Ownership Dynamics and Firm Policies with a Large Shareholder*

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Abstract

We develop a dynamic theory of large shareholders and firm policies, where a blockholder engages with the firm to improve asset productivity and influence decisions on investment, financing, and executive compensation. In equilibrium, the blockholder's ownership stake may grow or shrink over time driven by gains from trade that result from the relationship between ownership, corporate policies, and control. A feedback loop emerges: as the blockholder increases their ownership, it boosts investment and the firm's debt capacity, while higher returns from debt and investment incentivize the blockholder to increase their stake. While firm policies need not maximize dispersed shareholder value when the blockholder is in control, limiting blockholder influence ultimately reduces blockholder ownership and firm value.

Keywords: Activism, limited commitment, strategic trading, price impact, leverage, corporate investment, payout

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Large shareholders, or blockholders, play a central role in corporate governance and firm outcomes. While blockholder ownership is associated with changes in firm policies and performance, it is also endogenous and evolves dynamically over time. This inherent endogeneity poses a major challenge to understanding the role and impact of blockholders, impeding both empirical analysis and policy design. Despite its importance, a comprehensive framework capturing the interaction between blockholders, their ownership, and corporate policies is still lacking. This paper attempts to fill this gap by developing a tractable dynamic theory of blockholder ownership and firm policies, in which the blockholder's stake and the firm's investment, financing, and compensation policies are jointly determined.

Our framework builds upon the model of DeMarzo and Urošević (2006) by introducing a large, strategic blockholder who influences firm outcomes through four distinct channels. Unlike the original model, where the blockholder's role is limited to exerting effort to increase firm productivity, we incorporate additional strategic dimensions: (*i*) contracting with management under moral hazard, (*ii*) influencing investment decisions, and (*iii*) shaping financing decisions. We show that these novel elements fundamentally alter equilibrium ownership behavior. They generate ownership dynamics in which the blockholder may either build or unwind its stake over time, leading to persistently high or low long-run ownership. This richer structure also yields new insights that cannot arise in the baseline model.

First, blockholder ownership enhances the firm's borrowing capacity, creating gains from trade and making blockholder ownership and debt financing *complements*. Second, managerial agency conflicts crowd out blockholder engagement for a given level of ownership, but can lead to an increase blockholder ownership and engagement in the long run. Third, investment also increases with blockholder ownership, but the blockholder chooses an inefficiently low level of investment from the perspective of dispersed shareholders. Although limiting blockholder control can boost investment at a given ownership level, it ultimately reduces both blockholder ownership and involvement, leading to lower investment and firm value in the long run. Fourth, when the firm's investment opportunities are sufficiently strong or credit market conditions are favorable, the blockholder may take over the entire firm via a public-to-private leveraged buyout, accompanied by a rise in firm leverage.

In our framework, the blockholder's ownership stake serves as a state variable that influences both the level of engagement and the firm's policies. Starting from an initial position, the blockholder can adjust their stake through strategic buying and selling, while competitive, dispersed shareholders behave as price takers. An important feature is that any gains from trade are reflected in the equilibrium prices. When such gains are present, the blockholder faces higher marginal costs to acquire additional shares, meaning they experience price impact in their trades. To minimize this impact and capture gains from trade, the blockholder adjusts its position gradually, allowing it to earn returns on its existing (inframarginal) holdings while buying shares.

The blockholder's optimal trading strategy reflects three opposing forces. First, holding costs lead the blockholder to sell more (or buy less), as in DeMarzo and Urošević (2006).[1] A similar dampening effect arises when share supply from dispersed shareholders is inelastic. Second, a larger stake strengthens the blockholder's incentives to exert costly effort, enhancing productivity and firm value. Third, blockholder ownership influences corporate policies by affecting firm value through effort. Together, these mechanisms generate gains from trade both directly—through higher effort that increases firm value—and indirectly—by affecting corporate policies such as investment and debt financing. This indirect channel is distinctive to our framework and interacts with the direct one.

Formally, the gains from trade arising from a specific policy depend on two components: (A) the marginal impact of changes in the policy on the blockholder's value function, and (B) the marginal impact of blockholder ownership on that policy. Component (B) operates through blockholder effort, which increases with ownership and determines the magnitude, though not the sign, of the overall gains from trade. The strength of this channel depends on the severity of managerial agency conflicts—that is, the manager's ability to divert cash flows—which crowd out blockholder incentives and weaken (B).

Component (A), in turn, depends on whether the relevant policy is chosen to maximize blockholder value or is constrained by frictions or shaped by the objectives of dispersed shareholders. When a policy already maximizes the blockholder's objective, it has no incentive to adjust its stake. In contrast, when frictions, constraints, or limited control distort the policy away from the blockholder's preferred choice, the blockholder has an incentive to trade in order to influence that policy or relax those frictions—buying or selling, depending on the adjustment needed to move the policy in the desired direction.

This mechanism, component (A), operates in the model through an (1) incentive channel, a (2) debt-financing channel, and a (3) control channel. The *incentive channel* arises because managerial agency conflicts lead to suboptimally low blockholder effort, inducing

[1] DeMarzo and Urošević (2006) study a large shareholder with CARA preferences facing normally distributed shocks. Their setup implies a certainty-equivalent representation of preferences—the expected payoff minus a risk adjustment—which is akin to the holding cost in our model.

the blockholder to increase its stake and strengthen effort incentives.

The *debt-financing channel* arises from a commitment friction as firm owners cannot commit to repaying debt: The firm may strategically default, triggering inefficient liquidation. Firm value and the blockholder's valuation of its holdings determine whether default occurs and, in turn, the amount of debt the firm can issue for a given level of default risk. By increasing ownership and effort, the blockholder raises firm value and the firm's debt capacity, increasing the net benefits of debt. Higher blockholder ownership increases the benefits of debt on both margins, facilitating greater borrowing for a given level of default risk (extensive margin) and reducing default risk for a given debt level (intensive margin). Notably, the debt financing channel arises both in our baseline—where debt is risk-free and blockholder ownership expands the maximum debt level the firm can commit to repay—and in an extension with large cash flow shocks and risky debt, with greater blockholder ownership reducing default risk.

Last, in the baseline model, the blockholder controls and optimally chooses investment, implying that there are no gains from trade associated with investment and that investment policy does not interact with the blockholder's optimal trading strategy. In this scenario, investment is inefficiently low from the perspective of dispersed shareholders. When, instead, investment is set to maximize the value of (majority) dispersed shareholders, it ends up being too high for the blockholder. As the blockholder's ownership increases, both the stock price and investment rise, causing investment to diverge further from the blockholder's preferred level. This *control channel* creates an incentive for the blockholder to sell.

In the baseline model, where the blockholder controls corporate policies, the dynamics of ownership concentration reflect the blockholder's holding costs, and the incentive and debt financing channels. Importantly, the blockholder's propensity to acquire shares increases with ownership, as larger stakes amplify the benefits of engagement and debt capacity. As a result, starting from a high initial stake, the blockholder tends to accumulate further until reaching a relatively high target level—potentially full ownership. Conversely, with a smaller stake, the blockholder has weaker incentives to buy and may instead sell, converging toward a low target. Formally, there exist upper and lower thresholds for the blockholder's stake such that the blockholder (*i*) exits when its stake falls below the lower threshold, and (*ii*) otherwise increases its holdings toward the upper threshold.

That is, our model generates reinforcing ownership dynamics: firms ultimately exhibit either high or low levels of blockholder ownership, with intermediate concentrations being relatively rare—intuitively, the distribution of ownership concentration is bimodal. This pattern aligns with evidence from U.S. public firms, where Edmans and Holderness (2017) document that blockholder stakes tend to cluster at relatively low levels. At the same time, a growing share of large firms is privately held, featuring highly concentrated ownership by private equity (PE) funds. We also increasingly observe public-to-private transactions, in which PE funds expand their stakes in public firms over time to take them private (e.g., through a leveraged buyout)—a pattern that can emerge in our model.

Indeed, when debt market conditions are sufficiently favorable, the firm's investment opportunities are strong, and share supply is sufficiently elastic, the blockholder's target ownership converges toward full ownership. Under these circumstances, the model generates ownership dynamics that resemble a public-to-private leveraged buyout. Specifically, the blockholder gradually increases its stake up to a threshold, at which point it stochastically acquires the entire firm at once, effectively taking it private. This acquisition is accompanied by a jump in leverage, with the proceeds distributed as a leveraged payout.

In general, we find that blockholder ownership—and blockholder engagement—depends on firm characteristics and the blockholder's ability to influence firm policies. The blockholder is more likely to increase its stake, and less likely to exit, when debt provides greater benefits, blockholder effort has a stronger impact on firm productivity, the firm has attractive investment opportunities, and managerial agency conflicts are not too severe. In particular, debt financing and blockholder ownership are complements: access to debt financing induces the blockholder to increase its stake, while firm leverage rises with blockholder ownership.

Our analysis also has important implications for policies that limit blockholder control, such as governance structures like staggered boards (Bebchuk, Cohen, and Ferrell, 2009), which aim to shift decisions toward maximizing value for dispersed shareholders. We demonstrate that restricting blockholder control over investment can actually decrease the value for dispersed shareholders, due to the blockholder's endogenous response. While firms may invest more at a given level of blockholder ownership, the blockholder will, in turn, reduce their stake, ultimately leading to lower long-term investment. This dynamic arises from the interaction between control, corporate policies, and the blockholder's trading decisions, with the blockholder decreasing their holdings due to the control channel described earlier.

Our setting is flexible enough to incorporate several additional realistic features. First, while debt is risk-free in the baseline model, we extend the framework to accommodate large shocks, introducing the possibility for default and risky debt. We show that our main findings remain qualitatively similar in this model variant. Moreover, we find that default risk and credit spreads are negatively related to blockholder ownership, again suggesting that blockholder ownership and debt financing are complements.

Second, we extend the model to incorporate stochastic holding costs—liquidity shocks—for the blockholder. Liquidity shocks shape ownership dynamics by affecting the timing of trades: when a liquidity shock occurs after the blockholder has built a large position, it does not trigger exit, whereas an early shock following entry induces the blockholder to unwind its stake. These findings reflect again the reinforcing ownership dynamics, which, in the long run, lead to either very high or low blockholder ownership.

Related Literature. Our main contribution is to develop a tractable, comprehensive framework for understanding the interactions between blockholder ownership and firm policies. Accordingly, our paper primarily relates to the literature on shareholder activism and blockholders (see, e.g., Maug (1998); Edmans and Manso (2011); Khanna and Mathews (2012); Dasgupta and Piacentino (2015); Levit (2019); Marinovic and Varas (2025)). Most closely related to our paper, Admati, Pfleiderer, and Zechner (1994), DeMarzo and Urošević (2006), and Back, Collin-Dufresne, Fos, Li, and Ljungqvist (2018) study the trading of a large shareholder that influences firm performance via costly effort. In these models, blockholders have no direct influence over firm policies (there is no investment or debt financing), limiting the scope of engagement. Our model differs from these papers by allowing blockholders to actively shape firm policies while dynamically adjusting their stake, thereby capturing the interplay between trading behavior, corporate governance, and firm strategy.

We extend DeMarzo and Urošević (2006) to a large, strategic trader who influences firm outcomes through four channels: (*i*) effort; (*ii*) contracting with management; and influencing (*iii*) investment and (*iv*) borrowing decisions. Due to the novel elements (*ii*)-(*iv*), our model generates reinforcing and rich ownership dynamics, in which the blockholder may gradually increase its stake, take over the firm, or exit. Our setting also yields testable predictions linking firm policies to the size of the blockholder's stake and highlighting how control rights affect blockholder trading and firm value. These predictions shed light on the endogeneity and two-way feedback between blockholder ownership and corporate policies—the central challenge in empirical analysis.

Another strand of the literature on trading and activism related to our paper is the "governance by exit" literature, including the papers by Admati and Pfleiderer (2009), Edmans and Manso (2011), Dasgupta and Piacentino (2015), and Levit (2019). In these models, a blockholder has access to private information about firm value and may sell her block in response to negative information. The focus of these papers is on trading by an insider who has private information about firm value that is exogenous to her trading. These models have a single round of trading and cannot analyze feedback from prices to blockholder actions. In contrast, in our framework, firm value—and thus the stock price—is endogenously shaped by the blockholder's engagement and influence on corporate policies, while also being forward-looking and reflecting anticipated trading behavior.

Finally, we solve for the Markov-perfect equilibrium using a methodology similar to DeMarzo and Urošević (2006), adopted in other contexts too. For instance, Daley and Green (2020) study dynamic bargaining with adverse selection. DeMarzo and He (2021), Hu and Varas (2025), and DeMarzo, He, and Tourre (2023) respectively study corporate leverage, loan sales, and the dynamics of government debt under limited commitment.

1 Model Setup

We develop a dynamic model in which a blockholder (i.e., large shareholder) invests in a firm and influences its policies to improve firm value. The blockholder does so by (*i*) exerting private effort to increase asset productivity, (*ii*) incentivizing the firm's manager with an optimal contract, and (*iii*) shaping firm investment and financing. The blockholder can dynamically increase or decrease their ownership stake in the firm by purchasing or selling shares, leading to changes in effort, investment, and financing (i.e., leverage). When trading, the blockholder faces dispersed shareholders—also referred to as investors—who take prices as given. Trading decisions are observable to investors, who rationally take this information into account when determining their demand for shares. All policies—except payouts to management through the optimal contract—are chosen without commitment.

Technology. Time $t \in [0, \infty)$ is continuous and infinite. There are three risk-neutral agents with common discount rate $\rho > 0$: A blockholder (large shareholder), a continuum of dispersed shareholders (investors), and a manager. We consider a single firm run by the manager. The firm is financed with equity and debt, with $\theta_t \in [0, 1]$ denoting the fraction of the firm's equity held by the blockholder (with $1 - \theta_t$ held by dispersed shareholders).

Firm cash flows depend on the endogenous capital stock $K_t > 0$, blockholder effort $b_t \geq 0$, and managerial cash diversion $m_t \in \{0, 1\}$. Over any time period $[t, t + dt]$, incremental cash flows are given by $K_t dX_t$, where dX_t is the scaled cash flow per unit of capital with:

$$dX_t = \alpha_1 dt + dN_t(1 - m_t), \tag{1}$$

where $\alpha_1 \geq 0$. In (1), $dN_t \in \{0, 1\}$ is a jump process with $\mathbb{E}[dN_t] = (\alpha_2 + b_t)dt$ for $\alpha_2 > 0$.[2] This specification implies that large positive cash flows arrive infrequently, while the cash flow rate is α_1 otherwise. (While the baseline model features positive cash flow shocks and risk-free debt, Section 5 introduces negative cash flow shocks and risky debt.) Investors and the blockholder observe the realization of cash flows dX_t, but do not observe dN_t or m_t. When $dN_t = 1$ and additional cash flows of K_t are realized, the manager can divert from the cash flows by setting $m_t = 1$, in which case she gains a private benefit λK_t with $\lambda \in [0, 1)$ (i.e., diversion is inefficient). In this case, investors observe $dX_t = \alpha_1 dt$ and thus cannot discern cash flow diversion upon a shock from no shock occurring. We define

$$\alpha = \alpha_1 + \alpha_2, \tag{2}$$

so that expected (scaled) cash flows are given by $\mathbb{E}[dX_t] = (\alpha + b_t)dt$.

Blockholder effort. As in DeMarzo and Urošević (2006), firm cash flows depend on unobservable blockholder effort $b_t \geq 0$, which entails a private flow cost $\frac{1}{2}\kappa K_t b_t^2$ to the blockholder for a cost parameter $\kappa > 0$. The blockholder's private effort captures its engagement with the firm, for instance, by appointing key personnel and board members, providing industry connections, or developing strategies and proposals that increase the return on invested capital (such as better management of working capital, costs, overheads, purchases, or product mix). This effort choice cannot be contracted on and endogenously depends on the stake of the blockholder, with a blockholder who cannot commit to future trades.

Moral Hazard and Short-Term Contracting. To deal with managerial cash diversion, firm shareholders write a contract with the manager. Since firm policies and trading are chosen without commitment, we also preclude commitment to long-term contracts, and thus restrict the contracting space to short-term contracts over $[t, t + dt]$.[3] We focus on incentive-

[2] For simplicity, we adopt a specification where cash flows are always positive and the firm does not need to issue shares to cover losses. Our findings would exactly the same if we assumed $dX_t = (\alpha + b_t - m_t)dt + \sigma dZ_t$, where dZ_t is the increment of a Brownian motion and $\sigma > 0$, as in DeMarzo and Urošević (2006). However, under the specification with Brownian shocks dZ_t, the firm would frequently issue new shares to cover negative cash flows, a scenario we regard as less realistic. Nonetheless, the two specifications are isomorphic, and cash flows are not a state variable for the blockholder's problem under either specification.

[3] The blockholder holds an equity stake and, by assumption, cannot write a contract with dispersed share-

compatible, short-term contracts that implement $m_t = 0$.

As shown in Appendix A.2, which formalizes the contracting problem, the contract $\mathcal{C}$ stipulates performance-sensitive payments $K_t dC_t$ to the manager over $[t, t+dt]$, with $dC_t = \beta_t(dX_t - (\alpha + b_t)dt)$ in equilibrium and pay-performance sensitivity β_t. To derive the incentive condition, consider a cash flow shock $dN_t = 1$. If the manager diverts the generated cash flow and sets $m_t = 1$, she derives private benefit λK_t. Otherwise, if she does not divert cash flow, the manager gets $\beta_t K_t$. Thus, the manager does not divert cash flow if and only if:

$$\beta_t \geq \lambda. \tag{3}$$

As will become clear, both the stock price and the blockholder payoff decrease in β_t, so setting $\beta_t = \lambda$ is optimal and the incentive condition (3) is tight. In our framework, it does not matter whether the contract is set by dispersed shareholders or the blockholder: both parties find it optimal to choose the contract that minimizes the manager's cash flow exposure among the set of incentive-compatible contracts.

Capital Investment. The firm chooses the investment rate $i_t \geq 0$ against convex investment cost $\frac{1}{2}K_t i_t^2$ to grow its capital stock, which evolves according to

$$dK_t = (\mu i_t - \delta)K_t dt. \tag{4}$$

In (4), $\mu \geq 0$ captures the efficiency of investment and $\delta \geq 0$ is the depreciation rate.

Debt Financing. Blockholders, such as private equity funds or hedge funds, often initiate capital structure changes after investing in firms. To capture the effects of debt financing on blockholder trading, we introduce dynamic short-term debt issuance as in e.g. Abel (2018), Hu, Varas, and Ying (2025), or Bolton, Wang, and Yang (2023). As in these models, the firm issues at any time t short-term debt with endogenous face value L_t that matures at time $t + dt$. The benefits of debt are captured in reduced form by assuming that creditors discount cash flows at a rate $r \leq \rho$.[4] If the firm defaults on its debt, it is liquidated, with the liquidation value normalized to zero.

holders. That is, the blockholder's contract with the firm is exogenous and restricted to equity. Moreover, shareholders cannot side-contract with management, i.e., the manager contracts with the firm as a whole, and any payments to/from the manager are borne by shareholders in proportion to their stake.

[4]This modeling is standard in dynamic financing and investment models (e.g., DeAngelo, DeAngelo, and Whited (2011) and Geelen, Hajda, Morellec, and Winegar (2024)), and is equivalent to assuming tax benefits of debt $r = (1 - \tau)\rho$, where $\tau \in (0, 1)$ is the corporate tax rate.

Timing. In our model, blockholders make decisions in two main areas: (i) corporate policies and (ii) trading. The timing over a short period $[t, t + dt]$ can be summarized as follows, with Appendix A.1 providing a more detailed timeline. First, corporate policies and blockholder effort are chosen, taking blockholder ownership $\theta_t = \theta$ and capital stock $K_t = K$ as given. Second, trading occurs at the end of the period and the capital stock updates, determining next-period stake $\theta_{t+dt} = \theta_t + d\theta_t$ and capital stock K_{t+dt}. Both the blockholder and dispersed shareholders anticipate how trading will influence future corporate decisions. In our continuous-time model, periods are infinitesimally small and, therefore, the exact timing—while useful in guiding intuition—does not matter. Specifically, in the chosen timing convention, the blockholder trades after debt is repaid. However, as shown in Section 2.3, trading could occur both before and after debt repayment without altering the results.

At the beginning of time t, after debt is repaid and before new debt is issued, we denote the blockholder's value function by V_t and equity value (i.e., firm value) by P_t. As we show later in Lemma 1, the firm optimally defaults on debt just after issuance when $V_t < \theta_t L_t$ or $P_t < L_t$, irrespective of whether the blockholder or investors control the default decision. Clearly, creditors find it suboptimal to extend an amount of debt L_t that triggers immediate default. As in Abel (2018), this implies an endogenous borrowing constraint[5]

$$\theta_t L_t \leq V_t \quad \text{and} \quad L_t \leq P_t. \tag{5}$$

This borrowing constraint states that the blockholder and dispersed shareholders are protected by limited liability, i.e., they cannot commit to a particular ownership stake and can always choose to exit and obtain a weakly positive payoff. Therefore, in equilibrium, their continuation payoffs must remain positive at all times. Given the dynamics of cash flows in equation (1), constraint (5) implies that debt is risk-free, so that the fair interest rate on debt is $r_t = r$ and the default time T equals $+\infty$. Section 5 introduces large jump shocks to the firm's capital stock, leading to default and rendering debt risky.

Holding Costs and Benefits. We make two assumptions that affect the blockholder's and dispersed shareholders' incentives to trade. First, as in prior models of blockholder investors (DeMarzo and Urošević, 2006; Marinovic and Varas, 2025) and dynamic trading models (Du and Zhu, 2017; Duffie and Zhu, 2017), the blockholder incurs a flow holding cost $\frac{1}{2}\pi\theta_t^2 \geq 0$, that scales with the size of its investment in the firm as captured by firm size K_t.

[5] A similar constraint arises as part of an optimal long-term contract in Hartman-Glaser, Mayer, and Milbradt (2025). Lemma 1 will establish this borrowing constraint formally.

As discussed in Duffie and Zhu (2017), this disutility flow may reflect, in reduced form, the blockholder's financial or capital constraints or higher cost of capital.

Second, we incorporate a holding benefit, $\eta(\theta)$, that dispersed investors derive from holding the stock. Appendix B.7 provides a micro-foundation for the holding benefit, where we derive the functional form (our findings go through under different functional forms):[6] $\eta(\theta) = \pi^I[\theta - \widetilde{\theta}]^+$, with $[\cdot]^+ = \max\{0, \cdot\}$, for parameters $\pi^I > 0$ and $\widetilde{\theta} \in [0, 1]$. This holding benefit introduces inelasticity in supply (Koijen, Richmond, and Yogo, 2024; Haddad, Huebner, and Loualiche, 2025), making it costly for the blockholder to build up large stakes.[7] Section 4 shows that without such holding benefit, we can solve for an equilibrium with qualitatively similar properties.

Blockholder Payoffs. We next specify the payoffs of the blockholder and dispersed investors, where we assume for now that the blockholder controls managerial contracts and financing and investment decisions. Note that we could equivalently assume that the manager chooses investment i_t and/or debt. Because K_t is observable and dK_t is deterministic, investment i_t and debt level could be contracted upon.

Assuming that the blockholder chooses the level of debt L_t and the manager's contract $\mathcal{C}_t$ is without loss in generality because, as we demonstrate later, both the blockholder and the dispersed investors agree on the optimal level of debt and the managerial contract. However, the allocation of control rights over investment decisions affects equilibrium outcomes, as we discuss in more detail in Section 3.

At time $t = 0$, the firm starts with zero debt, i.e., $L_{0^-} = 0$, and there is a potentially discrete debt issuance $dL_0 > 0$. When the blockholder holds a stake in the firm, it chooses its effort b_t, trading $d\theta_t$, debt issuance dL_t, investment i_t, and contract $\mathcal{C}_t$ to maximize:

$$V_0 = \max_{(b_t, d\theta_t, dL_t, i_t, \mathcal{C}_t)_{t\geq 0}} \mathbb{E}_0\left[\int_0^T e^{-\rho t}\left\{\theta_t\left[K_t dX_t - \frac{K_t i_t^2}{2}dt - K_t dC_t + L_t(\rho - r)dt\right] - K_t\left(\frac{\pi\theta_t^2}{2} + \frac{\kappa b_t^2}{2}\right)du - (P_t + dP_t)d\theta_t\right\}\right]. \tag{6}$$

[6] This micro-foundation links the holding benefit to heterogeneous demand (Koijen and Yogo, 2019) among certain dispersed investors—such as index investors—who assign significant value to the stock.

[7] This assumption ensures the existence of a unique Markov equilibrium with continuous prices and trading in our model. As we show in Section 4, without such holding benefit, we can solve for an equilibrium with qualitatively similar properties, but featuring both smooth and lumpy trading, as it may become optimal for the blockholder to conduct large trades, notably in the context of public-to-private transactions. By contrast, the blockholder's holding cost, while affecting trading dynamics, is not essential for our formal results, notably for the existence and uniqueness of an equilibrium.

Because the blockholder owns a fraction θ_t of the firm, it collects a fraction θ_t of dividends $(K_t dX_t - \frac{K_t i_t^2}{2}dt - K_t dC_t + L_t(\rho - r)dt)$, while incurring the full cost of effort $\frac{\kappa b_t^2}{2}$ and the flow holding cost $\frac{\pi\theta_t^2}{2}$. The term $-(P_t + dP_t)d\theta_t$ captures the payoff from trading over a short time period $[t, t + dt]$. The blockholder has price impact and trades over $[t, t + dt]$ at "end-of-period" price $P_{t+dt} = P_t + dP_t$. Although we do not explicitly account for it in our notation, the blockholder's expectation in (6) is taken under its information set, which involves its own effort choice b_t. All controls are chosen without commitment.

To understand how debt issuance affects the blockholder's payoff, note that issuing an additional dollar of debt at time t and paying it out as a dividend, while repaying debt and interest at time $t + dt$, changes the blockholder's payoff over $[t, t + dt]$ by $\theta_t 1 - \theta(1 - \rho dt)(1 + rdt) = \theta_t(\rho - r)dt$, since terms of order $(dt)^2$ are negligible in continuous time. As such, the level of debt changes the blockholder's flow payoff by $\theta_t(\rho - r)L_t dt$.

Investors' Payoff and Stock Price. Due to the blockholder's private effort and holding costs as well as dispersed shareholders' holding benefit, the value of the firm *for dispersed shareholders* differs from V_0 (scaled by θ) and equals

$$P_0 = \mathbb{E}_0 \left[\int_0^T e^{-\rho t} \left(K_t \left(dX_t - \frac{i_t^2}{2}dt \right) - K_t dC_t + (\rho - r)L_t dt + K_t \eta(\theta_t) dt \right) \right], \tag{7}$$

where investors anticipate the level of blockholder effort and managerial cash diversion when forming expectations; in optimum, the information sets and expectations of the manager, the blockholder, and dispersed investors coincide. As shown by (7), the stock price is the expected discounted value of all future dividends, including the holding benefit $K_t\eta(\theta_t)$.

2 Model Solution and Markov Equilibrium

To ensure that valuations remain finite and that $\mu i < r + \delta$ always holds in equilibrium, both with and without blockholder ownership, we assume that

$$(r + \delta)^2 > \frac{\mu^2(1 + 2\kappa\alpha)}{\kappa}. \tag{8}$$

Second, to ensure that some blockholder ownership is optimal and occurs in equilibrium, we assume that holding costs are not prohibitively large, in that

$$\pi < \frac{1-\lambda^2}{\kappa}. \tag{9}$$

If this condition is not satisfied, the blockholder does not invest in the firm ($\theta_t = 0$), leading to the dispersed ownership benchmark discussed in the following proposition.

Proposition 1 (Dispersed ownership benchmark)**.** *In the dispersed ownership benchmark, policies are chosen to maximize firm value. The managerial contract satisfies $\beta_t = \lambda$ while blockholder effort is zero. Model quantities scale linearly in $K_t = K$ with debt issuance satisfying $L_t^0 = K_t p^0$ for $\rho > r$ and $L_t^0 \in [0, K_t p^0]$ for $\rho = r$, and scaled firm value satisfying $P_t^0 = K_t p^0$ with $p^0 = \frac{r+\delta-\sqrt{(r+\delta)^2-2\,\alpha\,\mu^2}}{\mu^2}$ and $i^0 = \mu p^0$.*

In the dispersed ownership benchmark, model quantities, including debt, scale with K_t or are constant, as is the case for effort or the investment rate. Any dynamics in the model will arise from the blockholder's trading, with the blockholder stake emerging as key state variable in equilibrium.

2.1 Continuous, Scaled Markov Equilibrium

We now turn to solving the model with a large shareholder. Our model features a dynamic trading game in which the blockholder acts as a large strategic player that internalizes the impact of its trades on prices. The blockholder faces a continuum of competitive, dispersed shareholders who take prices as given. We solve for a Markov Equilibrium with the blockholder's ownership stake θ_t and capital K_t as state variables. In particular, we focus on equilibria with the following properties:

1. **Markovian Payoffs with Scaling Property and Continuity.** The value function of the blockholder satisfies (6) and the firm's stock price satisfies (7) given equilibrium policies. Payoffs scale with the capital stock K_t, in that $V_t = K_t v(\theta_t)$, $L_t = K_t \ell(\theta)$, and $P_t = K_t p(\theta_t)$ for continuous functions $v(\theta)$ and $p(\theta)$.

2. **Markovian Equilibrium Policies.** The blockholder takes the *current* state $(K_t, \theta_t) = (K, \theta)$ as given when optimizing its payoff according to (6), i.e., equilibrium policies are Markov processes. Equilibrium efforts $(a_t, m_t)_{t \geq 0}$, investment $(i_t)_{t \geq 0}$, debt $(L_t)_{t \geq 0}$ are

Markov processes chosen according to (6). In particular, the trading strategy $(d\theta_t)_{t\geq 0}$, maximizing (6), is a Markov process that determines, given $\theta_t = \theta \in [0,1]$, the value of $\lim_{s\downarrow t} \theta_s = \theta_t + d\theta_t$ where $d\theta_t \in [-\theta_t, 1-\theta_t]$. The trading strategy solves for any $t \geq 0$:

$$d\theta_t \in \arg \max_{\Delta \in [-\theta_t, 1-\theta_t]} K_t \left\{ v(\theta_t + \Delta) - \Delta p(\theta_t + \Delta) \right\}, \tag{10}$$

given the price $P(K, \theta) = Kp(\theta)$ and the value function $V(K, \theta) = Kv(\theta)$.

We refer to equilibria satisfying these properties as *continuous, scaled Markov equilibria*. We will demonstrate the existence of a unique *continuous, scaled Markov equilibrium*, provided that $\eta(1) = \pi^I(1 - \widetilde{\theta})$ satisfies condition (A.1). This condition states that investors' demand for the stock is sufficiently inelastic, so that it is too costly for the blockholder to take over the entire firm. Given this, the blockholder's equilibrium trading is smooth, in that $d\theta = \dot{\theta}dt$ for an endogenous trading rate $\dot{\theta}$. For simplicity, we will hereafter refer to $p(\theta)$, $v(\theta)$, and $\ell(\theta)$ as the stock price, value function, and debt, respectively, omitting the term "scaled" for convenience. Unless otherwise mentioned, we assume that (A.1) holds. In Section 4, we relax this assumption and show that our findings remain qualitatively unchanged, yet the blockholder may conduct large trades, e.g., in the context of a full takeover.

2.2 Effort Choice and Contracting

A first channel through which the blockholder influences firm value is through their effort. The following proposition, proven in Appendix A.2, characterizes blockholder effort and the manager's pay performance sensitivity β.

Proposition 2. *The manager's pay performance sensitivity satisfies $\beta = \lambda$. Blockholder effort satisfies*

$$b = \frac{\theta(1-\lambda)}{\kappa}. \tag{11}$$

As neither the blockholder nor dispersed shareholders benefit from increasing managerial incentives above λ, it holds that $\beta = \lambda$. Blockholder effort in (11) increases with ownership θ and decreases with the severity of managerial moral hazard, i.e., $\beta = \lambda$. Indeed, since blockholder effort is unobservable and cannot be contracted with the manager, a fraction $\beta = \lambda$ of the benefits of effort are shared with management through the incentive contract, reducing blockholder effort. In addition, when the blockholder only owns part of the firm's equity, the benefits of higher effort are shared with other investors. That is, when $\theta < 1$ or

$\lambda > 0$, the blockholder incurs the full cost of effort but captures only part of its benefits. As we will show, managerial agency conflicts (λ) crucially affect the blockholder's trading decisions and ownership dynamics.

2.3 Trading and Default

We next turn to the decision to default on debt and the blockholder's decision to trade. We define the blockholder's (scaled) valuation of the firm:

$$y(\theta) := \frac{v(\theta)}{\theta}, \tag{12}$$

which will play a key role in the analysis. As we show, the blockholder's valuation of the firm is generally below that of dispersed shareholders. The blockholder's valuation of the firm's equity is then $y(\theta) - \ell$, dispersed shareholders' valuation of equity reads $p(\theta) - \ell$, and the borrowing constraint (5) can be rewritten as $\ell \leq \min\{y(\theta), p(\theta)\}$. The following Lemma shows that there is no default if and only if (5) holds.

Lemma 1. *Consider state (K, θ) with $\theta > 0$ reached after time $t > 0$ with positive probability and suppose $p(0) < y(\theta) \leq p(\theta)$, which is satisfied in equilibrium. The following holds:*

1. *The firm does not default if and only if constraint (5) holds, irrespective of whether the blockholder or dispersed shareholders control the default decision.*

2. *As long as (5) holds, the gains from trade are the same, irrespective of whether the blockholder trades just before or after debt is repaid (or at both times).*

Intuitively, if (5) holds, both dispersed shareholders and the blockholder are better off not defaulting. On the other hand, if (5) does not hold, we show in the proof of Lemma 1 that the blockholder would optimally sell its entire stake (at any weakly positive price), causing the stock price to drop and making it optimal for dispersed shareholders to default. Lemma 1 implies that, regardless of the allocation of control rights, the firm defaults immediately after debt issuance whenever $\ell > \min\{y(\theta), p(\theta)\}$. Thus, constraint (5) must hold in equilibrium and pins down the maximum amount of debt the firm can issue.

With (5) being met, there is no default. The second part of Lemma 1 shows that, when (5) holds, the blockholder realizes the same payoff and chooses the same next-period stake θ_{t+dt}, irrespective of whether it trades just before or after debt is repaid (or at both times).[8]

[8] Taking the limit $\hat{\theta} \to \theta$, this argument also applies to infinitesimal trades.

Thus, the blockholder is indifferent between the timing of trade over $[t, t+dt]$, and optimally trades after debt repayment. Likewise, dispersed shareholders are also indifferent regarding the blockholder's timing of trade, i.e., the pricing of the stock is not affected by the exact timing of trade.[9] For convenience and without loss of generality, we consider below that the blockholder *only* trades *after* debt is repaid.

2.4 Blockholder Value Function

We next want to understand how trading and corporate investment and financing interact. To do so, we first show in Appendix A.7 that the blockholder optimally trades smoothly in equilibrium, i.e., $d\theta = \dot{\theta}dt$ and that its value function satisfies the Hamilton-Jacobi-Bellman (HJB) equation:

$$(\rho+\delta)v(\theta) = \max_{\ell,\dot{\theta},i}\left[\theta\left(\alpha+b-\frac{i^2}{2}+(\rho-r)\ell\right)-\frac{\pi\theta^2}{2}-\frac{\kappa b^2}{2}+\mu i v(\theta)+\dot{\theta}\left(v'(\theta)-p(\theta)\right)\right], \tag{13}$$

which is solved subject to the borrowing constraint $\ell \leq \min\{p(\theta), y(\theta)\}$ derived in Lemma 1 and the incentive condition (11) derived in Proposition 2. In (13), the term $\dot{\theta}\left(v'(\theta)-p(\theta)\right)$ represents the gains associated with smooth trading. For an interior solution where $\dot{\theta} \in (-\infty, \infty)$, it must hold that:

$$p(\theta) = v'(\theta). \tag{14}$$

Indeed, the blockholder is willing to pay $v'(\theta)$ dollars for an additional unit of stock. The cost of purchasing this additional unit equals the market price of the stock $p(\theta)$ (i.e., dispersed investors' valuation). In equilibrium, the marginal benefit of purchasing an additional share equals the marginal cost, leaving the blockholder indifferent between trading and not trading.

Two key points emerge from this condition. First, while the blockholder earns no gains on the marginal trade itself (since $v'(\theta) = p(\theta)$), it can still realize gains on its infra-marginal holdings when trading smoothly. These infra-marginal gains are embedded in the value function $v(\theta)$, which plays a central role in the analysis. In particular, we will use this to characterize the total gains from trade, determine the portion that the blockholder can capture, and identify the equilibrium trading rate. Second, because (14) holds in equilibrium, the value function $v(\theta)$ can be determined "as if" the blockholder could not trade and $v(\theta)$

[9] This finding holds more generally. However, it is immediate to see that, because the blockholder will trade smoothly in equilibrium, the timing of trade does not affect shareholders' payoff and the stock price.

coincides with the payoff $\hat{v}(\theta)$ that would prevail in the absence of trading opportunities. In other words, $\hat{v}(\theta)$ is the solution to (13) when $\dot{\theta} = 0$, which can be solved explicitly, as shown in equation (A.6) in the Appendix.

The following lemma characterizes the blockholder's value function and the stock price under smooth trading:

Lemma 2. *The following holds when the blockholder trades smoothly in equilibrium:*

1. *The value function of the blockholder satisfies $V(K, \theta) = Kv(\theta)$, with $v(\theta)$ solving (13) and satisfying $v(\theta) = \hat{v}(\theta)$ where $\hat{v}(\theta)$ is available in closed form in (A.6). The stock price satisfies $P(\theta, K) = Kp(\theta)$ for a function $p(\theta)$ satisfying $p(\theta) = v'(\theta)$ and solving (16) below; $p(\theta)$ is available in closed form in (A.7) with $p(0) = p^0$.*

2. *The value function of the blockholder $v(\theta)$ is increasing and convex in its stake θ, in that $v'(\theta), v''(\theta) > 0$ with $v(0) = 0$. As such, the stock price $p(\theta) = v'(\theta) > y(\theta)$ increases with the equity stake of the blockholder.*

As established in Lemma 2, the value function of the blockholder is increasing and convex in θ. Together with (14) and $v(0) = 0$, this implies $y(\theta) < p(\theta)$. That is, the blockholder's marginal valuation $v'(\theta)$ of an additional unit of stock coincides with the stock price $p(\theta)$. However, its valuation of the firm $y(\theta)$ lies below the marginal valuation and the stock price. As a result, the borrowing constraint (5) reduces to $\ell \leq y(\theta)$. Since the blockholder's scaled payoff increases in ℓ, it is optimal to issue as much debt as its borrowing constraint allows. Therefore, the borrowing constraint binds. This leads to the following result:

Corollary 1. *Book leverage, $\ell(\theta) = y(\theta)$, and investment, $i(\theta) = \mu y(\theta)$ increase with the blockholder's ownership stake θ.*

Larger blockholder ownership and thus engagement via effort enhance firm value, relaxing the firm's borrowing constraints and increasing the marginal value of capital and investment. That is, blockholder ownership (and trading) affects corporate policies. We next show that corporate policies lead to gains from trade and affect the blockholder's trading rate.

2.5 Equilibrium Trading

To characterize equilibrium trading, we first determine the blockholder's valuation of an additional unit of stock. Differentiating (13) and using $p(\theta) = v'(\theta) = \frac{dv(\theta)}{d\theta}$ gives:

$$\underbrace{(\rho+\delta-\mu i)p(\theta)}_{(\rho+\delta-\mu i)\frac{dv(\theta)}{d\theta}} = \underbrace{\alpha + b - \frac{i^2}{2} + (\rho - r)\ell(\theta) - \pi\theta +}_{(\rho+\delta-\mu i)\frac{\partial v(\theta)}{\partial \theta}} \underbrace{\frac{\theta\lambda(1-\lambda)}{\kappa}}_{(\rho+\delta-\mu i)\left(\frac{\partial v(\theta)}{\partial b}\frac{\partial b}{\partial \theta}\right)} + \underbrace{\theta(\rho - r)\ell'(\theta)}_{(\rho+\delta-\mu i)\left(\frac{\partial v(\theta)}{\partial \ell}\frac{\partial \ell}{\partial \theta}\right)}. \tag{15}$$

In addition to (15) (derived from the valuation equation of blockholders), $p(\theta)$ satisfies the pricing equation of dispersed price-taking shareholders (resulting from (7)):

$$(\rho+\delta-\mu i)p(\theta) = \alpha + b - \frac{i^2}{2} + (\rho - r)\ell(\theta) + p'(\theta)\dot{\theta} + \eta(\theta). \tag{16}$$

Combining (15) and (16) yields the blockholder's equilibrium trading rate:

$$\dot{\theta} = \frac{1}{p'(\theta)}\left[\underbrace{\theta(\rho - r)\ell'(\theta)}_{\substack{\text{Debt financing}\\ \text{channel}}} + \underbrace{\frac{\theta\lambda(1-\lambda)}{\kappa}}_{\substack{\text{Effort-incentive}\\ \text{channel}}} - \underbrace{(\pi\theta + \eta(\theta))}_{\substack{\text{Effective}\\ \text{holding cost}}}\right]. \tag{17}$$

Corporate Policies and Gains from Trade. Equation (17) can be more generally written as:

$$\dot{\theta} = \frac{1}{p'(\theta)}\left[(\rho+\delta-\mu i)\sum_{x\in\mathcal{X}}\frac{\partial v(\theta)}{\partial x}\frac{\partial x}{\partial \theta} - \big(\pi\theta + \eta(\theta)\big)\right], \tag{18}$$

where $\mathcal{X} := \{b, \ell, i\}$ is the set of policies chosen by the blockholder. As expected, the trading rate decreases with both the blockholder's holding costs and the holding benefits enjoyed by dispersed investors (capturing inelastic share supply). Equations (17) and (18) also show that gains from trade can arise through changes in corporate policies x. These gains are governed by the product of (A) the marginal effect of policy x on the blockholder's value, $\frac{\partial v(\theta)}{\partial x}$, and (B) the marginal effect of ownership on that policy, $\frac{\partial x}{\partial \theta}$.

Component (A) reflects whether the policy is chosen optimally from the blockholder's perspective or is distorted by frictions or by other agents' objectives. If a policy already maximizes the blockholder's value—in that the first-order condition holds, $\frac{\partial v(\theta)}{\partial x} = 0$—the blockholder has no incentive to adjust its stake, and there are no gains from trade associated with that policy. By contrast, when a policy is distorted away from the blockholder's preferred level, so that $\frac{\partial v(\theta)}{\partial x} \neq 0$, the blockholder trades to influence the policy in the desired direction. In particular, a positive (negative) $\frac{\partial v(\theta)}{\partial x}$ provides incentives to buy (sell).

Component (B) operates through blockholder effort, which determines the magnitude, but not the sign, of the overall gains from trade. Other policies depend on blockholder ownership only indirectly through this effort channel, with (B) capturing the strength of this pass-through. Blockholder effort is pinned down by incentive-compatibility condition $b(\theta) = \frac{(1-\lambda)\theta}{\kappa}$. Effort therefore increases with ownership, in that $\frac{\partial b}{\partial \theta} = \frac{1-\lambda}{\kappa}$. The pass-through from ownership to corporate policies (B) is weaker when managerial agency conflicts λ are higher or when the cost of effort κ is larger. It vanishes when either $\lambda \to 1$ or $\kappa \to \infty$.

Effort-Incentive Channel. The origins of gains from trade arising from the product of (A) and (B) can be transparently illustrated for $x = b$, where $\frac{\partial b}{\partial \theta} = \frac{1-\lambda}{\kappa}$ and $(\frac{\lambda\theta}{\rho+\delta-\mu i})\frac{\partial v(\theta)}{\partial b} = \lambda\theta$. As discussed in Section 2.2, managerial agency conflicts crowd out blockholder incentives and the resulting blockholder effort is inefficiently low. The gains from additional effort, $\frac{\lambda\theta}{\rho+\delta-\mu i}$, increase with managerial agency frictions λ, which distort blockholder effort, and with the size of the blockholder's holdings θ, on which the gains are realized.

Together, these forces generate gains from trade $\frac{\theta\lambda(1-\lambda)}{\kappa}$, which are hump-shaped in λ. When agency frictions are low (λ small), there is little scope for improvement; when they are very high (λ large), ownership has little effect on effort b. At intermediate levels of moral hazard, the product $\lambda(1-\lambda)$ peaks, producing the strongest incentive for the blockholder to expand its ownership stake in order to bring effort closer to the desired level.

Debt Financing Channel. For $x = \ell$, we have $(\rho + \delta - \mu i)\frac{\partial v(\theta)}{\partial \ell} = \theta(\rho - r) > 0$, and $\frac{\partial \ell}{\partial \theta} = y'(\theta) > 0$. The first effect captures the additional benefits from debt ($\theta(\rho-r)$) that the blockholder enjoys on its existing holdings. The latter effect—that debt financing increases with ownership—operates through effort: Higher ownership θ induces greater effort, which raises firm value and thereby relaxes the (binding) borrowing constraint (5). This effect is proportional to $\frac{\partial b}{\partial \theta} = \frac{1-\lambda}{\kappa}$ and thus vanishes when either $\lambda \to 1$ or $\kappa \to \infty$.[10] Together, these effects generate positive gains from trade associated with debt financing.

Formally, this debt-financing channel arises from a commitment friction: firm owners cannot commit to repaying debt, and the firm may strategically default, triggering inefficient liquidation. The blockholder's valuation of its stake determines the likelihood of default and, in turn, the amount of debt the firm can issue for a given level of default risk. In our baseline, default risk is zero (one) when $\ell(\theta) \leq y(\theta)$ ($\ell(\theta) > y(\theta)$), while the firm exhausts its (risk-

[10] When $\mu = \pi = 0$, this relation is explicit as $\frac{\partial \ell}{\partial \theta} = \frac{1-\lambda}{\kappa}\frac{1+\lambda}{2(r+\delta)}$, but the result holds more generally.

free) borrowing capacity, $\ell(\theta) = y(\theta)$. Greater blockholder ownership strengthens effort incentives, and raises firm value and borrowing capacity, unlocking greater benefits of debt. This channel also operates when the firm is subject to large cash flow shocks (Section 5).[11]

Investment. When the blockholder controls investment, investment solves the first-order condition $\frac{\partial v}{\partial i} = 0$, so there are no direct gains from trade associated with investment. However, investment affects firm value and thus the strength of the debt-financing channel.

Overall Gains from Trade. The sign of the trading rate $\dot{\theta}$ in (17) is determined by the balance between the trading gains associated with corporate policies and the effective holding costs. These trading gains scale with the blockholder's stake and vanish at $\theta = 0$, while the holding cost dominates near $\theta = 1$, so $\dot{\theta}$ starts at zero, becomes negative at high ownership, and can cross zero only finitely many times. The following proposition characterizes the resulting stationary points and the global dynamics of θ.

Proposition 3 (Dynamics of the blockholder stake)**.** *The equilibrium trading rate $\dot{\theta} = \dot{\theta}(\theta)$ is given in (17), and satisfies $\dot{\theta}(0) = 0$ and $\dot{\theta}(1) < 0$. Define (for $\mu > 0$ and indicator $\mathbb{I}\{\cdot\}$):*[12]

$$\underline{\Theta} := \frac{\kappa}{\mu^2 (1 - \lambda^2 - \kappa\pi)} \left[(r+\delta)^2 - 2\mu^2\alpha - \frac{(\rho - r)^2 (1 - \lambda^2 - \kappa\pi)^2}{4 (\kappa\pi - \lambda(1-\lambda))^2} \right] \mathbb{I}\{\kappa\pi > \lambda(1-\lambda)\}. \quad (19)$$

There exist thresholds $0 \leq \underline{\theta} \leq \overline{\theta}$ such that $\dot{\theta}(\overline{\theta}) = \dot{\theta}(\underline{\theta}) = 0$ and

1. *When $\underline{\Theta}$ is negative, then $\underline{\theta} = 0 < \overline{\theta} < 1$. Then, $\dot{\theta} > 0$ for all $\theta \in (0, \overline{\theta})$ and $\dot{\theta} < 0$ for all $\theta \in (\overline{\theta}, 1)$. For any initial state $\theta_0 > 0$, the blockholder stake converges to $\overline{\theta}$.*

2. *When $\underline{\Theta}$ takes intermediate values (i.e., $\underline{\Theta} \in (0, \widetilde{\theta})$), then $\underline{\theta} = \underline{\Theta} < \overline{\theta} < 1$. Then, $\dot{\theta} > 0$ for all $\theta \in (\underline{\theta}, \overline{\theta})$ and $\dot{\theta} < 0$ for all $\theta \in (0, \underline{\theta})$ and $\theta \in (\overline{\theta}, 1)$. For initial stakes $\theta_0 > \underline{\theta}$, the blockholder stake converges to $\overline{\theta}$. Initial stakes $\theta_0 < \underline{\theta}$ lead to gradual exit.*

3. *When $\underline{\Theta}$ is large (i.e., $\underline{\Theta} > \widetilde{\theta}$), then $\underline{\theta} = \overline{\theta} = 0$ and $\dot{\theta} < 0$ for all $\theta \in [0, 1]$. For any initial state $\theta_0 > 0$, the blockholder gradually divests and exits.*

The term $\underline{\Theta}$ inversely captures the blockholder's incentives to hold a "large" ownership stake, with larger $\underline{\Theta}$ being linked to lower target ownership. As shown by equation (19), this term depends on model parameters that capture the blockholder's impact (effort cost)

[11] Section 5, we show that higher ownership reduces the firm's default probability for a given level of debt and, conversely, the amount of debt the firm can issue holding default risk fixed.

[12] The case $\mu = 0$ is nested by taking the limit $\mu \to 0$.

or reflect corporate policies, such as the interest on deb r or the efficiency of investment μ. Corollary 3 in Section 2.7 analyzes these effects in greater detail.

2.6 Equilibrium Summary, Entry, and Ownership Dynamics

We summarize the unique continuous, scaled Markov equilibrium in Proposition 4.

Proposition 4 (Existence and uniqueness of equilibrium)**.** *There exists a unique continuous, scaled Markov equilibrium with state variables (K, θ). In this Markov equilibrium:*

1. *All payoffs scale with K in that $V(K, \theta) = Kv(\theta)$, $P(K, \theta) = Kp(\theta)$, and $L(K, \theta) = K\ell(\theta)$. The blockholder's scaled value function satisfies (13), i.e., $v(\theta) = \hat{v}(\theta)$ and the stock price satisfies $p(\theta) = \hat{v}'(\theta)$.*
2. *The blockholder trades smoothly at rate $\dot{\theta}$ given in (17).*
3. *There exist thresholds $0 \leq \underline{\theta} \leq \overline{\theta} \leq 1$, characterized in Proposition 3, such that $\dot{\theta} = 0$ for $\theta = 0, \underline{\theta}, \overline{\theta}$ (where $\underline{\theta} = \overline{\theta}$ is possible). The trading rate $\dot{\theta}$ is negative for small $\theta \in (0, \underline{\theta})$ and large $\theta \in (\overline{\theta}, 1]$. The trading rate $\dot{\theta}$ is positive for $\theta \in (\underline{\theta}, \overline{\theta})$.*

While we solve for the equilibrium given an initial blockholder stake, we can also endogenize entry. Blockholder entry increases firm value. However, if this value creation is fully reflected in the price at which the blockholder initially acquires their equity stake, due to a free-rider problem among dispersed shareholders, the blockholder cannot capture the gains from activism and has no incentive to invest in the first place. In practice, blockholders can accumulate sizable ownership shares before disclosing them (Collin-Dufresne and Fos, 2016). We thus assume that the blockholder can acquire a fraction $\varphi \in [0, 1]$ of the firm at the price P_0^0 that prevails under dispersed ownership.[13] The remainder $\theta_0 - \varphi$ is bought at a price P_0 that reflects the gains from activism. Then, the blockholder pays $R(\theta_0)K_0$ dollars to acquire a stake θ_0, with $R(\theta_0) := \eta p(0) + \max\{0, \theta_0 - \varphi\}p(\theta_0)$, and enters if and only if

$$E(\theta_0) := \max_{\theta_0 \in [0,1]} K_0 \big[v(\theta_0) - R(\theta_0)\big] \geq 0. \tag{20}$$

The following proposition characterizes the blockholder's entry decision and initial stake.

[13] In the U.S., owners of more than 5% of the equity of a public firm are required to file a report with the SEC, at which point the price adjusts to reflect this information. Collin-Dufresne and Fos (2015) report that the average blockholder holds 7.51% of the target's shares when making its first public disclosure.

Proposition 5. *The blockholder's initial stake satisfies* $\theta_0 = \varphi$.

When the blockholder enters, it only acquires the stake φ, which it can buy at a discount. To gain some intuition, note that—whether at time $t = 0$ or afterward—the blockholder optimally trades smoothly, and large lumpy trades are strictly sub-optimal. Choosing $\theta_0 > \varphi$ is equivalent to acquiring first a stake φ in the firm and immediately buying $\theta_0 - \varphi > 0$ units of the firm equity in a large lumpy trade. However, this is sub-optimal.

We close this section by characterizing long-run ownership dynamics, which directly follows from Proposition 3 but, for a better overview, is stated in the following form:

Corollary 2 (Long-run Ownership Dynamics)**.** *In the limit* $t \to \infty$*, we have* $\theta_\infty := \lim_{t\to\infty} \theta_t \in \{0, \underline{\theta}, \overline{\theta}\}$*. When* $\theta_0 > \underline{\theta}$*, then* $\theta_\infty = \overline{\theta}$*. When* $\theta_0 = \underline{\theta}$*, then* $\theta_\infty = \underline{\theta}$*. When* $\theta_0 < \underline{\theta}$*, then* $\theta_\infty = 0$*.*

The model generates reinforcing ownership dynamics that lead to either relatively low or high blockholder ownership in the long-run. In particular, abstracting from knife-edge cases, blockholder ownership either approaches the relatively high target level $\overline{\theta}$ or zero. These long-run ownership dynamics are consistent with evidence from US capital markets, where Edmans and Holderness (2017) document that blockholder stakes in public firms tend to cluster at relatively low levels. At the same time, a growing share of large firms are privately held, featuring highly concentrated ownership by PE funds.

In this context, we later show in Section 4 how our model can produce public-to-private transactions and a full takeover, with $\overline{\theta} \to 1$. This occurs when, for instance, the benefits to debt are large or investment opportunities are sufficiently strong. Finally, note that in our baseline model, parameters are constant, while the state variables (K, θ) evolve over time. Because all value functions and prices scale with K, the meaningful dynamics in our model stem solely from the blockholder's trading. That is, model dynamics are entirely endogenous, and given the parameters, one can perfectly predict whether the blockholder increases its stake towards a stationary point (potentially acquiring the entire firm) or exits as $t \to \infty$. Section 6 extends the model to accommodate stochastic dynamics in the blockholder's ownership stake, making the dynamics of corporate policies also stochastic.

2.7 The Determinants of Blockholder Ownership

We now use the long-run dynamics characterized above to study how firm characteristics shape blockholder ownership. As shown in Proposition 2, there are three possible scenarios concerning blockholder entry and subsequent trading behavior. First, the blockholder chooses not to enter when the costs of exerting effort (κ), holding the stock (π), or dealing with managerial agency conflicts (λ) are too high. Second, the blockholder enters to capitalize on the lower acquisition price at entry, but eventually exits the firm by gradually divesting its stake. Third, the blockholder enters the firm and steadily increases its ownership, potentially acquiring the entire firm over time.

Corollary 3 determines how firm characteristics—such as managerial agency conflicts λ or the efficiency of investment μ—or the availability of debt financing—captured by r—affect blockholder ownership in the long run.

Corollary 3 (Long-run ownership and comparative statics)**.** *The following holds:*

1. *The blockholder is more likely to buy (and less likely to sell) when μ is larger, or r and κ are lower. When $\underline{\theta} > 0$, then $\underline{\theta}$ decreases in μ, and increases in r and κ. When $\overline{\theta} > 0$, then $\overline{\theta}$ increases in μ, and decreases in r and κ.*

2. *Suppose $\rho - r = 0$. If $4\kappa\pi > 1$, we have $\theta_\infty = \lim_{t\to\infty} \theta_t = 0$. If $4\kappa\pi \leq 1$, we have $\theta_\infty > 0$ whenever $\lambda \in [\lambda_-, \lambda_+]$ and $\theta_\infty = 0$ otherwise. Thus, the blockholder engages with the firm over the long term at intermediate levels of agency conflict λ, but exits when λ is too large or too small.*

Blockholder Ownership and Debt as Complements. Corollary 3 shows that blockholder ownership is larger when the net benefits of debt are large (r is low). At the same time, Lemma 2 and Corollary 1 show that firm debt (book leverage) $\ell(\theta)$ increases with the blockholder's stake. In particular, the formation of a blockholder stake should be associated with increased leverage. Taken together, blockholder ownership and debt levels exhibit a positive association. That is, they are complements in the firm's capital structure.

Investment. We also find that, while the level of investment increases with blockholder ownership, better investment opportunities μ (indirectly) generate gains from trade, leading to higher blockholder ownership. Similar to the relation between debt and blockholder ownership, there is a two-way feedback between blockholder ownership and investment.

Managerial Agency Conflicts. In addition, Corollary 3 shows that the trading rate is generally hump-shaped in the level of agency conflicts λ. When agency conflicts are high or low, the blockholder has no incentives to increase its stake and exert more effort, as either the benefits of effort are captured by management (high λ) or the moral hazard problem is weak (low λ), implying that the effort channel in (17) is quantitatively small. Instead, for

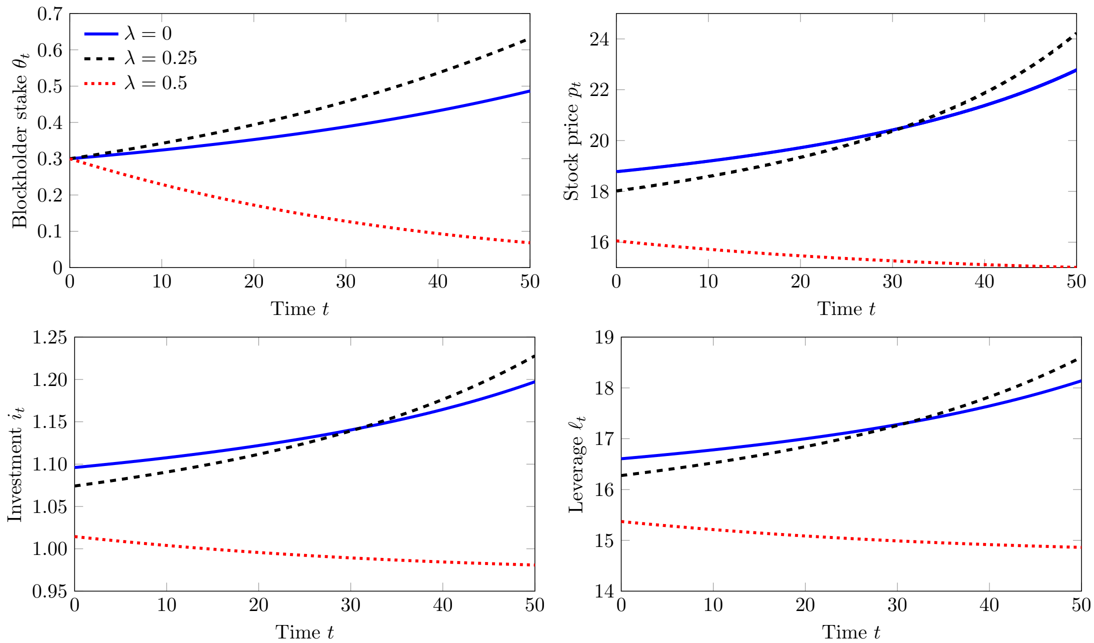

Figure 1: **Agency Conflicts and Model Dynamics.** We numerically calculate the equilibrium under varying levels of agency conflicts λ. The parameters are $\alpha = \kappa = 1$, $\rho+\delta = 0.2$, $r + \delta = 0.1$, $\mu = 0.066$, and $\pi = 0.6$, as well as $\widetilde{\theta} = 0.99$. We set $\theta_0 = 0.3$.

intermediate λ, blockholder effort is inefficiently low for any given θ and can be increased by increasing θ. Overall, the model predicts larger blockholder ownership in firms with intermediate levels of agency conflicts: blockholders tend to avoid firms that are either efficiently governed (low agency) or poorly governed with high managerial entrenchment.

These effects are illustrated in Figure 1. The upper left panel shows that the trading rate is non-monotonic in λ, with the blockholder's stake increasing rapidly when $\lambda = 0.25$, more slowly when $\lambda = 0$, and declining over time when $\lambda = 0.5$. The remaining panels show how the interaction between agency frictions (λ) and blockholder trading (θ) shapes firm investment, stock prices, and the debt-to-asset ratio.

Holding Costs. Lastly, as expected, Corollary 3 shows that the blockholder is more likely to acquire a larger stake in the firm—and thus blockholder ownership tends to be larger—if it has a lower cost of effort or a lower holding cost.

3 Control Rights over Financing and Investment

In our baseline model, the blockholder acts as a controlling shareholder and determines corporate policies. This section studies an alternative regime in which policies are chosen to maximize dispersed shareholder value. This assumption could capture governance structures such as anti-takeover or entrenchment provisions (Bebchuk et al., 2009) that limit blockholder control.

Our key result is that the blockholder and dispersed shareholders agree on the level of debt financing (i.e., it does not matter who chooses it), while they do not for investment. Consequently, investment policies depend on who is in control of corporate policies. Control, in turn, influences the blockholder's stake in equilibrium.

3.1 Financing and Managerial Compensation

Note that the blockholder and dispersed shareholders agree on the optimal managerial contract and choose the contract that minimizes agency rents and subject to (3), leading to $\beta_t = \lambda$ (as in the baseline). Intuitively, either party's payoff is maximized for $\beta_t = \lambda$, while raising β_t reduces blockholder incentives and the payoff of all shareholders.

In addition, the blockholder agree on the level of debt financing, which satisfies, as in the baseline, $L = K\ell(\theta)$ for $\ell(\theta) = y(\theta)$. The reason is the following. Conditional on no default and ignoring the borrowing constraint (5), both the blockholder's payoff $v(\theta)$ and the stock price $p(\theta)$ increase in ℓ. However, even if dispersed shareholders had control over debt policy, they could not increase leverage beyond $y(\theta)$. Indeed, $\ell > y(\theta)$ violates (5) and implies a negative private valuation for the blockholder that would trigger its immediate exit. This exit would in turn lead to a downward jump in the stock price (since $p'(\theta) > 0$), leading to negative equity value and making it optimal for dispersed shareholders to default, as shown in Lemma 1. That is, even if given full control, dispersed shareholders cannot raise ℓ above $y(\theta)$ due to the blockholder's limited commitment to a particular equity stake and to the threat of immediate exit.

3.2 Investment Control and Blockholder Exit

Similarly, in the baseline model, the blockholder controls investment and chooses the investment rate $i = \mu y(\theta)$. That is, the blockholder's valuation for the firm pins down investment. Given θ and smooth trading, dispersed shareholder value would be maximized under the investment rate $i = \mu p(\theta) > \mu y(\theta)$.[14] In essence, there is an under-investment problem that reduces dispersed shareholder value.

[14]Given θ, dispersed shareholders' value is maximized according to $i = \arg\max_{\hat{i}\geq 0}(\mu\hat{i}p(\theta) - \hat{i}^2/2)$.

To address this issue, suppose that the firm sets a governance structure that prevents the blockholder from gaining control and that dispersed shareholders control investment as long as $\theta < 1$. Thus, $i = \mu p(\theta)$, which increases investment and the stock price in a given state, i.e., holding θ fixed. However, the blockholder cannot commit to maintaining a given ownership stake and given that the change in investment reduces its valuation for the firm, it lowers its stake in the firm. As we show, this effect ultimately reduces stock prices and dispersed shareholder value, as well as investment in the long run.

To establish this result, we again solve for a continuous, scaled Markov equilibrium.[15] Appendix B.3 and Proposition 6 demonstrate that a unique continuous, scaled Markov equilibrium in which the blockholder trades smoothly exists.[16] When dispersed shareholders control investment, the smooth trading rate likewise satisfies (18), which now becomes:

$$\dot{\theta} = \frac{1}{p'(\theta)} \left[\theta(\rho - r)\ell'(\theta) + \frac{\theta\lambda(1-\lambda)}{\kappa} - \pi\theta - \eta(\theta) + \theta\mu i'(\theta) \left(y(\theta) - p(\theta) \right) \right], \tag{21}$$

and involves the term

$$\theta\mu i'(\theta) \left(y(\theta) - p(\theta) \right) = (\rho + \delta - \mu i) \left(\frac{\partial v(\theta)}{\partial i} \frac{\partial i}{\partial \theta} \right) < 0.$$

This term is absent in the baseline model where investment solves the first-order condition $\frac{\partial v(\theta)}{\partial i} = 0$. This term is negative, implying that the blockholder is more likely to sell its shares over time when dispersed shareholders control investment. This reflects the blockholder's lower valuation of the firm, as it internalizes that purchasing additional shares drives up the firm's stock price and investment, further worsening inefficiencies from its perspective. In other words, the blockholder exhibits an increased propensity to acquire more of the firm rather than to exit when it controls investment. This leads to Proposition 6:

Proposition 6. *When dispersed investors control investment and $i = \mu p(\theta)$, a continuous, scaled Markov equilibrium with smooth trading exists and is unique. In this equilibrium:*

1. *The blockholder's scaled value function $v(\theta)$ is strictly convex and solves* (B.6) *subject to* (B.9). *The stock price satisfies $p(\theta) = v'(\theta)$ with $y(\theta) = \frac{v(\theta)}{\theta} > p(\theta)$.*

[15] The equilibrium is defined as in the baseline model, except that we impose parameter condition (B.4) instead of (A.1) and the investment rate is chosen to maximize dispersed shareholder value.

[16] Admittedly, this uniqueness result is slightly less general than uniqueness among all continuous, scaled Markov equilibria, which we established for the baseline. Although we cannot prove it, we believe that our equilibrium is also unique among continuous, scaled Markov equilibria.

2. *The smooth trading rate $\dot{\theta}$ satisfies (18), which becomes (21).*

3. *The blockholder's value function is lower than in the baseline where the blockholder controls investment, i.e., $v(\theta) < \hat{v}(\theta)$.*

We can additionally characterize analytically the effects of investor control on the trading rate of the blockholder where there are no benefits of debt ($\rho = r$) and no holding costs or firm-level moral hazard ($\pi = \lambda = 0$). By continuity, the takeaways also apply as long as $\rho - r > 0$, $\pi > 0$, and $\lambda > 0$ are positive but sufficiently close to zero.

Corollary 4 (Investment control and blockholder exit)**.** *Suppose that $\pi = \lambda = 0$ and $\rho = r$. Suppose that, given dispersed shareholder control over investment, θ_0 satisfies $\theta_0 \in (0, \widetilde{\theta})$. Then, the following holds:*

1. *When dispersed shareholders control investment, we have $\dot{\theta} < 0$ and the blockholder eventually exits with $\lim_{t\to\infty} \theta_t = \theta_\infty = 0$ as well as $\lim_{t\to\infty} p(\theta_t) = \hat{v}'(0)$.*

2. *When the blockholder controls investment, we have $d\theta_t = 0$ for all $t \geq 0$, and θ_t and $p(\theta_t)$ remain constant at $\theta_0 > 0$ and $p(\theta_0) = \hat{v}'(\theta_0) > \hat{v}'(0)$ respectively.*

Our analysis shows that restricting blockholder control has distinct effects in the short versus long run, as it influences current corporate policies and the blockholder's trading rate, which, in turn, affects future ownership levels and firm decisions. Notably, allocating control to dispersed shareholders increases investment at a given level of blockholder ownership but reduces both the blockholder's trading rate and future investment. Thus, as shown in Corollary 4, whether exit can be part of an optimal strategy when there is no holding cost ($\pi = 0$) uniquely depends on the allocation of control rights. Exit is not optimal when the blockholder controls investment and financing, but can be part of the optimal strategy when the blockholder does not control investment.

Numerical Illustration. Figure 2 shows an example of equilibrium dynamics under two governance structures: blockholder control (dotted blue line) and dispersed shareholder control (solid red line), both starting from the same initial stake, θ_0. Shifting control rights from blockholders to dispersed shareholders—i.e., transitioning from the dotted blue to the solid black line—lowers the blockholder's valuation of the firm. This, in turn, leads to a gradual reduction in their ownership stake. The top left panel shows that when blockholders control investment, they increase their stake over time. Conversely, under dispersed shareholder

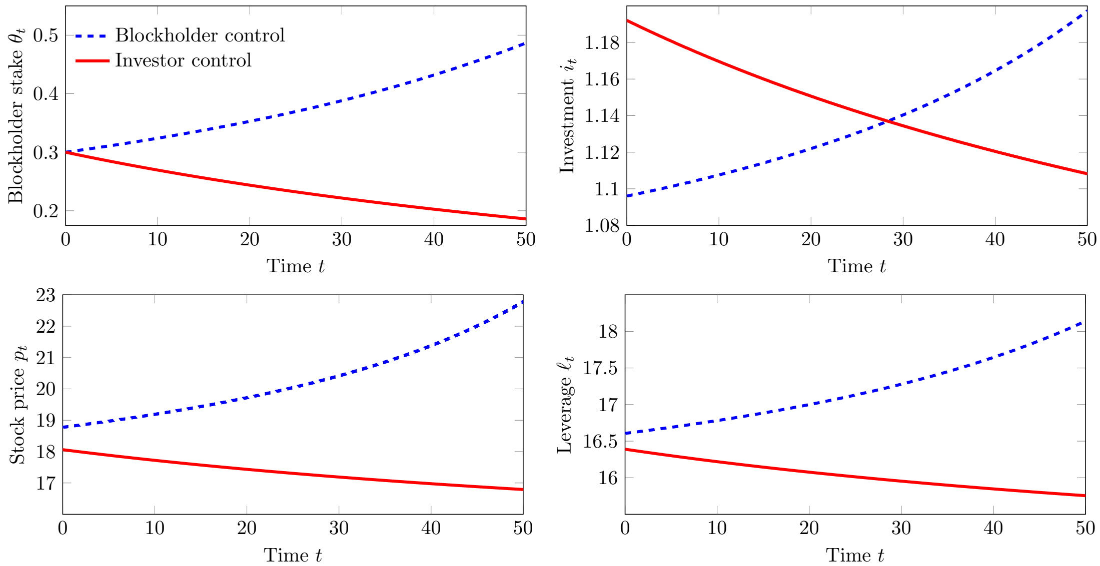

Figure 2: **The Dynamic Effects of Control over Corporate Policies.** We numerically calculate the equilibrium dynamics, both under the baseline (solid blue line) and with dispersed investor control (solid red line). The parameters are as in Figure 1.

control, blockholders steadily divest. As a result, blockholder control fosters rising levels of effort, firm value, debt, and investment, whereas all of these metrics decline under dispersed investor control. Since blockholders contribute positively to firm value, their reduced ownership under dispersed investor control diminishes the firm's stock price and its debt capacity (see the bottom panels). Thus, while investor control leads to higher initial investment at a given ownership level, the blockholder's divestment causes investment to decline over time.

4 Large Trades and Public-to-Private Transactions

In our baseline model, the blockholder faces dispersed shareholders with varying willingness to sell their stock. Some shareholders are reluctant to sell beyond a certain threshold, as indicated by condition (A.1). We now demonstrate that if condition (A.1) is not satisfied, we can similarly construct a Markov equilibrium in which quantities scale with K, i.e., $P(K,\theta) = Kp(\theta)$ and $V(K,\theta) = Kv(\theta)$.

With (A.1) not satisfied, we now characterize an equilibrium that follows the definition from Section 2.1 but without the requirement that $p(\theta)$ is continuous. We focus on the equilibrium that is closest to the baseline—specifically, the one with the fewest price discontinuities. A discontinuity in $p(\theta)$ can arise, for instance, when it becomes optimal for the blockholder to initiate a takeover bid for all remaining shares upon reaching an upper threshold θ^* from below, even if such a bid is not mandatory. Within the class of equilibria where $p(\theta)$ has at most one discontinuity—corresponding to the lumpy trade associated with a successful takeover—we obtain uniqueness. In this equilibrium, the choice of control variables (ℓ, i, b) and the smooth trading rate $\dot{\theta}$ are as in the baseline.

Proposition 7 (Equilibrium with takeovers)**.** *When (A.1) does not hold, there exists a scaled Markov equilibrium with state variables (K, θ), so that $P(K, \theta) = Kp(\theta)$ and $V(K, \theta) = Kv(\theta)$. The choice of control variables (ℓ, i, b) is as in the baseline: $b(\theta) = \frac{(1-\lambda)}{\kappa}$, $\ell(\theta) = y(\theta)$, and $i(\theta) = \mu y(\theta)$. Further, there exists a threshold θ^* (defined in (B.24)) such that:*

1. *For $\theta \in [0, \theta^*)$, the blockholder trades smoothly at rate $\dot{\theta}$ given in (17), where $\dot{\theta} > 0$ ($\dot{\theta} < 0$) for $\theta > \underline{\theta}$ ($\theta < \underline{\theta}$) and $\underline{\theta} = \underline{\Theta}$. Further, we have $v(\theta) = \hat{v}(\theta)$ and $p(\theta) = \hat{v}'(\theta)$.*

2. *For $\theta \in (\theta^*, 1)$, the blockholder conducts an immediate lumpy trade toward one (acquires the entire firm) and thereafter stops trading. Its value function is $v(\theta) = \hat{v}(1) - (1 - \theta)\hat{p}(1)$ where $\hat{p}(1)$ is the stock prevailing when the blockholder maintains perpetually full ownership of the firm. The stock price satisfies $p(\theta) = \hat{p}(1)$ where $\hat{p}(1) < \hat{v}'(1)$.*

3. *Suppose that θ reaches θ^* from below, i.e., $\lim_{\theta \to \theta^*} \dot{\theta}(\theta) > 0$. Then, once $\theta = \theta^*$, the blockholder randomizes between not trading at all or buying the entire firm at once at an endogenous rate $\gamma^* > 0$, given in (B.25). When $\lim_{\theta \to \theta^*} \dot{\theta}(\theta) \leq 0$, the blockholder trades smoothly at $\theta = \theta^*$ at rate $\dot{\theta}$ from (17). The blockholder's value function satisfies $\hat{v}(\theta^*) = \hat{v}(1) - (1 - \theta^*)\hat{p}(1)$, which pins down the threshold θ^*.*

Proposition 7 reveals a novel trading pattern that is similar to the baseline but now features both small and large trades, and potentially full ownership by the blockholder. In particular, the blockholder may gradually increase its stake up to an upper threshold θ^*, at which point it randomizes between acquiring the entire firm at once and not trading.[17] When $\theta_0 \geq \theta^*$, the blockholder acquires the entire firm immediately and never trades smoothly.

Proposition 7 shows that all policy choices remain as in the baseline model. Consequently, the blockholder's accumulation of a large stake should coincide with an immediate increase in both financial leverage and the investment rate, accompanied by a one-time payout to shareholders following the issuance of new debt. In particular, when the blockholder acquires the

[17] The discontinuity at $\theta = \theta^*$, when the blockholder shifts from smooth trading to an immediate acquisition, mirrors real-world takeover tactics, for instance, Elon Musk's acquisition of X (formerly Twitter) or Michael Dell's buyout of Dell Technologies. In both cases, the acquisitions were not abrupt. Instead, there was a clear threshold beyond which the acquirers decided to go all-in. Before reaching that critical point, they gradually increased their ownership stakes, preparing the market for a possible full buyout.

entire firm at once while issuing debt that is distributed as a dividend, the transaction closely resembles a *leveraged buyout*—where a public firm is taken private through full acquisition by the blockholder, resulting in a *public-to-private transaction*.

Public-to-private Transactions and Leveraged Buyouts. As in the baseline, the blockholder is more likely to buy—and in this case to take full ownership of the firm—when investment opportunities are good (μ is large), or the net benefits of debt are large (r is low). Thus, the model predicts a larger occurrence of public-to-private transactions, whereby public firms are acquired through an LBO, when firms have better investment opportunities or when debt financing is relatively cheap (e.g., the interest rate in the economy is low). Likewise, we have shown that the blockholder is more likely to acquire firms with intermediate levels of λ, suggesting that public-to-private transactions are less likely to occur for firms with either very low managerial agency conflicts (e.g., due to efficient governance) or very high managerial agency conflicts (e.g., due to poor governance).

5 Large Shocks, Default, and Risky Debt

The specification of cash flows in equation (1) implies that the firm never defaults on its debt when (5) is satisfied, which is always the case in equilibrium. We now extend the model to incorporate large, persistent shocks to the capital shock K as in, e.g., Bolton et al. (2023). Since cash flows scale with K, these shocks can also be interpreted as persistent shocks to cash flows. In particular, we assume that a fraction $1 - S$ of the capital stock is destroyed at a Poisson rate $\Lambda \geq 0$ where, for simplicity, S is uniformly distributed on $[0, 1]$. In the event of default following the shock, the firm is liquidated.

We now assume that, in liquidation, creditors recover the liquidation value RSK. Here, $R \geq 0$ is sufficiently small that the liquidation value does not fully repay creditors in default under equilibrium debt levels (and equity value falls to zero). In particular, $R < p^0$ where p^0 is the scaled stock price under dispersed ownership from Proposition 1. The capital shock is not influenced by the manager and is not contractible. As in the baseline, the blockholder chooses debt and investment, and the contract of the manager features a base wage of zero and an exposure $\beta_t = \lambda$ to cash flow shocks.

The solution to the model under this cash flow specification is presented in Appendix B.5. In the following, we provide a heuristic characterization of the continuous, scaled Markov equilibrium. We start by clarifying the timing of events and actions within a time interval $[t, t+dt]$, with pre-shock capital $K = K_{t^-} = \lim_{s\uparrow t} K_t$ and where use the left-limit notation to emphasize that K_{t^-} is the capital stock *before* realization of the jump shock. First, given θ, the blockholder chooses the managerial contract, the investment rate, and the amount of debt. Second, the blockholder chooses its effort level, the shock dN_t realizes and, observing dN_t, the manager chooses diversion m_t. Then, cash flows dX_t realize, and the manager receives its promised payments. Then, uncertainty regarding the capital shock unfolds: The capital stock remains at K if no shock realizes and drops to SK if a shock realizes. Third, shareholders repay debt or default. In the absence of default, cash flows net of managerial compensation, investment costs, and debt repayments are distributed as dividends to shareholders. As in the baseline model, the blockholder can trade before or after debt is repaid. Finally, the capital stock adjusts for investment and depreciation in the case of no default.

A negative shock to capital reduces the blockholder's payoff. If the blockholder's post-shock payoff, given by $K(Sv(\theta) - \theta\ell)$, becomes negative, the firm defaults. Default occurs either because the blockholder directly triggers it or because it sells its entire stake (at a positive price), lowering the equity value below the debt level and prompting dispersed shareholders to default. Due to $y(\theta) < p(\theta)$, it suffices to look at the blockholder's incentives to default, which are stronger than those of dispersed shareholders. As in the baseline, the blockholder's valuation of the firm $y = y(\theta)$ pins down default, regardless of who ultimately controls the decision. We can rewrite the default condition as

$$Sy(\theta) < \ell = \ell(\theta) \quad \Longleftrightarrow \quad S < \frac{\ell(\theta)}{y(\theta)} = \frac{\ell}{y}.$$

Conditional on a shock occurring, the probability of default is given by $\Delta := \frac{\ell}{y}$. In default, creditors recover in expectation $\frac{\ell}{2y}RK$ dollars or $\frac{R}{2y}$ dollars per unit of debt. The fair interest rate on risky debt is thus

$$\hat{r} := r + \Lambda\Delta\left(1 - \frac{R}{2y}\right), \tag{22}$$

with a credit spread $\hat{r} - r = \Lambda\Delta\left(1 - \frac{R}{2y}\right)$. The following proposition, proven in Appendix B.5, characterizes equilibrium outcomes.

Proposition 8. *In a continuous, scaled Markov equilibrium, the blockholder's valuation of the firm $y(\theta)$ increases in θ and satisfies $y(\theta) < p(\theta)$. Furthermore:*

1. *The trading rate satisfies*

$$\dot{\theta} = \frac{1}{p'(\theta)}\left[\theta(\rho - r)y'(\theta)\Delta + \frac{\theta\lambda(1-\lambda)}{\kappa} - \pi\theta - \eta(\theta)\right].$$

2. *Scaled debt $\ell = \ell(\theta)$ and the probability of default $\Lambda\Delta$ satisfy*

$$\ell = \ell(\theta) = \min\left\{y(\theta), \frac{(\rho - r)y(\theta)^2}{\Lambda\big(y(\theta) - R\big)}\right\} \quad \text{and} \quad \Lambda\Delta = \min\left\{\Lambda, \frac{(\rho - r)y(\theta)}{y(\theta) - R}\right\}.$$

3. *The default probability and credit spreads decrease in θ. Scaled debt ℓ increases in θ when $\frac{y(\theta)}{2} \geq R$ and decreases in θ when $\frac{y(\theta)}{2} < R$. When $R \leq \frac{p^0}{2}$, $\frac{y(\theta)}{2} \geq R$ for all $\theta \geq \underline{\theta}$ and ℓ increases in θ. When $R \in \left(\frac{p^0}{2}, p^0\right)$, ℓ decreases in a right-neighborhood of $\underline{\theta}$.*

We note that the baseline model and results are obtained when the arrival intensity of capital shocks tends to zero, i.e., $\Lambda \to 0$. Under these circumstances, we have $\ell \to y(\theta)$, $\Delta \to 1$ for any θ, while the default intensity $\Lambda\Delta$ vanishes, rendering debt risk-free.

In this model with large shocks, the choice of debt reflects a standard trade-off, whereby firm leverage balances the benefits of debt with the cost of default. One can interpret RK as the firm's tangible or collateralizable assets, which can be seized by creditors in default. All else equal, higher R increases creditors' recovery in default and therefore lowers the interest rate, making it attractive to issue more debt and raising scaled debt ℓ and default risk.

Blockholder engagement—that is, the value that the blockholder adds to the firm—is an intangible, non-pledgeable asset, whose value increases with θ. As the blockholder's stake in the firm increases, the blockholder adds greater value through engagement, which raises the firm's going-concern value and incentives to avoid default. Therefore, an increase in ownership concentration has two effects. First, holding the level of debt fixed, a higher θ reduces default risk and credit spreads. Second, unless R is large, the firm takes advantage of the lower spreads and raises more debt as θ increases, thereby increasing credit risk. When R is large, debt is high to begin with, and the firm finds it attractive to mitigate default risk by lowering debt as θ increases and default becomes more costly. Overall, the first effect dominates the second one, in that credit spreads and default risk decrease with θ. Book debt increases in θ, unless R is large; for larger R, book debt is U-shaped in θ.

As in the baseline model, the trading rate $\dot{\theta}$ reflects the gains from trade associated with debt issuance, increasing the blockholder's propensity to acquire a larger stake in the firm.

First, when $\Delta = 1$ and debt choice is constrained by the borrowing constraint (5) in that $\ell = y(\theta)$, then $\frac{\partial v(\theta)}{\partial \ell} > 0$ and $\frac{\partial \Delta}{\partial \theta} = 0$. As in the baseline, the gains from trade term associated with debt is proportional to $\frac{\partial v(\theta)}{\partial \ell}\frac{\partial \ell}{\partial \theta} > 0$. Specifically, the blockholder internalizes that by buying additional shares, it increases the firm's debt capacity, which increases firm value.

Second, when $\Delta < 1$, the debt choice is not constrained by the borrowing constraint $\ell \leq y(\theta)$ but solves the first-order condition $\frac{\partial v(\theta)}{\partial \ell} = 0$. In this case, the blockholder, acting as a large strategic trader, internalizes that buying additional shares reduces credit risk and the cost of default. This channel generates gains from trade, as dispersed investors do not account for this when valuing the firm's stock, given by $\theta(\rho - r)y'(\theta)\Delta = (\rho + \delta + \Lambda - \mu i)\left(\frac{\partial v(\theta)}{\partial \Delta}\frac{\partial \Delta}{\partial \theta}\right) > 0$.

Finally, note that in a given state θ, the blockholder chooses too high leverage relative to dispersed investors' preferences. The intuition is that, upon default, the blockholder loses $y(\theta)$ per unit of equity, while dispersed shareholders lose $p(\theta) > y(\theta)$. Thus, the blockholder associates a lower cost with default than do dispersed shareholders. Appendix B.5.6 shows how the allocation of control rights affects blockholder trading in this model variant.

6 Stochastic Trading and Liquidity Shocks

In the benchmark model, we abstract from elements that would introduce exogenous trading dynamics and are not key to our main findings. As a result, given the parameters, one can predict whether the blockholder will maintain its stake, acquire the entire firm, or exit as $t \to \infty$. Our framework can be extended by allowing model parameters to evolve over time, leading to stochastic trading and long-run ownership. For instance, preference parameters π, ρ, r, or technological parameters such as μ, κ, could follow a Markov-switching process, introducing exogenous randomness into the blockholder's equilibrium trading behavior that would complement the endogenous dynamics highlighted in the baseline framework.

To illustrate this in the simplest possible setting, suppose the holding cost π is initially zero but can jump to $\pi^+ = \pi$ at a Poisson arrival time with intensity ϕ. Both before and after the jump, condition (A.1) holds, so that we obtain an equilibrium with smooth trading.

We sketch the equilibrium in this setting. The equilibrium after the jump is similar to that described in Proposition 4, with blockholder value function $v(\theta) = \hat{v}(\theta)$, stock price $p(\theta) = \hat{v}'(\theta)$, and smooth trading given in (17). Prior to the jump, we denote the blockholder value function by $v_{Pre}(\theta)$ and the stock price by $p_{Pre}(\theta)$. For $\theta \in [0,1]$, $v_{Pre}(\theta)$ solves:

$$(\rho+\delta+\phi)v_{Pre}(\theta) = \max_{\ell,i}\left[\theta\left(\alpha+b+(\rho-r)\ell-\frac{i^2}{2}\right)-\frac{\kappa b^2}{2}+\mu i v_{Pre}(\theta)+\phi v(\theta)\right], \quad (23)$$

where $\phi(v(\theta)-v_{Pre}(\theta))$ captures the effects of a jump and b satisfies (11).

It is clear that for $y_{Pre}(\theta) = \frac{v_{Pre}(\theta)}{\theta}$, we obtain $\ell_{Pre}(\theta) = y_{Pre}(\theta)$, $i_{Pre}(\theta) = \mu y_{Pre}(\theta)$, and $b_{Pre}(\theta) = b(\theta)$. Also, we have that $v_{Pre}(\theta) > v(\theta)$ as an upward jump in π reduces the blockholder's valuation of the firm. Next, note that $p_{Pre}(\theta) = v'_{Pre}(\theta)$ and $v'(\theta) = p(\theta)$. We then can solve for the trading rate as

$$\dot{\theta}_{Pre} = \frac{1}{p'_{Pre}(\theta)}\left[\theta(\rho-r)\ell'_{Pre}(\theta)+\frac{\theta\lambda(1-\lambda)}{\kappa}-\eta(\theta)\right]. \quad (24)$$

Note that $\eta(\theta) = 0$ and $\dot{\theta}_{Pre} > 0$ for $\theta < \widetilde{\theta}$. Thus, the pre-shock upper stationary point $\overline{\theta}_{Pre}$ is larger than $\widetilde{\theta}$, while $\underline{\theta}$ is smaller than $\widetilde{\theta}$. Thus, given $\theta_0 \in (0, \widetilde{\theta})$, two cases can happen

1. First, $\underline{\theta} < \theta_0 < \overline{\theta}_{Pre}$. Then, the blockholder trades up to $\overline{\theta}_{Pre}$, at which point it stops trading. Once the shock hits, it will trade toward the target ownership level $\overline{\theta}$. Either way, we have $\theta_\infty = \overline{\theta}$.

2. Second, $\theta_0 \leq \underline{\theta} < \overline{\theta}_{Pre}$. Again, the blockholder gradually trades up to $\overline{\theta}_{Pre}$. If the liquidity shock hits early before θ surpasses $\underline{\theta}$, the blockholder will exit and $\theta_\infty = 0$. If, on the other hand, the liquidity shock hits late only after $\theta > \underline{\theta}$, the blockholder will remain invested in this firm and trade to $\overline{\theta}$. Thus, the long-run outcome is stochastic.

Overall, early realizations of negative liquidity shocks can lead to exit. In contrast, late realizations are less impactful and often do not prevent long-run acquisition by the blockholder. This highlights the critical role of timing in liquidity shocks, shaping both the short- and long-run evolution of blockholder ownership.

7 Conclusion and Implications

We examine how blockholders affect firm dynamics by developing a tractable dynamic model in which blockholder ownership and firm policies are jointly determined. In the model, a blockholder can exert costly effort to enhance firm productivity, contract with management to curb agency problems, and influence investment and financing choices. We derive a closed-form equilibrium that characterizes the blockholder's dynamic trading strategy, entry and exit decisions, and the firm's compensation, financing, and investment policies—all without assuming commitment by the blockholder. We then use the model to analyze the joint evolution of firm outcomes and ownership structure, and to assess how the allocation of control rights shapes dispersed shareholder value and firm growth.

The trading strategy of the blockholder exhibits features commonly observed in practice. Notably, after acquiring an endogenous toehold, the blockholder may follow one of several paths depending on firm characteristics: fully exit the firm, gradually converge to a stable target ownership level, initiate a takeover bid, or exhibit a bifurcation pattern where small initial stakes lead to exit while sufficiently large stakes result in continued engagement.

The ownership stake of the blockholder, which is endogenous and evolves dynamically, shapes blockholder engagement and firm policies. Our model shows that blockholder entry leads to an increase in firm value, book leverage, and long-run investment. In addition, both the rate of investment and leverage are predicted to increase with the stake of the blockholder. However, both policies are suboptimal from the perspective of dispersed investors in any given state, with investment rates that are too low and debt levels that are too high relative to levels that would maximize dispersed shareholder value.

While firm policies do not maximize dispersed shareholder value when the blockholder is in control, our analysis shows that granting dispersed shareholders control paradoxically reduces their value. This outcome arises due to the impact on the blockholder's trading behavior and engagement, and the blockholder's inability to commit to a specific stake. Notably, a key aspect of our model is the endogenous and dynamic nature of the blockholder's stake, which leads to divergent effects on shareholder welfare in static versus dynamic settings. Specifically, dispersed shareholder control boosts investment at a given level of blockholder ownership but diminishes the blockholder's trading activity and future investment potential. Moreover, in our model, exit is not optimal when the blockholder controls corporate policy but becomes optimal when they lack control.

Our results underscore the importance of the blockholder's dynamic and endogenous stake in shaping the implications of corporate control and highlight the need to consider how shifts in control affect not only current firm decisions but also the blockholder's ongoing and future engagement with the firm. They also demonstrate the two-way relationship between firm policies and ownership concentration, highlighting the need to account for feedback effects:

firm policies influence the incentives for large shareholders to engage or exit, while changes in ownership structure, in turn, reshape firm behavior and strategic choices.

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A Proofs for Main Model

In this Appendix, we assume that dispersed investors derive a holding benefit from holding the stock at $\theta = 1$ that is large enough that

$$\eta(1) = \pi^I(1 - \widetilde{\theta}) > \frac{(\rho - r)\,(1 - \lambda^2 - \kappa\pi)}{2\sqrt{\kappa\,\left(\kappa\,(r+\delta)^2 - \mu^2(1 - \lambda^2 - \kappa\pi) - 2\kappa\,\mu^2\alpha\right)}} + \frac{\lambda(1-\lambda)}{\kappa} - \pi. \quad \text{(A.1)}$$

As we show in Appendix A.7, this condition implies that the optimal trading rate of the blockholder is negative at $\theta = 1$ and ensures the existence of a unique continuous Markov equilibrium in which the blockholder trades smoothly according to $d\theta = \dot{\theta}dt$.

A.1 Heuristic Timing

We characterize the heuristic timing of the model within a time interval $[t, t + dt]$.

1. At the beginning of $[t, t + dt]$ and after previous-period debt has been repaid and previous-period trading, the blockholder's stake equals $\theta = \theta_t$ (i.e., θ is a predictable process). Given θ, shareholders choose the managerial contract (specifically, $\beta_t \geq \lambda$), the investment rate $i_t \geq 0$, and the amount of short-term debt L_t, where the proceeds from debt issuance are distributed as dividends.

2. The blockholder chooses its effort level, the shock dN_t realizes and, observing dN_t, the manager chooses diversion m_t. Then, the blockholder chooses its effort level, the shock dN_t realizes and, observing dN_t, the manager chooses diversion m_t.

3. At the end of $[t, t + dt]$, debt matures: The firm either repays creditors $(1 + r_t dt)L_t$ dollars, where r_t is the endogenous interest rate, or defaults. If the firm does not default, cash flows net of investment costs, managerial compensation, and debt repayments are distributed as dividends to shareholders.

4. The blockholder can trade and chooses $d\theta_t$, determining next-period stake $\theta_{t+dt} = \theta_t + d\theta_t$. Finally, the capital stock is updated with $K_{t+dt} = K_t\big[1 + (\mu i_t - \delta)dt\big]$.

Note that one can net out the "special" dividends from debt issuance with the debt repayment. Moreover, the change in the capital stock is of order dt and its effect on other quantities of order dt is therefore negligible. Hence, the exact timing of capital stock updates does not matter for trading, default, or contracting. To avoid tedious notation, we assume it occurs at the very end of the period.

Thus, the timing can be concisely summarized as follows. First, corporate policies and efforts over $[t, t+dt]$ are chosen, taking the ownership state $\theta_t = \theta$ and capital stock $K_t = K$ as given. Second, the blockholder trades at the end of the period, and the capital stock adjusts, determining the next-period stake θ_{t+dt} and the capital stock K_{t+dt}.

In our continuous-time model, periods are infinitesimal and, therefore, the exact timing—while useful in guiding intuition—does not matter. In particular, in the heuristic timing, the blockholder trades after debt is repaid. However, as we demonstrate in Section 2.3, this assumption is without loss in generality in that we could allow trading before and after the repayment of debt without changing the results.

A.2 Optimal Contracting and Effort (Proof of Proposition 2)

We follow the formulation of the short-term contracting problem in He and Krishnamurthy (2011). We focus on incentive-compatible contracts that implement $m_t = 0$.

To formalize the contracting problem, denote the anticipated blockholder effort by $\hat{b}_t$. In optimum, we have $\hat{b}_t = b_t$, i.e., anticipated and actual effort levels coincide. When the blockholder does not invest, then $\theta_t = 0$ and $b_t = \hat{b}_t = 0$. The short-term contract $\mathcal{C}_t = (\beta_t, c_t)$ stipulates a flow (base) wage c_t, in addition to a payout $\beta_t K_t\big(dX_t - (\alpha + \hat{b}_t)dt\big)$ which depends on the realization of dX_t relative to its expected mean $(\alpha + \hat{b}_t)dt$. Without loss of generality, we normalize the manager's reservation utility to zero, so that its expected payoff from the contract must be positive. We denote the total payment to the manager over $[t, t+dt]$, which is contingent on the realization of dX_t, by $K_t dC_t = K_t\big[c_t dt + \beta_t\big(dX_t - (\alpha + \hat{b}_t)dt\big)\big]$.

Consider a cash flow shock $dN_t = 1$. If the manager diverts the generated cash flow and sets $m_t = 1$, she derives private benefit λK_t. Otherwise, if she does not divert cash flow, the manager gets $\beta_t K_t$. Thus, the manager does not divert cash flow if and only if: $\beta_t \geq \lambda$ which is (3). As will become clear, both the stock price and the blockholder payoff decrease in β_t, so setting $\beta_t = \lambda$ is optimal and the incentive condition (3) is tight. The manager's participation constraint requires $K_t \mathbb{E}_t^m[dC_t]$ to be weakly positive, where $\mathbb{E}_t^m$ denotes the time-t expectation under the manager's information set, i.e., conditional on m_t and an anticipated level of blockholder effort $\hat{b}_t$ (which coincides in optimum with actual effort b_t). In optimum, shareholders design the optimal contract, which maximizes their own value, and the manager's participation constraint binds. Setting $\mathbb{E}_t^m[dC_t]$ to zero, leads to $c_t = 0$.

Optimal Effort. When choosing effort b, the blockholder takes the manager's contract (β, c) as given. Indeed, since b is not observable or contractible, the contract with the manager cannot condition on b. The blockholder's scaled expected flow payoff related to effort and contracting with management can be written as

$$\max_{\beta \geq \lambda, b} \left\{ \theta \mathbb{E}^b \left[dX - dC\right] - \frac{\kappa b^2}{2} dt \right\} = \theta \alpha dt + \max_{\beta \geq \lambda, b} \left\{ \left(\theta b - \theta(b - \hat{b})\beta\right) - \frac{\kappa b^2}{2} \right\} dt, \qquad \text{(A.2)}$$

where the expectation $\mathbb{E}^b$ is under the blockholder's information set, which includes the choice of b. Optimizing over b yields $b = \frac{\theta(1-\beta)}{\kappa}$. Inserting optimal b into (A.2) and imposing $\hat{b} = b$ shows that the blockholder's flow payoff decreases in β. As such, it is optimal to set $\beta = \lambda$. Likewise, setting higher managerial incentives over $[t, t+dt]$ reduces b and therefore harms dispersed shareholders. As neither the blockholder nor dispersed shareholders benefit from increasing managerial incentives above λ, it holds that $\beta = \lambda$ and $b = \frac{\theta(1-\lambda)}{\kappa}$.

A.3 Proof of Proposition 1

When the blockholder does not invest and $\theta = 0$, dispersed shareholders dynamically choose the managerial contract, debt issuance, and investment to maximize (with $\eta(0) = 0$):

$$P_0^0 = \max_{(dL_t, i_t, \mathcal{C}_t)_{t \geq 0}} \mathbb{E}_t \left[\int_0^\infty e^{-\rho t} \left(K_t dX_t - \frac{K_t i_t^2}{2} dt - K_t dC_t + (\rho - r) L_t dt \right) \right], \qquad \text{(A.3)}$$

subject to $L_t \leq P_t^0$ where P_t^0 is the stock price with dispersed shareholders owning the firm. Conjecture that $P_t = Kp^0$ and $L_t = \ell^0 K$ for $K_t = K$, the firm's stock of capital. The HJB equation then becomes (using $\mathbb{E}_t[dC_t] = 0$):

$$\rho p^0 = \max_{\ell^0 \leq p^0, i} \left\{ \alpha + (\rho - r)\ell^0 - \frac{i^2}{2} + (\mu i - \delta) p^0 \right\}.$$

Optiming over ℓ^0 yields $\ell^0 = p^0$. Optimizing over i yields $i = \mu p^0$. Inserting these into the above HJB equation, we get $(r + \delta)p^0 = \alpha + \frac{\mu^2 (p^0)^2}{2}$ which we solve for p^0.

A.4 Proof of Lemma 1

We focus on a Markov equilibrium in which all quantities scale with K. To analyze trading, we thus consider scaled payoffs, using that $V(K, \theta) = Kv(\theta)$, $P(K, \theta)$, and $L(K, \theta) = K\ell(\theta)$. To analyze trading and default, we consider the firm in a state $(K_t, \theta_t) = (K, \theta)$ that is attained in equilibrium after time 0, i.e., we abstract from states that are never reached.[18]

Claim 1. The "if part" is immediate. Indeed, if (5) holds, both the dispersed shareholders and the blockholder are better off not defaulting.

It remains to prove the "only if" part for $\theta > 0$. We consider the firm at time $t > 0$ in state $(K_t, \theta_{t^-}) = (K, \theta)$, which, by assumption, is attainable. Attainability of (K, θ) requires that there are no profitable deviations in state (K, θ), in that there does not exist a trade toward any other state $(K, \hat{\theta})$ that yields *strictly* higher payoff for the blockholder. That is, any trade toward another state $(K, \hat{\theta})$ must yield weakly lower payoff for the blockholder than state (K, θ). Otherwise, the blockholder would immediately exit that state, i.e., the state would not be attained.

Suppose to the contrary that there exists a stake $\hat{\theta}$ for which the firm does not default and for which $v(\theta) < v(\hat{\theta}) - (\hat{\theta} - \theta)p(\hat{\theta})$ holds. If this were the case, the blockholder would realize a strictly positive payoff by trading toward state $\hat{\theta}$ just before time t, at time $t^- = \lim_{s \uparrow t} s$. As such, state (K, θ) is not attainable. Thus, $v(\theta) \geq v(\hat{\theta}) - (\hat{\theta} - \theta)p(\hat{\theta})$ for all $\hat{\theta} \in [0, 1]$.

Consider now $\ell = \ell(\theta) > \frac{v(\theta)}{\theta}$. Suppose the firm does not default after the blockholder trades toward stake $\hat{\theta}$ just before debt repayment. As default would be optimal post-trade for both types of shareholders when $\ell > p(\hat{\theta}) \geq \frac{v(\hat{\theta})}{\hat{\theta}}$, it suffices to consider $p(\hat{\theta}) \geq \ell$. Then, the gains from trading toward state $\hat{\theta}$ just before debt repayment read

$$G^-(\theta, \hat{\theta}) := \left[v(\hat{\theta}) - \hat{\theta}\ell\right] - \left[v(\theta) - \theta\ell\right] - (\hat{\theta} - \theta)(p(\hat{\theta}) - \ell) = v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta}) \leq 0 \quad \text{(A.4)}$$

for all $\hat{\theta} \in [0, 1]$ that do not lead to default. The last inequality used that by attainability of state (K, θ), we have $v(\theta) \geq v(\hat{\theta}) - (\hat{\theta} - \theta)p(\hat{\theta})$ for all $\hat{\theta} \in [0, 1]$, as shown above. Thus, conditional on no default occurring, the blockholder's continuation payoff is (weakly) smaller than $v(\theta) - \theta\ell < 0$ and therefore strictly negative.

[18] As discussed below, attainability of a state (K, θ) in equilibrium implies that there exists no large trade toward another state $(K, \hat{\theta})$ that leads to *strictly* higher equilibrium payoffs. If such a trade toward $\hat{\theta}$ would yield strictly higher payoff, the blockholder would never remain in state (K, θ) and just immediately trade toward $(K, \hat{\theta})$, rendering (K, θ) unattainable.

However, the blockholder can do strictly better than that by selling its entire stake at any positive price, which yields a weakly positive payoff for the blockholder; indeed, due to the option to default, the firm's stock price must be weakly positive at any time. Note that selling the entire stake yields a weakly higher payoff than forcing immediate default, which is possible if the blockholder is in control of the default decision and yields a payoff of zero. Consequently, it suffices to prove the claim assuming that dispersed shareholders decide on default and default at time t whenever $p_t < \ell_t$.

When the blockholder sells its entire stake θ, it trades toward (post-trade) stake $(K, \hat{\theta})$ with $\hat{\theta} = 0$. Due to $p(0) \leq \frac{v(\theta)}{\theta} < \ell$, the firm optimally defaults post-trade for $\hat{\theta} = 0$.

By the above arguments, any other trade toward $\hat{\theta}$ that does not trigger default yields gains from trade $G^{-}(\theta, \hat{\theta}) \leq 0$ and therefore strictly negative post-trade payoff for the blockholder. In contrast, selling the entire stake triggers default and yields zero payoff. Thus, when $v(\theta) < \theta\ell$, the blockholder's optimal trading necessarily triggers default. Since the stock price in default is zero, the blockholder cannot do better than selling its entire stake once $v(\theta) < \theta\ell$.

Claim 2. With (5) being met before and after trading, there is no default. Just before debt is repaid and given a debt level ℓ, the blockholder's gain from trading toward state $\hat{\theta}$ reads

$$G^{-}(\theta, \hat{\theta}) := (v(\hat{\theta}) - \hat{\theta}\ell) - (v(\theta) - \theta\ell) - (\hat{\theta} - \theta)(p(\hat{\theta}) - \ell) = v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta}).$$

Thus, gains from trade just before debt is repaid are equal to gains from trade just after debt is repaid,

$$G(\theta, \hat{\theta}) := v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta}).$$

Taking the limit $\hat{\theta} \to \theta$ (from either side) implies that this argument also applies to infinitesimal trades. Since gains from trade do not depend on the timing of the trade (conditional on no default), the next period stake satisfies $\theta_{t+dt} = \arg\max_{\hat{\theta}} G^{-}(\theta, \hat{\theta}) = \arg\max_{\hat{\theta}} G(\theta, \hat{\theta})$.

This proves that when (5) holds, the gains from trade are the same, irrespective of whether the blockholder trades just before or after debt is repaid (or at both times). Hence, the exact timing of the blockholder's trading is immaterial.

A.5 Proof of Lemma 2

For a Markov equilibrium with states (K, θ), we conjecture and verify that $V(K, \theta) = Kv(\theta)$, $P(K, \theta) = Kp(\theta)$, and $L(K, \theta) = K\ell(\theta)$. Given these conjectures, we can rewrite the borrowing constraints from (5) as $\ell = \ell(\theta) \leq \min\{\frac{v(\theta)}{\theta}, p(\theta)\}$. Moreover, the partial derivatives of the value function then read $V_K(K, \theta) = v(\theta)$ and $V_\theta(K, \theta) = Kv'(\theta)$.

By the dynamic programming principle, the integral representation (6) implies that the blockholder's value function solves (under smooth trading) the HJB equation:

$$\rho V(K, \theta) = \max_{\ell, \dot{\theta}, i} \Bigg\{ K \left[\theta \left(\alpha + b + (\rho - r)\ell - \frac{i^2}{2} \right) - \frac{\kappa b^2}{2} - \frac{\pi \theta^2}{2} \right] \\ + V_K(K, \theta) K(\mu i - \delta) + \dot{\theta} \big[V_\theta(K, \theta) - P(K, \theta) \big] \Bigg\},$$

subject to (11) and the borrowing constraint, where we used that, in equilibrium, expected payouts to the manager are zero, i.e., $\mathbb{E}[dC] = \lambda\mathbb{E}[dX - (\alpha + b)dt] = \lambda\sigma\mathbb{E}[dZ] = 0$. Using $V(K,\theta) = Kv(\theta)$, $P(K,\theta) = Kp(\theta)$, $V_K(K,\theta) = v(\theta)$, and $V_\theta(K,\theta) = Kv'(\theta)$ and rearranging and canceling $K > 0$ on both sides, we obtain

$$(\rho+\delta)v(\theta) = \max_{\ell,\dot{\theta},i}\left[\theta\left(\alpha + b + (\rho - r)\ell - \frac{i^2}{2}\right) - \frac{\kappa b^2}{2} - \frac{\pi\theta^2}{2} + \mu i v(\theta) + \dot{\theta}[v'(\theta) - p(\theta)]\right], \tag{A.5}$$

which is (13). This verifies that $V(K,\theta) = Kv(\theta)$. Since the right-hand side of (13) and the borrowing constraint do not depend on K, it follows that (scaled) control variables do not depend on K either and are functions of θ only. We thus obtain $L = \ell(\theta)K$ in optimum.

Next, note that by (7), a dispersed shareholder's valuation for the firm's stock, i.e., the firm's stock price, satisfies:

$$\rho P(K,\theta) = K\big(\alpha + b - \frac{i^2}{2} + (\rho - r)\ell + \eta(\theta)\big) + P_K(K,\theta)K(\mu i - \delta) + P_\theta(K,\theta)\dot{\theta},$$

where we used that in optimum $\mathbb{E}[dC] = 0$. Using $P(K,\theta) = Kp(\theta)$, $P_\theta(K,\theta) = Kp'(\theta)$, and $P_K(K,\theta) = p(\theta)$, we obtain after simplifications

$$(\rho + \delta - \mu i)p(\theta) = \alpha + b + (\rho - r)\ell(\theta) - \frac{i^2}{2} + \eta(\theta) + p'(\theta)\dot{\theta},$$

which is (16), which verifies our conjecture $P(K,\theta) = Kp(\theta)$.

To derive closed-form expressions for $v(\theta)$ and $p(\theta)$, first conjecture that $p(\theta) \geq \frac{v(\theta)}{\theta}$, which we verify at the end of the proof. We now solve the optimization in (13). First, since the right-hand-side of (13) increases in ℓ, optimal scaled debt satisfies $\ell = \ell(\theta) = \frac{v(\theta)}{\theta}$.

Thus, the borrowing constraint $\ell = \ell(\theta) \leq \min\{\frac{v(\theta)}{\theta}, p(\theta)\} = \frac{v(\theta)}{\theta}$ binds in optimum. Second, the optimization with respect to the investment rate i yields $i = i(\theta) = \frac{\mu v(\theta)}{\theta}$. Third, notice that the right-hand side of (13) is linear in $\dot{\theta}$. As such, for smooth trading, i.e., $\dot{\theta} \in (-\infty, +\infty)$ to be optimal, it must be that $v'(\theta) = p(\theta)$.

Inserting these relations back into (13) and using (11), we can solve

$$v(\theta) = \hat{v}(\theta) := \frac{\theta\,\left(\kappa(r+\delta) - \sqrt{\kappa\,\left(\kappa\,(r+\delta)^2 - \mu^2(1-\lambda^2-\kappa\pi)\,\theta - 2\kappa\,\mu^2\alpha\right)}\right)}{\kappa\,\mu^2}. \tag{A.6}$$

Note that (8) implies that the term under the square root is positive. We can then calculate the stock price via

$$p(\theta) = \hat{v}'(\theta) = \frac{\theta\,(1-\lambda^2-\kappa\pi)}{2\sqrt{\kappa\,\left(\kappa\,(r+\delta)^2 - \mu^2(1-\lambda^2-\kappa\pi)\,\theta - 2\kappa\,\mu^2\alpha\right)}} + \frac{\hat{v}(\theta)}{\theta}. \tag{A.7}$$

Due to condition (9), we have that $p(\theta) > \frac{v(\theta)}{\theta} = y(\theta)$. Finally, calculate

$$\hat{v}''(\theta) = \frac{\kappa\,(1-\lambda^2-\kappa\,\pi)\,(4\,\kappa\,r^2 - 3\,\mu^2\,\theta + 3\,\lambda^2\,\mu^2\,\theta - 8\,\alpha\,\kappa\,\mu^2 + 3\,\kappa\,\mu^2\,\pi\,\theta)}{4\,(\kappa\,(\kappa\,r^2 - \mu^2\,\theta + \lambda^2\,\mu^2\,\theta - 2\,\alpha\,\kappa\,\mu^2 + \kappa\,\mu^2\,\pi\,\theta))^{3/2}}.$$

Conditions (8) and (9) imply that $\hat{v}''(\theta) > 0$.

A.6 Proof of Proposition 3

Define:

$$\overline{\theta} = \inf\left\{\theta \in [0,1] : \dot{\theta}(\theta') < 0 \quad \text{for all} \quad \theta' \in (\theta, 1)\right\} \tag{A.8}$$

$$\underline{\theta} = \inf\left\{\theta \in [0,\overline{\theta}) : \dot{\theta}(\theta') > 0 \quad \text{for all} \quad \theta' \in (\theta, \overline{\theta})\right\}. \tag{A.9}$$

Consider an equilibrium with smooth trading. Under smooth trading, we have $\hat{v}(\theta) = v(\theta)$ and $p(\theta) = v'(\theta)$, and the trading rate $\dot{\theta}$ satisfies (17). In addition, $\ell(\theta) = \frac{\hat{v}(\theta)}{\theta}$ and thus $\ell'(\theta) = \frac{\theta\hat{v}'(\theta) - \hat{v}(\theta)}{\theta^2} = \frac{1}{\theta}\left(\hat{v}'(\theta) - \frac{\hat{v}(\theta)}{\theta}\right)$. Using the last expression together with (A.7), we get:

$$\ell'(\theta) = \frac{(1-\lambda^2-\kappa\pi)}{2\sqrt{\kappa\,(\kappa\,(r+\delta)^2 - \mu^2(1-\lambda^2-\kappa\pi)\,\theta - 2\kappa\,\mu^2\alpha)}}. \tag{A.10}$$

For $\theta = 0$, we have $\dot{\theta} = 0$. Otherwise, for $\theta > 0$, the sign of $\dot{\theta} = \dot{\theta}(\theta)$ is determined by

$$\begin{aligned}\mathcal{D}(\theta) &:= \frac{\lambda(1-\lambda)}{\kappa} + (\rho - r)\ell'(\theta) - \pi - \frac{\eta(\theta)}{\theta} \\ &= \frac{(\rho-r)\,(1-\lambda^2-\kappa\pi)}{2\sqrt{\kappa\,(\kappa\,(r+\delta)^2 - \mu^2(1-\lambda^2-\kappa\pi)\,\theta - 2\kappa\,\mu^2\alpha)}} + \frac{\lambda(1-\lambda)}{\kappa} - \pi - \pi^I\left[1 - \frac{\widetilde{\theta}}{\theta}\right]^+,\end{aligned} \tag{A.11}$$

where $\eta(\theta)$ is defined in (B.26). Observe that by parameter condition (A.1), $\mathcal{D}(1) < 0$ and therefore $\dot{\theta}(1) < 0$. This implies that there exists a left-neighborhood of one in which we have $\dot{\theta} < 0$, which, in turn, implies $\overline{\theta} < 1$. More generally, note that $\dot{\theta} = \frac{\theta\mathcal{D}(\theta)}{p'(\theta)}$.

For $\theta \neq \widetilde{\theta}$, $\mathcal{D}(\theta)$ is differentiable and we obtain

$$\mathcal{D}'(\theta) = \frac{(\rho-r)\kappa\mu^2\,(1-\lambda^2-\kappa\pi)^2}{4\,[\kappa\,(\kappa\,(r+\delta)^2 - \mu^2(1-\lambda^2-\kappa\pi)\,\theta - 2\kappa\,\mu^2\alpha)]^{3/2}} - \left(\frac{\pi^I\widetilde{\theta}}{\theta^2}\right)\mathbb{I}\{\theta > \widetilde{\theta}\}.$$

Thus, when $\theta < \widetilde{\theta}$, we have $\mathcal{D}'(\theta) \geq 0$ with the inequality being strict if $\mu > 0$. Further, for $\theta \neq \widetilde{\theta}$, we have $\mathcal{D}''(\theta) > 0$.

This implies the following. First, for $\theta \in [0, \widetilde{\theta}]$, $\mathcal{D}(\theta)$ has at most one root. Moreover, on the interval $(\widetilde{\theta}, 1]$, $\mathcal{D}(\theta)$ also has at most one root, since $\mathcal{D}(1) < 0$. It has precisely one root on this interval if and only if $\mathcal{D}(\tilde{\theta}) > 0$. Taken together, the function $\mathcal{D}(\theta)$ has at most two roots on the interval $[0,1]$. Since $\dot{\theta} \propto \theta\mathcal{D}(\theta)$, there exist at most three stationary points, i.e.,

$\dot{\theta}(\theta)$ has (weakly) more than one but (weakly) less than three roots on $[0, 1]$.

We also note that when $\lambda(1-\lambda) > \kappa\pi$, then $\mathcal{D}(\theta) > 0$ for all $\theta \in (0, \widetilde{\theta})$.

There are three cases to distinguish.

1. Suppose $\mathcal{D}(\theta)$ has zero roots on $[0, 1]$. Then, $\mathcal{D}(\theta) < 0$ and $\dot{\theta} < 0$ for all $\theta \in [0, 1]$. Then, the definition (A.8) implies $\underline{\theta} = \overline{\theta} = 0$ and note that $\dot{\theta}(0) = 0$.

2. Suppose that $\mathcal{D}(\theta)$ has precisely one root, denoted θ^C, on $[0, 1]$. Then, it must be that this root θ^C lies in $[\widetilde{\theta}, 1]$. To see this, suppose to the contrary that $\theta^C \in (0, \widetilde{\theta})$. Then, $\mathcal{D}(\widetilde{\theta}) > 0$; thus, there exists a second root on $(\widetilde{\theta}, 1]$, a contradiction.

   By definition of $\overline{\theta}$ in (A.8), the root equals $\overline{\theta}$, i.e., $\theta^C = \overline{\theta}$, with $\overline{\theta} \in [\widetilde{\theta}, 1]$.

   We then have $\mathcal{D}(\theta) > 0$ and $\dot{\theta} > 0$ for all $\theta < \overline{\theta}$, as well as $\mathcal{D}(\theta) < 0$ and $\dot{\theta} < 0$ for all $\theta \in (\widetilde{\theta}, 1)$. The definition of $\underline{\theta}$ implies $\underline{\theta} = 0$. Overall, $\dot{\theta}(\underline{\theta}) = \dot{\theta}(\overline{\theta}) = 0$.

   We note that this case prevails whenever $\lambda(1-\lambda) > \kappa\pi$.

3. Suppose that $\mathcal{D}(\theta)$ has two roots, denoted $\theta_1^C < \theta_2^C$, on $[0, 1]$. Then, it must be that $\mathcal{D}(0), \mathcal{D}(1) < 0 < \mathcal{D}(\widetilde{\theta})$, and the smaller (larger) root satisfies $\theta_1^C \in (0, \widetilde{\theta})$ ($\theta_2^C \in (\widetilde{\theta}, 1)$). By definition of the thresholds in (A.8), we then have $\theta_1^C = \underline{\theta}$ and $\theta_2^C = \overline{\theta}$. It follows that $\dot{\theta} < 0$ for $\theta \in (0, \underline{\theta})$ or $\theta \in (\overline{\theta}, 1)$, while $\dot{\theta} > 0$ for $\theta \in (\underline{\theta}, \overline{\theta})$. Moreover, $\dot{\theta}(\underline{\theta}) = \dot{\theta}(\overline{\theta}) = 0$. Then, whenever $\mu > 0$ and $\lambda(1-\lambda) < \kappa\pi$, we can solve $\mathcal{D}(\underline{\theta}) = 0$ for $\underline{\theta} < \widetilde{\theta}$, yielding $\underline{\theta} = \underline{\Theta}$ for

$$\underline{\Theta} := \frac{\kappa}{\mu^2 (1 - \lambda^2 - \kappa\pi)} \left[(r+\delta)^2 - 2\mu^2\alpha - \frac{(\rho - r)^2 (1 - \lambda^2 - \kappa\pi)^2}{4 (\kappa\pi - \lambda(1-\lambda))^2} \right] \mathbb{I}\{\lambda(1-\lambda) < \kappa\pi\},$$

   which becomes (19), as desired. Observe that we account for the condition $\lambda(1-\lambda) < \kappa\pi$ through the indicator $\mathbb{I}\{\lambda(1-\lambda) < \kappa\pi\}$.

The expression $\underline{\Theta}$ determines the number of roots of $\mathcal{D}(\theta)$. **First**, when $\underline{\Theta} \geq \widetilde{\theta}$, then $\mathcal{D}(\theta)$ has zero roots, and $\underline{\theta} = \overline{\theta} = 0$. **Second**, when $\underline{\Theta} \in (0, \widetilde{\theta})$, then $\mathcal{D}(\theta)$ has two roots, i.e., $\underline{\theta}$ and $\overline{\theta}$, with $\underline{\theta} = \underline{\Theta}$ and $\overline{\theta} \in (\widetilde{\theta}, 1)$. **Third**, when $\underline{\Theta} \leq 0$, then $\mathcal{D}(\theta)$ has a single root $\overline{\theta} \in (\widetilde{\theta}, 1)$, and $\underline{\theta} = 0$ — this case prevails if $\lambda(1-\lambda) > \kappa\pi$. In particular, whenever $\overline{\theta} > 0$, we have $\overline{\theta} \in (\widetilde{\theta}, 1)$.

A.7 Proof of Proposition 4

We prove the existence of a unique scaled, continuous Markov equilibrium in which the price satisfies the scaling property, in that $P(K, \theta) = Kp(\theta)$. The proof proceeds in several parts. Part I presents the blockholder's HJB equation (A.16)— allowing for general trading processes and lumpy trading—and presents general results regarding the blockholder's optimal trading and equilibrium conditions—these conditions must hold in any Markov equilibrium where the price satisfies $P(K, \theta) = Kp(\theta)$ and they are also valid in other model variants. Part II verifies the optimality of the proposed trading strategy: Conjecturing the equilibrium (scaled) price $p(\theta)$ from Proposition 4, it shows that smooth trading is strictly optimal and the (scaled) value function from Proposition 4 solves the HJB equation. Part III argues that the equilibrium is unique. Unlike Part I, which presents general equilibrium conditions, the other parts present proofs that only apply to our baseline.

A.7.1 Part I: Equilibrium Properties

We allow for continuous and lumpy trading by specifying the dynamics of the blockholder's stake as

$$d\theta_t = \dot{\theta}_t dt + dI_t, \tag{A.12}$$

where $\dot{\theta}_t$ is the drift of $d\theta_t$ and dI_t captures solely lumpy trading, in that $I_t = \int_0^t dI_s$ is constant except for a countable number of times t. Since we focus on a Markov equilibrium, we will depress time subscripts in the remainder of the proof.

Specifically, we consider that at an endogenous (state-dependent) intensity $\gamma \in [0, \infty]$, the blockholder conducts a lumpy trade toward state $\hat{\theta} \in [0, 1]$, where $\hat{\theta}$ is optimally chosen by the blockholder and thus endogenous. With a slight abuse of notation, $\gamma = +\infty$ corresponds to a lumpy trade that occurs with some atom of probability (possibly with probability one). That is, we can write

$$d\theta = \dot{\theta} dt + (\hat{\theta} - \theta) dN, \tag{A.13}$$

where $dN \in \{0, 1\}$ is a jump process with $\mathbb{E}[dN] = \gamma dt$. We introduce the convention that whenever $\hat{\theta} = \theta$ in the aforementioned process, we set $\gamma = 0$.

Given a price $P(K, \theta) = Kp(\theta)$ for an increasing, continuous function $p(\theta)$, we conjecture and verify that $V(K, \theta) = Kv(\theta)$ in the Markov equilibrium, where $v(\theta)$ and $p(\theta)$ are almost everywhere differentiable (i.e., they are differentiable on $[0, 1]$ except at countably many points). Thus, for any $\theta \in (0, 1)$, the limits $\lim_{x\uparrow\theta} \omega(\theta)$ and $\lim_{x\downarrow\theta} \omega(\theta)$ exist and are well-defined for $\omega \in \{v', p\}$.

We characterize the blockholder's dynamic optimization, given the conjectured equilibrium price $P(K, \theta) = Kp(\theta)$—we note that the scaled price function $p(\theta)$ need not be the one proposed in Proposition 4 and our results of this part of the proof hold in any equilibrium.

To allow for generality and to encompass the model variant of Section 3 or B.3, we impose that the investment i_t must satisfy a constraint $i_t \in \mathcal{I}_t$. In Section 3, we set $\mathcal{I} = \{\mu p(\theta)\}$, while the baseline has $\mathcal{I}_t = [0, \infty)$. We restrict $i_t = i(\theta)$ to be Markovian.

HJB Equation. In state $(K_t, \theta_t) = (K, \theta)$ at time t, the value function equals

$$V(K, \theta) = \max_{(dL_s, d\theta_s, i_s \in \mathcal{I}_s)_{s\geq t}} \mathbb{E}_t \Bigg[\int_0^\infty e^{-\rho(s-t)} \Bigg\{ \theta_s \big[K_s dX_s - \frac{K_s i_s^2}{2} ds + L_s(\rho - r) ds \big] \tag{A.14}$$
$$- K_s \left(\frac{\pi \theta_s^2}{2} + \frac{\kappa b(\theta_s)^2}{2} \right) ds - d\theta_s K_s p(\theta_s + d\theta_s) \Bigg\} \Bigg| (K_t, \theta_t) = (K, \theta) \Bigg],$$

where we impose the optimal contract (with $\mathbb{E}[dC] = 0$), the optimal effort choice from (11), $b_s = b(\theta_s)$, the default time of $T = \infty$ and the borrowing constraint $L_s \leq \min\{\frac{V(K_s, \theta_s)}{\theta_s}, P(K_s, \theta_s)\}$.

By the dynamic programming principle and the integral expression (A.14), the blockholder's value function solves the HJB equation:

$$\rho V(K,\theta)dt = \max_{\ell,d\theta,i\in\mathcal{I}} \Bigg\{ K\left[\theta\left(\alpha + b + (\rho - r)\ell - \frac{i^2}{2}\right)dt - \frac{\pi\theta^2}{2} - \frac{\kappa b^2}{2}dt\right] \tag{A.15}$$
$$+ V_K(K,\theta)K(\mu i - \delta)dt + V(K,\theta + d\theta) - d\theta P(K,\theta + d\theta)\Big]\Bigg\},$$

where trading $d\theta$ follows the endogenous process (A.13), b is given in (11), and the borrowing constraint becomes $K\ell \leq \min\{\frac{V(K,\theta)}{\theta}, P(K,\theta)\}$.

Using the conjecture $V(K,\theta) = Kv(\theta)$ and $P(K,\theta) = Kp(\theta)$, we note that the gains from trade $V(K,\theta + d\theta) - d\theta P(K,\theta + d\theta) = K\big[v(\theta + d\theta) - d\theta p(\theta + d\theta)\big]$ are linear in K. Specifically, if, in state (K,θ), the blockholder changes its stake by $\hat{\theta} - \theta$, then its total payoff changes by $V(K,\hat{\theta}) - (\hat{\theta} - \theta)P(K,\theta) - V(K,\theta) = K\big[v(\theta) - (\hat{\theta} - \theta)p(\theta)\big]$. It is therefore without loss of generality to work with scaled payoffs when characterizing optimal trading. Using (A.13), one can show—similar to the derivation of (A.5) in Lemma 2—that the scaled value function satisfies (in equilibrium) the following HJB equation:

$$(\rho + \delta)v(\theta) = \max_{\dot{\theta},\gamma,\hat{\theta},\ell,i\in\mathcal{I}} \Bigg\{ \theta\left(\alpha + b + (\rho - r)\ell - \frac{i^2}{2}\right) - \frac{\pi\theta^2}{2} - \frac{\kappa b^2}{2} + \mu i v(\theta)$$
$$+ \dot{\theta}\Big[\mathbb{I}\{\dot{\theta} \geq 0\} \lim_{x\downarrow\theta}\big(v'(x) - p(x)\big) + \mathbb{I}\{\dot{\theta} < 0\} \lim_{x\uparrow\theta}\big(v'(x) - p(x)\big)\Big]$$
$$+ \gamma\big[v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta})\big]\Bigg\}, \tag{A.16}$$

subject to incentive constraint (11) for b and the borrowing constraint $\ell \leq \min\{\frac{v(\theta)}{\theta}, p(\theta)\}$.

HJB equation (A.16) generalizes (A.5) to account for the possibility of lumpy trades and randomized lumpy trades. It is well-defined for $\dot{\theta} \in (-\infty, +\infty)$ and $\gamma \in [0, +\infty)$. When $\gamma = +\infty$, the HJB equation becomes with some abuse of notation: $v(\theta) = \max_{\hat{\theta}\in[0,1]} v(\hat{\theta}) - (\hat{\theta} - \theta)p(\hat{\theta})$. Moreover, we regard an infinite trading rate $\dot{\theta} \in \{-\infty, +\infty\}$ as equivalent to a lumpy trade, $\gamma = +\infty$. Also note that according to (A.13), the optimization in (A.16) implies that optimal trading solves the equilibrium condition (10).

Gains From Trade and Equilibrium Trading Condition. For an interior solution $\dot{\theta} \in (-\infty, +\infty)$ and $\gamma \in [0, \infty)$ to be optimal, it must be

$$\lim_{x\downarrow\theta}\big(v'(x) - p(x)\big) \leq 0, \quad \text{and} \quad \lim_{x\uparrow\theta}\big(v'(x) - p(x)\big) \geq 0,$$
$$\max_{\hat{\theta}\in[0,1]}\big[v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta})\big] \leq 0. \tag{A.17}$$

We define the endogenous region

$$\mathcal{S} = \{\theta \in [0,1] : \dot{\theta} \in (-\infty, \infty) \text{ and } \gamma \in [0, \infty)\}. \tag{A.18}$$

Thus, $\mathcal{S}$ is defined as the set of states in which either smooth trading or lumpy trading at finite intensity ($\gamma < +\infty$) is optimal. As in the definition of the trading process in (A.13), we follow the convention that whenever $\hat{\theta} = \theta$, we set $\gamma = 0$. That is, whenever there is no lumpy trade in state θ, then $\gamma = \gamma(\theta) = 0$.

Note that for all $\theta \in \mathcal{S}$, the condition (A.17) holds. Moreover, $\mathcal{S}$ encompasses all states that may be attained in equilibrium, in that for any $\theta \in \mathcal{S}$, the blockholder does not immediately trade away from that state.

For $\theta \notin \mathcal{S}$, by definition, an immediate lumpy trade is optimal so that $\max_{\hat{\theta}\in[0,1]} \big[v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta})\big] = 0$. Either way, we have

$$v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta}) \leq 0 \quad \text{for all} \quad \theta, \hat{\theta} \in [0,1]. \tag{A.19}$$

Whenever $v(0) = 0$, we can set $\theta = 0$ in (A.19) to obtain $p(\hat{\theta}) \geq \frac{v(\hat{\theta})}{\hat{\theta}}$. That is, when $v(0) = 0$, then $\frac{v(\theta)}{\theta} \leq p(\theta)$ holds for $\theta \in [0,1]$.

Next, consider $\theta \in \mathcal{S}$. Clearly, $\max_{\hat{\theta}\in[0,1]} \big[v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta})\big] \leq 0$, which implies $\gamma\big[v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta})\big] = 0$ for any $\hat{\theta}$. Likewise, it is optimal to set $\dot{\theta} \leq 0$ when $\lim_{x\downarrow\theta} \big(v'(x) - p(x)\big) < 0$ and $\dot{\theta} \geq 0$ when $\lim_{x\uparrow\theta} \big(v'(x) - p(x)\big) > 0$. Overall, whenever $\dot{\theta} \in (-\infty, +\infty)$ and $\gamma \in [0, \infty)$ is optimal:

$$\max_{\dot{\theta},\gamma,\hat{\theta}} \Big\{ \dot{\theta}\Big[\mathbb{I}\{\dot{\theta} \geq 0\} \lim_{x\downarrow\theta} \big(v'(x) - p(x)\big) + \mathbb{I}\{\dot{\theta} < 0\} \lim_{x\uparrow\theta} \big(v'(x) - p(x)\big)\Big] \tag{A.20}$$
$$+ \gamma\big[v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta})\big] \Big\} = 0.$$

Plugging condition (A.20) back into (A.16), we obtain for the value function on $\mathcal{S}$:

$$(\rho + \delta)v(\theta) = \max_{\ell,i\in\mathcal{I}} \left\{ \theta \left(\alpha + b + (\rho - r)\ell - \frac{i^2}{2} \right) - \frac{\pi\theta^2}{2} - \frac{\kappa b^2}{2} + \mu i v(\theta) \right\}, \tag{A.21}$$

with b characterized in (11) and $\ell \leq \min\{\frac{v(\theta)}{\theta}, p(\theta)\}$. Optimal baseline investment is $i = \frac{v(\theta)}{\theta}$.

Baseline solution and $v(0) = 0$. Conjecture that $v(0) = 0$. Then, (A.19) implies $\frac{v(\theta)}{\theta} \leq p(\theta)$. Thus, the optimization in (A.21) implies $\ell = \frac{v(\theta)}{\theta}$, and we can solve (A.21) for $v(\theta) = \hat{v}(\theta)$ for all $\theta \in \mathcal{S}$. We verify that $v(0) = 0$. When $0 \in \mathcal{S}$, then (A.21) immediately implies that $v(0) = 0$. $v(0)$ solves (A.21) and the claim follows. When $0 \notin \mathcal{S}$, then an immediate lumpy trade toward some state $\hat{\theta} \in \mathcal{S}$ is optimal and $v(0) = \max_{\hat{\theta}\in[0,1]} G(\hat{\theta}) := \max_{\hat{\theta}\in[0,1]} \Big[\hat{v}(\hat{\theta}) - \hat{\theta}p(\hat{\theta})\Big]$. Note that $p(\hat{\theta}) = \hat{v}'(\hat{\theta})$, and $G'(\hat{\theta}) = -\hat{\theta}v''(\hat{\theta}) < 0$. In addition, $G(0) = 0$ and, because $G(\hat{\theta})$ decreases, we obtain $v(0) = 0$.

A.7.2 Part II: Optimality of Smooth Trading and Trading Strategy

We verify the optimality of the proposed trading strategy under the proposed price function satisfying $p(\theta) = \hat{v}'(\theta)$. That is, we conjecture the equilibrium price $p(\theta) = \hat{v}'(\theta)$ and verify that, given this price, smooth trading is optimal and constitutes an equilibrium strategy. This part then establishes the existence of a continuous, scaled Markov equilibrium.

Optimality of Smooth Trading. Consider state $\theta \in \mathcal{S}$, so that (A.17) holds and $v(\theta) = \hat{v}(\theta)$ (as shown before). Because the price is continuous, (A.17) implies that $v(\theta)$ is differentiable, with $p(\theta) = v'(\theta)$.

We now solve the blockholder's optimal trading and value function given the conjectured price function $p(\theta) = \hat{v}'(\theta)$. For this sake, we solve (10) or, equivalently, the choice of trading in the HJB equation (A.16). We show that smooth trading is optimal—that is,

$$\dot{\theta}dt \in \arg \max_{\Delta \in [-\theta, 1-\theta]} K_t \left\{ v(\theta + \Delta) - \Delta p(\theta + \Delta) \right\} \tag{A.22}$$

for finite trading rate $\dot{\theta} \in (-\infty, +\infty)$. Having established the optimality of smooth trading, the HJB equation (A.16) then implies $v(\theta) = \hat{v}(\theta)$.

Note that for any finite trading rate $d\theta = \dot{\theta}dt$, we have

$$\begin{aligned} v(\theta + \dot{\theta}dt) - \dot{\theta}dt \cdot p(\theta + \dot{\theta}dt) &= \hat{v}(\theta) + \hat{v}'(\theta)\dot{\theta}dt - \dot{\theta}dt \cdot (p(\theta) + p'(\theta)\dot{\theta}dt) + o((dt)^2) \\ &= \hat{v}(\theta) + (\hat{v}'(\theta) - p(\theta)\dot{\theta}dt = \hat{v}(\theta), \end{aligned}$$

where the first equality used a Taylor expansion, as well as $v(\theta) = \hat{v}(\theta)$. The second line uses $\hat{v}'(\theta) = p(\theta)$ and discards terms of order $(dt)^2$ or higher (which are negligible in continuous time), that is, $o((dt)^2) = 0$. Thus, smooth trading at any finite rate $\dot{\theta}$ yields continuation payoff $\hat{v}(\theta)$. The blockholder is indifferent across any finite trading rates and, in particular, $\hat{v}(\theta)$ is also the payoff when the blockholder does not trade at all.

In contrast, consider a lumpy trade toward state $\hat{\theta}$ from any state θ. We assume that $\hat{\theta} \in \mathcal{S}$, as otherwise the lumpy trade would be followed immediately by another lumpy trade and we could consolidate these trades. Thus, the blockholder chooses $d\theta = \hat{\theta} - \theta$ in state θ, which yields (scaled) payoff (given θ):

$$G(\hat{\theta}) := \hat{v}(\hat{\theta}) - (\hat{\theta} - \theta)p(\hat{\theta}) = \hat{v}(\hat{\theta}) - (\hat{\theta} - \theta)\hat{v}'(\hat{\theta}).$$

Note that $G(\theta) = v(\theta)$ and $G'(\hat{\theta}) = -(\hat{\theta} - \theta)\hat{v}''(\hat{\theta})$. Thus, $G(\hat{\theta})$ obtains its maximum for $\theta = \hat{\theta}$. Thus, given the price $p(\theta) = \hat{v}'(\theta)$, a lumpy trade is never optimal. It follows that smooth trading (including not trading at all) strictly dominates any lumpy trade with $\hat{\theta} \neq \theta$.

By definition, for all $\theta \notin \mathcal{S}$, the blockholder would find it optimal to conduct a lumpy trade toward a point $\hat{\theta} \in \mathcal{S}$. Above argument applies for both $\theta \in \mathcal{S}$ and $\theta \notin \mathcal{S}$. However, we have shown that the blockholder would be better off not trading at all. Thus, the set $[0, 1] - \mathcal{S}$ must be empty or, equivalently, $\mathcal{S} = [0, 1]$.

Thus, given the conjectured price function, smooth trading is optimal for all $\theta \in [0, 1]$ and yields $v(\theta) = \hat{v}(\theta)$. In particular, $v(\theta) = \hat{v}(\theta)$ solves the HJB equation (A.16).

Solving the Trading Rate. To show that $p(\theta) = \hat{v}'(\theta)$ is indeed an equilibrium price under smooth trading and to establish that the proposed equilibrium indeed exists, we need to solve for the equilibrium trade that is consistent with the price $p(\theta) = \hat{v}'(\theta)$ for all $\theta \in [0, 1]$.

To determine trading rate, we differentiate the closed-form expression for $\hat{v}(\theta)$ with respect to θ and using $\hat{v}'(\theta) = p(\theta)$, we obtain

$$(\rho + \delta)p(\theta) = \alpha + b + \mu i p(\theta) - \frac{i^2}{2} + (\rho - r)\ell(\theta) - \pi\theta + \frac{\theta\lambda(1 - \lambda)}{\kappa} + \theta(\rho - r)\ell'(\theta). \tag{A.23}$$

In addition to satisfying (A.23), $p(\theta)$ satisfies the pricing equation of dispersed shareholders

$$(\rho + \delta - \mu i)p(\theta) = \alpha + b - \frac{i^2}{2} + (\rho - r)\ell(\theta) + \eta(\theta) + p'(\theta)\dot{\theta}, \tag{A.24}$$

where $p(\theta)$ increases with θ, i.e., $p'(\theta) > 0$. Combining (A.23) and (A.24) yields (17).

Finally, as shown by Proposition 3, the trading rate $\dot{\theta} = \dot{\theta}(\theta)$ satisfies $\dot{\theta}(0) = 0$ and $\dot{\theta}(1) < 0$. Thus, the smooth trading is such that θ adheres to the "feasibility" constraint $\theta[0, 1]$. Specifically, the process $d\theta = \dot{\theta}dt$ with aforementioned trading rate stays within $[0, 1]$.

A.7.3 Part III: Equilibrium Uniqueness

Having shown that a continuous, scaled Markov equilibrium exists, we now argue that it must be unique. To see this, first recall that in equilibrium the condition (A.17) must hold for all $\theta \in \mathcal{S}$. As the price function is continuous, then (A.17) implies that in equilibrium, $v(\theta)$ is differentiable at θ with $v'(\theta) = p(\theta)$ for all $\theta \in \mathcal{S}$. Thus, on $\mathcal{S}$, we get that $v(\theta) = \hat{v}(\theta)$ and $p(\theta) = \hat{v}'(\theta)$. Given this price, we have shown that the blockholder's trading rate is uniquely determined according to (17). Thus, on the set $\mathcal{S}$, the price is uniquely determined as $p(\theta) = \hat{v}'(\theta)$, while the value function satisfies $v(\theta) = \hat{v}(\theta)$.

This leaves the possibility open that there might be an equilibrium with continuous scaled price $p(\theta)$ which features $\mathcal{S} \neq [0, 1]$. However, repeating arguments made before, one can then consider $\theta \in [0, 1] - \mathcal{S}$. In state $\theta \notin \mathcal{S}$, it is then, by definition of the set $\mathcal{S}$, optimal to lumpily trade toward state $\hat{\theta} \in \mathcal{S}$ (where $v(\hat{\theta}) = \hat{v}(\hat{\theta})$ and $p(\hat{\theta}) = \hat{v}'(\hat{\theta})$). This trade yields continuation payoff $\hat{v}(\hat{\theta}) - (\hat{\theta} - \theta)\hat{v}'(\hat{\theta}) < \hat{v}(\theta)$. Thus, the blockholder would prefer not trading at all in $\theta \notin \mathcal{S}$. Thus, the set $[0, 1] - \mathcal{S}$ must be empty in any continuous, scaled trading equilibrium, while the equilibrium is uniquely determined on $\mathcal{S}$.

Overall, the continuous, scaled Markov equilibrium we have characterized is unique.

B Other Results

B.1 Proof of Corollary 3

We recall for the purpose of this proof $\mathcal{D}(\theta)$ defined in (A.11), satisfying

$$\mathcal{D}(\theta) = \frac{(\rho - r)\,(1 - \lambda^2 - \kappa\pi)}{2\sqrt{\kappa\,\left(\kappa\,(r+\delta)^2 - \mu^2(1 - \lambda^2 - \kappa\pi)\,\theta - 2\kappa\,\mu^2\alpha\right)}} + \frac{\lambda(1-\lambda)}{\kappa} - \pi - \pi^I\left(1 - \frac{\widetilde{\theta}}{\theta}\right)^+ .$$

Also recall the definition of $\underline{\Theta}$ from (19). Furthermore, we recall from the proof of Proposition 3 in Appendix A.6: Whenever $\overline{\theta} > 0$, we have $\overline{\theta} \in (\widetilde{\theta}, 1)$.

Also observe that, as $t \to \infty$, the state θ drifts to one of the stationary points — in particular, 0, $\underline{\theta}$, and $\overline{\theta}$. The dynamics of θ are deterministic, so that $\theta_\infty \in \{0, \underline{\theta}, \overline{\theta}\}$ is deterministic too. We prove claims (1), and (2) of the Corollary in the respective order.

1. We start by proving the claims regarding $\underline{\Theta}$, assuming $\underline{\Theta} > 0$. Note that $\mathcal{D}(\underline{\Theta}) = 0$

with $\mathcal{D}'(\underline{\Theta}) > 0$. Thus,

$$\frac{\partial \underline{\Theta}}{\partial x} = -\frac{1}{\mathcal{D}'(\underline{\theta})} \frac{\partial \mathcal{D}(\underline{\Theta})}{\partial x}, \tag{B.1}$$

which has the opposite sign as $\frac{\partial \mathcal{D}(\underline{\theta})}{\partial x}$. Thus, $\underline{\Theta}$ decreases in μ, increases in r, and increases in κ. Next, note that $\mathcal{D}(\theta)$ decreases in r for any θ, so that $\overline{\theta}$ decreases in r.

Next, we prove the properties of $\overline{\theta}$. Suppose that $\overline{\theta} > 0$. Then, $\overline{\theta} > \widetilde{\theta}$ satisfies $\mathcal{D}(\overline{\theta}) = 0$. The function $\mathcal{D}(\theta)$ is differentiable at $\overline{\theta} \in (\widetilde{\theta}, 1)$, satisfying $\mathcal{D}'(\overline{\theta}) < 0$. Totally differentiating $\mathcal{D}(\overline{\theta})$ with respect to an arbitrary model parameter x, we get from the implicit function theorem:

$$\frac{\partial \overline{\theta}}{\partial x} = -\frac{1}{\mathcal{D}'(\overline{\theta})} \frac{\partial \mathcal{D}(\overline{\theta})}{\partial x}, \tag{B.2}$$

which has the same sign as $\frac{\partial \mathcal{D}(\overline{\theta})}{\partial x}$. Clearly, $\mathcal{D}(\theta)$ increases in μ for any θ. Thus, $\overline{\theta}$ increases in μ. Next, note that $\mathcal{D}(\theta)$ decreases in r for any θ, so that $\overline{\theta}$ decreases in r. Finally, $\mathcal{D}(\theta)$ decreases in κ for any θ, so that $\overline{\theta}$ decreases in κ too.

2. Finally, consider the case $\rho - r = 0$. Then, $\mathcal{D}(\theta)$ simplifies to

$$\mathcal{D}(\theta) = \frac{\lambda(1-\lambda)}{\kappa} - \pi - \pi^I \left(1 - \frac{\widetilde{\theta}}{\theta}\right)^+$$

and is independent of θ for $\theta < \widetilde{\theta}$. One verifies that $\frac{\lambda(1-\lambda)}{\kappa} - \pi \geq 0$ if and only if $\lambda \in [\lambda_-, \lambda_+]$ for $\lambda_\pm = \frac{1 \pm \sqrt{1-4\kappa\pi}}{2}$, provided $1 \geq 4\kappa\pi$. Thus, when $\lambda \in [\lambda_-, \lambda_+]$, we have $\mathcal{D}(\theta) \geq 0$ for $\theta < \widetilde{\theta}$, leading to $\theta_\infty > 0$ (assuming $\theta_0 > 0$).

When $1 < 4\kappa\pi$, then $\mathcal{D}(\theta) < 0$ and $\theta_\infty = 0$.

Likewise, when $\lambda < \lambda_-$ or $\lambda > \lambda_+$, then $\mathcal{D}(\theta) < 0$ for all $\theta \in [0,1]$, which implies $\theta_\infty = 0$.

B.2 Proof of Proposition 5

For $\theta_0 \in [0,1]$, the blockholder's scaled entry payoff reads

$$e(\theta_0) := v(\theta_0) - R(\theta_0) = v(\theta_0) - \theta_0 p(\theta_0) + \min\{\varphi, \theta_0\}(p(\theta_0) - p(0)), \tag{B.3}$$

where $v(\theta_0) = \hat{v}(\theta_0)$ and $p(\theta_0) = \hat{v}'(\theta_0)$, as well as $p(0) = \hat{v}'(0)$.

Consider $\theta_0 \leq \varphi$. Then, $e(\theta_0) = \hat{v}(\theta_0) - \theta_0 \hat{v}'(0)$. Note that $e(0) = 0$ and $e'(\theta_0) = \hat{v}'(\theta_0) - \hat{v}'(0) = 0$, i.e., $e(\theta_0)$ is increasing and positive for $\theta_0 \in (0, \varphi)$. This implies $e(\varphi) > 0$.

Next, consider $\theta_0 \geq \varphi$. Then, $e(\theta_0) = \hat{v}(\theta_0) - \theta_0 \hat{v}'(\theta_0) + \varphi\big[\hat{v}'(\theta_0) - \hat{v}'(0)\big]$. Note that $e(\varphi) > 0$, while $e'(\theta_0) = -(\theta_0 - \varphi)\hat{v}''(\theta_0) < 0$. Thus, $e(\theta_0)$ decreases for $\theta_0 > \varphi$.

Therefore, $e(\theta_0)$ is maximized for $\theta_0 = \varphi$.

B.3 Details for Section 3 and Proof of Proposition 6

We now provide the solution details for the model variant with other, dispersed shareholders controlling investment. Unless otherwise mentioned, all assumptions remain as in the baseline. We assume $\mu > 0$, as otherwise there is no investment and the baseline model applies.

In particular, we assume that instead of the blockholder, the dispersed shareholders choose investment whenever $\theta < 1$; for $\theta = 1$, the blockholder chooses investment. We look for a scaled, continuous Markov equilibrium with state variables $(K_t, \theta_t) = (K, \theta)$, where $L(K,\theta) = K\ell(\theta)$, $V(K,\theta) = Kv(\theta)$, and $P(K,\theta) = K_t p(\theta)$ with continuous and increasing price function $p(\theta)$. Besides investment choice, all other elements, including debt choice and the contracting with management, remain unchanged relative to the baseline. When writing down the payoffs, we already impose the optimal structure of the optimal contract with management, which is the same as in the baseline. In addition, as in the baseline, it can be shown that $y(\theta) < v'(\theta) = p(\theta)$.

Formally, dispersed shareholders choose the investment rate to maximize the stock price $P(K,\theta)$, taking the blockholder's trading $d\theta$ and other controls as given. That is, dispersed shareholders choose investment rate i to maximize

$$\rho P(K,\theta) = \max_{i \geq 0} \left\{ K\left(\alpha + b - \frac{i^2}{2} + \eta(\theta)\right) + (\rho - r)L(K,\theta) + \frac{\mathbb{E}[dP(K,\theta)]}{dt} \right\},$$

Due to $P(K,\theta) = Kp(\theta)$ and $\frac{dK}{K} = (\mu i - \delta)dt$, this optimization can be rewritten as

$$(\rho + \delta)p(\theta) = \max_{i \geq 0} \left\{ \alpha + b + \mu i p(\theta) - \frac{i^2}{2} + (\rho - r)\ell + \eta(\theta) + \frac{\mathbb{E}[dp(\theta)]}{dt} \right\},$$

where, as we show, equilibrium effort b satisfies $b = \frac{\theta(1-\lambda)}{\kappa}$ and debt ℓ satisfies $\ell = \ell(\theta) = \frac{v(\theta)}{\theta}$. The optimization with respect to i yields $i = i(\theta) = \mu p(\theta)$ for all $\theta \in (0,1)$.

To focus on non-trivial cases and to guarantee the existence of a unique continuous, scaled Markov equilibrium, we make the following assumptions. First, as in the baseline, we assume (8), (9), and $\mu i < r + \delta$. Second, analogously to (A.1), we assume that

$$\eta(1) = \pi^I(1 - \widetilde{\theta}) > (\rho - r)\sqrt{\frac{1 - \lambda^2 - \kappa\pi + 2\alpha\kappa}{\kappa\mu^2}} + \frac{\lambda(1-\lambda)}{\kappa} - \pi. \tag{B.4}$$

The condition (B.4) slightly differs from (A.1), but has similar economic meaning. It will ensure $\dot{\theta}(1) < 0$ and $\theta < 1$, i.e., the blockholder never acquires the entire firm. Likewise, as shown in Proposition 3, this outcome also prevailed in the baseline. Since the blockholder never acquires the entire firm or, more generally, a sufficiently large stake (unless it is born with it), both with and without control over investment, one could assume that the blockholder controls investment for ownership levels θ sufficiently close to one, without changing the key findings and equilibrium properties.

We note that the equilibrium properties of Appendix A.7.1—that is, (A.16), (A.19), or (A.21)—are also valid in this model variant and are used in the following proofs. In particular, Appendix A.7.1 derives equilibrium properties that must hold in any continuous, scaled Markov equilibrium under any Markovian investment policy.

We solve for a continuous, scaled Markov equilibrium where the blockholder trades smoothly at rate $\dot{\theta} = \dot{\theta}(\theta)$ for all $\theta \in [0, 1]$, in that the set $\mathcal{S}$—defined in (A.18)—satisfies $\mathcal{S} = [0, 1]$. In what follows, we suppose $\mathcal{S} = [0, 1]$, solve for the equilibrium, and then verify in Appendix B.4.2 that $\mathcal{S} = [0, 1]$ holds in equilibrium. Appendix B.4.2 also establishes the uniqueness of the continuous, scaled Markov equilibrium with smooth trading. We note that this uniqueness result is slightly less general than uniqueness among all continuous, scaled Markov equilibria (which we established for the baseline).

B.3.1 Optimization and HJB Equation

In a continuous, scaled Markov equilibrium, (A.16) must hold whenever $\dot{\theta} \in (-\infty, \infty)$ and a lumpy trade is not strictly optimal, in that gains from trade are zero in equilibrium. By continuity of the price function, we have $p(\theta) = v'(\theta)$. Analogously to (A.21), the blockholder's scaled value function $v(\theta)$ solves the HJB equation:

$$(\rho + \delta)v(\theta) = \max_{\ell} \left[\theta \left(\alpha + b + (\rho - r)\ell - \frac{i^2}{2} \right) - \frac{\pi\theta^2}{2} - \frac{\kappa b^2}{2} + \mu i v(\theta) \right], \tag{B.5}$$

subject to (5) and $b = \frac{\theta(1-\lambda)}{\kappa}$. We used that the gains from trade are zero, and the blockholder's value function is determined "as if" it could not trade at all (i.e., $\dot{\theta} = 0$). The choice of scaled debt satisfies, as in the baseline, $\ell(\theta) = y(\theta) = \frac{v(\theta)}{\theta}$. The only and key difference to (13) and (A.21) is that the blockholder does not choose investment, which, instead, equals $i = \mu p(\theta) = \mu v'(\theta)$ and statically maximizes equity value $p(\theta)$, i.e., $i = \arg\max_{\hat{i} \geq 0} \left[\mu \hat{i} p(\theta) - \frac{\hat{i}^2}{2} \right] = \mu p(\theta)$.

We can insert optimal debt policy, $\ell(\theta) = y(\theta)$, and the investment policy, $i = \mu p(\theta) = \mu v'(\theta)$, into (B.5) to obtain the following first-order ODE for $v(\theta)$:

$$(r + \delta)v(\theta) = \theta \left(\alpha + b - \frac{(\mu v'(\theta))^2}{2} \right) - \frac{\kappa b^2}{2} - \frac{\pi\theta^2}{2} + \mu^2 v'(\theta) v(\theta). \tag{B.6}$$

Using $y(\theta) = \frac{v(\theta)}{\theta}$ for $\theta \in (0, 1)$, we obtain from (B.6):

$$(r + \delta)y(\theta) = \alpha + \frac{\theta(1 - \lambda^2)}{2\kappa} - \frac{\pi\theta}{2} + \mu^2 v'(\theta) \left(y(\theta) - \frac{1}{2} v'(\theta) \right) \tag{B.7}$$

Note that when dispersed investors choose investment, the investment choice generally does not maximize the blockholder's scaled value function. As such, the blockholder's scaled value function is lower than that in the baseline, in that $v(\theta) < \hat{v}(\theta)$ and $y(\theta) < \frac{\hat{v}(\theta)}{\theta}$ for all $\theta \in (0, 1)$.

B.3.2 ODE for Value Function and Stock Price

First note that according to (A.19), we have

$$v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta}) \leq 0 \quad \text{for all} \quad \theta, \hat{\theta} \in [0,1].$$

We conjecture and verify that $v(0) = 0$ (see below). Setting $\theta = 0$ and using $v(0) = 0$, we have $p(\hat{\theta}) \geq \frac{v(\hat{\theta})}{\hat{\theta}}$. Thus, $y(\theta) = \frac{v(\theta)}{\theta} \leq p(\theta) = v'(\theta)$ holds for all $\theta \in [0,1]$.

Using $v'(\theta) \geq y(\theta)$ for $\theta \in (0,1)$, we can solve (B.6) for $v'(\theta)$ to obtain:

$$v'(\theta) = \sqrt{\frac{\theta(1-\lambda^2-\kappa\pi) + 2\,\alpha\,\kappa\, + y(\theta)^2\,\kappa\,\mu^2 - 2y(\theta)\,\kappa\,(r+\delta)}{\kappa\mu^2}} + y(\theta). \tag{B.8}$$

The ODE (B.8) is solved on $(0,1)$ subject to the boundary condition

$$\lim_{\theta\to 0} \frac{v(\theta)}{\theta} = \lim_{\theta\to 0} y(\theta) = \lim_{\theta\to 0} \frac{\hat{v}(\theta)}{\theta}, \tag{B.9}$$

where $\hat{v}(\theta)$ is the value function under the baseline (i.e., when blockholder controls investment) given in (A.6). The boundary condition (B.9) implies in particular, $\lim_{\theta\to 0} v(\theta) = \lim_{\theta\to 0} \hat{v}(\theta) = 0$, i.e., $v(0) = 0$. The boundary condition reflects that in the limit $\theta \to 0$, the blockholder's value function approaches the value under the baseline.

We now prove that the term under the square root in (B.8) is strictly positive and that $v'(\theta) > y(\theta)$ in the continuous, scaled Markov equilibrium. Since we have already shown $v'(\theta) \geq y(\theta)$, we suppose to the contrary there exists $\theta \in (0,1)$ such that $v'(\theta) = y(\theta)$ for $\theta > 0$. Thus, the baseline case effectively prevails where $i = \mu p(\theta) = \mu y(\theta)$ maximizes the blockholder's value function. Then, (B.6) reduces to (A.21) with $i = \frac{\mu v(\theta)}{\theta}$ and $\ell(\theta) = \frac{v(\theta)}{\theta}$. In this case, we could solve (B.6) or, equivalently, (B.7) for

$$v(\theta) = \hat{v}(\theta) = \frac{\theta\,\left(\kappa(r+\delta) - \sqrt{\kappa\,\left(\kappa\,(r+\delta)^2 - \mu^2(1-\lambda^2-\kappa\pi)\,\theta - 2\kappa\,\mu^2\alpha\right)}\right)}{\kappa\,\mu^2}$$

where above expression for $\hat{v}(\theta)$ is taken from (A.6). Thus, $y(\theta) = \frac{\hat{v}(\theta)}{\theta}$. Note that when $v(\theta) = \hat{v}(\theta)$, the term under the square root in (B.7) becomes zero.

The ODE (B.8) and $\lim_{\theta\to 0} v'(\theta) = \lim_{\theta\to 0} y(\theta)$ imply then $y(\hat{\theta}) = v'(\hat{\theta})$ and $v(\hat{\theta}) = \hat{v}(\hat{\theta})$ for all $\hat{\theta} \in [0,\theta]$. However, $v'(\hat{\theta}) = \hat{v}'(\hat{\theta}) > \frac{\hat{v}(\hat{\theta})}{\hat{\theta}} = y(\hat{\theta})$ (see (A.7) and (A.6)), a contradiction. The claim follows, i.e., $v'(\theta) > y(\theta)$ in a continuous, scaled Markov equilibrium for $\theta \in (0,1)$.

We note that $\lim_{\theta\to 0} \frac{\hat{v}(\theta)}{\theta} = p(0)$ where $p(0) = p^0$ is defined in Proposition 1. For $\theta \to 0$, we obtain that the term under the square root in (B.8) approaches zero, so that $\lim_{\theta\to 0} v'(\theta) = \lim_{\theta\to 0} y(\theta) = p(0)$, i.e., $v'(0) = p^0$.

B.3.3 Existence and Uniqueness of ODE Solution

Together, ODE (B.8) and boundary condition (B.9), as well as $p(\theta) = v'(\theta)$ and the trading strategy (discussed below), characterize the continuous, scaled Markov equilibrium. A solution to (B.8) exists, but may not be unique. We argue now that $v(\theta)$ is uniquely determined.

Note that the right-hand side of (B.8) is not Lipschitz continuous in $v(\theta)$ or $y(\theta)$, as the term under the square root approaches zero as $y(\theta)$ approaches $v'(\theta)$. Thus, standard uniqueness results do not apply, and there could be a solution to (B.8) where $v'(\theta) = y(\theta)$ on an interval. However, such a solution is ruled out, as we have shown $v'(\theta) > y(\theta)$ for all $\theta \in (0, 1)$.

That is, $v(\theta)$ is the unique solution to (B.8) subject to (B.9) which satisfies $v'(\theta) > y(\theta)$ for all $\theta \in (0, 1)$. This solution can also be obtained by solving an auxiliary ODE and invoking standard uniqueness results. Specifically, for $\varepsilon > 0$ and $y_\varepsilon(\theta) = \frac{v_\varepsilon(\theta)}{\theta}$, we can solve

$$v_\varepsilon'(\theta) = \sqrt{\max\left\{\varepsilon, \frac{\theta(1-\lambda^2-\kappa\pi) + 2\,\alpha\,\kappa\, + y_\varepsilon(\theta)^2\,\kappa\,\mu^2 - 2y_\varepsilon(\theta)\,\kappa\,(r+\delta)}{\kappa\mu^2}\right\}} + y_\varepsilon(\theta)$$

subject to

$$\lim_{\theta\to 0} \frac{v_\varepsilon(\theta)}{\theta} = \lim_{\theta\to 0} y_\varepsilon(\theta) = \lim_{\theta\to 0} \frac{\hat{v}(\theta)}{\theta}.$$

The solution $v_\varepsilon(\theta)$ to this ODE exists and is unique for any $\varepsilon > 0$, satisfying $v_\varepsilon'(\theta) > y_\varepsilon(\theta)$. We then obtain $v(\theta) = \lim_{\varepsilon\to 0} v_\varepsilon(\theta)$ for all $\theta \in [0, 1]$, satisfying $v'(\theta) > y(\theta)$ for $\theta \in (0, 1)$.

B.4 Strict Convexity of Value Function

We start by differentiating both sides of the ODE (B.6) with respect to θ to obtain

$$(r+\delta)v'(\theta) = \left(\alpha + b - \pi\theta - \frac{(\mu v'(\theta))^2}{2}\right) + \frac{\theta\lambda(1-\lambda)}{\kappa} + \mu^2(v'(\theta))^2 + \mu^2(v(\theta) - \theta v'(\theta))v''(\theta).$$

Thus, provided $\theta > 0$, we can solve for

$$v''(\theta) = \frac{1}{\mu^2\theta\big(v'(\theta) - y(\theta)\big)} \underbrace{\left[\alpha + b - \pi\theta + \frac{(\mu v'(\theta))^2}{2} + \frac{\theta\lambda(1-\lambda)}{\kappa} - (r+\delta)v'(\theta)\right]}_{=:\mathcal{A}(\theta)}. \quad \text{(B.10)}$$

By (B.8), it follows that $v'(\theta) > y(\theta)$ and $\mu^2\theta\big[v'(\theta) - y(\theta)\big] > 0$. Thus, to show that $v''(\theta) > 0$, we need to show that the term in square brackets in (B.10)—which we denote by $\mathcal{A}(\theta)$—is positive. Generally, (B.10) implies that on $(0, 1)$, $v''(\theta)$ has the same sign as $\mathcal{A}(\theta)$.

Recall that, as discussed above, the boundary condition (B.9) implies $v'(0) = p(0)$ where $p(0) = p^0$ is characterized in Proposition 1. Thus, inserting $v'(0) = p(0)$ into our expression for $\mathcal{A}(\theta)$, we get $\mathcal{A}(0) = 0$.

Next, calculate

$$\mathcal{A}'(\theta) = \frac{1-\lambda^2-\kappa\pi}{\kappa} - (r+\delta)v''(\theta) - \mu^2 v'(\theta)v''(\theta) = \frac{1-\lambda^2-\kappa\pi}{\kappa} - (r+\delta-\mu i)v''(\theta). \quad \text{(B.11)}$$

where we used $i = \mu v'(\theta)$. Note that $r + \delta - \mu i > 0$ (finite valuations requirement ensured by parameter condition (8)) and that $1 - \lambda^2 - \kappa\pi > 0$ due to parameter condition (9).

We now prove that $\mathcal{A}(\theta) > 0$ (i.e., $v''(\theta) > 0$) in a right-neighborhood of zero. Suppose to the contrary that there exists $\varepsilon > 0$ such that $\mathcal{A}(\theta) < 0$ (i.e., $v''(\theta) < 0$) for all $\theta \in (0, \varepsilon)$. However, $v''(\theta) < 0$ implies $\mathcal{A}'(\theta) > 0$ owing to (B.11). Thus, due to $\mathcal{A}(0) = 0$, we must have $\mathcal{A}(\theta) > 0$ on this interval, a contradiction. Thus, $\mathcal{A}(\theta) > 0$ and $v''(\theta) > 0$ in a right-neighborhood of zero.

Last, suppose to the contrary that there exists $\theta' \in (0, 1)$ such that $v''(\theta') < 0$, i.e., $\mathcal{A}(\theta') < 0$. Then, there exits—by continuity of $v''(\theta)$ and $\mathcal{A}(\theta)$—a value $\theta'' \in (0, \theta')$ where $v''(\theta'') = 0$, i.e., $\mathcal{A}(\theta'') = 0$, as well as $\mathcal{A}'(\theta'') < 0$. That is, because $v''(\theta) > 0$ and $\mathcal{A}(\theta) > 0$ in a right-neighborhood of zero and $v''(\theta'), \mathcal{A}(\theta') < 0$, there must exists a root θ'' of $v''(\theta)$ and $\mathcal{A}(\theta)$, whereby $\mathcal{A}(\theta)$ crosses zero from above with $\mathcal{A}'(\theta'') < 0$. However, inserting $v''(\theta'') = 0$ into (B.11) we obtain $\mathcal{A}'(\theta'') > 0$, a contradiction. It follows that $v''(\theta) > 0$ on $(0, 1)$.

B.4.1 Smooth Trading Rate

We determine the trading rate $\dot{\theta} = \dot{\theta}(\theta)$. Differentiating both sides of (B.5) with respect to θ and using $p(\theta) = v'(\theta)$, we get

$$\begin{aligned}(\rho + \delta - \mu i)p(\theta) = \alpha + b - \frac{i^2}{2} + (\rho - r)\ell(\theta) - \pi\theta \\ + \frac{\theta\lambda(1-\lambda)}{\kappa} + \theta(\rho - r)\ell'(\theta) + i'(\theta)\theta\mu\left(\frac{v(\theta)}{\theta} - p(\theta)\right).\end{aligned} \tag{B.12}$$

where $\ell = \ell(\theta) = \frac{v(\theta)}{\theta}$ and $i = i(\theta) = \mu p(\theta) = \mu v'(\theta)$.

In analogy to (A.24) in the benchmark model, $p(\theta)$ satisfies the fair pricing equation of dispersed shareholders:

$$(\rho + \delta - \mu i)p(\theta) = \alpha + b - \frac{i^2}{2} + (\rho - r)\ell(\theta) + \eta(\theta) + p'(\theta)\dot{\theta}.$$

Combining (B.12) with the fair pricing equation, we can solve for the trading rate:

$$\dot{\theta} = \dot{\theta}(\theta) = \frac{1}{p'(\theta)}\left[\frac{\theta\lambda(1-\lambda)}{\kappa} + \theta(\rho - r)\ell'(\theta) - \pi\theta - \eta(\theta) + \theta\mu i'(\theta)\left(y(\theta) - p(\theta)\right)\right]. \tag{B.13}$$

Due to $v''(\theta) > 0$ and $v'(\theta)\theta > v(\theta)$, we get that $\ell'(\theta) = \frac{1}{\theta}\left(p(\theta) - y(\theta)\right) > 0$ as well as $i'(\theta) = \mu v''(\theta) > 0$, so that

$$\theta\mu i'(\theta)\left(\frac{v(\theta)}{\theta} - p(\theta)\right) = \theta\mu^2 v''(\theta)\big(y(\theta) - v'(\theta)\big) < 0.$$

Finally, we verify that the proposed trading is such that θ stays within $[0, 1]$, which boils down to showing $\dot{\theta}(0) \geq 0$ and $\dot{\theta}(1) \leq 0$. First, inserting $\theta = 0$ into (B.13), we note that

$\dot{\theta}(0) = 0$. Second, we can calculate

$$\dot{\theta}(1) = \frac{1}{p'(1)} \left[\frac{\lambda(1-\lambda)}{\kappa} + (\rho - r)(p(\theta) - y(\theta)) - \pi - \eta(1) + \underbrace{\mu i'(1)\,(y(1) - p(1))}_{\leq 0} \right]$$
$$\leq \frac{1}{p'(1)} \left[\frac{\lambda(1-\lambda)}{\kappa} + (\rho - r)\sqrt{\frac{(1-\lambda^2-\kappa\pi) + 2\,\alpha\,\kappa}{\kappa\mu^2}} - \pi - \eta(1) \right] < 0,$$

where we used (B.8) to transition from the first to the second line, and we used (B.4) for the last inequality. In addition, we used that

$$\sqrt{\frac{\theta(1-\lambda^2-\kappa\pi) + 2\,\alpha\,\kappa\, + y(\theta)^2\,\kappa\,\mu^2 - 2y(\theta)\,\kappa\,(r+\delta)}{\kappa\mu^2}} \leq \sqrt{\frac{(1-\lambda^2-\kappa\pi) + 2\,\alpha\,\kappa}{\kappa\mu^2}}.$$

To see this, define

$$\mathcal{B}(y) := \frac{\theta(1-\lambda^2-\kappa\pi) + 2\,\alpha\,\kappa\, + y^2\,\kappa\,\mu^2 - 2y\,\kappa\,(r+\delta)}{\kappa\mu^2}.$$

Observe that $\mathcal{B}(0) > 0$ and $\mathcal{B}'(y) \propto -(r+\delta-\mu^2 y)$. For $y \leq v'(\theta)$, we get $r+\delta-\mu^2 y > r+\delta-\mu i$, which is positive under the assumption of finite valuations (see (8)).

B.4.2 Optimality of Smooth Trading and Uniqueness

We show that lumpy trades are strictly sub-optimal, in that

$$0 = \arg\max_{\Delta} \left[v(\theta + \Delta) - \Delta p(\theta + \Delta) \right] \tag{B.14}$$

for any $\Delta \in [-\theta, 1-\theta]$. The derivative with respect to Δ of the term in square brackets reads $\mathcal{G}(\Delta) := v'(\theta+\Delta) - p(\theta+\Delta) - \Delta p'(\theta+\Delta)$. We have $p(\theta+\Delta) = v'(\theta+\Delta)$, so $\mathcal{G}(\Delta) = -\Delta v''(\theta+\Delta)$. Condition (B.14) holds if $v''(\theta) > 0$ holds for all $\theta \in (0,1)$. We have shown that $v(\theta)$ is strictly convex, establishing the optimality of smooth trading. Indeed, given $v''(\theta) > 0$, it follows that smooth trading dominates lumpy trading in any given state, also verifying our conjecture that $\mathcal{S} = [0,1]$ holds in our equilibrium.

Uniqueness of the equilibrium—within the class of continuous, scaled Markov equilibrium with smooth trading—follows from the uniqueness of the solution $v(\theta)$ to (B.8) subject to (B.9) — which must hold in any continuous, scaled Markov equilibrium with $\mathcal{S} = [0,1]$ — combined with the pricing condition $p(\theta) = v'(\theta)$.

Thus, $v(\theta)$ and $p(\theta)$ are uniquely pinned down in continuous, scaled Markov equilibrium with smooth trading (which implies $\mathcal{S} = [0,1]$). As we have shown previously, given $v(\theta)$ and $p(\theta)$, the trading rate is uniquely pinned down too, whereby trading is such that $\dot{\theta}(0) \geq 0$ and $\dot{\theta}(1) < 0$, i.e., the equilibrium trading is consistent the constraint $\theta \in [0,1]$.

Thus, a continuous, scaled Markov equilibrium with smooth trading exists and is unique.

B.4.3 Proof of Corollary 4

First, the assumptions imply that under investor control over investment, the trading rate satisfies according to (21):

$$\dot{\theta} = \frac{1}{p'(\theta)} \left[\theta \mu i'(\theta) \left(y(\theta) - p(\theta)\right)\right] < 0.$$

Thus, $\lim_{t\to\infty} \theta_t = \hat{\theta}$ as well as $\lim_{t\to\infty} p(\theta_t) = p(\hat{\theta})$ for some value $\hat{\theta} \in [0, \theta_0]$. Suppose, to the contrary, that $\hat{\theta} > 0$. Then, $\lim_{\theta \downarrow \hat{\theta}} \dot{\theta} = 0$ which implies $\lim_{\theta \downarrow \hat{\theta}} y(\theta) = \lim_{\theta \downarrow \hat{\theta}} p(\theta)$, which contradicts that $y(\theta) < p(\theta)$ on $(0, 1)$. Therefore, $\lim_{t\to\infty} \theta_t = 0$, which implies—by virtue of (B.9)—that $\lim_{t\to\infty} p(\theta_t) = \hat{v}'(0)$.

Now, consider the baseline model. that the blockholder obtains the baseline, so the trading rate in the continuous, scaled Markov equilibrium is given in (17). Under our assumptions $\pi = \lambda = \rho - r = 0$, (17) readily implies $\dot{\theta} = 0$, so that θ_t and $p(\theta_t)$ remain constant at $\theta_t = \theta_0$ and $p(\theta_t) = p(\theta_0)$, where $p(\theta_0) > p(0) = p^0$.

B.5 Risky Debt: Details and Proof of Proposition 8

Suppose that the firm experiences a downward jump in capital stock at Poisson rate Λ, in that K_t follows

$$\frac{dK_t}{K_{t^-}} = (\mu i_t - \delta)dt - (1 - S_t)dN_t^K,$$

where $K_{t^-} = \lim_{s\uparrow t} K_s$ denotes the left-limit of capital, i.e., the capital stock at time t just before the shock to capital is realized. Here, $dN_t^K \in \{0, 1\}$ is the increment of a Poisson process with intensity $\Lambda = \frac{\mathbb{E}[dN_t^K]}{dt} > 0$ and S_t (or equivalently $1 - S_t$) is uniformly distributed over $[0, 1]$ and i.i.d across time t. That is, each time t, there is a new random draw of S_t from $U([0, 1])$ which is not observable unless $dN_t^K = 1$ and in particular not observable at time t^-. Instantaneous cash flows equal $K_{t^-}dX_t$. Intuitively, a capital shock over $[t, t + dt]$ does not affect current cash flows dX_t, but "next-period" cash flows.

Throughout, we assume that the blockholder chooses debt and investment. Furthermore, we assume that the large capital shock, specifically dN_t^K and S_t are not contractible with management; they are also not influenced by management, which only affects the cash flows $K_{t^-}dX_t$. That is, the manager's contract takes the same form as in the baseline, featuring a base wage c_t and an exposure β_t to cash flow realizations. By this assumption, we rule out the possibility that the firm could write an insurance contract with management in which the manager provides a large cash payment to repay debt following the shock—such a contract is counterfactual. Thus, the manager's contract $\mathcal{C}_t = (c_t, \beta_t)$ is only contingent on cash flows $K_{t^-}dX_t$. We already use that the flow wage equals zero, i.e., $c_t = 0$, and that the sensitivity satisfies $\beta_t = \lambda$.

In the following, we omit time subscripts and the left-limit notation, unless necessary. Specifically, fraction $1 - S$ of the capital is destroyed, where S is uniformly distributed on $[0, 1]$. Hence, when pre-shock capital stock equals K, then post-shock capital stock equals SK. In the event that the firm defaults following this shock, creditors recover the firm's liquidation value or RSK, where $R < p(0)$ is sufficiently small to ensure that liquidation is inefficient and creditors cannot be repaid in full in the event of default in equilibrium. See Section B.5.5 for a characterization of $p(0)$—since $p(0)$ is generally not available in closed-form, there is unfortunately no explicit condition on parameters that ensures $R < p(0)$.

The heuristic timing of the model with large shocks to capital within a time interval $[t, t+dt]$ is as follows. Initially, the blockholder's stake equals θ_t and capital stock equals K_{t^-}. First, at the beginning of $[t, t+dt]$, given a blockholder stake θ_t, shareholders choose the managerial contract $\mathcal{C}_t = (c_t, \beta_t)$, the investment rate $i_t \geq 0$, and the amount of short-term debt L_t, where we assume that the proceeds from debt issuance are distributed as dividends (one can net out these dividends with the debt repayment of the previous instant). Second, the blockholder chooses its effort; then dN_t realized and, observing dN_t, the manager chooses diversion m_t. Then, cash flows dX_t and capital shock $dN_t^K(1-S_t)$ are realized; the manager receives its promised payments. At this point in time, capital stock equals $K_{t^-}\big[1-(1-S_t)dN_t^K\big]$. Third, at the end of $[t, t+dt]$, debt matures and shareholders repay debtholders $(1+r_t dt)L_t$, where r_t is the endogenous interest rate, or default, in which case it is liquidated and the blockholder's continuation payoff and firm value become zero. In case of no default, cash flows net of investment cost, managerial compensation, and debt repayment are distributed as dividends to shareholders. Fourth, after cash flows and capital shock are realized, the blockholder can trade and chooses $d\theta_t$, determining the next-period stake $\theta_{t+dt} = \theta_t + d\theta_t$. Notably, the blockholder can trade just before or just after debt is repaid. Finally, investment and deprecation materialize, so $K_{t+dt} = K_{t^-}\big[1-(1-S_t)dN_t^K\big] + K_{t^-}\big[\mu i_t - \delta\big]dt$.[19]

We characterize a continuous, scaled Markov equilibrium with state variables $(K_{t^-}, \theta_{t^-}) = (K, \theta)$, where $V_{t^-} = K_{t^-}v(\theta_{t^-})$, $P_{t^-} = K_{t^-}p(\theta_t)$, and $L_{t^-} = K_{t^-}\ell(\theta_t)$—we omit time subscripts unless necessary. For brevity, we omit a formal proof of existence and uniqueness and sketch the arguments here. However, one could adapt our baseline arguments to prove the existence and uniqueness of a scaled, continuous Markov equilibrium, as well as to establish the optimality of smooth trading.

Following the same arguments as in the baseline, we obtain $v'(\theta) = p(\theta)$ as well as the strict convexity of $v(\theta)$, i.e., $v''(\theta) > 0$ at points of differentiability, as necessary conditions for the optimality of smooth trading. Moreover, we use that $y = y(\theta) = \frac{v(\theta)}{\theta}$ increases in θ, i.e., $\theta v'(\theta) - v(\theta) > 0$. Likewise, we already use that optimal investment satisfies $\frac{\mu v(\theta)}{\theta}$, as well as that blockholder effort satisfies $b = \frac{\theta(1-\lambda)}{\kappa}$. All these properties can be proven by repeating the same arguments as in the baseline; we omit them for the sake of brevity.

Default and Credit Spreads We start by characterizing endogenous default and credit spreads. Consider the firm just after it has issued scaled debt of ℓ in state (K, θ). When a shock of size S realizes, the blockholder's (unscaled) payoff drops from $K(v(\theta) - \ell)$ pre-shock to $K\big[Sv(\theta) - \theta\ell\big]$ post-shock.

When the blockholder's post-shock payoff $K[Sv(\theta) - \theta\ell]$ (before debt repayment) is negative, i.e., $\frac{Sv(\theta)}{\theta} < \ell$, the firm defaults following the shock. Analogous to the reasoning in

[19] The change in the capital stock due to investment and depreciation is of order dt and its effect on other quantities of order dt therefore negligible. As such, the exact timing of when the capital stock is updated does not matter for trading or default. For convenience and to avoid tedious notation, we assume it occurs at the very end of the period.

Lemma 1, which shows that the firm defaults whenever the blockholder's payoff is strictly negative, default following the shock occurs in one of two ways. Either the blockholder has the authority to force default, or it sells its entire stake (at a positive price), thereby reducing the firm's equity value below the level of outstanding debt and making default optimal for dispersed shareholders. Otherwise, when $\frac{Sv(\theta)}{\theta} \geq \ell$, the firm does not default following a shock.

Recall the definition (12), that is, $y(\theta) := \frac{v(\theta)}{\theta}$. In the Appendix, we may occasionally suppress the dependence of y on θ. As argued above, following a shock with size S, the firm defaults if and only if

$$Sy(\theta) < \ell = \ell(\theta) \quad \Longleftrightarrow \quad S < \frac{\ell(\theta)}{y(\theta)}.$$

Conditional on a shock occurring, the probability of default is therefore given by $\Delta := \frac{\ell}{y}$— which is a function of θ (dependence suppressed for convenience). Thus, the probability of default over an instant $[t, t + dt]$ is $\Lambda\Delta dt$.

The recovery value in default following a size S-shock is RKS. Conditional on default, i.e., $S < \frac{\ell}{y}$, creditors recover in expectation $\frac{\ell}{2y}RK$ dollars, i.e., $\frac{R}{2y}$ dollars per unit of debt (with face value of one dollar). When risk-neutral creditors with discount rate r lend one dollar, they require an expected repayment at time $t + dt$ of $1 + rdt$ dollars, in that

$$1 + rdt = \Lambda\Delta dt \cdot \frac{R}{2y} + (1 - \Lambda\Delta dt)(1 + \hat{r}dt),$$

where $\hat{r}$ is the endogenous interest rate on debt.

To understand the above equality, note that over $[t, t + dt]$, the creditors are repaid the dollar plus interest $\hat{r}dt$ in case the firm does not default, which happens with probability $1 - \Lambda\Delta dt$. With probability $\Lambda\Delta dt$, the firm defaults and creditors recover in expectation $\frac{R}{2y} < 1$ dollars per dollar lent. We can solve the above expression for the fair interest rate

$$\hat{r} := r + \Lambda\Delta \left(1 - \frac{R}{2y}\right),$$

which coincides with (22). Therefore, the credit spread equals $\hat{r} - r = \Lambda\Delta \left(1 - \frac{R}{2y}\right)$.

B.5.1 Payoffs

The following derivations turn out to be convenient in characterizing the blockholder's payoff function. Conditional on a shock occurring and no default — that is, $Sv(\theta) \geq \theta\ell \iff S \geq \frac{\ell}{y}$ — the average value of S equals:

$$\mathbb{E}_t^S \left[S \middle| Sv(\theta) \geq \theta\ell; \theta_t = \theta\right] = \frac{1}{2}\left(\frac{\ell}{y} + 1\right) = \frac{1 + \Delta}{2}.$$

Thus, scaled continuation payoff post-shock but before debt repayment equals

$$\mathbb{E}_t^S\left[Sv(\theta) - \theta\ell \middle| dN_t = 1; Sv(\theta) \geq \theta\ell; \theta_t = \theta\right] = \frac{v(\theta)}{2}\left(\frac{\ell}{y} + 1\right) - \theta\ell = \frac{\theta y}{2}(1+\Delta) - \theta\ell.$$

for the blockholder. It equals

$$\mathbb{E}_t^S\left[Sp(\theta) - \ell \middle| dN_t = 1; Sv(\theta) \geq \theta\ell; \theta_t = \theta\right] = \frac{p(\theta)}{2}\left(\frac{\ell}{y} + 1\right) - \ell = \frac{p(\theta)}{2}(1+\Delta) - \ell$$

for the dispersed investors. Given a level of debt ℓ, we can calculate the expected continuation payoff for the blockholder conditional on a shock occurring as:

$$\begin{aligned} E^b(\theta) &:= \mathbb{E}_t^S\left[\max\{0, Sv(\theta) - \theta\ell\} \middle| dN_t = 1; \theta_t = \theta\right] \\ &= (1-\Delta)\left(\frac{v(\theta)}{2}(1+\Delta) - \theta\ell\right) = \left(\frac{\theta y}{2}\right)\left[1 - \Delta^2\right] - \ell(1-\Delta), \end{aligned}$$

where the $\mathbb{E}_t^S$ is taken with respect to the random variable S *conditional* on $dN_t^K = 1$. Likewise, the expected continuation payoff for the dispersed shareholders, i.e., the expected stock price, conditional on a shock occurring becomes

$$\begin{aligned} E^p(\theta) &:= \mathbb{E}_t^S\left[\left(Sp(\theta) - \ell\right)\mathbb{I}\{Sv(\theta) \geq \theta\ell\} \middle| dN_t = 1; \theta_t = \theta\right] \\ &= (1-\Delta)\left(\frac{p(\theta)}{2}(1+\Delta) - \ell\right) = \left(\frac{p(\theta)}{2}\right)\left[1 - \Delta^2\right] - \ell(1-\Delta), \end{aligned} \tag{B.15}$$

where the indicator $\mathbb{I}\{Sv(\theta) \geq \theta\ell\}$ equals one if and only if the firm does not default following the shock, that is, if and only if $S \geq \frac{\ell}{y}$.

We heuristically derive the blockholder's HJB equation under optimal debt and investment choice, starting in state (K, θ) with zero debt outstanding. Recall that in optimum $i = \mu y$. Importantly, in equilibrium, gains from trade are zero and $v'(\theta) = p(\theta)$, so the blockholder's payoff is "as if" it could not trade and θ remains constant. Thus, consider the hypothetical scenario that θ remains constant over $[t, t + dt]$. First, recall that the firm receives a capital shock with probability Λdt, in which case it defaults with probability Δdt and does not default otherwise. The scaled payoff from issuing $K\ell$ dollars of debt maturing at $t + dt$, with proceeds paid out as dividends, satisfies:

$$\begin{aligned} Kv(\theta) = \theta K\ell &+ K\left(\theta\left(\alpha + b - \frac{i^2}{2}\right) - \frac{\pi\theta^2}{2} - \frac{\kappa b^2}{2}\right)dt \\ &+ e^{-\rho dt}(1 - \Lambda dt)\left\{[K + (\mu i - \delta)dt]v(\theta) - \theta K\ell(1 + \hat{r}dt)\right\} \\ &+ e^{-\rho dt}\Lambda(1-\Delta)dt\left\{K[\tilde{S} + (\mu i - \delta)dt]v(\theta) - \theta K\ell(1 + \hat{r}dt)\right\}, \end{aligned}$$

where $\tilde{S} := \mathbb{E}\left[S \middle| S \geq \frac{\ell}{y}\right] = \frac{1}{2} + \frac{\ell}{2y}$ for $y = \frac{v(\theta)}{\theta}$, $\hat{r} = r + \Lambda\Delta\left(1 - \frac{R}{2y}\right)$, and $\Delta = \frac{\ell}{y}$.

Taking the limit $dt \to 0$, ignoring terms of order $(dt)^2$, and doing some algebra, we can derive that the blockholder's value function solves

$$(\rho+\delta+\Lambda-\mu i)v(\theta) = \theta\left(\alpha+b-\frac{i^2}{2}+(\rho+\Lambda-\hat{r})\ell\right)-\frac{\pi\theta^2}{2}-\frac{\kappa b^2}{2}+\Lambda E^b(\theta).$$

Note that the shock arrival rate Λ effectively augments the blockholder's discount rate, i.e., the "effective" discount rate becomes $\rho+\Lambda$. Inserting the interest rate $\hat{r}$ from (22) into above equation, we can rearrange to obtain for the blockholder's value function:

$$(\rho+\delta-\mu i+\Lambda)v(\theta) = \theta\left(\alpha+b+(\rho-r)\ell-\frac{i^2}{2}\right)-\frac{\pi\theta^2}{2}-\frac{\kappa b^2}{2}+\frac{\theta\Lambda}{2}\big[y-\Delta^2(y-R)\big], \tag{B.16}$$

given the controls.

B.5.2 Optimal Debt Choice

Taking the derivative of the right-hand-side of (B.16) with respect to ℓ, we get

$$\frac{\partial v(\theta)}{\partial \ell}\propto \theta(\rho-r)-\frac{\theta\Lambda\Delta(y-R)}{y}\propto\frac{(\rho-r)}{\Lambda}-\frac{\ell(y-R)}{y^2},$$

where the proportionality sign $\propto$ indicates the omission of positive scaling constants. If $\ell<\frac{v(\theta)}{\theta}$, i.e. the borrowing constraint (5) does not bind, the optimal choice of debt ℓ solves $\frac{\partial v(\theta)}{\partial \ell}=0$. As such, we can solve for

$$\ell=\ell(\theta)=\min\left\{y,\frac{(\rho-r)y^2}{\Lambda(y-R)}\right\}, \tag{B.17}$$

where, as we recall, $y=y(\theta)=\frac{v(\theta)}{\theta}$. We also note that $y=y(\theta)\geq p(0)=p^0$, so the denominator of above expression is strictly positive.

As such, the default probability, conditional on a shock occurring, becomes

$$\Delta(\theta)=\frac{\ell(\theta)}{y(\theta)}=\min\left\{1,\frac{(\rho-r)y}{\Lambda(y-R)}\right\}.$$

B.5.3 Stock Price and Trading Rate

The stock price satisfies the pricing equation

$$(\rho+\delta+\Lambda-\mu i)p(\theta) = \alpha+b-\pi\theta-\frac{i^2}{2}+(\rho+\Lambda-\hat{r})\ell-\frac{\kappa b^2}{2}+\Lambda E^p(\theta),$$

where $E^p(\theta)$ is the expected continuation stock price conditional on a shock occurring from (B.15). Here, Λ augments the discount rate of dispersed shareholders. Upon a shock occurring with probability Λ, the dispersed shareholders realize the expected continuation stock price $E^p(\theta)$. We can rewrite the pricing equation as follows:

$$(\rho+\delta+\Lambda)p(\theta) = \alpha + b + (\rho - r)\ell - \frac{i^2}{2} + \mu i p(\theta) + \eta(\theta) + \frac{\Lambda\big[p(\theta) - \Delta^2(p(\theta) - R)\big]}{2} + p'(\theta)\dot{\theta}. \tag{B.18}$$

Next, we derive the blockholder's valuation of an additional unit of stock, i.e., $v'(\theta)$, which, in equilibrium, must equal the stock price $p(\theta)$. We distinguish two cases, (i) $\ell < 1$ and (ii) $\ell = y$.

Borrowing Constraint $\ell \leq y(\theta)$ is slack. First, suppose that $\ell < y = y(\theta)$, which implies $\Delta < 1$. Then, the choice of debt ℓ solves the first-order condition $\frac{\partial v(\theta)}{\partial \ell} = 0$, and so does investment, i.e., $\frac{\partial v(\theta)}{\partial i} = 0$. We now invoke the envelope theorem and differentiate both sides of (B.16) with respect to θ. After some algebra, we obtain that the marginal valuation of an additional unit of stock from the blockholder perspective satisfies $v'(\theta) = p(\theta)$ with

$$(\rho+\delta+\Lambda)p(\theta) = \alpha + b - \pi\theta + (\rho - r)\ell - \frac{i^2}{2} + \mu i p(\theta) + \frac{\theta\lambda(1-\lambda)}{\kappa} + \frac{\Lambda\big[p(\theta) - \Delta^2(p(\theta) - R)\big]}{2} + \theta\Lambda y'(\theta)\frac{\ell^2(y-R)}{y^3}.$$

The last term $\theta\Lambda y'(\theta)\frac{\ell^2(y-R)}{y^3}$ is strictly positive due to $y'(\theta) > 0$ and can be rewritten as follows:

$$\theta\Lambda y'(\theta)\frac{\ell^2(y-R)}{y^3} = \theta\Lambda y'(\theta)\frac{\ell(y-R)}{y^2}\Delta = \theta\Lambda y'(\theta)\frac{(\rho-r)y}{\Lambda(y-R)}\frac{y-R}{y^2}\Delta = \theta(\rho-r)y'(\theta)\Delta.$$

Borrowing Constraint $\ell \leq y(\theta)$ is binding. Second, consider $\ell = \ell(\theta) = y(\theta)$, i.e., $\Delta = 1$. Then, the choice of debt is constrained by the borrowing constraint $\ell \leq y$ and does not solve a first-order condition. Again, we differentiate both sides of (B.16) with respect to θ. We then obtain that the marginal valuation of an additional unit of stock from the blockholder perspective satisfies $v'(\theta) = p(\theta)$ with

$$(\rho+\delta+\Lambda)p(\theta) = \alpha + b - \pi\theta + (\rho - r)\ell - \frac{i^2}{2} + \mu i p(\theta) + \frac{\theta\lambda(1-\lambda)}{\kappa} + \frac{\Lambda\big[p(\theta) - \Delta^2(p(\theta) - R)\big]}{2} + \theta(\rho - r)\ell'(\theta).$$

Due to $\ell(\theta) = y(\theta)$ whenever $\Delta = 1$, we have

$$\theta(\rho-r)\ell'(\theta) = \theta(\rho-r)y'(\theta)\Delta = (\rho-r)[p(\theta) - y(\theta)] > 0.$$

In both scenarios, we can combine above equations for $p(\theta) = v'(\theta)$ with (B.18) to solve for the trading rate:

$$\dot{\theta} = \frac{1}{p'(\theta)} \left[\frac{\theta\lambda(1-\lambda)}{\kappa} - \pi\theta - \eta(\theta) + \theta(\rho - r)y'(\theta)\Delta \right]. \tag{B.19}$$

Here, $\theta(\rho - r)y'(\theta)\Delta = (\rho - r)\Delta\big[p(\theta) - y(\theta)\big] > 0$ captures the gains from trade associated with risky debt.

B.5.4 Dynamics of Debt, Default Probabilities, and Credit Spreads

Suppose that in state (K, θ), optimal debt satisfies $\ell = \frac{(\rho-r)y(\theta)^2}{\Lambda(y(\theta)-R)} < y$. Then,

$$\ell'(\theta) = \frac{y'(\theta)(\rho - r)}{\Lambda} \frac{2y(y-R) - y^2}{(y-R)^2} = \frac{y'(\theta)(\rho - r)}{\Lambda} \frac{y(y-2R)}{(y-R)^2}.$$

If $R \leq \frac{p(0)}{2}$, $y - 2R$ is unambiguously positive and debt ℓ increases with θ. Otherwise, when $\frac{p(0)}{2} < R < p(0)$, then there exists a right-neighborhood of 0 in which $\ell'(\theta)$. Then, $\ell(\theta)$ is generally U-shaped in θ, i.e., first decreases and then increases in θ.

Next, the default probability $\Delta(\theta)$ satisfies

$$\Delta'(\theta) = \frac{y'(\theta)}{\Lambda} \frac{(y(\theta) - R)(\rho - r) - (\rho - r)y(\theta)}{(y(\theta) - R)^2} = -R\frac{y'(\theta)}{\Lambda} \frac{(\rho - r)}{(y(\theta) - R)^2} < 0.$$

Thus, the default probability decreases with θ.

Finally, calculate the credit spread $\hat{r} - r = \frac{(\rho-r)y}{y-R} - \frac{R(\rho-r)}{2(y-R)} = (\rho - r)\frac{2y-R}{2(y-R)}$ which decreases with θ.

B.5.5 Dispersed Ownership Benchmark

Under (perpetual) dispersed shareholder ownership, the stock price satisfies $Kp(0)$ and investment satisfies $i = i^0 \mu p(0)$, as in the baseline. Debt issuance $\ell = \ell^0$ is subject to the borrowing constraint $\ell \leq p(0)$, which ensures that equity value remains positive and precludes immediate default. Moreover, following a capital shock of size S, the firm defaults under dispersed ownership whenever $Sp(0) < \ell$, leading to the probability of default $\Delta = \Delta^0 = \frac{\ell}{p(0)}$.

Analogously to the pricing equation (B.18)—which holds for $\theta > 0$—the scaled passive-ownership price satisfies

$$(\rho + \delta + \Lambda)p(0) = \max_{\ell} \left(\alpha + (\rho - r)\ell - \frac{i^2}{2} + \mu i p(0) + \frac{\Lambda\big[p(0) - \Delta^2(p(0) - R)\big]}{2} \right). \tag{B.20}$$

It can be obtained similar to (B.18) upon setting $\dot{\theta} = 0$, $\eta(\theta) = 0$, and $\theta = 0$, while optimizing the right-hand-side over debt.

Debt choice satisfies under dispersed ownership solves the first-order condition (if interior):

$$(\rho - r) - \frac{\Lambda \ell (p(0) - R)}{(p(0))^2} = 0,$$

leading to

$$\ell = \ell^0 = \min \left\{ p(0), \frac{(\rho - r)(p(0))^2}{\Lambda (p(0) - R)} \right\}$$

One can insert this expression for $\ell = \ell^0$ back into (B.20), but there is no closed-form solution for $p(0)$; one needs to solve $p(0)$ numerically. Overall, $p(0)$ will be a function of R. We impose $p(0) > R$, which leads to an implicit parameter condition on R.

Note that $p(0) > R$ implies that in equilibrium, the recovery value is insufficient to repay creditors in full in the event of default, making debt is risky. To see this, note that the firm default in state $\theta = 0$ whenever $S < \frac{\ell^0}{p(0)}$. Upon default, creditors then recover SRK dollars. Due to $SR < Sp(0) < \ell^0$ — where we used $R < p(0)$ and $S < \frac{\ell^0}{p(0)}$ — this amount is insufficient to repay creditors in full. Likewise, in state θ, the firm defaults whenever $S < \frac{\ell}{y}$, in which case creditors recover SRK dollars. Due to $SR < \frac{p(0)}{y} \ell < \ell$, this amount is insufficient to repay creditors in full.

B.5.6 Investor Control

We now briefly discuss how the solution changes when dispersed investors are in control of corporate policies, allowing them to choose both investment and debt. In a continuous, scaled Markov equilibrium with smooth trading, we have $p(\theta) = v'(\theta)$. Under investor control, debt and investment are determined according to:

$$(\rho + \delta + \Lambda) p(\theta) = \max_{i,\ell} \left\{ \alpha + b + \eta(\theta) + (\rho - r)\ell - \frac{i^2}{2} + \mu i p(\theta) + \frac{\Lambda \big[p(\theta) - \Delta^2 (p(\theta) - R) \big]}{2} + p'(\theta)\dot{\theta} \right\}. \tag{B.21}$$

where $\Delta = \frac{\ell}{y}$ and $\ell \leq y$. This leads to $i = i^I = \mu p(\theta) = \mu v'(\theta)$. Debt choice satisfies under investor control the first-order condition (if interior)

$$(\rho - r) - \frac{\Lambda \ell [p(\theta) - R]}{y^2} = 0,$$

which we solve for

$$\ell = \ell^I = \min \left\{ y, \frac{(\rho - r) y^2}{\Lambda [p(\theta) - R]} \right\}.$$

Given these policies, the blockholder value function satisfies (B.16), that is:

$$(\rho + \delta - \mu i + \Lambda) v(\theta) = \theta \left(\alpha + b + (\rho - r)\ell - \frac{i^2}{2} \right) - \frac{\pi \theta^2}{2} - \frac{\kappa b^2}{2} + \frac{\theta \Lambda}{2} \big[y - \Delta^2 (y - R) \big],$$

where the default probability $\Delta = \Delta^I = \frac{\ell^I}{y}$ is a non-linear function of $v'(\theta) = p(\theta)$, provided $\ell^I < y$; $\ell(\theta) = \ell^I(\theta)$ and $i(\theta) = i^I(\theta)$ are functions of θ too. This ODE is solved subject to $\lim_{\theta \to 0} \frac{v(\theta)}{\theta} = p(0)$—with $p(0)$ characterized in the dispersed ownership benchmark.

The determination of the continuous, scaled Markov equilibrium becomes similar to that in Section 3, but the ODE is now significantly more complicated and cannot even be explicitly solved for $v'(\theta)$. We leave the further analysis of this model variant.

Trading Rate with Investor Control. Yet, we can solve for the trading rate, without further characterizing the ODE for $v(\theta)$ or having a closed-form solution for $v(\theta)$ or $p(\theta)$. For this sake, assume that the borrowing constraint does not bind in that $\ell^I < y(\theta)$. Then, we can differentiate both sides of (B.16) with respect to θ to get that $p(\theta) = v'(\theta)$ satisfies:

$$(\rho + \delta + \Lambda - \mu i)p(\theta) = \alpha + b - \pi\theta + (\rho - r)\ell - \frac{i^2}{2} + \frac{\theta\lambda(1-\lambda)}{\kappa} + \frac{\Lambda\big[p(\theta) - \Delta^2(p(\theta) - R)\big]}{2}$$
$$+ \theta\Lambda\frac{y'(\theta)\ell^2}{y^3}(y - R) + (\rho + \delta + \Lambda - \mu i)\Big(\underbrace{\frac{\partial v(\theta)}{\partial \ell}\frac{\partial \ell^I}{\partial \theta}}_{>0}\Big) + \underbrace{i'(\theta)\theta\mu(y(\theta) - p(\theta))}_{<0}.$$

The term $\theta\Lambda\frac{y'(\theta)\ell^2}{y^3}(y - R)$ satisfies

$$\theta\Lambda\frac{y'(\theta)\ell^2}{y^3}(y - R) = \theta\Lambda\Delta^2\frac{y'(\theta)[y - R]}{y} = \theta y'(\theta)\left(\frac{(\rho - r)^2 y}{\Lambda(y - R)}\right) = \theta(\rho - r)y'(\theta)\Delta.$$

Combining these with (B.21), we can solve for the trading rate through $\dot{\theta} = \frac{\mathcal{A}(\theta)}{p'(\theta)}$ where

$$\mathcal{A}(\theta) = \frac{\theta\lambda(1-\lambda)}{\kappa} - \pi\theta - \eta(\theta) + \theta(\rho - r)y'(\theta)$$
$$+ (\rho + \delta + \Lambda - \mu i)\Big(\underbrace{\frac{\partial v(\theta)}{\partial \ell}\frac{\partial \ell^I}{\partial \theta}}_{>0}\Big) + \underbrace{i'(\theta)\theta\mu(y(\theta) - p(\theta))}_{<0}.$$

To gain some intuition, note that if dispersed investors are in control, they choose too high investment $i = i^I = \mu p(\theta)$ and too low debt $\ell = \ell^I$ from the blockholder's perspective. Therefore, a marginal increase in investment (debt) reduces (increases) the blockholder's value function, in that $\frac{\partial v(\theta)}{\partial i} < 0$ and $\frac{\partial v(\theta)}{\partial \ell} > 0$. The blockholder internalizes that when increasing its ownership stake marginally, it induces dispersed shareholders to choose higher levels of investment and debt, which affects its own payoff. This effect influences the blockholder's valuation of an additional unit of equity, and therefore the gains from trade and trading rate $\dot{\theta}$. Consequently, as also shown in Section 3, delegating investment decisions to dispersed shareholders reduces the blockholder's propensity to acquire additional shares and encourages exit. In contrast, delegating debt decisions to dispersed shareholders increases the blockholder's propensity to acquire additional shares.

B.6 Large Trades and Proof of Proposition 7

We characterize a scaled Markov equilibrium where $P(K,\theta) = p(\theta)$ and $V(K,\theta) = Kv(\theta)$.

For convenience and to abstract from knife-edge cases, we assume that the reverse inequality of (A.1) holds strictly (i.e., condition (A.1) is "strictly" violated), in that

$$\eta(1) = \pi^I(1-\widetilde{\theta}) < \frac{(\rho - r)\,(1-\lambda^2-\kappa\pi)}{2\sqrt{\kappa\,\left(\kappa\,(r+\delta)^2 - \mu^2(1-\lambda^2-\kappa\pi) - 2\kappa\,\mu^2\alpha\right)}} + \frac{\lambda(1-\lambda)}{\kappa} - \pi. \quad \text{(B.22)}$$

We note that in this case, it is not possible to construct a scaled Markov equilibrium where the price function is continuous. Instead, we will construct a Markov equilibrium where $p(\theta)$ exhibits one discontinuity at the endogenous threshold θ^*. Here, we note that the results from Appendix Section A.7.1 also apply in this context, since this Appendix establishes generalized equilibrium properties that hold in any scaled Markov equilibrium, i.e., so long as $P(K,\theta) = Kp(\theta)$ and $V(K,\theta) = Kv(\theta)$. All state dependent controls $b = b(\theta)$, the managerial contract, $i = i(\theta)$, and $\ell = \ell(\theta)$ are analogous to the baseline.

Finally, note that n the equilibrium of Proposition 7, the price exhibits one discontinuity (upward jump) at $\theta = \theta^*$. While there can be multiple equilibria and, in particular, numerous equilibria with price discontinuities, we characterize the equilibrium closest to the one from the baseline with a minimum number of price discontinuities, specifically, a single discontinuity. Within the class of equilibria where $p(\theta)$ has at most one discontinuity, we obtain uniqueness.

B.6.1 Preliminaries

In the following, it will be convenient to work with $\hat{p}(\theta)$ defined via

$$\hat{p}(\theta) := \frac{\alpha + b(\theta) + (\rho - r)\ell(\theta) - \frac{i(\theta)^2}{2} + \eta(\theta)}{\rho + \delta - \mu i(\theta)}, \quad \text{(B.23)}$$

with $\ell(\theta) = \frac{\hat{v}(\theta)}{\theta}$, $i(\theta) = \frac{\mu\hat{v}(\theta)}{\theta}$, and $b = b(\theta)$ from (11). Note that $\hat{p}(\theta)$ is the hypothetical scaled price that would prevail if the blockholder perpetually maintained ownership θ.

One can show that whenever (A.1) does not hold, we have $\hat{v}'(1) \geq \hat{p}(1)$, i.e., by construction, the condition (A.1) is *equivalent* to $\hat{v}'(1) < \hat{p}(1)$ — which implies, by $\hat{v}''(\theta) > 0$, that $\hat{v}'(\theta) < \hat{p}(1)$.

Next, define

$$J_H(\theta) = \hat{v}(1) - (1-\theta)\hat{p}(1) - \hat{v}(\theta),$$

where $\hat{p}(1)$ is the hypothetical price that prevails if the blockholder perpetually owns the entire firm with $\theta = 1$. Note that $J_H'(\theta) = \hat{p}(1) - \hat{v}'(\theta)$ and $J_H''(\theta) = -\hat{v}''(\theta) < 0$. Thus, $J_H'(1) < 0$. Since $J_H(1) = 0$, it follows that $J_H(\theta)$ has maximally one root on $(0,1)$, while $J_H(\theta) < 0$ for $\theta > 1$.

We define

$$\theta^* = \inf\{\theta \geq 0 : J_H(\theta) \geq 0\} \quad \text{(B.24)}$$

Due to $J_H''(\theta) < 0$ and $J_H(1) = 0$ as well as $J_H'(1) < 0$, there exist a left-neighborhood of 1 on which $J_H(\theta) > 0$. Thus, $\theta^* \in (0,1)$. If $J_H(\theta)$ has a root on $(0,1)$, then θ^* is this root, i.e.,

$J_H(\theta^*) = 0$. Otherwise, $J_H(\theta) \geq 0$ for all $\theta \in (0,1)$ and $\theta^* = 0$.

We note that $\theta^* \in (0,1)$ if and only if $J_H(0) < 0$, i.e., $\hat{v}(1) - \hat{p}(1) < 0$. If $J_H(0) \geq 0$, the equilibrium is trivial, with the blockholder immediately acquiring the entire firm and perpetually maintaining full ownership ($\theta = 1$).

We therefore assume (B.22) — that is, $J_H(0) = \hat{v}'(1) - \hat{p}(1) < 0$ — in what follows to obtain an equilibrium with meaningful trading dynamics. This implies $\theta^* \in (0,1)$. Last, we discuss the knife-edge case. If $\eta(1)$ is equal to the right-hand-side of (A.1) — that is, both (A.1) and (B.22) are violated — the threshold θ^* simply equals one and we get an equilibrium with smooth trading, i.e., the equilibrium from the baseline.

B.6.2 Conjecturing the Equilibrium

We conjecture the following equilibrium, where the blockholder optimally trades smoothly on the interval $[0, \theta^*)$ according to rate $\dot{\theta} = \dot{\theta}(\theta)$ characterized in (17). The trading rate can be positive or negative, as in the baseline, with $\dot{\theta}(0) = 0$. On $[0, \theta^*]$, the (scaled) value function satisfies $v(\theta) = \hat{v}(\theta)$ and the price satisfies $p(\theta) = \hat{v}'(\theta)$.

When $\hat{v}'(\theta^*) > \hat{p}(\theta^*)$, the trading rate $\dot{\theta}(\theta)$ is positive in a left-neighborhood of θ^* and θ reaches θ^* from below, in that $\lim_{\theta \to \theta^*} \dot{\theta}(\theta) > 0$. Once θ reaches θ^*, the blockholder is indifferent and randomizes between buying the entire firm at once (i.e., $d\theta = 1 - \theta^*$) and not trading at all (i.e., $d\theta = 0$). The rate at which the blockholder buys the entire is denoted $\gamma^* > 0$ and will be characterized later. The state θ remains at θ^* until the blockholder buys the entire firm.

When $\hat{v}'(\theta^*) \leq \hat{p}(\theta^*)$, the blockholder trades smoothly in state θ^* at rate $\dot{\theta}(\theta^*) \leq 0$. We then have $\lim_{\theta \to \theta^*} \dot{\theta}(\theta) \leq 0$, and θ^* is either absorbing or θ drifts away and below θ^*.

The threshold θ^* satisfies $J_H(\theta^*)$, with $\theta^* \in (0,1)$. At θ^*, we have $v(\theta^*) = \hat{v}(\theta^*)$ and $p(\theta^*) = \hat{v}'(\theta^*)$.

When $\theta = 1$, the blockholder stops trading and maintains perpetually full ownership of the firm, with value function $v(1) = \hat{v}(1)$ and price $p(1) = \hat{p}(1)$.

When $\theta \in (\theta^*, 1)$, the blockholder immediately buys the entire firm at price $\hat{p}(1)$, i.e., $d\theta = 1 - \theta$. The price equals $p(\theta) = \hat{p}(1)$ and the value function equals $v(\theta) = \hat{v}(1) - (1 - \theta)\hat{p}(1) > \hat{v}(\theta)$.

Thus, in this equilibrium, the value function is smooth and strictly convex on $[0, \theta^*)$ with $\hat{v}''(\theta) > 0$, while the price is continuous and increasing on this interval. At θ^*, the price exhibits an upward jump while the value function exhibits a kink.

For the following analysis, we recall from Appendix section A.7.1 that $v(\theta) = \hat{v}(\theta)$ on the set $\mathcal{S} = \{\theta \in [0,1] : \dot{\theta} \in (-\infty, +\infty) \quad \text{and} \quad \gamma \in [0, \infty)\}$, defined analogously in (A.18).

In this model variant, the set $\mathcal{S}$ is conjectured as $\mathcal{S} \in [0, \theta^*] \cup \{1\}$—with θ^* defined in (B.24). For $\theta \notin \mathcal{S}$, the blockholder finds it optimal to immediately conduct a lumpy trade, which, without loss of generality, brings θ into the set $\mathcal{S}$.

B.6.3 Optimality of the Trading Strategy

We verify the optimality of the proposed trading strategy. To do so, we show that, given the conjectured equilibrium price, the blockholder's scaled value function $v(\theta)$ solves the HJB equation (A.16). If $v(\theta)$ solves the HJB equation (A.16), it follows that the proposed trading strategy is optimal, since this strategy yields payoff $v(\theta)$. To show that $v(\theta)$ solves the HJB equation, we first need to characterize the optimal trading in different parts of the state space under the conjectured equilibrium price $p(\theta)$ and continuation payoff $v(\theta)$. Then, one can verify that the HJB equation holds by invoking the optimal trading strategy.

We start with preliminary findings, and distinguish the following cases and characterize the trading in different parts of the state space.

Result: Any Lumpy Trade must be onto the edges of $\mathcal{S}$. As a lumpy trade outside of $\mathcal{S}$ would be immediately followed by another lumpy trade, it is without loss of generality to assume that a lumpy trade brings the state θ into the set $\mathcal{S}$.

In the interior of $\mathcal{S}$, i.e., in $int(\mathcal{S})$, we have $v(\theta) = \hat{v}(\theta)$ and $p(\theta) = \hat{v}'(\theta)$. Consider state θ and suppose to the contrary that the blockholder conducts a trade from state θ toward state $\hat{\theta} \in int(\mathcal{S})$. While not trading at all yields continuation payoff $\hat{v}(\theta)$, the trade toward $\hat{\theta}$ yields continuation payoff

$$G(\hat{\theta}) = \hat{v}(\hat{\theta}) - (\hat{\theta} - \theta)\hat{v}'(\hat{\theta})$$

Note that $G(\theta) = \hat{v}(\theta)$, equal to the payoff of not trading at all. Calculate $G'(\hat{\theta}) = -(\hat{\theta} - \theta)\hat{v}''(\hat{\theta})$. Due to strict convexity of $\hat{v}(\theta)$ it follows that $G(\hat{\theta})$ is optimized for $\hat{\theta} = \theta$. Thus, a lumpy trade toward state $\hat{\theta} \in int(\mathcal{S})$ is strictly dominated by not trading at all. Hence, any lumpy trade must bring θ onto the edges of $\mathcal{S}$, i.e., $\hat{\theta} \in \{0, 1, \theta^*\}$

State $\theta \in (\theta^*, 1)$. Note that $\hat{v}(1) \leq v(1)$, as the blockholder always has the option not to trade at all in state $\theta = 1$ yielding a payoff $\hat{v}(1)$.

Next, suppose to the contrary that $\hat{\theta} \in (-\infty, \infty)$ and $\gamma \in [0, \infty)$ is optimal. Then, $\theta \in \mathcal{S}$ and $v(\theta) = \hat{v}(\theta)$. But, as $\theta \in (\theta^*, 1)$, we have $\hat{v}(\theta) < \hat{v}(1) + (1-\theta)\hat{p}(1) \leq v(1) + (1-\theta)\hat{p}(1)$ and the blockholder could attain strictly higher payoff through a lumpy trade toward 1, i.e., $d\theta = 1 - \theta$, a contradiction. As a consequence, $\theta \notin \mathcal{S}$.

Thus, the blockholder conducts a lumpy trade. According to our findings above, this lumpy trade brings θ onto the edge of $\mathcal{S}$, so the blockholder trades toward $\hat{\theta} \in \{0, 1, \theta^*\}$. Trading toward $\hat{\theta} = 1$ yields $v(1) - (1-\theta)\hat{p}(1) \geq \hat{v}(1) + (1-\theta)\hat{p}(1)$. Trading toward $\hat{\theta} = \theta^*$ yields

$$\begin{aligned}\hat{v}(\theta^*) + (\theta - \theta^*)p(\theta^*) &= \hat{v}(1) - (1-\theta^*)\hat{p}(1) + (\theta - \theta^*)p(\theta^*) \\ &< \hat{v}(1) - (1-\theta^*)\hat{p}(1) + (\theta - \theta^*)\hat{p}(1) \leq v(1) - (1-\theta)\hat{p}(1),\end{aligned}$$

where we have used that $\hat{v}(\theta^*) = \hat{v}(1) - (1-\theta^*)\hat{p}(1)$ and $p(\theta^*) < \hat{p}(1)$ for $\theta^* < 1$. Thus, trading toward $\hat{\theta} = 1$ strictly dominates trading toward $\hat{\theta} = \theta^*$.

A trade toward 0 yields payoff $\hat{p}(0)\theta < \hat{v}(\theta)$ and thus is strictly dominated by not trading at all.

In state $\theta \in (\theta^*, 1)$, immediately trading toward $\hat{\theta} = 1$, i.e., $d\theta = (1-\theta)$, is thus strictly optimal, delivering payoff $v(\theta) = v(1) - (1-\theta)\hat{p}(1)$. Inserting $\hat{\theta} = 1$ and $\gamma = +\infty$ into (A.16), we can see that (A.16) holds.

State $\theta = 1$. If the blockholder conducts a lumpy trade, then this trade is toward state $\hat{\theta} \in \{0, \theta^*\}$ on the edges of $\mathcal{S}$. Relative to not trading and collecting payoff $\hat{v}(1)$, a trade toward $\theta^* \in (0,1)$ changes payoff by

$$\hat{v}(\theta^*) + (1-\theta^*)p(\theta^*) - \hat{v}(1) < \hat{v}(\theta^*) + (1-\theta^*)\hat{p}(1) - \hat{v}(1) = 0,$$

where we have used that $p(\theta^*) < \hat{p}(1)$ (for $\theta^* < 1$) and that, by definition of θ^*, $\hat{v}(\theta^*) + (1-\theta^*)\hat{p}(1) - \hat{v}(1) = 0$ for $\theta^* \in (0,1)$.

A trade toward $\hat{\theta} = 0$ delivers payoff $\theta\hat{p}(0) < \hat{v}(\theta)$, and thus is strictly suboptimal too.

Next, suppose to the contrary that smooth trading in state $\theta = 1$ is optimal. Thus, $\dot{\theta} < 0$. But, as we have shown, in any state $\theta \in (\theta^*, 1)$, it is strictly optimal to trade toward one. As such, smooth trading $\dot{\theta} < 0$ in state $\theta = 1$ cannot be, because it would be immediately followed by a lumpy trade toward one. As a result, state $\theta = 1$ is absorbing, yielding continuation payoff $\hat{v}(1)$, which solves the HJB equation (A.16) under $d\theta = 0$.

State $\theta \in [0, \theta^*)$. Any lumpy trade must bring θ onto the edges of $\mathcal{S}$, as shown before, i.e., onto $\{0, \theta^*, 1\}$. The value function on $[0, \theta^*)$ satisfies $v(\theta) = \hat{v}(\theta)$.

First, a lumpy trade toward zero yields payoff $\theta\hat{p}(0) < \hat{v}(\theta)$ (for $\theta > 0$), and thus is strictly suboptimal.

A lumpy trade toward θ^* yields payoff $\hat{v}(\theta^*) - (\theta^* - \theta)\hat{v}'(\theta^*) < \hat{v}(\theta)$.

By definition of θ^*, we have $\hat{v}(\theta) > \hat{v}(1) - (1-\theta)\hat{p}(1)$ for $\theta \in [0, \theta^*)$, hence a lumpy trade toward one is strictly suboptimal too.

This verifies that any lumpy trade in state $\theta \in [0, \theta^*)$ is strictly suboptimal. Thus, $\frac{d\theta}{dt} = \dot{\theta} \in (-\infty, +\infty)$ and $\gamma = 0$ are optimal, and $v(\theta) = \hat{v}(\theta)$ satisfies (A.16).

One can verify that in state $\theta = 0$, the blockholder stops trading, in that $\dot{\theta}(0)$ from (17) is zero. By our arguments, it is optimal to stop trading in state $\theta = 0$.

State $\theta = \theta^*$. In state $\theta = \theta^*$, the blockholder is indifferent between not trading, yielding payoff $\hat{v}(\theta^*)$, or buying the entire firm at once, yielding payoff $\hat{v}(1) - (1-\theta^*)\hat{p}(1)$. Thus, it is weakly optimal to randomize between these two options. This implies that it is optimal to set $\dot{\theta} \leq 0$, as $\dot{\theta} > 0$ is akin to an immediate lumpy trade. A strictly positive trading rate $\dot{\theta} > 0$ would bring θ into the region $(\theta^*, 1)$ which would trigger an immediate lumpy trade toward one. Thus, in state $\theta = \theta^*$, it is optimal for the blockholder to either (i) randomize between buying the entire firm and not trading (i.e., $\dot{\theta} = 0$ and $\gamma \in [0, \infty)$ with $\hat{\theta} = 1$) or (ii) trade smoothly at negative (finite) rate $\dot{\theta} < 0$ which also delivers payoff $\hat{v}(\theta^*)$. Either way, the blockholder's payoff in state θ^* is $\hat{v}(\theta^*)$, solving (A.16) under $\dot{\theta} = 0$ and $\gamma \geq 0$.

B.6.4 Trading Rate

We now determine trading rate and randomized trading to ensure that the conjectured price $p(\theta)$ under the blockholder's proposed trading is consistent with dispersed shareholders' valuation of the firm, thereby verifying that $p(\theta)$ is the scaled equilibrium price. Recall that the blockholder stops trading in states 0 and 1, as shown above. Further, it is strictly optimal to trade towards 1 when $\theta \in (\theta^*, 1)$, i.e., $d\theta(\theta) = (1-\theta)$ for $\theta \in (\theta^*, 1]$.

State $\theta \in [0, \theta^*)$. For $\theta \in [0, \theta^*)$, the optimality condition for trading (A.17) implies $v(\theta) = \hat{v}(\theta)$ and $p(\theta) = \hat{v}'(\theta)$. Furthermore, it is optimal to trade smoothly, i.e., $\gamma = 0$. We determine the trading rate as in the baseline to obtain $\dot{\theta}$ from (17); see Appendix A.7.2 for details. The trading rate is uniquely determined in this region under smooth trading.

State $\theta = \theta^*$. Consider state $\theta = \theta^*$. When $\hat{v}'(\theta^*) < \hat{p}(\theta^*)$, then $\dot{\theta}(\theta) < 0$ in a left-neighborhood of θ^* and $\lim_{\theta \to \theta*} \dot{\theta}(\theta) \le 0$. Then, θ drifts into the interior of $[0, \theta^*]$ and no time is spent in state θ^*. In this case, the price satisfies $p(\theta) = \hat{v}'(\theta)$ and $\dot{\theta}(\theta)$ satisfies (17); there is no randomization over lumpy trading in that $\gamma = 0$. In the knife-edge case $\hat{v}'(\theta^*) = \hat{p}(\theta^*)$, we have $\dot{\theta} = \gamma = 0$ at $\theta = \theta^*$.

Next, consider that $\hat{v}'(\theta^*) > \hat{p}(\theta^*)$. Then, $\dot{\theta} = \dot{\theta}(\theta) > 0$ in a left-neighborhood of θ^*, in that θ reaches θ^* from below and $\lim_{\theta \to \theta^*} \dot{\theta}(\theta) > 0$. Because we have in addition that $\dot{\theta} \le 0$ at θ^* from (B.24), it must be that $\dot{\theta} = 0$ at θ^*. By definition of θ^*, the blockholder is indifferent between not trading at all and buying the entire firm at once in state θ^*. Once θ reaches one, the endogenous stock price becomes $p(1)$. As the blockholder stops trading once θ reaches one, we have $p(1) = \hat{p}(1)$. Because $p(\theta) = \hat{v}'(\theta)$ on $[0, \theta^*)$ and because θ reaches θ^* from below, we have $p(\theta^*) = \hat{v}'(\theta^*)$.

At $\theta = \theta^*$ where $\dot{\theta} = 0$, the randomization rate γ^* is such that $p(\theta^*) = \hat{v}'(\theta^*)$. Here, $p(\theta^*)$ satisfies dispersed investors' pricing equation:

$$(\rho + \delta - \mu i(\theta^*))p(\theta^*) = \underbrace{\alpha + b + (\rho - r)\ell(\theta^*) - \frac{i(\theta^*)^2}{2} + \eta(\theta^*)}_{=\hat{p}(\theta^*)} + \gamma^*(\hat{p}(1) - p(\theta^*)),$$

which can be rewritten as $p(\theta^*) = \frac{(\rho+\delta-\mu i(\theta^*))\hat{p}(\theta^*)+\gamma^*\hat{p}(1)}{\rho+\delta-\mu i(\theta^*)+\gamma^*}$. As a consequence,

$$\begin{aligned}\gamma^* &= \frac{[\rho + \delta - \mu i(\theta^*)]\left[\hat{p}(\theta^*) - p(\theta^*)\right]}{p(\theta^*) - \hat{p}(1)} \\ &= \frac{1}{\hat{p}(1) - p(\theta^*)}\left[\frac{\theta^*\lambda(1-\lambda)}{\kappa} + \theta^*(\rho - r)\ell'(\theta^*) - \pi - \eta(\theta^*)\right] > 0. \end{aligned} \tag{B.25}$$

The term in square brackets is precisely the numerator of (17) evaluated at θ^*.

Note that $\dot{\theta} > 0$ for θ close to θ^*, so $\left[\frac{\theta^*\lambda(1-\lambda)}{\kappa} + \theta(\rho - r)\ell'(\theta^*) - \pi - \eta(\theta^*)\right] > 0$. Moreover, $\hat{p}(\theta) < p(\theta)$ in a left-neighborhood of θ^*, so $\hat{p}(\theta^*) < p(\theta^*)$. Because $\hat{v}(\theta)$ is strictly convex and $\hat{v}(\theta^*) = \hat{v}(1) - (1 - \theta^*)\hat{p}(1)$, we have $p(\theta^*) < \hat{p}(1)$. As a result, the randomization rate γ^* from (B.25) is well-defined and strictly positive.

More generally, the trading rate γ is well-defined as long as (A.1) does not hold. As argued earlier, whenever (A.1) does not hold (in the sense of (B.22)), we have $\hat{v}'(1) \ge \hat{p}(1)$, i.e., by construction, the condition (A.1) is *equivalent* to $\hat{v}'(1) < \hat{p}(1)$ — which implies, by $\hat{v}''(\theta) > 0$, that $\hat{v}'(\theta) < \hat{p}(1)$.

B.7 Micro-Foundation of Holding Benefit

We micro-found the holding benefit and its functional form by modeling dispersed investors as consisting of a large mass of standard dispersed investors, who value the firm solely based on the present value of its dividends, and a unit mass of index investors, who apply the same valuation but aim to hold a fixed fraction θ^*_{Index} of the firm's equity. Neither type of investor can short-sell the firm's stock.

All index investors act symmetrically, holding a fraction $\theta_{t,Index}$ of the firm. When deviating from their desired ownership allocation θ^*_{Index}, index investors incur a disutility flow cost $K_t C(\theta_{t,Index})$, where $C(\cdot)$ is convex and differentiable, satisfying $C(\theta^*_{Index}) = C'(\theta^*_{Index}) = 0$. The negative of the disutility flow is a holding benefit. The disutility flow or holding benefit scale with the firm's capital stock. In what follows, we work with scaled quantities (by K_t).

Although not explicitly modeled, the interpretation is that index investors allocate a given amount of wealth to the stock market and aim to invest in the firm in proportion to its market weight.[20] Deviating from their desired ownership level may be costly for index investors, for example, because it reduces diversification or because these investors represent funds or institutional investors aiming to minimize tracking error.

When the blockholder's stake satisfies $\theta_t + \theta^*_{Index} > 1 \iff \theta_t > 1 - \theta^*_{Index}$, index investors hold at most a fraction $\theta_{t,Index} \leq 1 - \theta_t < \theta^*_{Index}$ of the firm. Thus, they hold strictly less than their desired ownership fraction θ^*_{Index} and, accordingly, value an additional unit of stock more than standard financial investors do, since they could reduce their disutility by increasing their ownership stake. Thus, in equilibrium, standard dispersed investors have zero ownership stake and index investors hold $\theta_{t,Index} = 1 - \theta_t$, yielding a disutility of $C(1 - \theta_t) > 0$ — the disutility is positive, since index investors deviate from their desired allocation. This disutility increases in θ, in that $\frac{d}{d\theta} C(1 - \theta_t) = -C'(1 - \theta_t) > 0$ and $C'(1 - \theta_t) < 0$.

The blockholder trades against index investors who are marginal: A marginal reduction in the blockholder's stake θ, resulting in a marginal increase of index investors' stake, changes index investors' cost by $-\frac{d}{d\theta} C(1 - \theta_t) = C'(1 - \theta_t) < 0$. This marginal reduction in blockholder stake thus generates a marginal holding benefit to index investors of $\eta(\theta_t) := -C'(1 - \theta_t) > 0$. Hence, $\eta(\theta_t) = -C'(1 - \theta_t)$ is the dispersed investors' holding benefit from an additional unit of ownership under these circumstances.

When $\theta_t + \theta^*_{Index} < 1 \iff \theta_t < 1 - \theta^*_{Index}$, then index investors hold their desired allocation $\theta_{t,Index} = \theta^*_{Index}$ and other dispersed investors hold fraction $1 - \theta_t - \theta_{t,Index} > 0$. In this scenario, the blockholder trades against the other dispersed investors who are marginal and adjust their holdings when θ_t changes, while index investors remain at their desired allocation. Index investors' disutility cost is zero and not affected by small changes in the stake of the blockholder: In this case, dispersed investors derive zero marginal holding benefit (i.e., zero disutility cost reduction) from increasing their stake, in that $\eta(\theta_t) = 0$.

In sum, the holding benefit is the negative of the marginal disutility flow of dispersed investors from deviating from their desired allocation, i.e., $\eta(\theta_t) = -C'(1 - \theta_t)$ for $\theta_t > 1 - \theta^*_{Index}$, while zero otherwise. We assume quadratic disutility:

$$C(\theta_{t,Index}) = \frac{\pi^I}{2}(\theta_{t,Index} - \theta^*_{Index})^2 \quad \text{for} \quad \pi^I > 0.$$

[20] That is, if the firm represents $\chi_t\%$ of the stock market index (e.g., S&P 500) and index investors allocate W_t dollars to this stock market index, they would aim to hold $\theta^*_{Index} = \chi_t\% \times W_t$ dollars in the firm, corresponding to an ownership share of $\chi_t\% \times \frac{W_t}{P_t}$. Assuming that $\frac{\chi_t W_t}{P_t}$ remains approximately constant over time, index investors would aim for a roughly constant ownership stake in the firm. This case prevails, for instance, when stock market capitalization equals M_t and wealth invested in the stock market is proportional to market capitalization, i.e., $W_t \propto M_t$, and the firm's stock price is also proportional to market capitalization, i.e., $P_t \propto M_t$, we obtain $\chi_t\% = \frac{P_t}{M_t}$ = constant and consequently, $\frac{\chi_t W_t}{P_t}$ = constant. Instead of explicitly modeling the dynamics of $\frac{\chi_t W_t}{P_t}$ and index investors' investment decisions, we simplify the analysis by assuming that they target a specific, constant fraction of ownership, θ^*_{Index}. Our results would remain qualitatively similar if we allowed θ^*_{Index} to vary over time.

For $\theta_t > 1 - \theta^*_{Index}$ where $\theta_{t,Index} = 1 - \theta_t < \theta^*_{Index}$, we obtain

$$C'(\theta_{t,Index}) = C'(1 - \theta_t) = \pi^I(\theta_{t,Index} - \theta^*_{Index}) = \pi^I(1 - \theta_t - \theta^*_{Index}) < 0.$$

Then, holding benefit takes the functional form $\eta(\theta_t) = -C'(1 - \theta_t) = \pi^I\left[\theta_t - (1 - \theta^*_{Index})\right]$. For $\theta_t \leq 1 - \theta^*_{Index}$, the holding benefit $\eta(\theta_t)$ equals zero. Hence, the holding benefit satisfies:

$$\eta(\theta_t) = \pi^I[\theta_t - \widetilde{\theta}]^+, \tag{B.26}$$

for parameters $\pi^I > 0$ and $\widetilde{\theta} = 1 - \theta^*_{Index} \in [0, 1]$. Here, $[\cdot]^+ = \max\{0, \cdot\}$.

Thus, the holding benefit reflects the demand for the stock from certain dispersed investors who are reluctant to sell beyond a certain point and thus attach a high value to the stock, such as index investors. This holding benefit implies that when θ_t is large, the blockholder must pay a high price because they are buying from investors who are reluctant to sell and place a high valuation on the firm. When π^I is sufficiently large, the blockholder will never acquire the entire firm, although θ_t may become arbitrarily close to one. Acquiring the entire firm would be hard or prohibitively costly in practice, as index investors would always hold some of the firm's ownership. When the blockholder buys the firm's stock, it would push up the price and thus increase the firm's weight in the stock market index, further stimulating demand from index investors, which in turn makes it harder to buy more of the stock. Motivated by our micro-foundation, we interpret $\pi^I > 0$ as sufficiently large and $\widetilde{\theta}$ as close to one, implying $\theta_t < 1$ in equilibrium. In particular, we impose condition (A.1).