Exit Markets and Venture Capital Financing*

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Abstract

We develop a model of VC financing in which startups build transferable and human capital. An IPO requires both, while acquisitions and acquihires allow earlier exits. Because entrepreneurs can leave with their human capital, they control acquihires, while VCs control acquisitions. The optimal contract uses state-contingent deadlines and milestone and IPO payments to provide effort incentives and strengthen the seller's bargaining position with outside buyers. When entrepreneurs control exit, the contract grants them a larger stake through deferred compensation and makes no acquihire-contingent payment. When VCs control exit, it may share acquisition proceeds with entrepreneurs. Stronger acquisition markets raise VC payoffs and expand financing. Stronger acquihire markets initially expand financing and lengthen runway; beyond a threshold, they increase entrepreneurial rents, reduce financing, and shorten runway.

Keywords: acquihires, acquisitions, dynamic contracting, exit markets, human capital, IPOs, startups, venture capital.

JEL Classifications: C78, D86, G24, G34, J24, L26.

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When Google acquired DeepMind in 2014 for a reported \$400–650 million, the deal largely aimed to secure its research team: DeepMind had no revenue and little transferable technology. The founders joined Google and received substantial compensation, while VC investors earned only modest returns.[1] Similar transactions have since become common. Microsoft paid Inflection AI some \$650 million in 2024, while hiring its co-founders and most of its staff. Inflection kept its technology and continued to operate: only the team changed hands, and investors recovered barely more than their initial investment.[2]

These examples illustrate that VC-backed startups can exit not only through IPOs and conventional acquisitions, but also through acquihires, in which buyers primarily seek entrepreneurial talent rather than the firm's assets. These transactions differ fundamentally from traditional acquisitions because the entrepreneur, rather than investors, controls the key asset being sold.[3] As a result, acquihires can redistribute value toward founders and away from venture capitalists. This raises a central question: how does control over different exit opportunities shape venture capital contracts and startup financing?

We address this question in a dynamic contracting model in which startup value is generated by two complementary forms of capital: transferable capital, such as technology or intellectual property, and inalienable human capital embodied in the entrepreneur. Developing either type requires costly, unobservable effort by the entrepreneur, creating a moral hazard problem. An IPO requires both forms of capital to be developed, while intermediate exits require only one. An acquisition transfers the startup's transferable capital and is therefore controlled by the venture capitalist. An acquihire transfers human capital, which cannot be contractually assigned and remains under the entrepreneur's control. Thus, the nature of the asset being sold determines who controls intermediate exit.

The central insight of the paper is that the optimal VC financing contract serves two objectives. As in standard dynamic agency models, it provides incentives for entrepreneurial

[1] The price was not disclosed; contemporaneous reports placed it above \$500 million, while later accounts cite \$650 million. Facebook also considered acquiring DeepMind, reportedly proposing relatively little for the shares but large signing bonuses for founders and key employees. See "The Inside Story of the Greatest Deal Google Ever Made: Buying DeepMind", *Wall Street Journal*, 25 March 2026.

[2] Reported as \$620 million for the license and \$30 million to waive claims over the hiring, with investors receiving \$1.10–\$1.50 per dollar invested. See "Microsoft pays Inflection \$650 mln in licensing deal while poaching top talents", *Reuters*, 21 March 2024.

[3] We use "acquihire" to include both acquisitions of the startup and "reverse acquihires", in which the buyer hires the team without acquiring the firm. The distinction is immaterial in our model: in both cases, the relevant asset is inalienable human capital and the entrepreneur controls whether it is transferred to the buyer. Even when the firm itself is acquired, consideration can be directed primarily to founders and employees rather than shareholders, making the economic outcome similar to a reverse acquihire.

effort. But once an outside sale becomes possible, it also shapes the bargaining position of the party controlling exit. Because the buyer's payment comes from outside the VC–entrepreneur relationship, a contract that strengthens this party's threat point can increase the total surplus available to both the entrepreneur and the investor. Contract design therefore interacts with bargaining in the exit market. How the contract does so depends on who controls the startup's intermediate exit route, which in turn depends on whether outside buyers value the startup for its human or transferable capital.

When the startup has already developed human capital but lacks the transferable capital for an IPO, the entrepreneur controls intermediate exit and may leave through an acquihire. Under these circumstances, compensation is optimally deferred. Keeping compensation inside the startup raises the entrepreneur's value from staying and therefore her threat point when bargaining with an outside buyer. She becomes more willing to reject low offers and extracts a higher acquihire price. A payment from the VC for completing the acquihire would work in the opposite direction: it lowers the price the buyer must offer to make her accept, so part of every dollar the VC promises ends up with the buyer rather than with her. The contract therefore never rewards an acquihire directly.

When the startup has developed transferable but not human capital, the VC controls intermediate exit through a potential acquisition and the above logic reverses. The entrepreneur's continuation value is not the VC's threat point. Instead, the contract may promise the entrepreneur part of the acquisition proceeds. This makes the VC less willing to accept a low offer and thereby raises the negotiated acquisition price. At the optimum, this commitment can induce the buyer to pay its full valuation.

Thus, entrepreneur control over exit shifts compensation toward deferred and IPO-contingent pay, whereas VC control shifts it toward immediate and acquisition-contingent pay. In both cases, the contract is designed so that an outside buyer finances part of the entrepreneur's compensation. The optimal dynamic contract specifies how progress is rewarded. Before the first milestone, the entrepreneur receives no payout, and an endogenous deadline provides incentives. Once either human or transferable capital is developed, the deadline is extended or even removed, raising the startup's financing runway. The entrepreneur may also receive a milestone payment or a larger stake in the startup through higher IPO-contingent compensation. Reaching a milestone quickly yields a larger continuation utility and thus more generous subsequent terms, potentially including a milestone payment and a longer financing horizon.

The type of milestone matters as well. Human-capital development is rewarded relatively more through deferred compensation, including a higher IPO-contingent payment, whereas transferable-capital development is rewarded relatively more through immediate and acquisition-contingent payments. Financing runway also depends on which type of capital was developed. After a human-capital milestone, an attractive acquihire opportunity makes it harder to sustain effort toward an IPO, creating a motive for tighter deadlines.

The asymmetry in control has sharp implications for startup financing. A stronger acquisition market raises the value captured by the VC and therefore increases financing capacity. A stronger acquihire market has two opposing effects. Before human capital is developed, the prospect of a buyer-funded payoff helps reward the entrepreneur for developing it, reducing the cost of providing incentives. After human capital has been developed, however, the same outside opportunity makes retaining the entrepreneur and incentivizing continued effort toward an IPO more costly.

As a result, VC financing—the maximum amount the VC is willing to provide while breaking even—is hump-shaped in the strength of the acquihire market, measured by outside buyers' valuation of the entrepreneur's human capital. Moderate acquihire opportunities expand financing because outside buyers help reward effort. Once these opportunities become sufficiently attractive, however, they increase the entrepreneur's rents and reduce the VC's payoff. By contrast, stronger acquisition markets expand financing monotonically. Better markets for entrepreneurial talent and better exit markets for investors thus have fundamentally different implications for startup financing.

Dynamic contracting also changes how exit-market conditions map into realized exit outcomes. In a stationary contract without deadlines, changes in acquihire values affect payoffs but not the probabilities of different exits. Under the optimal dynamic contract, they change termination decisions and therefore which development states startups reach. Initially, a stronger acquihire market makes reaching the human-capital milestone more valuable and increases the incidence of acquihires.

However, as the acquihire market becomes sufficiently attractive, retaining and incentivizing the entrepreneur after a human-capital milestone becomes harder, calling for tighter deadlines and a shorter runway. The probability of an acquihire can therefore eventually fall even as outside buyers place greater value on entrepreneurial human capital. Overall, stronger acquihire markets have a non-monotonic effect on startup financing: they initially lengthen runway and expand financing capacity, but beyond a threshold, they reduce both.

Taken together, these results recast the role of exit in venture finance. Because control over exit follows the type of capital developed, exit determines who bargains with outside buyers, how the contract rewards development, and how much capital can be raised to finance innovation. Our model highlights how different exit routes shape the availability of VC financing and startup runway: better IPO and acquisition prospects can expand financing and lengthen runway, while sufficiently attractive acquihire opportunities can have the opposite effect.

Our paper contributes to three strands of literature. First, we contribute to the literature on venture capital contracting, control rights, and exit (Aghion and Bolton, 1992; Berglöf, 1994; Hellmann, 1998; Casamatta, 2003; Chemla, Habib, and Ljungqvist, 2007; Ewens, Gorbenko, and Korteweg, 2022; Fulghieri, Hu, and Varas, 2025). This literature studies how contracts allocate cash-flow and control rights between entrepreneurs and investors and how these rights shape exit. Notably, Hellmann (2006) studies the allocation of cash-flow and control rights across acquisitions and IPOs. Chemla et al. (2007) analyze contractual provisions governing exit and transfers of control. Cumming (2008) documents a relation between VC control rights and exit outcomes. Our starting point is different: control over a given exit route is constrained by the nature of the asset being sold. Human capital remains under the entrepreneur's control, whereas transferable capital can be assigned to investors. We show how the optimal contract responds to this allocation of control and uses compensation to strengthen the bargaining position of the party negotiating with an outside buyer.

Second, we contribute to the literature on dynamic contracting and inalienable human capital (DeMarzo and Sannikov, 2006; Biais, Mariotti, Plantin, and Rochet, 2007; He, 2009; DeMarzo, Fishman, He, and Wang, 2012; Ai and Li, 2015; Hartman-Glaser, Lustig, and Xiaolan, 2019; Gryglewicz, Mayer, and Morellec, 2020; Feng, Taylor, Westerfield, and Zhang, 2024; Chen, Li, Thakor, and Ward, 2026). Most closely related, Bolton, Wang, and Yang (2019), Shi (2023), and Chemla, Rivera, and Shi (2025) study dynamic contracting when human capital is inalienable or agents cannot commit to remain with the firm. We introduce both inalienable human capital and transferable capital, together with outside buyers at intermediate development stages. Because the type of capital developed determines who controls the relevant exit opportunity, dynamic incentive provision also shapes bargaining with outside buyers. The optimal contract therefore serves not only to provide effort incentives, but also to increase the value extracted at exit.

Finally, we relate to the literature on acquihires and human-capital-driven acquisitions (Ouimet and Zarutskie, 2020; Chen, Gao, and Ma, 2021; Chen, Hshieh, and Zhang, 2024; Bar-Isaac, Johnson, and Nocke, 2025; Benkert, Letina, and Liu, 2025). This literature emphasizes the role of acquisitions as a channel for transferring or acquiring human capital and studies the incentives surrounding acquihires. We focus instead on how the possibility of an acquihire feeds back into the financing contract before exit. Because human capital is inalienable, a stronger market for entrepreneurial talent can help finance incentives before human capital is developed but increase the cost of retaining the entrepreneur afterward. This generates the non-monotonic effect of acquihire markets on VC financing (i.e., the maximum amount the VC is willing to provide while breaking even).

1 Model

Time is continuous and runs over $[0, +\infty)$. We consider a model of a penniless entrepreneur who can develop a profitable startup. There are three types of risk-neutral agents: a venture capitalist (the principal), the entrepreneur (the agent), and potential acquirers, all discounting at rate $\rho > 0$. The venture capitalist (VC) finances the startup, which is run by the entrepreneur. The financing terms and the entrepreneur's compensation are jointly determined by an optimal long-term contract. Launching the startup requires an initial outlay $F > 0$, which is provided by the VC. Because this outlay is paid at $t = 0$ and does not affect the subsequent technology, we solve the contracting problem in terms of payoffs gross of F.

The startup evolves along two dimensions–human capital and transferable capital–which jointly determine exit opportunities. Specifically, exit may occur through an acquihire, in which the acquirer values only the entrepreneur's human capital; through an acquisition, in which the acquirer purchases the startup's transferable capital; or through an IPO, in which investors jointly value both forms of capital. The two types of capital differ in one key respect. Transferable capital—such as physical capital, patents, proprietary technology, or an established user base—exists independently of the entrepreneur. It can thus be contractually assigned to the VC and retains its value even after the relationship ends. Human capital, by contrast, is embodied in the entrepreneur: it cannot be transferred by contract and departs with her if she leaves the startup. This fundamental distinction determines control over each exit route and, as we show, shapes every component of the optimal contract.

Figure 1 summarizes the timeline and state dynamics; we next provide the detailed model description.

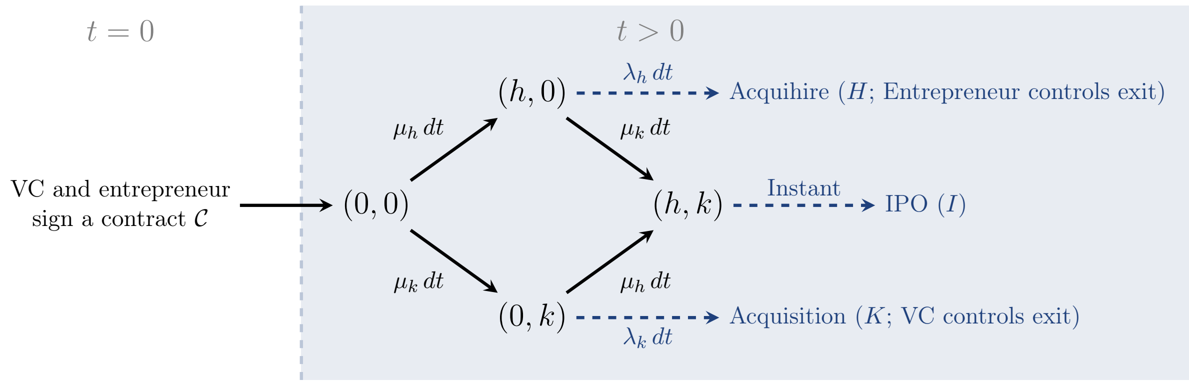

Figure 1: **Model timeline and state dynamics.** At $t = 0$, the VC and the entrepreneur sign a long-term contract. The startup then begins in state $(0, 0)$ and develops along the human-capital and transferable-capital dimensions according to the transition intensities shown. In states $(h, 0)$ and $(0, k)$, the startup may exit through an acquihire or an acquisition. Reaching state (h, k) results in an immediate IPO. Control over each intermediate exit follows the type of capital that drives it: the entrepreneur controls the acquihire, the VC controls the acquisition.

Technology and Exit Routes. At each date $t \geq 0$ following its creation, the startup is characterized by two forms of capital: (i) inalienable human capital embodied in the entrepreneur, $h_t \in \{0, h\}$, and (ii) transferable capital, $k_t \in \{0, k\}$, consisting of assets that can be redeployed independently of the entrepreneur. Each type of capital is either undeveloped or developed, so the startup's state is summarized by the tuple $(h_t, k_t) \in \{0, h\} \times \{0, k\}$. Failure or technological obsolescence destroys all value at rate $\delta \geq 0$, which we absorb into the effective discount rate $r \equiv \rho + \delta$.

Inalienability and Control over Exit. Transferable capital can be contractually assigned to the VC, who therefore controls its sale. Human capital is inalienable and remains under the entrepreneur's control. Thus, the entrepreneur controls exits that derive value from her human capital, whereas the VC controls exits based on transferable capital. This allocation of control is the central friction in the model.

When $(h_t, k_t) = (0, 0)$, the startup has no value to outside buyers and cannot exit. In each remaining state, the startup has access to one of three exit opportunities.

1. *Acquihire.* In state $(h, 0)$, the entrepreneur meets a potential acquirer at rate $\lambda_h > 0$.

The acquirer values only the entrepreneur's human capital, generating value to the acquirer of $\Pi_h > 0$, and assigns no value to transferable assets. The entrepreneur and acquirer bargain bilaterally over the transaction, with the entrepreneur receiving Nash bargaining weight $\theta_h \in (0, 1]$. The resulting transaction price is denoted by H_t. Because the entrepreneur controls this exit, the VC receives no proceeds from the sale, although the entrepreneur may also receive any payment specified by the contract.

2. *Acquisition.* In state $(0, k)$, the VC meets a potential acquirer at rate $\lambda_k > 0$. The acquirer values only the startup's transferable capital, generating value to the acquirer of $\Pi_k > 0$, and assigns no value to the entrepreneur's human capital. The VC and acquirer bargain bilaterally, with the VC receiving Nash bargaining weight $\theta_k \in (0, 1]$. The negotiated acquisition price is denoted by K_t. The VC gets the acquisition proceeds while the entrepreneur receives only the payment specified by the contract.

3. *IPO.* In state (h, k), the startup can immediately undertake an IPO, generating proceeds I.[4] The VC receives the IPO proceeds and the entrepreneur receives her contractual payout. Because the entrepreneur retains control over her inalienable human capital, she may instead reject the IPO and leave the relationship, obtaining her outside option R_h, defined below.

Outside Options and Liquidation Values. Following Hart and Moore (1994), human capital is inalienable in the sense that a contract cannot bind the entrepreneur's skill and knowledge to the startup. She therefore cannot commit to remain and may leave at any time. The VC, by contrast, can commit to the long-term contract and terminates only as prescribed by it. Upon termination, human capital remains with the entrepreneur and transferable capital with the VC. Each can then search for a buyer, who arrives at rate $\alpha_i\lambda_i$, $\alpha_i \in [0, 1)$, with bargaining as before. Hence, for $i \in \{h, k\}$, the outside value is $R_i = \frac{\alpha_i\theta_i\lambda_i\Pi_i}{r+\alpha_i\theta_i\lambda_i}$. We assume α_i is sufficiently small that termination is inefficient relative to continuation.

Table 1 summarizes the differences between the entrepreneur's human capital and the startup's transferable capital.

Effort and Moral Hazard. The development of human and transferable capital depends on the entrepreneur's private effort. At each instant, the entrepreneur chooses effort along

[4] Allowing the IPO opportunity to arrive at Poisson rate λ_i instead simply replaces I with its present value, $\tilde{I} = \frac{\lambda_i I}{r+\lambda_i}$, leaving the contracting problem unchanged.

	Human capital	Transferable capital
Examples	founders' talent and expertise	patents, technology, user base
Transferable by contract	no	yes
Intermediate exit route	acquihire	acquisition
Negotiates with the buyer	entrepreneur (θ_h)	VC (θ_k)
Receives the negotiated price	entrepreneur (H)	VC (K)
Value upon termination	R_h, to the entrepreneur	R_k, to the VC

Table 1: **Two types of capital.**

both dimensions, $e_t^h, e_t^k \in \{0,1\}$. Effort is costly: $e_t^h = 1$ and $e_t^k = 1$ forgo flow private benefits $\gamma_h > 0$ and $\gamma_k > 0$, respectively. Because effort is hidden and costly, the model features moral hazard. If $h_t = 0$ and the entrepreneur exerts effort $e_t^h = 1$, human capital develops with intensity $\mu_h > 0$, transitioning to the state $h_t = h$. Transferable capital develops analogously with intensity $\mu_k > 0$ under effort $e_t^k = 1$, transitioning to $k_t = k$.

Once a type of capital has been developed, no further effort along that dimension is needed and, for simplicity, no further private benefits can be collected. Thus, the entrepreneur can derive γ_h only in states $(0,0)$ and $(0,k)$, and γ_k only in states $(0,0)$ and $(h,0)$—in each case only when shirking. One micro-foundation is that the VC allocates financing to the firm to develop each type of capital, from which the entrepreneur can divert resources. Once the respective capital has been developed, funding is no longer provided.

Contracting Problem. At the time of financing, the VC offers the entrepreneur a long-term contract $\mathcal{C} = (Y, T_D)$, where $Y = \{Y_t\}_{t\geq 0}$ is the cumulative compensation process and T_D is a contractual stopping time.[5] We denote incremental payments by dY_t, allowing for both continuous and lump-sum payments. At T_D, the relationship is terminated. After termination, the entrepreneur receives no more payments and cannot derive any private benefits from shirking. The entrepreneur is protected by limited liability, in that $dY_t \geq 0$.

The contract may condition on publicly observed development states (h_t, k_t) and transitions, accepted exit events, and the path to IPO. Bargaining with outside buyers, however, is private, and the contract cannot condition on buyer meetings, rejected transactions, or the negotiated transfers. That is, although acquisition prices may be publicly observed in practice, bargaining parties can always implement hidden side payments, so effective transfers (H_t, K_t) remain non-contractible. Consequently, the party controlling a bilateral exit

[5] We allow the contract to randomize over the entrepreneur's continuation utility. The Appendix shows that the resulting value functions are concave and non-degenerate randomization is never strictly optimal.

cannot commit ex ante to complete it: acceptance must be optimal when a buyer arrives.

We break ties in favor of completing an available bilateral exit: whenever the party controlling an acquihire or acquisition is indifferent between completing the transaction and continuing, it completes the transaction. Because the IPO is a contractible terminal exit and does not rely on bilateral bargaining with a specific outside buyer, we treat the IPO differently and assume that the VC can commit through the long-term contract to undertake an IPO once state (h, k) is reached. The entrepreneur cannot be forced to participate in the IPO and may instead leave with her human capital.

We focus on incentive-compatible contracts that (i) induce effort, that is, $e_t^h = 1$ whenever $h_t = 0$ and $e_t^k = 1$ whenever $k_t = 0$, (ii) induce the entrepreneur not to quit prior to contractual termination or exit and to participate in the IPO upon reaching state (h, k), and (iii) induce the party controlling an exit to complete it whenever an acquirer arrives.

Parameter Restrictions. We focus on a parameter region in which all three exit routes are economically relevant. First, the two forms of capital are complementary,

$$I > \Pi_h + \Pi_k,$$

so that once both have been developed, the IPO is the most valuable exit. Second, once one type of capital has been developed, the corresponding intermediate exit is efficient whenever a buyer arrives:

$$\Pi_k > \frac{\mu_h I}{r + \mu_h} \qquad \text{and} \qquad \Pi_h > \frac{\mu_k I}{r + \mu_k}.$$

Thus, acquisitions and acquihires are not dominated by continuation toward the IPO. These assumptions ensure that each exit route plays a role in the relevant development state.

Appendix A states the remaining conditions on outside values, pledgeable income, and regularity used to characterize the optimal contract.

Objectives and Financing. The entrepreneur chooses her effort, exit participation, and quitting decision to maximize her expected discounted payoff from contractual compensation, private benefits, acquihire proceeds, and her outside value upon termination. The VC has full bargaining power at the financing stage, and we normalize the entrepreneur's ex ante reservation utility to zero. Subject to the entrepreneur's incentive and participation constraints and the sequential optimality of acquisition decisions, the VC therefore chooses the contract to maximize its expected discounted payoff from acquisition and IPO proceeds and the liquidation value of transferable capital, net of compensation paid to the entrepreneur.

We denote the resulting initial payoffs of the entrepreneur and the VC by U_0 and V_0, respectively, where V_0 is measured gross of the initial outlay F. The startup can be financed under a contract if and only if $V_0 \geq F$. Thus, V_0 represents the startup's "pledgeable income" in the spirit of Holmstrom and Tirole (1997): an increase in V_0 raises the largest initial outlay that the VC is willing to finance. Under the optimal contract characterized below, $U_0 = U_{00}$ and $V_0 = V_{00}(U_{00})$. Allowing the entrepreneur to have bargaining power at the financing stage would not change the continuation contract conditional on her initial promise, but would change the initial division of surplus and hence the amount pledgeable to the VC.

Appendix A formally defines the VC's and entrepreneur's objective functions.

Tie-Breaking Rule for Payouts. Because the entrepreneur and the VC discount at the same rate, the precise timing of some payouts is payoff-irrelevant—a given continuation utility can sometimes be delivered either immediately or after a delay. To resolve this indeterminacy, we break ties in favor of earlier payouts and set acquisition, acquihire, and IPO payments at their lowest optimal level.[6]

2 Model Solution

We solve the contracting problem recursively across development states. The entrepreneur's continuation utility U_t summarizes the contract's relevant history and, together with the development state (h_t, k_t), is an endogenous state variable. The VC's value function can therefore be written as $V_t = V_{h_t k_t}(U_t)$, with the continuation contract depending on U_t in each non terminal state.

We suppress time subscripts below. We first solve the contracting problem in the intermediate states $(h, 0)$ and $(0, k)$, in which exit is feasible, taking as given the continuation utility with which each state is entered. We then turn to the initial state $(0, 0)$, in which the promises made upon reaching a milestone provide effort incentives. The Appendix contains all proofs and expressions for the value functions.

Throughout, when the IPO state is reached from $(h, 0)$, the entrepreneur is paid Y_h; when it is reached from $(0, k)$, she is paid Y_k. Upon an acquihire, the entrepreneur receives a

[6] Suppose instead that the entrepreneur discounts at rate $r + \epsilon$ and the VC at rate r. For every $\epsilon > 0$, front-loading strictly reduces the cost of delivering a given continuation utility, so the earliest-payout contract is uniquely optimal. Our convention to deliver early payments corresponds to the limit as $\epsilon \to 0$.

contractual payment X_h from the VC in addition to the price H negotiated with the acquirer. Upon an acquisition, she receives a contractual payment X_k, whereas the VC collects the negotiated price K. All other payouts are summarized by $dy \geq 0$.

The optimal contract pursues two objectives. First, as in standard dynamic agency models, it provides effort incentives at minimum cost. Second, and specific to our setting, it maximizes the value extracted from external buyers upon exit. The second objective matters because the buyer's payment comes from outside the VC–entrepreneur relationship: every dollar extracted from a buyer can be split between the VC and the entrepreneur, making both parties better off. The interaction between these two objectives determines the timing and form of compensation state by state.

2.1 State $(h, 0)$: Acquihires and Deferred Compensation

Consider first state $(h, 0)$, in which human capital has been developed, and the entrepreneur must still develop transferable capital to reach an IPO. Under full effort and acceptance of exit opportunities, the entrepreneur's continuation utility evolves according to

$$dU = \dot{U}_{h0}dt + (Y_h - U)dN^{\mu_k} + (H + X_h - U)dN^{\lambda_h} - dy,$$

where dN^x denotes an independent Poisson process with intensity x. The continuation utility drifts at rate $\dot{U}_{h0}$, jumps to the IPO payment Y_h upon a breakthrough (second term), jumps to the total acquihire payoff $H + X_h$ upon an acquihire (third term), and falls one-for-one with any payout (last term). Promise keeping requires that, in expectation, the change in continuation utility plus payouts compensates the entrepreneur for her time preference, $\mathbb{E}[dU + dy] = rUdt$, which pins down the drift rate in the entrepreneur's continuation utility:

$$\dot{U}_{h0} = rU - \mu_k(Y_h - U) - \lambda_h(H + X_h - U). \tag{1}$$

Two constraints discipline the IPO payment Y_h. First, effort is incentive compatible only if the expected jump in the entrepreneur's payoff due to a breakthrough exceeds the forgone private benefit:

$$\mu_k(Y_h - U) \geq \gamma_k. \tag{IC-h}$$

Second, because human capital is inalienable, the entrepreneur can always decline the IPO and leave with her outside option, so the IPO payment must satisfy

$$Y_h \geq R_h. \tag{IC-IPO-h}$$

For the same reason, the entrepreneur's continuation utility cannot fall below her outside option at any time: the contract is terminated when U first reaches R_h, at which point she leaves and searches for an acquihire on her own.

Consider next the bargaining that determines the acquihire price. If bargaining breaks down, the entrepreneur retains U and the acquirer obtains zero. If the deal is completed at price H, the entrepreneur gets $H + X_h$, and the acquirer gets $\Pi_h - H$. Nash bargaining with entrepreneur bargaining power θ_h implies that the entrepreneur's gain from the deal, $H + X_h - U$, equals a fraction θ_h of the bilateral deal surplus, $\Pi_h + X_h - U$. Hence, the negotiated price is

$$H = U - X_h + \theta_h\big(\Pi_h + X_h - U\big), \tag{2}$$

and the entrepreneur accepts the deal whenever the deal surplus is positive,

$$\Pi_h + X_h \geq U. \tag{Acq-H}$$

Equation (2) reveals the central force in state $(h, 0)$. The entrepreneur's inside utility U is her threat point in the negotiation: whenever $\theta_h < 1$, the price she extracts from the acquirer rises with her inside utility U and falls with any payment X_h she is promised upon leaving. The contract should therefore keep her continuation utility high—by deferring compensation—and should not reward her for leaving.

In the continuation region, the VC's value function solves the HJB equation

$$(r + \mu_k + \lambda_h)V_{h0}(U) = \max_{Y_h, X_h \geq 0} \Big\{\mu_k(I - Y_h) - \lambda_h X_h + V'_{h0}(U)\dot{U}_{h0}\Big\},$$

subject to (IC-h), (IC-IPO-h), and (Acq-H), and the boundary condition $V_{h0}(R_h) = 0$. The first two terms in the brackets are the VC's expected payoffs upon an IPO and an acquihire, and the last term captures the effect of the evolution of its promise to the entrepreneur with $\dot{U}_{h0}$ given in (1).

The value function satisfies $V'_{h0}(U) \geq -1$ because the VC can always make an immediate payment to the entrepreneur. The marginal benefits of increasing Y_h and X_h are respectively

$$-\mu_k(1 + V'_{h0}(U)) \leq 0, \qquad \text{and} \qquad -\lambda_h(1 + \theta_h V'_{h0}(U)) \leq 0,$$

and both payments are set at their lowest feasible levels. The acquihire-contingent payment X_h is particularly inefficient: a dollar paid by the VC raises the entrepreneur's total acquihire payoff by only θ_h, because the negotiated price H falls in response. The acquihire participation constraint (Acq-H) bounds the acquisition payment at $X_h(U) = [U - \Pi_h]^+$. As the next proposition establishes, the optimal contract keeps $U \leq \Pi_h$, so $X_h = 0$ on path. Since $U \geq R_h$, the effort constraint (IC-h) implies the IPO participation constraint (IC-IPO-h), and the lowest feasible IPO payment is $Y_h(U) = U + \gamma_k/\mu_k$.

Proposition 1 (Optimal Contract in State $(h, 0)$)**.** *On path, the IPO payment is $Y_h(U) = U + \frac{\gamma_k}{\mu_k}$ and the acquihire payment is $X_h(U) = 0$. There are three regions:*

1. *For $U \in [R_h, \hat{U}_{h0})$, the entrepreneur's continuation utility drifts downward, and the contract is terminated when U reaches R_h.*
2. *For $U \in (\hat{U}_{h0}, \overline{U}_{h0})$, the continuation utility drifts upward.*
3. *For $U > \overline{U}_{h0}$, a payout $dy = U - \overline{U}_{h0}$ is made, bringing U back to $\overline{U}_{h0}$.*

The stationary point and the payout boundary are:

$$\hat{U}_{h0} = \frac{\gamma_k + \theta_h \lambda_h \Pi_h}{r + \theta_h \lambda_h} \qquad \text{and} \qquad \overline{U}_{h0} = \begin{cases} \Pi_h, & \theta_h < 1, \\ \hat{U}_{h0}, & \theta_h = 1. \end{cases}$$

The value function $V_{h0}(U)$ is concave and continuously differentiable with $V'_{h0}(U) \geq -1$; its closed-form expression is given in Proposition 7.[7]

Figure 2 displays the resulting dynamics: the entrepreneur's continuation utility drifts away from the stationary point $\hat{U}_{h0}$, down toward termination on the left and up toward the payout boundary on the right. The contract defers compensation to increase the value extracted from acquirers. By withholding payouts and setting $X_h = 0$, the contract keeps

[7] When $\theta_h = 1$, the entrepreneur extracts the full valuation Π_h regardless of her inside stake, so deferral no longer raises the price: $V'_{h0}(U) = -1$ on all of $[\hat{U}_{h0}, \Pi_h]$, the deferral region degenerates, and the tie-breaking rule in favor of early payouts selects the payout boundary $\hat{U}_{h0}$.

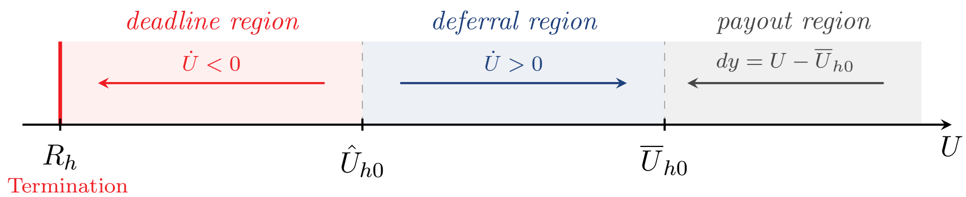

Figure 2: **State $(h, 0)$: continuation utility dynamics.** The continuation utility drifts away from $\hat{U}_{h0}$: down to termination below $\hat{U}_{h0}$, implying a finite deadline, and up to the payout boundary $\overline{U}_{h0}$ above it. The figure depicts $\theta_h < 1$, for which $\overline{U}_{h0} = \Pi_h$; when $\theta_h = 1$, the deferral region disappears and $\overline{U}_{h0} = \hat{U}_{h0}$.

the entrepreneur's inside stake high, making her more willing to reject unattractive offers and letting her extract a higher price H. For $\theta_h < 1$, payouts are deferred up to the boundary $\overline{U}_{h0} = \Pi_h$, the point at which the acquihire price is $H = \Pi_h = U$, and she is indifferent between accepting the acquihire and staying in the firm when a buyer arrives. The entrepreneur receiving the entire acquihire price is not a loss for the VC. Because the payoff is funded by an external buyer, it substitutes for other, costlier forms of compensation.

The continuation utility therefore plays two roles. Below $\hat{U}_{h0}$ it drifts down toward termination. With a small promise, the payoffs at a breakthrough and an acquihire can be delivered only if the promise erodes absent either event, and under limited liability it can erode only toward termination. Above $\hat{U}_{h0}$ it drifts up toward $\overline{U}_{h0}$, which buys the entrepreneur a higher price from the buyer. When the entry continuation utility exceeds $\overline{U}_{h0}$, full deal surplus is extracted from the acquirer and the contract pays out the excess immediately as a milestone payment. The continuation utility then stays at the boundary, where, for $\theta_h < 1$, the upward drift is absorbed by continuing payouts and the entrepreneur captures the buyer's full valuation.

Corollary 1 and Figure 3 show how the IPO payment Y_h and the acquihire price H vary with the continuation utility, and therefore how they move over time in each of the two cases.

Corollary 1 (Delay Costs in State $(h, 0)$)**.** *In state $(h, 0)$, the IPO payment Y_h is strictly increasing in U and the acquihire price H is weakly increasing in U, strictly so when $\theta_h < 1$. When the state is entered below $\hat{U}_{h0}$, both decline over time absent a breakthrough and reach their minimum at the deadline. When the state is entered above $\hat{U}_{h0}$, there is no deadline and both weakly increase over time.*

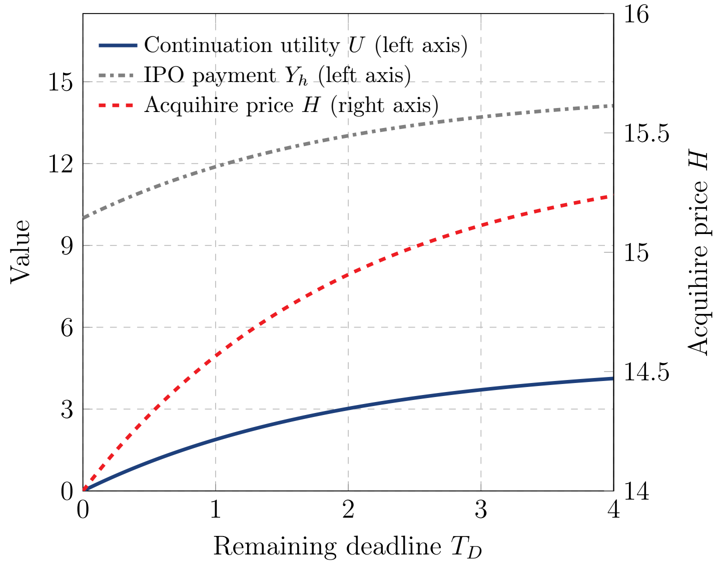

Figure 3: **Contract Instruments and Deadline in State** $(h, 0)$**.** The figure plots the entrepreneur's continuation utility U and the IPO payment Y_h (left axis) and the acquihire price H (right axis) as functions of the remaining deadline T_D. Calendar time runs from right to left; every reward reaches its minimum at termination ($T_D = 0$). Baseline parameters: $r = 0.40$, $\mu_h = 0.10$, $\mu_k = 0.03$, $\gamma_h = \gamma_k = 0.30$, $\lambda_h = \lambda_k = 0.15$, $\theta_h = \theta_k = 0.7$, $\Pi_h = \Pi_k = 20$, $I = 60$, $\alpha_h = \alpha_k = 0$.

2.2 State $(0, k)$: Acquisitions and Contingent Payments

Consider next state $(0, k)$, in which transferable capital has been developed and control over exit rests with the VC. The continuation utility now jumps to Y_k upon a breakthrough and to the contractual payment X_k upon an acquisition, since the entrepreneur receives nothing from the acquirer. Similarly to state $(h, 0)$, promise keeping under full effort and the acceptance of exit opportunities yields the drift of the entrepreneur's continuation utility[8]

$$\dot{U}_{0k} = rU - \mu_h(Y_k - U) - \lambda_k(X_k - U). \tag{3}$$

As before, effort requires

$$\mu_h(Y_k - U) \geq \gamma_h. \tag{IC-k}$$

By the time of an IPO, the entrepreneur has developed her human capital and can leave with outside value R_h, so the IPO payment must satisfy

$$Y_k \geq R_h. \tag{IC-IPO-k}$$

[8] As in state $(h, 0)$, we have $dU = \dot{U}_{0k}dt + (Y_k - U)dN^{\mu_h} + (X_k - U)dN^{\lambda_k} - dy$ under full effort. Promise keeping, $\mathbb{E}[dU + dy] = rUdt$, pins down the drift (3).

Unlike in state $(h, 0)$, the entrepreneur has not yet developed human capital while the startup remains in state $(0, k)$, so quitting yields zero. The contract is therefore terminated when U first reaches zero, at which point the VC recovers the liquidation value R_k.

The VC now negotiates with the acquirer. If bargaining breaks down, the VC retains its continuation value $V_{0k}(U)$, and the acquirer obtains zero. If the deal is completed at price K, the VC obtains $K - X_k$, and the acquirer obtains $\Pi_k - K$. Nash bargaining with VC bargaining power θ_k implies that the VC's gain from the deal, $K - X_k - V_{0k}(U)$, equals a fraction θ_k of the bilateral deal surplus, $\Pi_k - X_k - V_{0k}(U)$. Hence, the negotiated price is

$$K = X_k + V_{0k}(U) + \theta_k\big(\Pi_k - X_k - V_{0k}(U)\big), \tag{4}$$

and the VC accepts the deal whenever the bilateral deal surplus is positive,

$$\Pi_k - X_k \geq V_{0k}(U). \tag{Acq-K}$$

Compared with state $(h, 0)$, the exit-participation constraint reverses direction: acquihire participation places a lower bound on X_h, whereas acquisition participation places an upper bound, $X_k \leq \Pi_k - V_{0k}(U)$. This upper bound is related to the bargaining role of the acquisition-contingent payment X_k. From (4), a larger X_k makes completing the deal less attractive to the VC and thereby raises the negotiated price. In particular, a one-dollar increase in X_k raises K by $1 - \theta_k$, so its net cost to the VC is only θ_k. At the upper bound, the VC is indifferent between selling and continuing and extracts the acquirer's full valuation, $K = \Pi_k$.

In the continuation region, the VC's value function solves

$$(r + \mu_h + \lambda_k)V_{0k}(U) = \max_{Y_k, X_k \geq 0} \Big\{\mu_h(I - Y_k) + \lambda_k(K - X_k) + V_{0k}'(U)\dot{U}_{0k}\Big\},$$

subject to (IC-k), (IC-IPO-k), (Acq-K), and the boundary condition $V_{0k}(0) = R_k$. The first two terms in the brackets are the VC's expected payoff upon an IPO and acquisition, respectively, while the last term captures the effect of changes in the entrepreneur's continuation utility on the VC's value.

As in state $(h, 0)$, the value function is concave and satisfies $V_{0k}'(U) \geq -1$. The marginal benefits of increasing Y_k and X_k are respectively

$$-\mu_h(1 + V'_{0k}(U)) \leq 0, \qquad \text{and} \qquad -\lambda_k(\theta_k + V'_{0k}(U)).$$

The IPO payment Y_k is therefore set at its lowest feasible level determined by (IC-k) and (IC-IPO-k). Under the additional regularity condition $R_h \leq \gamma_h/\mu_h$, stated formally in Appendix A, the IPO participation constraint is implied by the effort constraint. The optimal Y_k therefore sits at the incentive lower bound, $Y_k(U) = U + \frac{\gamma_h}{\mu_h}$.

The acquisition payment X_k, by contrast, need not be minimized. As shown above, a dollar of X_k costs the VC only θ_k net of the induced increase in the negotiated price, whereas carrying an additional dollar of continuation utility costs $-V'_{0k}(U)$. Because the HJB equation is linear in X_k, the optimal payment is bang-bang: the contract sets $X_k = 0$ when $-V'_{0k}(U) < \theta_k$ and sets it at its upper bound, $X_k = \Pi_k - V_{0k}(U)$, when $-V'_{0k}(U) > \theta_k$. Because V_{0k} is concave, the switch occurs at a threshold $U_X = \inf\{U \geq 0 \mid V'_{0k}(U) \leq -\theta_k\}$.

Proposition 2 (Optimal Contract in State $(0, k)$)**.** *The IPO payment is $Y_k(U) = U + \frac{\gamma_h}{\mu_h}$. There is a threshold $U_X = \inf\{U \geq 0 | V'_{0k}(U) \leq -\theta_k\}$ and three regions:*

1. *For $U \in [0, U_X)$, there is no acquisition payment, $X_k = 0$. The continuation utility drifts downward and reaches the termination boundary at zero absent an exit.*

2. *For $U \in (U_X, \overline{U}_{0k})$, the entrepreneur is paid $X_k(U) = \Pi_k - V_{0k}(U)$ upon an acquisition. The continuation utility drifts downward.*

3. *For $U > \overline{U}_{0k}$, a payout $dy = U - \overline{U}_{0k}$ brings U back to $\overline{U}_{0k}$, where it remains constant.*

If $\theta_k < 1$, $\overline{U}_{0k} = \frac{\gamma_h + \lambda_k(\Pi_k - S_k)}{r}$ where $S_k = \frac{\mu_h I + \lambda_k \Pi_k}{r + \mu_h + \lambda_k}$ is the total surplus in state $(0, k)$.[9] *The value function $V_{0k}(U)$ is concave and continuously differentiable with $V'_{0k}(U) \geq -1$; its semi-closed-form expression is in Proposition 8.*

Figure 4 displays the resulting dynamics: the entrepreneur's continuation utility drifts downward everywhere below the payout boundary. The distinctive instrument in this state is the acquisition payment. When the contract sets $X_k = \Pi_k - V_{0k}(U)$, the negotiated price reaches the acquirer's full valuation, $K = \Pi_k$. The VC then receives its continuation

[9] When $\theta_k = 1$, the VC already extracts Π_k without an acquisition payment. Hence the acquisition-payment region is empty and $U_X = \overline{U}_{0k} = \gamma_h/(r + \lambda_k)$, the stationary point under $X_k = 0$.

value $V_{0k}(U)$, while the remaining proceeds finance the entrepreneur's acquisition-contingent compensation. Thus, X_k allows the contract to shift part of the cost of compensating the entrepreneur onto the acquirer. Even so, the contract uses this instrument only when it is sufficiently cheap. Because X_k rewards an acquisition rather than the effort-induced transition to an IPO, it weakens the entrepreneur's incentive to develop human capital. The contract therefore uses X_k only when continuation utility is high—that is, when the milestone was reached quickly—and the marginal cost of carrying additional continuation utility to the IPO stage is correspondingly high.

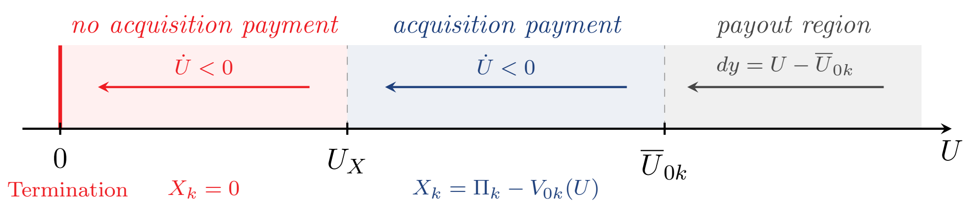

Figure 4: **State $(0, k)$: continuation utility dynamics.** The entrepreneur's continuation utility drifts down below $\overline{U}_{0k}$, implying a finite deadline. The acquisition payment switches on at U_X, while cash is paid only upon entry at $\overline{U}_{0k}$. The figure depicts $\theta_k < 1$. When $\theta_k = 1$, the acquisition-payment region is empty and $\overline{U}_{0k} = U_X$.

The timing of payouts also differs from state $(h, 0)$. There, deferring payouts keeps U high and thereby strengthens the entrepreneur's bargaining position. Here, continuation utility is not itself the bargaining threat point. The contract can instead strengthen the VC's bargaining position through the acquisition-contingent payment X_k, so there is no analogous bargaining motive to keep U high by deferring ordinary payouts. Accordingly, the payout boundary $\overline{U}_{0k}$ coincides with the stationary point of the continuation utility dynamics. It can therefore be reached only upon entering the state, not by subsequent drift. When the entry continuation utility exceeds $\overline{U}_{0k}$, the excess is paid immediately as a milestone payment, so the state is entered at the payout boundary.

Corollary 2 (Delay Costs in State $(0, k)$)**.** *In state $(0, k)$, the IPO payment Y_k and the acquisition payment X_k are weakly increasing in U. Consequently, along a no-event continuation path with a finite deadline, both payments weakly decline over time and reach their minimum at the deadline.*

2.3 State $(0,0)$: Deadlines and Milestone Promises

In the initial state, no capital has been developed, exit is infeasible, and the entrepreneur must exert effort along both dimensions. Upon a breakthrough, the startup transitions to $(h,0)$ or $(0,k)$, and the entrepreneur's continuation utility jumps to the milestone promise u_{h0} or u_{0k}—the continuation utility with which the corresponding intermediate state is entered. The milestone promises summarize the continuation contracts following each breakthrough, transmitting the exit opportunities, IPO payments, milestone payments, and post-milestone deadlines of states $(h,0)$ and $(0,k)$ back to the initial-stage incentive problem. Promise keeping yields

$$\dot{U}_{00} = rU - \mu_h(u_{h0} - U) - \mu_k(u_{0k} - U), \tag{5}$$

and effort along both dimensions is incentive compatible if and only if

$$\mu_h(u_{h0} - U) \geq \gamma_h \quad \text{and} \quad \mu_k(u_{0k} - U) \geq \gamma_k. \tag{IC-hk}$$

In the continuation region, the VC's value function solves

$$(r + \mu_h + \mu_k)V_{00}(U) = \max_{u_{h0}, u_{0k}} \Big\{\mu_h V_{h0}(u_{h0}) + \mu_k V_{0k}(u_{0k}) + V_{00}'(U)\dot{U}_{00}\Big\}, \tag{6}$$

subject to (IC-hk) and the boundary condition $V_{00}(0) = 0$. The first two terms are the VC's expected continuation values following the two possible breakthroughs, while the last term captures the effect of the evolution of the entrepreneur's continuation utility.

Proposition 3 (Optimal Contract in State $(0,0)$)**.** *The optimal contract has the following features:*

1. *Initial continuation utility U_{00} maximizes $V_{00}(U)$, is interior, and satisfies $V_{00}'(U_{00}) = 0$.*

2. *The entrepreneur receives no payouts in state $(0,0)$, $dy = 0$, and absent a breakthrough, the contract terminates at a finite deadline when U reaches zero.*

3. *Upon a breakthrough, the continuation utility jumps to the milestone promise for that branch. Absent a binding effort constraint, the optimal promises u_{h0} and u_{0k} satisfy*

$$V_{h0}'(u_{h0}) = V_{00}'(U), \qquad V_{0k}'(u_{0k}) = V_{00}'(U).$$

If either solution lies below its incentive requirement, the corresponding constraint binds:

$$u_{h0} = U + \frac{\gamma_h}{\mu_h}, \qquad u_{0k} = U + \frac{\gamma_k}{\mu_k}.$$

If the milestone promise exceeds the payout boundary of the new state, the excess is paid out immediately as a milestone payment.

The value function $V_{00}(U)$ is concave and continuously differentiable.

Three features deserve comment. First, the entrepreneur receives no payouts in state $(0, 0)$. Along the equilibrium path $U \leq U_{00}$, the VC's value is weakly increasing in U, so an immediate payout—which lowers U one-for-one—cannot increase the VC's value and brings termination closer. Compensation is therefore deferred until a milestone or later.

Second, the finite deadline is endogenous rather than imposed. The VC could start the entrepreneur with a sufficiently large promise to avoid termination, but doing so requires conceding greater rents. Limited liability rules out negative transfers, so allowing the continuation utility to drift toward zero and terminating after sufficiently prolonged lack of progress provides discipline. The optimal initial promise U_{00} balances the rent cost of a larger promise against the expected loss from prematurely terminating a viable startup. Figure 5 displays the resulting dynamics.

Third, the milestone promises balance their value after a breakthrough against their cost through promise keeping before the breakthrough. Consider the human-capital branch. Raising u_{h0} marginally changes the VC's continuation value after the breakthrough at rate $V'_{h0}(u_{h0})$, but it also requires delivering a larger contingent promise from the current state, whose shadow value is $V'_{00}(U)$. When the effort constraint is slack, the optimum therefore satisfies $V'_{h0}(u_{h0}) = V'_{00}(U)$, and analogously for u_{0k}. If the unconstrained promise is too low to provide effort incentives, the corresponding incentive floor binds.

The terms of entry into the intermediate states depend on when the breakthrough arrives. Early in the initial stage, when U is high, the contract grants generous milestone promises, translating into long deadline extensions, possibly milestone payouts, and, in state $(0, k)$, acquisition-contingent payments. As the initial deadline approaches and U falls, milestone promises move toward their incentive floors and these rewards shrink.

Corollary 3 (Milestone Promises and the Timing of Breakthroughs)**.** *For each intermediate state, the milestone promise, the milestone payment, and the post-milestone deadline are*

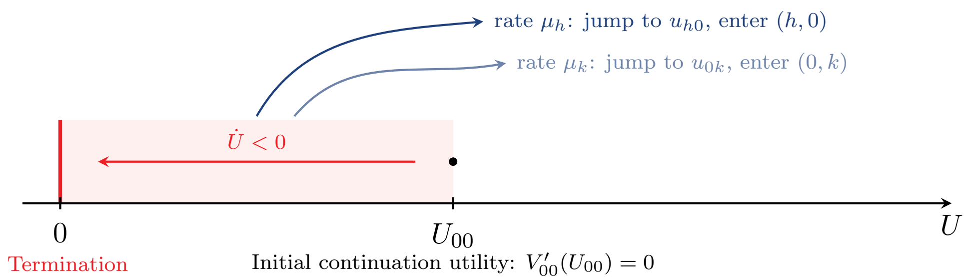

Figure 5: **State $(0,0)$: continuation utility dynamics.** The contract starts at the continuation utility U_{00}. Without a breakthrough, continuation utility drifts down to the termination boundary, so the initial stage always has a finite deadline. At the first breakthrough, the startup enters the corresponding state, and the continuation utility jumps to the milestone promise u_{h0} or u_{0k}.

weakly increasing in the continuation utility U at the time of the breakthrough, and therefore weakly decreasing in the time taken to reach the milestone.

3 Analysis and Implications

Having characterized the optimal contract, we now turn to its economic implications. We proceed in five steps. First, we isolate the role of control over exit by comparing the two intermediate states when their technologies and exit markets are otherwise symmetric. Second, we examine how changes in exit-market conditions affect contract terms and post-milestone runway. Third, we study how exit markets affect pledgeable income and VC financing. Fourth, we study the impact of acquihire markets on exit outcomes. Finally, we describe the model's empirical implications.

3.1 Control over Exit and Contract Terms

The distinction between inalienable and transferable capital determines who controls an intermediate exit. In state $(h, 0)$, the entrepreneur owns the human capital sought by the acquirer and controls the acquihire exit. In state $(0, k)$, transferable capital belongs to the VC, who controls the acquisition exit. To isolate the implications of this difference in control, we assume in this subsection that the technology and exit markets are otherwise symmetric:

$$\begin{aligned} \mu_h = \mu_k = \mu, \qquad \gamma_h = \gamma_k = \gamma, \qquad \lambda_h = \lambda_k = \lambda, \\ \theta_h = \theta_k = \theta \in (0,1), \qquad \Pi_h = \Pi_k = \Pi, \qquad \alpha_h = \alpha_k = \alpha, \end{aligned} \tag{7}$$

and that

$$\mu(I - \Pi) > \gamma. \tag{8}$$

The restriction in equation (8) is natural and requires that the value of effort exerted to exit via an IPO instead of an intermediate exit exceeds the incentive payment needed to induce that effort. We impose this restriction only in this subsection; the optimal contract characterized in Sections 1 and 2 and further results do not require it.

As a first step, consider how the contract shapes the bargaining position of the party that controls an outside sale. Under symmetry, the bargaining prices derived in Section 2 read

$$H = \theta\Pi + (1-\theta)(U - X_h) \qquad \text{and} \qquad K = \theta\Pi + (1-\theta)\big[V_{0k}(U) + X_k\big].$$

When the entrepreneur controls the sale, keeping her compensation inside the startup raises her threat point and therefore the price she can negotiate with an acquihire buyer. Paying her upon an acquihire has the opposite effect, so the optimal contract sets $X_h = 0$. When the VC controls the sale, by contrast, promising the entrepreneur a payment upon acquisition makes the VC less willing to accept a low offer and thereby raises the acquisition price. Thus, the contract uses different forms of compensation to strengthen the bargaining position of the party controlling exit. This difference in bargaining positions determines how compensation is structured following the first development milestone.

Proposition 4 (Control over Exit and Contract Terms)**.** *Suppose the primitives of the two intermediate states are symmetric as in equation (7), with $\theta < 1$, and that $\mu(I - \Pi) > \gamma$. Fix the on-path continuation utility U in state $(0,0)$ when a milestone is reached, and evaluate all contract terms upon entry into the intermediate state after any associated milestone payment.*

1. *The entrepreneur is promised weakly more continuation utility following the development of human capital than following the development of transferable capital, both before and after any immediate milestone payment:*

$$u_{0k} \leq u_{h0}, \qquad \textit{and} \qquad \min\{u_{0k}, \overline{U}_{0k}\} \leq \min\{u_{h0}, \overline{U}_{h0}\}.$$

2. *Immediate milestone payments are weakly smaller following the development of human capital,*

$$B_h(U) = \big[u_{h0} - \overline{U}_{h0}\big]^+ \leq \big[u_{0k} - \overline{U}_{0k}\big]^+ = B_k(U).$$

IPO-contingent payments, by contrast, are weakly larger: $Y_k \leq Y_h < I$*.*

3. *The optimal acquihire-contingent payment is zero,* $X_h = 0$*, whereas the entrepreneur receives a strictly positive acquisition-contingent payment,* $X_k > 0$*. Consequently, the VC extracts the buyer's full valuation upon an acquisition,* $K = \Pi$*.*

Proposition 4 shows that control over exit changes how the contract rewards the first development milestone, even when technologies and exit markets are otherwise identical. When human capital is developed first, continuation utility both rewards the entrepreneur and strengthens her bargaining position in a subsequent acquihire. The contract therefore promises her more continuation utility and defers more compensation toward the IPO, while avoiding acquihire-contingent payments that would weaken her bargaining position.

When transferable capital is developed first, the entrepreneur's continuation utility is not the VC's bargaining threat point in an acquisition. The contract instead relies relatively more on immediate and acquisition-contingent compensation. In particular, promising the entrepreneur part of the acquisition proceeds makes the VC more willing to reject a low offer, raising the negotiated acquisition price. At the optimum, this commitment allows the VC to extract the buyer's full valuation.

Thus, holding technologies and exit-market opportunities fixed, entrepreneur control shifts compensation toward deferred and IPO-contingent pay, whereas VC control shifts compensation toward immediate and acquisition-contingent pay. Control over exit therefore shapes not only the division of proceeds at exit, but also the timing and form of compensation used to reward development.

3.2 Exit Markets and Contract Terms

We now relax the symmetry imposed in Section 3.1 and examine how exit-market conditions affect the contract following a development milestone. We focus on two objects that summarize the continuation contract: the payout boundary, which determines when compensation is paid immediately, and the stationary point of continuation utility, which determines whether continuation carries a finite deadline. We vary the value Π_i, arrival rate λ_i, and bargaining power θ_i associated with each outside-sale opportunity.

Proposition 5 (Exit Markets, Milestone Terms, and Deadlines)**.** *Suppose* $\theta_h, \theta_k < 1$*. Then:*

1. *In state $(h, 0)$, the deadline threshold $\hat{U}_{h0}$ is strictly increasing in Π_h, λ_h, and θ_h, whereas the payout boundary $\overline{U}_{h0}$ is strictly increasing in Π_h and independent of λ_h and θ_h.*

2. *In state $(0, k)$, the deadline threshold and payout boundary coincide, $\overline{U}_{0k} = \hat{U}_{0k}$. This common threshold is strictly increasing in Π_k and λ_k and is independent of θ_k.*

Proposition 5 shows that stronger exit opportunities generally expand the range of promises associated with a finite deadline and reduce the range triggering an immediate milestone payment. When the entrepreneur controls exit, the deadline threshold responds to the acquihire value, arrival rate, and her bargaining power, while the payout boundary depends only on the buyer's valuation. When the VC controls exit, the deadline and payout thresholds coincide and depend on the acquisition value and arrival rate, but not on the VC's bargaining power.

Conditional Post-Milestone Runway. Exit markets also affect the length of a finite deadline. Let $T_D^{h0}(U)$ denote the remaining deadline in state $(h, 0)$ when the state is entered with continuation utility U. Holding this entry promise fixed,

$$\frac{\partial T_D^{h0}(U)}{\partial \Pi_h} < 0, \qquad \frac{\partial T_D^{h0}(U)}{\partial \lambda_h} < 0, \qquad \frac{\partial T_D^{h0}(U)}{\partial \theta_h} < 0, \qquad U \in (R_h, \hat{U}_{h0}).$$

Thus, stronger acquihire opportunities shorten post-milestone runway, holding the entry promise fixed: they make outside exit more attractive and require continuation utility to erode more rapidly along the no-event path. Acquisition-market effects are less uniform because the entrepreneur benefits from an acquisition only when acquisition-contingent compensation is used; Appendix A.6 provides the full comparative statics. We next turn to the ex ante financing effects, which incorporate the endogenous response of the entire contract.

3.3 Exit Markets and VC Financing

A stronger acquihire market has two opposing effects that operate at different stages of development. Before human capital is developed, a more valuable future acquihire lowers the cost of providing incentives to develop it. After human capital is developed, however, the same opportunity strengthens the entrepreneur's outside option and makes retaining her until an IPO more costly. A more valuable acquisition does not create the same retention problem because the VC controls the sale. We first isolate this tradeoff analytically in the stationary benchmark and then show that the same non-monotonicity arises under the optimal long-term contract.

Stationary Benchmark. A stationary contract conditions payments on the current development state and observable transitions or exits, but not on elapsed time or the history within a state. The benchmark thus retains moral hazard, bargaining, and limited commitment while removing payment dynamics and the use of termination. Appendix B characterizes the optimal stationary contract.

Proposition 6 (Exit Values and VC Financing). *Let V_{00}^s denote the VC's initial payoff under the optimal stationary contract. Define*

$$\overline{\Pi}_h = \frac{\gamma_h + \gamma_k}{r} + \frac{\gamma_h(r + \theta_h\lambda_h + \mu_h)}{\theta_h\lambda_h\mu_h}. \tag{9}$$

Under this contract:

1. *The VC's initial payoff satisfies:*

$$\frac{\partial V_{00}^s}{\partial \Pi_h} \begin{cases} > 0, & \Pi_h < \overline{\Pi}_h, \\ < 0, & \Pi_h > \overline{\Pi}_h. \end{cases}$$

   The VC's initial payoff is therefore hump-shaped in Π_h and peaks at $\Pi_h = \overline{\Pi}_h$.

2. *The VC's initial payoff is strictly increasing in the acquisition value:*

$$\frac{\partial V_{00}^s}{\partial \Pi_k} > 0.$$

3. *The entrepreneur's initial payoff is constant in Π_h below $\overline{\Pi}_h$ and strictly increasing in Π_h above it. It is independent of Π_k.*

The threshold $\overline{\Pi}_h$ separates two regimes. When $\Pi_h < \overline{\Pi}_h$, the incentive requirement binds. A higher Π_h makes achieving the human-capital milestone in state $(0, 0)$ more valuable without raising the entrepreneur's initial utility: the future acquihire buyer supplies some of the value needed to reward human-capital development. Pledgeable income therefore rises. When $\Pi_h > \overline{\Pi}_h$, the acquihire opportunity provides more continuation value than incentives

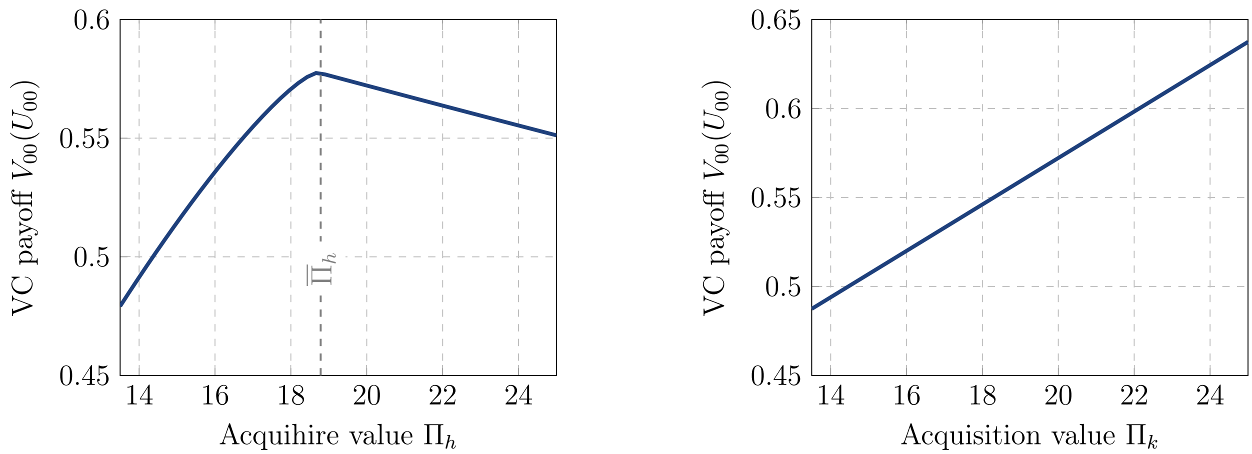

Figure 6: **Exit Values and VC Payoff.** The figure plots the VC's initial payoff $V_{00}(U_{00})$ in the optimal contract against the acquihire value Π_h (left panel) and the acquisition value Π_k (right panel). The vertical line marks the stationary-contract threshold $\overline{\Pi}_h$ given by equation (9). Baseline parameters: $r = 0.40$, $\mu_h = 0.10$, $\mu_k = 0.03$, $\gamma_h = \gamma_k = 0.30$, $\lambda_h = \lambda_k = 0.15$, $\theta_h = \theta_k = 0.7$, $\Pi_h = \Pi_k = 20$, $I = 60$, $\alpha_h = \alpha_k = 0$.

require. Further increases in Π_h then raise the rent required to retain the entrepreneur after human capital has been developed in state $(h, 0)$, reducing the VC's payoff. Accordingly, $\overline{\Pi}_h$ is higher when incentive provision is more demanding and lower when acquihire opportunities arrive more frequently or the entrepreneur has greater bargaining power.

Acquisition markets do not generate the same tradeoff because the VC controls the sale. A higher Π_k raises the value captured upon acquisition without increasing the entrepreneur's initial utility, so pledgeable income increases monotonically with Π_k.

The non-monotonic effect of acquihire values is not specific to the stationary restriction. Figure 6 shows that, under the optimal long-term contract, the VC payoff $V_{00}(U_{00})$ first increases and then decreases with Π_h, while it increases with Π_k. The same hump-shaped pattern is robust across alternative parameter specifications, although the location of the turning point changes. Deadlines and state-contingent milestone promises can shift the turning point relative to the stationary benchmark, but the underlying tradeoff remains the same: a stronger market for entrepreneurial talent initially facilitates financing and, once sufficiently attractive, reduces it.

3.4 Acquihire Markets and Exit Outcomes

We now ask how acquihire markets affect the way startups ultimately exit. To do so, we solve the model and calculate the probabilities of six possible outcomes for the startup: obsolescence, termination at a contractual deadline in state $(0, 0)$ or in a later state, acquisition, acquihire, and IPO. Appendix C derives semi-closed-form solutions for exit probabilities. Figure 7 reports these probabilities as a function of the acquihire value Π_h, together with the incidence and length of endogenous deadlines.

The stationary benchmark isolates the role of dynamic contracting. Without deadlines, changes in Π_h affect payoffs but not the transition or buyer arrival rates, and therefore leave exit probabilities unchanged. Under the optimal long-term contract, by contrast, deadlines respond endogenously to Π_h, changing both which development states startups reach and how long they remain there. As shown in Proposition 6 and Figure 6, initial increases in Π_h make reaching state $(h, 0)$ more valuable and therefore increase the cost of terminating the startup before the human-capital milestone. The optimal contract responds by extending the initial deadline in state $(0, 0)$. More startups then reach state $(h, 0)$, increasing the probability of an acquihire. In this region, the deadline in state $(h, 0)$ is generally long and may be infinite.

As Π_h rises further, the second force behind the hump shape in Proposition 6 and Figure 6 becomes important. A more valuable acquihire makes exit increasingly attractive to the entrepreneur after human capital has been developed, weakening her incentives to continue toward an IPO. The optimal contract responds by introducing and then tightening the deadline in state $(h, 0)$. Termination after the milestone becomes more likely, eventually reducing the probability that an acquihire occurs. Thus, the relation between the value and frequency of acquihires is non-monotonic: sufficiently attractive acquihire opportunities can ultimately make acquihires less likely because the incentive problems they create induce the contract to terminate the startup sooner.

3.5 Empirical Implications

The model yields empirical predictions for compensation, milestone timing, runway, and VC financing. The central distinction is whether startup value resides primarily in human or transferable capital, which determines who controls exit. This distinction is empirically relevant: acquisitions are used to transfer skilled workers (Ouimet and Zarutskie, 2020), respond to shortages of high-skilled labor (Chen et al., 2024), and are more common when hiring workers directly is more difficult (Chen et al., 2021).

Capital Development and Compensation. Conditional on otherwise similar startups,

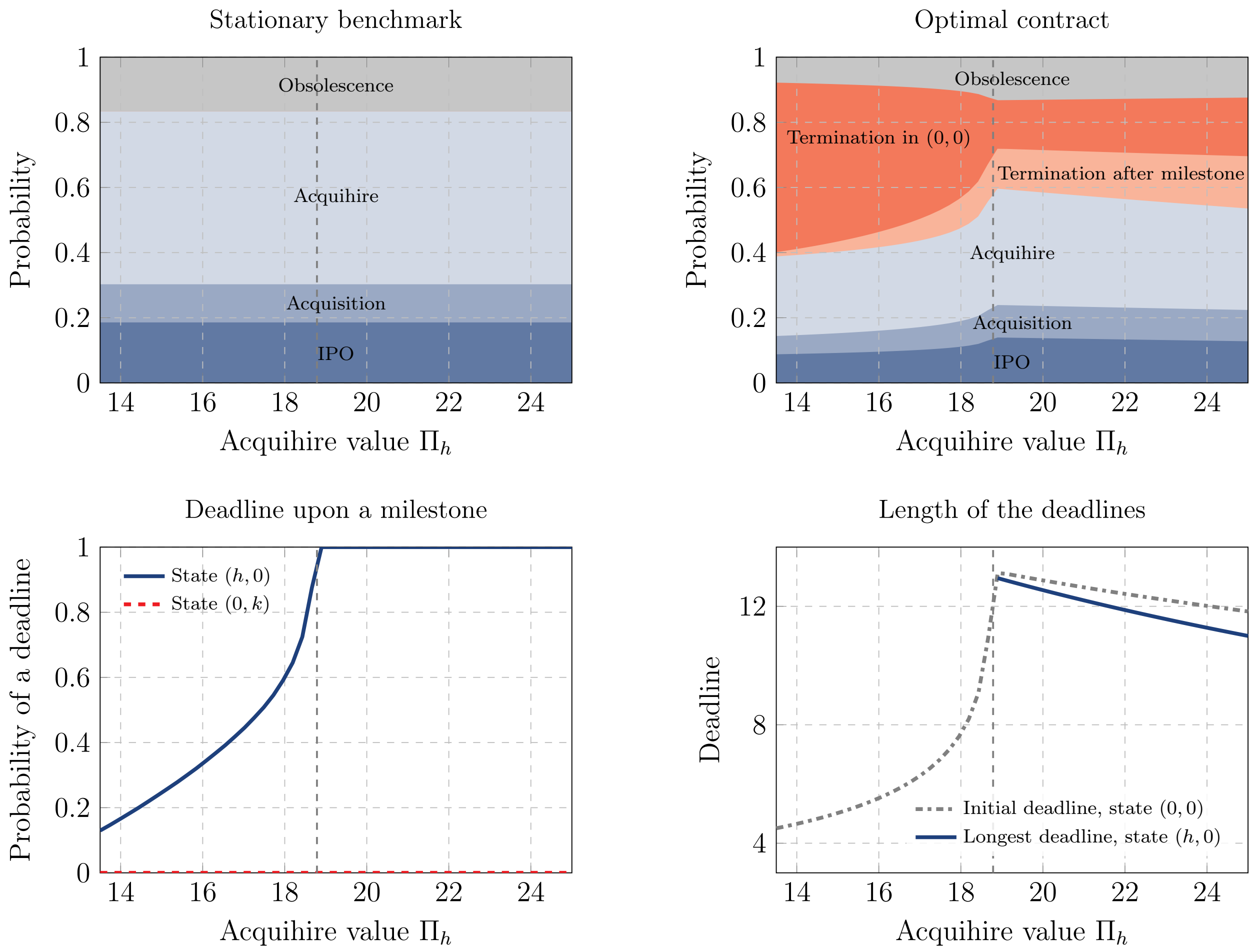

Figure 7: **Exit Outcomes, Deadlines, and the Acquihire Market.** Top panels: how the relationship ends under the stationary benchmark (left) and the optimal contract (right) as a function of the acquihire value Π_h. Bottom left: probability of a finite deadline upon entering state $(h,0)$ or $(0,k)$. Bottom right: the initial deadline in state $(0,0)$ and the deadline in state $(h,0)$ when the human-capital milestone is reached at $U = U_{00}$, which is infinite to the left of the threshold. The dashed line marks $\overline{\Pi}_h$ from equation (9). Baseline parameters: $r = 0.40$, $\delta = 0.015$, $\mu_h = 0.10$, $\mu_k = 0.03$, $\gamma_h = \gamma_k = 0.30$, $\lambda_h = \lambda_k = 0.15$, $\theta_h = \theta_k = 0.7$, $\Pi_h = \Pi_k = 20$, $I = 60$, $\alpha_h = \alpha_k = 0$.

the type of capital developed first should predict subsequent compensation. Following a human-capital milestone, compensation should be relatively deferred and more heavily tied to a future IPO. Empirically, such deferral may take the form of retained founder equity, vesting, or future equity grants. Consistent with this prediction, founder cash compensation is low early in a startup's life (Hall and Woodward, 2010; Ewens, Nanda, and Stanton, 2024), while vesting is prevalent in early-stage and pre-revenue firms (Kaplan and Strömberg, 2003).

Following a transferable-capital milestone, by contrast, compensation should be relatively more immediate and more likely to be contingent on an acquisition. A marketable product or technology makes the firm's assets transferable and shifts control over exit toward investors. Consistent with this prediction, founder cash compensation rises as startups develop tangible, marketable products (Ewens et al., 2024), and VC contracts condition cash-flow and control rights on development milestones (Kaplan and Strömberg, 2003).

The model further predicts that acquisition-contingent compensation should be most common when a transferable-capital milestone is reached quickly. Absent further progress, the entrepreneur's continuation promise declines and the acquisition payment eventually switches off. Thus, founder bonuses or carve-outs in VC-controlled sales should be concentrated among startups that reach development milestones relatively early.

Milestone Timing and Runway. Milestones reached earlier should be followed by larger continuation promises, larger milestone payments, and longer subsequent runway. Exit-market conditions generate a distinct prediction. Stronger acquihire markets shorten runway after a human-capital milestone because they make outside exit more attractive to the entrepreneur. At the same time, they can lengthen the initial runway by making human-capital development more valuable. Stronger acquihire markets should therefore be associated with longer runway before human capital is developed but shorter runway afterward.

Exit Markets and VC Financing. The financing implications again depend on who controls exit. Stronger acquisition markets increase pledgeable income and should therefore be associated with a greater prevalence of VC financing and larger investments. This prediction is consistent with evidence that VC investment increases with M&A activity and with institutional changes that facilitate takeovers (Phillips and Zhdanov, 2024).

Markets for entrepreneurial talent generate a different prediction. A stronger acquihire market initially facilitates financing because the prospect of a buyer-funded payoff helps reward the entrepreneur for developing human capital. Once acquihire opportunities become sufficiently attractive, however, they generate rents beyond what is needed for incentives and make retaining the entrepreneur after the human-capital milestone more costly. VC financing should therefore be hump-shaped in the strength of the market for entrepreneurial talent. The turning point should occur at stronger acquihire markets when entrepreneurial effort is more costly to induce or human-capital development is slower, and at weaker markets when buyers are more plentiful or entrepreneurs have greater bargaining power.

4 Conclusion

Who controls a startup exit depends on what is being sold. Transferable capital can be assigned to investors; human capital cannot be transferred by contract and leaves with the entrepreneur. Control over exit therefore follows the type of capital that drives the transaction rather than being freely allocated by contract.

This allocation of control shapes the optimal VC contract. In addition to providing effort incentives, contract design strengthens the bargaining position of the party controlling exit. When the entrepreneur controls exit, compensation is deferred, raising her value from remaining with the startup and hence her threat point in an acquihire. When the VC controls exit, the entrepreneur may instead receive acquisition-contingent compensation, making the VC less willing to sell cheaply. In both cases, contractual compensation is structured to extract more value from outside buyers.

The distinction has different implications for startup financing. Stronger acquisition markets increase pledgeable income because the VC controls the asset being sold and captures the resulting gains. Acquihire markets initially facilitate financing because buyer-funded payoffs help reward the development of human capital. But sufficiently attractive acquihire opportunities make retaining the entrepreneur more costly and shift rents away from investors. Financing is therefore hump-shaped in the strength of the market for entrepreneurial talent, even though it increases monotonically with the value of conventional acquisitions.

The broader implication is that the effect of exit markets depends on who controls the asset being sold. When financiers control exit, stronger outside markets translate directly into pledgeable value. When the relevant asset is inalienable, outside markets can initially help finance incentives but eventually strengthen the entrepreneur's outside option and generate rents. Better markets for talent and better markets for startup financing are therefore not the same thing.

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Appendix

This appendix collects the proofs. Appendix A solves the optimal long-term contract by backward induction on the development state, characterizing the VC's value function first in the intermediate states $(h, 0)$ and $(0, k)$, where exit is feasible, and then in the initial state $(0, 0)$, which inherits the two branch value functions through the milestone promises. Two lemmas complete the state-$(0, 0)$ characterization: Lemma 1 shows that the promise can drift up only where the VC's value function is steep enough to make an acquisition payment worthwhile, and Lemma 2 uses this to establish that the promise drifts strictly down below the initial promise, so that state $(0, 0)$ always carries a finite deadline. Appendix A.5 reads the optimal contract off these value functions, Appendix A.6 derives the deadlines, Appendix A.7 derives the results for the case in which exit markets are symmetric, and Appendix A.8 derives additional comparative statics with respect to the stationary point and payout boundary. Appendix B defines and characterizes the optimal stationary contract, which carries no deadlines and therefore admits closed forms. Appendix C derives the exit probabilities.

Randomization Over Continuation Utility. We allow the optimal long-term contract to randomize over the entrepreneur's continuation utility, and we show that doing so is never strictly optimal. Randomization enters each Hamilton–Jacobi–Bellman equation as the second derivative of the VC's value function: wherever that function is locally convex, a mean-preserving spread of the promise leaves promise keeping intact and makes the VC strictly better off. Ruling out randomization is therefore an output of the verification, not a restriction on the contracting space. In states $(h, 0)$ and $(0, k)$ we verify that the candidate value function is concave. In state $(0, 0)$ the logic runs in the opposite direction: the ability to randomize is what delivers concavity in the first place, which we then use to show that no non-degenerate randomization is ever strictly optimal.

A Optimal Long-Term Contracting: Proofs

A.1 Objective Functions

This appendix describes the VC's and Entrepreneur's objective functions. We denote by T_h, T_k, and T_i the times of acquihire, acquisition, and IPO, and by T_q the time at which the entrepreneur leaves with her human capital, which she may do at any point when $h_t = h$. The VC's acceptance probability $a = \{a_t\}_{t\geq 0}$, with $a_t \in [0, 1]$, records whether it completes an acquisition in state $(0, k)$, and T_k is the time of the first offer it accepts. We let $T = \min\{T_h, T_k, T_i, T_D, T_q\}$ denote the end of the contractual relationship. If the relationship ends without an exit or failure, that is, if $T \in \{T_D, T_q\}$, the entrepreneur gets R_h when $h_T = h$ and zero otherwise, while the VC gets R_k when $k_T = k$ and zero otherwise.

Let IC be the set of contracts $\mathcal{C}$ and acquisition acceptance decisions a that induce the entrepreneur to exert effort, stay with the firm, and accept exit opportunities. The VC's initial payoff, gross of the financing cost, is

$$V_0 = \sup_{(\mathcal{C},a)\in IC} \mathbb{E}\left[e^{-rT}\left(\mathbb{I}_{\{T=T_k\}}K_T + \mathbb{I}_{\{T=T_i\}}I + \mathbb{I}_{\{T=T_D\}}\mathbb{I}_{\{k_T=k\}}R_k\right) - \int_0^T e^{-rt}dY_t\right],$$

where the first term is the proceeds from an acquisition, an IPO, or a termination in state $(0,k)$ and the second term is the cumulative compensation paid to the entrepreneur.[10] The expectation is taken under the measure induced by the acceptance rule a and by the entrepreneur exerting effort, staying with the firm, and accepting exit opportunities.

Fix a pair $(\mathcal{C},a)$ and a strategy σ for the entrepreneur, consisting of the effort processes $\{e_t^h, e_t^k\}_{t\geq 0}$, her decisions to accept an acquihire and to undertake an IPO, and her quitting time T_q. Her payoff is

$$U_0(\sigma|\mathcal{C},a) = \mathbb{E}^{\sigma}\left[e^{-rT}\left(\mathbb{I}_{\{T=T_h\}}H_T + \mathbb{I}_{\{T\in\{T_D,T_q\}\}}\mathbb{I}_{\{h_T=h\}}R_h\right) + \int_0^T e^{-rt}(dY_t + \Gamma_t dt)\right],$$

where $\Gamma_t = \mathbb{I}_{\{h_t=0\}}(1-e_t^h)\gamma_h + \mathbb{I}_{\{k_t=0\}}(1-e_t^k)\gamma_k$ is the flow of private benefits. The three terms in the square brackets are, respectively, the proceeds from an acquihire, the value of her human capital if she leaves without an exit, and the compensation and private benefits collected until the relationship ends. The expectation is taken under the measure induced by σ, holding fixed the contract $\mathcal{C}$ and the VC's decisions to complete an acquisition a. Let $\sigma^\star$ denote the strategy under which the entrepreneur exerts full effort, accepts every acquihire offer, undertakes an IPO in state (h,k), and never quits. The pair $(\mathcal{C},a)$ is incentive compatible if $\sigma^\star$ attains $\sup_\sigma U_0(\sigma|\mathcal{C},a)$ and a is optimal for the VC at every arrival given $\sigma^\star$. We then write $U_0 = U_0(\sigma^\star|\mathcal{C},a)$.

A.2 State $(h,0)$

The relevant assumptions made in the main text are:

Assumption 1 (Assumptions $(h,0)$)**.** *Assume that*

- *An IPO yields a larger payoff than an acquihire* $I > \Pi_h$.
- *The entrepreneur's outside option is sufficiently weak* $R_h < I - \frac{\gamma_k}{\mu_k}$.
- *Acquihires are efficient,* $\Pi_h > \frac{\mu_k I + \lambda_h \Pi_h}{r+\mu_k+\lambda_h}$.
- *Effort is efficient* $\frac{\mu_k I}{r+\mu_k} > \frac{\gamma_k}{r}$.
- *The entrepreneur has some bargaining power* $\theta_h \in (0,1]$.

Under Assumption 1, the following holds:

[10] The transfers (K_t, H_t) are determined through bilateral bargaining with the acquirer, which we characterize next. For simplicity, we do not fold the associated optimization into the formulation below.

Proposition 7 (VC's Value Function in State $(h, 0)$)**.** *The VC's value function in state $(h, 0)$ is C^1, concave, and given by*

$$V_{h0}(U) = \begin{cases} V^p_{h0}(U) - V^p_{h0}(R_h) \left(\dfrac{\hat{U}_{h0} - U}{\hat{U}_{h0} - R_h} \right)^{p_h}, & U \in [R_h, \hat{U}_{h0}], \\ V^p_{h0}(U) + \left[V_{h0}(\Pi_h) - V^p_{h0}(\Pi_h) \right] \left(\dfrac{U - \hat{U}_{h0}}{\Pi_h - \hat{U}_{h0}} \right)^{p_h}, & U \in [\hat{U}_{h0}, \Pi_h], \\ V_{h0}(\Pi_h) - (U - \Pi_h), & U > \Pi_h, \end{cases}$$

where $A_h := r + \mu_k + \lambda_h$, $a_h := r + \theta_h \lambda_h$, $p_h := \frac{A_h}{a_h}$, and

$$\hat{U}_{h0} = \frac{\gamma_k + \theta_h \lambda_h \Pi_h}{a_h},$$
$$V^p_{h0}(U) = \frac{\mu_k (I - \hat{U}_{h0}) - \gamma_k}{A_h} + \frac{\mu_k (\hat{U}_{h0} - U)}{A_h - a_h},$$
$$V_{h0}(\Pi_h) = \frac{\mu_k I + \lambda_h \Pi_h}{A_h} - \Pi_h.$$

We write $S_h := \frac{\mu_k I + \lambda_h \Pi_h}{A_h} = V_{h0}(\Pi_h) + \Pi_h$ for the full surplus in state $(h, 0)$.

Proof of Proposition 7. **VC's Problem.** In state $(h, 0)$, by standard verification arguments (see, e.g., Biais, Mariotti, Rochet, and Villeneuve, 2010), if a C^1 and piecewise C^2 function $V_{h0}(U)$ of at most linear growth solves the HJB equation

$$0 = \max \Bigg\{ -(r + \mu_k + \lambda_h) V_{h0}(U) + \max_{Y_h, X_h \geq 0} \Bigg(\mu_k (I - Y_h) - \lambda_h X_h \\ + V'_{h0}(U) \big[(r + \mu_k + \theta_h \lambda_h) U - \mu_k Y_h - \theta_h \lambda_h (\Pi_h + X_h) \big] \Bigg), -V'_{h0}(U) - 1, V''_{h0}(U) \Bigg\}, \tag{HJB_h}$$

subject to

1. The IC constraint: $\mu_k (Y_h - U) \geq \gamma_k$.
2. The IPO participation constraint: $Y_h \geq R_h$.
3. The acquihire participation constraint: $\Pi_h + X_h \geq U$,

then this function $V_{h0}(U)$ is the VC's value function.

The outer maximum in the (HJB_h) equation compares continuation (the first term) with an immediate payout (the second term) and randomization over continuation utility (the third term). In the continuation term, the term multiplying $V'_{h0}(U)$ is the drift of the continuation utility $\dot{U}$, see equation (1). For the VC, the marginal benefit of making an immediate payout (decreasing U) is given by $-V'_{h0}(U)$, while the marginal cost is 1. For the VC, if $V''_{h0}(U) > 0$, then by Jensen's inequality the VC would be strictly better off randomizing over the entrepreneur's continuation utility while keeping the expected continuation utility fixed. Because the entrepreneur can walk away and get her reservation utility R_h, we can focus throughout on continuation utilities $U \geq R_h$.

Acquihire Payments. The derivative of the HJB objective with respect to X_h is $-\lambda_h\big(1 + \theta_h V'_{h0}(U)\big)$. We conjecture (and verify below) that the value function satisfies $V'_{h0}(U) \geq -1$. This result in combination with $\theta_h \in (0,1]$ implies that $1 + \theta_h V'_{h0}(U) \geq 1 - \theta_h \geq 0$, so the objective is weakly decreasing in X_h. The optimal acquihire payment is therefore the lowest payment that ensures acceptance of the offer, $X_h = \max\{U - \Pi_h, 0\}$.

IPO Payments. We conjecture (and verify below) that the value function satisfies $V'_{h0}(U) \geq -1$. Therefore, the objective is weakly decreasing in Y_h. As a consequence, an optimal payment upon IPO is the lowest payment that ensures acceptance of the IPO offer and effort: $Y_h = \max\{R_h, U + \frac{\gamma_k}{\mu_k}\} = U + \frac{\gamma_k}{\mu_k}$ since $U \geq R_h$.

Value Function Conjecture. We conjecture (and verify below) that the value function solves a differential equation and boundary conditions for $U \in [R_h, \Pi_h]$ and is of the form $V_{h0}(U) = V_{h0}(\Pi_h) - (U - \Pi_h)$ for $U > \Pi_h$.

$U \leq \Pi_h$ **Region.** We conjecture that in this region the value function solves the differential equation

$$\begin{aligned} 0 &= -(r + \mu_k + \lambda_h)\, V_{h0}(U) + \mu_k(I - U) - \gamma_k + V'_{h0}(U)\big[(r + \theta_h\lambda_h)U - \gamma_k - \theta_h\lambda_h\Pi_h\big] \\ &= -A_h V_{h0}(U) + \mu_k(I - U) - \gamma_k + V'_{h0}(U) a_h\big[U - \hat{U}_{h0}\big], \end{aligned} \tag{A.1}$$

where A_h, a_h, and $\hat{U}_{h0}$ are defined in the proposition.

Our assumptions imply $\Pi_h > \hat{U}_{h0} > R_h$. For the first inequality, note that acquihire efficiency, $\Pi_h > \frac{\mu_k I + \lambda_h \Pi_h}{A_h}$, rearranges to $\Pi_h > \frac{\mu_k I}{r + \mu_k}$, which combined with effort efficiency, $\frac{\mu_k I}{r+\mu_k} > \frac{\gamma_k}{r}$, yields $\Pi_h > \frac{\gamma_k}{r}$. Writing $\hat{U}_{h0} = w\frac{\gamma_k}{r} + (1-w)\Pi_h$ with $w = \frac{r}{a_h} \in (0,1]$, we obtain $\hat{U}_{h0} < \Pi_h$ since $w > 0$. For the second inequality, let $f(x) = \frac{\gamma_k + x\Pi_h}{r+x}$ and $g(x) = \frac{x\Pi_h}{r+x}$, so that $\hat{U}_{h0} = f(\theta_h\lambda_h)$ and $R_h = \frac{\alpha_h\theta_h\lambda_h\Pi_h}{r+\alpha_h\theta_h\lambda_h} = g(\alpha_h\theta_h\lambda_h)$. It follows that

$$R_h = g(\alpha_h\theta_h\lambda_h) \leq g(\theta_h\lambda_h) < f(\theta_h\lambda_h) = \hat{U}_{h0},$$

since g is increasing, $\alpha_h \leq 1$, and $f > g$ pointwise because $\gamma_k > 0$.

The drift of the continuation utility if the firm remains in state $(h, 0)$ is

$$a_h\big[U - \hat{U}_{h0}\big].$$

For $U = \hat{U}_{h0}$, the continuation utility U does not drift. For $U < \hat{U}_{h0}$, U drifts downwards. For $U > \hat{U}_{h0}$, U drifts upwards. There are therefore two regions.

$U \in [R_h, \hat{U}_{h0}]$ **Region.** In this region, U drifts downwards and the boundary condition at $U = R_h \geq 0$ is $V_{h0}(R_h) = 0$, since the contract is terminated at the entrepreneur's outside option R_h. We show that $V'_{h0}(U) \geq -1$ on this interval, which guarantees optimality of $Y_h = U + \frac{\gamma_k}{\mu_k}$ and $X_h = 0$ and therefore ensures that $V_{h0}(U)$ solves the (HJB_h) equation.

Since the differential equation (A.1) is linear, its general solution is the affine particular solution $V^p_{h0}(U)$ plus the one-parameter homogeneous family $C(\hat{U}_{h0}-U)^{p_h}$. The boundary condition $V_{h0}(R_h)=0$ selects $C=-\frac{V^p_{h0}(R_h)}{(\hat{U}_{h0}-R_h)^{p_h}}$. Therefore,

$$V_{h0}(U)=V^p_{h0}(U)-V^p_{h0}(R_h)\left(\frac{\hat{U}_{h0}-U}{\hat{U}_{h0}-R_h}\right)^{p_h},$$

with $p_h=\frac{A_h}{a_h}$ and where direct substitution shows that

$$V^p_{h0}(U)=\frac{\mu_k(I-\hat{U}_{h0})-\gamma_k}{A_h}+\frac{\mu_k(\hat{U}_{h0}-U)}{A_h-a_h}$$

is a particular solution of the differential equation (A.1).

Since $V^p_{h0}(U)$ is affine with $(V^p_{h0})'(U)=-\frac{\mu_k}{A_h-a_h}$, differentiating the value function and using $1-\frac{\mu_k}{A_h-a_h}=\frac{(1-\theta_h)\lambda_h}{A_h-a_h}$ gives

$$V'_{h0}(U)+1=\frac{(1-\theta_h)\lambda_h}{A_h-a_h}+V^p_{h0}(R_h)\frac{p_h}{\hat{U}_{h0}-R_h}\left(\frac{\hat{U}_{h0}-U}{\hat{U}_{h0}-R_h}\right)^{p_h-1}. \tag{A.2}$$

Differentiating the value function twice gives

$$V''_{h0}(U)=-V^p_{h0}(R_h)\frac{p_h(p_h-1)}{(\hat{U}_{h0}-R_h)^2}\left(\frac{\hat{U}_{h0}-U}{\hat{U}_{h0}-R_h}\right)^{p_h-2}.$$

The value function $V_{h0}(U)$ is concave if and only if $V^p_{h0}(R_h)\geq 0$ since $p_h=\frac{A_h}{a_h}>1$ and $U\leq\hat{U}_{h0}$. Observe that

$$\begin{aligned}
0 &\leq V^p_{h0}(R_h),\\
0 &\leq \frac{A_h-a_h}{\mu_k}V^p_{h0}(R_h),\\
0 &\leq \frac{A_h-a_h}{\mu_k}\left(\frac{\mu_k(I-\hat{U}_{h0})-\gamma_k}{A_h}+\frac{\mu_k(\hat{U}_{h0}-R_h)}{A_h-a_h}\right),\\
R_h &\leq \hat{U}_{h0}+\frac{A_h-a_h}{A_h}\left(I-\hat{U}_{h0}-\frac{\gamma_k}{\mu_k}\right),
\end{aligned}$$

which is true given our bound on R_h. Therefore, $V_{h0}(U)$ is concave. This concavity in combination with

$$V'_{h0}(\hat{U}_{h0})+1=\frac{(1-\theta_h)\lambda_h}{A_h-a_h}\geq 0,$$

implies that

$$V'_{h0}(U)\geq -1.$$

This result implies that $Y_h=U+\frac{\gamma_k}{\mu_k}$ and $X_h=0$ are optimal on $[R_h,\hat{U}_{h0}]$. Therefore the first term of the (HJB_h) equation coincides with the differential equation (A.1) and equals zero by construction, while $V'_{h0}(U) \geq -1$ ensures the second term of the (HJB_h) equation is weakly less than zero. Furthermore, because $V_{h0}(U)$ is concave the third term in the (HJB_h) is non-positive. Therefore, the (HJB_h) equation is satisfied on $[R_h, \hat{U}_{h0}]$.

$U \in [\hat{U}_{h0}, \Pi_h]$ **Region.** On the region $U \in [\hat{U}_{h0}, \Pi_h]$, the continuation utility drifts upwards and the boundary condition at $U = \Pi_h$ is

$$V_{h0}(\Pi_h) = \frac{\mu_k I + \lambda_h \Pi_h}{r + \mu_k + \lambda_h} - \Pi_h, \tag{A.3}$$

because the entrepreneur's contract will not be terminated (since $\Pi_h > \hat{U}_{h0}$, established above, and therefore U drifts upwards at Π_h) and the VC's value is therefore the value from completing the project minus the promised payment to the entrepreneur.

The solution to equation (A.1) and the boundary condition at Π_h for $U \in [\hat{U}_{h0}, \Pi_h]$ is

$$V_{h0}(U) = V^p_{h0}(U) + [V_{h0}(\Pi_h) - V^p_{h0}(\Pi_h)] \left(\frac{U - \hat{U}_{h0}}{\Pi_h - \hat{U}_{h0}} \right)^{p_h},$$

where p_h and $V^p_{h0}(U)$ are defined in the proposition. Differentiating the value function with respect to U yields,

$$V'_{h0}(U) = -\frac{\mu_k}{A_h - a_h} + [V_{h0}(\Pi_h) - V^p_{h0}(\Pi_h)] \frac{p_h}{\Pi_h - \hat{U}_{h0}} \left(\frac{U - \hat{U}_{h0}}{\Pi_h - \hat{U}_{h0}} \right)^{p_h - 1}.$$

A direct calculation shows

$$[V_{h0}(\Pi_h) - V^p_{h0}(\Pi_h)] \frac{p_h}{\Pi_h - \hat{U}_{h0}} = -\frac{(1-\theta_h)\lambda_h}{A_h - a_h},$$

so the derivative reduces to

$$V'_{h0}(U) = -\frac{\mu_k}{A_h - a_h} - \frac{(1-\theta_h)\lambda_h}{A_h - a_h} \left(\frac{U - \hat{U}_{h0}}{\Pi_h - \hat{U}_{h0}} \right)^{p_h - 1}.$$

Adding one and using $1 - \frac{\mu_k}{A_h - a_h} = \frac{(1-\theta_h)\lambda_h}{A_h - a_h}$,

$$V'_{h0}(U) + 1 = \frac{(1-\theta_h)\lambda_h}{A_h - a_h} \left[1 - \left(\frac{U - \hat{U}_{h0}}{\Pi_h - \hat{U}_{h0}} \right)^{p_h - 1} \right] \geq 0, \tag{A.4}$$

because $(1-\theta_h)\lambda_h \geq 0$ and $\frac{U - \hat{U}_{h0}}{\Pi_h - \hat{U}_{h0}} \in [0,1]$ with $p_h - 1 > 0$. Hence $V'_{h0}(U) \geq -1$ throughout the growth region, with equality at $U = \Pi_h$ and everywhere when $\theta_h = 1$. Therefore, the first term of the (HJB_h) equation and the differential equation (A.1) coincide and equal zero by construction. Furthermore, $V'_{h0}(U) \geq -1$ ensures that the second term of the (HJB_h)

equation is weakly less than zero. Furthermore, $V'_{h0}(U)+1$ is weakly decreasing on $[\hat{U}_{h0},\Pi_h]$ since $p_h = \frac{A_h}{a_h} > 1$ and therefore $V_{h0}(U)$ is concave on this region and the third term in (HJB_h) is non-positive. Therefore, (HJB_h) is satisfied on $[\hat{U}_{h0},\Pi_h]$.

$U > \Pi_h$ **Region.** For $U > \Pi_h$ set $V_{h0}(U) = V_{h0}(\Pi_h) - (U - \Pi_h)$, so that $V'_{h0}(U) = -1$ and the payout term $-1 - V'_{h0}(U)$ in the (HJB_h) equation is zero. The value function is linear in U and therefore concave, with $V''_{h0}(U) = 0$. It remains to verify the first term in the (HJB_h) equation. The optimal acquihire payment is $X_h = U - \Pi_h = \max\{U - \Pi_h, 0\}$ and the optimal IPO payment is $Y_h = U + \frac{\gamma_k}{\mu_k}$. Observe that ensuring that the entrepreneur accepts the acquihire offer $\Pi_h + X_h \geq U$ is optimal for the VC at the arrival as well as socially optimal since $\Pi_h > \frac{\mu_k I + \lambda_h \Pi_h}{r+\mu_k+\lambda_h}$. Substituting $V'_{h0}(U) = -1$, $Y_h = U + \frac{\gamma_k}{\mu_k}$, $X_h = U - \Pi_h$, and $V_{h0}(U) + U = V_{h0}(\Pi_h) + \Pi_h$ into the first term of (HJB_h) yields

$$ODE_U = -A_h\big[V_{h0}(\Pi_h) + \Pi_h\big] + \mu_k I + \lambda_h \Pi_h = 0$$

where the last equality follows from equation (A.3). Hence $V_{h0}(U) = V_{h0}(\Pi_h) - (U - \Pi_h)$ solves (HJB_h) for $U > \Pi_h$.

Given that the first, second, and third terms in the (HJB_h) all equal zero, the VC is indifferent between a lump-sum payment, a higher IPO payment, delayed payments, or randomization over any of these.[11]

Continuity and Differentiability. On each of the three regions $[R_h, \hat{U}_{h0}]$, $[\hat{U}_{h0}, \Pi_h]$, and (Π_h, ∞), the value function $V_{h0}(U)$ is continuously differentiable: on the first two it equals $V^p_{h0}(U) + C|U - \hat{U}_{h0}|^{p_h}$, with $V^p_{h0}(U)$ affine, $p_h = \frac{A_h}{a_h} > 1$, and a different constant C on each region, while on the third it is affine. It therefore remains to check that the pieces join in a C^1 fashion at the two junctions $\hat{U}_{h0}$ and Π_h.

At $U = \hat{U}_{h0}$ the power term $C|U - \hat{U}_{h0}|^{p_h}$ vanishes, so both adjacent pieces equal $V^p_{h0}(\hat{U}_{h0})$ and V_{h0} is continuous. Its contribution to the slope, $\pm C p_h |U - \hat{U}_{h0}|^{p_h - 1}$, also vanishes as $U \to \hat{U}_{h0}$ because $p_h - 1 > 0$. Hence both one-sided derivatives equal $V'_{h0}(\hat{U}_{h0}) = (V^p_{h0})'(\hat{U}_{h0}) = -\frac{\mu_k}{A_h - a_h}$, so V_{h0} is C^1 at $\hat{U}_{h0}$.

At $U = \Pi_h$ the growth-region piece satisfies the boundary condition $V_{h0}(\Pi_h)$ in equation (A.3), which is also the value taken by the affine piece at Π_h, so V_{h0} is continuous. The $[\hat{U}_{h0}, \Pi_h]$-region derivative satisfies $V'_{h0}(\Pi_h) = -1$, see equation (A.4), which equals the slope of the affine piece on (Π_h, ∞). Hence V_{h0} is C^1 at Π_h.

Therefore, $V_{h0}(U)$ is C^1 on $[R_h, \infty)$. Moreover, $V_{h0}(U)$ is globally concave: $V'_{h0}(U)$ is continuous by the above and non-increasing on the interior of each of the three regions, and is therefore non-increasing on $[R_h, \infty)$. The fact that the value function we constructed solves (HJB_h), is concave, and is C^1 ensures that it is the optimal VC value function.

□

[11] It is not costly to delay payments at $U > \Pi_h$ because at an acquihire the entrepreneur is indifferent between accepting and rejecting, so no surplus is split.

A.3 State $(0, k)$

The relevant assumptions made in the main text are:

Assumption 2 (Assumptions $(0, k)$)**.** *Assume that*

- *IPOs yield larger payoffs than acquisitions* $I > \Pi_k$.
- *Acquisitions are efficient* $\Pi_k > \frac{\mu_h I + \lambda_k \Pi_k}{r + \mu_h + \lambda_k}$.
- *The entrepreneur's IPO outside option is sufficiently weak* $R_h \leq \frac{\gamma_h}{\mu_h}$.
- *The human-capital milestone arrives fast enough,* $\mu_h > (1 - \theta_k)\lambda_k$.
- *The VC can commit to an IPO in state* (h, k) *but not to an acquisition in state* $(0, k)$.

Under Assumption 2, the following holds:

Proposition 8 (VC's Value Function in State $(0, k)$)**.** *The VC's value function in state* $(0, k)$ *is* C^1*, concave, and given by*

$$V_{0k}(U) = \begin{cases} V_N^p(U) + C_N(\hat{U}_{0k}^{X=0} - U)^{p_{k,N}}, & U \in [0, U_X), \\ S_k - U - L_k(U), & U \in [U_X, \hat{U}_{0k}], \\ S_k - U, & U > \hat{U}_{0k}, \end{cases}$$

where $a_k := r + \lambda_k$, $A_{k,N} := r + \mu_h + \theta_k \lambda_k$, $A_k := r + \mu_h + \lambda_k$, $p_{k,N} := \frac{A_{k,N}}{a_k} > 1$, $q_k := \frac{r}{A_k} \in (0, 1)$*, and the full surplus, the stationary promise, and the drift-stationary point of the no-acquisition-payment dynamics are*

$$S_k := \frac{\mu_h I + \lambda_k \Pi_k}{A_k}, \qquad \hat{U}_{0k} := \frac{\gamma_h + \lambda_k(\Pi_k - S_k)}{r}, \qquad \hat{U}_{0k}^{X=0} := \frac{\gamma_h}{a_k}.$$

On the first region $[0, U_X)$*, we have*

$$C_N = \frac{R_k - V_N^p(0)}{(\hat{U}_{0k}^{X=0})^{p_{k,N}}},$$

$$V_N^p(U) = \frac{\mu_h(I - \hat{U}_{0k}^{X=0}) - \gamma_h + \theta_k \lambda_k \Pi_k}{A_{k,N}} + \frac{\mu_h(\hat{U}_{0k}^{X=0} - U)}{A_{k,N} - a_k},$$

and the switching point is

$$U_X = \inf\{U \geq 0 \mid V_{0k}'(U) \leq -\theta_k\} = \max\left\{ \hat{U}_{0k}^{X=0} - \left(\frac{\theta_k - \frac{\mu_h}{\mu_h - (1-\theta_k)\lambda_k}}{p_{k,N} C_N} \right)^{\frac{1}{p_{k,N} - 1}}, 0 \right\}.$$

On the second region $[U_X, \hat{U}_{0k}]$, *the function* $L_k(U) := S_k - U - V_{0k}(U) \geq 0$ *satisfies* $L_k(U) = 0$ *when* $\theta_k = 1$ *and otherwise is characterized implicitly by*

$$\hat{U}_{0k} - U = C_X L_k(U)^{q_k} + \frac{\lambda_k}{\mu_h + \lambda_k} L_k(U),$$

with

$$C_X = \frac{\hat{U}_{0k} - U_X - \frac{\lambda_k}{\mu_h + \lambda_k} L_k(U_X)}{L_k(U_X)^{q_k}} > 0,$$

$$L_k(U_X) = \begin{cases} S_k - U_X - V_{0k}(U_X^-), & \text{if } U_X > 0, \\ S_k - R_k, & \text{if } U_X = 0. \end{cases}$$

Proof of Proposition 8. **VC's Problem.** In state $(0, k)$, by standard verification arguments (see, e.g., Biais et al., 2010), if a C^1 and piecewise C^2 function $V_{0k}(U)$ of at most linear growth solves the HJB equation

$$0 = \max\Big\{ -(r + \mu_h + \theta_k\lambda_k)V_{0k}(U) + \max_{Y_k, X_k \geq 0}\Big(\mu_h(I - Y_k) + \theta_k\lambda_k(\Pi_k - X_k) \quad (HJB_k)$$
$$+ V_{0k}'(U)\big[(r + \mu_h + \lambda_k)U - \mu_h Y_k - \lambda_k X_k\big]\Big), -V_{0k}'(U) - 1, V_{0k}''(U)\Big\},$$

subject to

1. The IC constraint: $\mu_h(Y_k - U) \geq \gamma_h$.
2. The IPO participation constraint: $Y_k \geq R_h$.
3. The acquisition acceptance constraint: $\Pi_k - X_k \geq V_{0k}(U)$.

then this function $V_{0k}(U)$ is the VC's value function.

The outer maximum in the (HJB_k) equation compares continuation (first term) with an immediate payout (second term) and randomization over continuation utility (third term). In the continuation term, the term multiplying $V_{0k}'(U)$ is the drift of the continuation utility $\dot{U}$, see equation (3). The marginal benefit of making an immediate payout (i.e. decreasing U) for the VC is given by $-V_{0k}'(U)$ while the marginal cost is 1. For the VC, if $V_{0k}''(U) > 0$ then by Jensen's inequality the VC would be strictly better off randomizing over the entrepreneur's continuation utility while keeping the expected continuation utility fixed.

The entrepreneur's contract is terminated at the limited-liability floor, so we focus on $U \geq 0$. Upon termination the VC retains the transferable capital and collects the outside option R_k, leading to the boundary condition $V_{0k}(0) = R_k$.

Acquisition Payments. The admissible acquisition payments are bounded by the limited liability constraint $X_k \geq 0$ and the acquisition acceptance constraint $X_k \leq \Pi_k - V_{0k}(U)$. Accepting the acquisition offer is efficient and thus $\Pi_k > \frac{\mu_h I + \lambda_k \Pi_k}{r + \mu_h + \lambda_k} \geq V_{0k}(U)$ for any U, where the last inequality follows from the fact that the VC cannot receive more than all firm's cash flows.[12] Therefore, we have $\Pi_k - V_{0k}(U) > 0$ and the non-empty admissible interval for X_k is $[0, \Pi_k - V_{0k}(U)]$.

The derivative of the first-term of the (HJB_k) equation with respect to X_k is $-\lambda_k\big(\theta_k + V'_{0k}(U)\big)$, which is non-negative if and only if $V'_{0k}(U) \le -\theta_k$. The optimum is therefore bang-bang and lies at one of the two endpoints of the admissible interval $[0, \Pi_k - V_{0k}(U)]$:[13]

$$X_k(U) = \begin{cases} 0, & V'_{0k}(U) > -\theta_k, \\ X_k \in [0, \Pi_k - V_{0k}(U)], & V'_{0k}(U) = -\theta_k, \\ \Pi_k - V_{0k}(U), & V'_{0k}(U) < -\theta_k. \end{cases} \tag{A.5}$$

IPO Payments. The derivative of the first-term of the (HJB_k) equation with respect to Y_k is $-\mu_h\big(1 + V'_{0k}(U)\big)$. We conjecture (and verify below) that the VC's value function satisfies $V'_{0k}(U) \ge -1$, so the objective is weakly decreasing in Y_k and the optimal IPO payment is the lowest payment ensuring effort and IPO participation,

$$Y_k(U) = \max\left\{U + \frac{\gamma_h}{\mu_h}, R_h\right\} = U + \frac{\gamma_h}{\mu_h},$$

where the second equality follows from the fact that $R_h \le \frac{\gamma_h}{\mu_h}$.

Value Function Conjecture. We conjecture (and verify below) that the VC makes the maximum acquisition payment $X_k = \Pi_k - V_{0k}(U) > 0$ if $U > U_X$ and $X_k = 0$ otherwise.[14] Furthermore, we conjecture that the value function is of the form $V_{0k}(U) = V_{0k}(\hat{U}_{0k}) - (U - \hat{U}_{0k})$ for $U \ge \hat{U}_{0k}$ with $U_X < \hat{U}_{0k}$.

No-Acquisition-Payment Region $U \in [0, U_X)$. With $X_k = 0$ and $Y_k = U + \frac{\gamma_h}{\mu_h}$, the first term in the (HJB_k) equation reduces to the differential equation

$$\begin{aligned} 0 &= -(r + \mu_h + \theta_k\lambda_k)V_{0k}(U) + \mu_h(I - U) - \gamma_h + \theta_k\lambda_k\Pi_k + V'_{0k}(U)\big[(r + \lambda_k)U - \gamma_h\big] \\ &= -A_{k,N}V_{0k}(U) + \mu_h(I - U) - \gamma_h + \theta_k\lambda_k\Pi_k + V'_{0k}(U)a_k\big[U - \hat{U}_{0k}^{X=0}\big], \end{aligned} \tag{A.6}$$

where a_k, $A_{k,N}$, and $\hat{U}_{0k}^{X=0}$ are defined in the proposition.

[12] The VC receives at most I upon a breakthrough, which arrives at rate μ_h; at most the acquirer's valuation Π_k upon an acquisition, which arrives at rate λ_k; and R_k upon termination at the deadline T_D, while its payments to the entrepreneur are non-negative. Discounting these receipts gives, pathwise and hence in expectation,

$$V_{0k}(U) \le \int_0^{T_D} e^{-(r+\mu_h+\lambda_k)t}\big(\mu_h I + \lambda_k\Pi_k\big)dt + e^{-(r+\mu_h+\lambda_k)T_D}R_k \le S_k,$$

where the last inequality uses $R_k < S_k$ since $\alpha_k\theta_k\lambda_k \le \lambda_k$ and $I > \Pi_k$.

[13] When $V'_{0k}(U) = -\theta_k$ the objective is independent of X_k, so any value in $[0, \Pi_k - V_{0k}(U)]$ is optimal.

[14] If $V'_{0k}(U_X) = -\theta_k$ then the policy is not uniquely defined at U_X since the VC is indifferent between any $X_k \in [0, \Pi_k - V_{0k}(U)]$.

The drift of the entrepreneur's promised utility in this region is

$$a_k\big[U - \hat{U}_{0k}^{X=0}\big]. \tag{A.7}$$

Since the no-acquisition-payment region is $[0, U_X)$ with $U_X \leq \hat{U}_{0k}^{X=0}$ (as established below), the promise drifts strictly downward throughout this region.

The particular solution of the differential equation (A.6) is

$$V_N^p(U) = \frac{\mu_h(I - \hat{U}_{0k}^{X=0}) - \gamma_h + \theta_k\lambda_k\Pi_k}{A_{k,N}} + \frac{\mu_h(\hat{U}_{0k}^{X=0} - U)}{A_{k,N} - a_k},$$

and the general solution satisfying $V_{0k}(0) = R_k$ is

$$V_{0k}(U) = V_N^p(U) + C_N\left(\hat{U}_{0k}^{X=0} - U\right)^{p_{k,N}}, \tag{A.8}$$

where $p_{k,N} = \frac{A_{k,N}}{a_k} > 1$ (because $\mu_h > (1-\theta_k)\lambda_k$) and

$$C_N = \frac{R_k - V_N^p(0)}{\left(\hat{U}_{0k}^{X=0}\right)^{p_{k,N}}}.$$

Furthermore, $C_N < 0$ since $\hat{U}_{0k}^{X=0} > 0$ and

$$R_k - V_N^p(0) \leq R_k - \frac{\mu_h I + \theta_k\lambda_k\Pi_k}{r + \mu_h + \theta_k\lambda_k} < R_k - \frac{\mu_h\Pi_k + \theta_k\lambda_k\Pi_k}{r + \mu_h + \theta_k\lambda_k} < 0.$$

The first-order and second-order derivatives of V_{0k} with respect to $U < \hat{U}_{0k}^{X=0}$ are

$$\begin{aligned} V_{0k}'(U) &= -\frac{\mu_h}{A_{k,N} - a_k} - p_{k,N}C_N(\hat{U}_{0k}^{X=0} - U)^{p_{k,N}-1}, \\ V_{0k}''(U) &= p_{k,N}(p_{k,N} - 1)C_N(\hat{U}_{0k}^{X=0} - U)^{p_{k,N}-2} < 0 \end{aligned} \tag{A.9}$$

since $C_N < 0$ and $p_{k,N} > 1$. This implies that $V_{0k}(U)$ is strictly concave on $[0, U_X)$.

At the stationary point $U = \hat{U}_{0k}^{X=0}$, this first-order derivative becomes

$$V_{0k}'(\hat{U}_{0k}^{X=0}) = -\frac{\mu_h}{A_{k,N} - a_k} = -\frac{\mu_h}{\mu_h - (1-\theta_k)\lambda_k} \leq -\theta_k. \tag{A.10}$$

The fact that the function in (A.9) is strictly decreasing for $U < \hat{U}_{0k}^{X=0}$ with $V_{0k}'(\hat{U}_{0k}^{X=0}) \leq -\theta_k$ implies that

$$\begin{aligned} -\theta_k &= -\frac{\mu_h}{\mu_h - (1-\theta_k)\lambda_k} - p_{k,N}C_N(\hat{U}_{0k}^{X=0} - \tilde{U})^{p_{k,N}-1}, \\ \tilde{U} &= \hat{U}_{0k}^{X=0} - \left(\frac{\theta_k - \frac{\mu_h}{\mu_h - (1-\theta_k)\lambda_k}}{p_{k,N}C_N}\right)^{\frac{1}{p_{k,N}-1}}, \end{aligned}$$

where $\tilde{U} \le \hat{U}_{0k}^{X=0}$ can be negative. Therefore,

$$\begin{aligned} U_X &= \inf\{U \ge 0 \mid V_{0k}'(U) \le -\theta_k\} \\ &= \max\{\tilde{U}, 0\} \\ &= \max\left\{ \hat{U}_{0k}^{X=0} - \left(\frac{\theta_k - \frac{\mu_h}{\mu_h - (1-\theta_k)\lambda_k}}{p_{k,N} C_N} \right)^{\frac{1}{p_{k,N}-1}}, 0 \right\}. \end{aligned}$$

If $U_X = 0$ then $[0, U_X)$ is empty and the construction proceeds directly to the acquisition-payment region below, with boundary condition $V_{0k}(0) = R_k$ at termination; here $V_{0k}'(0) \le -\theta_k$ holds by the definition of U_X. If $U_X > 0$ then $U_X = \tilde{U}$ and $V_{0k}'(U_X) = -\theta_k$. Strict concavity of $V_{0k}(U)$ on $[0, \hat{U}_{0k}^{X=0})$ then makes $V_{0k}'(U)$ strictly decreasing, so for $U \in [0, U_X)$, $V_{0k}'(U) > V_{0k}'(U_X) = -\theta_k \ge -1$. Hence $X_k = 0$ and $Y_k = U + \frac{\gamma_h}{\mu_h}$ are optimal on $[0, U_X)$: $V_{0k}(U)$ in (A.8) solves the first-term of (HJB_k) by construction, and $V_{0k}'(U) \ge -1$ makes the second term non-positive, while concavity $V_{0k}''(U) \le 0$ makes the third term non-positive so the (HJB_k) equation is satisfied on $[0, U_X)$.

Acquisition-Payment Region $U \in [U_X, \hat{U}_{0k}]$. With $X_k = \Pi_k - V_{0k}(U)$ and $Y_k = U + \frac{\gamma_h}{\mu_h}$, the first-term in the (HJB_k) equation reduces to the ordinary differential equation

$$0 = -(r + \mu_h)V_{0k}(U) + \mu_h(I - U) - \gamma_h + V_{0k}'(U)\big[(r + \lambda_k)U - \gamma_h - \lambda_k(\Pi_k - V_{0k}(U))\big]. \quad \text{(A.11)}$$

We first locate the stationary point $\hat{U}_{0k}$ for the continuation utility U. This stationary point solves

$$0 = (r + \lambda_k)\hat{U}_{0k} - \gamma_h - \lambda_k(\Pi_k - V_{0k}(\hat{U}_{0k})).$$

At this point, (A.11) reduces to

$$0 = -(r + \mu_h)V_{0k}(\hat{U}_{0k}) + \mu_h(I - \hat{U}_{0k}) - \gamma_h.$$

These are two linear equations in $\hat{U}_{0k}$ and $V_{0k}(\hat{U}_{0k})$. Solving these equations yields

$$\hat{U}_{0k} = \frac{\gamma_h + \lambda_k(\Pi_k - S_k)}{r}, \qquad V_{0k}(\hat{U}_{0k}) = S_k - \hat{U}_{0k}, \qquad S_k = \frac{\mu_h I + \lambda_k \Pi_k}{r + \mu_h + \lambda_k}, \quad \text{(A.12)}$$

where S_k is the full surplus in state $(0, k)$. Efficiency of the acquisition implies

$$\hat{U}_{0k} > \frac{\gamma_h}{r} \ge \frac{\gamma_h}{a_k} = \hat{U}_{0k}^{X=0} \ge U_X,$$

so that the region $[U_X, \hat{U}_{0k}]$ is non-empty.

To solve (A.11), define the loss from the full-surplus benchmark

$$L_k(U) = S_k - U - V_{0k}(U) \ge 0,$$

so that $V_{0k}'(U) = -1 - L_k'(U)$. The drift of the continuation utility can be written as

$$\begin{aligned}(r+\lambda_k)U - \gamma_h - \lambda_k(\Pi_k - V_{0k}(U)) &= (r+\lambda_k)U - \gamma_h - \lambda_k(\Pi_k - (S_k - U - L_k(U)))\\ &= rU - (\gamma_h + \lambda_k(\Pi_k - S_k)) - \lambda_k L_k(U)\\ &= -r\big(\hat{U}_{0k} - U\big) - \lambda_k L_k(U). \end{aligned} \tag{A.13}$$

The first term is strictly negative for $U < \hat{U}_{0k}$ and the second non-positive, so the promise drifts strictly downward everywhere below the stationary point $\hat{U}_{0k}$.

This downward drift on $[U_X, \hat{U}_{0k})$ means that value matching applies at U_X, $V_{0k}(U_X^+) = V_{0k}(U_X^-)$, where U_X^- and U_X^+ denote the left and right limits at U_X.

If $U_X > 0$, then $V_{0k}'(U_X^-) = -\theta_k$ from the no-acquisition-payment region. At this slope the acquisition term X_k in (HJB_k) enters with coefficient $-\lambda_k(\theta_k + V_{0k}') = 0$, so the pair $(V_{0k}(U_X^-), V_{0k}'(U_X^-)) = (V_{0k}(U_X^-), -\theta_k)$ solves both the differential equations (A.6) and (A.11) at U_X. Value matching $V_{0k}(U_X^+) = V_{0k}(U_X^-)$ and the non-zero continuation utility drift coefficient in the differential equation (A.11) (since $\hat{U}_{0k}$ is the unique stationary point of (A.11) and $U_X < \hat{U}_{0k}$) ensure that $V_{0k}'(U_X^+)$ is uniquely pinned down and therefore $V_{0k}'(U_X^+) = -\theta_k$. Hence V_{0k} is C^1 at U_X.

If $U_X = 0$, then the solution to the boundary condition $V_{0k}(0) = R_k$ and the differential equation (A.6) satisfies $V_{0k}'(0) \le -\theta_k$. The differential equation (A.6) at $U_X = 0$ can be written as

$$A_{k,N} R_k = \mu_h I - \gamma_h + \theta_k \lambda_k \Pi_k - \gamma_h V_{0k}'(0).$$

Solving for the slope and using $\gamma_h > 0$,

$$V_{0k}'(0) = \frac{\mu_h I - \gamma_h + \theta_k \lambda_k \Pi_k - A_{k,N} R_k}{\gamma_h} \le -\theta_k \quad \iff \quad A_{k,N} R_k \ge \mu_h I + \theta_k \lambda_k \Pi_k - (1-\theta_k)\gamma_h. \tag{A.14}$$

The differential equation (A.11) at $U = 0$ with the boundary condition $V_{0k}(0) = R_k$ reads

$$(r+\mu_h)R_k = \mu_h I - \gamma_h + V_{0k}'(0)\big[-\gamma_h - \lambda_k(\Pi_k - R_k)\big].$$

Since $\gamma_h + \lambda_k(\Pi_k - R_k) > 0$, solving for the slope and imposing $V_{0k}'(0) \le -\theta_k$ yields

$$V_{0k}'(0) = \frac{\mu_h I - \gamma_h - (r+\mu_h)R_k}{\gamma_h + \lambda_k(\Pi_k - R_k)} \le -\theta_k \quad \iff \quad A_{k,N} R_k \ge \mu_h I + \theta_k \lambda_k \Pi_k - (1-\theta_k)\gamma_h,$$

which is exactly the condition in (A.14). Therefore, both the no-acquisition-payment solution and the acquisition-payment solution of the value function satisfy the slope condition $V_{0k}'(0) \le -\theta_k$ under the identical primitive condition (A.14). As a consequence, if $U_X = 0$ then $V_{0k}'(0) \le -\theta_k$ for the solution $V_{0k}(U)$ of the differential equation (A.11).

Rewriting (A.11) using (A.13) yields:

$$L_k'(U) = -\frac{A_k L_k(U)}{r(\hat{U}_{0k} - U) + \lambda_k L_k(U)}, \tag{A.15}$$

with $A_k = r + \mu_h + \lambda_k$. There are two cases:

1. Assume $U_X = \hat{U}_{0k}^{X=0} > 0$. From equation (A.10) it follows that $\theta_k = 1$. From the differential equation (A.6), $\theta_k = 1$, and the definition of $\hat{U}_{0k}^{X=0}$ it follows that

$$\begin{aligned}
V_{0k}(U_X^-) &= \frac{\mu_h(I - U_X) - \gamma_h + \lambda_k \Pi_k + V_{0k}'(U_X^-) a_k \left[U_X - \hat{U}_{0k}^{X=0}\right]}{A_{k,N}} \\
&= \frac{\mu_h(I - U_X) - \gamma_h + \lambda_k \Pi_k}{r + \mu_h + \lambda_k} \\
&= \frac{\mu_h I + \lambda_k \Pi_k}{r + \mu_h + \lambda_k} - \frac{\mu_h \hat{U}_{0k}^{X=0} + a_k \hat{U}_{0k}^{X=0}}{r + \mu_h + \lambda_k} \\
&= S_k - \hat{U}_{0k}^{X=0}.
\end{aligned}$$

This is equivalent to no surplus being lost, $L_k(U_X^-) = 0$. Therefore, for $U \in [U_X, \hat{U}_{0k}]$ no surplus is lost, $L_k(U) = 0$, or written differently $V_{0k}(U) = S_k - U$. Substituting $V_{0k}(U) = S_k - U$ and $V_{0k}'(U) = -1$ into the differential equation (A.11) gives

$$\mu_h I + \lambda_k \Pi_k - (r + \mu_h + \lambda_k) S_k = 0,$$

which holds by the definition of S_k. The conjecture therefore solves the differential equation (A.11) and $V_{0k}'(U) = -1$ for $U \in [U_X, \hat{U}_{0k}]$. The value function is also concave for $U \in [U_X, \hat{U}_{0k}]$ since $V_{0k}''(U) = 0$.

2. Assume $U_X < \hat{U}_{0k}^{X=0}$, which by equation (A.10) corresponds to $\theta_k < 1$. The proof has several steps. We first establish that $L_k(U_X) > 0$, which allows us to rewrite the differential equation (A.15) and get an implicit solution. We then derive an additional result that bounds $L_k(U)$ and helps us establish that $L_k(U)$ is convex and decreasing.

   Positivity of $L_k(U_X)$. We first establish that $L_k(U_X) > 0$. Suppose first that $U_X > 0$. On $[0, U_X)$ we have $V_{0k}'(U) > -\theta_k \geq -1$, so by continuity $V_{0k}'(U_X^-) \geq -1$. Evaluating equation (A.6) at U_X, multiplying $V_{0k}'(U_X^-) \geq -1$ by the strictly negative drift $a_k(U_X - \hat{U}_{0k}^{X=0})$, and using $a_k \hat{U}_{0k}^{X=0} = \gamma_h$ and $\mu_h + a_k = A_k$ gives

   $$A_{k,N} V_{0k}(U_X) \leq \mu_h(I - U_X) - \gamma_h + \theta_k \lambda_k \Pi_k + a_k(\hat{U}_{0k}^{X=0} - U_X) = \mu_h I + \theta_k \lambda_k \Pi_k - A_k U_X.$$

   Combining this with $A_{k,N} S_k = \mu_h I + \lambda_k \Pi_k - (1 - \theta_k)\lambda_k S_k$, which follows from $A_k S_k = \mu_h I + \lambda_k \Pi_k$ and $A_{k,N} = A_k - (1 - \theta_k)\lambda_k$, yields

   $$L_k(U_X) = S_k - U_X - V_{0k}(U_X) \geq \frac{(1 - \theta_k)\lambda_k(\Pi_k + U_X - S_k)}{A_{k,N}} > 0,$$

   where the final inequality uses $\theta_k < 1$ and $\Pi_k > S_k$. If instead $U_X = 0$, then $L_k(0) = S_k - R_k$, which is strictly positive: since $I > \Pi_k$ and $\alpha_k \theta_k \leq 1$, $S_k > \frac{(\mu_h + \lambda_k)\Pi_k}{A_k} \geq R_k$.

   Observe that while $U \in [U_X, \hat{U}_{0k})$ no surplus is lost since the entrepreneur exerts effort, there is an IPO when there is a breakthrough, and the acquirer pays $V_{0k}(U) + X_k + \theta_k(\Pi_k - V_{0k}(U) - X_k) = \Pi_k$ (given $X_k = \Pi_k - V_{0k}(U)$) for the acquisition and the VC accepts the acquisition. This observation in combination with the Poisson exit events implies that for $U \in [U_X, \hat{U}_{0k})$ we can write the loss of surplus $L_k(U)$ as

$$L_k(U) = e^{-(r+\mu_h+\lambda_k)T(U)} L_k(U_X),$$

where $T(U)$ is the time it takes to reach U_X starting from U assuming no exit events happen. This result implies $L_k(U) > 0$ for all $U \in [U_X, \hat{U}_{0k})$. Furthermore, $L_k(\hat{U}_{0k}) = 0$ since $T(U) = \infty$ as $\hat{U}_{0k}$ is a stationary point and so U does not drift down. Equation (A.15) then gives $L_k'(U) < 0$, so L_k is strictly decreasing and $U \mapsto L_k(U)$ is invertible on this interval.

Implicit solution of $L_k(U)$. We can therefore write $Z = \hat{U}_{0k} - U$ and treat Z as a function of L_k, which turns the differential equation (A.15) into the linear ordinary differential equation

$$\frac{dZ}{dL_k} = \frac{r}{A_k}\frac{Z}{L_k} + \frac{\lambda_k}{A_k}.$$

Multiplying by the integrating factor $L_k^{-q_k}$ with $q_k = \frac{r}{A_k}$ and using $A_k(1-q_k) = \mu_h + \lambda_k$ gives the implicit closed-form solution[15]

$$\hat{U}_{0k} - U = C_X L_k(U)^{q_k} + \frac{\lambda_k}{\mu_h + \lambda_k} L_k(U). \tag{A.16}$$

Sign of C_X and bound on $L_k(U)$. The constant C_X is pinned down by value matching at the switch point U_X:

$$C_X = \frac{\hat{U}_{0k} - U_X - \frac{\lambda_k}{\mu_h+\lambda_k} L_k(U_X)}{L_k(U_X)^{q_k}}. \tag{A.17}$$

To sign C_X, recall that the solution to the differential equation (A.6) at U_X satisfies $V_{0k}'(U_X^-) = -\theta_k$ when $U_X > 0$, and $V_{0k}'(0) \leq -\theta_k$ when $U_X = 0$. Evaluating the differential equation (A.6) at U_X and using that the continuation drift is negative

[15] In standard form the equation is $\frac{dZ}{dL_k} - \frac{q_k}{L_k} Z = \frac{\lambda_k}{A_k}$, with integrating factor $L_k^{-q_k}$. Multiplying through makes the left side an exact derivative, which we integrate from the switching point U_X to U:

$$\begin{aligned}
&\frac{d}{dL_k}\left(L_k^{-q_k} Z\right) = L_k^{-q_k}\frac{dZ}{dL_k} - q_k L_k^{-q_k-1} Z = \frac{\lambda_k}{A_k} L_k^{-q_k},\\
&L_k(U)^{-q_k} Z(U) - L_k(U_X)^{-q_k} Z(U_X) = \frac{\lambda_k}{A_k}\int_{L_k(U_X)}^{L_k(U)} L_k^{-q_k}\, dL_k = \frac{\lambda_k}{\mu_h+\lambda_k}\left(L_k(U)^{1-q_k} - L_k(U_X)^{1-q_k}\right),\\
&Z(U) = C_X L_k(U)^{q_k} + \frac{\lambda_k}{\mu_h+\lambda_k} L_k(U), \qquad C_X = \frac{\hat{U}_{0k} - U_X - \frac{\lambda_k}{\mu_h+\lambda_k} L_k(U_X)}{L_k(U_X)^{q_k}},
\end{aligned}$$

where the last line multiplies through by $L_k(U)^{q_k}$ and collects the terms proportional to $L_k(U)^{q_k}$ into C_X, using $Z(U_X) = \hat{U}_{0k} - U_X$.

(since $U_X < \hat{U}_{0k}^{X=0}$),

$$\begin{aligned}
&A_{k,N} V_{0k}(U_X^-) \\
&= \mu_h(I - U_X) - \gamma_h + \theta_k \lambda_k \Pi_k + V_{0k}'(U_X^-)\big[(r+\lambda_k)U_X - \gamma_h\big] \\
&\geq \mu_h(I - U_X) - \gamma_h + \theta_k \lambda_k \Pi_k - \theta_k\big[(r+\lambda_k)U_X - \gamma_h\big] \\
&= \mu_h I + \theta_k \lambda_k \Pi_k - (1-\theta_k)\gamma_h - \big[\theta_k(r+\lambda_k) + \mu_h\big]U_X \\
&= \mu_h I + \theta_k \lambda_k \Pi_k - (1-\theta_k)(r\hat{U}_{0k} - \lambda_k(\Pi_k - S_k)) - \big[A_{k,N} - (1-\theta_k)r\big]U_X \\
&= \mu_h I + \lambda_k \Pi_k - (1-\theta_k)(r\hat{U}_{0k} + \lambda_k S_k) - \big[A_{k,N} - (1-\theta_k)r\big]U_X \\
&= (A_{k,N} + (1-\theta_k)\lambda_k)S_k - (1-\theta_k)(r\hat{U}_{0k} + \lambda_k S_k) - \big[A_{k,N} - (1-\theta_k)r\big]U_X \\
&= A_{k,N}(S_k - U_X) - (1-\theta_k)r(\hat{U}_{0k} - U_X) \\
&= A_{k,N}\big(S_k - U_X - w_k(\hat{U}_{0k} - U_X)\big),
\end{aligned}$$

where the inequality uses $V_{0k}'(U_X^-) \leq -\theta_k$ together with the negative drift of the continuation utility, and the last equality uses S_k, $\hat{U}_{0k}$ from equation (A.12), $A_k = A_{k,N} + (1-\theta_k)\lambda_k$, and $w_k := \frac{(1-\theta_k)r}{A_{k,N}} \in [0,1)$. Dividing by $A_{k,N}$ and using value matching at U_X yields

$$L_k(U_X) = S_k - U_X - V_{0k}(U_X) \leq w_k(\hat{U}_{0k} - U_X).$$

Therefore, the numerator of equation (A.17) satisfies

$$\hat{U}_{0k} - U_X - \frac{\lambda_k}{\mu_h + \lambda_k} L_k(U_X) \geq \left(1 - \frac{\lambda_k}{\mu_h + \lambda_k} w_k\right)(\hat{U}_{0k} - U_X) > 0,$$

since $\frac{\lambda_k}{\mu_h+\lambda_k} < 1$, $w_k < 1$, and $\hat{U}_{0k} > U_X$. This result in combination with $L_k(U_X) > 0$ implies $C_X > 0$. Therefore, the right-hand side of equation (A.16) is strictly increasing in $L_k(U)$ and so a unique solution exists.

Concavity of $V_{0k}(U)$. Differentiating the differential equation (A.11) with respect to U yields

$$\begin{aligned}
0 = &- (r+\mu_h)V_{0k}'(U) - \mu_h + V_{0k}''(U)\left[(r+\lambda_k)U - \gamma_h - \lambda_k(\Pi_k - V_{0k}(U))\right] \\
&+ V_{0k}'(U)\big[(r+\lambda_k) + \lambda_k V_{0k}'(U)\big].
\end{aligned}$$

Solving for $V_{0k}''(U)$ gives

$$V_{0k}''(U) = \frac{(1 + V_{0k}'(U))(\mu_h - \lambda_k V_{0k}'(U))}{\left[(r+\lambda_k)U - \gamma_h - \lambda_k(\Pi_k - V_{0k}(U))\right]}. \tag{A.18}$$

We know that

$$(r+\lambda_k)U - \gamma_h - \lambda_k(\Pi_k - V_{0k}(U)) < 0$$

because $U < \hat{U}_{0k}$ and $U = \hat{U}_{0k}$ is the unique stationary point of the continuation utility drift given $X_k = \Pi_k - V_{0k}(U) > 0$. Using $V_{0k}'(U) = -1 - L_k'(U)$ and the differential equation (A.15) yields

$$\mu_h - \lambda_k V_{0k}'(U) = (\mu_h + \lambda_k) + \lambda_k L_k'(U) = \frac{r\big[(\mu_h + \lambda_k)(\hat{U}_{0k} - U) - \lambda_k L_k(U)\big]}{r(\hat{U}_{0k} - U) + \lambda_k L_k(U)}.$$

The numerator is positive because equation (A.16) gives

$$\frac{\lambda_k}{\mu_h + \lambda_k} L_k(U) = (\hat{U}_{0k} - U) - C_X L_k(U)^{q_k} < \hat{U}_{0k} - U,$$

since $C_X > 0$ and $L_k(U)^{q_k} > 0$. The denominator is positive because $r(\hat{U}_{0k} - U) > 0$ and $L_k(U) > 0$. Hence $\mu_h - \lambda_k V_{0k}'(U) > 0$. This result in combination with $L_k'(U) = -1 - V_{0k}'(U) < 0$ implies that both factors in the numerator of (A.18) are strictly positive while the denominator is strictly negative, so $V_{0k}''(U) < 0$ and $V_{0k}(U)$ is strictly concave on $(U_X, \hat{U}_{0k})$.

Slope bounds. Strict concavity pins down the slope. Since $V_{0k}'(U)$ is strictly decreasing and $V_{0k}'(U_X) \le -\theta_k$, we have $V_{0k}'(U) \le -\theta_k$ for every $U \in [U_X, \hat{U}_{0k})$, so $X_k > 0$ is indeed optimal throughout this region.

Using equation (A.16), it follows that

$$\lim_{U \to \hat{U}_{0k}} \frac{L_k(U)}{\hat{U}_{0k} - U} = \lim_{U \to \hat{U}_{0k}} \frac{L_k(U)^{1-q_k}}{C_X + \frac{\lambda_k}{\mu_h + \lambda_k} L_k(U)^{1-q_k}} = 0$$

since $\lim_{U \to \hat{U}_{0k}} L_k(U) = 0$. Using equation (A.15), this implies that

$$\lim_{U \to \hat{U}_{0k}} L_k'(U) = \lim_{U \to \hat{U}_{0k}} -\frac{\frac{A_k L_k(U)}{\hat{U}_{0k} - U}}{r + \lambda_k \frac{L_k(U)}{\hat{U}_{0k} - U}} = 0$$

Therefore, $\lim_{U \to \hat{U}_{0k}} V_{0k}'(U) = \lim_{U \to \hat{U}_{0k}} -1 - L_k'(U) = -1$. The strict concavity of $V_{0k}(U)$ in combination with $V_{0k}'(U_X) \le -\theta_k$ and $V_{0k}'(\hat{U}_{0k}) = -1$ implies that $V_{0k}'(U) \in [-1, -\theta_k]$ on $[U_X, \hat{U}_{0k}]$.

We have shown that $V_{0k}'(U) \in [-1, -\theta_k]$ on $[U_X, \hat{U}_{0k}]$. Therefore, $X_k = \Pi_k - V_{0k}(U)$ and $Y_k = U + \frac{\gamma_h}{\mu_h}$ are optimal on $[U_X, \hat{U}_{0k}]$. By construction, $V_{0k}(U)$ ensures that the first-term in (HJB_k) equals zero. Furthermore, the second-term is also satisfied since $V_{0k}'(U) \ge -1$ and the third-term is satisfied since $V_{0k}''(U) \le 0$.

$U > \hat{U}_{0k}$ **Region.** For $U > \hat{U}_{0k}$, we conjecture that the value function has the form $V_{0k}(U) = V_{0k}(\hat{U}_{0k}) - (U - \hat{U}_{0k}) = S_k - U$ since $L_k(\hat{U}_{0k}) = S_k - \hat{U}_{0k} - V_{0k}(\hat{U}_{0k}) = 0$. By construction, the second-term in (HJB_k) satisfies $-V_{0k}'(U) - 1 = 0$. The value function is linear in U and therefore concave $V_{0k}''(U) = 0$ and the third-term is satisfied. It remains to check the first term. Since $V_{0k}'(U) = -1 \le -\theta_k$, the optimal acquisition payment is $X_k = \Pi_k - V_{0k}(U) = (\Pi_k - S_k) + U > 0$ and the optimal IPO payment is $Y_k = U + \frac{\gamma_h}{\mu_h}$. The first-term of the (HJB_k) equation then becomes

$$ODE_U = -(r+\mu_h)V_{0k}(U) + \mu_h(I-U) - \gamma_h + V'_{0k}(U)\big[(r+\lambda_k)U - \gamma_h - \lambda_k(\Pi_k - V_{0k}(U))\big].$$

Substituting $V_{0k}(U) = S_k - U$ and $V'_{0k}(U) = -1$ yields

$$\begin{aligned} ODE_U =& -(r+\mu_h)(S_k - U) + \mu_h(I-U) - \gamma_h - \big[(r+\lambda_k)U - \gamma_h - \lambda_k(\Pi_k - S_k + U)\big] \\ =& \mu_h I + \lambda_k \Pi_k - (r+\mu_h+\lambda_k)S_k = 0, \end{aligned}$$

All three terms in the (HJB_k) equation therefore equal zero, so the VC is indifferent between paying out, delaying payment, or randomizing.

Continuity and differentiability. The solution is described on three regions: the no-acquisition-payment region $[0, U_X)$, where V_{0k} solves the differential equation (A.6) subject to $V_{0k}(0) = R_k$; the acquisition-payment region $[U_X, \hat{U}_{0k}]$, where V_{0k} solves the differential equation (A.11) subject to a boundary condition at U_X and is given implicitly by equation (A.16); and the payout region $U > \hat{U}_{0k}$, where $V_{0k}(U) = V_{0k}(\hat{U}_{0k}) - (U - \hat{U}_{0k})$. Each branch solves a smooth ordinary differential equation and is C^1 on its subinterval, so it remains only to glue the branches at U_X and at $\hat{U}_{0k}$.

At U_X, value matching $V_{0k}(U_X^+) = V_{0k}(U_X^-)$ holds by construction. When $U_X > 0$, we have shown above that $V'_{0k}(U_X^+) = V'_{0k}(U_X^-) = -\theta_k$, so V_{0k} is C^1 at U_X; when $U_X = 0$, there is no left region and the acquisition-payment branch starts at the floor with $V_{0k}(0) = R_k$.

At $\hat{U}_{0k}$, value matching holds because the payout branch is defined to equal $V_{0k}(\hat{U}_{0k})$ there. The branches also join C^1: the acquisition-payment branch satisfies $\lim_{U \to \hat{U}_{0k}} V'_{0k}(U) = -1$, shown above, which equals the slope -1 of the payout branch on $U > \hat{U}_{0k}$.

Hence V_{0k} is continuous and C^1 on $[0, \infty)$. Moreover, $V_{0k}(U)$ is globally concave: $V'_{0k}(U)$ is continuous by the above and non-increasing on the interior of each of the three regions, and is therefore non-increasing on $[0, \infty)$. The value function we constructed solves (HJB_k), is concave, and is C^1, which ensures that it is the optimal value function for the VC. □

A.4 State $(0, 0)$

Throughout this subsection we write $\ell_h(U) = U + \frac{\gamma_h}{\mu_h}$ and $\ell_k(U) = U + \frac{\gamma_k}{\mu_k}$ for the incentive floors on the milestone promises in (IC-hk), and $\overline{U}_h := \overline{U}_{h0}$ and $\overline{U}_k := \overline{U}_{0k}$ for the payout boundaries of the two intermediate states.

The relevant assumptions made in the main text are:

Assumption 3 (Assumptions $(0,0)$)**.** *Assume that*

- *Assumptions for state $(h, 0)$ and state $(0, k)$ hold true.*
- *The VC wants to sign the contract for some U:*

$$\max_{u_h \geq \ell_h(0), u_k \geq \ell_k(0)} \Big(\mu_h V_{h0}(u_h) + \mu_k V_{0k}(u_k) \Big) > 0.$$

Under Assumption 3, the following holds:

Proposition 9 (VC's Value Function in State $(0,0)$)**.** *The value function $V_{00}(U)$ is C^1 and concave. There exists a threshold $\overline{U}_{00} = \inf\{U \geq 0 \,|\, V_{00}'(U^+) \leq -1\} \in (0,\infty)$ such that for $U \in (0, \overline{U}_{00})$, $V_{00}(U)$ solves the differential equation*

$$\begin{aligned}0 =& -(r+\mu_h+\mu_k)V_{00}(U) \\ &+ \max_{u_h \geq \ell_h(U), u_k \geq \ell_k(U)} \Big(\mu_h V_{h0}(u_h) + \mu_k V_{0k}(u_k) + V_{00}'(U)\big[(r+\mu_h+\mu_k)U - \mu_h u_h - \mu_k u_k\big]\Big)\end{aligned}$$

with boundary condition $V_{00}(0) = 0$. For $U \geq \overline{U}_{00}$, $V_{00}(U) = \overline{J} - U$, where

$$S_h := \frac{\mu_k I + \lambda_h \Pi_h}{r+\mu_k+\lambda_h}, \qquad S_k := \frac{\mu_h I + \lambda_k \Pi_k}{r+\mu_h+\lambda_k}, \qquad \overline{J} := \frac{\mu_h S_h + \mu_k S_k}{r+\mu_h+\mu_k} > 0. \tag{A.19}$$

Proof of Proposition 9. **VC's Problem.** In state $(0,0)$, if the VC's value function is C^2 on some region then it solves

$$\begin{aligned}0 = \max\Big\{ &-(r+\mu_h+\mu_k)V_{00}(U) + \max_{u_h,u_k \geq 0}\Big(\mu_h V_{h0}(u_h) + \mu_k V_{0k}(u_k) \\ &+ V_{00}'(U)\big[(r+\mu_h+\mu_k)U - \mu_h u_h - \mu_k u_k\big]\Big), -V_{00}'(U^-) - 1, V_{00}''(U)\Big\},\end{aligned} \tag{HJB_0}$$

subject to

1. The IC-h constraint: $\mu_h(u_h - U) \geq \gamma_h$.
2. The IC-k constraint: $\mu_k(u_k - U) \geq \gamma_k$.
3. The entrepreneur's state h outside option: $u_h \geq R_h$.

In the (HJB_0) equation, the outer maximum compares continuation with an immediate payout, whose marginal benefit is $-V_{00}'(U^-)$ against a marginal cost of 1, and randomization.

We know that the IC-h constraint can be rewritten as $u_h \geq U + \frac{\gamma_h}{\mu_h}$ and by the assumption in state $(0,k)$ that $\frac{\gamma_h}{\mu_h} \geq R_h$. Combining these two yields for $U \geq 0$ that

$$u_h \geq U + \frac{\gamma_h}{\mu_h} \geq \frac{\gamma_h}{\mu_h} \geq R_h.$$

Hence the IC-h constraint ensures that the entrepreneur's outside option is always satisfied.

Concavity. Let $V_{00}(U)$ be the value function for the VC. The ability for the VC to randomize the continuation utility U implies that the value function must satisfy

$$V_{00}(\kappa U_1 + (1-\kappa)U_2) \geq \kappa V_{00}(U_1) + (1-\kappa)V_{00}(U_2) \qquad \kappa \in [0,1].$$

Therefore, the value function $V_{00}(U)$ is concave, which ensures that $V_{00}(U)$ is continuous and left and right differentiable on $(0,\infty)$ (see Chapter 6 of De la Fuente, 2000). Since the VC can always make an immediate lump-sum payment, $V_{00}(U) \geq V_{00}(U-\Delta) - \Delta$ for all $0 \leq \Delta \leq U$, so the one-sided derivatives of V_{00} satisfy $V_{00}'(U^{\pm}) \geq -1$. Finally, $V_{00}(U)$ is continuous at

$U = 0$. First, $V_{00}(U) \geq -U$, since an immediate payout of U followed by termination is feasible. Second, fix any admissible contract with termination time T_D and write $V^{\mathcal{C}}$ for the VC's payoff under it. Promise keeping, together with limited liability ($dY_t \geq 0$ and $U_t \geq 0$) and the incentive floors $\mu_h u_{h,t} + \mu_k u_{k,t} \geq (\mu_h + \mu_k)U_t + \gamma_h + \gamma_k \geq \gamma_h + \gamma_k$, implies

$$U \geq \mathbb{E}\left[\int_0^{T_D} e^{-(r+\mu_h+\mu_k)t}\big(\mu_h u_{h,t} + \mu_k u_{k,t}\big)dt\right] \geq (\gamma_h + \gamma_k)\mathbb{E}\left[\int_0^{T_D} e^{-(r+\mu_h+\mu_k)t}dt\right].$$

Payments between the VC and the entrepreneur are zero sum and termination yields zero to both parties, so the joint payoff collects the total branch value at the first transition; since $V_{h0}(u) + u \leq S_h$ and $V_{0k}(u) + u \leq S_k$ for every admissible milestone promise by Propositions 7 and 8, it follows that

$$V^{\mathcal{C}} + U \leq (\mu_h S_h + \mu_k S_k)\mathbb{E}\left[\int_0^{T_D} e^{-(r+\mu_h+\mu_k)t}dt\right] \leq (\mu_h S_h + \mu_k S_k)\frac{U}{\gamma_h + \gamma_k}.$$

Taking the supremum over admissible contracts yields $V_{00}(U) + U \leq (\mu_h S_h + \mu_k S_k)\frac{U}{\gamma_h+\gamma_k}$, and therefore $\lim_{U \to 0} V_{00}(U) = V_{00}(0) = 0$.

Region $U \in [\overline{U}_{00}, \infty)$. Recall from the proposition that $\overline{U}_{00} = \inf\{U \geq 0 | V'_{00}(U^+) \leq -1\} \in [0, \infty]$. Since $V_{00}(U)$ is concave, $V'_{00}(U^+)$ is non-increasing in U, which in combination with the bound $V'_{00}(U^{\pm}) \geq -1$ implies that $V'_{00}(U^+) = -1$ for every $U > \overline{U}_{00}$ whenever $\overline{U}_{00} < \infty$.

First, we establish $\overline{U}_{00} > 0$. From Assumption 3 and continuity of the value functions $V_{h0}(U)$ and $V_{0k}(U)$ (Propositions 7 and 8) and constraints $\ell_h(U)$ and $\ell_k(U)$ in U it follows that there exists an interval $[0, \tilde{U})$ such that

$$M(U) = \max_{u_h \geq \ell_h(U), u_k \geq \ell_k(U)} \Big(\mu_h V_{h0}(u_h) + \mu_k V_{0k}(u_k)\Big) > 0.$$

We know that the optimizers $u_h^M(U)$ and $u_k^M(U)$ are finite because above the payout boundaries $V'_{h0}(U) = V'_{0k}(U) = -1$. Shrink $\tilde{U}$ if necessary so that $\tilde{U} \in (0, \frac{\gamma_h+\gamma_k}{r})$. On $U \in [0, \tilde{U})$, the continuation utility then drifts down since

$$\dot{U} = (r + \mu_h + \mu_k)U - \mu_h u_h^M(U) - \mu_k u_k^M(U) \leq rU - \gamma_h - \gamma_k < 0,$$

which uses the IC constraints ($\mu_h(u_h - U) \geq \gamma_h$ and $\mu_k(u_k - U) \geq \gamma_k$). Since the policy $(u_h^M(U), u_k^M(U))$ satisfies the IC constraints, it is feasible, and therefore

$$V_{00}(\tilde{U}) \geq \int_0^{T_d} e^{-(r+\mu_h+\mu_k)t} M(U_t)dt$$

where U_t starts at $U_0 = \tilde{U}$ and T_d is the time it hits zero given the policy $(u_h^M(U), u_k^M(U))$ and no transition. We then know that $V_{00}(\tilde{U}) > 0$ since $M(U) > 0$ on $U \in [0, \tilde{U})$ and $T_d > 0$. This result in combination with concavity implies that $V'_{00}(0^+) > 0$ and $\overline{U}_{00} > 0$.

Second, we establish the upper bound $V_{00}(U) + U \leq \overline{J}$. From Propositions 7 and 8, the slopes of the branch value functions satisfy $V'_{h0}(u) \geq -1$ and $V'_{0k}(u) \geq -1$, so the total branch values $V_{h0}(u) + u$ and $V_{0k}(u) + u$ are non-decreasing in u; since $V_{h0}(u) = S_h - u$ for $u \geq \overline{U}_h$ and $V_{0k}(u) = S_k - u$ for $u \geq \overline{U}_k$, it follows that

$$V_{h0}(u) + u \leq S_h \quad \text{and} \quad V_{0k}(u) + u \leq S_k \quad \text{for any admissible } u, \tag{A.20}$$

with equality on the respective payout regions. In state $(0, 0)$ no exit is feasible, payments between the VC and the entrepreneur are zero sum, and termination yields zero to both parties, so total value is created only at the first transition, which arrives at rate $\mu_h + \mu_k$. Fix any admissible contract $\mathcal{C}$ with milestone promises $(u_{h,t}, u_{k,t})$ and termination time T_D, and write $V^{\mathcal{C}}$ for the VC's payoff under it. Then

$$\begin{aligned} V^{\mathcal{C}} + U &= \mathbb{E}\left[\int_0^{T_D} e^{-(r+\mu_h+\mu_k)t}\big(\mu_h\left(V_{h0}(u_{h,t}) + u_{h,t}\right) + \mu_k\left(V_{0k}(u_{k,t}) + u_{k,t}\right)\big)\,dt\right] \\ &\leq \mathbb{E}\left[\int_0^{T_D} e^{-(r+\mu_h+\mu_k)t}\big(\mu_h S_h + \mu_k S_k\big)\,dt\right] \leq \int_0^{\infty} e^{-(r+\mu_h+\mu_k)t}\big(\mu_h S_h + \mu_k S_k\big)\,dt = \overline{J}, \end{aligned}$$

where the first inequality follows from equation (A.20) and the second from the fact that $\mu_h S_h + \mu_k S_k > 0$. Taking the supremum over admissible contracts yields $V_{00}(U) + U \leq \overline{J}$.

With $\overline{U}_h$ and $\overline{U}_k$ the payout boundaries defined in the proposition, let

$$D(u_h, u_k|U) := (r+\mu_h+\mu_k)U - \mu_h u_h - \mu_k u_k, \qquad \underline{D}(U) := D\big(\max\{\ell_h(U), \overline{U}_h\}, \max\{\ell_k(U), \overline{U}_k\} \,\big|\, U\big). \tag{A.21}$$

Resolving each maximum in equation (A.21) in its two possible ways gives four affine functions of U, one for each pattern of binding incentive floors,

$$\begin{aligned} f_1(U) &= rU - \gamma_h - \gamma_k, & f_2(U) &= (r + \mu_k)U - \gamma_h - \mu_k\overline{U}_k, \\ f_3(U) &= (r + \mu_h)U - \gamma_k - \mu_h\overline{U}_h, & f_4(U) &= (r + \mu_h + \mu_k)U - \mu_h\overline{U}_h - \mu_k\overline{U}_k, \end{aligned}$$

and $\underline{D}(U) = \min_i f_i(U)$, since the maxima enter with negative coefficients. Each f_i has slope at least $r > 0$, so $\underline{D}(U)$ is a minimum of strictly increasing functions and is itself strictly increasing, and it has a unique zero at the largest of the four zeros of f_1, f_2, f_3, and f_4,

$$\zeta := \max\left\{\frac{\gamma_h + \gamma_k}{r}, \ \frac{\gamma_h + \mu_k\overline{U}_k}{r + \mu_k}, \ \frac{\gamma_k + \mu_h\overline{U}_h}{r + \mu_h}, \ \frac{\mu_h\overline{U}_h + \mu_k\overline{U}_k}{r + \mu_h + \mu_k}\right\} < \infty,$$

with $\underline{D}(U) < 0$ for $U < \zeta$, because the f_i attaining the largest zero is still negative there, and $\underline{D}(U) \geq 0$ for $U \geq \zeta$, because every f_i is.

Consider the contract that, starting from $U_0 = U \geq \zeta$, makes no payments, never terminates, and sets the milestone promises at $u_h = \max\{\ell_h(U_t), \overline{U}_h\}$ and $u_k = \max\{\ell_k(U_t), \overline{U}_k\}$. Promise keeping gives $\dot{U}_t = \underline{D}(U_t) \geq 0$, so $U_t \geq U_0 \geq \zeta$ for all t and the termination boundary is never reached; the exit rate $\mu_h + \mu_k > 0$ ensures transversality,[16] so U_t is the entrepreneur's

[16] Observe that

$$U_0 = \int_0^T e^{-(r+\mu_h+\mu_k)t}\left(\mu_h u_h(U_t) + \mu_k u_k(U_t)\right)dt + e^{-(r+\mu_h+\mu_k)T} U_T.$$

continuation utility and the contract delivers exactly U. Moreover, $u_h \geq \overline{U}_h$ and $u_k \geq \overline{U}_k$, so each transition delivers the full branch surplus, $V_{h0}(u_h) + u_h = S_h$ and $V_{0k}(u_k) + u_k = S_k$. Total value under this contract is therefore $\int_0^\infty e^{-(r+\mu_h+\mu_k)t}(\mu_h S_h + \mu_k S_k)dt = \overline{J}$, and since the entrepreneur receives U, the VC receives $\overline{J} - U$. Hence $V_{00}(U) \geq \overline{J} - U$, which combined with the upper bound $V_{00}(U) + U \leq \overline{J}$ yields $V_{00}(U) = \overline{J} - U$ on $[\zeta, \infty)$ and, in particular, $\overline{U}_{00} \leq \zeta < \infty$.

Finally, since $V'_{00}(U^+) = -1$ for every $U > \overline{U}_{00}$, the function V_{00} is affine with slope -1 on $[\overline{U}_{00}, \infty)$; as it coincides with $\overline{J} - U$ on $[\zeta, \infty)$, it equals $\overline{J} - U$ on all of $[\overline{U}_{00}, \infty)$.

Optimality of Delay for $U \in [\overline{U}_{00}, \infty)$. On this region $V_{00}(U) = \overline{J} - U$, so $V'_{00}(U) = -1$ and the payout term of the (HJB_0) equation, $-V'_{00}(U) - 1$, vanishes; a payout is therefore weakly optimal. It remains to show that the continuation term also vanishes so that delay is weakly optimal as well and the VC is indifferent between the two.

Substituting $V_{00}(U) = \overline{J} - U$ and $V'_{00}(U) = -1$ into the first-term of the (HJB_0) equation and grouping the promises into total branch values gives

$$-(r + \mu_h + \mu_k)(\overline{J} - U) + \max_{u_h \geq \ell_h(U), u_k \geq \ell_k(U)} \Big(\mu_h\big(V_{h0}(u_h) + u_h\big) + \mu_k\big(V_{0k}(u_k) + u_k\big)\Big) - (r + \mu_h + \mu_k)U$$

$$= -(r + \mu_h + \mu_k)\overline{J} + \mu_h S_h + \mu_k S_k$$
$$= 0,$$

where the first equality follows from the fact that the maximum equals $\mu_h S_h + \mu_k S_k$ because the inequalities in equation (A.20) bind whenever $u_h = \max\{\overline{U}_h, \ell_h(U)\}$ and $u_k = \max\{\overline{U}_k, \ell_k(U)\}$, which are feasible promises. The final equality is the definition of $\overline{J}$ in equation (A.19).

Both the payout term and the continuation term of the (HJB_0) equation are therefore zero on $[\overline{U}_{00}, \infty)$, so the VC is indifferent between paying the promise down immediately and delaying payment.

Region $U \in (0, \overline{U}_{00})$. Define the drift of the promise under milestone promises (u_h, u_k) and the continuation term[17]

$$\begin{aligned}
D(u_h, u_k|U) :=& (r + \mu_h + \mu_k)U - \mu_h u_h - \mu_k u_k, \\
\Phi(U) :=& -(r + \mu_h + \mu_k)V_{00}(U) \\
&+ \max_{u_h \geq \ell_h(U), u_k \geq \ell_k(U)} \Big(\mu_h V_{h0}(u_h) + \mu_k V_{0k}(u_k) + V'_{00}(U^-)D^- + V'_{00}(U^+)D^+\Big),
\end{aligned} \tag{A.22}$$

The contract delivers U_0 if and only if the terminal term vanishes. Here $\dot{U}_t = \underline{D}(U_t) \leq rU_t - \gamma_h - \gamma_k \leq rU_t$, so $U_t \leq U_0 e^{rt}$ and

$$e^{-(r+\mu_h+\mu_k)T}U_T \leq U_0 e^{-(\mu_h+\mu_k)T} \xrightarrow[T\to\infty]{} 0.$$

[17]The maximum is attained: for u_i above the payout boundary $V'_i(u_i) = -1 < V'_{00}(U^{\pm})$, so lowering u_i strictly increases the objective and the maximizers are bounded, while $V_{h0}(U)$ and $V_{0k}(U)$ are continuous by Propositions 7 and 8.

where $D^+ := \max\{D(u_h, u_k|U), 0\}$ and $D^- := \min\{D(u_h, u_k|U), 0\}$. We show that $\Phi(U) = 0$ at every point of differentiability in $(0, \overline{U}_{00})$ and then at any non-differentiable point, so that $V_{00}(U)$ solves the differential equation in the proposition and neither payout nor randomization is strictly optimal on this region.

Preliminary observations. We make four preliminary observations:

1. The value function satisfies the dynamic programming principle over the admissible contracts. The value at every $U > 0$ is therefore supported by one of three actions: an immediate payout that is value-neutral, a non-degenerate mean-preserving randomization that is value-neutral, or continuation under some feasible pair (u_h, u_k), evaluated at the one-sided derivative selected by the sign of that pair's drift. In the third case the maximum in equation (A.22) is attained: for $u_i > \max\{\ell_i(U), \overline{U}_i\}$ we have $V_i'(u_i) = -1 \leq V_{00}'(U^\pm)$, so lowering u_i weakly increases the objective and the maximizers lie in the compact set $[\ell_i(U), \max\{\ell_i(U), \overline{U}_i\}]$, over which the objective is continuous by Propositions 7 and 8.

2. Optimality of the value function implies $\Phi(U) \leq 0$ for every U.

3. Immediate payout is strictly suboptimal on $[0, \overline{U}_{00})$ since $V_{00}'(U^-) \geq V_{00}'(U^+) > -1$ there.

4. If a non-degenerate randomization of the continuation utility is weakly optimal then V_{00} is affine on the convex support of this randomization.

Points of Differentiability. Suppose toward a contradiction that $\Phi(U_0) < 0$ at some $U_0 \in (0, \overline{U}_{00})$ where V_{00} is differentiable. Observe that payout is suboptimal for $U_0 \in (0, \overline{U}_{00})$ since $V_{00}'(U_0^-) > -1$ by construction. Therefore, it must be that randomization takes place at U_0 as otherwise $\Phi(U_0) = 0$ by construction. Let $[U_1, U_2]$ be the maximal interval containing U_0 on which $V_{00}(U)$ is affine with slope $m > -1$. Since the randomization at U_0 is non-degenerate, $U_0 \in (U_1, U_2)$. We know that $U_2 \leq \overline{U}_{00} < \infty$.

Let the map $U \mapsto \Phi^m(U)$ evaluate the maximum in equation (A.22) at the fixed slope m,

$$\Phi^m(U) := -(r+\mu_h+\mu_k)V_{00}(U) + \max_{u_h \geq \ell_h(U), u_k \geq \ell_k(U)} \Big(\mu_h V_{h0}(u_h) + \mu_k V_{0k}(u_k) + mD(u_h, u_k|U)\Big).$$

Since V_{00} is differentiable at U_0 with slope m, $\Phi^m(U_0) = \Phi(U_0) < 0$. We show $\Phi^m(U_0) \geq \Phi^m(U_2)$. Fix $U \in [U_0, U_2]$ and let (u_h^U, u_k^U) be the optimizer of $\Phi^m(U)$. This pair (u_h^U, u_k^U) is feasible at U_0 since $\ell_i(U) \geq \ell_i(U_0)$. Furthermore, $V_{00}(U) = V_{00}(U_0) + m(U - U_0)$ and $D(u_h^U, u_k^U|U) = D(u_h^U, u_k^U|U_0) + (r + \mu_h + \mu_k)(U - U_0)$. Therefore,

$$\begin{aligned}
\Phi^m(U_0) \geq& -(r + \mu_h + \mu_k)V_{00}(U_0) + \mu_h V_{h0}(u_h^U) + \mu_k V_{0k}(u_k^U) + mD(u_h^U, u_k^U|U_0) \\
=& -(r + \mu_h + \mu_k)(V_{00}(U_0) + m(U - U_0)) + \mu_h V_{h0}(u_h^U) + \mu_k V_{0k}(u_k^U) \\
&+ m\big(D(u_h^U, u_k^U|U_0) + (r + \mu_h + \mu_k)(U - U_0)\big) \\
=& -(r + \mu_h + \mu_k)V_{00}(U) + \mu_h V_{h0}(u_h^U) + \mu_k V_{0k}(u_k^U) + mD(u_h^U, u_k^U|U) \\
=& \Phi^m(U).
\end{aligned}$$

Hence $\Phi^m(U) \le \Phi^m(U_0) < 0$ for $U \in [U_0, U_2]$, and since $V'_{00}(U_2^-) = m$, $\Phi(U_2^-) = \Phi^m(U_2) < 0$.

We now examine how the value at U_2 is supported. A payout is strictly suboptimal since $V'_{00}(U_2^-) = m > -1$; a randomization is strictly suboptimal since U_2 is an endpoint of the affine piece; and continuation with weakly negative drift is priced at the left derivative m, in which case we would have $0 = \Phi(U_2) = \Phi(U_2^-) = \Phi^m(U_2) < 0$, a contradiction. The value at U_2 is therefore supported by continuation with a non-negative drift: there is a feasible pair (u_h^2, u_k^2) with $D(u_h^2, u_k^2|U_2) \ge 0$ attaining the maximum at $V'_{00}(U_2^+)$, so that $\Phi(U_2) = \Phi(U_2^+) = 0$.

The fact that $\Phi(U_0) < 0$ implies that

$$\begin{aligned}
0 >& \Phi(U_0) \\
=& \Phi^m(U_0) \\
\ge& -(r + \mu_h + \mu_k)V_{00}(U_0) + \mu_h V_{h0}(u_h^2) + \mu_k V_{0k}(u_k^2) + mD(u_h^2, u_k^2|U_0) \\
=& -(r + \mu_h + \mu_k)V_{00}(U_2) + \mu_h V_{h0}(u_h^2) + \mu_k V_{0k}(u_k^2) + mD(u_h^2, u_k^2|U_2) \\
=& -(r + \mu_h + \mu_k)V_{00}(U_2) + \mu_h V_{h0}(u_h^2) + \mu_k V_{0k}(u_k^2) + V'_{00}(U_2^+)D(u_h^2, u_k^2|U_2) \\
&+ \big(m - V'_{00}(U_2^+)\big)D(u_h^2, u_k^2|U_2) \\
=& \Phi(U_2) + \big(m - V'_{00}(U_2^+)\big)D(u_h^2, u_k^2|U_2) \\
=& \big(m - V'_{00}(U_2^+)\big)D(u_h^2, u_k^2|U_2) \\
\ge& 0,
\end{aligned}$$

a contradiction. The first inequality holds because (u_h^2, u_k^2) is feasible at U_0 (as $\ell_i(U_0) \le \ell_i(U_2)$) but need not maximize the slope-m objective, so it lower-bounds the maximum. The second equality uses the affine identity $V_{00}(U_0) = V_{00}(U_2) + m(U_0 - U_2)$ together with $D(u_h^2, u_k^2|U_0) = D(u_h^2, u_k^2|U_2) + (r + \mu_h + \mu_k)(U_0 - U_2)$. The third equality adds and subtracts $V'_{00}(U_2^+)D(u_h^2, u_k^2|U_2)$. The fourth equality uses that (u_h^2, u_k^2) maximizes $\Phi(U_2)$. The fifth equality uses $\Phi(U_2) = 0$. The final inequality uses $m \ge V'_{00}(U_2^+)$ since the value function is concave and $D(u_h^2, u_k^2|U_2) \ge 0$. This all leads to a contradiction, and hence $\Phi(U_0) = 0$ at every point where $V_{00}(U)$ is differentiable.

Points of Non-Differentiability. Consider $\tilde{U} > 0$ with $m^- := V'_{00}(\tilde{U}^-) > V'_{00}(\tilde{U}^+) =: m^+$. A payout is strictly suboptimal since $m^- > -1$, and a randomization is strictly suboptimal since the kink makes V_{00} strictly concave at $\tilde{U}$. Therefore, delay is optimal and $\Phi(\tilde{U}) = 0$.

Continuously Differentiable. Consider $\tilde{U} > 0$ with $m^- := V'_{00}(\tilde{U}^-) > V'_{00}(\tilde{U}^+) =: m^+$. The same arguments as above imply that delay is optimal and $\Phi(\tilde{U}) = 0$.

We next show that the delay policy $(\tilde{u}_h, \tilde{u}_k)$ supporting $\tilde{U}$ satisfies $D(\tilde{u}_h, \tilde{u}_k|\tilde{U}) = 0$. Suppose instead $D(\tilde{u}_h, \tilde{u}_k|\tilde{U}) > 0$. Then the promise drifts strictly up under $(\tilde{u}_h, \tilde{u}_k)$, so the value at $\tilde{U}$ is priced at the right derivative and $\Phi(\tilde{U}) = 0$. For $U \in (\tilde{U} - \epsilon, \tilde{U})$ with $\epsilon > 0$ and small, the pair $(\tilde{u}_h, \tilde{u}_k)$ remains feasible, since $\ell_i(U) < \ell_i(\tilde{U}) \le \tilde{u}_i$, so

$$0 \ge \Phi(U) \ge -(r + \mu_h + \mu_k)V_{00}(U) + \mu_h V_{h0}(\tilde{u}_h) + \mu_k V_{0k}(\tilde{u}_k) + V'_{00}(U^+)D(\tilde{u}_h, \tilde{u}_k|U),$$

where the drift $D(\tilde{u}_h, \tilde{u}_k|U)$ is affine in U and hence strictly positive since U is close to $\tilde{U}$, so the right derivative prices this pair. Letting $U \uparrow \tilde{U}$ and using the fact that $D(\tilde{u}_h, \tilde{u}_k|\tilde{U}) > 0$, $V_{00}'(U^+) \geq m^- > m^+$, and $\Phi(\tilde{U}) = 0$ yields

$$\begin{aligned}
0 &\geq -(r+\mu_h+\mu_k)V_{00}(\tilde{U}) + \mu_h V_{h0}(\tilde{u}_h) + \mu_k V_{0k}(\tilde{u}_k) + m^- D(\tilde{u}_h, \tilde{u}_k|\tilde{U}) \\
&= \Phi(\tilde{U}) + (m^- - m^+)D(\tilde{u}_h, \tilde{u}_k|\tilde{U}) \\
&> 0,
\end{aligned}$$

a contradiction. Therefore, $D(\tilde{u}_h, \tilde{u}_k|\tilde{U}) \leq 0$.

Suppose instead $D(\tilde{u}_h, \tilde{u}_k|\tilde{U}) < 0$. Then the promise drifts strictly down under $(\tilde{u}_h, \tilde{u}_k)$, so the value at $\tilde{U}$ is priced at the left derivative and $\Phi(\tilde{U}) = 0$. For $\epsilon > 0$, the shifted pair $(\tilde{u}_h + \epsilon, \tilde{u}_k + \epsilon)$ is feasible at $\tilde{U} + \epsilon$, since $\ell_i(\tilde{U} + \epsilon) = \ell_i(\tilde{U}) + \epsilon \leq \tilde{u}_i + \epsilon$, and its drift is

$$D(\tilde{u}_h + \epsilon, \tilde{u}_k + \epsilon|\tilde{U} + \epsilon) = D(\tilde{u}_h, \tilde{u}_k|\tilde{U}) + r\epsilon,$$

which remains strictly negative for ϵ small, so the left derivative prices this pair. Hence

$$0 \geq \Phi(\tilde{U}+\epsilon) \geq -(r+\mu_h+\mu_k)V_{00}(\tilde{U}+\epsilon)+\mu_h V_{h0}(\tilde{u}_h+\epsilon)+\mu_k V_{0k}(\tilde{u}_k+\epsilon)+V_{00}'((\tilde{U}+\epsilon)^-)\big[D(\tilde{u}_h, \tilde{u}_k|\tilde{U})+r\epsilon\big].$$

Letting $\epsilon \downarrow 0$ and using continuity of $V_{00}(U)$, $V_{h0}(U)$, and $V_{0k}(U)$, and the fact that $V_{00}'((\tilde{U} + \epsilon)^-) \leq m^+ < m^-$, $D(\tilde{u}_h, \tilde{u}_k|\tilde{U}) < 0$, and $\Phi(\tilde{U}) = 0$ yields

$$\begin{aligned}
0 &\geq -(r+\mu_h+\mu_k)V_{00}(\tilde{U}) + \mu_h V_{h0}(\tilde{u}_h) + \mu_k V_{0k}(\tilde{u}_k) + m^+ D(\tilde{u}_h, \tilde{u}_k|\tilde{U}) \\
&= \Phi(\tilde{U}) + (m^+ - m^-)D(\tilde{u}_h, \tilde{u}_k|\tilde{U}) \\
&> 0,
\end{aligned}$$

a contradiction. Therefore, $D(\tilde{u}_h, \tilde{u}_k|\tilde{U}) = 0$.

It remains to rule out kinks. Consider $\tilde{U} > 0$ with $m^- := V_{00}'(\tilde{U}^-) > V_{00}'(\tilde{U}^+) =: m^+$, and let $(\tilde{u}_h, \tilde{u}_k)$ denote the supporting pair, which we have shown has zero drift, $D(\tilde{u}_h, \tilde{u}_k|\tilde{U}) = 0$. The continuation term therefore attains the maximum with the derivative term vanishing,

$$(r+\mu_h+\mu_k)V_{00}(\tilde{U}) = \mu_h V_{h0}(\tilde{u}_h) + \mu_k V_{0k}(\tilde{u}_k). \tag{A.23}$$

We compare this identity with the value at two nearby promises, approaching $\tilde{U}$ from below in Step 1 and from above in Step 2. Each comparison bounds the same combination of branch slopes, from opposite sides, and the two bounds are incompatible with $m^- > m^+$.

Step 1: approaching from below. Fix $\epsilon > 0$, let $U_\epsilon := \tilde{U} - \epsilon$, and consider the pair $(\tilde{u}_h - \epsilon, \tilde{u}_k - \epsilon)$, which is feasible at U_ϵ because $\ell_i(U_\epsilon) = \ell_i(\tilde{U}) - \epsilon$, and whose drift is $D(\tilde{u}_h - \epsilon, \tilde{u}_k - \epsilon|U_\epsilon) = -r\epsilon < 0$, so it is priced at $m_\epsilon := V_{00}'(U_\epsilon^-)$. Optimality of the value function, $\Phi(U_\epsilon) \leq 0$, applied to this feasible but possibly suboptimal pair gives

$$(r+\mu_h+\mu_k)V_{00}(U_\epsilon) \geq \mu_h V_{h0}(\tilde{u}_h - \epsilon) + \mu_k V_{0k}(\tilde{u}_k - \epsilon) - m_\epsilon r\epsilon.$$

Subtracting this from equation (A.23), gives

$$(r+\mu_h+\mu_k)(V_{00}(\tilde{U})-V_{00}(U_\epsilon)) \leq \mu_h(V_{h0}(\tilde{u}_h)-V_{h0}(\tilde{u}_h-\epsilon))+\mu_k(V_{0k}(\tilde{u}_k)-V_{0k}(\tilde{u}_k-\epsilon))+m_\epsilon r\epsilon.$$

Bounding $V_{00}(\tilde{U}) - V_{00}(U_\epsilon) \geq m^-\epsilon$ and $V_i(\tilde{u}_i) - V_i(\tilde{u}_i - \epsilon) \leq V_i'(\tilde{u}_i - \epsilon)\epsilon$ by concavity, and dividing by ϵ yields

$$(r + \mu_h + \mu_k)m^- \leq \mu_h V_{h0}'(\tilde{u}_h - \epsilon) + \mu_k V_{0k}'(\tilde{u}_k - \epsilon) + rm_\epsilon.$$

Letting $\epsilon \downarrow 0$ and using that $V_{h0}(U)$ and $V_{0k}(U)$ are C^1 by Propositions 7 and 8, and that $m_\epsilon \to m^-$ because the left derivative of a concave function is left-continuous, gives[18]

$$(\mu_h + \mu_k)m^- \leq \mu_h V_{h0}'(\tilde{u}_h) + \mu_k V_{0k}'(\tilde{u}_k). \tag{A.24}$$

Step 2: approaching from above. Fix $\epsilon > 0$, let $U^\epsilon := \tilde{U}+\epsilon$, and consider the pair $(\tilde{u}_h+\epsilon, \tilde{u}_k+\epsilon)$, which is feasible at U^ϵ because $\ell_i(U^\epsilon) = \ell_i(\tilde{U})+\epsilon$, and whose drift is $D(\tilde{u}_h+\epsilon, \tilde{u}_k+\epsilon|U^\epsilon) = r\epsilon > 0$, so it is priced at $m^\epsilon := V_{00}'((U^\epsilon)^+)$. Optimality of the value function, $\Phi(U^\epsilon) \leq 0$, applied to this feasible but possibly suboptimal pair gives

$$(r + \mu_h + \mu_k)V_{00}(U^\epsilon) \geq \mu_h V_{h0}(\tilde{u}_h + \epsilon) + \mu_k V_{0k}(\tilde{u}_k + \epsilon) + m^\epsilon r\epsilon.$$

Subtracting equation (A.23) from this gives

$$(r+\mu_h+\mu_k)(V_{00}(U^\epsilon)-V_{00}(\tilde{U})) \geq \mu_h(V_{h0}(\tilde{u}_h+\epsilon)-V_{h0}(\tilde{u}_h))+\mu_k(V_{0k}(\tilde{u}_k+\epsilon)-V_{0k}(\tilde{u}_k))+m^\epsilon r\epsilon.$$

Bounding $V_{00}(U^\epsilon) - V_{00}(\tilde{U}) \leq m^+\epsilon$ and $V_i(\tilde{u}_i + \epsilon) - V_i(\tilde{u}_i) \geq V_i'(\tilde{u}_i + \epsilon)\epsilon$ by concavity, and dividing by ϵ yields

$$(r + \mu_h + \mu_k)m^+ \geq \mu_h V_{h0}'(\tilde{u}_h + \epsilon) + \mu_k V_{0k}'(\tilde{u}_k + \epsilon) + rm^\epsilon.$$

Letting $\epsilon \downarrow 0$ and using that $V_{h0}(U)$ and $V_{0k}(U)$ are C^1 by Propositions 7 and 8, and that $m^\epsilon \to m^+$ because the right derivative of a concave function is right-continuous, gives[19]

$$(\mu_h + \mu_k)m^+ \geq \mu_h V_{h0}'(\tilde{u}_h) + \mu_k V_{0k}'(\tilde{u}_k). \tag{A.25}$$

[18]Since $V_{00}'(\cdot^-)$ is non-increasing, $m_\epsilon \geq m^-$ and m_ϵ is non-decreasing in ϵ, so $\lim_{\epsilon\to 0} m_\epsilon$ exists. Concavity implies that for any $w < U_\epsilon$

$$m^- = V_{00}'(\tilde{U}^-) \leq V_{00}'(U_\epsilon^-) = m_\epsilon \leq \frac{V_{00}(U_\epsilon) - V_{00}(w)}{U_\epsilon - w}.$$

Fixing $w < \tilde{U}$ and letting $\epsilon \to 0$ gives

$$m^- \leq \lim_{\epsilon\to 0} m_\epsilon \leq \frac{V_{00}(\tilde{U}) - V_{00}(w)}{\tilde{U} - w}.$$

Letting $w \to \tilde{U}$, the right-hand side converges to $V_{00}'(\tilde{U}^-) = m^-$, so $m_\epsilon \to m^-$.

[19]The argument for right continuity of the derivative is symmetric to the one in footnote 18, with all inequalities reversed.

Combining equations (A.24) and (A.25),

$$(\mu_h + \mu_k)m^- \leq \mu_h V'_{h0}(\tilde{u}_h) + \mu_k V'_{0k}(\tilde{u}_k) \leq (\mu_h + \mu_k)m^+,$$

so that $m^- \leq m^+$, contradicting $m^- > m^+$. Hence $V_{00}(U)$ has no kink for $U > 0$.

The proof establishes that $V_{00}(U)$ is concave and differentiable on $(0, \infty)$, so that $V'_{00}(U^\pm) = V'_{00}(U)$ and the continuation term $\Phi(U)$ in equation (A.22) coincides with the maximum in the proposition evaluated at $V'_{00}(U)$. Since $\Phi(U) = 0$ on $(0, \overline{U}_{00})$, the value function solves the differential equation of the proposition with boundary condition $V_{00}(0) = 0$. Furthermore, the proof shows that $\overline{U}_{00} \in (0, \infty)$ and that $V_{00}(U) = \overline{J} - U$ on $[\overline{U}_{00}, \infty)$, which establishes the proposition. □

Lemma 1 (Non-Negative Continuation Utility Drift Implies $V'_{00}(\overline{U}) \leq -\theta_k$)**.** *Let $\overline{U} > 0$ be a point at which some feasible pair $(\overline{u}_h, \overline{u}_k)$ with drift $D(\overline{u}_h, \overline{u}_k|\overline{U}) \geq 0$ attains the maximum in equation (A.22) with $\Phi(\overline{U}) = 0$. Then $V'_{00}(\overline{U}) \leq -\theta_k$. In particular, since $\theta_k > 0$, no point with $V'_{00}(\overline{U}) \geq 0$ is supported by continuation with non-negative drift.*

Proof of Lemma 1. We proceed in three steps. We first show that the k-branch slope is at most $-\theta_k$ at every feasible milestone promise. We then bound the h-branch slope by the slope of V_{00}. We finally combine the two bounds using a difference quotient. Throughout we use that $V_{00}(U)$ is C^1 by Proposition 9, so that the continuation term $\Phi(U)$ is evaluated at $V'_{00}(U)$.

Step 1: $V'_{0k}(\overline{u}_k) \leq -\theta_k$. Since $\overline{u}_h \geq \ell_h(\overline{U})$ and $\overline{u}_k \geq \ell_k(\overline{U})$,

$$0 \leq D(\overline{u}_h, \overline{u}_k|\overline{U}) \leq (r + \mu_h + \mu_k)\overline{U} - \mu_h \ell_h(\overline{U}) - \mu_k \ell_k(\overline{U}) = r\overline{U} - \gamma_h - \gamma_k,$$

so $\overline{U} \geq \frac{\gamma_h + \gamma_k}{r}$ and hence

$$\overline{u}_k \geq \ell_k(\overline{U}) > \overline{U} \geq \frac{\gamma_h + \gamma_k}{r} > \frac{\gamma_h}{r + \lambda_k} = \hat{U}^{X=0}_{0k} \geq U_X.$$

The definition of $\hat{U}^{X=0}_{0k}$ and the fact that $\hat{U}^{X=0}_{0k} \geq U_X$ follow from the proof of Proposition 8. Furthermore, from Proposition 8 it also follows that $V'_{0k}(U_X) \leq -\theta_k$ by construction. Since $V_{0k}(U)$ is concave, $V'_{0k}(U)$ is non-increasing, so $V'_{0k}(\overline{u}_k) \leq -\theta_k$.

Step 2: $V'_{h0}(\overline{u}_h) \leq V'_{00}(\overline{U})$. The pair $(\overline{u}_h, \overline{u}_k)$ attains the maximum in equation (A.22) at $\overline{U}$. Since $D(u_h, u_k|\overline{U})$ is additively separable in u_h and u_k, this maximum separates across branches, so $\overline{u}_h$ maximizes $\mu_h\big(V_{h0}(u_h) - V'_{00}(\overline{U})u_h\big)$ subject to $u_h \geq \ell_h(\overline{U})$. The constraint bounds u_h from below only, so increasing $\overline{u}_h$ is always feasible and the VC cannot be strictly better off by doing so,

$$0 \geq \mu_h\big(V'_{h0}(\overline{u}_h) - V'_{00}(\overline{U})\big),$$

which gives $V'_{h0}(\overline{u}_h) \leq V'_{00}(\overline{U})$.

Step 3: Difference quotients. Fix $\epsilon > 0$ small, let $U_\epsilon := \overline{U} - \epsilon$, and consider the pair $(\overline{u}_h - \epsilon, \overline{u}_k - \epsilon)$, which is feasible at U_ϵ because $\ell_i(U_\epsilon) = \ell_i(\overline{U}) - \epsilon$, and whose drift is

$D(\overline{u}_h-\epsilon,\overline{u}_k-\epsilon|U_\epsilon)=D(\overline{u}_h,\overline{u}_k|\overline{U})-r\epsilon$. Optimality of the value function, $\Phi(U_\epsilon)\leq 0$, applied to this feasible but possibly suboptimal pair $(\overline{u}_h-\epsilon,\overline{u}_k-\epsilon)$ gives

$$(r+\mu_h+\mu_k)V_{00}(U_\epsilon)\geq \mu_h V_{h0}(\overline{u}_h-\epsilon)+\mu_k V_{0k}(\overline{u}_k-\epsilon)+V_{00}'(U_\epsilon)\big[D(\overline{u}_h,\overline{u}_k|\overline{U})-r\epsilon\big], \quad (A.26)$$

whereas at $\overline{U}$ the supporting pair $(\overline{u}_h,\overline{u}_k)$ attains the maximum with equality,

$$(r+\mu_h+\mu_k)V_{00}(\overline{U})=\mu_h V_{h0}(\overline{u}_h)+\mu_k V_{0k}(\overline{u}_k)+V_{00}'(\overline{U})D(\overline{u}_h,\overline{u}_k|\overline{U}). \quad (A.27)$$

Subtracting equation (A.26) from equation (A.27) and using $\big(V_{00}'(\overline{U})-V_{00}'(U_\epsilon)\big)D(\overline{u}_h,\overline{u}_k|\overline{U})\leq 0$, which holds because $V_{00}'(U)$ is non-increasing and $D(\overline{u}_h,\overline{u}_k|\overline{U})\geq 0$, gives

$$(r+\mu_h+\mu_k)\big[V_{00}(\overline{U})-V_{00}(U_\epsilon)\big]\leq \mu_h\big[V_{h0}(\overline{u}_h)-V_{h0}(\overline{u}_h-\epsilon)\big]+\mu_k\big[V_{0k}(\overline{u}_k)-V_{0k}(\overline{u}_k-\epsilon)\big]+V_{00}'(U_\epsilon)r\epsilon.$$

Bounding $V_{00}(\overline{U})-V_{00}(U_\epsilon)\geq V_{00}'(\overline{U})\epsilon$ and $V_i(\overline{u}_i)-V_i(\overline{u}_i-\epsilon)\leq V_i'(\overline{u}_i-\epsilon)\epsilon$ by concavity, and dividing by ϵ, yields

$$(r+\mu_h+\mu_k)V_{00}'(\overline{U})\leq \mu_h V_{h0}'(\overline{u}_h-\epsilon)+\mu_k V_{0k}'(\overline{u}_k-\epsilon)+rV_{00}'(U_\epsilon).$$

Letting $\epsilon\downarrow 0$ and using that $V_{h0}(U)$, $V_{0k}(U)$, and $V_{00}(U)$ are C^1 by Propositions 7, 8, and 9 gives

$$(\mu_h+\mu_k)V_{00}'(\overline{U})\leq \mu_h V_{h0}'(\overline{u}_h)+\mu_k V_{0k}'(\overline{u}_k)\leq \mu_h V_{00}'(\overline{U})-\mu_k\theta_k,$$

where the second inequality uses Steps 1 and 2. Hence $V_{00}'(\overline{U})\leq -\theta_k$, completing the proof. □

Lemma 2 (State-$(0,0)$ deadline). *The smallest maximizer $U_{00}=\inf\{\arg\max_{U\geq 0}V_{00}(U)\}$ is interior. Write $(u_h^\star(U),u_k^\star(U))$ for any pair attaining the maximum in equation (A.22) at U and $D^\star(U):=D(u_h^\star(U),u_k^\star(U)|U)$ for its drift. There exists $c>0$ such that the drift $D^\star(U)\leq -c$ for every $U\in(0,U_{00}]$. Consequently, absent a development transition the promise reaches the termination boundary from U_{00} no later than time $\frac{U_{00}}{c}$, and state $(0,0)$ always carries a finite deadline.*

Proof of Lemma 2. We proceed in three steps. We first show that the smallest maximizer of $V_{00}(U)$ is interior and that $V_{00}'(U)\geq 0$ below it. Lemma 1 then implies that the promise drifts strictly downward throughout this region. Finally, we strengthen this pointwise result to a uniform bound, which delivers a finite deadline.

Step 1. Under Assumption 3, the proof of Proposition 9 shows that $V_{00}(0)=0$ and $V_{00}'(0^+)>0$, while $V_{00}(U)=\overline{J}-U$ for $U\geq\overline{U}_{00}$. The slope $V_{00}'(U)$ is continuous by Proposition 9, positive near zero, and equal to -1 on $[\overline{U}_{00},\infty)$, so it crosses zero in $(0,\overline{U}_{00})$ and by concavity every maximizer lies in that interval; we select the smallest one,

$$U_{00}=\inf\{\arg\max_{U\geq 0}V_{00}(U)\}.$$

Since $V_{00}(U)$ is C^1 and U_{00} is an interior maximizer, $V_{00}'(U_{00})=0$. Concavity makes $V_{00}'(U)$ non-increasing, so $V_{00}'(U)\geq 0$ on $(0,U_{00}]$.

Step 2. The proof of Proposition 9 shows that $\Phi(U) = 0$ on $(0, \overline{U}_{00})$ and that the maximum in equation (A.22) is attained, so at each $U \in (0, U_{00}]$ every pair $(u_h^\star(U), u_k^\star(U))$ is feasible, attains the maximum, and satisfies $\Phi(U) = 0$. Lemma 1 together with $V_{00}'(U) \geq 0 > -\theta_k$ then rules out every supporting pair with non-negative drift on $(0, U_{00}]$, so the continuation utility drift under the optimal milestone promises $D^\star(U)$ satisfies $D^\star(U) < 0$ there.

Step 3. Step 2 establishes that the promise drifts strictly down at every point of $(0, U_{00}]$, but pointwise negativity does not yet deliver a finite deadline.[20] We therefore need to find a $c > 0$ with $D^\star(U) \leq -c$ for every $U \in (0, U_{00}]$. We establish the uniform bound on two subintervals, split at $\underline{U} := \min\{\frac{\gamma_h+\gamma_k}{2r}, U_{00}\}$.

Consider first $U \in (0, \underline{U}]$. The IC constraints give

$$D^\star(U) \leq rU - \gamma_h - \gamma_k \leq -\frac{\gamma_h + \gamma_k}{2} =: -c_1 < 0.$$

Consider next $U \in [\underline{U}, U_{00}]$ and define $c_2 := -\sup_{[\underline{U}, U_{00}]} D^\star(U)$, which satisfies $c_2 \geq 0$ by Step 2. Suppose toward a contradiction that $c_2 = 0$, so that $\sup_{[\underline{U}, U_{00}]} D^\star(U) = 0$. By definition of the supremum there is a sequence $U_n \in [\underline{U}, U_{00}]$ with $D^\star(U_n) \to 0$.

The optimal promises are bounded, $u_i^\star(U_n) \in [\ell_i(U_n), \max\{\ell_i(U_n), \overline{U}_i\}]$, because for $u_i > \max\{\ell_i(U_n), \overline{U}_i\}$, we have $V_i'(u_i) = -1 < 0 \leq V_{00}'(U_n)$, so lowering u_i strictly increases the objective in equation (A.22). Since U_n lies in $[\underline{U}, U_{00}]$ and $u_i^\star(U_n)$ lies in a bounded set, by Bolzano–Weierstrass, there exists a subsequence along which $(U_n, u_h^\star(U_n), u_k^\star(U_n)) \to (\overline{U}, \overline{u}_h, \overline{u}_k)$. Furthermore, continuity implies that $\overline{u}_h \geq \ell_h(\overline{U})$ and $\overline{u}_k \geq \ell_k(\overline{U})$ and therefore $(\overline{u}_h, \overline{u}_k)$ is feasible at $\overline{U}$. Continuity of $D(u_h, u_k|\overline{U})$ gives $D(\overline{u}_h, \overline{u}_k|\overline{U}) = \lim_n D^\star(U_n) = 0$.

Each $U_n \in [\underline{U}, U_{00}]$ is supported by continuation, so

$$(r + \mu_h + \mu_k)V_{00}(U_n) = \mu_h V_{h0}(u_h^\star(U_n)) + \mu_k V_{0k}(u_k^\star(U_n)) + V_{00}'(U_n)D^\star(U_n).$$

By Step 1 and concavity, $V_{00}'(U_n) \in [0, V_{00}'(\underline{U})]$ is bounded, so as n gets large the last term vanishes since $D^\star(U_n) \to 0$, and continuity of $V_{00}(U)$, $V_{h0}(U)$, and $V_{0k}(U)$ yields

$$(r + \mu_h + \mu_k)V_{00}(\overline{U}) = \mu_h V_{h0}(\overline{u}_h) + \mu_k V_{0k}(\overline{u}_k).$$

The pair (u_h, u_k) is feasible at U with zero drift and, by the preceding equality, delivers a continuation value of zero in equation (A.22). Since $\Phi(\overline{U}) \leq 0$, it follows that this pair attains the maximum and $\Phi(\overline{U}) = 0$. The pair therefore satisfies the hypothesis of Lemma 1 at $\overline{U}$ with $D(\overline{u}_h, \overline{u}_k|\overline{U}) = 0 \geq 0$, giving $V_{00}'(\overline{U}) \leq -\theta_k$, while Step 1 gives $V_{00}'(\overline{U}) \geq 0$, a contradiction. Therefore $c_2 > 0$ and $D^\star(U) \leq -c_2$ on $[\underline{U}, U_{00}]$.

Setting $c := \min\{c_1, c_2\} > 0$ gives $D^\star(U) \leq -c$ on $(0, U_{00}]$. Since continuation is weakly optimal on this region, the promise never exits it upward, so absent a development transition it satisfies $\dot{U}_t \leq -c$ and reaches zero from U_{00} no later than time $\frac{U_{00}}{c}$. State $(0, 0)$ therefore carries a finite deadline, completing the proof. $\square$

[20] Pointwise negativity of $D^\star(U)$ does not rule an infinite deadline out, because $D^\star(U)$ need not be continuous: if, for example, $D^\star(U) = -(U - \underline{U}) - \mathbb{I}_{\{U = \underline{U}\}}$, then $D^\star(U) < 0$ for every U yet $\sup_{[\underline{U}, U_{00}]} D^\star(U) = 0$ and the deadline will not be finite.

A.5 Implementation of the Optimal Contract

The relevant assumptions made in the main text are:

Assumption 4 (Optimal Contract Tie-Breaking)**.** *When indifferent, the VC*

1. *Pays out immediately rather than delaying payouts.*

2. *Sets the exit-contingent payments X_h, X_k, Y_h, and Y_k and the milestone promises u_h and u_k at their lowest optimal levels.*

3. *Does not randomize over the continuation utility.*

4. *The party controlling a bilateral exit completes it when indifferent.*

The optimal long-term contract follows from Propositions 7, 8, and 9 and Lemmas 1 and 2. It is implemented through the entrepreneur's continuation utility U_t, which evolves according to the promise-keeping equations, jumps at development transitions, is reflected at state-contingent payout boundaries, and terminates at the corresponding lower boundaries. We prove the three propositions of Section 2 in turn. Throughout, let $\ell_h(U) = U + \frac{\gamma_h}{\mu_h}$ and $\ell_k(U) = U + \frac{\gamma_k}{\mu_k}$ denote the incentive floors in (IC-hk), and write $\overline{U}_h := \overline{U}_{h0}$ and $\overline{U}_k := \overline{U}_{0k}$ for the intermediate-state payout boundaries.

Three facts recur in the two intermediate states, and we record them once. First, Propositions 7 and 8 establish that $V_{h0}(U)$ and $V_{0k}(U)$ are concave, C^1, and satisfy $V_i'(U) \geq -1$ for $i \in \{h, k\}$, where $V_h(U) = V_{h0}(U)$ and $V_k(U) = V_{0k}(U)$. Second, a payout is optimal if and only if $-V_i'(U) - 1 \geq 0$, so the payout boundary $\overline{U}_i$ is the smallest promise at which $V_i'(U) = -1$, and Assumption 4 releases any excess immediately, $dy = [U - \overline{U}_i]^+$. Third, $V_i'(U) \geq -1$ gives $MB_{Y_h} = -\mu_k(1 + V_{h0}'(U)) \leq 0$ and $MB_{Y_k} = -\mu_h(1 + V_{0k}'(U)) \leq 0$. The IPO participation constraints are implied, by $U \geq R_h$ in state $(h, 0)$ and by $R_h \leq \frac{\gamma_h}{\mu_h}$ in state $(0, k)$, so the IPO payment sits at its effort floor, $Y_h(U) = U + \frac{\gamma_k}{\mu_k}$ from (IC-h) and $Y_k(U) = U + \frac{\gamma_h}{\mu_h}$ from (IC-k). Each proof below therefore treats only what is distinctive to its state: the exit payment, the location of the payout boundary, and the drift of the promise.

Proof of Proposition 1. **Acquihire payment.** Since $V_{h0}'(U) \geq -1$ and $\theta_h \in (0, 1]$, we have $MB_{X_h} = -\lambda_h(1 + \theta_h V_{h0}'(U)) \leq 0$, so the acquihire payment sits at the acceptance floor (Acq-H), $X_h(U) = [U - \Pi_h]^+$, by either strict optimality when $\theta_h V_{h0}'(U) > -1$ or by Assumption 4 when $\theta_h V_{h0}'(U) = -1$. The third step shows that the promise never exceeds $\overline{U}_h \leq \Pi_h$ on path, so the floor is zero and $X_h = 0$.

Payout boundary. Consider first $\theta_h < 1$. Equations (A.4) and (A.2), together with $V_{h0}^p(R_h) > 0$ under Assumption 1, imply that $V_{h0}'(U) > -1$ for $U < \Pi_h$, with equality at Π_h. Payouts are therefore strictly suboptimal below Π_h and weakly optimal at and above it, so $\overline{U}_h = \Pi_h$. When $\theta_h = 1$, equation (A.4) gives $V_{h0}'(U) = -1$ on $[\hat{U}_{h0}, \Pi_h]$, while (A.2) gives $V_{h0}'(U) > -1$ below $\hat{U}_{h0}$. Assumption 4 therefore selects the earliest payout, $\overline{U}_h = \hat{U}_{h0}$.

Promise dynamics. Substituting the IPO payment, $X_h = 0$, and the Nash price (2) into the promise-keeping equation (1) gives $\dot{U}_{h0} = a_h(U - \hat{U}_{h0})$, which is strictly negative below

$\hat{U}_{h0}$ and strictly positive above it. The proof of Proposition 7 also establishes $R_h < \hat{U}_{h0} < \Pi_h$. Thus, all three regions are non-empty when $\theta_h < 1$, while the middle region degenerates when $\theta_h = 1$. A promise below $\hat{U}_{h0}$ erodes to the termination boundary R_h in finite time, at which point the entrepreneur leaves with her human capital; Lemma 3 gives the deadline in closed form. A promise above $\hat{U}_{h0}$ instead climbs to $\overline{U}_h$, where the reflecting flow payout absorbs the drift, while a promise above $\overline{U}_h$ is first paid down by the lump sum $U - \overline{U}_h$. Since $U_t \leq \overline{U}_h \leq \Pi_h$ on path, the acceptance constraint (Acq-H) holds with $X_h = 0$, with equality at Π_h, where Assumption 4 selects the acquihire. □

Proof of Corollary 1. By Proposition 1, $Y_h(U) = U + \frac{\gamma_k}{\mu_k}$ and $X_h(U) = 0$ on path, so equation (2) gives $H(U) = \theta_h \Pi_h + (1-\theta_h)U$. Hence Y_h is strictly increasing in U, and H is weakly increasing, strictly so when $\theta_h < 1$. If the state is entered below $\hat{U}_{h0}$, the drift $\dot{U}_{h0} = a_h(U - \hat{U}_{h0})$ is strictly negative, so the promise declines to R_h at the deadline of Lemma 3 and both rewards decline over time and attain their minimum at termination. If the state is entered above $\hat{U}_{h0}$, the promise either drifts up to $\overline{U}_{h0}$ or is paid down to $\overline{U}_{h0}$ on entry and held there, so both rewards weakly increase over time. □

Proof of Proposition 2. **Acquisition payment.** The marginal benefit $MB_{X_k} = -\lambda_k(\theta_k + V_{0k}'(U))$ is constant in X_k, so the acquisition payment is bang-bang between the limited-liability floor and the acceptance ceiling (Acq-K), as in equation (A.5). Concavity makes $V_{0k}'(U)$ non-increasing, so the switch occurs at the single threshold U_X of Proposition 8. The contract sets $X_k = 0$ for $U < U_X$, and at U_X, when $U_X > 0$, the VC is indifferent and Assumption 4 selects $X_k = 0$. When $\theta_k < 1$, strict concavity on $(U_X, \overline{U}_{0k})$ ensures that $V_{0k}'(U) < -\theta_k$ and $X_k(U) = \Pi_k - V_{0k}(U) > 0$. When $\theta_k = 1$ then $U_X = \overline{U}_{0k}$ and the set $(U_X, \overline{U}_{0k})$ is empty. On the acquisition-payment region the acceptance constraint (Acq-K) binds, so equation (4) gives $K = \Pi_k$ and the VC extracts the acquirer's full valuation.

Payout boundary. Consider first $\theta_k < 1$. The proof of Proposition 8 establishes that $V_{0k}'(U) > -1$ below $\hat{U}_{0k}$ with $\lim_{U \to \hat{U}_{0k}} V_{0k}'(U) = -1$, so payouts are strictly suboptimal below $\hat{U}_{0k}$ and weakly optimal above, and $\overline{U}_k = \hat{U}_{0k}$. Consider next $\theta_k = 1$. Equation (A.10) gives $U_X = \hat{U}_{0k}^{X=0}$, and the first case in the proof of Proposition 8 shows that no surplus is lost above U_X, so $V_{0k}'(U) = -1$ from U_X on and Assumption 4 selects the earliest payout, $\overline{U}_k = \hat{U}_{0k}^{X=0}$. The promise then never exceeds U_X on path, so the acquisition-payment region is empty and $X_k = 0$ throughout when $\theta_k = 1$.

Promise dynamics. The drifts below and above U_X are equations (A.7) and (A.13) of the proof of Proposition 8, evaluated at the optimal payments, and both are strictly negative below the payout boundary. Both drifts are non-decreasing in U, the first being affine with slope a_k and the second having slope $r - \lambda_k L_k'(U) > 0$ because $L_k(U)$ is decreasing, so along a decreasing path the drift never exceeds its strictly negative value at entry, and the promise reaches the termination boundary $U = 0$ in finite time, at which point the contract is terminated and the VC collects R_k; Lemma 4 gives the corresponding deadline. The drift vanishes exactly at $\overline{U}_k$: at $\hat{U}_{0k}$ when $\theta_k < 1$, because $L_k(\hat{U}_{0k}) = 0$, and at $\hat{U}_{0k}^{X=0}$ when $\theta_k = 1$. The payout boundary can therefore be reached only upon entering the state, in which case the payout $dy = U - \overline{U}_k$ is made once and the promise remains constant thereafter. This establishes the three regions and the proposition. $\square$

Proof of Corollary 2. By Proposition 2, $Y_k(U) = U + \frac{\gamma_h}{\mu_h}$ is strictly increasing in U, while $X_k(U) = 0$ for $U \leq U_X$ and $X_k(U) = \Pi_k - V_{0k}(U)$ for $U > U_X$. On $(U_X, \overline{U}_{0k}]$ Proposition 8 gives $V_{0k}'(U) \leq -\theta_k < 0$, so $\Pi_k - V_{0k}(U)$ is strictly increasing there, and X_k jumps up at U_X because $\Pi_k - V_{0k}(U_X) > 0$. Hence X_k is weakly increasing in U. By Lemma 4 the deadline is finite if and only if the state is entered strictly below the payout boundary $\overline{U}_{0k}$, in which case Proposition 2 gives a strictly negative drift throughout $[0, \overline{U}_{0k})$. Absent a breakthrough the promise therefore declines monotonically to the termination boundary, and both payments weakly decline with it, attaining their minimum at the deadline. $\square$

Proof of Proposition 3. **Initial promise and deadline.** The VC has full bargaining power and the entrepreneur's outside option at the contracting stage is normalized to zero, so the VC offers the promise that maximizes its own payoff; when indifferent, Assumption 4 selects the smallest such promise, $U_{00} = \inf\{\arg\max_{U \geq 0} V_{00}(U)\}$. Proposition 9 and its proof establish that $V_{00}(U)$ is concave and C^1 on $(0, \infty)$ with $V_{00}(0) = 0$, $V_{00}'(0^+) > 0$, and $V_{00}(U) = \overline{J} - U$ for $U \geq \overline{U}_{00}$, so the maximizer is interior and the first-order condition holds, $V_{00}'(U_{00}) = 0$. Lemma 2 establishes that there exists $c > 0$ such that the drift under the optimal milestone promises satisfies $D^\star(U) \leq -c$ for every $U \in (0, U_{00}]$. Absent a development transition, the promise therefore reaches the termination boundary from U_{00} no later than time $\frac{U_{00}}{c}$, so the initial deadline is finite; at that point the contract is terminated and, since no capital has been developed, both parties receive nothing.

No interim payouts. Since $V_{00}'(U_{00}) = 0 > -1$ and $V_{00}'(U)$ is non-increasing by concavity, $V_{00}'(U) \geq 0 > -1$ for every $U \in (0, U_{00}]$. The payout term of the (HJB_0) equation, $-V_{00}'(U) - 1$, is therefore strictly negative on the on-path region, so an immediate payout is strictly suboptimal and $dy = 0$. Randomization is likewise never strictly optimal, because $V_{00}(U)$ is concave.

Milestone promises. Fix $U \in (0, U_{00}]$, so that $V_{00}'(U) \geq 0 > -1$ by the second step. Given the slope $V_{00}'(U)$, the inner maximization in the (6) equation separates across branches,

$$\max_{u_h \geq \ell_h(U)} \Big(\mu_h \big[V_{h0}(u_h) - V_{00}'(U) u_h \big] \Big) + \max_{u_k \geq \ell_k(U)} \Big(\mu_k \big[V_{0k}(u_k) - V_{00}'(U) u_k \big] \Big),$$

and each objective is concave in its own promise. The maximizer is finite because $V_i'(u) = -1 < 0 \leq V_{00}'(U)$ for every $u \geq \overline{U}_i$, and the smallest one selected by Assumption 4 is

$$u_i^\star(U) = \inf\{u_i \geq \ell_i(U) | V_i'(u_i) \leq V_{00}'(U)\},$$

which is the expression in the proposition. Notice that the floor need not bind: whenever $V_i'(\ell_i(U)) > V_{00}'(U)$, the VC promises strictly more than incentives require, because a marginal promise is worth more in the branch than at the current promise.

Milestone payouts. If $\ell_i(U) \geq \overline{U}_i$, then $V_i'(\ell_i(U)) = -1 \leq V_{00}'(U)$, so the incentive floor itself satisfies the inequality defining $u_i^\star(U)$ and hence $u_i^\star(U) = \ell_i(U)$. If instead $\ell_i(U) < \overline{U}_i$, then $V_i'(\overline{U}_i) = -1 \leq V_{00}'(U)$ and hence $u_i^\star(U) \leq \overline{U}_i$. The milestone promise therefore exceeds the branch payout boundary only when the incentive floor does, and then coincides with it. For promises above $\overline{U}_i$ we have $V_i'(U) = -1$, so the VC is indifferent between carrying the promise and paying it down, and Assumption 4 selects the immediate milestone payout in the first case, whereas no payout is made in the second. In both cases

$$B_i(U) = \left[u_i^\star(U) - \overline{U}_i\right]^+ = \left[\ell_i(U) - \overline{U}_i\right]^+,$$

and the branch is entered at $\min\{u_i^\star(U), \overline{U}_i\}$. Collecting the initial promise, the deadline, the absence of interim payouts, and the milestone promises and payouts establishes the proposition. □

Proof of Corollary 3. Write $\iota_i(q) = \inf\{u \geq \mathbb{I}_{i=h} R_h \,|\, V_i'(u) \leq q\}$. Since $V_i(u)$ is concave, Proposition 3 gives

$$u_i^\star(U) = \max\left\{\ell_i(U),\, \iota_i(V_{00}'(U))\right\},$$

so it suffices to show that each argument of the maximum is non-decreasing in U. The first is, because $\ell_i(U) = U + \frac{\gamma_i}{\mu_i}$ is strictly increasing. For the second, $\iota_i(q)$ is non-increasing in q because $V_i'(u)$ is non-increasing in u, while $V_{00}'(U)$ is non-increasing in U by concavity of $V_{00}(U)$; composing the two, $\iota_i(V_{00}'(U))$ is non-decreasing in U. Hence $u_i^\star(U)$ is weakly increasing in U, and so are the milestone payment $B_i(U) = [\ell_i(U) - \overline{U}_i]^+$ and the entry promise $\min\{u_i^\star(U), \overline{U}_i\}$. The deadlines of Lemmas 3 and 4 are increasing in the entry promise where they are finite, and are infinite at and above the state's stationary promise, so they too are weakly increasing in U. Lemma 2 establishes that the promise drifts strictly down on $(0, U_{00}]$, so U is strictly decreasing in the time taken to reach the milestone, which establishes the corollary. □

A.6 Deadlines

A deadline is the longest a relationship can survive without progress. In each development state, the relationship ends when the entrepreneur's promise first reaches a state-contingent lower boundary $\underline{U}_{h_t k_t}$, equal to zero in states $(0,0)$ and $(0,k)$, where the VC dismisses her, and to R_h in state $(h,0)$, where she leaves with her human capital. We define the deadline $T_D^{h_t k_t}(U)$ in state (h_t, k_t) as the time the continuation utility, starting at U, takes to reach $\underline{U}_{h_t k_t}$ absent any obsolescence, development transition, acquihire, or acquisition. Here U is the continuation utility with which the state is actually entered, after any immediate milestone payment when applicable. Therefore, it is the financing horizon the entrepreneur faces on entering the state with continuation utility U.

Between Poisson events and below the payout boundaries, the promise drifts deterministically and evolves as

$$dU_t = \dot{U}_{h_t k_t}(U_t)dt,$$

with the drift $\dot{U}_{h_t k_t}$ given by the promise-keeping equations (1), (3), and (5) at the optimal payments. Wherever the drift is strictly negative, $t \mapsto U_t$ is a strictly decreasing bijection from $[0, T_D^{h_tk_t}(U))$ onto $(\underline{U}_{h_tk_t}, U]$, so we may change the variable of integration in

$$T_D^{h_tk_t}(U) = \int_0^{T_D^{h_tk_t}(U)} dt = \int_U^{\underline{U}_{h_tk_t}} \frac{dU'}{\dot{U}_{h_tk_t}(U')} = \int_{\underline{U}_{h_tk_t}}^{U} \frac{dU'}{-\dot{U}_{h_tk_t}(U')}, \tag{A.28}$$

the last equality flipping the limits so the integrand is positive. In the two intermediate states, the deadline $T_D^{h_tk_t}(U)$ is finite if and only if the promise U lies strictly below the state's smallest stationary promise, where the drift is negative and the promise reaches the termination boundary $\underline{U}_{h_tk_t}$ in finite time. In state $(0,0)$, we establish finiteness only on the on-path region $(0, U_{00}]$, which is all the contract uses. The explicit deadlines below make this precise state by state.

In state $(h,0)$ the promise erodes toward the outside option R_h, and integrating that drift gives the deadline in closed form.

Lemma 3 (Deadline in state $(h,0)$)**.** *Let $a_h = r + \theta_h \lambda_h$. The deadline is*

$$T_D^{h0}(U) = \begin{cases} \frac{1}{a_h} \ln\left(\dfrac{\hat{U}_{h0} - R_h}{\hat{U}_{h0} - U}\right), & U \in [R_h, \hat{U}_{h0}), \\ \infty, & U \geq \hat{U}_{h0}. \end{cases}$$

Proof of Lemma 3. On the deadline region $[R_h, \hat{U}_{h0})$ the optimal payments are $Y_h = U + \frac{\gamma_k}{\mu_k}$ and $X_h = 0$ by Proposition 1, so the drift $\dot{U}_{h0}$ reduces to $a_h(U - \hat{U}_{h0})$, which is strictly negative for $U < \hat{U}_{h0}$. Substituting into equation (A.28) with $\underline{U}_{h0} = R_h$ and solving the integral yields

$$T_D^{h0}(U) = \int_{R_h}^{U} \frac{dU'}{a_h(\hat{U}_{h0} - U')} = \frac{1}{a_h} \ln\left(\frac{\hat{U}_{h0} - R_h}{\hat{U}_{h0} - U}\right).$$

For $U \geq \hat{U}_{h0}$ and $\theta_h < 1$, the drift is non-negative, the promise never falls to R_h and the deadline is infinite. When $\theta_h = 1$ the payout boundary is $\overline{U}_{h0} = \hat{U}_{h0}$, the promise is paid down to $\hat{U}_{h0}$ on entry and held there by the vanishing drift, and the deadline is infinite. $\square$

State $(0,k)$ splits into the no-payment region $[0, U_X)$ and the payment region $[U_X, \hat{U}_{0k})$ of Proposition 8. On the payment region the deadline is semi-closed form only, since L_k solves the implicit equation (A.16).

Lemma 4 (Deadline in state $(0,k)$)**.** *Let $a_k = r + \lambda_k$ and $A_k = r + \mu_h + \lambda_k$, and let $L_k(U)$ be as in Proposition 8. When $\theta_k < 1$, the deadline is*

$$T_D^{0k}(U) = \begin{cases} \frac{1}{a_k} \ln\left(\frac{\hat{U}_{0k}^{X=0}}{\hat{U}_{0k}^{X=0} - U}\right), & U \in [0, U_X), \\ \dfrac{1}{A_k} \ln\left(\frac{L_k(U_X)}{L_k(U)}\right) + \frac{1}{a_k} \ln\left(\frac{\hat{U}_{0k}^{X=0}}{\hat{U}_{0k}^{X=0} - U_X}\right), & U \in [U_X, \hat{U}_{0k}), \\ \infty, & U \geq \hat{U}_{0k}. \end{cases}$$

When $\theta_k = 1$, the payment region is degenerate and

$$T_D^{0k}(U) = \begin{cases} \frac{1}{a_k} \ln\left(\frac{\hat{U}_{0k}^{X=0}}{\hat{U}_{0k}^{X=0} - U}\right), & U < \hat{U}_{0k}^{X=0}, \\ \infty, & U \geq \hat{U}_{0k}^{X=0}. \end{cases}$$

Proof of Lemma 4. Assume $\theta_k < 1$. Below U_X, Proposition 2 gives $Y_k = U + \frac{\gamma_h}{\mu_h}$ and $X_k = 0$, so the drift of the continuation utility is

$$\dot{U}_{0k}(U) = a_k(U - \hat{U}_{0k}^{X=0})$$

and therefore

$$T_D^{0k}(U) = \int_0^U \frac{dU'}{a_k(\hat{U}_{0k}^{X=0} - U')} = \frac{1}{a_k} \ln\left(\frac{\hat{U}_{0k}^{X=0}}{\hat{U}_{0k}^{X=0} - U}\right).$$

For $U \in [U_X, \hat{U}_{0k})$, Proposition 2 gives $X_k(U) = \Pi_k - V_{0k}(U)$ and the drift is

$$\dot{U}_{0k}(U) = -r(\hat{U}_{0k} - U) - \lambda_k L_k(U) = \frac{A_k L_k(U)}{L_k'(U)},$$

where the second equality follows from the differential equation (A.15). Therefore,

$$T_D^{0k}(U) - T_D^{0k}(U_X) = \int_{U_X}^U \frac{dU'}{-\dot{U}_{0k}(U')} = -\frac{1}{A_k} \int_{U_X}^U \frac{L_k'(U')}{L_k(U')} dU' = \frac{1}{A_k} \ln\left(\frac{L_k(U_X)}{L_k(U)}\right),$$

and since the time from U_X down to zero is $T_D^{0k}(U_X) = \frac{1}{a_k} \ln\left(\frac{\hat{U}_{0k}^{X=0}}{\hat{U}_{0k}^{X=0} - U_X}\right)$ by the first case, the result follows. For $U \geq \hat{U}_{0k}$, the promise is paid down to the payout boundary $\overline{U}_{0k} = \hat{U}_{0k}$ on entry, where the drift vanishes, so it never falls to zero and the deadline is infinite.

When $\theta_k = 1$, then $U_X = \hat{U}_{0k}^{X=0}$, and for $U \in [0, \hat{U}_{0k}^{X=0})$ the deadline is derived exactly as for $U < U_X$ above. For $U \geq \hat{U}_{0k}^{X=0}$, the promise is paid down to $\overline{U}_{0k} = \hat{U}_{0k}^{X=0}$ on entry, where the drift vanishes, so the deadline is again infinite. □

State $(0, 0)$ has no closed-form milestone promises, so we give the deadline in integral form. The termination boundary is $\underline{U}_{00} = 0$, and the drift is

$$\dot{U}_{00}(U) = A_0 U - \mu_h u_h^\star(U) - \mu_k u_k^\star(U),$$

where $A_0 = r + \mu_h + \mu_k$ and $(u_h^\star(U), u_k^\star(U))$ are the optimal milestone promises. Substituting into equation (A.28) gives for $U \in (0, U_{00}]$

$$T_D^{00}(U) = \int_0^U \frac{dU'}{\mu_h u_h^\star(U') + \mu_k u_k^\star(U') - A_0 U'}.$$

The integral is well-defined because the drift is strictly negative on the on-path region: Lemma 2 shows $A_0 U - \mu_h u_h^\star(U) - \mu_k u_k^\star(U) \leq -c < 0$ for $U \in (0, U_{00}]$, so the promise reaches zero in finite time and $T_D^{00}(U) \leq U/c$.

The following comparative statics with respect to the deadline can be established.

Proposition 10 (Conditional Post-Milestone Runway)**.** *Fix the continuation utility U in the intermediate state. All derivatives are evaluated within the indicated region.*

1. *Suppose state $(h,0)$ is entered in its deadline region, $U \in (R_h, \hat{U}_{h0})$. Its remaining runway $T_D^{h0}(U)$ is strictly decreasing in the acquihire value, arrival rate, and entrepreneur bargaining power:*

$$\frac{\partial T_D^{h0}(U)}{\partial \Pi_h} < 0, \qquad \frac{\partial T_D^{h0}(U)}{\partial \lambda_h} < 0, \qquad \frac{\partial T_D^{h0}(U)}{\partial \theta_h} < 0.$$

2. *Suppose state $(0,k)$ is entered in the no-acquisition-payment region, $U \in (0, U_X)$. Then*

$$\frac{\partial T_D^{0k}(U)}{\partial \Pi_k} = \frac{\partial T_D^{0k}(U)}{\partial \theta_k} = 0, \qquad \frac{\partial T_D^{0k}(U)}{\partial \lambda_k} > 0.$$

By contrast, if $\theta_k < 1$ and U is sufficiently close to $\hat{U}_{0k}$ from below then

$$\frac{\partial T_D^{0k}(U)}{\partial \Pi_k} < 0, \qquad \frac{\partial T_D^{0k}(U)}{\partial \lambda_k} < 0.$$

Proof of Proposition 10. We solve state $(h,0)$ and $(0,k)$ separately.

State $(h,0)$: Consider first state $(h,0)$ with $R_h < U < \hat{U}_{h0}$. By Lemma 3 and using the general representation (A.28) with the drift $a_h(U - \hat{U}_{h0})$ (from the proof of Proposition 7),

$$T_D^{h0}(U) = \int_{R_h}^{U} \frac{dU'}{\gamma_k - rU' + \theta_h\lambda_h(\Pi_h - U')}.$$

The denominator is positive throughout the integration region since $U < \hat{U}_{h0}$. Moreover, $U < \hat{U}_{h0} < \Pi_h$, so $\Pi_h - U' > 0$ for every $U' \in [R_h, U]$. Hence increasing Π_h, λ_h, or θ_h strictly increases the denominator. In addition, the lower integration bound $R_h = \frac{\alpha_h\theta_h\lambda_h\Pi_h}{r+\alpha_h\theta_h\lambda_h}$ is nondecreasing in each of these parameters, so the lower integration bound moves in the same direction. Therefore,

$$\frac{\partial T_D^{h0}(U)}{\partial \Pi_h} < 0, \qquad \frac{\partial T_D^{h0}(U)}{\partial \lambda_h} < 0, \qquad \frac{\partial T_D^{h0}(U)}{\partial \theta_h} < 0.$$

State $(0,k)$: Consider next state $(0,k)$ with $0 < U < U_X$. By Lemma 4 and using the general representation (A.28) with the drift (A.7),

$$T_D^{0k}(U) = \int_{0}^{U} \frac{dU'}{\gamma_h - (r+\lambda_k)U'}.$$

The denominator is positive throughout the integration region since $U < U_X \leq \hat{U}_{0k}^{X=0} = \frac{\gamma_h}{r+\lambda_k}$.

Neither the integrand nor the integration bounds depend on Π_k or θ_k, so

$$\frac{\partial T_D^{0k}(U)}{\partial \Pi_k} = \frac{\partial T_D^{0k}(U)}{\partial \theta_k} = 0.$$

Increasing λ_k strictly decreases the denominator for every $U' \in (0, U]$, while the lower integration bound 0 is unaffected. Therefore,

$$\frac{\partial T_D^{0k}(U)}{\partial \lambda_k} > 0.$$

Finally, suppose $\theta_k < 1$ and consider $U < \widehat{U}_{0k}$ sufficiently close to $\widehat{U}_{0k}$. Since $U_X < \widehat{U}_{0k}$, such U lies in the acquisition-payment region. Fix $U_0 \in (U_X, \widehat{U}_{0k})$ below U. By Lemma 4,

$$T_D^{0k}(U) - T_D^{0k}(U_0) = \frac{1}{A_k} \log\left(\frac{L_k(U_0)}{L_k(U)}\right), \qquad A_k = r + \mu_h + \lambda_k.$$

Proposition 8 characterizes $L_k(U)$ in this region by

$$\widehat{U}_{0k} - U = C_X L_k(U)^{q_k} + \frac{\lambda_k}{\mu_h + \lambda_k} L_k(U), \qquad q_k = \frac{r}{A_k} \in (0, 1).$$

As $U \uparrow \widehat{U}_{0k}$, $L_k(U) \downarrow 0$. Since $q_k < 1$, the $L_k(U)^{q_k}$ term dominates the linear term, and hence

$$L_k(U)^{q_k} = \frac{\widehat{U}_{0k} - U}{C_X}(1 + o(1)).$$

Taking logs gives

$$\log L_k(U) = \frac{1}{q_k}\left[\log(\widehat{U}_{0k} - U) - \log C_X\right] + o(1).$$

Therefore, for $p \in \{\Pi_k, \lambda_k\}$, holding U fixed,

$$\frac{\partial \log L_k(U)}{\partial p} = \frac{1}{q_k(\widehat{U}_{0k} - U)} \frac{\partial \widehat{U}_{0k}}{\partial p} + o\left(\frac{1}{\widehat{U}_{0k} - U}\right).$$

Indeed, the terms arising from the parameter dependence of C_X are bounded, while those arising from q_k grow at most logarithmically and are therefore of lower order than $(\widehat{U}_{0k} - U)^{-1}$.

Differentiating the deadline expression, all terms other than $-\log L_k(U)/A_k$ are likewise of lower order. In particular, when $p = \lambda_k$, differentiating $1/A_k$ contributes only a term of order $|\log L_k(U)|$. Since $A_k q_k = r$, it follows that

$$\frac{\partial T_D^{0k}(U)}{\partial p} = -\frac{1}{r(\widehat{U}_{0k} - U)} \frac{\partial \widehat{U}_{0k}}{\partial p} + o\left(\frac{1}{\widehat{U}_{0k} - U}\right).$$

For $\theta_k < 1$,

$$\widehat{U}_{0k} = \frac{\gamma_h}{r} + \frac{\lambda_k\big((r + \mu_h)\Pi_k - \mu_h I\big)}{r(r + \mu_h + \lambda_k)}.$$

Thus,

$$\frac{\partial \widehat{U}_{0k}}{\partial \Pi_k} = \frac{\lambda_k(r+\mu_h)}{r(r+\mu_h+\lambda_k)} > 0$$

and

$$\frac{\partial \widehat{U}_{0k}}{\partial \lambda_k} = \frac{(r+\mu_h)\big((r+\mu_h)\Pi_k - \mu_h I\big)}{r(r+\mu_h+\lambda_k)^2} > 0,$$

where the second inequality follows from acquisition efficiency. Hence, for U sufficiently close to $\widehat{U}_{0k}$ from below,

$$\frac{\partial T_D^{0k}(U)}{\partial \Pi_k} < 0, \qquad \frac{\partial T_D^{0k}(U)}{\partial \lambda_k} < 0.$$

□

A.7 Symmetric Setting

Proof of Proposition 4. The incentive constraint for developing transferable capital requires

$$u_{0k} \geq U + \frac{\gamma}{\mu}.$$

Symmetry, complementarity, and intermediate-exit efficiency imply $I > 2\Pi$ and $\Pi > \frac{\mu I}{r+\mu}$, and hence $r > \mu$. Hence, since $U \geq 0$,

$$U + \frac{\gamma}{\mu} \geq \frac{\gamma}{\mu} > \frac{\gamma}{r+\lambda}.$$

Proposition 8 implies that $U_X \leq \hat{U}_{0k}^{X=0} = \frac{\gamma}{r+\lambda}$. Therefore, every feasible milestone promise satisfies $u_{0k} > U_X$. Since every feasible milestone promise satisfies $u_{0k} > U_X$, concavity of $V_{0k}(U)$ and $V_{0k}'(U_X) \leq -\theta < 0$ imply $V_{0k}'(u_{0k}) < 0$ for every feasible u_{0k}. Along the optimal state-$(0,0)$ path, Proposition 3 implies $V_{00}'(U) \geq 0$. The objective $\mu\big(V_{0k}(u_{0k}) - u_{0k}V_{00}'(U)\big)$ is therefore strictly decreasing in u_{0k} on the feasible set, so its maximizer is the incentive floor, $u_{0k} = U + \frac{\gamma}{\mu}$, for every U on path. The human-capital milestone is subject to the same lower bound, so

$$u_{0k} \leq u_{h0}.$$

For $\theta < 1$, Propositions 1 and 2 give

$$\overline{U}_{h0} = \Pi \qquad \text{and} \qquad \overline{U}_{0k} = \hat{U}_{0k} = \frac{\gamma + \lambda(\Pi - S)}{r},$$

where $S = \frac{\mu I + \lambda\Pi}{r+\mu+\lambda}$. Substituting the definition of S gives $\Pi - S = \frac{(r+\mu)\Pi - \mu I}{r+\mu+\lambda}$, where the numerator is strictly positive by intermediate-exit efficiency. The payout boundary in state $(0,k)$ therefore satisfies

$$r\overline{U}_{0k} = \gamma + \lambda(\Pi - S) = \gamma + \frac{\lambda}{r+\mu+\lambda}\big[(r+\mu)\Pi - \mu I\big] < \gamma + (r+\mu)\Pi - \mu I = r\Pi - \big[\mu(I-\Pi) - \gamma\big],$$

where the inequality uses $\frac{\lambda}{r+\mu+\lambda} < 1$ and positivity of the bracket. The restriction $\mu(I-\Pi) > \gamma$ makes the final bracket strictly positive, so $r\overline{U}_{0k} < r\Pi$ and therefore

$$\overline{U}_{0k} < \Pi = \overline{U}_{h0}.$$

Therefore,

$$\min\{u_{h0}, \overline{U}_{h0}\} \geq \min\{u_{0k}, \overline{U}_{0k}\}$$

and the entrepreneur enters state $(h, 0)$ with weakly more continuation utility than state $(0, k)$. As a consequence

$$Y_k = \min\{u_{0k}, \overline{U}_{0k}\} + \frac{\gamma}{\mu} \leq \min\{u_{h0}, \overline{U}_{h0}\} + \frac{\gamma}{\mu} = Y_h \leq \Pi + \frac{\gamma}{\mu} < I$$

where the last inequality follows from the fact that $\mu(I - \Pi) > \gamma$.

For the milestone payments there are two cases. First, if $u_{h0} = U + \frac{\gamma}{\mu}$ then

$$B_h(U) = [u_{h0} - \overline{U}_{h0}]^+ = \left[U + \frac{\gamma}{\mu} - \overline{U}_{h0}\right]^+ \leq \left[U + \frac{\gamma}{\mu} - \overline{U}_{0k}\right]^+ = [u_{0k} - \overline{U}_{0k}]^+ = B_k(U).$$

Second, if $u_{h0} > U + \frac{\gamma}{\mu}$ then $V'_{h0}(u_{h0}) = V'_{00}(U) \geq 0$ but this result combined with concavity of $V_{h0}(U)$ and $V'_{h0}(\Pi) = -1$ means that $u_{h0} < \Pi$. Therefore,

$$B_h(U) = [u_{h0} - \overline{U}_{h0}]^+ = 0 \leq [u_{0k} - \overline{U}_{0k}]^+ = B_k(U).$$

Finally, Proposition 1 gives $X_h = 0$ on path. We have already shown that $u_{0k} > U_X$, and $\Pi - S > 0$ together with Proposition 8 implies

$$U_X \leq \hat{U}_{0k}^{X=0} < \hat{U}_{0k} = \overline{U}_{0k}.$$

Hence the entry promise $\min\{u_{0k}, \overline{U}_{0k}\}$ lies in $(U_X, \overline{U}_{0k}]$ whether or not an immediate milestone payment is made. The proof of Proposition 8 shows that, for $\theta < 1$, $V_{0k}(U)$ is strictly concave on $(U_X, \hat{U}_{0k})$ with $V'_{0k}(U_X) \leq -\theta$ and $V'_{0k}(\hat{U}_{0k}) = -1 < -\theta$. It follows that $V'_{0k}(U) < -\theta$ at the entry promise, so by equation (A.5) the maximal acquisition payment is strictly optimal, $X_k = \Pi - V_{0k}(\cdot)$. This payment is strictly positive because $V_{0k}(U) \leq S < \Pi$. The acceptance constraint (Acq-K) then binds and equation (4) yields $K = \Pi$. □

A.8 Additional Results

Proof of Proposition 5. We treat the two intermediate states in turn. In each, the deadline threshold is the smallest stationary promise, below which Lemmas 3 and 4 give a finite deadline, and the payout boundary is the promise above which Propositions 1 and 2 pay out any excess on entry. Throughout we assume $\theta_h, \theta_k < 1$.

State $(h, 0)$**.** By Proposition 1 and Lemma 3, the deadline threshold and the payout boundary are

$$\hat{U}_{h0} = \frac{\gamma_k + \theta_h \lambda_h \Pi_h}{r + \theta_h \lambda_h} \qquad \text{and} \qquad \overline{U}_{h0} = \Pi_h.$$

The payout boundary is strictly increasing in Π_h and does not depend on λ_h or θ_h. For the deadline threshold, the parameters λ_h and θ_h enter only through the product $x := \theta_h \lambda_h$, which is strictly increasing in each of them. Writing

$$\hat{U}_{h0} = \Pi_h - \frac{r\Pi_h - \gamma_k}{r + x},$$

the threshold is strictly increasing in x because $r\Pi_h > \gamma_k$, which follows from acquihire efficiency and effort efficiency in Assumption 1, and it is strictly increasing in Π_h because $\frac{\partial \hat{U}_{h0}}{\partial \Pi_h} = \frac{x}{r+x} > 0$. Hence $\hat{U}_{h0}$ is strictly increasing in Π_h, λ_h, and θ_h, which establishes the first part.

State $(0, k)$**.** By Proposition 2 and Lemma 4, the payout boundary and the deadline threshold coincide at

$$\overline{U}_{0k} = \hat{U}_{0k} = \frac{\gamma_h + \lambda_k(\Pi_k - S_k)}{r}, \qquad S_k = \frac{\mu_h I + \lambda_k \Pi_k}{r + \mu_h + \lambda_k}.$$

Substituting the definition of S_k gives $\Pi_k - S_k = \frac{(r+\mu_h)\Pi_k - \mu_h I}{r+\mu_h+\lambda_k}$, so that

$$\hat{U}_{0k} = \frac{\gamma_h}{r} + \frac{\lambda_k\big[(r+\mu_h)\Pi_k - \mu_h I\big]}{r(r+\mu_h+\lambda_k)}. \tag{A.29}$$

Acquisition efficiency in Assumption 2, $\Pi_k > S_k$, is equivalent to $(r+\mu_h)\Pi_k > \mu_h I$, so the bracket in equation (A.29) is strictly positive. Differentiating equation (A.29) yields

$$\frac{\partial \hat{U}_{0k}}{\partial \Pi_k} = \frac{\lambda_k(r+\mu_h)}{r(r+\mu_h+\lambda_k)} > 0 \qquad \text{and} \qquad \frac{\partial \hat{U}_{0k}}{\partial \lambda_k} = \frac{(r+\mu_h)\big[(r+\mu_h)\Pi_k - \mu_h I\big]}{r(r+\mu_h+\lambda_k)^2} > 0,$$

where the second inequality uses positivity of the bracket. Finally, θ_k does not appear in equation (A.29), so the common threshold is independent of the VC's bargaining power. This establishes the second part and completes the proof. □

B Stationary Benchmark: Proofs

We define a stationary contract as follows:

Definition 1 (Stationary Contract)**.** *A stationary contract is a contract in which all payments are only a function of the state* (h_t, k_t) *or a state transition, hence* $\dot{U}_t = 0$ *between transitions.*[21] *We study stationary contracts in which there is no termination, the entrepreneur exerts effort, and all exit opportunities are taken.*

[21] A state transition is when (h_t, k_t) changes or when the entrepreneur gets an acquihire offer or the VC an acquisition offer.

We impose the same parameter and indifference restrictions as before (Assumption 1, 2, 3, and 4). We can then show that:

Proposition 11 (Optimal Stationary Contract)**.** *Payments in the optimal stationary contract:*

- *IPO payment in state $i \in \{h, k\}$: $Y_i = U_i + \frac{\gamma_{i'}}{\mu_{i'}}$ where i' is the other state.*
- *No payment to the entrepreneur in an acquihire: $X_h = 0$.*
- *Acquisition payment to the entrepreneur only with bargaining: $X_k = 0$ if $\theta_k = 1$ and $X_k = \min\left\{\frac{(r+\lambda_k)U_k - \gamma_h}{\lambda_k}, \Pi_k - S_k + U_k\right\} > 0$ if $\theta_k < 1$.*
- *Milestone payout upon reaching state $i \in \{h, k\}$: $B_i = \left[U_{00} + \frac{\gamma_i}{\mu_i} - \overline{U}_i\right]^+$.*
- *Flow payment to the entrepreneur only in state h with bargaining $\theta_h < 1$: $\frac{dY_t}{dt} = U_h r - \gamma_k - \theta_h\lambda_h(\Pi_h - U_h)^+ \geq 0$.*

where U_i is the entrepreneur's continuation utility in state $i \in \{h, k\}$, $\hat{U}_i$ the stationary promise with no flow payment and no acquisition/acquihire payment ($dY_t = 0$ and $X_{h,k} = 0$) in state $i \in \{h, k\}$, and $\overline{U}_i$ the payout boundary in state $i \in \{h, k\}$. The entrepreneur's initial promise U_{00} is the largest of two candidate promises,

$$U_{00} = \max\left\{\frac{\gamma_h + \gamma_k}{r},\ \frac{\mu_h\hat{U}_h + \gamma_k}{r + \mu_h}\right\}.$$

The state-$(0, k)$ surplus is $S_k = \frac{\mu_h I + \lambda_k \Pi_k}{r + \mu_h + \lambda_k}$, and

$$\hat{U}_h = \hat{U}_{h0}, \qquad \hat{U}_k = \hat{U}_{0k}^{X=0}, \qquad \overline{U}_h = \begin{cases} \Pi_h, & \theta_h < 1, \\ \hat{U}_{h0}, & \theta_h = 1, \end{cases} \qquad \overline{U}_k = \begin{cases} \hat{U}_{0k}, & \theta_k < 1, \\ \hat{U}_{0k}^{X=0}, & \theta_k = 1. \end{cases}$$

Proof of Proposition 11. **State** $(h, 0)$**.** For a contract to be stationary, we must have that $\mathbb{E}[dU_t + dY_t] = rU_t dt$ and $\dot{U} = 0$, which implies that

$$\frac{dY_t}{dt} = rU - \mu_k(Y_h - U) - \theta_h\lambda_h(\Pi_h + X_h - U).$$

To ensure limited liability, we must have $\frac{dY_t}{dt} \geq 0$. For any $U < \hat{U}_{h0}$,

$$\frac{dY_t}{dt} \leq rU - \gamma_k - \theta_h\lambda_h(\Pi_h - U) < r\hat{U}_{h0} - \gamma_k - \theta_h\lambda_h(\Pi_h - \hat{U}_{h0}) = 0,$$

where the first inequality uses the effort constraint $\mu_k(Y_h - U) \geq \gamma_k$ and limited liability constraint $X_h \geq 0$, and the second inequality uses that $(r + \theta_h\lambda_h)U - \gamma_k - \theta_h\lambda_h\Pi_h$ is strictly increasing in U. The equality follows from the fact that $\hat{U}_{h0} = \frac{\gamma_k + \theta_h\lambda_h\Pi_h}{r + \theta_h\lambda_h}$. Thus no promise below $\hat{U}_{h0}$ can be held stationary without violating limited liability and every stationary promise U satisfies $U \geq \hat{U}_{h0}$. This also guarantees that the entrepreneur does not walk away since $U \geq \hat{U}_{h0} > R_h$.

Let $V^s_{h0}(U|X_h, Y_h)$ be the VC's value under the stationary contract with promise U. Substituting the flow payment $\frac{dY_t}{dt} = (r + \mu_k + \theta_h\lambda_h)U - \mu_k Y_h - \theta_h\lambda_h(\Pi_h + X_h)$ into the VC's stationary valuation equation gives

$$\begin{aligned} V^s_{h0}(U|X_h, Y_h) &= \frac{\mu_k(I - Y_h) - \lambda_h X_h - \frac{dY_t}{dt}}{r + \mu_k + \lambda_h} \\ &= \frac{\mu_k I + \theta_h\lambda_h\Pi_h - (r + \mu_k + \theta_h\lambda_h)U - \lambda_h(1 - \theta_h)X_h}{r + \mu_k + \lambda_h}. \end{aligned}$$

The VC chooses (Y_h, X_h) to maximize

$$V^s_{h0}(U) = \max_{Y_h, X_h \geq 0} V^s_{h0}(U|X_h, Y_h)$$

subject to

1. The IC constraint: $\mu_k(Y_h - U) \geq \gamma_k$.
2. The IPO participation constraint: $Y_h \geq R_h$.
3. The acquihire participation constraint: $\Pi_h + X_h \geq U$.
4. Limited liability of the flow payment constraint: $\frac{dY_t}{dt} \geq 0$

We know that $U \geq \hat{U}_{h0}$ and the IC constraint then implies

$$Y_h \geq \hat{U}_{h0} + \frac{\gamma_k}{\mu_k} = \frac{\gamma_k + \theta_h\lambda_h\Pi_h}{r + \theta_h\lambda_h} + \frac{\gamma_k}{\mu_k} > \frac{\theta_h\lambda_h\Pi_h}{r + \theta_h\lambda_h} \geq \frac{\alpha_h\theta_h\lambda_h\Pi_h}{r + \alpha_h\theta_h\lambda_h} = R_h$$

and so if the IC constraint is satisfied then the IPO constraint is also satisfied.

Given U the value is independent of Y_h, since a larger IPO payment is offset one-for-one by a smaller flow payment; the tie-break favoring the lowest IPO payment (Assumption 4) sets Y_h at its floor that ensures that the IC constraint is satisfied $Y_h = U + \frac{\gamma_k}{\mu_k}$, and delivers the residual as flow. The value is weakly decreasing in X_h because $-\lambda_h(1 - \theta_h) \leq 0$, and the tie-break favoring the lowest acquihire payment (Assumption 4) selects the smallest feasible $X_h = [U - \Pi_h]^+$ that ensures acquihire participation. Therefore,

$$V^s_{h0}(U) = \begin{cases} \frac{\mu_k I + \theta_h\lambda_h\Pi_h - (r + \mu_k + \theta_h\lambda_h)U}{r + \mu_k + \lambda_h}, & U \in [\hat{U}_{h0}, \Pi_h], \\ \frac{\mu_k I + \lambda_h\Pi_h - (r + \mu_k + \lambda_h)U}{r + \mu_k + \lambda_h}, & U > \Pi_h. \end{cases}$$

We just need to verify limited liability of the flow payment given $U \geq \hat{U}_{h0}$

$$\begin{aligned}\frac{dY_t}{dt} &= rU - \mu_k(Y_h - U) - \theta_h\lambda_h(\Pi_h + X_h - U) \\ &= rU - \gamma_k - \theta_h\lambda_h(\Pi_h - U)^+ \\ &\geq r\hat{U}_{h0} - \gamma_k - \theta_h\lambda_h(\Pi_h - \hat{U}_{h0})^+ \\ &\geq 0,\end{aligned}$$

where the second equality follows from the fact that $\mu_k(Y_h - U) = \gamma_k$ and $X_h = (U - \Pi_h)^+$ (or $\Pi_h + X_h - U = (\Pi_h - U)^+$), the first inequality from the fact that the function is increasing in U, and the last inequality follows from the fact that $\hat{U}_{h0} = \frac{\gamma_k + \theta_h\lambda_h\Pi_h}{r + \theta_h\lambda_h} < \Pi_h$. This also implies that the flow payment is positive for $U > \hat{U}_{h0}$.

Taking the derivative of the value function with respect to U yields

$$(V^s_{h0})'(U) = \begin{cases} -\frac{r+\mu_k+\theta_h\lambda_h}{r+\mu_k+\lambda_h}, & U \in [\hat{U}_{h0}, \Pi_h], \\ -1, & U > \Pi_h. \end{cases}$$

A milestone payout upon reaching state $(h, 0)$ is optimal only when $(V^s_{h0})'(U) \leq -1$ since in that case the marginal net benefit from lowering U through a milestone payment is

$$-(V^s_{h0})'(U) - 1 \geq 0.$$

When indifferent, the VC prefers to make payments early. Therefore, a milestone payout is made as $(V^s_{h0})'(U) = -1$ for $U > \Pi_h$ when $\theta_h < 1$ and for $U > \hat{U}_{h0}$ when $\theta_h = 1$. The VC's value function $V^s_{h0}(U)$ for $U \geq \hat{U}_{h0}$ is also the VC's value function upon reaching state $(h, 0)$ but before making the milestone payment since $(V^s_{h0})'(U) \geq -1$.

State $(0, k)$**.** For a contract to be stationary, we must have that $\mathbb{E}[dU_t + dY_t] = rU_t dt$ and $\dot{U} = 0$, which implies that

$$\frac{dY_t}{dt} = rU - \mu_h(Y_k - U) - \lambda_k(X_k - U).$$

To ensure limited liability, we must have $\frac{dY_t}{dt} \geq 0$. For any $U < \hat{U}^{X=0}_{0k}$,

$$\frac{dY_t}{dt} \leq rU - \gamma_h - \lambda_k(X_k - U) < r\hat{U}^{X=0}_{0k} - \gamma_h - \lambda_k(X_k - \hat{U}^{X=0}_{0k}) = -\lambda_k X_k \leq 0,$$

where the first inequality uses $\mu_h(Y_k - U) \geq \gamma_h$, the second uses that $(r + \lambda_k)U - \gamma_h - \lambda_k X_k$ is strictly increasing in U, the equality uses $\hat{U}^{X=0}_{0k} = \frac{\gamma_h}{r+\lambda_k}$, and the last inequality uses $X_k \geq 0$. Thus no promise below $\hat{U}^{X=0}_{0k}$ can be held stationary without violating limited liability, and every stationary promise satisfies $U \geq \hat{U}^{X=0}_{0k}$.

Let $V^s_{0k}(U|X_k, Y_k)$ be the VC's value under the stationary contract with promise U. Substituting the flow payment $\frac{dY_t}{dt} = (r + \mu_h + \lambda_k)U - \mu_h Y_k - \lambda_k X_k$ into the VC's stationary valuation equation gives

$$
\begin{aligned}
V_{0k}^s(U|X_k, Y_k) &= \frac{\mu_h(I - Y_k) + \theta_k\lambda_k(\Pi_k - X_k) - \frac{dY_t}{dt}}{r + \mu_h + \theta_k\lambda_k} \\
&= \frac{\mu_h I + \theta_k\lambda_k\Pi_k - (r + \mu_h + \lambda_k)U + \lambda_k(1 - \theta_k)X_k}{r + \mu_h + \theta_k\lambda_k}.
\end{aligned}
$$

The VC chooses (Y_k, X_k) to maximize

$$
V_{0k}^s(U) = \max_{Y_k, X_k \geq 0} V_{0k}^s(U|X_k, Y_k)
$$

subject to

1. The IC constraint: $\mu_h(Y_k - U) \geq \gamma_h$.
2. The IPO participation constraint: $Y_k \geq R_h$.
3. The acquisition acceptance constraint: $\Pi_k - X_k \geq V_{0k}^s(U|X_k, Y_k)$.
4. Limited liability of the flow payment constraint: $\frac{dY_t}{dt} \geq 0$

We know that the IC constraint implies

$$
Y_k \geq U + \frac{\gamma_h}{\mu_h} \geq U + R_h \geq R_h
$$

since $R_h \leq \frac{\gamma_h}{\mu_h}$ and so if the IC constraint is satisfied then the IPO constraint is also satisfied.

Observe that the acquisition acceptance constraint can be rewritten as $\Pi_k - V_{0k}^s(U|X_k, Y_k) \geq X_k$. For any contract in this class, $\Pi_k - V_{0k}^s(U|X_k, Y_k) > 0$ since $\Pi_k > S_k = \frac{\mu_h I + \lambda_k \Pi_k}{r + \mu_h + \lambda_k} \geq V_{0k}^s(U|X_k, Y_k)$ where the last inequality follows from the fact that the VC can never collect more than all the cash flows the firm generates.

We can rewrite the limited liability of flow payment constraint as

$$
\begin{aligned}
\frac{dY_t}{dt} &= rU - \mu_h(Y_k - U) - \lambda_k(X_k - U) \geq 0, \\
\mu_h Y_k + \lambda_k X_k &\leq (r + \mu_h + \lambda_k)U.
\end{aligned}
$$

Therefore, we try to maximize $V_{0k}^s(U|X_k, Y_k)$ subject to the constraints

$$
\begin{aligned}
Y_k &\geq U + \frac{\gamma_h}{\mu_h}, \\
X_k &\leq \Pi_k - V_{0k}^s(U|X_k, Y_k), \\
\mu_h Y_k + \lambda_k X_k &\leq (r + \mu_h + \lambda_k)U.
\end{aligned}
$$

$V_{0k}^s(U|X_k, Y_k)$ is independent of Y_k as a larger IPO payment is offset one-for-one by a smaller flow payment. Furthermore, the first constraint only depends on Y_k and not on X_k while the second constraint is independent of Y_k (since $V_{0k}^s(U|X_k, Y_k)$ is), and the third constraint becomes weakly more slack as Y_k goes down. Therefore, $Y_k = U + \frac{\gamma_h}{\mu_h}$ yields the largest feasible set for X_k. This implies that the limited liability of the flow payment constraint becomes

$$X_k \leq \frac{(r+\lambda_k)U - \gamma_h}{\lambda_k}$$

where $\frac{(r+\lambda_k)U-\gamma_h}{\lambda_k} \geq 0$ for $U \geq \hat{U}_{0k}^{X=0}$.

The VC's value $V_{0k}^s(U|X_k, Y_k)$ is weakly increasing in X_k because $\lambda_k(1-\theta_k) \geq 0$. When $\theta_k < 1$ it is strictly increasing, so we set X_k as large as feasible; when $\theta_k = 1$ the value is independent of X_k, and the tie-break favoring the lowest acquisition payment selects $X_k = 0$ and the acquisition acceptance constraint is slack. When $\theta_k < 1$ then the acquisition acceptance and limited liability of the flow payment constraint imply that

$$X_k = \min\left\{\frac{(r+\lambda_k)U-\gamma_h}{\lambda_k}, \Pi_k - V_{0k}^s(U|X_k, Y_k)\right\} \geq 0.$$

Observe that $\Pi_k - V_{0k}^s(U|X_k, Y_k)$ is weakly decreasing in X_k, which guarantees that the solution is unique.

Plugging the solutions for X_k and Y_k into $V_{0k}^s(U|X_k, Y_k)$ yields

$$V_{0k}^s(U) = \begin{cases} \frac{\mu_h I + \theta_k\lambda_k\Pi_k - (r+\mu_h+\lambda_k)U + (1-\theta_k)((r+\lambda_k)U-\gamma_h)}{r+\mu_h+\theta_k\lambda_k}, & X_k = \frac{(r+\lambda_k)U-\gamma_h}{\lambda_k}, \\ S_k - U, & X_k = \Pi_k - V_{0k}^s(U). \end{cases}$$

for every $\theta_k \in (0,1]$.[22] Given that $V_{0k}^s(U|X_k, Y_k)$ is weakly increasing in X_k, we have that

$$V_{0k}^s(U) = \min\left\{\frac{\mu_h I + \theta_k\lambda_k\Pi_k - (r+\mu_h+\lambda_k)U + (1-\theta_k)((r+\lambda_k)U-\gamma_h)}{r+\mu_h+\theta_k\lambda_k}, S_k - U\right\}.$$

Furthermore,

$$(V_{0k}^s)'(U) = \begin{cases} \frac{-\mu_h - \theta_k(r+\lambda_k)}{r+\mu_h+\theta_k\lambda_k}, & X_k = \frac{(r+\lambda_k)U-\gamma_h}{\lambda_k}, \\ -1, & X_k = \Pi_k - V_{0k}^s(U). \end{cases}$$

Therefore, $(V_{0k}^s)'(U) \geq -1$. This implies that

$$\frac{\partial \frac{(r+\lambda_k)U-\gamma_h}{\lambda_k}}{\partial U} > 1 = \frac{\partial \Pi_k - S_k + U}{\partial U} \geq \frac{\partial \Pi_k - V_{0k}^s(U)}{\partial U}.$$

Therefore, there exists a threshold $\hat{U}$ such that below $\hat{U}$ $X_k = \frac{(r+\lambda_k)U-\gamma_h}{\lambda_k}$ while above it $X_k = \Pi_k - V_{0k}^s(U)$. Furthermore, $\hat{U} > \hat{U}_{0k}^{X=0}$ since

$$\frac{(r+\lambda_k)\hat{U}_{0k}^{X=0} - \gamma_h}{\lambda_k} = 0 < \Pi_k - S_k < \Pi_k - V_{0k}^s(\hat{U}_{0k}^{X=0}),$$

since $V_{0k}^s(\hat{U}_{0k}^{X=0}) \leq S_k - \hat{U}_{0k}^{X=0}$. At $\hat{U}$ since $X_k = \Pi_k - V_{0k}^s(\hat{U})$ we have that $V_{0k}^s(\hat{U}) = S_k - \hat{U}$ and therefore $\Pi_k - V_{0k}^s(\hat{U}) = \Pi_k - S_k + \hat{U}$. As a result, below $\hat{U}$ $\Pi_k - S_k + U > \frac{(r+\lambda_k)U-\gamma_h}{\lambda_k}$

[22] When $\theta_k = 1$, the VC's value is independent of X_k and the tie-break selects $X_k = 0$. Both branches then coincide: the first reduces to $\frac{\mu_h I + \lambda_k\Pi_k - (r+\mu_h+\lambda_k)U}{r+\mu_h+\lambda_k} = S_k - U$, so $V_{0k}^s(U) = S_k - U$ for every $U \geq \hat{U}_{0k}^{X=0}$.

while above $\Pi_k - V_{0k}^s(U) = \Pi_k - S_k + U < \frac{(r+\lambda_k)U-\gamma_h}{\lambda_k}$. Therefore,

$$X_k = \min\left\{\frac{(r+\lambda_k)U-\gamma_h}{\lambda_k}, \Pi_k - S_k + U\right\}.$$

At $\hat{U}$, the two bounds on X_k coincide

$$\frac{(r+\lambda_k)\hat{U}-\gamma_h}{\lambda_k} = \Pi_k - V_{0k}^s(\hat{U}),$$
$$V_{0k}^s(\hat{U}) = \Pi_k - \frac{(r+\lambda_k)\hat{U}-\gamma_h}{\lambda_k}.$$

Furthermore, at $\hat{U}$ we have that $V_{0k}^s(\hat{U}) = S_k - \hat{U}$ and therefore,

$$S_k - \hat{U} = \Pi_k - \frac{(r+\lambda_k)\hat{U}-\gamma_h}{\lambda_k},$$
$$\hat{U} = \frac{\lambda_k}{r}\left(\Pi_k - S_k + \frac{\gamma_h}{\lambda_k}\right) = \hat{U}_{0k}.$$

Acquisition efficiency $\Pi_k > S_k$ implies $\hat{U}_{0k} = \frac{\gamma_h+\lambda_k(\Pi_k-S_k)}{r} > \frac{\gamma_h}{r} \geq \frac{\gamma_h}{r+\lambda_k} = \hat{U}_{0k}^{X=0}$, so the region $[\hat{U}_{0k}^{X=0}, \hat{U}_{0k}]$ is non-degenerate.

As a consequence,

$$V_{0k}^s(U) = \begin{cases} \frac{\mu_h I + \theta_k\lambda_k\Pi_k - (r+\mu_h+\lambda_k)U + (1-\theta_k)((r+\lambda_k)U-\gamma_h)}{r+\mu_h+\theta_k\lambda_k}, & U \in [\hat{U}_{0k}^{X=0}, \hat{U}_{0k}], \\ S_k - U, & U > \hat{U}_{0k}, \end{cases}$$

and the derivative with respect to U is

$$(V_{0k}^s)'(U) = \begin{cases} \frac{-\mu_h - \theta_k(r+\lambda_k)}{r+\mu_h+\theta_k\lambda_k}, & U \in [\hat{U}_{0k}^{X=0}, \hat{U}_{0k}], \\ -1, & U > \hat{U}_{0k} \end{cases}$$

with $(V_{0k}^s)'(U) \in [-1, 0)$. When $\theta_k = 1$ then the slope for $U \in [\hat{U}_{0k}^{X=0}, \hat{U}_{0k}]$ is $(V_{0k}^s)'(U) = -1$ while if $\theta_k < 1$ then for $U \in [\hat{U}_{0k}^{X=0}, \hat{U}_{0k}]$ $(V_{0k}^s)'(U) > -1$.

The derivative of the value function with respect to U is,

$$(V_{0k}^s)'(U) \geq -1$$

for $U \geq \hat{U}_{0k}^{X=0}$. When the slope equals -1, the VC is indifferent between paying and holding, and the tie-break favoring early payment pays the promise down to $\overline{U}_k$. Hence a milestone payout is made whenever the entry promise exceeds $\overline{U}_k = \hat{U}_{0k}^{X=0}$ when $\theta_k = 1$ and $\overline{U}_k = \hat{U}_{0k}$ when $\theta_k < 1$. We have that there are no flow payments as $\frac{dY_t}{dt} = 0$ because the milestone payments bring U back to $\hat{U}_{0k}^{X=0}$ when $\theta_k = 1$ and $\hat{U}_{0k}$ when $\theta_k < 1$. Therefore, after the milestone payment $U \in [\hat{U}_{0k}^{X=0}, \hat{U}_{0k}]$ and $X_k = \frac{(r+\lambda_k)U-\gamma_h}{\lambda_k}$ and therefore the limited liability flow payment constraint binds and $\frac{dY_t}{dt} = 0$. The VC's value function $V_{0k}^s(U)$ for $U \geq \hat{U}_{0k}^{X=0}$

is also the VC's value function upon reaching state $(0,k)$ but before making the milestone payment since $(V^s_{0k})'(U) \geq -1$.

State $(0,0)$**.** No exit is feasible; the startup transitions to $(h,0)$ at rate μ_h with milestone promise u_h and to $(0,k)$ at rate μ_k with milestone promise u_k. For a contract to be stationary, we must have that $\mathbb{E}[dU_t + dY_t] = rU_t dt$ and $\dot{U} = 0$, which implies that

$$\frac{dY_t}{dt} = rU - \mu_h(u_h - U) - \mu_k(u_k - U).$$

Let $V^s_{00}(U|u_h, u_k)$ be the VC's value under the stationary contract with promise U and milestone promises (u_h, u_k). Substituting the flow payment $\frac{dY_t}{dt} = (r+\mu_h+\mu_k)U - \mu_h u_h - \mu_k u_k$ into the VC's stationary valuation equation gives

$$\begin{aligned} V^s_{00}(U|u_h, u_k) &= \frac{\mu_h V^s_{h0}(u_h) + \mu_k V^s_{0k}(u_k) - \frac{dY_t}{dt}}{r + \mu_h + \mu_k} \\ &= \frac{\mu_h[V^s_{h0}(u_h) + u_h] + \mu_k[V^s_{0k}(u_k) + u_k]}{r + \mu_h + \mu_k} - U. \end{aligned}$$

The VC chooses (u_h, u_k, U_{00}) to maximize

$$V^s_{00} = \max_{u_h, u_k, U_{00} \geq 0} V^s_{00}(U_{00}|u_h, u_k)$$

subject to

1. The IC constraints: $\mu_h(u_h - U_{00}) \geq \gamma_h$ and $\mu_k(u_k - U_{00}) \geq \gamma_k$.
2. The entrepreneur's state h outside option: $u_h \geq R_h$.
3. Successor stationarity: $u_h \geq \hat{U}_{h0}$ and $u_k \geq \hat{U}^{X=0}_{0k}$
4. Limited liability of the flow payment: $\frac{dY_t}{dt} \geq 0$.

We know that the IC-h constraint can be rewritten as $u_h \geq U + \frac{\gamma_h}{\mu_h}$ and by the assumption in state $(0,k)$ that $\frac{\gamma_h}{\mu_h} \geq R_h$. Combining these two yields

$$u_h \geq U + \frac{\gamma_h}{\mu_h} \geq \frac{\gamma_h}{\mu_h} \geq R_h,$$

and therefore the IC-h constraint ensures that the entrepreneur's outside option is always satisfied.

Observe that lowering U makes the IC constraints more slack and does not affect the successor stationarity constraint while the limited liability of the flow payment becomes tighter. Furthermore, holding (u_h, u_k) fixed, the value function $V^s_{00}(U_{00}|u_h, u_k)$ is strictly decreasing in U_{00} with slope -1, so the VC is better off lowering the continuation utility U_{00} until the limited liability of the flow payment binds, $\frac{dY_t}{dt} = 0$, giving $U_{00} = \frac{\mu_h u_h + \mu_k u_k}{r+\mu_h+\mu_k}$ and

$$V^s_{00} = \frac{\mu_h V^s_{h0}(u_h) + \mu_k V^s_{0k}(u_k)}{r + \mu_h + \mu_k}.$$

Therefore, the VC strictly prefers offering a contract with U_{00} over any $U > U_{00}$. Since given U_{00} $\frac{dY_t}{dt} = 0$ no flow payment is made in state $(0,0)$. Furthermore, no lump-sum payment is made in state $(0,0)$ since doing so would cause U to drop below U_{00}, which would violate limited liability since for any $U < U_{00}$ we have that $\frac{dY_t}{dt} < 0$.

We know that $V^s_{h0}(U)$ and $V^s_{0k}(U)$ are strictly decreasing in U (with slopes in $[-1,0)$ on their stationary regions). Furthermore, lowering a milestone promise lowers U_{00} and thereby relaxes the IC constraint in the other state while the successor stationarity constraints are unaffected. Therefore, the VC minimizes each milestone promise subject to the IC and successor stationarity lower bounds, evaluated at $U_{00} = \frac{\mu_h u_h + \mu_k u_k}{r+\mu_h+\mu_k}$. The promises are

$$u_h = \max\left\{U_{00} + \frac{\gamma_h}{\mu_h}, \hat{U}_{h0}\right\}, \qquad u_k = \max\left\{U_{00} + \frac{\gamma_k}{\mu_k}, \hat{U}^{X=0}_{0k}\right\},$$

where the first argument is the IC floor and the second the successor floor, and U_{00} is the fixed point of

$$F(U) = \frac{\mu_h \max\{U + \frac{\gamma_h}{\mu_h}, \hat{U}_{h0}\} + \mu_k \max\{U + \frac{\gamma_k}{\mu_k}, \hat{U}^{X=0}_{0k}\}}{r + \mu_h + \mu_k}, \tag{A.30}$$

which has a unique solution since $F(0) > 0$ and $F'(U) < 1$.

The k-successor floor never binds at the fixed point. Since $F(U) \geq \frac{\mu_h(U+\gamma_h/\mu_h)+\mu_k(U+\gamma_k/\mu_k)}{r+\mu_h+\mu_k}$ for all U, evaluating at the fixed point gives $(r+\mu_h+\mu_k)U_{00} \geq (\mu_h+\mu_k)U_{00} + \gamma_h + \gamma_k$, that is, $U_{00} \geq \frac{\gamma_h+\gamma_k}{r}$. Therefore,

$$U_{00} + \frac{\gamma_k}{\mu_k} \geq \frac{\gamma_h + \gamma_k}{r} + \frac{\gamma_k}{\mu_k} > \frac{\gamma_h}{r + \lambda_k} = \hat{U}^{X=0}_{0k},$$

so $u_k = U_{00} + \frac{\gamma_k}{\mu_k}$ in every case. Only the inner maximum on the h-branch of (A.30) remains, and resolving it in each of its two ways gives the two candidate fixed points: $C_1 = \frac{\gamma_h+\gamma_k}{r}$ when the h-IC floor binds so

$$(r + \mu_h + \mu_k)C_1 = \mu_h C_1 + \gamma_h + \mu_k C_1 + \gamma_k,$$

and $C_2 = \frac{\mu_h \hat{U}_{h0}+\gamma_k}{r+\mu_h}$ when the h-successor floor binds so

$$(r + \mu_h + \mu_k)C_2 = \mu_h \hat{U}_{h0} + \mu_k C_2 + \gamma_k.$$

Write $F_1(U)$ and $F_2(U)$ for the two resolved branches, so that $F(U) = \max\{F_1(U), F_2(U)\}$ and $F_j(C_j) = C_j$. Each $F_j(U) - U$ is strictly decreasing, so if $C_1 \geq C_2$ then $F_2(C_1) \leq C_1 = F_1(C_1)$ and $F(C_1) = C_1$, and symmetrically $F(C_2) = C_2$ if $C_2 \geq C_1$. Therefore, the fixed point of (A.30) is the larger of the two candidates,

$$U_{00} = \max\{C_1, C_2\} = \max\left\{\frac{\gamma_h + \gamma_k}{r}, \frac{\mu_h \hat{U}_{h0} + \gamma_k}{r + \mu_h}\right\}.$$

Milestone payouts and $X_k > 0$. The milestone payout on branch $i \in \{h, k\}$ is

$$B_i = \left[U_{00} + \frac{\gamma_i}{\mu_i} - \overline{U}_i\right]^+.$$

For the k-branch this is immediate since $u_k = U_{00} + \frac{\gamma_k}{\mu_k}$. For the h-branch, if the h-IC floor binds then $u_h = U_{00} + \frac{\gamma_h}{\mu_h}$; if instead the successor floor binds, then $u_h = \hat{U}_{h0} \le \overline{U}_h$ and $U_{00} + \frac{\gamma_h}{\mu_h} \le \hat{U}_{h0} \le \overline{U}_h$, so both expressions equal zero. Finally, for $\theta_k < 1$ $U_k = \min\{u_k, \overline{U}_k\} > \hat{U}_{0k}^{X=0}$ and therefore $X_k > 0$ since $\frac{(r+\lambda_k)U-\gamma_h}{\lambda_k} > 0$ for $U > \hat{U}_{0k}^{X=0}$ and $\Pi_k - V_{0k}^s(U) \ge \Pi_k - S_k + U > 0$. $\square$

Proof of Proposition 6. **VC's Value Function V_{00}^s.** From Proposition 11, upon reaching state $i \in \{h, k\}$ but before making the milestone payment, the VC's value functions are

$$V_{h0}^s(U) = \begin{cases} \dfrac{\mu_k I + \theta_h \lambda_h \Pi_h - (r + \mu_k + \theta_h \lambda_h)U}{r + \mu_k + \lambda_h}, & U \in [\hat{U}_{h0}, \Pi_h], \\ \dfrac{\mu_k I + \lambda_h \Pi_h}{r + \mu_k + \lambda_h} - U, & U > \Pi_h, \end{cases} \tag{A.31}$$

and

$$\begin{aligned} V_{0k}^s(U) &= \min\left\{\frac{\mu_h I + \theta_k \lambda_k \Pi_k - (1-\theta_k)\gamma_h - \big(\mu_h + \theta_k(r + \lambda_k)\big)U}{r + \mu_h + \theta_k \lambda_k}, S_k - U\right\} \qquad \text{(A.32)} \\ &= \begin{cases} \frac{\mu_h I + \theta_k \lambda_k \Pi_k - (1-\theta_k)\gamma_h - \big(\mu_h + \theta_k(r+\lambda_k)\big)U}{r + \mu_h + \theta_k \lambda_k}, & U \in [\hat{U}_{0k}^{X=0}, \hat{U}_{0k}], \\ S_k - U, & U > \hat{U}_{0k}. \end{cases} \end{aligned}$$

Both $V_{h0}^s(U)$ and $V_{0k}^s(U)$ are continuous and strictly decreasing in U, with slopes in $[-1, 0)$. The initial payoff is

$$V_{00}^s = \frac{\mu_h V_{h0}^s(u_h) + \mu_k V_{0k}^s(u_k)}{r + \mu_h + \mu_k}, \tag{A.33}$$

where, by Proposition 11, the milestone promises and the initial promise are

$$u_h = \max\left\{U_{00} + \frac{\gamma_h}{\mu_h}, \hat{U}_{h0}\right\}, \qquad u_k = U_{00} + \frac{\gamma_k}{\mu_k}, \qquad U_{00} = \max\{C_1, C_2\},$$

with $C_1 = \frac{\gamma_h + \gamma_k}{r}$, $C_2 = \frac{\mu_h \hat{U}_{h0} + \gamma_k}{r + \mu_h}$, and $\hat{U}_{h0} = \frac{\gamma_k + \theta_h \lambda_h \Pi_h}{r + \theta_h \lambda_h}$. Note that $u_k \ge C_1 + \frac{\gamma_k}{\mu_k} > \hat{U}_{0k}^{X=0}$, so $V_{0k}^s(u_k)$ in equation (A.32) is well defined. Since C_1, C_2, $\hat{U}_{h0}$, and the two branches of (A.31) and (A.32) are continuous in (Π_h, Π_k), so are U_{00}, u_h, u_k, and hence V_{00}^s.

Binding regime and the threshold $\overline{\Pi}_h$. C_1 is independent of Π_h while C_2 is affine and strictly increasing in Π_h. Therefore, $C_2 = C_1$ at exactly one point $\overline{\Pi}_h$, with $C_1 > C_2$ below it and $C_2 > C_1$ above it. Solving $C_2 = C_1$ for Π_h gives

$$\begin{aligned}
\frac{\gamma_h+\gamma_k}{r} &= \frac{\mu_h\hat{U}_{h0}+\gamma_k}{r+\mu_h},\\
\frac{r+\mu_h}{\mu_h}\frac{\gamma_h+\gamma_k}{r} - \frac{\gamma_k}{\mu_h} &= \hat{U}_{h0} = \frac{\gamma_k+\theta_h\lambda_h\overline{\Pi}_h}{r+\theta_h\lambda_h},\\
\overline{\Pi}_h &= \frac{r+\theta_h\lambda_h}{\theta_h\lambda_h}\left(\frac{r+\mu_h}{\mu_h}\frac{\gamma_h+\gamma_k}{r} - \frac{\gamma_k}{\mu_h}\right) - \frac{\gamma_k}{\theta_h\lambda_h},\\
\overline{\Pi}_h &= \frac{\gamma_h+\gamma_k}{r} + \frac{\gamma_h(r+\theta_h\lambda_h+\mu_h)}{\theta_h\lambda_h\mu_h}.
\end{aligned}$$

Using $(r+\mu_h)C_2 = \mu_h\hat{U}_{h0}+\gamma_k$ and $rC_1 = \gamma_h+\gamma_k$, direct computation gives

$$\hat{U}_{h0} - \left(C_1 + \frac{\gamma_h}{\mu_h}\right) = \frac{(r+\mu_h)(C_2-C_1)}{\mu_h}, \qquad \hat{U}_{h0} - \left(C_2 + \frac{\gamma_h}{\mu_h}\right) = \frac{r(C_2-C_1)}{\mu_h},$$

so the IC floor attains the maximum in u_h when $C_1 \geq C_2$ and the successor floor attains it when $C_2 \geq C_1$. So for $\Pi_h < \overline{\Pi}_h$ we have $U_{00} = C_1$ and $u_h = C_1 + \frac{\gamma_h}{\mu_h}$, while for $\Pi_h > \overline{\Pi}_h$ we have $U_{00} = C_2$ and $u_h = \hat{U}_{h0}$.

Derivatives Π_h. Differentiating equation (A.33) in Π_h and applying the chain rule,

$$\frac{\partial V_{00}^s}{\partial \Pi_h} = \frac{\mu_h}{r+\mu_h+\mu_k}\frac{\partial V_{h0}^s}{\partial \Pi_h}\Big|_{u_h} + \frac{\mu_h}{r+\mu_h+\mu_k}(V_{h0}^s)'(u_h)\frac{\partial u_h}{\partial \Pi_h} + \frac{\mu_k}{r+\mu_h+\mu_k}(V_{0k}^s)'(u_k)\frac{\partial u_k}{\partial \Pi_h}, \tag{A.34}$$

where $V_{0k}^s(U)$ does not depend on Π_h directly, so the k-branch contributes only through u_k.

For $\Pi_h < \overline{\Pi}_h$, $U_{00} = C_1 = \frac{\gamma_h+\gamma_k}{r}$ and hence u_h and u_k are independent of Π_h, so only the direct channel remains and equation (A.31) yields

$$\frac{\partial V_{00}^s}{\partial \Pi_h} = \begin{cases} \dfrac{\mu_h}{r+\mu_h+\mu_k}\dfrac{\theta_h\lambda_h}{r+\mu_k+\lambda_h} > 0, & u_h \leq \Pi_h,\\[2ex] \dfrac{\mu_h}{r+\mu_h+\mu_k}\dfrac{\lambda_h}{r+\mu_k+\lambda_h} > 0, & u_h > \Pi_h. \end{cases} \tag{A.35}$$

For $\Pi_h > \overline{\Pi}_h$, $U_{00} = C_2 = \frac{\mu_h\hat{U}_{h0}+\gamma_k}{r+\mu_h}$, $u_h = \hat{U}_{h0} = \frac{\gamma_k+\theta_h\lambda_h\Pi_h}{r+\theta_h\lambda_h}$, and $u_k = C_2 + \frac{\gamma_k}{\mu_k}$, with

$$\frac{\partial u_h}{\partial \Pi_h} = \frac{\theta_h\lambda_h}{r+\theta_h\lambda_h}, \qquad \frac{\partial u_k}{\partial \Pi_h} = \frac{\partial C_2}{\partial \Pi_h} = \frac{\mu_h}{r+\mu_h}\frac{\theta_h\lambda_h}{r+\theta_h\lambda_h} > 0.$$

Since $u_h = \hat{U}_{h0} < \Pi_h$ because $\frac{\gamma_k}{r} < \Pi_h$, the first case of equation (A.31) applies, so $\frac{\partial V_{h0}^s}{\partial \Pi_h}\Big|_{u_h} =$

$\frac{\theta_h\lambda_h}{r+\mu_k+\lambda_h}$ and $(V^s_{h0})'(u_h) = -\frac{r+\mu_k+\theta_h\lambda_h}{r+\mu_k+\lambda_h}$. The first two terms in equation (A.34) combine to

$$\begin{aligned}&\frac{\mu_h\theta_h\lambda_h}{(r+\mu_h+\mu_k)(r+\mu_k+\lambda_h)}\Big[1-\frac{r+\mu_k+\theta_h\lambda_h}{r+\theta_h\lambda_h}\Big]\\&=-\frac{\mu_h\mu_k\theta_h\lambda_h}{(r+\mu_h+\mu_k)(r+\theta_h\lambda_h)(r+\mu_k+\lambda_h)}<0.\end{aligned}$$

The third term is strictly negative, because $\frac{\partial u_k}{\partial \Pi_h} > 0$ and $(V^s_{0k})'(u_k) \in [-1, 0)$ on both branches of equation (A.32). Therefore, $\frac{\partial V^s_{00}}{\partial \Pi_h} < 0$ for $\Pi_h > \overline{\Pi}_h$. Together with equation (A.35) and continuity of V^s_{00} at $\overline{\Pi}_h$, V^s_{00} is strictly increasing for $\Pi_h < \overline{\Pi}_h$ and strictly decreasing for $\Pi_h > \overline{\Pi}_h$.

Derivatives Π_k. Neither branch of equation (A.31) depends on Π_k, and C_1, C_2, and $\hat{U}_{h0}$ are independent of Π_k, so U_{00}, u_h, and u_k do not vary with Π_k. The only channel is the direct dependence of $V^s_{0k}(u_k)$ on Π_k in equation (A.32). At fixed u_k, both arguments of the minimum in equation (A.32) are affine and strictly increasing in Π_k. Therefore, $V^s_{0k}(u_k)$ is strictly increasing in Π_k and V^s_{00} is strictly increasing in Π_k.

Entrepreneur's payoff. The VC has full bargaining power and the entrepreneur's outside option at the contracting stage is zero, so $U_0 = U_{00} = \max\{C_1, C_2\}$. The candidate C_1 is independent of Π_h, whereas $\frac{\partial C_2}{\partial \Pi_h} = \frac{\mu_h\theta_h\lambda_h}{(r+\mu_h)(r+\theta_h\lambda_h)} > 0$, so $U_{00} = C_1 = \frac{\gamma_h+\gamma_k}{r}$ for $\Pi_h < \overline{\Pi}_h$ and $U_{00} = C_2 = \frac{\gamma_k(r+\theta_h\lambda_h+\mu_h)+\theta_h\lambda_h\mu_h\Pi_h}{(r+\theta_h\lambda_h)(r+\mu_h)}$ for $\Pi_h > \overline{\Pi}_h$, which is constant below the threshold and strictly increasing above it. Neither candidate involves Π_k, so U_{00} is constant in Π_k, which establishes the proposition. □

C Exit Probabilities

This appendix describes how we compute the probabilities of the six outcomes reported in Figure 7: obsolescence, termination at a deadline in state $(0, 0)$, termination at a deadline after the first milestone, acquisition, acquihire, and IPO.

Throughout, obsolescence arrives at rate δ in every development state and is absorbed into the effective discount rate $r = \rho + \delta$ in the contracting problem, but it affects the probability of each outcome. We define the total event intensities

$$\Lambda_0 := \delta + \mu_h + \mu_k, \qquad \Lambda_h := \delta + \mu_k + \lambda_h, \qquad \Lambda_k := \delta + \mu_h + \lambda_k,$$

in states $(0, 0)$, $(h, 0)$, and $(0, k)$, respectively. We treat termination as terminal.

C.1 Intermediate States

Consider state $(h, 0)$ entered with continuation utility u, after any immediate milestone payment, so that $u \le \overline{U}_{h0}$. Let $T^{h0}_D(u)$ denote the deadline of Lemma 3, which is finite if and only if $u < \hat{U}_{h0}$. Three random events can end the state: obsolescence, a transferable-capital breakthrough leading to an IPO, and the arrival of a buyer leading to an acquihire, driven by the independent Poisson processes N^{δ}, N^{μ_k}, and N^{λ_h} with constant intensities δ, μ_k, and λ_h. Absent all three, the relationship is terminated at $T_D^{h0}(u)$.

Let τ denote the first arrival time among the three Poisson processes. Because the processes are independent with constant intensities, the probability that none has arrived by time t is

$$P(\tau > t) = P(N_t^{\delta} = 0)P(N_t^{\mu_k} = 0)P(N_t^{\lambda_h} = 0) = e^{-\delta t}e^{-\mu_k t}e^{-\lambda_h t} = e^{-\Lambda_h t},$$

so τ is exponentially distributed with parameter Λ_h.

The probability that the first arrival occurs in $[t, t+dt]$ and is an acquihire offer is the probability that no event has arrived by t times the probability that the acquihire process jumps in $[t, t+dt]$, namely $e^{-\Lambda_h t}\lambda_h dt$, and likewise $e^{-\Lambda_h t}\mu_k dt$ for an IPO, and $e^{-\Lambda_h t}\delta dt$ for obsolescence.

An acquihire occurs if and only if the first event is an acquihire offer and it arrives before the deadline. Integrating the joint density over $[0, T_D^{h0}(u)]$ gives

$$P_h(\text{acquihire}|u) = \int_0^{T_D^{h0}(u)} \lambda_h e^{-\Lambda_h t} dt = \frac{\lambda_h}{\Lambda_h}\big(1 - e^{-\Lambda_h T_D^{h0}(u)}\big),$$

and the same integral with μ_k or δ in place of λ_h gives the IPO and obsolescence probabilities. The relationship is terminated at the deadline if and only if no event has arrived by $T_D^{h0}(u)$, which has probability $P(\tau > T_D^{h0}(u)) = e^{-\Lambda_h T_D^{h0}(u)}$.

Collecting terms, the conditional outcome probabilities are

$$\begin{aligned} P_h(\text{acquihire}|u) &= \frac{\lambda_h}{\Lambda_h}\Big(1 - e^{-\Lambda_h T_D^{h0}(u)}\Big), & P_h(\text{IPO}|u) &= \frac{\mu_k}{\Lambda_h}\Big(1 - e^{-\Lambda_h T_D^{h0}(u)}\Big), \\ P_h(\text{obsolescence}|u) &= \frac{\delta}{\Lambda_h}\Big(1 - e^{-\Lambda_h T_D^{h0}(u)}\Big), & P_h(\text{termination}|u) &= e^{-\Lambda_h T_D^{h0}(u)}, \end{aligned} \tag{A.36}$$

which sum to one since the three rate shares sum to one. When $T_D^{h0}(u) = \infty$, which is the case for $u \geq \hat{U}_{h0}$, termination has probability zero and the three probabilities reduce to the rate shares λ_h/Λ_h, μ_k/Λ_h, and δ/Λ_h.

State $(0, k)$ is solved in the same way with the roles of the two forms of capital reversed. The three competing events are obsolescence, a human-capital breakthrough leading to an IPO, and the arrival of a buyer leading to an acquisition, with constant intensities δ, μ_h, and λ_k. Therefore,

$$\begin{aligned} P_k(\text{acquisition}|u) &= \frac{\lambda_k}{\Lambda_k}\Big(1 - e^{-\Lambda_k T_D^{0k}(u)}\Big), & P_k(\text{IPO}|u) &= \frac{\mu_h}{\Lambda_k}\Big(1 - e^{-\Lambda_k T_D^{0k}(u)}\Big), \\ P_k(\text{obsolescence}|u) &= \frac{\delta}{\Lambda_k}\Big(1 - e^{-\Lambda_k T_D^{0k}(u)}\Big), & P_k(\text{termination}|u) &= e^{-\Lambda_k T_D^{0k}(u)}. \end{aligned} \tag{A.37}$$

The deadline is finite in state $(0, k)$ if and only if $U < \overline{U}_{0k}$.

C.2 Initial State

The contract starts in state $(0, 0)$ at the initial promise U_{00} with deadline $T_D^{00}(U_{00})$. Absent an event, the promise follows the deterministic path U_t solving $dU_t = \dot{U}_{00}(U_t)dt$ with $U_0 = U_{00}$, and it reaches zero at $T_D^{00}(U_{00})$. Since the three events arrive at constant rates μ_h, μ_k, and δ, the probability that the state is still active at time t is $e^{-\Lambda_0 t}$, and the density of a human-capital breakthrough at time t is $\mu_h e^{-\Lambda_0 t}$ on $[0, T_D^{00}(U_{00})]$, and likewise for transferable capital $\mu_k e^{-\Lambda_0 t}$ and for obsolescence $\delta e^{-\Lambda_0 t}$.

A breakthrough at time t transmits the current promise U_t into the intermediate state through the milestone promise of Proposition 3. The entry promise after the milestone payment is $\overline{u}_{h0}(U) := \min\{u_{h0}(U), \overline{U}_{h0}\}$, $\quad \overline{u}_{0k}(U) := \min\{u_{0k}(U), \overline{U}_{0k}\}$, where $u_{h0}(U)$ and $u_{0k}(U)$ are the optimal milestone promises.

Collecting the two stages, the unconditional probability of each outcome ω is

$$\begin{aligned} P(\omega) =& \mathbb{I}_{\{\omega=\text{obsolescence}\}} \frac{\delta}{\Lambda_0}\left(1 - e^{-\Lambda_0 T_D^{00}(U_{00})}\right) + \mathbb{I}_{\{\omega=\text{termination in }(0,0)\}} e^{-\Lambda_0 T_D^{00}(U_{00})} \\ &+ \int_0^{T_D^{00}(U_{00})} e^{-\Lambda_0 t}\Big[\mu_h P_h(\omega|\overline{u}_{h0}(U_t)) + \mu_k P_k(\omega|\overline{u}_{0k}(U_t))\Big]dt, \end{aligned} \tag{A.38}$$

where $p_h(\omega|u)$ and $p_k(\omega|u)$ are given by equations (A.36) and (A.37), set to zero for outcomes that cannot occur in the respective state. Termination after the first milestone collects the two branch termination terms, obsolescence collects its state-$(0, 0)$ term and the two branch terms, and the IPO probability collects the two branch IPO terms. The six probabilities sum to one by construction.

C.3 Stationary Benchmark

In the stationary benchmark no deadline exists, so every exponential term containing a deadline in equations (A.36), (A.37), and (A.38) vanishes and the integral in equation (A.38) runs to infinity. As a consequence, the probabilities reduce to products of rate shares,

$$P^s(\text{acquihire}) = \frac{\mu_h}{\Lambda_0}\frac{\lambda_h}{\Lambda_h}, \qquad P^s(\text{acquisition}) = \frac{\mu_k}{\Lambda_0}\frac{\lambda_k}{\Lambda_k}, \qquad P^s(\text{IPO}) = \frac{\mu_h}{\Lambda_0}\frac{\mu_k}{\Lambda_h} + \frac{\mu_k}{\Lambda_0}\frac{\mu_h}{\Lambda_k},$$

with $1 - P^s(\text{acquihire}) - P^s(\text{acquisition}) - P^s(\text{IPO})$ assigned to obsolescence.