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Dynamic Contracting with Intermediation: Operational, Governance, and Financial Engineering

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ABSTRACT

Private equity funds intermediate investment and affect portfolio firm performance by actively engaging in operational, governance, and financial engineering. We study this type of intermediation in a dynamic agency model in which an active intermediary raises funds from outside investors and invests in a firm run by an agent. Optimal contracting addresses moral hazard at the intermediary and firm levels. The intermediary's incentives to affect firm performance are strongest after poor performance, while the agent's incentives are strongest after good performance. We also show how financial engineering, that is, financial contracting with outside investors, interacts with operational and governance engineering.

FINANCIAL INTERMEDIARIES—SUCH AS PRIVATE equity (PE) funds, hedge funds, and banks—play a crucial role for firms and their performance, as they affect corporate governance, monitor management's activities, and actively seek to improve firm operations. While these active intermediaries may possess unique abilities to improve firm performance and address firm-level agency conflicts, they are subject to agency frictions of their own. Thus, active intermediation is a double-agency problem, and the efficiency of governance, operating, and financing decisions can be compromised by these

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agency frictions. In this paper, we develop a unifying model to study this type of intermediation.

To do so, we formulate a dynamic contracting model in which an intermediary raises funds from a principal and invests in a firm run by an agent who has limited liability. The agent controls the firm's output via costly but unobservable effort, leading to firm-level moral hazard. The intermediary also affects the firm's output via costly effort capturing the intermediary's monitoring activity or his direct influence on firm performance. In addition, the intermediary offers compensation contracts to the agent and the principal. The intermediary therefore faces a two-task problem, namely, effort provision and contracting. Because both the intermediary's effort and the contract offered to the agent are unobservable to the principal, moral hazard arises at the intermediary level. The moral hazard problems interact. On the one hand, agency conflicts at the firm level make it harder to discern the impact of the intermediary's effort on the firm's output, which can exacerbate agency conflicts at the intermediary level. That is, moral hazard propagates from the firm to the intermediary level. On the other hand, because the intermediary determines the agent's contract, the intermediary's moral hazard affects the agent's incentives. That is, moral hazard propagates from the intermediary to the firm level.

The model can broadly represent various forms of active intermediation, but the setting in which it most directly applies is the leveraged buyout (LBO) sector of PE investment. The PE fund or the general partners (GPs) of the PE fund, representing the intermediary, raise funds from outside investors (the principal) and invest in a firm run by a manager (the agent). In practice, PE funds (i.e., PE firms) add value to their portfolio firms through operational, governance, and financial engineering (Kaplan and Strömberg (2009)). The intermediary is in charge of monitoring and governance, including compensation contracts of the manager (governance engineering), but can also directly influence the performance of the firm (operational engineering) and raise capital from outside investors to alter cash flows and capital structure (financial engineering). An extensive literature studies the incentive contracts of PE GPs (e.g., Axelson, Strömberg, and Weisbach (2009), Metrick and Yasuda (2010)) and of portfolio firm managers (e.g., Leslie and Oyer (2008), Cronqvist and Fahlenbrach (2013)), yet the two are commonly treated as separate and isolated incentives problems. Likewise, there is no unifying model explaining the joint determination of operational, governance, and financial engineering, which leaves several questions open: How do GP incentives affect the contracts of portfolio firm managers? When do PE firms engage in operational engineering, after good or after poor performance? And how does financial engineering affect operational and governance engineering?

To address these questions, it is essential to understand how the incentives of the agent and the intermediary interact with each other. The key feature of the optimal contracts that address the double moral hazard problem is compensation sensitive to observable firm performance. However, the incentive role of performance pay is different for the agent and the intermediary. The intermediary's performance pay motivates the intermediary to exert effort on his own and to incentivize the agent to exert effort. As such, the intermediary passes part of his incentives through the agent's contract, that is, incentives *trickle down* from the intermediary to the agent. The agent's performance pay incentivizes the agent's own effort, but it also indirectly affects the intermediary's incentives to exert effort via the *trickle-up* effect. As the intermediary increases effort, the resulting gains in the firm's output partially accrue to the agent due to the agent's performance pay. After good past performance when the agent's stake in the firm is large, this effect leads to an *agency overhang* problem in the sense that the intermediary is reluctant to exert effort as most of the benefits are reaped by the agent. Conversely, when the agent's stake in the firm is low after poor performance, this effect generates additional incentives for the intermediary to exert effort to avoid agency-induced distress and to save the firm. In short, the trickle-up effect generates additional incentives for the intermediary after poor performance, but reduces incentives after good performance. The optimal set of contracts for the investment relationship accounts for both the trickle-up and the trickle-down effects of incentives.

We analyze the optimal design of incentives addressing the double moral hazard problem and the intermediary's multitasking problem. To address the agent's moral hazard problem, the agent's stake in the firm increases after good firm performance and decreases after poor performance. After sufficiently bad performance, the agent's contract is terminated and the firm enters agency-induced distress. The sensitivity of the agent's stake to firm performance determines the agent's incentives to exert effort. It is costly to expose the agent to performance when in distress, and so accordingly, the trickle-down effect is weak, and the intermediary passes little incentives in the agent's contract. Conversely, it is relatively cheap to incentivize the agent away from distress, the trickle-down effect is strong, and the intermediary passes strong incentives in the agent's contract. As the sensitivity of compensation to performance increases after good performance, the agent's incentives are convex or option-like.

The shape of the intermediary's incentives is driven by the trickle-up effect of the agent's incentives on the intermediary. Because the sign of the trickle-up effect depends on the agent's stake, the trickle-up effect generates disincentives for the intermediary after good performance (due to the agency overhang problem) and positive incentives after poor performance (to mitigate agency-induced distress). To counter disincentives after good performance, the intermediary's contract with the principal is set to amplify the intermediary's exposure to cash flow shocks (akin to a leveraged position). To curb excessive trickle-up incentives, the intermediary's contract with the principal reduces the intermediary's direct exposure to cash flow shocks (akin to risk-sharing with the principal, but present without a risk-sharing motive). Taken together, the intermediary's direct exposure to cash flow shocks increases in firm performance. This means that the intermediary's direct exposure to firm performance is convex and exhibits option-like features. Remarkably, the intermediary's total incentives, which consist of the direct exposure to cash flow shocks and the indirect exposure via the trickle-up effect, are no longer convex in firm performance as they are highest under distress due to the strong positive trickle-up incentives. The mechanism is that the intermediary benefits from saving the firm from agency-induced distress, and this generates incentives without direct exposure to cash flows.

We next study the impact of the agent's and the intermediary's efforts on firm performance. Because the intermediary passes his incentives on to the agent, one may expect the two efforts to move in accord, that is, when the intermediary's contract strongly exposes the intermediary to firm performance, the intermediary would both exert high effort and pass strong incentives to exert effort in the agent's contract. Under this view, the concurrence of efforts (and idleness) could have a destabilizing effect on the firm. We show that this is not the case. The reason is that the trickle-up and trickle-down effects are time-varying and performance-sensitive. The agent's and the intermediary's efforts are proportional to the incentives as discussed above. Consequently, the agent exerts most effort after good performance, while the intermediary exerts most effort when the firm is in distress. The model thus implies that the intermediary is primarily active in the firm when the agent's role is diminished after poor performance, stepping back when the agent's role is increased. The interaction between the two incentive problems endogenously generates stability in firm performance. In the context of PE, our model therefore suggests that PE ownership stabilizes firm performance, consistent with the empirical findings in Bernstein et al. (2017) and Bernstein, Lerner, and Mezzanotti (2019).

The model also sheds light on the interconnected effects of operational, governance, and financial engineering in PE investment. In particular, we show that the intermediary engages in operational engineering and takes a more hands-on approach, especially after poor performance and when the firm is in distress, consistent with the findings in Bernstein, Lerner, and Mezzanotti (2019) and Gompers, Kaplan, and Mukharlyamov (2020). Under these circumstances, the role of the agent (i.e., manager) in firm operations is diminished. After good performance, however, the intermediary takes a more hands-off approach in operational engineering and provides stronger incentives to the agent. Overall, the combination of both governance and operational engineering leads to highly convex incentives for the manager (agent), consistent with the findings in Cronqvist and Fahlenbrach (2013).

The model implies that financial engineering can be understood as a function of operational and governance engineering rather than an independent source of value. In particular, our analysis suggests that financial engineering facilitates efficient operational and governance engineering, and is therefore complementary to other theories of financial engineering in PE investments, such as Malenko and Malenko (2015). We show that for active investors such as PE funds that engage in operational and governance engineering, it is optimal to use external financing and financial engineering. In other words, intermediated investment emerges as an optimal form of active ownership. The intuition for why a financing contract between the intermediary and the principal adds value despite no financing constraints or risk-sharing motives is related to its effect on incentives. The financing contract allows the agent's and the intermediary's incentives to be decoupled. In the absence of outside investors, the intermediary and the agent split the exposure to firm performance, so their incentives are tightly linked. With the financing contract, the agent's and the intermediary's incentives remain linked via the trickle-up and trickle-down effects, but there is an extra degree of freedom in the terms of the financing contract. In particular, financial engineering can reduce the distortions to the agent's and intermediary's incentives due to agency overhang and agency-induced distress.

We show that optimal contracting with the principal, that is, financial engineering, involves two parts: (i) granting the intermediary levered exposure to firm performance and (ii) debt-like financing provided by the principal. Taken together, financial engineering reduces the intermediary's incentives and effort (e.g., operational engineering or monitoring) after poor performance and when in distress, in which case the debt-like claim held by the principal reduces the intermediary's exposure to firm performance. In contrast, financial engineering increases the intermediary's incentives and effort (e.g., operational engineering or monitoring) after good performance, reflecting the increased convexity of the intermediary's incentives. We also show that via the trickle-down effect, financial engineering adds additional convexity and performance sensitivity to managerial compensation contracts in PE-owned firms.

We extend the model to consider the effects of increased investor participation in firm-level governance. Such increased participation means that investors more directly influence managerial contract terms, which could capture in PE the direct coinvestment of limited partners (LPs) as in Fang, Ivashina, and Lerner (2015). To capture this in our setting, we consider a variant of our model in which the contract between the intermediary and the agent is publicly observable and contractible, so that the principal can directly influence the agent's contract. We show that direct investor participation changes the level of incentives for the agent and the intermediary, especially after poor performance. In particular, the agent's pay sensitivity to performance increases and the intermediary's pay sensitivity decreases relative to the case with delegated contracting. This is because the intermediary—now facing only one task of his own effort—can be effectively insulated from the agent's incentive problem. Whereas agency-induced distress made the intermediary exert more of his own effort and delegate less in the agent's contract, this effect can be eliminated under direct investor participation. In the context of PE investment, the model predicts that direct coinvestment by LPs adds convexity to the incentives of the GPs, effectively reducing the GPs' incentives when the firm faces distress and increasing them after good performance. As a further consequence, direct coinvestment by LPs reduces the extent of operational engineering but increases the manager's incentives under distress.

Finally, we show how the model can be adapted to study delegated contracting, whereby the principal contracts with the intermediary and delegates contracting with the manager to the intermediary. The optimal contract to the intermediary can accommodate payouts to the intermediary that are nonnegative, with the intermediary not injecting funds into the firm except, possibly, for the initial investment. With such an assumption, the model fits other applications apart from our leading one of PE investment. In particular, the intermediary can represent boards of directors who monitor and set executives' contracts on behalf of shareholders. In this setting, a shift from delegated to direct contracting can represent the introduction of say-on-pay regulations that increase shareholder participation in determining executive compensation. The model predicts that say-on-pay regulations raise the level of executives' performance pay and increase the sensitivity of pay to poor realizations of performance, consistent with empirical evidence. Shareholders' say-on-pay is particularly beneficial after poor performance when delegated contracting via the board leads to the largest distortions in incentive provision. The delegated contracting model can also be applied to studying hierarchical agency within the firm, whereby the intermediary is the CEO and the agent is the division manager. In this context, the model predicts that the CEO takes a more hands-on approach when the firm experiences distress, whereas she takes a more hands-off approach after good performance and delegates more tasks to the division manager.

Our theory focuses on the monitoring and contracting functions of financial intermediaries and complements previous agency-based models of intermediation that consider other functions. Bhattacharya and Pfleiderer (1985) study delegated portfolio management within a one-period model with hidden information, while Ou-Yang (2003) studies portfolio management in a continuous-time model with moral hazard. Cuoco and Kaniel (2011), Kaniel and Kondor (2012), and Guerrieri and Kondor (2012) study the impact of delegated portfolio management on asset prices. Likewise, He and Krishnamurthy (2011, 2013) and Rampini and Viswanathan (2019) analyze financial intermediaries that facilitate access to assets in general equilibrium models.

The fact that the monitoring function of financial intermediaries is limited by their own moral hazard has been studied in the banking literature, starting with Diamond (1984). Other related contributions include Hellwig (2000) and Bond (2004).[1] More closely related to our paper is Holmström and Tirole (1997), who consider monitoring by financial intermediaries in an agency model. Their focus is on intermediaries' financial constraints and their effect on the provision of loans and on equilibrium interest rates. In contrast to these theories, our model is dynamic, and its objective is to examine the provision of incentives for both the intermediary and the agent (firm manager).

Our paper is also related to the more general literature on multilayered moral hazard problems. Strausz (1997) and Rahman (2012) study the optimality of delegating monitoring, but not contracting, to a supervisor. Macho-Stadler and Pérez-Castrillo (1998) compare a decentralized organizational structure with delegated contracting and a centralized structure (direct contracting with all agents) with possible side contracting between the agents. Baliga and Sjöström (1998) analyze the advantages of delegated contracting when the supervisor's effort is observable to the agent. Buffa, Liu, and White (2020) study when to delegate contracting in a model with two agents, in

[1] Bebchuk, Cohen, and Hirst (2017) discuss agency problems of various institutional investors other than banks, such as passive or active mutual funds and hedge funds.

which the principal cannot fully commit to privately observed contracts. In contrast to our paper, all of these models are static and focus primarily on the optimal choice of organizational structure rather than on incentives and their interactions along the hierarchy. More broadly, our paper is related to the literature on hierarchies in which the agency friction is adverse selection instead of moral hazard (see Mookherjee (2013) for a review). A large part of this literature focuses on contrasting various organizational forms and identifying conditions under which delegation can dominate centralized organizational structures (e.g., Faure-Grimaud, Laffont, and Martimort (2003), Mookherjee and Tsumagari (2004)). More closely related to our paper is the observation in Melumad, Mookherjee, and Reichelstein (1995) that in a static model of delegation under adverse selection, the middle agent tends to assign a higher production task to his favor at the expense of a lower production task assigned to the lower agent. This is parallel to incentives trickling down in our model of delegation under moral hazard.

These multilayered agency models and ours are distinct from two-sided agency problems in which both the agent and the principal are subject to moral hazard, as in the static model of Bhattacharyya and Lafontaine (1995) or in the dynamic model of Hori and Osano (2013). Our theory also relates to papers that consider optimal monitoring in two-player agency models in both dynamic settings (see, e.g., Piskorski and Westerfield (2016), Halac and Prat (2016), Dilmé and Garrett (2019), Malenko (2019), Varas, Marinovic, and Skrzypacz (2020)) and static settings (see, e.g., Lazear (2006), Eeckhout, Persico, and Todd (2010)). The main difference of our model is that the monitoring party is also an agent, one that exerts effort and contracts with the ultimate agent.

Our paper is part of the growing literature on dynamic contracting models including, among others, Holmström and Milgrom (1987), DeMarzo and Sannikov (2006), Biais et al. (2007), Sannikov (2008), Biais et al. (2010), He (2011), DeMarzo et al. (2012), Zhu (2012), Green and Taylor (2016), He et al. (2017), Varas (2018), Marinovic and Varas (2019), Szydlowski (2019), Gryglewicz, Mayer, and Morellec (2020), Mayer (2022), Feng and Westerfield (2021), and Feng et al. (2021). We contribute to this literature by adding an intermediary.

The paper is organized as follows. Section I introduces the model. Section II describes the model solution. Section III presents the model's implications for PE investment. Section IV analyzes the role of several constraints relevant to the contracting problem. Section V discusses applications to boards of directors and firm hierarchies. Section VI concludes.

I. Model

A. Setup

Time t is continuous on $[0, \infty)$. There are three players: the principal ("they" or "player P"), the intermediary ("he" or "player I"), and the agent ("she" or "player A"). All players are risk neutral, discount the future at rate $r > 0$, and have zero reservation value. The principal and the intermediary, as the firm's owners, have deep pockets and provide financing to the firm, which is run by the agent. The agent is penniless and has limited liability, which precludes negative payments to the agent.

The agent's limited liability also implies that at any point in time her continuation payoff exceeds her outside option, which is her reservation value of zero.[2] Similar to the agent, the principal and the intermediary, as the firm's owners, have limited commitment in the following sense. At any point in time t, total firm value net of the payouts to the agent must exceed the firm's liquidation value R. Otherwise, the principal and the intermediary would be better off liquidating the firm, terminating the agent, and seizing the liquidation value R.[3] Put differently, limited commitment requires firm owners' joint continuation payoff to exceed their effective outside option, which is the recovery value R. The type of limited commitment we assume is similar to limited commitment in Ai and Li (2015) and Bolton, Wang, and Yang (2019).

The agent affects firm performance by exerting effort a^A. The intermediary also contributes to firm performance by exerting effort a^I, which may capture the intermediary's monitoring activity or direct influence on firm performance. For theoretical clarity, we assume that the agent's effort and the intermediary's effort independently affect the firm's cash flows. That is, the cash flow process until firm liquidation (at endogenous time $\tau \in [0, \infty]$) is given by

$$dX_t = (a_t^A + a_t^I)dt + \sigma dZ_t, \tag{1}$$

where dZ_t is the increment of a standard Brownian motion and $\sigma > 0$ is constant volatility. Cash flows dX_t are publicly observable and contractible, whereas no player observes cash flow shocks dZ_t and cash flow shocks dZ_t are not contractible. Effort a_t^A is observed only by the agent, and effort a_t^I is observed only by the intermediary; neither effort is not contractible. Effort is costly in that the agent and the intermediary incur private flow costs of $g^A(a_t^A) := \frac{1}{2}\delta(a_t^A)^2$ and $g^I(a_t^I) := \frac{1}{2}\lambda(a_t^I)^2$, respectively. This specification gives rise to moral hazard at both the firm and the intermediary levels. For regularity purposes, we assume that both efforts are bounded, $a_t^A, a_t^I \in [0, A]$, with a constant $A > 0$. We focus on parameter configurations that lead to optimal interior effort levels $a_t^A, a_t^I \in (0, A)$ at all times $t \geq 0$ until firm liquidation.

B. Contracting Problem

A concrete application of our model is intermediated investment by PE firms.[4] A PE firm or, alternatively, the GPs of the PE fund (the intermediary) invest in a portfolio firm run by a manager (the agent). PE firms, and in

[2] The limited liability constraint readily implies that the agent's payoff at time $t = 0$ exceeds her reservation value, so that the agent is motivated to participate.

[3] In other words, the intermediary and the principal would find it profitable to collude to expropriate the agent.

[4] Our model applies to the buyout component of private capital markets. LBOs focus on more mature companies where the primary source of risk is the level of cash flows. This is in contrast

particular GPs, have a central role in the investment relationship: They are actively engaged in the governance and operations of portfolio firms. The PE firm also raises financing from outside investors (the principal), who receive claims on the portfolio firm's cash flows. Outside investors are not uniform and may include LPs in the PE fund as well as banks and other lenders.

Motivated by this application, the contracting problem is as follows. The intermediary is the residual claimant on total firm value and can extract all surplus from both the principal and the agent, reflecting that outside investors are competitive and the agent as a manager has little or no bargaining power. The intermediary collects the firm's cash flows dX_t and offers a contract (security) Π^P to the principal, specifying payouts to the principal, and a contract Π^A to the agent, specifying the agent's compensation. The intermediary can fully commit to any long-term contracts Π^A and Π^P as long as the limited liability and commitment constraints discussed above are met.

The terms of the contract Π^A offered to the agent are not observable to the principal and are not contractible between the principal and the intermediary. This reflects the assumption that the principal represents passive outside investors who do not engage in firm governance.[5]

The contract offered to the agent $\Pi^A = (w^A, \hat{a}^A, \hat{a}^I)$ specifies (i) prescribed effort $\hat{a}^A$ for the agent, (ii) prescribed effort $\hat{a}^I$ for the intermediary, and (iii) cumulative payouts (wages) w^A to the agent. Likewise, the contract offered to the principal $\Pi^P = (w^P, \bar{a}^A, \bar{a}^I)$ specifies (i) prescribed effort $\bar{a}^A$ for the agent, (ii) prescribed effort $\bar{a}^I$ for the intermediary, and (iii) cumulative payouts w^P to the principal. Throughout the paper, we consider *incentive compatible* contracts Π^A and Π^P that induce the intermediary and the agent to exert the prescribed effort levels (so that $\hat{a}^A = \bar{a}^A = a^A$ and $\hat{a}^I = \bar{a}^I = a^I$) and respect the agent's and firm owners' limited commitment.

Because the agent is protected by limited liability and has zero wealth, payouts to the agent must be positive, in that $dw_t^A \geq 0$. In contrast, payouts to the principal dw_t^P can be negative, in that the intermediary can raise new financing from investors without friction.[6] As in DeMarzo and Sannikov (2006), because the agent cannot be paid negative wages and is protected by limited liability, incentive provision may require that the agent's contract Π^A be terminated at some time τ, leading to firm liquidation and $dX_t = dw_t^A = dw_t^P = 0$ for $t > \tau$.[7] Firm liquidation is inefficient. Upon firm liquidation at time τ, the intermediary seizes the liquidation value worth R dollars. With a slight abuse

to venture capital funds, which focus on younger firms in which the risk is primarily about failing or achieving a breakthrough. In our model, there is no failure or breakthrough but instead cash flows are risky.

[5] Section III.C studies a version of the model in which the contract Π^A is publicly observable and contractible, for example, because the principal directly engages in the firm's governance.

[6] In the context of PE, $dw_t^P < 0$ can reflect the GPs calling capital from the LPs.

[7] Admittedly, the intermediary could continue running the firm without the agent and exert effort $a^I > 0$ after the agent's contract is terminated. For simplicity, we consider liquidation values that satisfy $R \geq R_L$, where $R_L = \frac{1}{2r\lambda}$. As we show in Appendix E, $R \geq R_L$ implies that the intermediary prefers liquidation over running the firm without the agent. The Internet Appendix discusses this assumption in more detail and argues that it has no qualitative effects on the model

of notation, we write $dX_\tau = R$ and $dX_t = 0$ for $t > \tau$, while dX_t follows (1) for $t < \tau$.

Given contract Π^A, the agent chooses effort a^A to maximize

$$v_0^A := \max_{a^A} \mathbb{E}\left[\int_0^\infty e^{-rt}\left(dw_t^A - g^A\left(a_t^A\right)dt\right)\right]. \tag{2}$$

The principal's payoff derived from the contract Π^P is equal to the expected discounted stream of future payouts dw_t^P:

$$v_0^P := \mathbb{E}\left[\int_0^\infty e^{-rt} dw_t^P\right].$$

The intermediary chooses his effort level a^I, the agent's contract Π^A, and the principal's contract Π^P to maximize

$$v_0^I := \max_{a^I,\Pi^A,\Pi^P} \mathbb{E}\left[\int_0^\infty e^{-rt}\left(dX_t - dw_t^P - dw_t^A - g^I(a_t^I)dt\right)\right] \tag{3}$$

subject to all relevant incentive compatibility, limited liability, commitment, and break-even constraints, which we discuss below. Note that the intermediary collects net dollar payoffs $dw_t^I := dX_t - dw_t^P - dw_t^A$ from financing the firm, which is the firm's cash flows after payouts to the principal and the agent.

In general, the principal, the intermediary, and the agent may have different private information and therefore apply potentially different probability measures to evaluate payoffs. For simplicity, we do not distinguish between these probability measures in the main text. We provide a formal discussion of this issue in Appendix A.

II. Model Solution

A. First-Best Benchmark

We start by considering the *first-best* benchmark in which efforts a^A and a^I are publicly observable and contractible. The first-best solution is as follows. As the principal and intermediary have identical preferences, there is no need to involve the principal, so without loss of generality we can set $dw_t^P = 0$. The intermediary then collects the cash flows dX_t, and compensates the agent for the flow costs of effort, that is, $dw_t^A = \frac{1}{2}\delta(a_t^A)^2dt$. Optimal efforts (a_t^A, a_t^I) maximize the firm's expected cash flows net the costs of effort,

$$a_t^A + a_t^I - \frac{\delta(a_t^A)^2}{2} - \frac{\lambda(a_t^I)^2}{2},$$

which leads to

$$a_t^A = a_{FB}^A \equiv \frac{1}{\delta} \quad \text{and} \quad a_t^I = a_{FB}^I \equiv \frac{1}{\lambda}. \tag{4}$$

solution. The Internet Appendix is available in the online version of the article on *The Journal of Finance* website.

Note that in the first-best benchmark, optimal payouts and efforts are constant over time and the firm is never liquidated. Moreover, the total firm value reads

$$F^{FB} = \max_{a_t^A, a_t^I} \left[\frac{1}{r} \left(a_t^A + a_t^I - \frac{\delta (a_t^A)^2}{2} - \frac{\lambda (a_t^I)^2}{2} \right) \right] = \frac{1}{2r} \left(\frac{1}{\delta} + \frac{1}{\lambda} \right). \tag{5}$$

The reason for the stationarity of the first-best solution is that the financially constrained agent is not exposed to firm risk. In the remainder of this section, we provide the solution for the full model with agency conflicts where the optimal contract requires exposing the intermediary, the agent, and the principal to cash flow shocks dX_t.[8]

B. Continuation Values and Optimal Effort

In the following, "player j" refers to "the intermediary or the agent," in that $j = A, I$. For any time $t < \tau$ and contracts Π^A and Π^P, we define player j's continuation value as

$$v_t^j = \mathbb{E}_t \left[\int_t^\infty e^{-r(s-t)} \left(dw_s^j - g^j(a_s^j) ds \right) \right] \tag{6}$$

for $j = A, I$. Using the martingale representation theorem, we can derive

$$dv_t^j = r v_t^j dt + g^j(a_t^j) dt - dw_t^j + \beta_t^j (dX_t - \hat{a}_t^A dt - \hat{a}_t^I dt), \tag{7}$$

where $dX_t - \hat{a}_t^A dt - \hat{a}_t^I dt = \sigma dZ_t$ when $\hat{a}_t^A = a_t^A$ and $\hat{a}_t^I = a_t^I$. Here, β_t^j captures the endogenous exposure to cash flows dX_t and is determined by the contracts Π^A and Π^P. Note that the intermediary affects the agent's continuation value through not only the choice of Π^A and β_t^A but also his unobservable and noncontractible effort a_t^I. For instance, by reducing his effort level a_t^I below the prescribed level $\hat{a}_t^I$ expected by the agent, the intermediary reduces the firm's realized cash flows dX_t by amount $(\hat{a}_t^I - a_t^I)dt$, thereby reducing the agent's continuation payoff v_t^A by amount $\beta_t^A (\hat{a}_t^I - a_t^I) dt$.

Likewise, for the principal's continuation payoff

$$v_t^P := \mathbb{E}_t \left[\int_t^\infty e^{-r(s-t)} dw_s^P \right], \tag{8}$$

we obtain

$$dv_t^P = r v_t^P dt - dw_t^P + \beta_t^P \left(dX_t - \bar{a}_t^A dt - \bar{a}_t^I dt \right). \tag{9}$$

In (9), β_t^P is the principal's endogenous exposure to cash flow shocks and $dX_t - \bar{a}_t^A dt - \bar{a}_t^I dt = \sigma dZ_t$ when $\bar{a}_t^A = a_t^A$ and $\bar{a}_t^I = a_t^I$. Note that an unexpected

[8] In another important benchmark of the model, the principal and the intermediary are combined in a single entity. We study this in Section III.B.2 as a special case of the full model.

change in the agent's or the intermediary's effort, that is, $a_t^j \neq \bar{a}_t^j$, changes the principal's continuation value by $\beta_t^P(a_t^A + a_t^I - \bar{a}_t^A - \bar{a}_t^I)dt$. Also note that (7) and (9) can be interpreted as "promise-keeping" constraints: Any payout to player j is accompanied by a commensurate decrease in continuation payoff.

The agent's limited liability requires that $v_t^A \geq 0$ at any time t because otherwise the agent would be better off leaving the firm. Also note that the intermediary and the principal, as the firm's owners, can always form a coalition and liquidate the firm, seize the recovery value of R dollars, and renege on the promised payouts to the agent. Limited commitment for the firm's owners therefore requires $v_t^I + v_t^P \geq R$ to hold at all times $t \geq 0$. The following lemma summarizes the limited liability and commitment constraints.

LEMMA 1: *At all times $t \geq 0$, the agent's limited liability requires $v_t^A \geq 0$, and the firm owners' limited commitment requires $v_t^I + v_t^P \geq R$.*

Next, we characterize the agent's and the intermediary's incentives to exert effort. Player j chooses effort a_t^j to maximize the (expected) change in continuation utility dv_t^j and incremental payouts dw_t^j net the costs of effort $g^j(a_t^j)dt$, so that

$$a_t^j = \arg\max_{a^j \in [0,\bar{a}]} \left(\beta_t^j a^j - g^j\left(a_t^j\right)\right)dt. \tag{10}$$

Raising effort by one unit over $[t, t + dt)$ increases cash flows dX_t by $1dt$, which increases the sum of continuation utility and incremental payouts by $\beta_t^j dt$. At the same time, player j incurs higher costs of effort, which reduces utility by $(g^j)'(a_t^j)dt$. As a result, the incentive condition (11) in the following lemma pins down the intermediary's and the agent's effort and ensures $\hat{a}_t^j = a_t^j$.

LEMMA 2: *Optimal effort is characterized by*

$$a_t^A = \hat{a}_t^A = \frac{\beta_t^A}{\delta} \quad \text{and} \quad a_t^I = \hat{a}_t^I = \frac{\beta_t^I}{\lambda}. \tag{11}$$

That is, the exposure to cash flow shocks β_t^j makes player j's continuation utility v_t^j sensitive to the firm's cash flows and therefore provides incentives to exert effort. The sensitivity β_t^j quantifies player j's *incentives*. As will become clear below, the principal's exposure to cash flow shocks β_t^P affects the intermediary's and the agent's exposure to firm performance and therefore also plays a key role in incentives.

C. Optimal Contracting

To solve for the optimal contracts Π^A and Π^P, we apply dynamic programming techniques to maximize the intermediary's payoff at time $t = 0$, $v_0^I + (-dw_0^P)$. Here, $-dw_0^P$ is the lump-sum cash payment that the intermediary receives from the principal at time $t = 0$ in exchange for the contract

Π^P, and with a slight abuse of notation, v_0^I is the intermediary's continuation payoff "just after" this lump-sum payment is made. Because the agent is penniless, there is no lump-sum payment from the agent to the intermediary at time $t = 0$. Because the intermediary has full bargaining power, he can extract all surplus from the principal, and the principal, who has zero reservation value, merely breaks even at time $t = 0$, so that $v_0^P = -dw_0^P$ (v_0^P again denotes the continuation payoff just after the initial lump-sum payment).[9]

Thus, at time $t = 0$, the intermediary dynamically maximizes the sum of the intermediary's and the principal's payoff $v_0^I + v_0^P$, which we refer to as total firm value (net of the payouts to the manager). The intermediary's dynamic maximization problem and time-t total firm value $v_t^I + v_t^P$ now depend on two state variables: (i) the agent's continuation value v_t^A and (ii) the principal's continuation value v_t^P. As a result, total firm value is a function of v_t^A and v_t^P, in that $v_t^I + v_t^P = \hat{F}(v_t^A, v_t^P)$. In what follows, we omit time subscripts to simplify notation.

To characterize the optimal contracts Π^A and Π^P, we proceed in three steps. First, we discuss the optimal timing of payouts to the principal and the agent. Second, we characterize the optimal provision of incentives to the agent β^A and the endogenous relationship between the agent's and the intermediary's incentives characterized by the trickle-down and trickle-up effects. Third, we apply the dynamic programming principle to derive the Hamilton-Jacobi-Bellman (HJB) equation for the intermediary's problem, which then pins down the optimal contracts that the intermediary offers the principal and the agent.

C.1. Optimal Payout Timing

Note that at any point in time, the intermediary can stipulate a transfer dw_t^P to the principal, which—according to the promise-keeping constraint in (9)—decreases the principal's continuation payoff by dw_t^P (i.e., $dv_t^P = -dw_t^P$) and hence leaves the principal's change in payoff $dv_t^P + dw_t^P$ unchanged. In addition, according to (7), the transfer dw_t^P increases the intermediary's continuation payoff by the same amount (i.e., $dv_t^I = dw_t^P$). As such, the transfer dw_t^P does not change the intermediary's and the principal's joint payoff and therefore leaves total firm value $F(v_t^A, v_t^P) = v_t^I + v_t^P$ unchanged but it does affect the split of total firm value. Intuitively, given both the principal and the intermediary have deep pockets, transfers from the intermediary to the principal (or the other way around) can implement any split of total firm value but do not change total firm value itself. This implies that $\hat{F}(v_t^A, v_t^P - dw_t^P) = \hat{F}(v_t^A, v_t^P)$ for any v_t^P and dw_t^P, so $\frac{\partial}{\partial v_t^P}\hat{F}(v_t^A, v_t^P) = 0$. Hence, $\hat{F}(v^A, v^P) = F(v^A)$ for some function $F(v^A)$. That is, the exact value of v^P is not payoff-relevant, and, as we will see, the principal's contract affects firm value only through the choice of β^P.

[9] Moreover, there is no lump-sum payment from the principal to the agent. That the agent cannot be paid negative wages already implies that the agent's payoff exceeds the outside option of zero, so no extra payment to the agent at $t = 0$ is needed to motivate participation.

Because payments to the agent dw_t^A must be nonnegative, it is always possible to decrease but not to increase the agent's deferred compensation v^A with payouts to the agent $dw^A \geq 0$. Thus, $\hat{F}(v^A - dw^A) - dw^A \leq \hat{F}(v^A)$. Taking $dw^A \to 0$, it follows that $F'(v^A) \geq -1$, with equality if $dw^A > 0$.

When v^A reaches zero and hits the agent's limited liability constraint, the agent's contract must be terminated. Termination of the agent's contract also implies firm liquidation, so that $\tau = \inf\{t \geq 0 : v_t^A = 0\}$ and

$$F(0) = R. \tag{12}$$

Because contract termination and liquidation are inefficient, the agent's payouts are delayed, which implies that the agent's continuation payoff v^A is positive for $t < \tau$.

Note that providing incentives β^A to the agent raises the volatility of v^A and therefore the risk of costly firm liquidation. To reduce these agency costs, the intermediary delays payouts to the agent until the firm's distance to liquidation is sufficiently large and v^A exceeds the payout boundary $\overline{v}$, so $dw^A = \max\{0, v^A - \overline{v}\}$. At the payout boundary, the smooth-pasting condition

$$F'(\overline{v}) = -1 \tag{13}$$

holds. Given all players discount at the same rate $r > 0$, delaying payouts to the agent is not costly for the intermediary and the principal but reduces the risk of firm liquidation, which is beneficial. Thus, firm owners optimally delay payouts to the agent as much as limited commitment allows, and the limited commitment constraint $F(v^A) \geq R$ binds at $v^A = \overline{v}$:[10]

$$F(\overline{v}) = R. \tag{14}$$

We conclude this section with the following lemma.

LEMMA 3: *The following hold:*

1. *Payouts to the agent satisfy cause v^A to reflect at $\overline{v}$, where $\overline{v}$ satisfies $F'(\overline{v}) - 1 = F(\overline{v}) - R = 0$.*
2. *The firm is liquidated when $v^A = 0$, leading to $F(0) = R$ and $\tau = \inf\{t \geq 0 : v_t^A = 0\}$.*

C.2. Direct versus Indirect Incentives

In this section, we discuss the mechanism that jointly determines the level of risk-sharing with the principal and the levels of incentives provided to the

[10] More generally, one could assume that when liquidating the firm as part of a contract, the liquidation value R is higher than the recovery value when walking away from the contract due to limited commitment. The difference, L, could be due to legal and reputation costs, in which case (14) would become $F(\overline{v}) = R - L$, and the remainder of the analysis would remain qualitatively unchanged.

agent and the intermediary. Specifically, we show that the interaction between the incentives of the agent and the intermediary can be understood as a combination of trickle-down and trickle-up effects, where the intermediary's incentives trickle down to the agent and the agent's incentives trickle up to influence those of the intermediary's.

Note that according to (7), $\beta^I\sigma$ is the volatility of the intermediary's instantaneous payoff $dw^I + dv^I$, which consists of instantaneous dollar payoffs $dw^I = dX - dw^P - dw^A$ and the change in future payoffs $dv^I = d(F(v^A) - v^P)$. When $v^A < \overline{v}$, there are no payouts to the agent (i.e., $dw^A = 0$) and the intermediary's instantaneous change in payoff is $dX + dF(v^A) - (dw^P + dv^P)$. According to (9), the volatility of $dw^P + dv^P$ is $\beta^P\sigma$. By Itô's Lemma, $dF(v^A)$ has volatility $F'(v^A)\beta^A\sigma$, where $F'(v^A)$ captures the sensitivity of total firm value to changes in the agent's compensation. Since dX has volatility σ, β^I can be decomposed as[11]

$$\beta^I = \underbrace{1}_{\substack{\text{Exogenous}\\\text{cash flow risk}}} - \underbrace{\beta^P}_{\substack{\text{Principal's}\\\text{exposure}}} + \underbrace{F'(v^A)\beta^A}_{\substack{\text{Endogenous}\\\text{cash flow risk}}}. \tag{15}$$

According to (15), the intermediary's incentives are determined by (i) the direct exposure to firm cash flow risk minus the risk shared with the principal and (ii) the endogenous exposure to cash flow risk through the agent's compensation contract. We interpret the first component, $1 - \beta^P$, as the intermediary's "direct incentives," which are determined by the financing contract between the principal and the intermediary and capture the cash flow risk that is not borne by the principal. The second component, $F'(v^A)\beta^A$, is determined by the contract between the intermediary and the agent and captures the intermediary's exposure to firm liquidation and the agent's payouts.

The endogenous and indirect exposure of the intermediary to cash flow risk via the agent's incentives means that incentives *trickle up* from the agent to the intermediary. The intuition behind this mechanism is as follows. By exerting more effort a^I, the intermediary increases the firm's cash flows dX. Because the intermediary's effort a^I is not observable to the agent and not contractible, the increase in cash flows dX raises the value of the agent's continuation payoff v^A, so part of the gains generated by the intermediary's effort accrue to the agent.[12] The increase in v^A has two opposing effects, as it reduces the firm's liquidation risk but increases the costs of compensating the agent. When $F'(v^A) > 0$ ($F'(v^A) < 0$), the first (second) effect dominates, leading to

[11] Note that (15) also holds at the payout boundary when $v^A = \overline{v}$ and $F'(\overline{v}) = -1$. Then, $dF(v^A) - dw^A$ has volatility $F'(\overline{v})\beta^A\sigma = -\beta^A\sigma$. Recall that the volatility of $dw^P + dv^P$ is $\beta^P\sigma$. The volatility of the intermediary's instantaneous payoff $dX + dF(v^A) - dw^A - (dw^P + dv^P)$ is therefore $\sigma(1 - \beta^P - \beta^A)$, which equals β^I, leading to (15).

[12] For a more formal argument, rewrite (7) for $j = A$ as $dv_t^A + dw_t^A = (rv_t^A + g^A(a_t^A))dt + \beta_t^A(a_t^I - \hat{a}_t^I)dt + \beta_t^A(dX_t - \hat{a}_t^A dt - a_t^I dt)$ and note that $\beta_t^A(dX_t - \hat{a}_t^A dt - a_t^I dt)$ has expectation zero under the intermediary's information, as the intermediary observes both the prescribed effort $\hat{a}_t^A$ and his own effort a_t^I. Thus, under the intermediary's information, $\frac{\partial}{\partial a_t^I}\mathbb{E}[dv_t^A + dw_t^A] = \beta_t^A dt$.

additional trickle-up *incentives* (*disincentives*) for the intermediary. Thus, $F'(v^A)$ quantifies the magnitude of the trickle-up effect.

The intermediary's exposure to changes in firm value and risk-sharing with the principal influence the level of incentives the intermediary provides to the agent, creating what we term the *trickle-down* effect. To quantify this effect, Lemma 4 below shows that at any point in time t, the intermediary chooses the agent's incentives β^A to maximize

$$\max_{\beta^A \geq 0} \left((1-\beta^P)(a^A + a^I) - \frac{\lambda (a^I)^2}{2} + F'(v^A)\left(rv^A + \frac{\delta (a^A)^2}{2} \right) + \frac{F''(v^A)(\beta^A \sigma)^2}{2} \right), \tag{16}$$

taking into account the effort incentive constraints (11) and the composition of incentives in (15). Intuitively, the first term of the objective in (16) reveals that the intermediary internalizes only fraction $1-\beta^P$ of *actual* expected output $(a^A + a^I)dt$ because the principal attributes changes in cash flows due to unobservable deviations in the choice of a^A or a^I as realizations of cash flow shocks, of which the principal receives fraction β^P. For instance, the intermediary can distort the principal's information set by secretly changing the agent's incentives and effort or his own effort, which are unobserved by the principal. The second term is the intermediary's cost of effort. The last two terms are the expected change in total firm value $\mathbb{E}[dF(v^A)]/dt$ by Itô's Lemma.

The following lemma shows that the agent's incentives solve problem (16) and characterizes the solution and the endogenous interdependence of the intermediary's and agent's incentives.

LEMMA 4: *The intermediary's incentives satisfy (15). The agent's incentives are the solution to (16). The agent's incentives satisfy* $\beta^A = (1-\beta^P)\pi^I$, *with*

$$\pi^I = \pi^I(v^A) := \frac{\overbrace{1}^{\text{Cash flows}}}{\underbrace{\frac{\delta (F'(v^A))^2}{\lambda}}_{=C(>0)} \underbrace{-F'(v^A)}_{=B} \underbrace{-\delta \sigma^2 F''(v^A)}_{=A(>0)}}. \tag{17}$$

The equation $\beta^A = (1-\beta^P)\pi^I$ *together with (15) also implies that*

$$\beta^I = (1 + F'(v^A)\pi^I)(1-\beta^P), \tag{18}$$

and after substituting $1-\beta^P = \frac{\beta^A}{\pi^I}$, *we have*

$$\beta^A = \beta^I \left(\frac{\pi^I}{1 + F'(v^A)\pi^I} \right). \tag{19}$$

The first part of the lemma states that the agent's incentives $\beta^A = (1-\beta^P)\pi^I$ increase with the intermediary's direct incentives $1-\beta^P$, in that incentives trickle down from the intermediary to the agent. The strength of this *trickle-down* effect is captured by π^I in (17). This coefficient reflects that by raising the agent's incentives, the intermediary boosts the agent's effort a^A and cash flows but also increases the risk of liquidation (term A) and the cost of compensating the agent (term B). Lastly, the choice of the agent's incentives affects the intermediary's incentives to exert effort via the trickle-up effect (term C). The trickle-up effect decreases π^I and undermines trickle-down incentives because it inadvertently affect the intermediary's incentives and effort, which in principle is costly. In other words, term C captures the shadow cost of constraint (15) linking the intermediary's and agent's incentives. The shadow cost is lower when δ/λ is low, that is, when the agent's effort is relatively cheaper and the intermediary focuses more on efficient incentive provision to the agent than on distortions to his own effort.

The combination of trickle-up and trickle-down incentives leads to a feedback loop between the agent's and the intermediary's incentives, and in turn to the fixed point provided in the second part of the lemma, equation (18). As a result, the intermediary's total incentives β^I reflect both the trickle-up and the trickle-down effects via $F'(v^A)\pi^I$. When $F'(v^A) > 0$, trickle-up and trickle-down incentives reinforce each other and amplify the intermediary's direct incentives $1-\beta^P$. When $F'(v^A) < 0$, incentives trickle down from the intermediary to the agent but generate trickle-up disincentives, dampening the intermediary's direct incentives $1-\beta^P$.

D. HJB Equation

We now derive the HJB equation for total firm value, $F(v^A)$. To begin, recall that the integral expressions for v_t^I in (6) and for v_t^P in (8) imply that total firm value at time t satisfies

$$F(v^A) = \mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)}\left(dX_s - dw_s^A - \frac{\lambda\left(a_s^I\right)^2}{2}ds\right)\middle| v_t^A = v^A\right].$$

Note that when $v^A \in [0,\overline{v}]$, there are no payouts to the agent (i.e., $dw^A = 0$), and thus firm owners' expected instantaneous payoff equals $\mathbb{E}[dX] - \frac{\lambda(a^I)^2}{2}dt$. By the dynamic programming principle, the expected instantaneous payoff $\mathbb{E}[dX] - \frac{\lambda(a^I)^2}{2}dt$ and the expected change in payoff $\mathbb{E}[dF(v^A)]$ must in optimum compensate firm owners for their time preference $rF(v^A)dt$. Using Itô's Lemma to calculate $\mathbb{E}[dF(v^A)]$, we can derive that, on $[0,\overline{v}]$, $F(v^A)$ solves the HJB equation

$$rF(v^A) = \max_{\beta^A,\beta^I}\left\{a^A + a^I - \frac{\lambda\left(a^I\right)^2}{2} + F'(v^A)\left(rv^A + \frac{\delta\left(a^A\right)^2}{2}\right) + \frac{F''\left(v^A\right)\left(\beta^A\sigma\right)^2}{2}\right\}, \tag{20}$$

subject to the boundary conditions $F(0) = R$ (liquidation), $F'(\overline{v}) = -1$ (smooth pasting), and $F(\overline{v}) = R$ (limited commitment). The choice of β^A and β^I is subject to the effort incentive constraint (11) and the characterization of the agent's incentives (19). Also note that when the incentive conditions (11) and (19) hold, prescribed and actual effort levels coincide.

As $\beta^I = (1 + F'(v^A))(1 - \beta^P)$, maximizing (20) over β^I by choosing the intermediary's optimal incentives is equivalent to maximizing (20) over β^P by choosing the optimal sensitivity β^P. This yields the intermediary's direct incentives

$$1 - \beta^P = \frac{\overbrace{\frac{1 + F'(v^A)\pi^I}{\lambda}}^{=A'} + \overbrace{\frac{\pi^I}{\delta}}^{=B'}}{\underbrace{\frac{(1 + F'(v^A)\pi^I)^2}{\lambda}}_{=C'} - \underbrace{\frac{F'(v^A)(\pi^I)^2}{\delta}}_{=D'} \underbrace{- F''(v^A)(\pi^I\sigma)^2}_{=E'}}, \tag{21}$$

with π^I defined in (17) and $\beta^I = (1 - \beta^P)(1 + F'(v^A)\pi^I)$. The optimal provision of incentives to the intermediary reflects the trickle-down and trickle-up effects of incentives in addition to the direct impact of incentives on the intermediary's effort. Reducing the principal's exposure β^P or, equivalently, raising the intermediary's direct incentives $1 - \beta^P$ increases the intermediary's actual incentives β^I and effort a^I, which increases cash flows (term A') but also increases the intermediary's required compensation for effort costs (term C'). Moreover, incentives trickle down to the agent, which increases cash flows through the agent's effort (term B') as well as the cost of compensating the agent (term D') and the firm's liquidation risk (term E').

Finally, combining the findings of this section with Lemmas 2 to 4, we can complete the characterization of the optimal contracts Π^A and Π^P with the following proposition.

PROPOSITION 1: *In optimum, the following hold:*

1. *The value of the intermediary's deferred payouts $F(v^A)$ solves (20) subject to the boundary conditions $F(0) - R = F'(\overline{v}) - 1 = F(\overline{v}) = R$.*
2. *Payouts dw^A occur at the payout boundary $\overline{v}$, and cause v^A to reflect at $\overline{v}$.*
3. *The function F is strictly concave, in that $F''(v^A) < 0$ for all $v^A \in [0, \overline{v}]$.*
4. *The sensitivities β^A, β^P, and β^I are characterized by (15), (19), and (21). Effort is characterized by (11).*

Figure 1 presents a numerical example of the solution under our baseline parameters. We use the discount rate $r = 0.05$ and we normalize volatility and agency cost parameters to one, that is, $\delta = \lambda = \sigma = 1$. The recovery value is set to $R = 12.5$. Panel A of Figure 1 shows that $F(v^A)$ is hump-shaped and concave. The concavity of $F(v^A)$ reflects that increasing the volatility of v^A by providing the agent stronger incentives β^A is costly because it increases the risk of firm liquidation. Also note that an increase in v^A has two opposing effects—it reduces the firm's liquidation risk but also increases the cost of compensating the agent. When v^A is small (large), the first (second) effect dominates

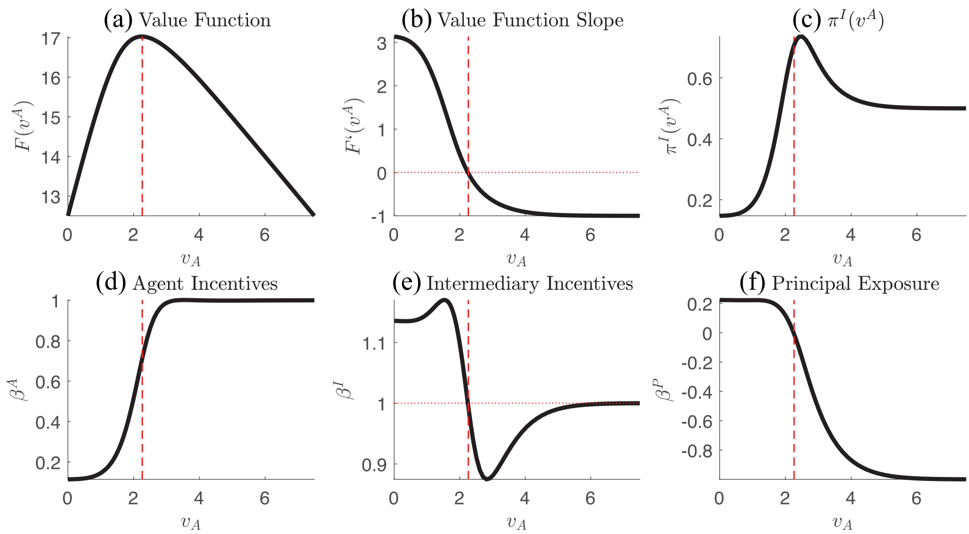

Figure 1. Numerical example of the model solution. The dashed line depicts the value of v^A at which $F'(v^A) = 0$ and $F(v^A)$ is at its peak. The parameters are $r = 0.05$, $\delta = \lambda = \sigma = 1$, and $R = 12.5$. (Color figure can be viewed at wileyonlinelibrary.com)

and $F'(v^A) > 0$ ($F'(v^A) < 0$). Note that $F'(v^A)$ switches sign exactly once when $v^A = v^*$, with $F'(v^*) = 0$ and $F(v^*)$ at its peak, which is depicted in Figure 1 by the dashed line.

III. Analysis and Implications

A. *Dynamics of Incentives*

We start the analysis of the model by examining the fundamental output of the model, namely, the dynamics of the agent's and the intermediary's incentives, β^A and β^I. By the incentive constraint (11), effort a^j is directly proportional to incentives β^j, and thus the dynamics of effort and incentives follow a similar pattern. Notice that the level of v^A increases (decreases) after positive (negative) cash flow realizations and therefore quantifies the firm's past performance as well as its distance to liquidation. When v^A is relatively small and close to zero, the risk of firm liquidation is high and the firm is in distress. When v^A is relatively large and close to $\overline{v}$, the risk of liquidation is low.

Panel D of Figure 1 illustrates the standard result that the agent's incentives increase after good performance (i.e., increase with v^A) or in other words, the agent's incentives are convex.[13] The reason is that when v^A is low, the firm's distance to liquidation is small. Therefore, the cost of providing incentives to

[13] Note that β^j measures the sensitivity of player j's value to firm performance. If the sensitivity to firm performance increases in firm performance, then incentives are option-like or convex.

the agent is high and the strength of the trickle-down incentives π^I is low, which reduces incentive provision to the agent.

Panel E of Figure 1 shows that the intermediary's incentives β^I (solid line) are in general not monotonic in v^A but, considering larger changes in firm performance, are higher for low values of v^A than for high values of v^A. For low values of v^A when $F'(v^A) > 0$, trickle-up incentives lead to strong intermediary incentives. Intuitively, when the firm experiences distress, the intermediary takes control and works hard to save the firm, whereas the agent's incentives to exert effort are weak under distress. Conversely, for larger values of v^A with $F'(v^A) < 0$, the trickle-up effect weakens the intermediary's incentives and leads to disincentives, reflecting an *agency overhang* problem. The intuition is that when v^A is large, the agent possesses a large stake in the firm. Hence, the gains generated by the intermediary's effort mostly accrue to the agent, undermining the intermediary's incentives β^I. Thus, after sufficiently good past performance, the intermediary takes a hands-off approach and has low incentives to exert effort, but the agent's incentives to exert effort are high. In light of these effects, the intermediary's total incentives β^I do not necessarily increase after good performance.

Panel F of Figure 1 depicts the principal's exposure to firm performance, β^P. Note that β^P decreases with v^A, and thus $1 - \beta^P$ increases with v^A. Following poor performance when v^A is low and $F'(v^A)$ is positive, the trickle-up effect implies strong intermediary incentives after poor performance. To curb the intermediary's excessive incentives, the intermediary offloads exposure to the principal, so that $\beta^P > 0$. Conversely, following good performance when v^A is high and $F'(v^A) < 0$, the trickle-up effect leads to agency overhang, which undermines the intermediary's incentives. The intermediary, in turn, stipulates $\beta^P < 0$ to increase his exposure to firm performance so as to boost his incentives in the presence of agency overhang. Taken together, the role of β^P is to partially undo the distortions to the intermediary's and agent's incentives due to agency-induced distress after poor performance and agency overhang after good performance. As a result, the principal's exposure to firm performance, as captured by β^P, decreases with v^A. This effect makes the intermediary's direct incentives $1 - \beta^P$ increase after good performance so that the intermediary's direct incentives are convex. Thus, while the intermediary's direct incentives $1 - \beta^P$ increase after good performance and are convex, the intermediary's total incentives β^I do not necessarily increase after good performance and are not convex.

B. Implications for PE

PE funds apply three types of changes to firms they acquire, which Kaplan and Strömberg (2009) categorize as governance, operational, and financial engineering. Our model captures these three channels as follows.

First, PE funds affect managerial incentives and contract terms via governance engineering (see Cronqvist and Fahlenbrach (2013) for direct evidence). In our model, the intermediary's choice of the agent's incentives β^A captures governance engineering. Second, PE funds actively and directly influence firm performance through operational engineering. These activities include applying industry or strategic expertise to improve firm operations (see Bernstein and Sheen (2016) for direct evidence). In our model, the intermediary's incentives β^I or, equivalently, the intermediary's effort a^I capture the extent to which the PE fund engages in operational engineering. Third, raising funds from outside investors alters the financing structure and incentives, which is equivalent to financial engineering. In our model, the exposure of investors to firm performance β^P describes the extent of such financial engineering.

B.1. The Effects of Governance and Operational Engineering

Our model predicts that the outcome of governance engineering is a high-powered convex incentive contract for the portfolio firm manager. To gauge if the convexity of the agent's incentives is *relatively* high, we compare the baseline model to a model without operational and financial engineering. The latter is obtained when the intermediary's effort cost goes to infinity as shown in the following corollary.

COROLLARY 1: *In the limit, as $\lambda \to \infty$, it holds that $a^I = 0$, $\beta^A = \pi^I$, $\beta^I = 1 + F'(v^A)\pi^I$, and $\beta^P = 0$.*

The result of the corollary means that without operational engineering, there is no financial engineering. In the model without operational and financial engineering, the intermediary is not active in the firm beyond offering a financing contract and in fact is not an intermediary anymore as there are no outside investors involved. In our setting, this benchmark is akin to firm ownership without PE involvement and when the firm owners are passive. In other words, comparing the baseline with the limit case $\lambda \to \infty$ allows us to identify the effects of PE ownership as predicted by our model.

Figure 2 plots the baseline model and the benchmark with $\lambda \to \infty$.[14] Panel B shows that the portfolio firm manager's incentives are convex even without operational and financial engineering, but they increase more steeply in firm performance in the model with operational and financial engineering. That is, our model predicts that PE ownership makes managerial incentives more convex. This pattern is broadly consistent with empirical studies, such as Leslie and Oyer (2008), Acharya et al. (2012), and Cronqvist and Fahlenbrach (2013), on the impact of PE investment on managerial incentives in portfolio firms. In particular, Cronqvist and Fahlenbrach (2013) document that the most significant effect of PE governance engineering is the use of performance-vesting in stock compensation, which implies highly convex incentives.

The analysis of Section III.A applied to the PE setting implies that operational engineering as captured by the intermediary's effort a^I is most intensive after poor performance (i.e., for low values of v^A) and tends to be lower after

[14] Note that the x-axis uses $v_A/\bar{v}$ to make the models with different optimal thresholds $\bar{v}$ comparable.

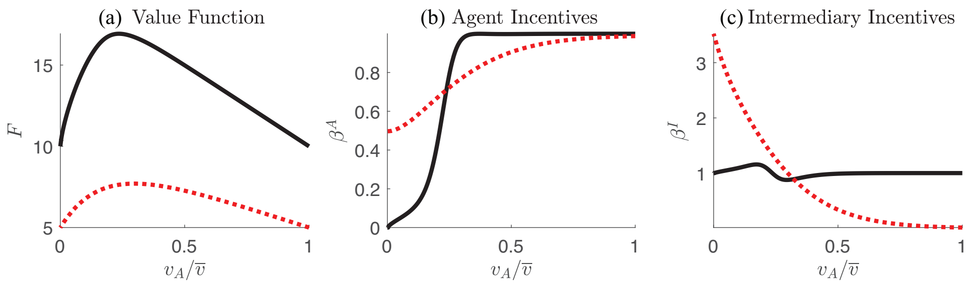

Figure 2. The effects of operational engineering. The figure compares the solution in the baseline model with $\lambda = 1$ (solid line) to the model solution without intermediary effort, that is, $\lambda = \infty$ and $a^I = 0$ (dotted line). To ensure comparability, in both scenarios the recovery value is set to $R = 0.5 \cdot F^{FB}$, where F^{FB} is given in (5) and depends on λ. For all other parameter values, we use our baseline parameters (see Figure 1). (Color figure can be viewed at wileyonlinelibrary.com)

good performance (i.e., for high values of v^A). The model therefore predicts that the GPs of the PE fund take more control and a more hands-on approach when the portfolio firm is in distress. Under these circumstances, the agent exerts relatively low effort and has little impact on firm performance. In contrast, after good performance, GPs have relatively weak incentives to affect firm performance directly and take a more hands-off approach, whereas the manager receives strong incentives and exerts high effort.

Overall, our findings suggest that PE funds take a more active role in their portfolio firms after poor performance and when the firm experiences distress, which is consistent with the results of several empirical studies. In a recent paper, Gompers, Kaplan, and Mukharlyamov (2020) show that engagement of PE funds increased during the COVID-19 pandemic, especially in firms more severely affected by the pandemic. Bernstein, Lerner, and Mezzanotti (2019) also find that PE investors take a more active role in portfolio firms and are more likely to engage in both operational and financial engineering during times of crisis. Hotchkiss, Smith, and Strömberg (2021) show that PE-backed firms respond more effectively to distress than other companies and attribute part of the effect to the engagement of PE sponsors in distress resolution. In addition, Cornelli, Kominek, and Ljungqvist (2013) find that PE-controlled boards monitor firm managers to discipline them after poor performance, suggesting that PE investors seek to improve portfolio firm performance after poor performance through monitoring.

As the next section shows, PE financial engineering, captured by β^P, interacts with both governance and operational engineering and is crucial for both the intermediary's and the agent's incentives.

B.2. The Effects of Financial Engineering

Financial engineering (i.e., contracting between the intermediary and principal) has opposing effects on the intermediary's incentives depending on past

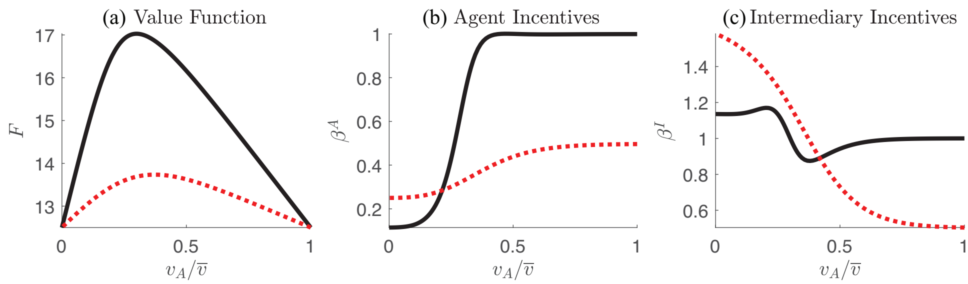

Figure 3. The effects of financial engineering. The figure compares the model with optimal β^P (solid line) to the model with $\beta^P = 0$ (dotted line). We use our baseline parameters (see Figure 1). (Color figure can be viewed at wileyonlinelibrary.com)

firm performance. First, when v^A is large, financial engineering stipulates $\beta^P < 0$ and implies a levered exposure to firm performance for the intermediary, boosting the intermediary's incentives. Second, when v^A is low, financial engineering involves exposing the principal to firm performance ($\beta^P > 0$), which reduces the intermediary's exposure and incentives. As a result, financial engineering weakens the intermediary's incentives after poor performance but strengthens them after good performance. In other words, financial engineering increases the extent of operational engineering (i.e., the intermediary's effort) after poor performance but decreases it after good performance. Via the trickle-down effect, the strength of the intermediary's incentives also affects the agent's incentives. Therefore, financial engineering weakens the agent's incentives after poor performance but strengthens them after good performance, thereby contributing additional convexity to the agent's compensation scheme.

These effects are illustrated in Figure 3, which compares the baseline model with optimal β^P (solid line) and a constrained model with $\beta^P = 0$ (dotted line), that is, without financial engineering. Note that constraining β_P to zero is equivalent to a model in which the principal and the intermediary are combined into one entity. This combined principal has all the functions of the intermediary in our baseline model, but cannot engage in financial engineering. Panel B of Figure 3 shows that financial engineering strengthens governance engineering by making the portfolio firm manager's incentives increase more steeply with performance and thus more convex. Note that in our model, financial engineering does not have a direct impact on the manager, but it works via the incentives of the PE fund (or the GPs of the PE fund), which passes its incentives in the manager's contract. Panel C of Figure 3 shows that, in contrast to the manager's incentives, the intermediary's incentives are flattened by financial engineering. This is because, in the absence of financial engineering, the intermediary is insufficiently incentivized after good performance and excessively incentivized after bad performance.

Panel A of Figure 3 shows that financial engineering adds value. Recall that in our model, the intermediary is not financially constrained, and there are no risk-sharing motives for external financing. The added value comes purely from the effect of external financing on incentives. Financial engineering is valuable because it allows the agent's and the intermediary's incentives to be decoupled. In the absence of outside investors and without financial engineering, the intermediary and the agent split the exposure to firm performance, so their incentives are tightly linked. With financial engineering, the agent's and the intermediary's incentives remain linked via the trickle-up and trickle-down effects, but there is an extra degree of freedom in the choice of β^P. The role of the principal, and thus of financial engineering, in our model resembles that in the multiple-agents model of Holmström (1982). Like in our paper, in Holmström (1982), closed contracts between agents without a principal can be inefficient as they require a budget constraint on total incentives. The principal can administer multiple-agent incentive schemes that do not need to balance the budget.

To illustrate the working of this mechanism, suppose the intermediary would like to incentivize the agent to exert a lot of effort after good performance. However, without external financing, the small complementary exposure of the intermediary due to agency overhang limits the intermediary's incentives and, via the trickle-down effect, the agent's incentives too, which might make it infeasible to incentivize high effort from the agent. For this reason, without financial engineering, the optimal contract implements inefficiently low levels of effort for both the manager and the intermediary after good performance (see the dotted curves for β^A and β^I at $v_A = \overline{v}$ in Figure 3). Symmetrically, the intermediary's incentives after poor performance are inefficiently high in the absence of external financing, which leads to inefficiently high effort incentives for the agent as well. Crucially, financial engineering (i.e., the financing contract with the principal) alleviates these distortions and inefficiencies in effort provision, as it boosts both efforts after good performance but curbs them after poor performance. Finally, our analysis highlights that financial engineering in PE investments facilitates efficient operational and governance engineering in PE investments, and therefore relates to Malenko and Malenko (2015), who find that financial and operational engineering in PE are complements.[15] In a broader context, our analysis implies that intermediated investment emerges as an optimal form of active ownership.

B.3. The Implementation of Financial Engineering

We now discuss how to link the financial contract with outside investors (the principal) to financial engineering in practice. Any implementation of the optimal contract offers claims to investors and the intermediary, exposing investors to firm cash flow shocks with total sensitivity β^P and leaving the intermediary with direct incentives $1 - \beta^P$. Recall that β^P decreases in firm performance, so that negative shocks to firm performance have the most negative impact on investors after bad performance, that is, when the firm experiences

[15] Different from Malenko and Malenko (2015), our findings derive from the multilayered agency conflicts of PE investors and firm management.

distress and v^A is low. This suggests that outside investors' claim has features of risky (long-term) debt with possible default when the firm is liquidated at $v_A = 0$. Additionally, β^P is negative when v_A is large, suggesting that $1 - \beta^P$, the intermediary's exposure to firm performance, is amplified after good performance. Consistent with these insights, we argue that financial engineering (i) grants the intermediary levered exposure to firm performance following good performance and (ii) involves debt-like financing from outside investors so that the intermediary's direct incentives decrease following poor performance.

To formalize this intuition, let us decompose the principal's exposure to cash flow shocks as follows:

$$\beta^P(v^A) = \beta^P_+(v^A) + \beta^P_-(v^A),$$

where $\beta^P_-(v^A) = \min\{\beta^P(v^A), 0\} \leq 0$ and $\beta^P_+(v^A) = \max\{\beta^P(v^A), 0\} \geq 0$ are the negative and positive parts of $\beta^P(v^A)$, respectively. Recall that the principal broadly describes different types of outside investors in PE financing, such as banks and private lenders, who in practice often provide long-term debt or credit line financing, or LPs, who provide equity financing. As such, the exposures β^P_+ and β^P_- could derive from different claims held by different types of outside investors.

Next, we argue that the positive part β^P_+ of the principal's exposure to firm performance could be generated by selling a claim to the principal whose payoff structure resembles that of risky long-term debt. Notice that $\beta^P_+(v^A)$ increases with v^A (i.e., following good performance) and vanishes for larger values of v^A. This means that, in effect, $\beta^P_+(v^A)$ moves the exposure to cash flows from the intermediary to the principal after poor performance when the firm experiences distress and v^A is low. Note that a claim with value $P(v^A)$, which satisfies $P'(v^A) = \beta^P_+(v^A)/\beta^A(v^A)$ and stipulates smooth payouts at rate $\mu^P(v^A)$, generates exposure to cash flow shocks $\beta^P_+(v^A)$ for its holders.[16] Figure 4 numerically solves the ordinarily differential equation (ODE) $P'(v^A) = \beta^P_+(v^A)/\beta^A(v^A)$ (subject to the boundary condition $P(0) = R$) for the value of this claim $P(v^A)$ (Panel A) and the payout rate $\mu^P(v^A)$ (Panel B) as a function of firm performance v^A. The payoff structure $P(v^A)$ is concave and increasing in performance v^A, and the slope is zero for large v^A. That is, the claim $P(v^A)$ resembles the payoff structure of risky long-term debt with default at $v^A = 0$. The payout rate $\mu^P(v^A)$ is constant in normal times (i.e., for larger values of v^A) and decreases in distress.[17]

[16] By construction, the claim has volatility $vol(dP(v^A)) = P'(v^A)\beta^A(v^A)\sigma = \beta^P_+(v^A)$. The payout rate is then determined according to the valuation equation

$$\mu^P(v^A) = rP(v^A) - P'(v^A)\left(rv^A + \frac{\delta(a^A)^2}{2}\right) - \frac{P''(v^A)(\beta^A\sigma)^2}{2}.$$

[17] Broadly, the claim with value $P(v^A)$ can be interpreted as risky long-term debt with coupons that decrease under distress, which can be related to the multilayered debt structure employed in PE with substantial amounts of subordinated and mezzanine debt (Axelson et al. (2013)). These

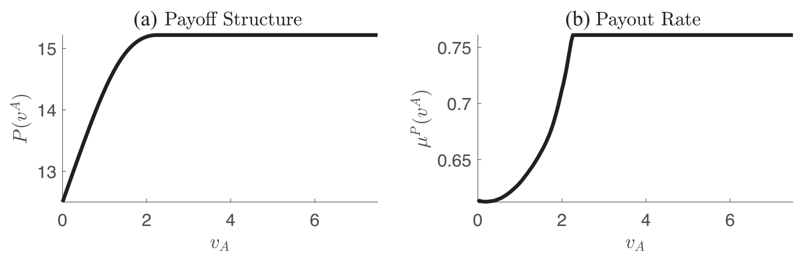

Figure 4. Numerical illustration of the claim $P(v^A)$. Panel A plots the value (i.e., payoff structure) of the claim, $P(v_A)$. Panel B plots the payout rate, $\mu^P(v_A)$. We use our baseline parameters (see Figure 1).

We now turn to the negative part, $\beta_-^P(v^A)$, which is strictly negative only for larger values of v^A, that is, after sufficiently good past performance, and zero otherwise. The negative exposure $\beta_-^P(v^A)$ exposes the principal's payoff negatively to performance and, on the flip side, generates additional incentives $-\beta_-^P(v^A) \geq 0$ for the intermediary. We argue that such negative exposure to performance for outside investors can be a consequence of the use of on-demand financing in PE deals, which can convexify and strengthen the intermediary's incentives. Outside investors, who provide on-demand financing, may have negative exposure to the performance of the PE fund or portfolio firms as sufficiently good past performance may limit the use of on-demand financing and hence restrict their participation in profitable investment.

On-demand debt is common in PE financing, both at the portfolio firm level as credit lines (Shive and Forster (2022)) and at the PE fund level, in which case they are known as subscription lines of credit (SLCs) (Albertus and Denes (2020), Schillinger, Braun, and Cornel (2020)). Typically, these credit facilities are drawn (repaid) following negative (positive) performance; see Albertus and Denes (2020) for evidence in the context of SLCs. Furthermore, default on SLCs is rare, so one can view SLCs as credit line debt with very low or negligible credit risk. These characteristics of SLCs imply that good performance at the fund or firm level (in our model, an increase in v^A) leads to the repayment of the SLC and thus reduces interest payments from the PE fund to credit line lenders, which harms credit line lenders. Conversely, worse performance has little effect on the default risk of the SLC but can increase the use of on-demand debt and benefit lenders. A similar logic applies to revolving credit lines at the portfolio firm level. Taken together, lenders who provide on-demand debt financing in PE may maintain negative exposure to firm or fund performance as captured by β_-^P in our model, at least when firm liquidation is distant and v^A is sufficiently far from zero.

high-yield debt tranches often include payment-in-kind interests, which may replace cash interests in certain circumstances, particularly when cash flows are low.

C. Observable Contract Terms and Direct Coinvestment by LPs

In this section, we consider the case in which the contract terms Π^A between the intermediary and the agent are publicly observable and contractible between the principal and the intermediary, and thus the principal can directly influence the agent's contract Π^A via the contract Π^P with the intermediary. Under these circumstances, financial engineering (i.e., contracting with outside investors) involves fewer frictions than in the baseline with unobservable contract terms Π^A. However, financial engineering is still subject to the moral hazard problem that the agent's effort and intermediary's effort are hidden.

Note that observable and contractible contract terms Π^A may reflect increased participation of investors (i.e., the principal) in the firm's governance. In practice, increased investor participation can represent various forms of investor activism. In the context of PE, this can arise when LPs directly coinvest in portfolio firms outside the fund structure. Investment within the fund structure gives LPs essentially no influence on the selection and monitoring of portfolio firms, whereas coinvestment involves LPs as direct equity holders. The trend toward increased coinvestment, documented in Fang, Ivashina, and Lerner (2015) and Braun, Jenkinson, and Schemmerl (2020), is viewed as evidence of disintermediation in PE investment.

When the agent's contract Π^A is observable and contractible, the value function—which we denote for simplicity, but with a slight abuse of notation, by $F(v^A)$—solves the HJB equation (20) too. However, as the agent's incentives β^A and her prescribed effort $\hat{a}^A$ are observable and contractible, the agent's and the intermediary's incentives β^A and β^I are no longer linked via the constraint (19) and therefore can be chosen independently to maximize $F(v^A)$ subject to the effort incentive constraints (11). Thus, the optimal incentives are obtained by solving the first-order conditions $\frac{\partial F(v^A)}{\partial \beta^A} = 0$ and $\frac{\partial F(v^A)}{\partial \beta^I} = 0$ (or, equivalently, $\frac{\partial F(v^A)}{\partial \beta^P} = 0$) taking into account (11). This leads to

$$\beta^A = \frac{1}{-F'(v^A) - \delta\sigma^2 F''(v^A)} \tag{22}$$

and

$$\beta^I = 1. \tag{23}$$

Combining (15) and (23), we are able to characterize optimal risk-sharing with the principal and as such the intermediary's direct exposure to cash flow risk,

$$1 - \beta^P = 1 - F'(v^A)\beta^A, \tag{24}$$

under direct contracting. Note that while the intermediary's total incentives β^I are constant under direct contracting, the agent's incentives β^A and the intermediary's direct incentives $1 - \beta^P$ are state-dependent.

Figure 5 plots the agent's incentives β^A, the intermediary's incentives β^I, and the principal's exposure β^P against v^A both in the baseline model (solid line) and under direct contracting (dotted line). Because β^A and β^I are no

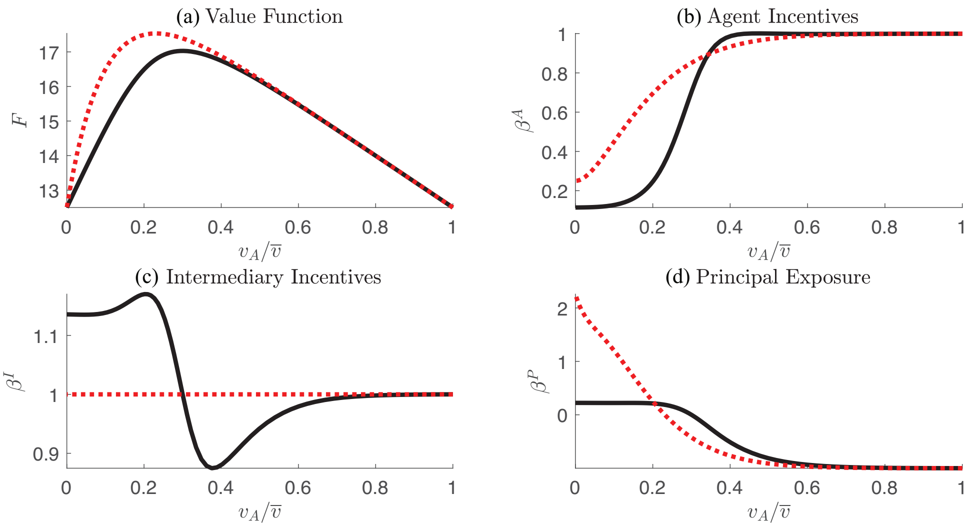

Figure 5. Model solution when the contract between the agent and the intermediary is publicly observed. The dashed lines represent the model with publicly observable and contractible Π^A, while the solid back lines represent the baseline model with noncontractible Π^A. We use our baseline parameters (see Figure 1). (Color figure can be viewed at wileyonlinelibrary.com)

longer linked via (19), incentives no longer trickle down from the intermediary to the agent. However, incentives continue to trickle up from the agent to the intermediary. Because it is optimal to implement a constant level of incentives β^I and effort a^I, risk-sharing between the principal and the intermediary (i.e., β^P) exactly offsets the trickle-up effect.

Note that the intermediary's incentives become stronger (weaker) relative to the baseline when v^A is low (high), while the agent's incentives become stronger relative to the baseline for low values of v^A.

In the context of PE, the model implies that the effect of increased participation of LPs in firm governance will depend on past performance and the state of the firm. When the portfolio firm is in financial distress, increased participation of LPs reduces GPs' incentives, transfers risk to LPs, and increases the incentives of the portfolio firm's manager. The model therefore predicts that GPs are less likely to engage in operational engineering in firms facing distress when there is coinvestment by LPs.

In contrast, when the portfolio firm is financially sound after strong performance, increased participation of LPs has little effect on the manager's incentives, yet it may increase GPs' incentives, possibly leading to more operational engineering. We conclude with the following proposition formalizing the results discussed in this section.

PROPOSITION 2: *When the contract* Π^A *is publicly observable and contractible, the following hold:*

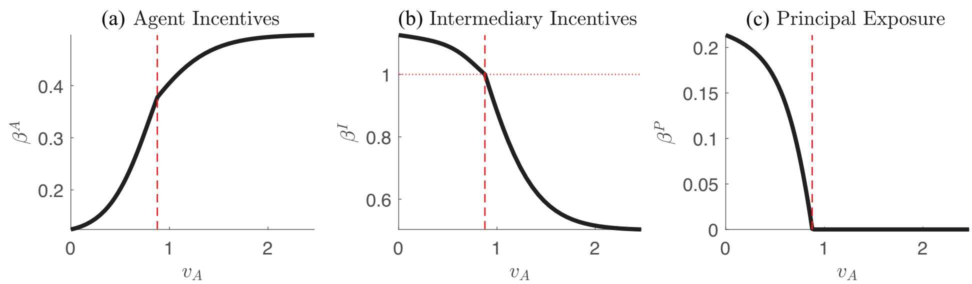

Figure 6. A numerical example of the model solution when $\beta^P \geq 0$ is imposed. The dashed line denotes the value of v^A where $F'(v^A) = 0$ and $F(v^A)$ has the peak. We use our baseline parameters (see Figure 1). (Color figure can be viewed at wileyonlinelibrary.com)

1. *The value of the intermediary's deferred payouts $F(v^A)$ solves (20) subject to the boundary conditions $F(0) - R = F'(\overline{v}) - 1 = F(\overline{v}) = R$. Payouts occur at the payout boundary $\overline{v}$ and lead v^A to reflect at $\overline{v}$. The function $F(v^A)$ is strictly concave.*
2. *The sensitivities β^A, β^I, and β^P, are characterized by (22), (23), and (24). Effort is characterized by (11).*
3. *In the limit, as $\frac{1}{\lambda} \to 0$, the intermediary's incentives and effort are lower relative to the baseline scenario with unobservable Π^A when $v^A < v^*$, and higher otherwise.*

IV. Additional Constraints on Contracting

In the baseline model, contracting between the principal and the intermediary is subject to two frictions: (i) the principal does not observe the agent's incentives and (ii) the principal does not observe the intermediary's effort. In this section, we discuss three other empirically relevant constraints that could limit the scope of contracting between the principal and the intermediary.

A. Monotonicity

The optimal contract features a negative sensitivity $\beta^P < 0$ in some states of the world, which implies that the principal is worse off after a positive cash flow shock $dZ > 0$. A usual assumption in the contracting literature is *monotonicity*, that is, the principal cannot benefit from negative shocks to firm performance.[18] In terms of our model, imposing $\beta^P \geq 0$ would be akin to imposing monotonicity. Figure 6 displays the solution and the sensitivities β^A, β^I, and β^P when the constraint $\beta^P \geq 0$ holds. Similar to the baseline case of the model, the

[18] The standard motivation for restricting contracts to be monotonic is to prevent earnings manipulation via hidden borrowing (Innes (1990)). Common financing contracts, such as debt and equity, are also monotonic. However, in practice, incentive contracts that stipulate convex exposure to agents can require nonmonotonic financing contracts for investors.

agent's incentives β^A and the intermediary's direct incentives $1-\beta^P$ increase with v^A, but the intermediary's total incentives do not. We also can implement the monotonic contract between the principal and the intermediary as in Section III.B.3. The restriction that $\beta^P \geq 0$ implies $\sigma^P(v^A)=0$, so payouts to the principal are smooth. The contract between the principal and the intermediary therefore consists only of the second claim discussed in Section III.B.3 and resembles long-term debt.

B. Noncontractible Cash Flows

So far we have assumed that actual cash flows dX_t are publicly observed and contractible. Depending on the model application, however, it might be more realistic to assume that $dX_t - dw_t^A$, that is, cash flows net of payouts to the agent, are observed by the principal and contractible between the principal and the intermediary. We now assume that under these circumstances, the intermediary could "secretly" pay the manager $\Delta_t^w dt$ dollars with $\Delta_t^w \geq 0$. Setting $\Delta_t^w > 0$ reduces the agent's continuation payoff by $\Delta_t^w dt$ and reduces cash flows observed by the principal by Δ_t^w. The principal attributes the lower net cash flow realization to adverse cash flow shocks. Notably, deviations in pay to the agent must be of order dt, as otherwise they could be detected by the principal observing $dX_t - dw_t^A$. Also note that setting $\Delta_t^w > 0$ can be interpreted as a diversion from the cash flows, which increases managerial payouts.

To characterize the intermediary's incentives not to secretly pay the agent, recall that the principal's continuation payoff follows (9). Suppose now that the intermediary deviates and secretly pays the agent a dollar, $1dt$, which decreases v^A by $1dt$ and changes total firm value by $-F'(v^A)dt$. This deviation reduces cash flows observed by the principal by $1dt$ and thus reduces the principal's payoff by $-\beta^P dt$ dollars, as the principal's exposure to observed cash flows is β^P. Thus, if the intermediary deviates to pay the agent an additional dollar, the principal will bear the marginal cost of β^P dollars of this extra compensation. As such, the intermediary does not deviate if and only if $1-\beta^P \geq -F'(v^A)$, or[19]

$$F'(v^A) \geq -1 + \beta^P. \tag{25}$$

[19] To make this argument more formal, define $d\hat{X}_t = dX_t - \Delta_t^w dt$ and note that the principal observes $d\hat{X}_t$ but not dX_t; $d\hat{X}_t$ but not dX_t is contractible between the principal and the intermediary. It can be then shown that the principal's continuation payoff evolves, similar to (9), according to

$$dv_t^P = rv_t^P - dw_t^P + \beta_t^P \left(d\hat{X}_t - \bar{a}_t^A dt - \bar{a}_t^I dt \right).$$

Thus, setting $\Delta_t^w = 1$ reduces $d\hat{X}_t$ by $1dt$ and, ceteris paribus, leads $dv_t^P + dw_t^P$ to fall by $\beta_t^P dt$. That is, $\frac{\partial}{\partial \Delta_t^w}(dv_t^P + dw_t^P) = -\beta_t^P dt$. For the intermediary's instantaneous change in payoff, we have $\frac{\partial}{\partial \Delta_t^w}(dX_t - dw_t^A - dw_t^P + dv_t^I) = -F'(v_t^A)dt - 1dt + \beta_t^P dt$, using $dv_t^I = dF(v_t^A) - dv_t^P$, $\frac{\partial}{\partial \Delta_t^w} dw_t^A = 1dt$ and $\frac{\partial}{\partial \Delta_t^w} dF(v_t^A) = -F'(v_t^A)dt$. Setting $\frac{\partial}{\partial \Delta_t^w}(dX_t - dw_t^A - dw_t^P + dv_t^I) = -F'(v_t^A)dt - 1dt + \beta_t^P dt \leq 0$ to ensure that the intermediary optimally chooses $\Delta_t^w = 0$ yields (25).

This constraint reveals that the temptation of the intermediary to secretly pay the agent is not very strong. Note that in the baseline model, $F'(v^A) \geq -1$ holds for all v^A, so (25) is a weaker constraint whenever $\beta^P < 0$. Notably, we verify numerically that in our baseline solution, this constraint is slack for all values $v^A \in [0, \overline{v}]$. Thus, the incentive constraint (25) is payoff-irrelevant under our baseline parameters.

C. Commitment and Renegotiation

Our analysis assumes that the intermediary can commit to long-term contracts (subject to limited commitment with respect to contracts falling behind the value of the outside option). The optimal long-term contract between the intermediary and the agent maximizes the ex ante value by providing incentives via termination, but it leaves the temptation to renegotiate ex post. In particular, as $F'(v^A) > 0$ for low values of v^A, it would be ex post optimal for the intermediary to renegotiate to raise v^A, thereby making both the intermediary and the agent better off. DeMarzo and Sannikov (2006) demonstrate how to construct a renegotiation-proof long-term contract when renegotiation is costless. This renegotiation-proof contract involves randomized termination, features a weakly decreasing value function, and transfers all surplus to the agent. The renegotiation-proof contract has some extreme characteristics. For instance, the intermediary, despite being the residual claimant, is never negatively affected by distress or liquidation, cannot obtain any surplus ex ante, and never benefits from good performance. It therefore seems reasonable to assume, as we do in the baseline model, that the intermediary will find mechanisms to commit to long-term contracts or make renegotiation costly. As long as renegotiation entails some costs, our model results are likely to carry through in a qualitative sense, in that we obtain a concave value function that is upward-sloping on some region in the state space.

Finally, note that many of our results are driven by the fact that the slope of the value function decreases after good performance (i.e., concavity of the value function), but do not strictly rely on the fact that the value function is upward-sloping. In other words, many results derive from the change but not the level of the slope of the value function. For instance, the trickle-up effect leads to severe disincentives due to agency overhang when $v^A = \overline{v}$ is large and $F'(v^A) = -1$, but implies higher intermediary incentives when v^A is lower and $F'(v^A)$ is higher (but not necessarily positive) due to concavity. Similarly, the result that the agent's incentives increase following good performance derives from the fact that the value function's concavity vanishes near the boundary. Nevertheless, it remains the case that our results would be weakened if the value function were only downward-sloping.

D. Risk Aversion

The Internet Appendix studies the impact of the agent's and the intermediary's risk aversion. In the case of constant absolute risk-aversion (CARA) utility functions, the model remains tractable and the optimal contracts are qualitatively comparable to the risk-neutral case.

V. Delegated Contracting and Alternative Model Applications

Our baseline model studies a contracting problem in which a "central intermediary" contracts with both a principal and an agent. Note that from a theory point of view, it is irrelevant whether the principal offers a contract Π^I to the intermediary and collects the firm's cash flows or the intermediary offers a contract to the principal Π^P and collects the firm's cash flows. The reason is that both the principal and the intermediary would like to minimize agency costs and maximize firm value. In particular, an equivalent formulation of the contracting problem is that the principal collects the firm's cash flows dX_t and offers the intermediary a contract $\Pi^I = (w^I, \bar{w}^A, \bar{a}^A, \bar{a}^I)$, stipulating cumulative (net) payouts w^I to the intermediary and recommended payouts $\bar{w}^A$ to the agent, while the intermediary offers a contract $\Pi^A = (w^A, \hat{a}^A)$ that stipulates recommended effort $\hat{a}^A$ and payouts w^A. Specifically, with $\Pi^P = (w^P, \bar{a}^A, \bar{a}^I)$ and $\Pi^A = (w^A, \hat{a}^A, \hat{a}^I)$ solving the intermediary's problem (3), one can define $\Pi^I = (w^I, w^A, \bar{a}^A, \bar{a}^I)$ with $dw_t^I = dX_t - dw_t^P - dw_t^A$. Then, Π^I is the optimal contract that the principal offers to the intermediary under the alternative formulation of the problem, while the agent is offered the contract Π^A. Hence, our model describes *delegated contracting*: The principal contracts with the intermediary and the intermediary contracts with the agent, so the principal effectively delegates contracting with the agent to the intermediary.

Our baseline model features an intermediary who continuously injects funds into the firm. This is a plausible assumption in the application to PE, and it allows us to isolate the incentive motive for external financing. Nevertheless, in other applications, intermediaries tend not to inject funds into the firm frequently. In one such important application, the intermediary represents a board of directors. To demonstrate that the model can be consistent with the board of directors application, we show that the optimal contracts can implement nonnegative payouts to the intermediary, in the sense that in optimum $dw_t^I = dX_t - dw_t^A - dw_t^P \geq 0$ at all times $t > 0$, while the intermediary possibly injects funds at time $t = 0$ at the initial round of financing (i.e., $dw_0^I < 0$). To do so, we build on the optimal contract of the baseline model and its feature that the total value function $F(v^A)$ is invariant to the composition of the intermediary and principal values. Specifically, $F(v^A)$ (total firm value net of payouts to the agent) equals $v^I + v^P$ by definition, but does not depend separately on v^I and v^P. We can then construct a contract that implements payouts to the intermediary and the principal such that the intermediary's payouts remain positive while not changing total firm value and its dynamics as well as the dynamics of incentives stipulated in Proposition 1. Under these circumstances, the intermediary effectively intermediates funds from the principal to the firm and the agent. The details of the solution of the above implementation of payouts are presented in Internet Appendix.

In what follows, we discuss two applications that fit the delegated contracting framework as well as related empirical implications.

A. Board of Directors as Intermediary and Say on Pay

In their traditional roles, shareholders delegate to boards both monitoring of and contracting with firm managers, as in the delegated contracting interpretation of the baseline model. When the intermediary is the board of directors, the model predicts that directors' incentives and monitoring activity (as captured by effort a^I) are high when the firm experiences financial distress and are low after good past performance. The traditional roles have changed with the adoption of various say-on-pay regulations, which increased shareholders' direct participation in arranging executive compensation. These regulations have shifted the shareholders-board-manager relationship toward the alternative setting of our model studied in Section III.C with observable and contractible contract terms Π^A in which the principal (i.e., shareholders) determines the agent's (i.e., manager's) compensation directly.

Adapting the results of Section III.C, the model has several implications with respect to say-on-pay regulations. First, say-on-pay regulations increase managerial incentives but decrease board members' incentives and monitoring activity under financial distress. With a decreased influence on managerial contracts, boards will also have a diminished monitoring function in distressed firms. Second, there is little effect on optimal incentives after strong past performance when agency frictions are temporally small. This implies that the benefit of more direct contracting under say-on-pay regulations accrues to firms in distress when the agency frictions are most severe. The predicted effects on managerial compensation are consistent with empirical evidence: Pay-on-say adoption increases pay-for-performance (Correa and Lel (2016), Iliev and Vitanova (2019)), in particular, it increases sensitivity of pay to poor realizations of performance (Ferri and Maber (2013), Alissa (2015)).

B. Hierarchical Agency within the Firm

We can also use our model to analyze hierarchical agency problems within the firm. In this context, the principal represents the firm's investors, the intermediary represents the firm's manager or CEO, and the agent represents a division or operations manager. Interpreted broadly, the agent's effort and incentives quantify the level of decision authority that the CEO or headquarters delegates to the division manager.

The model then predicts that the CEO possesses strong incentives when the firm experiences financial distress, while the CEO has lower incentives and "enjoys the quiet life" after good performance. When enjoying the quiet life, the CEO provides strong incentives to subordinate operations and division managers. In other words, the CEO tends to delegate more decision authority and tasks after good performance, while she takes a more hands-on approach and tends to delegate less when facing distress. In addition, the model results imply that owners' (the principal's) exposure to firm risk decreases after good performance and therefore is highest under firm distress. As we have argued, such a risk-sharing agreement is consistent with debt financing. Below we lay out how our model predictions relate to empirical studies on hierarchical agency within a firm and internal capital markets.

The severity of the division manager (agent) is high when v^A is low and the firm is in agency-induced stress. As such, v^A serves as a measure of the severity of (dynamic) agency, and the severity of agency conflicts (i.e., v^A) decreases following poor performance. In this context, our model predicts that the CEO tends to delegate less decision authority to the division manager when agency conflicts on the level of the division manager are severe and contracted effort a^A is low. This prediction is consistent with Hoang, Gatzer, and Ruckes (2021), who document that firms are relatively more likely to impose divisional spending limits that effectively restrict the division manager's control and that divisional budgets are smaller if agency problems at the divisional level are severe. Similarly, Graham, Harvey, and Puri (2015) find that delegation of decision authority to the division manager, as captured in our model by effort a^A and incentives β^A, and allocated capital tend to increase with the division manager's past performance.

In addition, our model predicts that the CEO tends to engage more in firm operations (i.e., the CEO has higher incentives to exert) after poor performance, when agency conflicts of the division manager are severe and v^A is low. This prediction is also consistent with Hoang, Gatzer, and Ruckes (2021), who document that top management tends to affect the firm's investment and budgeting policies more when agency problems at the divisional level are severe.

According to our model, the strength of the CEO's and the division manager's incentives are inversely related over time in the sense that the agent (intermediary) has relatively strong incentives to exert effort after good (poor) performance. Put differently, the CEO induces less effort by the division manager precisely when the CEO's own incentives are strong, which helps explain why CEOs tend to delegate less when they receive more incentive pay (i.e., variable compensation) as documented by Graham, Harvey, and Puri (2015).

VI. Conclusion

Financial intermediaries, such as PE funds, create value in portfolio firms as they affect corporate governance, monitor management's activities, and actively seek to improve firm operations. However, these active intermediaries are subject to agency problems of their own. To understand the complex agency problems inherent to active intermediation as well as their effects on governance, operating, and financing decisions, we analyze a dynamic agency model in which an active intermediary raises funds from outside investors and invests in a firm run by an agent. In our model, the intermediary affects firm performance by (i) determining the agent's contract terms (i.e., governance engineering), (ii) exerting effort herself (i.e., operational engineering), and (iii) contracting with outside investors and seeking outside financing (i.e., financial engineering). As such, our model provides a unifying framework to evaluate the effects of operational, governance, and financial engineering in PE financing.

The intermediary's and the agent's incentive problems are endogenously linked via trickle-down and trickle-up effects. In particular, the intermediary passes part of his incentives through to the agent, in that incentives trickle down from the intermediary to the agent. In addition, the agent's incentives also trickle up and affect intermediary incentives. We find that the intermediary's incentives to affect firm performance are strongest after poor performance, while the agent's incentives are strongest after good performance. Thus, the model helps explain why PE sponsors engage in operational engineering most when portfolio firms are in distress. Importantly, the agent's incentives are convex in that they increase following good performance, but the intermediary's incentives are not. The financing contract between the intermediary and outside investors, that is, financial engineering, facilitates more efficient incentive provision by reducing the intermediary's incentives after poor performance but increasing them after good performance. We show that financial engineering grants the intermediary levered exposure to firm performance and involves debt-like financing provided by the principal.

Our framework is sufficiently tractable and can be employed to analyze more involved moral hazard problems that arise within the triangular investors-intermediary-manager relationship. For instance, we show how our model can be applied to study the complex agency conflicts between a firm's shareholders, the board of directors, and management. In this context, the model can be used to assess the effects of say-on-pay regulations. Our framework also describes hierarchical agency conflicts within a firm, for instance, between a firm's investors, the CEO, and division managers. Future work can also extend the model to include institutional details from other specific intermediation settings.

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Appendix A: Preliminaries

Uncertainty is modeled via the complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ that satisfies the usual conditions and is equipped with the filtration $\mathbb{F} := (\mathcal{F}_t)_{t\geq 0}$. Here, $\mathcal{F}_t = \sigma(X_s : s \leq t)$ is the public information that is available at time t to all players, as all players observe cash flow realizations dX_t. Below, we work with four different probability measures, denoted by $\mathbb{P}$ and $\mathbb{P}^k$ for $k = A, I, P$. The expectation $\mathbb{E}_t$ is taken under the probability measure $\mathbb{P}$ conditional on time t information. The expectation $\mathbb{E}_t^k$ is taken under the probability measure $\mathbb{P}^k$ conditional on time t information. We discuss the four probability measures in more detail.

1. The measure $\mathbb{P}^A$ is induced by the efforts a^A and $\hat{a}^I$, so that $dZ_t^A = \frac{dX_t - a_t^A dt - \hat{a}_t^I dt}{\sigma}$ is the increment of a standard Brownian Motion under the

measure $\mathbb{P}^A$. Note that the agent observes her own effort a_t^A and the prescribed effort $\hat{a}_t^I$ via the contract Π^A, so the expectation $\mathbb{E}_t^A$ is taken under (i.e., conditional on) the agent's information.

2. The measure $\mathbb{P}^I$ is induced by the efforts $\hat{a}^A$ and a^I, so that $dZ_t^I = \frac{dX_t - \hat{a}_t^A dt - a_t^I dt}{\sigma}$ is the increment of a standard Brownian Motion under the measure $\mathbb{P}^I$. Note that the intermediary observes his own effort a_t^I and the prescribed effort $\hat{a}_t^A$ via the contract Π^A, so the expectation $\mathbb{E}_t^I$ is taken under (i.e., conditional on) on the intermediary's information.
3. The measure $\mathbb{P}^P$ is induced by the efforts $\bar{a}^A$ and $\bar{a}^I$, so that $dZ_t^P = \frac{dX_t - \bar{a}_t^A dt - \bar{a}_t^I dt}{\sigma}$ is the increment of a standard Brownian Motion under the measure $\mathbb{P}^P$. Note that the principal observes the prescribed efforts $\bar{a}_t^A, \bar{a}_t^I$ via the contract Π^P, so the expectation $\mathbb{E}_t^P$ is taken under (i.e., conditional on) the principal's information.
4. The measure $\mathbb{P}$ is induced by the efforts $\hat{a}^A$ and $\hat{a}^I$, so that $d\hat{Z}_t = \frac{dX_t - \hat{a}_t^A dt - \hat{a}_t^I dt}{\sigma}$ is the increment of a standard Brownian Motion under the measure $\mathbb{P}$. Note that the contract Π^A stipulates prescribed efforts $\hat{a}_t^j$ for $j = A, I$, so the expectation $\mathbb{E}_t$ is taken under (i.e., conditional on) public information $\mathcal{F}_t$ and the contract terms Π^A.

We focus on incentive-compatible contracts Π^A and Π^P, in that (in optimum) $a^j = \bar{a}^j = \hat{a}^j$ for $j = A, I$ and $\mathbb{P}$ is the probability measure induced by the efforts a^A and a^I. Note that in optimum, $\mathbb{P}$ coincides with $\mathbb{P}^k$, as prescribed and actual effort levels coincide. In the main text, we do not formally distinguish between prescribed and actual effort levels and carry out the arguments using the equilibrium probability measure $\mathbb{P}$. That is, in the main text, we do not formally distinguish between the different probability measures ($\mathbb{P}$, $\mathbb{P}^k$).

Throughout the paper and in all problems, we stipulate that the sensitivities β_t^k for $k \in \{A, I, P\}$, implicitly defined in (7) and (9), are bounded. That is, $|\beta_t^k| < M$ for all $t \geq 0$ and $M > 0$. This is merely a regularity condition, used in the verification argument, and we can pick $M < \infty$ sufficiently large to ensure that this constraint never binds in optimum.

Appendix B: Proof of Lemma 1

Follows from the arguments in the main text.

Appendix C: Proof of Lemma 2

A.1. Part I—Martingale Representation

Take player $j's$ continuation utility

$$v_t^j = \mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)}\left(dw_s^j - g^j\left(a_s^j\right) ds\right)\right]$$

evaluated under the measure $\mathbb{P}$, so the expectation $\mathbb{E}_t$ is taken under public information $\mathcal{F}_t$ and the contract terms Π^A. Define

$$A_t^j = \mathbb{E}_t\left[\int_0^\infty e^{-rs}\left(dw_s^j - g^j\left(a_s^j\right)ds\right)\right] = \int_0^t e^{-rs}\left(dw_s^j - g^j\left(a_s^j\right)ds\right) + e^{-rt}v_t^j. \quad \text{(C1)}$$

By construction, $A^j = \{A_t^j\}$ is a martingale under the probability measure $\mathbb{P}$. By the martingale representation theorem, there exists a stochastic process $\beta^j = \{\beta_t^j\}$ such that

$$e^{rt}dA_t^j = \beta_t^j\left(dX_t - \hat{a}_t^A dt - \hat{a}_t^I dt\right), \quad \text{(C2)}$$

where $d\hat{Z}_t = \frac{dX_t - \hat{a}_t^A dt - \hat{a}_t^I dt}{\sigma}$ is the increment of a standard Brownian Motion under the probability measure $\mathbb{P}$.

We differentiate (C1) with respect to time t to obtain an expression for dA_t^j, then plug this expression into (C2) and solve (C2) to get

$$dv_t^j = rv_t^j dt + g^j\left(a_t^j\right)dt - dw_t^j + \beta_t^j\left(dX_t - \hat{a}_t^A dt - \hat{a}_t^I dt\right), \quad \text{(C3)}$$

which is (7).

Next, we consider the principal's continuation payoff

$$v_t^P := \mathbb{E}_t^P\left[\int_t^\infty e^{-r(s-t)}dw_s^P\right] \quad \text{(C4)}$$

evaluated under the measure $\mathbb{P}^P$ that is taken under (i.e., conditional on) the principal's information. Define

$$A_t^P = \mathbb{E}_t^P\left[\int_0^\infty e^{-rs}dw_s^P\right] = \int_0^t e^{-rs}dw_s^P + e^{-rt}v_t^P. \quad \text{(C5)}$$

By construction, $A^P = \{A_t^P\}$ is a martingale under the probability measure $\mathbb{P}^P$. By the martingale representation theorem, there exists a stochastic process $\beta^P = \{\beta_t^P\}$ such that

$$e^{rt}dA_t^P = \beta_t^P\left(dX_t - \bar{a}_t^A dt - \bar{a}_t^I dt\right), \quad \text{(C6)}$$

where $dZ_t^P = \frac{dX_t - \bar{a}_t^A dt - \bar{a}_t^I dt}{\sigma}$ is the increment of a standard Brownian motion under the probability measure $\mathbb{P}^P$. We differentiate (C5) with respect to time t to obtain an expression for dA_t^P, then plug this expression into (C6) and solve (C6) to get

$$dv_t^P = rv_t^P dt - dw_t^P + \beta_t^P\left(dX_t - \bar{a}_t^A dt - \bar{a}_t^I dt\right),$$

which is (9).

A.2. Part II—Optimal Effort

We denote player j's proposed effort level by $\hat{a}_t^j$ and player j's actual effort level by a_t^j. Under player j's proposed effort level $\hat{a}_t^j$, player j's continuation payoff reads

$$v_t^j = \mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)}\left(dw_s^j - g^j\left(\hat{a}_s^j\right)ds\right)\right]. \tag{C7}$$

According to (7), with $g^j(a_t^j)$ replaced by $g^j(\hat{a}_t^j)$, we obtain the following dynamics of v_t^j:

$$dv_t^j = rv_t^j dt + g^j\left(\hat{a}_t^j\right)dt - dw_t^j + \beta_t^j\left(dX_t - \hat{a}_t^A dt - \hat{a}_t^I dt\right). \tag{C8}$$

By the dynamic programming principle, player j chooses effort a_t^j to maximize her expected change in payoff minus the cost of exerting effort, that is, at any point in time t, player j solves:

$$\max_{a_t^j\in[0,A]} \mathbb{E}_t^j\left[dv_t^j + dw_t^j - g^j\left(a_t^j\right)dt\right], \tag{C9}$$

where $dv_t^j + dw_t^j$ is characterized in (C8).

Note that the expectation $\mathbb{E}_t^j$ is taken under probability measure $\mathbb{P}^j$ (which is induced by efforts $(a^A, \hat{a}^I)$ if $j = A$, and by efforts $(\hat{a}^A, a^I)$ if $j = I$). As a result, we have

$$\mathbb{E}_t^A\left[dX_t - \hat{a}_t^A dt - \hat{a}_t^I dt\right] = \left(a_t^A - \hat{a}_t^A\right)dt, \tag{C10}$$

$$\mathbb{E}_t^I\left[dX_t - \hat{a}_t^A dt - \hat{a}_t^I dt\right] = \left(a_t^I - \hat{a}_t^I\right)dt. \tag{C11}$$

Using (C8), we can rewrite (C9) as

$$\max_{a_t^j\in[0,A]} \mathbb{E}_t^j\left[rv_t^j dt + g^j\left(\hat{a}_t^j\right)dt + \beta_t^j\left(dX_t - \hat{a}_t^A dt - \hat{a}_t^I dt\right) - g^j\left(a_t^j\right)dt\right]. \tag{C12}$$

For $j = A$, we can use (C10) to write (C12) as

$$\max_{a_t^A\in[0,A]} \left[rv_t^A dt + g^A\left(\hat{a}_t^A\right)dt + \beta_t^A\left(a_t^A - \hat{a}_t^A\right)dt - g^A\left(a_t^A\right)dt\right], \tag{C13}$$

where $g^A(a_t^A) = \frac{1}{2}\delta(a_t^A)^2$. Provided $a_t^A \in (0, A)$, optimal effort a_t^A must solve the first-order condition

$$\beta_t^A = \frac{\partial}{\partial a_t^A} g^A\left(a_t^A\right) \iff a_t^A = \frac{\beta_t^A}{\delta}.$$

The second-order condition for the optimization is (C13) is $-\frac{\partial^2}{\partial(a_t^A)^2}g^A(a_t^A) > 0$, so that the first-order condition is sufficient, and $a_t^A = \frac{\beta_t^A}{\delta}$ is the optimal effort level for the agent. Incentive compatibility then requires $a_t^A = \hat{a}_t^A = \frac{\beta_t^A}{\delta}$, as stated in Lemma 2.

Likewise, for $j = I$, we can use (C10) to write (C12) as

$$\max_{a_t^I \in [0,A]} \left[r v_t^I dt + g^I\left(\hat{a}_t^I\right) dt + \beta_t^I \left(a_t^I - \hat{a}_t^I\right) dt - g^I\left(a_t^I\right) dt \right], \tag{C14}$$

where $g^I(a_t^I) = \frac{1}{2}\lambda(a_t^I)^2$. Provided $a_t^I \in (0, A)$, optimal effort a_t^I must solve the first-order condition

$$\beta_t^I = \frac{\partial}{\partial a_t^I} g^I\left(a_t^I\right) \iff a_t^I = \frac{\beta_t^I}{\lambda}.$$

The second-order condition for the optimization is (C14) is $-\frac{\partial^2}{\partial(a_t^I)^2}g^I(a_t^I) > 0$, so that the first-order condition is sufficient, and $a_t^I = \frac{\beta_t^I}{\lambda}$ is the optimal effort level for the agent. Incentive compatibility then requires $a_t^I = \hat{a}_t^I = \frac{\beta_t^I}{\lambda}$, as stated in Lemma 2.

Appendix D: Proof of Lemma 3

Lemma 3—which states standard results—readily follows from the arguments presented in the main text.

Appendix E: Proofs of Lemma 4 and Proposition 1

Lemma 3, Lemma 4, and Proposition 1 jointly describe the solution to the intermediary's problem (3). In this section, we characterize the solution to the intermediary's problem (3) and thereby prove the claims of Lemma 4 and Proposition 1. The argument is split into several parts.

Part I puts structure on the intermediary's dynamic optimization problem (3) and argues that, by the dynamic programming principle, the intermediary's objective function (i.e., value function) solves the HJB equation (E7), which is a partial differential equation (PDE). Part II simplifies the dynamic optimization problem by reducing its dimensionality. In particular, Part II shows that the intermediary's objective can be characterized as a solution to an ODE. Part III characterizes the intermediary's incentives and payouts to the agent and establishes the relationship (15). Part IV characterizes the agent's optimal incentives, proving the claims of Lemma 4. Part V characterizes optimal risk-sharing with the principal and, as such, the intermediary's optimal incentives, establishing (21). Part VI proves the concavity of the value function and characterizes the payout boundary. Part VII provides the formal verification argument that the contracts Π^A and Π^P from Proposition 1 are indeed optimal and solve the intermediary's problem (3). Part VIII characterizes optimal firm liquidation.

A.1. Part I

The intermediary chooses effort a^I and the contracts Π^A, Π^P to solve (3) and to maximize his expected lifetime utility v_0^I. Recall that by Lemma 2, efforts a_t^A, a_t^I satisfy the incentive condition(s) (11) under any incentive compatible contracts Π^A, Π^P.

At time $t = 0$, the intermediary chooses contracts Π^A, Π^P to (dynamically) maximize

$$v_{0^-}^I = (-dw_0^P) + \left(-dw_0^A\right) + \underbrace{\mathbb{E}_0^I\left[\int_0^\infty e^{-rt}\left(dw_t^I - g^I\left(a_t^I\right)dt\right)\right]}_{=v_0^I}, \tag{E1}$$

where dw_0^P and dw_0^A denote initial lump-sum payments to the principal and the agent, respectively (i.e., if $dw_0^P < 0$, the intermediary receives a lump-sum payment of $(-dw_0^P) > 0$ dollars from the principal at time $t = 0$). To distinguish between the time before and after such lump-sum payments, we introduce time $t = 0^-$, where $t = 0^-$ denotes the time before these lump-sum payments are made and time $t = 0$ denotes the time "just after" these lump-sum payments are made.[20]

As the agent cannot be paid negative wages at any time, it follows that $dw_0^A = 0$. Since the principal has full bargaining power and can extract all surplus from the principal, the principal merely breaks even so that

$$-dw_0^P = \mathbb{E}_0^P\left[\int_0^\infty e^{-rt}dw_t^P\right] = v_0^P. \tag{E2}$$

As a result, the intermediary's total expected payoff reads

$$\begin{aligned} v_0^I + v_0^P &= \mathbb{E}_0^I\left[\int_0^\infty e^{-rt}\left(dX_t - dw_t^A - dw_t^P - g^I\left(a_t^I\right)dt\right)\right] + \mathbb{E}_0^P\left[\int_0^\infty e^{-rt}dw_t^P\right] \\ &= \mathbb{E}_0^I\left[\int_0^\infty e^{-rt}\left(dX_t - dw_t^A - g^I\left(a_t^I\right)dt\right)\right] + \Delta_0, \end{aligned} \tag{E3}$$

where we define

$$\Delta_t := \mathbb{E}_t^P\left[\int_0^\infty e^{-r(s-t)}dw_s^P\right] - \mathbb{E}_t^I\left[\int_0^\infty e^{-r(s-t)}dw_s^P\right].$$

Define

$$\tilde{v}_t^P = \mathbb{E}_t^I\left[\int_t^\infty e^{-r(s-t)}dw_s^P\right],$$

[20] To reduce notation, we do not introduce time $t = 0^-$ in the main text, and do not formally distinguish between time $t = 0$ and time $t = 0^-$.

which is the principal's expected payoff at time t evaluated under the intermediary's probability measure $\mathbb{P}^I$ induced by efforts $\hat{a}^A$ and a^I. Similar to (9), we obtain that $\tilde{v}_t^P$ has the dynamics

$$d\tilde{v}_t^P = r\tilde{v}_t^P dt - dw_t^P + \beta_t^P \left(dX_t - \hat{a}_t^A dt - a_t^I dt \right),$$

where $\frac{dX_t - \hat{a}_t^A dt - a_t^I dt}{\sigma}$ is the increment of a standard Brownian motion under the probability measure $\mathbb{P}^I$ induced by efforts $\hat{a}^A$ and a^I. Note that $\Delta_t = v_t^P - \tilde{v}_t^P$.

Recall that $v_t^P = \mathbb{E}_t^P[\int_t^\infty e^{-r(s-t)} dw_s^P]$ has the dynamics (9), where $\frac{dX_t - \bar{a}_t^A dt - \bar{a}_t^I dt}{\sigma}$ is the increment of a standard Brownian Motion under the probability measure $\mathbb{P}^P$.[21] As a result,

$$d\Delta_t = dv_t^P - d\tilde{v}_t^P = r\Delta_t dt + \beta_t^P \left(\hat{a}_t^A + a_t^I - \bar{a}_t^A - \bar{a}_t^I \right) dt, \tag{E4}$$

where we use $\Delta_t = v_t^P - \tilde{v}_t^P$. In optimum and under incentive compatible contracts, the probability measures $\mathbb{P}^I$ and $\mathbb{P}^P$ coincide, leading to $v_t^P = \tilde{v}_t^P$ and therefore $\Delta_t = 0$. We can integrate (E4) over time to obtain

$$\Delta_t = \int_t^\infty e^{-r(s-t)} \beta_s^P \left[\bar{a}_s^A + \bar{a}_s^I - \hat{a}_s^A - a_s^I \right] ds. \tag{E5}$$

Because the principal's payoff at time $t = 0$ satisfies (E3), the principal dynamically maximizes $v_t^I + v_t^P$, which we refer to as "total firm value" (net of the payouts to the agent). The dynamic optimization of $v_t^I + v_t^P$ depends on two state variables: (i) the principal's continuation payoff v_t^P and (ii) the agent's continuation utility v_t^A. As a result, using (E5) and $g^I(a_s^I) = \frac{\lambda (a_s^I)^2}{2}$, we can write the time-$t$ value of the objective

$$\begin{aligned} v_t^I + v_t^P &= \mathbb{E}_t^I \left[\int_t^\infty e^{-r(s-t)} \left(dX_s - dw_s^A - \frac{\lambda \left(a_s^I\right)^2}{2} ds \right) \right] + \Delta_t \\ &= \mathbb{E}_t^I \left[\int_t^\infty e^{-r(s-t)} \left(dX_s - dw_s^A - \frac{\lambda \left(a_s^I\right)^2}{2} ds + \beta_s^P \left[\bar{a}_s^A + \bar{a}_s^I - \hat{a}_s^A - a_s^I \right] ds \right) \right] \end{aligned} \tag{E6}$$

as function of v_t^A, v_t^P, in that $v_t^I + v_t^P = \tilde{F}(v_t^A, v_t^P)$. In the following, we suppress time subscripts and the argument of the function $\tilde{F}(\cdot)$ whenever confusion is not likely to arise.

[21] That is, by (9), we have $dv_t^P = rv_t^P dt + \beta_t^P (dX_t - \bar{a}_t^A dt - \bar{a}_t^I dt)$.

By the dynamic programming principle and the representation of total firm value $\tilde{F}$ in (E6), the function $\tilde{F}$ solves the HJB equation

$$r\tilde{F}dt = \max_{\beta^A,\beta^I,\beta^P,dw^P,dw^A\geq 0}\left\{\mathbb{E}^I[dX - dw^A] + \beta^P\Big[\bar{a}^A + \bar{a}^I - \hat{a}^A - a^I\Big]dt - \frac{\lambda\left(a^I\right)^2}{2}dt + \mathbb{E}^I[d\tilde{F}]\right\}, \tag{E7}$$

subject to incentive compatibility with respect to effort, (11), and the limited liability and commitment constraints (see Lemma 1). The term $\mathbb{E}^I[d\tilde{F}]$ can be expanded using Itô's Lemma, which yields that (E7) is a PDE. The expectation $\mathbb{E}^I$ is taken under the measure $\mathbb{P}^I$, induced by efforts $\hat{a}^A$ and a^I. That is, the expectation $\mathbb{E}^I$ is taken under the intermediary's information.

Providing a formal existence and uniqueness proof for a solution to (E7) is beyond the scope of the paper. Therefore, make the following assumption throughout the remainder of the paper.

ASSUMPTION E.1 (Existence, uniqueness, and smoothness): *The PDE (E7) admits a unique solution $\tilde{F}$ that is twice continuously differentiable.*

A.2. Part II

In this part, we conjecture and verify that $\tilde{F}(v^A, v^P)$ takes the form $\tilde{F}(v^A, v^P) = F(v^A)$ with a (twice continuously differentiable) function $F(v^A)$. To start, first recall that the expectation $\mathbb{E}^I$ is taken under the probability measure $\mathbb{P}^I$ (induced by efforts $\hat{a}^A$ and a^I) and hence is taken under the intermediary's information. We therefore have

$$\begin{aligned}
&\mathbb{E}^I[dX - \hat{a}^A dt - a^I dt] = 0,\\
&\mathbb{E}^I[dX - \hat{a}^A dt - \hat{a}^I dt] = \left(a^I - \hat{a}^I\right)dt,\\
&\mathbb{E}^I[dX - \bar{a}^A dt - \bar{a}^I dt] = \left(\hat{a}^A + a^I - \bar{a}^A - \bar{a}^I\right)dt.
\end{aligned} \tag{E8}$$

Second, because payouts dw^A to the agent cannot become negative and reduce v^A by amount dw^A (see (7)), and because the agent's limited liability requires $v^A \geq 0$, it is natural to conjecture that optimal payouts occur at some upper boundary $\overline{v}$ and reflect v^A back into the interior of the state space, as in DeMarzo and Sannikov (2006). We therefore conjecture that as stipulated in Proposition 1, optimal payouts to the agent take the form $dw^A = \max\{v^A - \overline{v}, 0\}$, with endogenous payout boundary $\overline{v}$. At the payout boundary $v^A = \overline{v}$, the smooth-pasting condition

$$F'(\overline{v}) = -1$$

must hold, as shown, for example, in Dumas (1991) and DeMarzo and Sannikov (2006). We verify the optimality of this payout strategy in Part VII of the proof, where we verify that the contracts Π^A, Π^P from Proposition 1 are indeed optimal.

Third, consider $v^A < \overline{v}$, so $dw^A = 0$. Notice that according to (E8) and (7) (with incentive compatible effort $\hat{a}^A = a^A$),

$$\mathbb{E}^I[dv^A] = \left(rv^A + \frac{\delta\left(\hat{a}^A\right)^2}{2}\right)dt + \beta^A\left(a^I - \hat{a}^I\right)dt. \tag{E9}$$

In addition, the quadratic variation of dv^A, denoted by $[dv^A]^2$, equals $(\beta^A\sigma)^2$. Using the conjecture $\tilde{F}(v^A, v^P) = F(v^A)$, Itô's Lemma, and (E9), we get

$$\begin{aligned}\mathbb{E}^I[d\tilde{F}] &= \mathbb{E}^I[dF\left(v^A\right)] = F'\left(v^A\right)\mathbb{E}[dv^A] + \frac{F''\left(v^A\right)[dv^A]^2}{2}\\ &= \left[F'\left(v^A\right)\left(rv^A + \frac{\delta\left(\hat{a}^A\right)^2}{2}\right) + \frac{F''\left(v^A\right)\left(\beta^A\sigma\right)^2}{2}\right]dt + F'\left(v^A\right)\beta^A\left(a^I - \hat{a}^I\right)dt.\end{aligned} \tag{E10}$$

Inserting (E10) and $\tilde{F} = F(v^A)$ into (E7), noting that $\mathbb{E}^I[dX] = (\hat{a}^A + a^I)dt$, and simplifying yields

$$\begin{aligned}rF\left(v^A\right) = \max_{\beta^A,\beta^I,\beta^P}\Bigg\{&\left(\hat{a}^A + a^I\right)(1-\beta^P) + \beta^P\left(\bar{a}^A + \bar{a}^I\right) + F'\left(v^A\right)\beta^A\left(a^I - \hat{a}^I\right) - \frac{\lambda\left(a^I\right)^2}{2}\\ &+ F'\left(v^A\right)\left(rv^A + \frac{\delta\left(\hat{a}^A\right)^2}{2}\right) + \frac{F''\left(v^A\right)\left(\beta^A\sigma\right)^2}{2}\Bigg\}.\end{aligned} \tag{E11}$$

The ODE (E11) is solved on $[0, \overline{v}]$ subject to incentive compatibility with respect to effort, (11), and the limited liability and commitment constraints as well as $F'(\overline{v}) = -1$. Notably, the right-hand side of (E11) depends only on v^A, and so do the left-hand side and the optimal controls. In particular, the right-hand side does not depend on v^P or dw^P, and hence neither do the left-hand side and the optimal controls β^A, β^I, β^P. This confirms the conjecture $\tilde{F}(v^A, v^P) = F(v^A)$.

A.3. Part III

This part of the proof characterizes the intermediary's incentives for effort β^I (compare (11)) and then characterizes optimal payouts to the principal. First, note that $vol(dw^I + dw^A) = vol(dX) - vol(dw^P) = \sigma - vol(dw^P)$, where $vol(\cdot)$ denotes the volatility of a stochastic process and $dw^I = dX - dw^A - dw^P$. By (7), the volatility of $dv^I - dw^A$ is $\beta^I\sigma - vol(dw^I + dw^A)$, which can be written as $\beta^I\sigma - \sigma + vol(dw^P)$.

On the other hand, as $v^I = F(v^A) - v^P$, Itô's Lemma implies that the volatility of $dv^I - dw^A$ is $F'(v^A)\beta^A\sigma - vol(dv^P)$, where $vol(dv^P)$ is the volatility of v^P and $vol(dv^A + dw^A) = \beta^A\sigma$ from (7). Note that (9) implies $vol(dv^P) = \beta^P\sigma - vol(dw^P)$. Thus,

$$\beta^I\sigma - \sigma + vol\left(dw^P\right) = F'\left(v^A\right)\beta^A\sigma - vol\left(dv^P\right) = F'\left(v^A\right)\beta^A\sigma - \beta^P\sigma + vol\left(dw^P\right),$$

which can be rewritten as

$$\beta^I \sigma - \left(1 - \beta^P\right) \sigma = F'\left(v^A\right) \beta^A \sigma$$

and is equivalent to (15), as stated in Lemma 4.

Importantly, (15) is independent of the choice of dw^P and v^P. Likewise, the right-hand side of (E11) is independent of dw^P and v^P, meaning that the choice of dw^P and v^P does not directly affect the intermediary's dynamic optimization in the HJB equation (E11). As a result, the intermediary's payoff and the dynamic optimization depend on the contract Π^P and on v^P and dw^P only via the sensitivity β^P, which determines the principal's exposure to cash flow shocks.

A.4. Part IV and Proof of Lemma 4

Part IV of the proof maximizes the HJB equation (E11) with respect to β^A and hence characterizes the agent's optimal incentives, taking into account the effort incentive constraint (11) and the relationship (15).

Note that (11) and (15) imply

$$\hat{a}^I = a^I = \frac{1 - \beta^P + F'\left(v^A\right)\beta^A}{\lambda} = \frac{\beta^I}{\lambda}. \tag{E12}$$

In addition, one can verify that maximizing the right-hand side of (E11) over a^I yields the same expression for optimal effort a^I as in (E12). Plugging $a^I = \hat{a}^I = \beta^I/\lambda$ and $a^A = \hat{a}^A = \beta^A/\delta$ into (E11) yields

$$rF\left(v^A\right) = \max_{\beta^A, \beta^P, \beta^I} \left\{ \left(\frac{\beta^I}{\lambda} + \frac{\beta^A}{\delta}\right)\left(1 - \beta^P\right) + \beta^P\left(\bar{a}^I + \bar{a}^A\right) - \frac{\left(\beta^I\right)^2}{2\lambda} \right.$$
$$\left. + F'\left(v^A\right)\left(rv^A + \frac{\left(\beta^A\right)^2}{2\delta}\right) + \frac{F''\left(v^A\right)\left(\beta^A\sigma\right)^2}{2} \right\}, \tag{E13}$$

which is solved subject to the limited liability and commitment constraints and subject to (15). Due to (15), one of the controls $(\beta^A, \beta^I, \beta^P)$ is redundant. In particular, maximizing (E13) over (β^A, β^I) is equivalent to maximizing (E13) over (β^A, β^P). Also, note that the agent's incentives β^A and prescribed effort $\hat{a}^A$ are not observable to the principal and not contractible between the principal and the intermediary. Accordingly, when maximizing (E13) with respect to β^A, the intermediary takes the principal's contract Π^P and so effort levels $\bar{a}^A$, $\bar{a}^I$ and sensitivity β^P as given.

Noting that $a^A = \beta^A/\delta$ and $a^I = \beta^I/\lambda$, the maximization of (E13) with respect to β^A is equivalent to

$$\max_{\beta^A \geq 0} \left(\left(1 - \beta^P\right)\left(a^A + a^I\right) - \frac{\lambda\left(a^I\right)^2}{2} + F'\left(v^A\right)\left(rv^A + \frac{\delta\left(a^A\right)^2}{2}\right) + \frac{F''\left(v^A\right)\left(\beta^A\sigma\right)^2}{2} \right), \tag{E14}$$

which equals (16) as stipulated in Lemma 4.

Taking the first-order condition in (E13) (or equivalently (E14)) with respect to β^A and noting that $\frac{\partial \beta^I}{\partial \beta^A} = F'(v^A)$ (due to (15)) yields

$$\frac{1-\beta^P}{\delta} + \frac{F'\left(v^A\right)\left(1-\beta^P\right)}{\lambda} - \left(\frac{1}{\lambda}\right)\left(\underbrace{1-\beta^P + F'\left(v^A\right)\beta^A}_{=\beta^I}\right)F'\left(v^A\right)$$
$$+ F'\left(v^A\right)\beta^A\left(\frac{1}{\delta}\right) + F''\left(v^A\right)\sigma^2\beta^A = 0. \tag{E15}$$

We can solve (E15) to get

$$\beta^A = \left(1-\beta^P\right)\pi^I, \tag{E16}$$

with

$$\pi^I := \frac{1}{\frac{\delta}{\lambda}\left(F'\left(v^A\right)\right)^2 - F'\left(v^A\right) - \delta\sigma^2 F''\left(v^A\right)},$$

as stated in Lemma 4.

Note that (E16) and (11) jointly imply $a^A = \hat{a}^A = \frac{(1-\beta^P)\pi^I}{\delta}$. That is, in optimum (under incentive compatible contracts), it must hold that $\bar{a}^A = a^A = \hat{a}^A = \frac{(1-\beta^P)\pi^I}{\delta}$. Effectively, (E16) is the intermediary's incentive condition for implementing $\hat{a}^A = \bar{a}^A = a^A = \frac{(1-\beta^P)\pi^I}{\delta}$. As long as $\bar{a}^A = \frac{(1-\beta^P)\pi^I}{\delta}$ holds, the intermediary finds it optimal to implement $\bar{a}^A = \hat{a}^A$.

Finally, we rewrite (15) as

$$\beta^A F'\left(v^A\right) = \beta^I - \left(1-\beta^P\right) \tag{E17}$$

and multiply both sides of (E16) by $F'(v^A)$ to get

$$\beta^A F'\left(v^A\right) = \left(1-\beta^P\right)\pi^I F'\left(v^A\right). \tag{E18}$$

Combining (E17) and (E18) yields

$$\beta^I = \left(1 + F'\left(v^A\right)\pi^I\right)\left(1-\beta^P\right), \tag{E19}$$

which is (18) (as stated in Lemma 4). Next, we rewrite (E16) to $1-\beta^P = \frac{\beta^A}{\pi^I}$ and insert this expression into (E19) to get

$$\beta^A = \beta^I\left(\frac{\pi^I}{1+F'\left(v^A\right)\pi^I}\right), \tag{E20}$$

which is (19), as stated in Lemma 4.

A.5. Part V

Part V of the proof maximizes the HJB equation (E11) with respect to β^I (or equivalently β^P), taking into account the effort incentive constraint (11), the relationship (15), and the characterization of the agent's incentives (i.e., (19) or (E20)).

Plugging $\bar{a}^A = a^A = \beta^A/\delta$ and $\bar{a}^I = a^I = \beta^I/\lambda$ into (E13) gives

$$rF\left(v^A\right) = \max_{\beta^A,\beta^P,\beta^I}\left\{\left(\frac{\beta^I}{\lambda} + \frac{\beta^A}{\delta}\right) - \frac{\left(\beta^I\right)^2}{2\lambda} + F'\left(v^A\right)\left(rv^A + \frac{\left(\beta^A\right)^2}{2\delta}\right) + \frac{F''\left(v^A\right)\left(\beta^A\sigma\right)^2}{2}\right\}. \tag{E21}$$

The optimization in (E21) is solved subject to the "incentive" condition (E16) (ensuring $\hat{a}^A = \bar{a}^A$) and (15). Also recall that (E16) and (15) jointly imply (E20).

Note that we can use (11) to replace sensitivities by efforts and to rewrite (E21) as

$$rF\left(v^A\right) = \max_{\beta^A,\beta^P,\beta^I}\left\{a^A + a^I - \frac{\lambda\left(a^I\right)^2}{2} + F'\left(v^A\right)\left(rv^A + \frac{\delta\left(a^A\right)^2}{2}\right) + \frac{F''\left(v^A\right)\left(\beta^A\sigma\right)^2}{2}\right\},$$

which is (20) (after dropping control β^P which is possible due to (15)). This shows that under the optimal contracts, the intermediary's value function solves the HJB equation (20), as stipulated by Proposition 1.

Inserting (E16) and (E19) into (E21) changes (E21) to

$$\begin{aligned} rF\left(v^A\right) = \max_{\beta^P}\Bigg\{ &\left(1-\beta^P\right)\left(\frac{1+F'\left(v^A\right)\pi^I}{\lambda} + \frac{\pi^I}{\delta}\right) - \left(1-\beta^P\right)^2\left(1+F'\left(v^A\right)\pi^I\right)^2\left(\frac{1}{2\lambda}\right) \\ &+ F'\left(v^A\right)rv^A + \left(1-\beta^P\right)^2F'\left(v^A\right)\left(\pi^I\right)^2\left(\frac{1}{2\delta}\right) + \frac{\left(1-\beta^P\right)^2F''\left(v^A\right)\left(\pi^I\sigma\right)^2}{2}\Bigg\}, \end{aligned} \tag{E22}$$

where we have dropped the redundant controls β^A, β^I. We can solve the optimization in (E22) to get

$$1-\beta^P = \frac{\frac{1+F'\left(v^A\right)\pi^I}{\lambda} + \frac{\pi^I}{\delta}}{\left(\frac{1}{\lambda}\right)\left(1+F'\left(v^A\right)\pi^I\right)^2 - F'\left(v^A\right)\left(\pi^I\right)^2\left(\frac{1}{\delta}\right) - F''\left(v^A\right)\left(\pi^I\sigma\right)^2}, \tag{E23}$$

as stated in Proposition 1.

Using (E19), it follows that (E23) is equivalent to

$$\beta^I = \frac{\left(1+F'\left(v^A\right)\pi^I\right)\left(\frac{1+F'\left(v^A\right)\pi^I}{\lambda} + \frac{\pi^I}{\delta}\right)}{\left(\frac{1}{\lambda}\right)\left(1+F'\left(v^A\right)\pi^I\right)^2 - F'\left(v^A\right)\left(\pi^I\right)^2\left(\frac{1}{\delta}\right) - F''\left(v^A\right)\left(\pi^I\sigma\right)^2}, \tag{E24}$$

as stated in Proposition 1. Due to (19) (or (E20)), the agent's optimal incentives are then

$$\beta^A = \frac{\pi^I\left(\frac{1+F'(v^A)\pi^I}{\lambda} + \frac{\pi^I}{\delta}\right)}{\left(\frac{1}{\lambda}\right)\left(1+F'\left(v^A\right)\pi^I\right)^2 - F'\left(v^A\right)\left(\pi^I\right)^2\left(\frac{1}{\delta}\right) - F''\left(v^A\right)\left(\pi^I\sigma\right)^2}. \tag{E25}$$

To summarize, the sensitivities β^A, β^I, and β^P that solve the maximization in (E11) satisfy (E25), (E24), and (E23), respectively. The effort incentive constraints (11) map sensitivities β^A, β^I to effort levels a^A, a^I.

A.6. Part VI—Properties of the Value Function

As shown in the previous part of the proof, the value function $F(v^A)$ under the contracts from Proposition 1 solves the HJB equation (20) subject to $F'(\overline{v}) = -1$. We demonstrate that $F(v^A)$ is strictly concave, in that $F''(v^A) < 0$ for all $v^A \in [0, \overline{v}]$.

Note that by definition the optimal payout boundary satisfies

$$\overline{v} = \inf\{v^A \geq 0 : F'\left(v^A\right) \leq -1\}.$$

Otherwise, payouts would be optimal once v^A reaches some value $v' < \overline{v}$ with $F'(v') \leq -1$, contradicting the fact that $\overline{v}$ is the payout boundary. If now $F''(\overline{v}) > 0$, then the smooth pasting condition, $F'(\overline{v}) = -1$, implies that there exists $v' < \overline{v}$ with $F'(v') < -1$, a contradiction. Thus, $F''(\overline{v}) \leq 0$.

Next, using the envelope theorem, we differentiate the HJB equation (20) evaluated under the optimal controls to get

$$F'''\left(v^A\right) = \frac{2}{\left(\beta^A\sigma^2\right)} F''\left(v^A\right)\left(rv^A + \frac{\delta\left(a^A\right)^2}{2}\right). \tag{E26}$$

Suppose that $F''(\overline{v}) = 0$. Then, $F'''(\overline{v}) = 0$ while $F'(\overline{v}) = -1$. As a result, the solution to (20) is affine and takes the form $F(v^A) = K - v^A$ for some constant K. Thus, there exists $v' < \overline{v}$ with $F'(v') = -1$, a contradiction. It follows that $F''(\overline{v}) < 0$. Because (E26) implies that $F''(v^A) < 0 \iff F'''(v^A) > 0$, it follows that $F''(v^A) < 0$ for all $v^A \in [0, \overline{v}]$.

Finally, we show that the payout boundary satisfies $F(\overline{v}) = R$, so that the limited commitment constraint binds at the payout boundary. When $F(\overline{v}) > R$, the choice of the payout boundary is not constrained by the limited commitment constraint $F(\overline{v}) \geq R$, in that it is possible to slightly increase or decrease the payout boundary without violating limited commitment. Without constraints, the payout boundary must satisfy the super-contact condition $F''(\overline{v}) = 0$ (as shown, e.g., in Dumas (1991)). However, we have shown that $F''(\overline{v}) < 0$. As a result, at the payout boundary $\overline{v}$, the limited commitment constraint must bind, in that $F(\overline{v}) = R$.

A.7. Part VII—Verification

Under the proposed strategy and contracts Π^A, Π^P from Proposition 1, the sensitivities $\beta_t^A, \beta_t^I, \beta_t^P$ solve the maximization in (E11) or, equivalently, the maximization in (E13). Payouts to the agent take the form $dw_t^A = \max\{v_t^A - \overline{v}, 0\}$. As shown in Parts IV and V of the proof, the sensitivities $\beta_t, \beta_t^I, \beta_t^P$ that solve the maximization in (E13) (or (E11)) satisfy (E25), (E24), and (E23). The incentive condition (11) maps sensitivities β_t^A, β_t^I to effort levels a_t^A, a_t^I. In what follows, we verify that the proposed strategy (i.e., the proposed contracts) from Proposition 1 yields a higher payoff than any other strategy (i.e., any other incentive compatible contracts) and thus is indeed optimal.

Take any time $t < \tau$. Suppose that the intermediary deviates from the proposed strategy and follows an alternative strategy up to time t (i.e., for times $s \leq t$), with sensitivities $\beta_s^A, \beta_s^I, \beta_s^P$ and payouts to the agent $dw_s^A \geq 0$. After time t (i.e., for times $s \geq t$), the intermediary follows the proposed strategy (contracts) from Proposition 1. Then, the intermediary's payoff at time 0 (under the deviation), which is equal to total firm value, can be written as (see (E6))

$$G_t = \int_0^t e^{-rs}\left(dX_s - dw_s^A - \frac{\lambda\left(a_s^I\right)^2}{2}ds + \beta_s^P\left[\bar{a}_s^A + \bar{a}_s^I - \hat{a}_s^A - a_s^I\right]ds\right) + e^{-rt}[F\left(v_t^A\right)]. \quad \text{(E27)}$$

We use (7), apply Itô's Lemma, and differentiate G_t with respect to time t to get

$$\begin{aligned} e^{rt}dG_t = &\left\{\left(\hat{a}_t^A + a_t^I\right)(1-\beta_t^P) + \beta_t^P\left(\bar{a}_t^I + \bar{a}_t^A\right) + F'\left(v_t^A\right)\beta_t^A\left(a_t^I - \hat{a}_t^I\right) - \frac{\lambda\left(a_t^I\right)^2}{2}\right. \\ &\left. + F'\left(v_t^A\right)\left(rv_t^A + \frac{\delta\left(\hat{a}_t^A\right)^2}{2}\right) + \frac{F''\left(v_t^A\right)\left(\beta_t^A\sigma\right)^2}{2} - rF\left(v_t^A\right)\right\}dt \\ &+ \left(1 + F'\left(v_t^A\right)\beta_t^A\right)[dX_t - \hat{a}_t^A dt - a_t^I dt] - dw_t^A\left(F'\left(v_t^A\right) + 1\right) \\ =&\mu_t^G dt + \left(1 + F'\left(v_t^A\right)\beta_t^A\right)[dX_t - \hat{a}_t^A dt - a_t^I dt] - dw_t^A\left(F'\left(v_t^A\right) + 1\right). \end{aligned}$$

By (E13) (or equivalently (E11)), the drift term in curly brackets, μ_t^G, is zero when β_t^A, β_t^I, and β_t^P solve the optimization in (E13) (or equivalently (E11)) subject to the incentive compatibility and limited commitment constraints. As shown in Parts IV and V of this proof, this is the case if and only if β_t^A, β_t^I, and β_t^P satisfy (E25), (E24), and (E23). Any other choice of β_t^A, β_t^I, and β_t^P makes the drift term μ_t^G weakly negative, in that $\mu_t^G \leq 0$.

In addition, recall that because $F(v^A)$ is strictly concave and because $F'(\overline{v}) = 1$, it follows that $F'(v_t^A) \geq -1$ with equality if and only if $v_t^A = \overline{v}$. As $dw_t^A \geq 0$ and $F'(v_t^A) \geq -1$, the term $-dw_t^A(F'(v_t^A) + 1)$ is weakly negative under any payout policy $dw_t^A \geq 0$ and zero under the proposed payout policy $dw_t^A = \max\{v_t^A - \overline{v}, 0\}$.

Next, recall that $\frac{dX_t - \hat{a}_t^A dt - a_t^I dt}{\sigma}$ is the increment of a standard Brownian motion under the probability measure $\mathbb{P}^I$ that is taken under the intermediary's information. Because the sensitivities β_t^j are by assumption bounded and $F(v^A)$ is twice continuously differentiable on $(0, \overline{v})$, $1 + F'(v_t^A)\beta_t^A$ is also bounded and it follows that

$$\mathbb{E}_0^I\left[\int_0^t e^{-rs}\left(1 + F'\left(v_s^A\right)\beta_s^A\right)\left(dX_s - \hat{a}_s^A ds - a_s^I ds\right)\right] = 0.$$

Thus, G_t, with

$$dG_t = e^{-rt}\left(\underbrace{\mu_t^G dt}_{\leq 0}\underbrace{-dw_t^A\left(F'\left(v_t^A\right)+1\right)}_{\leq 0}\right) + e^{-rt}\left(1 + F'\left(v_t^A\right)\beta_t^A\right)[dX_t - \hat{a}_t^A dt - a_t^I dt],$$

follows a super-martingale (i.e., decreases in expectation) under the measure $\mathbb{P}^I$. In turn,

$$G_0 \geq \mathbb{E}_0^I[G_t]. \tag{E28}$$

Because $a_t^j \in [0, A]$ with $A < \infty$ for $j = A, I$, it follows that total surplus is bounded from above by $2A/r$. As such, $F(v_t^A)$ is bounded from above by $2A/r$ and bounded from below by zero (due to limited liability) so that

$$0 \leq \lim_{t\to\infty} e^{-rt}[F\left(v_t^A\right)] \leq \lim_{t\to\infty} e^{-rt}\frac{2A}{r} = 0.$$

Thus, (E27) implies

$$\lim_{t\to\infty}\mathbb{E}_0^I[G_t] = \mathbb{E}_0^I\left[\int_0^\tau e^{-rs}\left(dX_s - dw_s^P - dw_s^A - \frac{\lambda\left(a_s^I\right)^2}{2}ds + \beta_s^P\left[\bar{a}_s^A + \bar{a}_s^I - \hat{a}_s^A - a_s^I\right]ds\right)\right] = \mathbb{E}_0^I[G_\tau].$$

Next, we take the limit $t \to \infty$ in (E28) to obtain

$$G_0 \geq \mathbb{E}_0^I[G_\tau] = \mathbb{E}_0^I\left[\int_0^\tau e^{-rs}\left(dX_s - dw_s^A - \frac{\lambda\left(a_s^I\right)^2}{2}ds + \beta_s^P\left[\bar{a}_s^A + \bar{a}_s^I - \hat{a}_s^A - a_s^I\right]ds\right)\right]. \tag{E29}$$

The inequality (E29) implies that the proposed contracts Π^A, Π^P from Proposition 1 are indeed optimal and solve the intermediary's problem (3), as (E29) holds in equality under the proposed contracts and the intermediary's expected payoff at time 0 (i.e., total firm value) under the proposed contracts is G_0.

A.8. Part VIII—Liquidation

We demonstrate the circumstances under which firm liquidation is indeed optimal when the agent's contract is terminated. Note that the agent's contract is terminated at time τ when $v_\tau^A = 0$, leading to $a_t^A = \beta_t^A = v_t^A = 0$ for $t \geq \tau$. If the intermediary does not liquidate the firm, the HJB equation (20) implies that (after inserting $a^A = \beta^A = v^A = 0$)

$$F(0) = \max_{\beta^I} \frac{1}{r}\left(a^I - \frac{\lambda\left(a^I\right)^2}{2}\right), \tag{E30}$$

which is solved subject to the effort incentive condition $a^I = \frac{\beta^I}{\lambda}$. Note that as the intermediary has deep pockets, he can run the firm forever without the agent after the agent's contract is terminated at time τ and doing so yields the payoff characterized in (E30).

The solution to the optimization problem in (E30) satisfies

$$\beta^I = 1 \quad \text{and} \quad a^I = \frac{\beta^I}{\lambda} = \frac{1}{\lambda}. \tag{E31}$$

Inserting the expressions (E31) into (E30) yields

$$F(0) = \frac{1}{r}\left(a^I - \frac{\lambda\left(a^I\right)^2}{2}\right) = \frac{1}{2r\lambda},$$

with a^I from (E31), which is the intermediary's payoff when he continues running the firm without the agent after time τ. It follows that when

$$R \geq R_L := \frac{1}{2r\lambda}, \tag{E32}$$

it is optimal for the intermediary to liquidate the firm when $v^A = 0$ and the agent's contract is terminated (instead of running the firm without the agent). Recall that we have made the assumption $R \geq R_L$. Thus, the intermediary optimally liquidates the firm when $v^A = 0$ and seizes the liquidation value R, so the boundary condition $F(0) = R$ applies.

Appendix F: Proof of Corollary 1

We use the expression for π^I in (17) and take the limit $\lambda \to \infty$ to obtain

$$\pi^I = \frac{1}{-F'\left(v^A\right) - \delta\sigma^2 F''\left(v^A\right)}. \tag{F1}$$

For simplicity and without loss of generality, we focus on the case of $\pi^I \neq 0$. Using the expression for $1 - \beta^P$ in (21) and taking the limit $\lambda \to \infty$, we have

$$1 - \beta^P = \frac{\frac{\pi^I}{\delta}}{-F'\left(v^A\right)\left(\pi^I\right)^2\left(\frac{1}{\delta}\right) - F''\left(v^A\right)\left(\pi^I\sigma\right)^2}. \tag{F2}$$

Combining (F1) and (F2), we obtain that $\beta^P = 0$.

Lemma 4 and $\beta^P = 0$ imply that $\beta^A = \pi^I(1-\beta^P) = \pi^I$ and $\beta^I = (1 + F'(v^A)\pi^I)(1-\beta^P) = 1 + F'(v^A)\pi^I$. Finally, we use $a^I = \beta^I/\lambda$ from (11) and take the limit $\lambda \to \infty$ to obtain that $a^I = 0$.

Appendix G: Proof of Proposition 2

The proof of Proposition 2 is analogous to the proof of Proposition 1. To highlight the difference and to facilitate comparison, we structure the proof of Proposition 2 along the lines of the proof of Proposition 1. As a result, the proof of Proposition 2 is split into several parts that correspond to the respective parts in the proof of Proposition 1.

Finally, statement 3 of the proposition is an immediate consequence of Corollary 1 and the fact that $\beta^I = 1$ when Π^A is contractible.

A.1. Parts I to III

Most of the steps are identical to Parts I to III of the proof of Proposition 1. Note that $\bar{a}^j = \hat{a}^j$ holds because the contract Π^A is observed by the principal and is contractible between the principal and the intermediary. The intermediary dynamically maximizes total firm value, $v_t^A + v_t^I$, which is characterized in (E6). Due to $\hat{a}^j = \bar{a}^j$, the expression (E6) simplifies to

$$v_t^I + v_t^P = \mathbb{E}_t^I\left[\int_t^\infty e^{-r(s-t)}\left(dX_s - dw_s^A - \frac{\lambda\left(a_s^I\right)^2}{2}ds + \beta_s^P\left[\hat{a}_s^I - a_s^I\right]ds\right)\right]. \tag{G1}$$

As before, the intermediary's value function can be expressed as a function of v^A, in that $v^I = F(v^A) - v^P$. Payouts to the agent take the form $dw^A = \max\{v^A - \overline{v}, 0\}$ with endogenous payout boundary $\overline{v}$.

On $[0, \overline{v}]$, the function $F(v^A)$ solves the HJB equation (E11) with $\bar{a}^j = \hat{a}^j$, which simplifies to

$$\begin{aligned} rF\left(v^A\right) = \max_{\beta^A,\beta^I,\beta^P}\Bigg\{&\hat{a}^A + \hat{a}^I + \left(1 - \beta^P + F'\left(v^A\right)\beta^A\right)\left(a^I - \hat{a}^I\right) - \frac{\lambda\left(a^I\right)^2}{2} \\ &+ F'\left(v^A\right)\left(rv^A + \frac{\delta\left(\hat{a}^A\right)^2}{2}\right) + \frac{F''\left(v^A\right)\left(\beta^A\sigma\right)^2}{2}\Bigg\}. \end{aligned} \tag{G2}$$

In addition, the incentive conditions for effort (11) and the relationship (15) hold, so that $\beta^I = 1 - \beta^P + F'(v^A)\beta^A$.

A.2. Parts IV and V

Recall that because Π^A is observable for the principal and is contractible between the principal and the intermediary, it follows that $\hat{a}^j = \bar{a}^j$ for $j = A, I$.

As a result, (11) implies

$$\hat{a}^A = \bar{a}^A = a^A = \frac{\beta^A}{\delta} \quad \text{and} \quad \hat{a}^I = \bar{a}^I = a^I = \frac{\beta^I}{\lambda}. \tag{G3}$$

In addition, one can verify that maximizing the right-hand side of (G2) over a^I yields the same expression for optimal effort a^I as in (G3). We insert $a^I = \hat{a}^I = \bar{a}^I = \beta^I/\lambda$ and $a^A = \hat{a}^A = \bar{a}^A = \beta^A/\delta$ as well as $\beta^I = (1-\beta^P) + F'(v^A)\beta^A$—that is, the relationship (15)—into (G2) to obtain

$$rF\left(v^A\right) = \max_{\beta^A,\beta^I}\left\{\left(\frac{\beta^I}{\lambda} + \frac{\beta^A}{\delta}\right) - \frac{\lambda\left(a^I\right)^2}{2} + F'\left(v^A\right)\left(rv^A + \frac{\left(\beta^A\right)^2}{2\delta}\right) + \frac{F''\left(v^A\right)\left(\beta^A\sigma\right)^2}{2}\right\}, \tag{G4}$$

where we drop the control β^P (which is possible due to (15)).

Also note that we can use (11) to rewrite (G4) as

$$rF\left(v^A\right) = \max_{\beta^A,\beta^I}\left\{a^A + a^I - \frac{\lambda\left(a^I\right)^2}{2} + F'\left(v^A\right)\left(rv^A + \frac{\delta\left(a^A\right)^2}{2}\right) + \frac{F''\left(v^A\right)\left(\beta^A\sigma\right)^2}{2}\right\},$$

which is (20). This shows that under the optimal contracts the intermediary's value function solves the HJB equation (20), as stipulated by Proposition 2.

The optimal values β^A and β^I, solving the maximization in (G4), must solve the first-order conditions

$$\frac{\partial F\left(v^A\right)}{\partial\beta^A} \propto \frac{1}{\delta} + F'\left(v^A\right)\left(\frac{\beta^A}{\delta}\right) + \beta^A F''\left(v^A\right)\sigma^2 = 0,$$

$$\frac{\partial F\left(v^A\right)}{\partial\beta^I} \propto \frac{1}{\lambda} - \frac{\beta^I}{\lambda} = 0.$$

We can solve these two equations to get

$$\beta^A = \frac{1}{-F'\left(v^A\right) - \delta\sigma^2 F''\left(v^A\right)}, \tag{G5}$$

which is (22), and

$$\beta^I = 1, \tag{G6}$$

which is (23). Using (15), we obtain

$$1 - \beta^P = \beta^I - F'\left(v^A\right)\beta^A = 1 - \frac{F'\left(v^A\right)}{-F'\left(v^A\right) - \delta\sigma^2 F''\left(v^A\right)}, \tag{G7}$$

which can be solved for

$$\beta^P = \frac{F'\left(v^A\right)}{-F'\left(v^A\right) - \delta\sigma^2 F''\left(v^A\right)}.$$

A.3. Part VI—Properties of the Value Function

This part is identical to Part VI of the proof for Proposition 1. That is, the payout boundary satisfies $F(\overline{v}) = R$ and the value function is strictly concave, in that $F''(\overline{v}) < 0$ for $v \in [0, \overline{v}]$.

A.4. Part VII—Verification

We provide the formal verification argument that the proposed strategy and contracts Π^A, Π^P from Proposition 2 are indeed optimal. Under the proposed contracts Π^A, Π^P, the sensitivities $\beta_t^A, \beta_t^I, \beta_t^P$ solve the maximization in (G4), while $\bar{a}_t^j = \hat{a}_t^j$ given that the contract Π^A is publicly observable and is contractible between the principal and the intermediary. Payouts to the agent are $dw_t^A = \max\{v_t^A - \overline{v}, 0\}$, with endogenous payout boundary $\overline{v}$. As shown in Parts IV and V of the proof, the sensitivities $\beta_t^A, \beta_t^I, \beta_t^P$ that solve the maximization in (G4) satisfy (G5), (G6), and (G7). The incentive condition (11) maps sensitivities β_t^A, β_t^I to effort levels a_t^A, a_t^I. In what follows, we verify that the proposed strategy (i.e., the proposed contracts) from Proposition 2 yields a higher payoff than any other strategy (i.e., any other incentive compatible contracts) and thus is indeed optimal.

Take any time $t < \tau$. Suppose that the intermediary deviates from the proposed strategy and follows an alternative strategy up to time t (i.e., for times $s \leq t$), with sensitivities $\beta_s^A, \beta_s^I, \beta_s^P$ and payouts to the agent $dw_s^A \geq 0$. After time t (i.e., for times $s \geq t$), the intermediary follows the proposed strategy (contracts) from Proposition 2. Then, total firm value at time 0 (under the deviation) can be written as (see (G1))

$$G_t = \int_0^t e^{-rs}\left(dX_s - dw_s^A - \frac{\lambda\left(a_s^I\right)^2}{2}ds + \beta_s^P[\hat{a}_s^I - a_s^I]ds\right) + e^{-rt}[F\left(v_t^A\right)]. \quad \text{(G8)}$$

We use (7) and Itô's Lemma and differentiate G_t with respect to time t to get

$$\begin{aligned} e^{rt}dG_t = \Bigg\{ &\left(\hat{a}_t^A + a_t^I\right) - \frac{\lambda\left(a_t^I\right)^2}{2} \\ &+ F'\left(v_t^A\right)\left(rv_t^A + \frac{\delta\left(\hat{a}_t^A\right)^2}{2} + F'\left(v_t^A\right)\beta_t^A\left(a_t^I - \hat{a}_t^I\right)\right) + \frac{F''\left(v_t^A\right)\left(\beta_t^A\sigma\right)^2}{2} - rF\left(v_t^A\right)\Bigg\}dt \\ &+ \left(1 + F'\left(v_t^A\right)\beta_t^A\right)[dX_t - \hat{a}_t^A dt - a_t^I dt] - dw_t^A\left(F'\left(v_t^A\right) + 1\right) \\ = {} &\mu_t^G dt + \left(1 + F'\left(v_t^A\right)\beta_t^A\right)[dX_t - \hat{a}_t^A dt - a_t^I dt] - dw_t^A\left(F'\left(v_t^A\right) + 1\right). \end{aligned}$$

Note that the incentive condition (11) and the observability (and contractibility) of Π^A imply $a_t^I = \hat{a}_t^I = \bar{a}_t^I = \beta_t^I/\lambda$ and $a_t^A = \hat{a}_t^A = \bar{a}_t^I = \beta_t^A/\delta$.

By (G4), the drift term in curly brackets, μ_t^G, is zero when β_t^A, β_t^I, and β_t^P solve the optimization in (G4) subject to the incentive compatibility constraint (11) and the limited liability and commitment constraints. As shown in Parts IV and V of this proof, this is the case if and only if β_t^A, β_t^I, and β_t^P satisfy (G5), (G6), and (G7). Any other choice of β_t^A, β_t^I, and β_t^P makes the drift term μ_t^G weakly negative, in that $\mu_t^G \leq 0$. In addition, note that because $F'(v_t^A) \geq -1$, with equality if and only $v_t^A = \overline{v}$, the term $-dw_t^A(F'(v_t^A)+1)$ is weakly negative under any payout policy $dw_t^A \geq 0$ and zero under the proposed payout policy $dw_t^A = \max\{v_t^A - \overline{v}, 0\}$.

Next, recall that $\frac{dX_t - \hat{a}_t^A dt - a_t^I dt}{\sigma}$ is the increment of a standard Brownian motion under the probability measure $\mathbb{P}^I$ that is taken under the intermediary's information. Because the sensitivity β_t^I is by assumption bounded and $F(v^A)$ is twice continuously differentiable on $(0, \overline{v})$, it follows that

$$\mathbb{E}_0^I\left[\int_0^t e^{-rs}\left(1 + F'\left(v_s^A\right)\beta_s^A\right)\left(dX_s - \hat{a}_s^A ds - a_s^I ds]\right)\right] = 0.$$

Thus, G_t, with

$$dG_t = e^{-rt}\left(\underbrace{\mu_t^G dt}_{\leq 0}\underbrace{-dw_t^A\left(F'\left(v_t^A\right)+1\right)}_{\leq 0}\right) + e^{-rt}\left(1 + F'\left(v_t^A\right)\beta_t^A\right)[dX_t - \hat{a}_t^A dt - a_t^I dt],$$

follows a super-martingale (i.e., decreases in expectation) under the measure $\mathbb{P}^I$. In turn,

$$G_0 \geq \mathbb{E}_0^I[G_t]. \tag{G9}$$

Because $a_t^j \in [0, A]$ with $A < \infty$ for $j = A, I$, it follows that total surplus is bounded from above by $2A/r$. As such, $F(v_t^A)$ is bounded from above by $2A/r$ and bounded from below by zero (due to limited liability), so that

$$0 \leq \lim_{t\to\infty} e^{-rt}[F\left(v_t^A\right)] \leq \lim_{t\to\infty} e^{-rt}\frac{2A}{r} = 0. \tag{G10}$$

Thus, (G8) implies

$$\lim_{t\to\infty}\mathbb{E}_0^I[G_t] = \mathbb{E}_0^I\left[\int_0^\tau e^{-rs}\left(dX_s - dw_s^A - \frac{\lambda\left(a_s^I\right)^2}{2}ds + \beta_s^P[\hat{a}_s^I - a_s^I]ds\right)\right] = \mathbb{E}_0^I[G_\tau].$$

Next, we take the limit $t \to \infty$ in (G9) to obtain

$$G_0 \geq \mathbb{E}_0^I[G_\tau] = \mathbb{E}_0^I\left[\int_0^\tau e^{-rs}\left(dX_s - dw_s^A - \frac{\lambda\left(a_s^I\right)^2}{2}ds + \beta_s^P[\hat{a}_s^I - a_s^I]ds\right)\right]. \tag{G11}$$

The inequality (G11) implies that the proposed contracts Π^A, Π^P from Proposition 2 are indeed optimal and solve the intermediary's problem, as (G11) holds in equality under the proposed contracts and the intermediary's expected payoff at time 0 under the proposed contracts is $G_0 = F(v_0^A)$.

A.5. Part VIII—Liquidation

This part is identical to Part VII of the proof for Proposition 1. That is, the intermediary optimally liquidates the firm at $v^A = 0$ when the agent's contract is terminated, if $R \geq R_L$ with R_L defined in (E32).

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.
Replication Code.