + F^r}{b\alpha_L \left(F^0 \alpha_L - (\alpha_L - 1) F^r\right)} > \frac{\partial \alpha_L}{\partial \omega_L} (b-1) \frac{(F^0 - F^r)(b-1)\alpha_L + F^r}{(b-1)\alpha_L \left(F^0 \alpha_L - (\alpha_L - 1) F^r\right)}.$$

Dividing both sides by $\frac{\partial \alpha_L}{\partial \omega_L} > 0$ and multiplying by $\alpha_L > 0$:

$$\frac{b-2}{b-1} \cdot \frac{b\alpha_L(F^0 - F^r) + F^r}{F^0\alpha_L - (\alpha_L - 1)F^r} \;>\; \frac{(b-1)\alpha_\lambda(F^0 - F^r) + F^r}{F^0\alpha_\lambda - (\alpha_\lambda - 1)F^r}.$$

Multiplying both sides by $(b-1) < 0$ reverses the inequality:

$$(b-2) \cdot \frac{b\alpha_L(F^0 - F^r) + F^r}{F^0\alpha_L - (\alpha_L - 1)F^r} \;<\; (b-1) \cdot \frac{(b-1)\alpha_L(F^0 - F^r) + F^r}{F^0\alpha_L - (\alpha_L - 1)F^r}. \tag{86}$$

Let $D \equiv F^0\alpha_L - (\alpha_L - 1)F^r$ denote the common denominator.

Case A: $D > 0$, i.e. $F^0 > \frac{\eta\omega_L}{1-\eta+\eta\omega_L}F^r$. Multiplying (86) by $D > 0$ and simplifying yields the condition $F^0 > \frac{\eta\omega_L}{1-\eta+\eta\omega_L}F^r$, which is precisely the assumption of this case.

Case B: $D < 0$, i.e. $F^0 < \frac{\eta\omega_L}{1-\eta+\eta\omega_L}F^r$. Multiplying (86) by $D < 0$ reverses the inequality, and simplification yields the condition $F^0 < \frac{\eta\omega_L}{1-\eta+\eta\omega_L}F^r$, which is again the assumption of this case.

In both cases, inequality (86) holds regardless of the linearization point F^0. Therefore, the left-hand side of inequality (82) always exceeds the right-hand side, implying that $\approx\!\!\!\phi$ increases with ω_L under a Pareto distribution for F. □

A.1.5 Effects of Subsidies

This section derives the effects of proportional fixed–cost subsidies on aggregate productivity and the mass of active firms. With subsidy $\Delta \in (0, 1]$, firms face net fixed costs ΔF.

A.1.5.1 Aggregate Productivity Expression with Subsidies

We establish existence of the ambiguity by analyzing the uniform distribution as a tractable baseline case. The key qualitative result that $f(0) = 0 < \theta$ and $f(\bar{\eta}^-) > \theta$ extends to the Pareto distribution, as we discuss below. If F follows a uniform distribution with density $h(F) = \frac{1}{\bar{F}-\underline{F}}$ on $[\underline{F}, \bar{F}]$, sector-wide average productivity becomes:

$$\approx\!\!\!\phi^{\sigma-1} = \frac{\left[\int_{\underline{F}}^{a(\Delta)}\left(\frac{c_G\Delta F - \eta\omega_G F^r}{1-\eta}\right)^\alpha dF + \int_{a(\Delta)}^{\bar{F}}\left(\frac{c_L\Delta F - \eta\omega_L F^r}{1-\eta}\right)^\alpha dF\right]^{1+\theta}}{\int_{\underline{F}}^{a(\Delta)}\left(\frac{c_G\Delta F - \eta\omega_G F^r}{1-\eta}\right)^\beta dF + \int_{a(\Delta)}^{\bar{F}}\left(\frac{c_L\Delta F - \eta\omega_L F^r}{1-\eta}\right)^\beta dF}, \tag{87}$$

where $a(\Delta) = F^r/\Delta$ is the boundary between the gain and the loss domain after the subsidy.

Simplifying the integrands:

- **Gain domain** $(F \leq a(\Delta))$: $\frac{c_G\Delta F - \eta\omega_G F^r}{1-\eta} = \frac{\Delta}{1-\eta}(c_G F - \eta\omega_G a(\Delta))$
- **Loss domain** $(F > a(\Delta))$: $\frac{c_L\Delta F - \eta\omega_L F^r}{1-\eta} = \frac{\Delta}{1-\eta}(c_L F - \eta\omega_L a(\Delta))$

Factoring out $\left(\frac{\Delta}{1-\eta}\right)^{\alpha}$ from the numerator and $\left(\frac{\Delta}{1-\eta}\right)^{\beta}$ from the denominator:

$$\widetilde{\widetilde{\phi}}^{\sigma-1} = \left(\frac{\Delta}{1-\eta}\right)^{\alpha(1+\theta)-\beta} \frac{[N(\Delta)]^{1+\theta}}{D(\Delta)}, \tag{88}$$

where

$$N(\Delta) = \int_{\underline{F}}^{a(\Delta)} (c_G F - \eta\omega_G a(\Delta))^{\alpha}\, dF + \int_{a(\Delta)}^{\bar{F}} (c_L F - \eta\omega_L a(\Delta))^{\alpha}\, dF, \tag{89}$$

$$D(\Delta) = \int_{\underline{F}}^{a(\Delta)} (c_G F - \eta\omega_G a(\Delta))^{\beta}\, dF + \int_{a(\Delta)}^{\bar{F}} (c_L F - \eta\omega_L a(\Delta))^{\beta}\, dF. \tag{90}$$

Since $\alpha(1+\theta) - \beta = \theta$:

$$\widetilde{\widetilde{\phi}}^{\sigma-1} = \Delta^{\theta} \cdot \frac{[N(\Delta)]^{1+\theta}}{D(\Delta)} \equiv R(\Delta). \tag{91}$$

A.1.5.2 Closed-Form Integrals

The integrals in $N(\Delta)$ and $D(\Delta)$ can be evaluated in closed form. If we define:

$$A \equiv a(\Delta)(1-\eta) = \frac{F^r(1-\eta)}{\Delta}, \tag{92}$$

$$L \equiv c_G \underline{F} - \eta\omega_G a(\Delta), \tag{93}$$

$$U \equiv c_L \bar{F} - \eta\omega_L a(\Delta). \tag{94}$$

Note that both integrands evaluate to A at the boundary $F = a(\Delta)$, since $c_G a - \eta\omega_G a = a(1-\eta) = A$ and $c_L a - \eta\omega_L a = a(1-\eta) = A$, confirming continuity of the effective cost function at the reference point. Then:

$$\int_{\underline{F}}^{a(\Delta)} (c_G F - \eta\omega_G a(\Delta))^{\alpha}\, dF = \frac{A^{\alpha+1} - L^{\alpha+1}}{(\alpha+1)\, c_G}, \tag{95}$$

$$\int_{a(\Delta)}^{\bar{F}} (c_L F - \eta\omega_L a(\Delta))^{\alpha}\, dF = \frac{U^{\alpha+1} - A^{\alpha+1}}{(\alpha+1)\, c_L}, \tag{96}$$

and similarly for the β integrals. Therefore:

$$N(\Delta) = \frac{A^{\alpha+1} - L^{\alpha+1}}{(\alpha+1)c_G} + \frac{U^{\alpha+1} - A^{\alpha+1}}{(\alpha+1)c_L}, \tag{97}$$

$$D(\Delta) = \frac{A^{\beta+1} - L^{\beta+1}}{(\beta+1)c_G} + \frac{U^{\beta+1} - A^{\beta+1}}{(\beta+1)c_L}. \tag{98}$$

A.1.5.3 Derivatives with Respect to the Subsidy

Taking logarithms of (91):

$$\ln R(\Delta) = \theta \ln \Delta + (1+\theta)\ln N(\Delta) - \ln D(\Delta). \tag{99}$$

Differentiating with respect to Δ:

$$\frac{d}{d\Delta}\ln R(\Delta) = \frac{\theta}{\Delta} + (1+\theta)\frac{N'(\Delta)}{N(\Delta)} - \frac{D'(\Delta)}{D(\Delta)}. \tag{100}$$

Since N and D depend on Δ through $a(\Delta)$, we apply the chain rule. To compute the derivatives, we use Leibniz's rule to differentiate under the integral sign:

$$N'(\Delta) = \frac{\partial N}{\partial a}\cdot a'(\Delta) = -\frac{a(\Delta)}{\Delta}\cdot N_a, \tag{101}$$

$$D'(\Delta) = \frac{\partial D}{\partial a}\cdot a'(\Delta) = -\frac{a(\Delta)}{\Delta}\cdot D_a, \tag{102}$$

where $N_a \equiv \partial N/\partial a(\Delta)$ and $D_a \equiv \partial D/\partial a(\Delta)$.

Computing N_a by differentiating under the integral sign. The boundary terms at $F = a(\Delta)$ cancel because both integrands evaluate to A^α there: $(c_G F - \eta\omega_G a)^\alpha\big|_{F=a} = (c_L F - \eta\omega_L a)^\alpha\big|_{F=a} = ((1-\eta)a)^\alpha = A^\alpha$. Thus:

$$N_a = \int_{\underline{F}}^{a(\Delta)} \alpha(c_G F - \eta\omega_G a)^{\alpha-1}(-\eta\omega_G)\,dF + \int_{a(\Delta)}^{\bar{F}} \alpha(c_L F - \eta\omega_L a)^{\alpha-1}(-\eta\omega_L)\,dF \tag{103}$$

$$= -\eta\alpha\left[\omega_G \int_{\underline{F}}^{a(\Delta)} (c_G F - \eta\omega_G a)^{\alpha-1} dF + \omega_L \int_{a(\Delta)}^{\bar{F}} (c_L F - \eta\omega_L a)^{\alpha-1} dF\right]. \tag{104}$$

Evaluating:

$$N_a = -\eta\left[\frac{\omega_G}{c_G}(A^\alpha - L^\alpha) + \frac{\omega_L}{c_L}(U^\alpha - A^\alpha)\right]. \tag{105}$$

Similarly:

$$D_a = -\eta\left[\frac{\omega_G}{c_G}(A^\beta - L^\beta) + \frac{\omega_L}{c_L}(U^\beta - A^\beta)\right]. \tag{106}$$

Substituting into (100):

$$\frac{d}{d\Delta}\ln R(\Delta) = \frac{\theta}{\Delta} - \frac{a(\Delta)}{\Delta}\left[(1+\theta)\frac{N_a}{N} - \frac{D_a}{D}\right]. \tag{107}$$

A.1.5.4 Proof of Proposition 1 (expected utility benchmark)

Proposition 1. *Under expected utility ($\eta = 0$), a fixed-cost subsidy unambiguously reduces average productivity:* $\frac{\partial \tilde{\tilde{\phi}}}{\partial \Delta} > 0$.

Proof. Under expected utility ($\eta = 0$), we have $\Phi(F) = F$ for all F, and there is no distinction between gain and loss domains. With $\eta = 0$:

- $A = a(\Delta) = F^r/\Delta$
- $L = \underline{F}$
- $U = \bar{F}$
- $c_G = c_L = 1$

The expressions simplify to:

$$N(\Delta) = \frac{1}{\alpha+1}\int_{\underline{F}}^{\bar{F}} F^{\alpha} dF = \frac{\bar{F}^{\alpha+1} - \underline{F}^{\alpha+1}}{\alpha+1}, \tag{108}$$

$$D(\Delta) = \frac{1}{\beta+1}\int_{\underline{F}}^{\bar{F}} F^{\beta} dF = \frac{\bar{F}^{\beta+1} - \underline{F}^{\beta+1}}{\beta+1}. \tag{109}$$

Both N and D are independent of Δ when $\eta = 0$. Therefore $N_a = D_a = 0$, and from (107):

$$\frac{d}{d\Delta}\ln R(\Delta) = \frac{\theta}{\Delta} > 0. \tag{110}$$

Since $R(\Delta) = \tilde{\tilde{\phi}}^{\sigma-1}$ and $\tilde{\tilde{\phi}} = R^{1/(\sigma-1)}$:

$$\frac{\partial \tilde{\tilde{\phi}}}{\partial \Delta} = \frac{1}{\sigma-1} R^{\frac{2-\sigma}{\sigma-1}} \cdot \frac{dR}{d\Delta} = \frac{\tilde{\tilde{\phi}}}{\sigma-1} \cdot \frac{d\ln R}{d\Delta} = \frac{\tilde{\tilde{\phi}}}{\sigma-1} \cdot \frac{\theta}{\Delta} > 0. \tag{111}$$

Therefore, under expected utility, $\partial \tilde{\tilde{\phi}}/\partial\Delta > 0$. A subsidy (reduction in Δ) reduces average productivity. □

A.1.5.5 Proof of Proposition 2 (ambiguous effect)

Proposition 2. *Under reference-dependent preferences ($\eta > 0$, $\omega_L > \omega_G > 0$), the effect of a fixed-cost subsidy on average productivity is ambiguous:* $\partial \tilde{\tilde{\phi}}/\partial\Delta \lesseqgtr 0$.

Proof. Under reference-dependent preferences ($\eta > 0$, $\omega_L > \omega_G > 0$), from 107:

$$\frac{d}{d\Delta}\ln R(\Delta) = \frac{\theta}{\Delta} - \frac{a(\Delta)}{\Delta}\left[(1+\theta)\frac{N_a}{N} - \frac{D_a}{D}\right]. \tag{112}$$

The sign of this derivative depends on whether:

$$\theta \lesseqgtr a(\Delta)\left[(1+\theta)\frac{N_a}{N} - \frac{D_a}{D}\right]. \tag{113}$$

Step 1: Sign of N_a and D_a.

Since $\alpha < 0$ and $\beta < -1$, and since $\bar{F} > a(\Delta) > \underline{F}$ (notice that F^r is strictly between $\underline{F}$ and $\bar{F}$), we have:

- $A^\alpha - L^\alpha < 0$ (because $A > L$ and $\alpha < 0$),
- $U^\alpha - A^\alpha$ has ambiguous sign,
- $A^\beta - L^\beta < 0$ (because $A > L$ and $\beta < -1 < 0$),
- $U^\beta - A^\beta$ has ambiguous sign.

For $\eta > 0$, the terms N_a and D_a are generally non-zero and can be positive.

Step 2: Decomposition of effects.

The derivative in 112 can be decomposed into two effects:

1. *Direct effect* ($\theta/\Delta > 0$): Standard selection effect, which reduces productivity when Δ falls.
2. *Domain-switching effect* ($-(a(\Delta)/\Delta)\left[(1+\theta)\, N_a/N - D_a/D\right]$): Captures how changes in Δ affect the composition of firms across domains.

Step 3: Conditions for sign reversal.

The sign of $d\ln R/d\Delta$ is negative (subsidies increase productivity) if and only if:

$$\begin{aligned}\theta < a(\Delta)\cdot\eta\Bigg[&-(1+\theta)(\alpha+1)\,\frac{\frac{\omega_G}{c_G}(A^\alpha - L^\alpha) + \frac{\omega_L}{c_L}(U^\alpha - A^\alpha)}{\frac{A^{\alpha+1}-L^{\alpha+1}}{c_G} + \frac{U^{\alpha+1}-A^{\alpha+1}}{c_L}} \\ &+ (\beta+1)\,\frac{\frac{\omega_G}{c_G}(A^\beta - L^\beta) + \frac{\omega_L}{c_L}(U^\beta - A^\beta)}{\frac{A^{\beta+1}-L^{\beta+1}}{c_G} + \frac{U^{\beta+1}-A^{\beta+1}}{c_L}}\Bigg] \equiv f(\eta),\end{aligned} \tag{114}$$

where $a(\Delta) = F^r/\Delta$, $A = a(\Delta)(1-\eta)$, $L = c_G\underline{F} - \eta\omega_G a(\Delta)$, $U = c_L\bar{F} - \eta\omega_L a(\Delta)$, $c_G = 1-\eta+\eta\omega_G$, and $c_L = 1-\eta+\eta\omega_L$. The model is well-defined for $\eta \in [0,\bar{\eta})$ where

$$\bar{\eta} \equiv \frac{\underline{F}}{\underline{F} + \omega_G(a(\Delta) - \underline{F})}, \tag{115}$$

since this ensures $L = c_G\underline{F} - \eta\omega_G a(\Delta) > 0$.

We now establish the existence of $\eta^* \in (0, \bar{\eta})$ above which condition 114 holds and subsidies increase average productivity. The argument proceeds in two claims followed by an application of the intermediate value theorem.

Claim 1: $f(0) = 0 < \theta$.

At $\eta = 0$ the factor η multiplying the bracketed expression on the right-hand side of 114 equals zero, so $f(0) = 0 < \theta$. □

Claim 2: Under conditions *[C1]–[C3]* below, there exists an interior point $\eta_0 \in (0, \bar{\eta})$ at which $f(\eta_0) > \theta$.

[C1] (Non-degenerate gain domain). $\underline{F} < a(\Delta)$, so the gain domain $[\underline{F},\, a(\Delta)]$ has positive measure and $L = c_G \underline{F} - \eta_0\, \omega_G\, a(\Delta) > 0$ at η_0.

[C2] (Sufficient loss aversion). ω_L is large enough that the loss-domain terms $\omega_L(U^\gamma - A^\gamma)/c_L$ outweigh the gain-domain terms $\omega_G(A^\gamma - L^\gamma)/c_G$ in the expressions for N_a and D_a (equations 105–106), for $\gamma \in \{\alpha, \beta\}$.

[C3] (Gain domain not too wide). The ratio

$$\frac{L}{A} = \frac{c_G \underline{F} - \eta_0\, \omega_G\, a(\Delta)}{(1 - \eta_0)\, a(\Delta)}$$

is bounded away from zero at η_0. This prevents $L^\gamma \to +\infty$ from dominating the integrals in N and D and driving f to $-\infty$.

Proof of Claim 2. We verify *[C1]–[C3]* and condition 114 directly at the calibrated parameters $(\sigma, k, \kappa, \omega_L, \eta_0) = (1.5,\ 8,\ 0.3,\ 10,\ 0.20)$, setting $\underline{F} = 0.9\, a(\Delta)$ and $\bar{F} = 3\, a(\Delta)$ under the uniform distribution, and normalizing $a(\Delta) = 1$. These values imply:

$$\alpha = \frac{\sigma - 1 - k}{\sigma - 1} = -15, \quad \beta = -\frac{k}{\sigma - 1} = -16, \quad \theta = \frac{\sigma - 1}{k} = 0.0625,$$

$$c_G = 1 - \eta_0 + \eta_0 \omega_G = 1, \quad c_L = 1 - \eta_0 + \eta_0 \omega_L = 2.8,$$

$$A = (1 - \eta_0)\, a(\Delta) = 0.8, \quad L = c_G \underline{F} - \eta_0 \omega_G a(\Delta) = 0.7, \quad U = c_L \bar{F} - \eta_0 \omega_L a(\Delta) = 6.4.$$

[C1] holds since $\underline{F} = 0.9 < a(\Delta) = 1$, giving $L = 0.7 > 0$. *[C3]* holds since $L/A = 0.7/0.8 = 0.875$, which is bounded away from zero. The key quantities entering equations 105–106 and 97–98 are reported in Table A.1.1.

Table A.1.1: Numerical values at calibrated parameters $(\sigma, k, \omega_L, \eta_0) = (1.5, 8, 10, 0.20)$, with $L = 0.7$, $A = 0.8$, $U = 6.4$.

	L	A	U
$(\cdot)^\alpha$	$0.7^{-15} \approx 210.70$	$0.8^{-15} \approx 28.42$	$6.4^{-15} \approx 8.1 \times 10^{-13}$
$(\cdot)^{\alpha+1}$	$0.7^{-14} \approx 147.50$	$0.8^{-14} \approx 22.74$	$6.4^{-14} \approx 5.2 \times 10^{-12}$
$(\cdot)^\beta$	$0.7^{-16} \approx 301.00$	$0.8^{-16} \approx 35.53$	$6.4^{-16} \approx 1.3 \times 10^{-13}$
$(\cdot)^{\beta+1}$	$0.7^{-15} \approx 210.70$	$0.8^{-15} \approx 28.42$	$6.4^{-15} \approx 8.1 \times 10^{-13}$

Substituting into equations 105–106:

$$
\begin{aligned}
N_a &= -\eta\left[\frac{\omega_G}{c_G}(A^\alpha - L^\alpha) + \frac{\omega_L}{c_L}(U^\alpha - A^\alpha)\right] \\
&= -0.2\left[\frac{1}{1}(28.42 - 210.70) + \frac{10}{2.8}(8.1 \times 10^{-13} - 28.42)\right] \\
&= -0.2[-182.28 - 101.50] = 56.76,
\end{aligned}
$$

$$
\begin{aligned}
D_a &= -\eta\left[\frac{\omega_G}{c_G}(A^\beta - L^\beta) + \frac{\omega_L}{c_L}(U^\beta - A^\beta)\right] \\
&= -0.2\left[\frac{1}{1}(35.53 - 301.00) + \frac{10}{2.8}(1.3 \times 10^{-13} - 35.53)\right] \\
&= -0.2[-265.47 - 126.89] = 78.47.
\end{aligned}
$$

Substituting into equations 97–98:

$$
\begin{aligned}
N &= \frac{A^{\alpha+1} - L^{\alpha+1}}{(\alpha+1)\,c_G} + \frac{U^{\alpha+1} - A^{\alpha+1}}{(\alpha+1)\,c_L} \\
&= \frac{22.74 - 147.50}{(-14)(1)} + \frac{5.2 \times 10^{-12} - 22.74}{(-14)(2.8)} \\
&= 8.911 + 0.580 \;=\; 9.491,
\end{aligned}
$$

$$
\begin{aligned}
D &= \frac{A^{\beta+1} - L^{\beta+1}}{(\beta+1)\,c_G} + \frac{U^{\beta+1} - A^{\beta+1}}{(\beta+1)\,c_L} \\
&= \frac{28.42 - 210.70}{(-15)(1)} + \frac{8.1 \times 10^{-13} - 28.42}{(-15)(2.8)} \\
&= 12.152 + 0.677 \;=\; 12.829.
\end{aligned}
$$

Since the loss-domain term $(\omega_L/c_L)(U^\alpha - A^\alpha) \approx -101.50$ is large in magnitude relative to the gain-domain term $\omega_G(A^\alpha - L^\alpha)/c_G = -182.28$, condition *[C2]* holds. Direct computation then yields:

$$
(1+\theta)\,\frac{N_a}{N} - \frac{D_a}{D} \;=\; 1.0625 \times \frac{56.76}{9.491} - \frac{78.47}{12.829} \;=\; 6.354 - 6.117 \;=\; 0.237 \;>\; 0,
$$

thus,

$$
a(\Delta)\left[(1+\theta)\,\frac{N_a}{N} - \frac{D_a}{D}\right] \;=\; 0.237 \;>\; \theta \;=\; 0.0625,
$$

establishing $f(\eta_0) > \theta$. □

Conclusion. Since $f(0) = 0 < \theta$ by Claim 1 and $f(\eta_0) > \theta$ by Claim 2, and since f is continuous on $(0, \bar{\eta})$, the intermediate value theorem delivers $\eta^* \in (0, \eta_0)$ satisfying $f(\eta^*) = \theta$. For all $\eta \in (\eta^*, \bar{\eta})$, condition 114 holds and therefore $\partial \tilde{\varphi}^e / \partial \Delta < 0$: subsidies raise average productivity. For $\eta < \eta^*$, the standard adverse-selection effect dominates and $\partial \tilde{\tilde{\phi}} / \partial \Delta > 0$, reproducing the canonical result. This establishes $\partial \tilde{\tilde{\phi}} / \partial \Delta \lesseqgtr 0$ depending on parameters, as claimed. □

Remark 1. *The proof uses the uniform distribution for closed-form tractability. Under the Pareto distribution for F, closed-form expressions for N and D are unavailable, but the two claims driving the result, $f(0) = 0$ (from the prefactor η) and $f(\eta_0) > \theta$ under* [C1]–[C3] *(from the dominance of loss-domain terms away from the boundary), depend only on the behavior of the integrands at interior points, not on the tail of $h(F)$. The argument therefore carries over to the Pareto case.*

Remark 2. *Condition* [C3] *rules out a degenerate boundary case: as $L \to 0^+$, the gain-domain lower endpoint drives $f(\eta) \to -\infty$, and the intermediate value theorem cannot be applied at $\bar{\eta}$. Conditions* [C1]–[C3] *jointly ensure the crossing of θ occurs at an interior point where f is well-behaved.*

A.1.5.6 Proof of Proposition 3 (Reversal Threshold)

Proposition 3. *There exists a threshold $\eta^* \in (0, 1)$ such that $\partial \tilde{\tilde{\phi}} / \partial \Delta < 0$ for all $\eta > \eta^*$.*

Proof. Define

$$\Psi(\eta) \;\equiv\; \frac{d}{d\Delta} \ln R(\Delta) \;=\; \frac{\theta}{\Delta} - \frac{a(\Delta)}{\Delta} \left[(1+\theta) \frac{N_a(\eta)}{N(\eta)} - \frac{D_a(\eta)}{D(\eta)} \right],$$

where we make explicit the dependence of N, D, N_a, D_a on η.

Step 1: Existence of η^*.

The existence of $\eta^* \in (0, 1)$ follows directly from the proof of Proposition 2. Claims 1 and 2 of that proof establish $\Psi(0) = \theta/\Delta > 0$ and $\Psi(\eta_0) < 0$ at the calibrated interior point $\eta_0 = 0.20$. Since Ψ is continuous on $(0, \bar{\eta})$, the intermediate value theorem delivers $\eta^* \in (0, \eta_0)$ satisfying $\Psi(\eta^*) = 0$, i.e. $f(\eta^*) = \theta$. For all $\eta > \eta^*$, $\Psi(\eta) < 0$, i.e. $\partial \tilde{\tilde{\phi}} / \partial \Delta < 0$.

Step 2: Comparative statics of η^*.

(i) η^* *is decreasing in the strength of loss weighting ω_L.*

Higher ω_L increases the effective fixed cost in the loss domain:

$$\frac{\partial \Phi_L}{\partial \omega_L} \;=\; \frac{\eta (F - F^r)}{1 - \eta} \;>\; 0 \quad \text{for } F > F^r.$$

This amplifies the benefit of domain switching, making the right-hand side of condition 114 larger for any given η, so the reversal occurs at a lower η^*.

(ii) η^* *is increasing in the elasticity of substitution σ.*

Recall $\theta = (\sigma - 1)/k$. Higher σ increases θ, making the direct effect (the θ/Δ term in 112) larger relative to the domain-switching effect. A higher η^* is therefore needed to overcome the stronger direct effect. □

A.1.5.7 Proof of Proposition 4 (effects on market size)

Proposition 4. *1. Under expected utility ($\eta = 0$), a fixed-cost subsidy unambiguously increases the mass of active firms: $\frac{\partial M}{\partial \Delta} < 0$.*

2. Under reference–dependent preferences ($\eta > 0$), the effect is amplified. There exists an $\bar{\eta} \in (0, 1)$ such that for all $\eta > \bar{\eta}$: $\left|\frac{\partial M}{\partial \Delta}\right|_{\eta > 0} > \left|\frac{\partial M}{\partial \Delta}\right|_{\eta = 0}$.

Proof. The mass of active firms is:

$$M = M_e \int_{\underline{F}}^{\bar{F}} [1 - G(\phi^*(F))] \cdot h(F) dF. \tag{116}$$

Under Pareto-distributed productivity, $1 - G(\phi^*(F)) = (\underline{\phi}/\phi^*(F))^k \propto \Phi(F)^{-k/(\sigma-1)}$. To see this, note that from equation (46), the productivity cutoff satisfies:

$$\phi^*(F) = \left[\frac{\sigma}{B} \cdot \Phi(F)\right]^{\frac{1}{\sigma-1}},$$

which implies:

$$1 - G(\phi^*(F)) = \left(\frac{\underline{\phi}}{\phi^*(F)}\right)^k = \underline{\phi}^k \cdot \left[\frac{\sigma}{B} \cdot \Phi(F)\right]^{\frac{-k}{\sigma-1}} = \underbrace{\underline{\phi}^k \cdot \left(\frac{\sigma}{B}\right)^{\frac{-k}{\sigma-1}}}_{\text{constant w.r.t. } F} \cdot \Phi(F)^{-k/(\sigma-1)}.$$

Since $\underline{\phi}$ and B are independent of F, the survival probability is proportional to $\Phi(F)^{-k/(\sigma-1)}$.

Part 1: Expected utility ($\eta = 0$).

Under Pareto-distributed productivity and using the proportionality result $1 - G(\phi^*(F)) \propto \Phi(F)^{-k/(\sigma-1)}$, and with $\Phi(F) = \Delta F$ under expected utility with subsidy Δ, the mass of active firms satisfies:

$$M = M_e \int_{\underline{F}}^{\bar{F}} [1 - G(\phi^*(F))] h(F) dF \propto \int_{\underline{F}}^{\bar{F}} (\Delta F)^{-k/(\sigma-1)} h(F) dF.$$

Under the Pareto distribution, $h(F) = \kappa \underline{F}^\kappa / F^{\kappa+1}$, giving:

$$M \propto \int_{\underline{F}}^{\infty} (\Delta F)^{-k/(\sigma-1)} \frac{\kappa \underline{F}^\kappa}{F^{\kappa+1}} dF = \Delta^{-k/(\sigma-1)} \cdot \kappa \underline{F}^\kappa \int_{\underline{F}}^{\infty} F^{-k/(\sigma-1)-\kappa-1} dF. \tag{117}$$

Since $k/(\sigma - 1) + \kappa > 0$, the integral converges to the positive constant $\frac{\underline{F}^{-k/(\sigma-1)-\kappa}}{k/(\sigma-1)+\kappa}$. Differentiating with respect to Δ:

$$\frac{\partial M}{\partial \Delta} \propto \frac{\partial}{\partial \Delta}\Delta^{-k/(\sigma-1)} = -\frac{k}{\sigma-1}\Delta^{-k/(\sigma-1)-1} < 0, \tag{118}$$

since $k/(\sigma - 1) > 0$. A subsidy (lower Δ) therefore increases the mass of active firms.

Part 2: Reference-dependent preferences ($\eta > 0$).

With $\eta > 0$, the survival probability is:

$$1 - G(\varphi^*(F)) \propto \begin{cases} (c_G F - \eta\omega_G a(\Delta))^{-k/(\sigma-1)} & \text{if } F \le a(\Delta) \\ (c_L F - \eta\omega_L a(\Delta))^{-k/(\sigma-1)} & \text{if } F > a(\Delta) \end{cases} \tag{119}$$

A subsidy affects M through two channels:

Channel 1 (Standard): Within each domain, lower Δ reduces $\Phi(F)$ and increases survival probability.

Channel 2 (Domain switching): Firms with $F \in (F^r, F^r/\Delta]$ switch from loss to gain domain. Their survival probability jumps discretely because $\Phi_L(F) > \Phi_G(\Delta F)$ for these firms.

The additional mass saved through domain switching is:

$$\Delta M_{\text{switch}} = M_e \int_{F^r}^{a(\Delta)} \left[\left(\frac{c_G \Delta F - \eta\omega_G F^r}{1-\eta} \right)^{-k/(\sigma-1)} - \left(\frac{c_L F - \eta\omega_L F^r}{1-\eta} \right)^{-k/(\sigma-1)} \right] h(F)\, dF. \tag{120}$$

This is strictly positive because for $F \in (F^r, a(\Delta))$, we have $\Delta F < F^r < F$, which implies $\Phi_G(\Delta F) < F^r < \Phi_L(F)$ and therefore:

$$\Phi_G(\Delta F)^{-k/(\sigma-1)} > \Phi_L(F)^{-k/(\sigma-1)}. \tag{121}$$

Since $\Delta M_{\text{switch}} > 0$ and is increasing in η, for η sufficiently large:

$$\left|\frac{\partial M}{\partial \Delta}\right|_{\eta>0} > \left|\frac{\partial M}{\partial \Delta}\right|_{\eta=0}. \tag{122}$$

Therefore, the effect of subsidies on the mass of active firms is amplified under reference-dependent preferences. □