A Unifying Theory of Shareholder Activism, Ownership Dynamics, and Firm Policies*

Sebastian Gryglewicz[†] Simon Mayer[‡] Erwan Morellec[§]

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Abstract

We develop a unifying theory of shareholder activism in which a blockholder actively influences key firm policies. The blockholder may adjust their ownership stake over time, shaping their level of engagement and the firm's decisions. We analytically characterize the blockholder's dynamic trading strategy, their entry and exit decisions, and the firm's financing, investment, and compensation policies. Our model shows that restricting blockholder control can benefit passive shareholders in the short run, but it endogenously reduces blockholder ownership and firm value in the long run.

Keywords: Activism, limited commitment, strategic trading, price impact, leverage, corporate investment, payout

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[†]Erasmus University Rotterdam. Email: gryglewicz@ese.eur.nl

[‡]Tepper School of Business, Carnegie Mellon University. E-mail: simonmay@andrew.cmu.edu.

[§]EPF Lausanne, Swiss Finance Institute, and CEPR. E-mail: erwan.morellec@epfl.ch.

Activist shareholders, who push for changes in company policies and seek to influence share prices, play a significant role in corporate governance. When trying to improve firm performance and capture the value they generate, activists must deal with complex and closely intertwined decisions related to the strategic trading of their shares and the level and form of their engagement. Despite its importance, a comprehensive framework for understanding the interplay among these decisions is still lacking. Key questions remain unanswered: How should activists adjust their holdings over time? In what ways does trading impact their engagement and influence the company's financial and strategic choices? And, ultimately, should activists exit their position by selling their shares or work towards taking the company private? Addressing these questions is critical to advancing our understanding of activist strategies and their impact on firm outcomes.

Our goal in this article is to develop a tractable dynamic theory of shareholder activism in which an activist's equity stake and the firm's investment, financing, payout, and compensation policies are jointly determined in the presence of agency conflicts. We model activism as a dynamic process involving endogenous entry, post-entry trading, and engagement. A large shareholder—or blockholder—shapes firm outcomes through three distinct mechanisms: (*i*) exerting effort to improve productivity, (*ii*) contracting with management to reduce agency costs, and (*iii*) directly influencing investment and financing decisions. Alongside these efforts and policy interventions, the blockholder can adjust their stake in the firm over time, affecting their level of engagement, the firm's share price, and firm policies.

Our model generalizes the framework of DeMarzo and Urošević (2006) to a strategic trader, the potential activist, who not only influences firm productivity through costly effort, but can also engage in contracting with management, impact firm investment and financing decisions, or choose to exit or take the firm private through dynamic trading. We derive the equilibrium in closed form, which includes the activist's dynamic trading strategy, entry and exit decisions, alongside the firm's compensation, investment, payout, and financing policies, all under the assumption of no commitment from the activist. The model highlights how the allocation of control rights over firm policies influences the activist's trading policy and shapes both their engagement and the firm's strategic decisions and value.

A key feature of the model is the endogenous and dynamic nature of the activist's ownership stake, which serves as a state variable influencing both engagement levels and firm policies. Beginning with an initial stake, the activist can adjust its holdings over time through dynamic trading. Post-entry, the activist trades strategically with price impact, recognizing how its actions influence market prices. The activist faces a continuum of dispersed, passive shareholders who take prices as given. In determining its optimal trading strategy, the activist balances several competing forces. First, a larger stake enhances engagement. Second, increased ownership boosts firm value and amplifies the benefits of debt by improving the firm's financing capacity. Third, passive shareholders may supply shares inelastically, leading to greater price impact.[1] Last, maintaining a large position imposes holding costs due to financial or capital constraints.

The fourth effect—similar to the mechanism in DeMarzo and Urošević (2006)—naturally leads the blockholder to sell more (or buy less).[2] The third effect highlights the costliness of building a large stake when prices react to trades. When this friction is removed, takeover bids and public-to-private transactions may arise in equilibrium. The second effect reflects how a larger ownership stake increases firm value, enabling more borrowing and unlocking greater debt-related benefits. As a large, strategic trader, the blockholder internalizes these gains, which boost the returns on their existing holdings. Passive shareholders, in contrast, ignore this firm-value effect when selling shares, leading to gains from trade. The first effect arises because blockholder effort is not contractible, requiring that effort at any given time remains incentive-compatible. Due to moral hazard, blockholder effort is inefficiently low. However, when the blockholder increases their equity stake, their incentive to exert effort improves, resulting in greater payoffs on their existing infra-marginal equity holdings. This incentive effect, like the debt benefit, is internalized by the activist but ignored by passive investors, further contributing to gains from trade. Whether the activist's stake rises or falls over time depends on the balance among these forces. Importantly, because the activist internalizes more strongly the benefits of improved effort and debt capacity when holding a larger stake, its propensity to buy (sell) increases as its stake becomes larger (lower).

A central result of the paper is that the activist's optimal trading policy involves a

[1] Our assumption that demand is inelastic reflects the growing evidence that shifts in demand significantly influence price levels, regardless of whether such trades contain information. See e.g. Koijen and Yogo (2019); Haddad, Huebner, and Loualiche (2025); Hartzmark and Solomon (2025) and the references therein.

[2] DeMarzo and Urošević (2006) model a large shareholder with CARA preferences that faces normally distributed shocks. As a result, its preferences can be expressed as a certainty equivalent equal to the mean payoff expected to be received less a risk adjustment, which corresponds to the holding cost in our model.

combination of a double-barrier policy and the continuous management of the ownership stake between the barriers through dynamic sales and purchases. We show that there exists endogenous upper and lower thresholds for the blockholder's equity stake—the endogenous state variable—such that it is optimal for the blockholder (i) to gradually sell and eventually exit when the initial stake falls below the lower threshold, and (ii) otherwise, to increase ownership until reaching the upper threshold, where the blockholder either maintains a constant stake or initiates a takeover bid. We demonstrate that the duration of activist engagement is endogenous and depends on corporate governance and the level of agency conflicts, with high and low levels more likely to lead to exit. We also show that the blockholder is more likely to increase its stake over time, and less likely to exit, when debt benefits are larger, when it is effective at improving firm productivity through effort, and when the firm faces good investment opportunities.

Additionally, we find that blockholder entry raises firm value and long-term investment, with investment rates increasing alongside the size of the blockholder's stake.[3] However, because the average valuation of capital for blockholders—which determines investment—is less than the average valuation of capital for passive shareholders, there is underinvestment from the perspective of passive shareholders in any given state. Blockholder entry also increases debt financing, with a book leverage ratio that grows with their stake. Yet, similar to investment, blockholders choose a debt level that differs from the passive investors' optimum, again due to their lower valuation of the firm—and thus, a lower perceived cost of default.

In our baseline model, the blockholder determines corporate policies. We then examine the consequences of limiting the blockholder's influence—restrictions that, in practice, may arise from governance structures such as anti-takeover or entrenchment provisions like a staggered board (Bebchuk, Cohen, and Ferrell, 2009). While blockholder-controlled policies may not maximize the value for passive shareholders, our analysis reveals a paradox: restricting blockholder control can ultimately reduce other shareholders' value. More specifically, we show that while the blockholder and passive investors are aligned on the choice of managerial compensation and leverage, they have divergent preferences over investment. Assigning investment control to dispersed shareholders leads the firm to allocate more resources to long-term investment for any given level of blockholder ownership. However, on average, the

[3] Lewellen and Lewellen (2022) provide direct evidence on institutions' financial incentives to be engaged shareholders by looking at the gains in management fees due to an increase in value of their stockholdings.

blockholder's stake is lower under passive investor control than under blockholder control, resulting in reduced long-term investment. This outcome arises because the blockholder's value function declines: it internalizes that purchasing additional shares raises the stock price and increases investment, thereby moving the investment policy further away from the blockholder's desired investment level. As a result, without the ability to change investment, the blockholder is less inclined to maintain a significant stake, less willing to acquire additional shares, and more likely to exit. These findings highlight the importance of the endogenous and dynamic nature of the blockholder's stake in corporate control, underscoring the need to consider how shifts in control influence not only current policy choices but also the blockholder's ongoing and future engagement with the firm.

Although the policies chosen by the blockholder do not maximize passive shareholder value, our analysis reveals a counterintuitive result: passive shareholder value can actually be lower when they choose investment and financing decisions. This occurs due to changes in the blockholder's trading behavior and level of engagement. Specifically, when dispersed shareholders control investment, the firm allocates more resources to long-term investment for any given level of blockholder ownership. However, on average, the blockholder's stake is lower under passive investor control than under blockholder control, leading to reduced long-term investment. This outcome arises because the blockholder's value function declines, as it internalizes that purchasing additional shares drives up the firm's stock price and investment, thereby moving the investment level further away from its desired investment level. As a result, the blockholder is less likely to maintain a significant stake and acquire additional shares and more likely to exit. We in fact demonstrate that whether exit can be part of an optimal strategy when there is no holding cost uniquely depends on the allocation of control rights. Exit is not optimal when the blockholder controls investment and financing. Exit becomes a feature of the optimal strategy when blockholder does not control investment. These findings emphasize the importance of the endogeneity and dynamic nature of blockholder stake when examining the implications of corporate control and the need to consider how shifts in control influence both immediate corporate policies and the blockholder's ongoing and future engagement with the firm.

Our main contribution is to develop a tractable, comprehensive framework for understanding the dynamic interactions between shareholder activism and firm policies and show

how the allocation of control rights shapes trading, engagement, and value creation. Therefore, our paper primarily relates to the literature on shareholder activism and blockholders (see, e.g., Maug (1998); Noe (2002); Edmans and Manso (2011); Khanna and Mathews (2012); Dasgupta and Piacentino (2015); Levit (2019); Marinovic and Varas (2025)).

Most closely related to our paper, Admati, Pfleiderer, and Zechner (1994), DeMarzo and Urošević (2006), and Back, Collin-Dufresne, Fos, Li, and Ljungqvist (2018) study the trading of a large shareholder that influences firm performance via costly effort. In these models, blockholders have no direct influence over firm decisions, limiting the scope of engagement. Our model allows blockholders to actively shape firm policies while dynamically adjusting their stake, thereby capturing the rich interplay between trading behavior, corporate governance, and firm strategy in a more realistic and tractable way. Under certain conditions (no managerial moral hazard, no debt, and fixed investment policy), our setting coincides with that in DeMarzo and Urošević (2006). However, by introducing agency conflicts and modeling firm policies explicitly, we obtain equilibrium outcomes that differ from those in DeMarzo and Urošević (2006). Notably, the blockholder may increase its stake over time, take over the firm, or exit. Our richer setting allows us to generate predictions relating firm policies to the size of the blockholder stake or regarding the duration of activist engagement. It also allows us to examine the effects of the allocation of control rights on the activist trading behavior and firm value and endogenize the duration of activist engagement, emerging from the interaction between dynamic trading behavior and firm policies.

More generally, our paper advances the literature by allowing the blockholder to affect firm performance not only via its own effort but also by contracting with management and by influencing the firm's financing and investment choices. A higher blockholder stake makes contracting more efficient, increases leverage and investment, and generates endogenous gains from trade, leading to the novel result that the blockholder may dynamically buy a larger stake despite holding costs. Methodologically, we advance the literature on shareholder activism by jointly and tractably modeling entry, dynamic trading, dynamic financing, and investment choices, and the interactions between the incentives of management and the blockholder. Notably, our modeling allows us to capture the key economic mechanisms and to analytically characterize the equilibrium in closed form.

We solve for the Markov-perfect equilibrium trading strategy using a methodology similar to DeMarzo and Urošević (2006), which has also proved useful in other contexts. For instance, Daley and Green (2020) study dynamic bargaining with adverse selection. Hu and Varas (2025), DeMarzo and He (2021), and DeMarzo, He, and Tourre (2023) respectively study loan sales, corporate leverage, and the dynamics of government debt under limited commitment.

Another strand of the literature on trading and activism that is related to our paper is the literature on "governance by exit," which includes the papers by Admati and Pfleiderer (2009), Edmans and Manso (2011), Dasgupta and Piacentino (2015), and Levit (2019). In these models, a blockholder has access to private information about firm value and may sell her block on negative information. The focus of these papers is on trading by an insider who has private information about firm value that is exogenous to her trading. These models have a single round of trading and cannot analyze feedback from prices to blockholder actions.

1 Model Setup

We develop a dynamic model of shareholder activism, in which a blockholder (i.e., activist shareholder) invests in a firm to alter its policies and improve firm value. The blockholder does so by (*i*) exerting private effort to improve asset productivity, (*ii*) incentivizing the firm's manager with an optimal contract, and (*iii*) shaping the firm's investment and financing policies. The blockholder can dynamically increase or decrease its ownership stake in the firm by purchasing or selling shares, leading to changes in effort, investment, and financing (i.e., leverage). Trading decisions are observable by investors, and investors rationally take into account this information when determining their demand for shares. All policies—except payouts to management through the optimal contract—are chosen without commitment.

Technology. Time $t \in [0, \infty)$ is continuous and infinite. There are three risk-neutral agents with common discount rate $\rho > 0$: A blockholder, a continuum of dispersed passive investors, and a manager. We consider a single firm run by the manager. The firm is financed with equity and debt.[4] The large shareholder's trading/ownership policy is given by the function θ, where $\theta_t \in [0, 1]$ denotes the fraction of the firm's equity held by the blockholder with the remaining $1 - \theta_t$ held by passive investors.

[4]In addition to these three agents sharing the same preferences, our model also features debtholders (with a different discount rate) who play a passive role and are introduced later on.

Firm cash flows depend on the endogenous capital stock $K_t > 0$, blockholder effort $b_t \geq 0$, and managerial cash diversion $m_t \in \{0, 1\}$. Over any period of time $[t, t + dt)$, incremental cash flows are $K_t dX_t$, where dX_t is the scaled cash flow per unit of capital with:[5]

$$dX_t = \alpha_1 dt + dN_t(1 - m_t), \tag{1}$$

where $\alpha_1 \geq 0$. In (1), $dN_t \in \{0, 1\}$ is a jump process with $\mathbb{E}[dN_t] = (\alpha_2 + b_t)dt$ for $\alpha_2 > 0$. This specification implies that large positive cash flows arrive infrequently, while the cash flow rate is α_1 otherwise. Investors (and the blockholder) observe the realization of cash flows dX_t, but do not observe dN_t or m_t. When $dN_t = 1$ and additional cash flows of K_t are realized, the manager can divert from the cash flows by setting $m_t = 1$, in which case she gains a private benefit of λK_t with $\lambda \in [0, 1)$ (diversion is inefficient). In this case, investors observe $dX_t = \alpha_1 dt$ and thus cannot discern cash flow diversion upon a shock from no shock occurring. We define

$$\alpha = \alpha_1 + \alpha_2 \tag{2}$$

so that expected (scaled) cash flows are given by $\mathbb{E}[dX_t] = (\alpha + b_t)dt$. Also, while the baseline model only allows for positive cash flow shocks and risk-free debt, Section 4 introduces negative cash flow shocks and risky debt.

As in DeMarzo and Urošević (2006), firm cash flows depend on unobservable blockholder effort $b_t \geq 0$, which entails a private flow cost $\frac{1}{2}\kappa K_t b_t^2$ to the blockholder for a cost parameter $\kappa > 0$. The blockholder's private effort captures its engagement with the firm, for instance, by appointing key personnel and board members, providing industry connections, or developing strategies and proposals that increase the return on invested capital (such as better management of working capital, costs, overheads, purchases, or product mix). This effort choice cannot be contracted on and endogenously depends on the size of the stake of the blockholder, with a blockholder that cannot commit to future trades.

[5] For simplicity, we adopt a specification where cash flows are always positive so the firm does not generate negative dividends, which would have to be covered via frequent share issuance. Crucially, our findings would also be entirely unchanged if we assumed $dX_t = (\alpha + b_t - m_t)dt + \sigma dZ_t$, where dZ_t is the increment of a Brownian motion and $\sigma > 0$, as in DeMarzo and Urošević (2006). However, under the specification with Brownian shocks dZ_t, cash flows would become negative, and the firm would frequently issue new shares to cover cash flow shortfalls, a scenario we regard as less realistic. Nonetheless, the two stipulations are isomorphic, and cash flows are not a state variable for the blockholder's problem under either specification.

Moral Hazard and Short-Term Contracting. To deal with managerial cash diversion, firm shareholders write a contract with the manager. Since corporate policies and trading are chosen without commitment, we also preclude commitment to long-term contracts, and thus restrict the contracting space to short-term contracts over $[t, t + dt)$.[6] We follow the formulation of the short-term contracting problem in continuous time in He and Krishnamurthy (2011). Notably, we focus on incentive-compatible contracts that implement $m_t = 0$. Under our assumptions, it does not matter whether the contract is set by passive shareholders or the blockholder. Loosely speaking, both parties find it optimal to choose the contract that provides the manager with the minimum cash flow exposure in the set of incentive-compatible contracts, thereby maximizing the residual exposure of shareholders.

To formalize the contracting problem, denote the anticipated blockholder effort by $\hat{b}_t$. In optimum, we have $\hat{b}_t = b_t$, i.e., anticipated and actual effort levels coincide. When the blockholder does not invest, then $\theta_t = 0$ and $b_t = \hat{b}_t = 0$. The short-term contract $\mathcal{C}_t = (\beta_t, c_t)$ stipulates a flow (base) wage c_t, in addition to a payout $\beta_t K_t\big(dX_t - (\alpha + \hat{b}_t)dt\big)$ which depends on the realization of dX_t relative to its expected mean $(\alpha + \hat{b}_t)dt$. With no loss in generality, we normalize the manager's reservation utility to zero, so that its expected payoff from the contract must be positive. We denote the total payment to the manager over $[t, t + dt)$, which is contingent on the realization of dX_t, by $K_t dC_t = K_t\big[c_t dt + \beta_t\big(dX_t - (\alpha + \hat{b}_t)dt\big)\big]$.

Consider a cash flow shock $dN_t = 1$. If the manager diverts the generated cash flow and sets $m_t = 1$, she derives private benefit λK_t. Otherwise, if she does not divert cash flow, the manager gets $\beta_t K_t$. Thus, the manager does not divert cash flow if the following incentive constraint holds:

$$\beta_t \geq \lambda. \tag{3}$$

As will become clear, both the stock price and the blockholder payoff decrease in β_t, so setting $\beta_t = \lambda$ is optimal and the incentive condition (3) is tight. The manager's participation constraint requires $K_t \mathbb{E}_t^m[dC_t]$ to be weakly positive, where $\mathbb{E}_t^m$ denotes the time-t expectation under the manager's information set, i.e., conditional on m_t and an anticipated level of blockholder effort $\hat{b}_t$ (which coincides in optimum with actual effort b_t). In optimum, share-

[6] The blockholder holds an equity stake in the firm and, by assumption, cannot write a contract with passive shareholders. That is, the blockholder's contract with the firm is exogenous and restricted to equity. Moreover, shareholders cannot side-contract with management, i.e., the manager contracts with the firm as a whole, and any payments to/from the manager are borne by shareholders in proportion to their stake.

holders design the optimal contract, which maximizes their own value, so that the manager's participation constraint binds. Setting $\mathbb{E}_t^m[dC_t]$ to zero, the base wage is zero, i.e., $c_t = 0$.

Capital Investment. The firm chooses the investment rate $i_t \geq 0$ against convex investment cost $\frac{1}{2}K_t i_t^2$ to grow its capital stock, which evolves according to

$$dK_t = (\mu i_t - \delta)K_t dt. \tag{4}$$

In (4), $\mu \geq 0$ captures the efficiency of investment and $\delta \geq 0$ is the constant rate of depreciation. In the baseline model, investment is chosen by the blockholder, provided it holds an ownership stake $\theta_t > 0$. Section 3 analyzes a model variant in which dispersed shareholders choose investment and shows that, via its feedback effect on blockholder trading and effort choice, dispersed shareholder control over investment reduces passive shareholder value.

Note that we could equivalently assume that the manager chooses investment i_t. Because K_t is observable and dK_t is deterministic, investment i_t could be contracted upon, i.e., there is no moral hazard with respect to investment.

Debt Financing. Blockholders, such as private equity funds or hedge funds, often initiate capital structure changes after investing in firms. To capture the effects of debt financing on blockholder trading and firm policies, we introduce dynamic short-term debt issuance as in e.g. Abel (2018), Hu, Varas, and Ying (2025), or Bolton, Wang, and Yang (2023). As in those models, the firm issues at any time t short-term debt with endogenous face value L_t that matures at time $t + dt$. The benefits of debt are captured in reduced form by assuming that creditors discount cash flows at a rate $r \leq \rho$.[7] If the firm defaults on its debt, it is liquidated. The liquidation value of assets given by RK, where $R \geq 0$, is sufficiently small to ensure that liquidation is inefficient and that creditors are not repaid in full in default. (A specific parameter condition is $R < p^0$, where p^0 is scaled firm value under passive ownership characterized in Proposition 1.) In default, the absolute priority of claims is enforced so that the firm's equity value and the blockholder's continuation payoff fall to zero.

Timing. We characterize the heuristic timing of the model within a time interval $[t, t + dt)$. First, at the beginning of $[t, t + dt)$ and after previous-period debt has been repaid and

[7]This modeling is standard in dynamic financing and investment models (e.g., DeAngelo, DeAngelo, and Whited (2011) and Geelen, Hajda, Morellec, and Winegar (2024)), and is equivalent to assuming tax benefits of debt $r = (1 - \tau)\rho$, where $\tau \in (0, 1)$ is the corporate tax rate.

previous-period trading, the blockholder's stake equals $\theta = \theta_t$ (i.e., θ is a predictable process). Given θ, shareholders choose the managerial contract $\mathcal{C}_t = (c_t, \beta_t)$, the investment rate $i_t \geq 0$, and the amount of (new) short-term debt L_t, where the proceeds from debt issuance are distributed as dividends. Second, the blockholder chooses its effort level, the shock dN_t realizes and, observing dN_t, the manager chooses diversion m_t. Then, cash flows dX_t are realized and observed, and the manager receives its promised payments. Third, at the end of $[t, t+dt)$, debt matures: Shareholders either repay creditors $(1+r_t dt)L_t$ dollars, where r_t is the endogenous interest rate, or default. If the firm does not default, cash flows net of investment cost, managerial compensation, and debt repayment are distributed as dividends to the shareholders. Fourth, the blockholder can trade and chooses $d\theta_t$, determining next-period stake $\theta_{t+dt} = \theta_t + d\theta_t$. Finally, the capital stock is updated with $K_{t+dt} = K_t\big[1+(\mu i_t - \delta)dt\big]$.[8]

Broadly speaking, our model features two types of blockholder decisions: (*i*) corporate policies and (*ii*) trading. We assume that corporate policies within an instant are chosen taking blockholder ownership as given, while trading occurs at the end of the period. Both the blockholder and passive investors anticipate how trading will influence future corporate decisions. In our continuous time model, periods are infinitesimal and, therefore, the exact timing—while useful in guiding intuition—does not matter. In particular, in the heuristic timing, the blockholder trades after debt is repaid. However, as we demonstrate in Section 2.5, this assumption is without loss in generality in that we could allow trading before and after the repayment of debt without changing the results.

At the beginning of time t, after debt is repaid and before new debt is issued, we denote the blockholder's value function by V_t and equity value (i.e., firm value) by P_t. As we show later in Lemma 1, the firm optimally defaults on debt just after issuance when $V_t < \theta_t L_t$ or $P_t < L_t$, irrespective of whether the blockholder or passive shareholders control the default decision. Clearly, creditors find it suboptimal to extend an amount of debt L_t that triggers immediate default. As in Abel (2018), this observation implies an endogenous borrowing constraint

$$\theta_t L_t \leq V_t \quad \text{and} \quad L_t \leq P_t. \tag{5}$$

[8]Note that one can net out the "special" dividends from debt issuance with the debt repayment. Moreover, the change in the capital stock is of order dt and its effect on other quantities of order dt is therefore negligible. Hence, the exact timing of when the capital stock is updated does not matter for trading, default, or contracting. To avoid tedious notation, we assume it occurs at the very end of the period.

This borrowing constraint states that the blockholder and passive shareholders are protected by limited liability, i.e., they cannot commit to a particular ownership stake, and they can always choose to exit and obtain a weakly positive payoff. Therefore, in equilibrium, their continuation payoffs must remain positive at all times. Given the dynamics of cash flows in equation (1), constraint (5) implies that debt is risk-free, so that the fair interest rate on debt is $r_t = r$ and the default time T equals $+\infty$. Section 4 introduces large jump shocks to the firm's capital stock, which leads to default and renders debt risky.

Preferences: Holding Costs and Benefits. We make two assumptions that affect the blockholder's and passive investors' incentives to trade. First, as in prior models of blockholder investors (DeMarzo and Urošević, 2006; Marinovic and Varas, 2025) and dynamic trading models (Du and Zhu, 2017; Duffie and Zhu, 2017), the blockholder incurs a flow holding cost $\frac{1}{2}\pi\theta_t^2 \geq 0$, that scales with the size of its investment in the firm as captured by firm size K_t. As discussed in Duffie and Zhu (2017), this disutility flow may reflect, in reduced form, the blockholder's financial or capital constraints or higher cost of capital. We emphasize that this cost, while affecting trading dynamics, is not essential for our formal results, notably for the existence and uniqueness of an equilibrium.

Second, our paper extends previous models by explicitly introducing a utility benefit, $\eta(\theta)$, that passive investors derive from holding the stock. This utility benefit leads to inelastic demand (Koijen, Richmond, and Yogo, 2024; Haddad et al., 2025), making it more difficult for the blockholder to acquire the entire firm. More broadly, it captures the growing evidence that trades influence prices, regardless of whether they convey information, as recently shown using event studies or estimation of demand systems (see Hartzmark and Solomon (2025) and references therein). Appendix E provides a micro-foundation for this utility benefit, linking it to heterogeneous demand (Koijen and Yogo, 2019) among certain passive investors—such as index investors—who assign a high value to the stock and are reluctant to sell beyond a certain threshold. In our model, this utility benefit implies the existence of a unique Markov equilibrium with continuous prices and trading. However, as we show in Section 5.1, without such utility benefit, we can solve for an equilibrium with qualitatively similar properties, but featuring both smooth and lumpy trading, as it may become optimal for the blockholder to make a takeover offer to acquire the entire firm.

Blockholder Payoffs. We next specify the payoffs of the blockholder and passive investors, where we assume that the blockholder controls managerial contracts and financing and investment decisions. Assuming that the blockholder chooses the level of debt L_t and the manager's contract $\mathcal{C}_t$ is without loss in generality, as the blockholder and passive investors agree on both the optimal level of debt and the managerial contract, as we show later. In contrast, control rights over investment matter for equilibrium outcomes; see Section 3.

At time $t = 0$, the firm starts with zero debt, i.e., $L_{0^-} = 0$, and there is a potentially discrete debt issuance $dL_0 > 0$. When the blockholder holds a stake in the firm, it chooses its effort b_t, trading $d\theta_t$, debt issuance dL_t, investment i_t, and contract $\mathcal{C}_t$ to maximize:

$$V_0 = \max_{(b_t, d\theta_t, dL_t, i_t, \mathcal{C}_t)_{t\geq 0}} \mathbb{E}_0\bigg[\int_0^T e^{-\rho t}\bigg\{\theta_t\big[K_t dX_t - \frac{K_t i_t^2}{2}dt - K_t dC_t + L_t(\rho - r)dt\big] - K_t\left(\frac{\pi\theta_t^2}{2} + \frac{\kappa b_t^2}{2}\right)du - (P_t + dP_t)d\theta_t\bigg\}\bigg], \tag{6}$$

subject to (4) and (5). Because the blockholder owns a fraction θ_t of the firm, it collects a fraction θ_t of dividends, while incurring the full cost of effort $\frac{\kappa b_t^2}{2}$ as well as the flow holding cost $\frac{\pi\theta_t^2}{2}$. The term $-d\theta_t(P_t + dP_t)$ captures the payoff that the blockholder collects from trading over a short time period $(t, t+dt)$. The blockholder has price impact and trades over $[t, t + dt)$ at "end-of-period" price $P_{t+dt} = P_t + dP_t$. Although we do not explicitly account for it in our notation, the blockholder's expectation in (6) is taken under its information set, which involves its own effort choice b_t. All controls are chosen without commitment.

To understand how debt issuance affects the blockholder's initial payoff and specifically its flow payoff, note that issuing an additional dollar of debt at time t and paying it out as a dividend, while repaying debt and interest at time $t + dt$, changes the blockholder's payoff over $[t, t+dt]$ by $\theta_t 1 - \theta(1 - \rho dt)(1 + r dt) = \theta_t(\rho - r)dt$, since terms of order $(dt)^2$ are negligible in continuous time. As such, the level of debt changes the blockholder's flow payoff by $\theta_t(\rho - r)L_t dt$. Lastly, in the baseline model without large negative shocks, (5) makes it optimal for both the blockholder and passive investors to avoid default and $T = \infty$.

Passive Investors' Payoff and Stock Price. Due to the blockholder's private effort and holding costs and passive shareholders' utility benefit, the value of the firm under blockholder ownership *from passive investor perspective* differs from V_0 (scaled by θ) and equals

$$P_0 = \mathbb{E}_0 \left[\int_0^T e^{-\rho t} \left(K_t \left(dX_t - \frac{i_t^2}{2} dt \right) - K_t dC_t + (\rho - r) L_t dt + K_t \eta(\theta_t) dt \right) \right], \tag{7}$$

where passive investors anticipate the level of blockholder effort and managerial cash diversion when forming expectations; in optimum, the information sets and expectations of the manager, the blockholder, and passive investors coincide. The stock price is the expected discounted value of all future dividends, including the utility benefit $K_t \eta(\theta_t)$. Appendix E provides a micro-foundation for the utility benefit, where we derive the functional form (our findings go through under different functional forms):

$$\eta(\theta) = \pi^I [\theta - \widetilde{\theta}]^+, \tag{8}$$

for parameters $\pi^I > 0$ and $\widetilde{\theta} \in [0, 1]$. Here, $[\cdot]^+ = \max\{0, \cdot\}$.

In summary, the blockholder and passive investors differ in three dimensions. First, the blockholder exerts private effort, while passive investors do not. Second, the blockholder acts as a large strategic trader, while passive shareholders take prices as given. Third, the blockholder incurs a flow holding cost, while passive investors get a flow utility benefit.

2 Model Solution and Markov Equilibrium

2.1 Parameter Conditions

In what follows, we impose several parameter conditions. First, we assume that

$$(r + \delta)^2 > \frac{\mu^2(1 + 2\kappa\alpha)}{\kappa}, \tag{9}$$

to ensure that valuations are finite and that $\mu i < r + \delta$ always holds in equilibrium, both under passive and blockholder ownership. Second, we assume that holding costs are not prohibitively large, in that

$$\pi < \frac{1 - \lambda^2}{\kappa}. \tag{10}$$

This condition ensures that some blockholder ownership is optimal and occurs in equilibrium. If this condition is not met, the equilibrium is trivial with the blockholder holding zero stake in the firm. Third and last, we assume that

$$\eta(1) = \pi^I(1-\widetilde{\theta}) > \frac{(\rho - r)\,(1-\lambda^2-\kappa\pi)}{2\sqrt{\kappa\,\left(\kappa\,(r+\delta)^2 - \mu^2(1-\lambda^2-\kappa\pi) - 2\kappa\,\mu^2\alpha\right)}} + \frac{\lambda(1-\lambda)}{\kappa} - \pi. \tag{11}$$

This condition ensures the existence of a unique continuous Markov equilibrium. In this equilibrium, the blockholder trades smoothly according to $d\theta = \dot{\theta}dt$. We relax this assumption in Section 5.1 and show that our findings remain qualitatively unchanged, yet the blockholder may conduct large trades (e.g., in the context of a takeover).

2.2 Passive Ownership Benchmark

We start by characterizing optimal effort, investment, and financing in the passive ownership benchmark. When $\theta = 0$, passive investors dynamically choose the managerial contract, debt issuance, and investment to maximize firm value (with $\eta(0) = 0$):

$$P_0^0 = \max_{(dL_t, i_t, C_t)_{t\geq 0}} \mathbb{E}_t\left[\int_0^\infty e^{-\rho t}\left(K_t dX_t - \frac{K_t i_t^2}{2}dt - K_t dC_t + (\rho - r)L_t dt\right)\right], \tag{12}$$

subject to $L_t \leq P_t^0$ where P_t^0 is the stock price with passive owners. This yields:

Proposition 1 (Passive ownership benchmark)**.** *In the passive ownership benchmark, the managerial contract satisfies $\beta_t = \lambda$ while blockholder effort is zero. Model quantities scale linearly in $K_t = K$ with debt issuance satisfying $L_t^0 = K_t p^0$ for $\rho > r$ and $L_t^0 \in [0, K_t p^0]$ for $\rho = r$, and scaled firm value satisfying $P_t^0 = K_t p^0$ with $p^0 = \frac{r+\delta-\sqrt{(r+\delta)^2 - 2\,\alpha\,\mu^2}}{\mu^2}$ and $i^0 = \mu p^0$.*

In the passive ownership benchmark, model quantities, including debt, scale with K_t or are constant, as is the case for effort or the investment rate. The firm issues debt up to the borrowing constraint (when $\rho > r$), with the choice of debt being indeterminate when $\rho = r$ and debt has no value effects (Modigliani and Miller (1958)). Since debtholders discount at a weakly lower rate r, shareholders sell the firm to debtholders (the firm is fully debt-financed) and firm value equals free cash flows discounted at r net of the cash flow growth rate.

2.3 Continuous, Scaled Markov Equilibrium

Our model features a dynamic trading game in which the blockholder acts as a large strategic player (or trader) that internalizes the impact of its trading on prices. The blockholder faces a continuum of dispersed, passive shareholders who take prices as given. As in DeMarzo and Urošević (2006), we solve for a Markov Perfect Equilibrium with the blockholder's ownership stake θ_t and capital K_t as state variables and trading with price impact (while this model assumes that prices only move due to new information, in our model they may also move due to inelastic supply). We focus on Markov equilibria with the following properties:

1. **Markovian Payoffs with Scaling Property and Continuity.** The value function of the blockholder satisfies (6) and the firm's stock price satisfies (7) given equilibrium policies. Payoffs scale with the capital stock K_t, in that $V_t = K_t v(\theta_t)$ and $P_t = K_t p(\theta_t)$ for increasing, continuous functions $v(\theta)$ and $p(\theta)$.

2. **Markovian Equilibrium Policies.** The blockholder takes the *current* state $(K_t, \theta_t) = (K, \theta)$ as given when optimizing its continuation payoff according to (6), i.e., equilibrium policies are Markov processes. Equilibrium efforts $(a_t, m_t)_{t \geq 0}$, investment $(i_t)_{t \geq 0}$, debt $(L_t)_{t \geq 0}$ are Markov processes chosen according to (6). In particular, the trading strategy $(d\theta_t)_{t \geq 0}$, maximizing (6), is a Markov process that determines, given the current state $\theta_t = \theta \in [0, 1]$, the value of $\lim_{s \downarrow t} \theta_s = \theta_t + d\theta_t$ where $d\theta_t \in [-\theta_t, 1 - \theta_t]$. The trading strategy solves for any $t \geq 0$:

$$d\theta_t \in \arg \max_{\Delta \in [-\theta_t, 1-\theta_t]} K_t \left\{ v(\theta_t + \Delta) - \Delta p(\theta_t + \Delta) \right\}. \tag{13}$$

   given the price $P(K, \theta) = Kp(\theta)$ and the value function $V(K, \theta) = Kv(\theta)$.

We refer to Markov perfect equilibria satisfying these properties as *continuous, scaled Markov equilibria*. We will show that a unique *continuous, scaled Markov equilibrium* exists. In this Markov equilibrium, the blockholder's trading is smooth, in that $d\theta = \dot{\theta} dt$ for an endogenous trading rate $\dot{\theta}$. Moreover, in this Markov equilibrium, optimal efforts $(a_t, m_t)_{t \geq 0}$, investment $(i_t)_{t \geq 0}$, and the trading strategy $(d\theta_t)_{t \geq 0}$ are functions of θ_t only and do not depend on K_t. In addition, the debt policy L_t scales with K_t—just like the stock price and value function—in that $L_t = K_t \ell(\theta_t)$ where $\ell(\theta_t)$ is the firm's scaled debt level. In what follows, we refer to $p(\theta)$, $v(\theta)$, and $\ell(\theta)$ simply as the stock price, value function, and debt, respectively, omitting for convenience the adjective "scaled".

2.4 Effort Choice and Contracting

A first channel through which the blockholder affects firm value is by providing effort. When choosing effort b, the blockholder takes the contract of the manager (β, c) as given. Indeed, since b is not observable or contractible, the contract with the manager cannot condition on the choice of b. The blockholder's scaled expected flow payoff related to effort and contracting with management can be written as

$$\max_{\beta \geq \lambda, b} \left\{ \theta \mathbb{E}^b \left[dX - dC \right] - \frac{\kappa b^2}{2} dt \right\} = \theta \alpha dt + \max_{\beta \geq \lambda, b} \left\{ \left(\theta b - \theta (b - \hat{b}) \beta \right) - \frac{\kappa b^2}{2} \right\} dt, \tag{14}$$

where the expectation $\mathbb{E}^b$ is under the blockholder's information set, which includes the choice of b. Optimizing over b yields $b = \frac{\theta(1-\beta)}{\kappa}$. Inserting optimal b into (14) and imposing $\hat{b} = b$ shows that the blockholder's flow payoff decreases in β. As such, it is optimal to set $\beta = \lambda$. Likewise, setting higher managerial incentives over $[t, t + dt)$ reduces b and therefore harms passive shareholders. As neither the blockholder nor passive shareholders benefit from increasing managerial incentives above λ, it holds that $\beta = \lambda$ in an optimal contract. With $\beta = \lambda$, the incentive condition of the blockholder becomes

$$b = \frac{\theta(1 - \lambda)}{\kappa}. \tag{15}$$

Blockholder effort increases with its equity stake and decreases with the severity of managerial moral hazard, i.e., the strength of managerial incentives, $\beta = \lambda$. When $\theta < 1$ and/or $\lambda > 0$, the blockholder incurs the full cost of effort but captures only part of its benefits. Indeed, because the blockholder only owns part of the firm's equity, the benefits of higher effort are shared with passive investors. In addition, since blockholder effort is unobservable and cannot be contracted with the manager, a fraction $\beta = \lambda$ of the benefits of effort are shared with management through the incentive contract. As a result, effort is inefficiently low and depends on θ and therefore on the trading strategy of the blockholder.

2.5 Trading and Default

Let us next turn to trading in state (K, θ), which, in principle, can happen just before debt is repaid (i.e., with scaled debt ℓ outstanding) or after debt is repaid (i.e., with zero debt outstanding). We consider the firm in a state $(K_t, \theta_t) = (K, \theta)$ that is attained in equilibrium after time 0, i.e., we abstract from states that are never reached.[9]

Before proceeding, we define the blockholder's (scaled) valuation of the firm:

$$y(\theta) := \frac{v(\theta)}{\theta}, \tag{16}$$

which will play a key role in the following analysis. As we show, the blockholder's valuation of the firm is generally below that of passive shareholders. The blockholder's valuation of the firm's equity is then $y(\theta) - \ell$, passive shareholders' valuation of equity reads $p(\theta) - \ell$, and the borrowing constraint (5) can be rewritten as $\ell \leq \min\{y(\theta), p(\theta)\}$.

First, we analyze the endogenous default decision and show that there is no default if and only if (5) holds, irrespective of whether the blockholder or passive investors control the default decision. Recall that (5) can be written as $\ell \leq \min\{y(\theta), p(\theta)\}$. Clearly, if (5) holds, both passive shareholders and the blockholder are better off not defaulting. To prove the "only if" part, we conjecture that $p(0) < y(\theta) \leq p(\theta)$, which, as we verify, holds in the Markov equilibrium for all levels of θ that are attained. As shown in the proof of Lemma 1, if $\ell > y(\theta)$ and the blockholder derived a negative continuation payoff from holding a stake θ, the blockholder would optimally sell its entire stake (at any weakly positive price) before repaying debt. Indeed, due to limited liability and the associated option of default, the stock price is always weakly positive. Selling the entire stake yields a weakly positive payoff and, therefore, is at least as good as forcing default if given the opportunity. Following this trade, the price would drop to $p(0) < \ell$ and equity value would become negative, making it optimal for passive shareholders to default. This leads to the following Lemma:

Lemma 1. *Consider any state (K, θ) with $\theta > 0$ reached after time $t > 0$ with positive probability and suppose $p(0) < y(\theta) \leq p(\theta)$, which is satisfied in equilibrium. Then, the firm*

[9] As discussed in greater detail in the proof of Lemma 1, attainability of a state (K, θ) in equilibrium implies that there exists no large trade toward another state $(K, \hat{\theta})$ that leads to *strictly* higher equilibrium payoffs. If such a trade toward $\hat{\theta}$ would yield strictly higher payoff, the blockholder would never remain in state (K, θ) and just immediately trade toward $(K, \hat{\theta})$, rendering (K, θ) unattainable.

does not default if and only if constraint (5) *holds, irrespective of whether the blockholder or passive shareholders control the default decision.*

Lemma 1 implies that, regardless of the allocation of control rights, the firm defaults immediately after debt issuance whenever $\ell > \min\{y(\theta), p(\theta)\}$. Although we do not explicitly model their behavior, creditors are rationally unwilling to extend an amount of debt that triggers immediate default. Thus, constraint (5) must hold in equilibrium and pins down the maximum amount of debt the firm can issue.

Second, with (5) being met before and after trading, there is no default. Just before debt is repaid and given a debt level ℓ, the blockholder's gain from trading toward state $\hat{\theta}$ reads

$$G^{-}(\theta,\hat{\theta}) := (v(\hat{\theta}) - \hat{\theta}\ell) - (v(\theta) - \theta\ell) - (\hat{\theta} - \theta)(p(\hat{\theta}) - \ell) = v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta}).$$

Thus, gains from trade just before debt is repaid are equal to gains from trade just after debt is repaid, $G(\theta,\hat{\theta}) := v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta})$. Clearly, since gains from trade do not depend on the timing of the trade (conditional on no default), the next period stake satisfies $\theta_{t+dt} = \arg\max_{\hat{\theta}} G^{-}(\theta,\hat{\theta}) = \arg\max_{\hat{\theta}} G(\theta,\hat{\theta})$. Consequently, conditional on no default, the blockholder realizes the same payoff and chooses the same next-period stake θ_{t+dt}, irrespective of whether it trades just before or after debt is repaid (or at both times). Taking the limit $\hat{\theta} \to \theta$, this argument also applies to infinitesimal trades.

Thus, the blockholder is indifferent between the timing of trade over $[t, t+dt]$, and optimally trades after debt repayment. Likewise, passive investors are also indifferent regarding the blockholder's timing of trade, i.e., the pricing of the stock is not affected by the exact timing of trade.[10] For convenience and without loss of generality, we consider below that the blockholder *only* trades *after* debt is repaid.

[10] This finding holds more generally. However, it is immediate to see that, because the blockholder will trade smoothly in equilibrium, the timing of trade does not affect shareholders' payoff and the stock price.

2.6 Blockholder Value Function

As shown in the Appendix, the blockholder optimally trades smoothly in equilibrium, i.e., $d\theta = \dot{\theta}dt$ and its value function satisfies the Hamilton-Jacobi-Bellman (HJB) equation:

$$(\rho+\delta)v(\theta) = \max_{\ell,\dot{\theta},i}\left[\theta\left(\alpha+b-\frac{i^2}{2}+(\rho-r)\ell\right)-\frac{\pi\theta^2}{2}-\frac{\kappa b^2}{2}+\mu i v(\theta)+\dot{\theta}\left(v'(\theta)-p(\theta)\right)\right] \tag{17}$$

which is solved subject to the borrowing constraint $\ell \le \min\{p(\theta), y(\theta)\}$ and the incentive condition (15). In (17), the term $\dot{\theta}\left(v'(\theta)-p(\theta)\right)$ captures the gains associated with (smooth) trading. For an interior solution $\dot{\theta} \in (-\infty, \infty)$, it must be that

$$p(\theta) = v'(\theta). \tag{18}$$

Indeed, the blockholder is willing to pay $v'(\theta)$ dollars for an additional unit of stock. The cost of purchasing this additional unit equals the market price (i.e., passive investors' valuation) of the stock $p(\theta)$. In equilibrium, the marginal benefit of buying equals the marginal cost, and the blockholder is indifferent between trading smoothly and not trading. Therefore, the value function $v(\theta)$ can be determined "as if" the blockholder could not trade at all and hence coincides with the payoff $\hat{v}(\theta)$ that would prevail absent any trading opportunities. That is, $\hat{v}(\theta)$ is the solution to (17) upon setting $\dot{\theta} = 0$, which can be solved for in closed form; see equation (A.3) in the Appendix. The following Lemma characterizes the blockholder's value function and the stock price under smooth trading:

Lemma 2. *The following holds when the blockholder trades smoothly in equilibrium:*

1. *The value function of the blockholder satisfies $V(K,\theta) = Kv(\theta)$, with $v(\theta)$ solving (17) and satisfying $v(\theta) = \hat{v}(\theta)$ where $\hat{v}(\theta)$ is available in closed form in (A.3). The stock price satisfies $P(\theta, K) = Kp(\theta)$ for a function $p(\theta)$ satisfying $p(\theta) = v'(\theta)$ and solving (22) below; $p(\theta)$ is available in closed form in (A.5) with $p(0) = p^0$.*

2. *The value function of the blockholder $v(\theta)$ is increasing and convex in its stake θ, in that $v'(\theta), v''(\theta) > 0$ with $v(0) = 0$. As such, the stock price $p(\theta) = v'(\theta) > y(\theta)$ increases with the equity stake of the blockholder.*

As established in Lemma 2, the value function of the blockholder is increasing and convex in θ. Together with (18) and $v(0) = 0$, this implies $y(\theta) < p(\theta)$. That is, the blockholder's marginal valuation $v'(\theta)$ of an additional unit of stock coincides with the stock price $p(\theta)$. However, its valuation of the firm $y(\theta)$ lies below the marginal valuation and the stock price. As a result, (5) reduces to $\ell \leq y(\theta)$. Since the blockholder's scaled payoff increases in ℓ, it is optimal to issue as much debt as its borrowing constraint allows. Therefore, the borrowing constraint binds in that

$$\ell = \ell(\theta) = y(\theta). \tag{19}$$

We can additionally solve for the firm's choice of investment as

$$i = \mu y(\theta). \tag{20}$$

Notice that a marginal increase in K increases the firm's stock price by $p(\theta)$ and the blockholder's value of the firm by $y(\theta) < p(\theta)$. As a result, the blockholder does not internalize the full benefits of investment to the firm and underinvests.

2.7 Equilibrium Trading

To characterize the blockholder's equilibrium trading behavior, we first obtain its valuation of an additional unit of stock. For this sake, we differentiate both sides of (17) and use $p(\theta) = v'(\theta) = \frac{dv(\theta)}{d\theta}$ to obtain:

$$\underbrace{(\rho+\delta-\mu i)p(\theta)}_{(\rho+\delta-\mu i)\frac{dv(\theta)}{d\theta}} = \underbrace{\alpha + b - \frac{i^2}{2} + (\rho - r)\ell(\theta) - \pi\theta +}_{(\rho+\delta-\mu i)\frac{\partial v(\theta)}{\partial\theta}} \underbrace{\frac{\theta\lambda(1-\lambda)}{\kappa}}_{(\rho+\delta-\mu i)\left(\frac{\partial v(\theta)}{\partial b}\frac{\partial b}{\partial \theta}\right)} + \underbrace{\theta(\rho - r)\ell'(\theta)}_{(\rho+\delta-\mu i)\left(\frac{\partial v(\theta)}{\partial \ell}\frac{\partial \ell}{\partial \theta}\right)}. \tag{21}$$

In addition to (21) (derived from the valuation equation of blockholders), $p(\theta)$ satisfies the pricing equation of passive investors (resulting from (7)) who are dispersed and price-takers:[11]

$$(\rho + \delta - \mu i)p(\theta) = \alpha + b - \frac{i^2}{2} + (\rho - r)\ell(\theta) + p'(\theta)\dot{\theta} + \eta(\theta). \tag{22}$$

[11]To derive (22), note that passive investors are risk-neutral and have a discount rate of ρ. Thus, expected dividends and price appreciation must yield their required rate of return ρ in equilibrium, in that $\rho P(K,\theta)dt = \mathbb{E}[K(dX - dC - 0.5i^2dt) + dP(K,\theta)]$. Using $P(K,\theta) = Kp(\theta)$, we get $\mathbb{E}[dP(K,\theta)] = K\big(p'(\theta)\dot{\theta} + (\mu i - \delta)p(\theta)\big)dt$. Using this relation, we can rewrite the pricing equation to (22).

Combining (21) and (22) allows us to derive the blockholder's equilibrium trading rate as

$$\dot{\theta} = \frac{1}{p'(\theta)} \left[\theta(\rho - r)\ell'(\theta) + \frac{\theta\lambda(1-\lambda)}{\kappa} - \pi\theta - \eta(\theta) \right]. \tag{23}$$

The trading rate in (23) balances three forces, each reflecting gains from trade. First, a larger stake increases the firm's debt capacity, generating gains from trade $\theta(\rho - r)\ell'(\theta)$ through the *debt capacity channel*. Second, an increase in the blockholder's stake mitigates the moral hazard problem and increases blockholder effort, resulting in gains from trade of $\frac{\theta\lambda(1-\lambda)}{\kappa}$ through the *effort channel*. Third, the blockholder gains from selling shares because of the marginal flow holding cost $\pi\theta$ and passive investors' utility benefit $\eta(\theta)$. The third effect, all else equal, causes the blockholder to sell more (or buy less). The first and second effects—which we explain in greater detail below—are positive and increase the trading rate.

Debt Capacity Channel. To understand the debt capacity channel, note that a marginal increase in the blockholder's stake θ relaxes the borrowing constraint $\ell \leq y(\theta)$, which increases firm value and the blockholder's payoff on its existing infra-marginal equity holdings. In particular, since the debt choice ℓ does not satisfy a first-order condition, we have $\frac{\partial v(\theta)}{\partial \ell} > 0$ in (17). Therefore, when the blockholder marginally increases its stake, it realizes an additional payoff due to the increased debt capacity of $(\rho + \delta - \mu i)\left(\frac{\partial v(\theta)}{\partial \ell}\frac{\partial \ell}{\partial \theta}\right) = \theta(\rho - r)\ell'(\theta) > 0$, which boosts its valuation of an additional unit of equity $v'(\theta)$. As a large, strategic trader, the blockholder internalizes this effect which increases its valuation of the firm's stock in (21) by $\theta(\rho - r)\ell'(\theta) = (\rho - r)\big(p(\theta) - y(\theta)\big) > 0$. In contrast, as shown in (22), dispersed passive shareholders do not internalize this value-enhancing effect when selling to the blockholder, and therefore, their valuation of the firm does not reflect it. This difference generates gains from trade, increasing the blockholder's incentive to acquire a larger stake in the firm.

Effort Channel. Recall that blockholder effort is unobservable and not contractible, while management is subject to moral hazard, creating a double moral hazard problem. As a result, any increase in cash flows due to higher effort partially benefits management through the incentive contract. Notably, when increasing its effort, the blockholder only captures a fraction $(1 - \lambda)\theta$ of the resulting increase in cash flows. Consequently, the blockholder chooses effort b to maximize its payoff given by $(1 - \lambda)\theta b - \frac{1}{2}\kappa b^2$, which leads to the incentive constraint (15). In contrast, if blockholder effort could be contracted upon, the blockholder would maximize its total payoff from effort for a given stake θ, that is, $\theta b - \frac{1}{2}\kappa b^2$, leading to a higher effort level of $\frac{\theta}{\kappa}$. Thus, due to moral hazard, blockholder effort is inefficiently low, even accounting for partial ownership (i.e., $\theta < 1$), because it does not maximize $\theta b - \frac{1}{2}\kappa b^2$. In particular, in the HJB equation (17), we have $\frac{\partial v(\theta)}{\partial b} > 0$ because b does not maximize the right-hand side of (17), but rather is pinned down by the incentive condition (15).

When the blockholder marginally increases its equity stake, its incentives to exert effort increase by $\frac{\partial b}{\partial \theta} = \frac{1-\lambda}{\kappa}$. This effect raises the blockholder's payoff from effort on its existing infra-marginal equity holdings by $\frac{\partial}{\partial \theta}\left(\theta b - \frac{1}{2}\kappa b^2\right) = b + \theta\lambda \frac{\partial b}{\partial \theta}$. In particular, $\theta\lambda\frac{\partial b}{\partial \theta} = (\rho + \delta - \mu i)\left(\frac{\partial v(\theta)}{\partial b}\frac{\partial b}{\partial \theta}\right) > 0$ reflects the additional payoff resulting from increased efficiency in effort provision and the alleviation of the double moral hazard problem due to a marginal increase in ownership. The blockholder, acting as a large strategic trader, internalizes this increased efficiency in effort provision when acquiring additional shares, which raises its valuation of the firm's equity in (21) by $\theta\lambda\frac{\partial b}{\partial \theta} = \frac{\theta\lambda(1-\lambda)}{\kappa}$. As shown in (22), dispersed passive investors do not internalize this effect when selling to the blockholder.

Investment. In contrast with debt and blockholder effort, investment is set efficiently for the blockholder and, as such, is not directly associated with gains from trade. Indeed, investment satisfies the first-order condition $\frac{\partial v(\theta)}{\partial i} = 0$, so that the gains from trade related to investment, $(\rho + \delta - \mu i)\left(\frac{\partial v(\theta)}{\partial i}\frac{\partial i}{\partial \theta}\right)$, are zero. However, as will become clear, investment opportunities and μ indirectly affect the gains from trade by changing the firm's debt capacity.

Overall Gains from Trade. Depending on the relative strength of the forces at play, the overall gains from trade, and thus the trading rate of the blockholder, can be positive or negative. Crucially, the extent to which the blockholder internalizes the increase in firm value from enhanced debt capacity and effort provision depends on the size of its equity stake. Notably, the blockholder internalizes these effects more when owning a larger fraction of the firm, which increases its propensity to buy (sell) when θ is high (low).

Lemma 3 (Dynamics of the blockholder stake)**.** *The trading rate $\dot{\theta} = \dot{\theta}(\theta)$ in a continuous, scaled Markov equilibrium is given in (23), and satisfies $\dot{\theta}(0) = 0$ and $\dot{\theta}(1) < 0$. Define the* *thresholds:*

$$\overline{\theta} = \inf \left\{ \theta \in [0,1] : \dot{\theta}(\theta') < 0 \quad \textit{for all} \quad \theta' \in (\theta, 1) \right\} \tag{24}$$

$$\underline{\theta} = \inf \left\{ \theta \in [0,\overline{\theta}) : \dot{\theta}(\theta') > 0 \quad \textit{for all} \quad \theta' \in (\theta, \overline{\theta}) \right\}. \tag{25}$$

We have $\dot{\theta}(\overline{\theta}) = \dot{\theta}(\underline{\theta}) = 0$, i.e., $\underline{\theta} \leq \overline{\theta}$ are stationary points in a continuous, scaled Markov equilibrium. Define (whenever $\mu > 0$)

$$\underline{\Theta} := \frac{\kappa}{\mu^2 (1-\lambda^2-\kappa\pi)} \left[(r+\delta)^2 - 2\mu^2\alpha - \frac{(\rho - r)^2 (1-\lambda^2-\kappa\pi)^2}{4 (\kappa\pi - \lambda(1-\lambda))^2} \right]. \tag{26}$$

The following cases can prevail:

1. *When $\underline{\Theta} < 0$, we have that $\underline{\theta} = 0$ and $\overline{\theta} \in (\widetilde{\theta}, 1)$. Then, $\dot{\theta} > 0$ for all $\theta \in (0, \overline{\theta})$ and $\dot{\theta} < 0$ for all $\theta \in (\overline{\theta}, 1)$.*

2. *When $\underline{\Theta} \in (0, \widetilde{\theta})$, we have that $\underline{\theta} = \underline{\Theta}$ and $\overline{\theta} \in (\widetilde{\theta}, 1)$. Then, $\dot{\theta} > 0$ for all $\theta \in (\underline{\theta}, \overline{\theta})$ and $\dot{\theta} < 0$ for all $\theta \in (0, \underline{\theta})$ and $\theta \in (\overline{\theta}, 1)$.*

3. *When $\underline{\Theta} > \widetilde{\theta}$, we have that $\dot{\theta} < 0$ for all $\theta \in [0,1]$, and $\underline{\theta} = \overline{\theta} = 0$.*

Lemma 3 shows that the equilibrium features at least one and at most three stationary points on $[0, 1]$, one of them being zero. Three cases can prevail. First, we can have $\underline{\theta} = \overline{\theta} = 0$ so that zero is the only stationary point. Then, from any $\theta > 0$, the blockholder divests its stake and eventually exits in that $\dot{\theta} < 0$ and θ converges toward zero. Second, we can have $\underline{\theta} = 0 < \overline{\theta}$ in which case zero and $\overline{\theta}$ are stationary points. Then, starting from a non-zero stake θ, the equity stake of the blockholder θ will always smoothly converge to the "target" ownership level $\overline{\theta}$. Third, when $0 < \underline{\theta} < \overline{\theta} \leq 1$, we have that for $\theta \in (0, \underline{\theta})$ and $\theta \in (\overline{\theta}, 1)$, the trading rate is negative, while for $\theta \in (\underline{\theta}, \overline{\theta})$, the trading rate is positive. Thus, for $\theta > \underline{\theta}$, we have convergence of θ toward $\overline{\theta}$; for $\theta < \underline{\theta}$, there is convergence toward zero and exit.

2.8 Equilibrium Summary and Endogenous Entry

We summarize the unique continuous, scaled Markov equilibrium in Proposition 2.

Proposition 2 (Existence and uniqueness of equilibrium)**.** *There exists a unique continuous, scaled Markov equilibrium with state variables (K, θ). In this Markov equilibrium:*

1. *All payoffs scale with K in that $V(K,\theta) = Kv(\theta)$, $P(K,\theta) = Kp(\theta)$, and $L(K,\theta) = K\ell(\theta)$. The blockholder's scaled value function satisfies (17), i.e., $v(\theta) = \hat{v}(\theta)$ and the stock price satisfies $p(\theta) = \hat{v}'(\theta)$.*

2. *The blockholder trades smoothly at rate $\dot{\theta}$ given in (23).*

3. *There exist thresholds $0 \leq \underline{\theta} \leq \overline{\theta} \leq 1$, characterized in Lemma 3, such that $\dot{\theta} = 0$ for $\theta = 0, \underline{\theta}, \overline{\theta}$ (where $\underline{\theta} = \overline{\theta}$ is possible). The trading rate is negative, i.e., $\dot{\theta} < 0$, for small $\theta \in (0, \underline{\theta})$ and large $\theta \in (\overline{\theta}, 1]$. The trading rate is positive, i.e., $\dot{\theta} > 0$, for $\theta \in (\underline{\theta}, \overline{\theta})$.*

While we solve for the equilibrium given an initial blockholder stake, we can also endogenize entry. Blockholder entry increases firm value. However, if this value creation is fully reflected in the price at which the blockholder initially acquires their equity stake, due to a free-rider problem among passive shareholders, the blockholder cannot capture the gains from activism and has no incentive to invest in the first place. In practice, blockholders can accumulate sizable ownership shares before disclosing them (Collin-Dufresne and Fos, 2016).[12] We thus assume that the blockholder can acquire a fraction $\eta \in [0, 1]$ of the firm at the price P_0^0 that prevails under passive ownership. The remainder $\theta_0 - \eta$ is bought at a price P_0 that reflects the gains from activism. Then, the blockholder pays $R(\theta_0)K_0$ dollars to acquire a stake θ_0, with $R(\theta_0) := \eta p(0) + \max\{0, \theta_0 - \eta\} p(\theta_0)$, and enters if and only if

$$E(\theta_0) := \max_{\theta_0 \in [0,1]} K_0 \big[v(\theta_0) - R(\theta_0) \big] \geq 0. \tag{27}$$

The following proposition characterizes the blockholder's entry decision and initial stake.

Proposition 3. *The blockholder's initial stake satisfies $\theta_0 = \eta$.*

When the blockholder enters, it only acquires the stake η, which it can buy at a discount. To gain some intuition, note that—whether at time $t = 0$ or afterward—the blockholder optimally trades smoothly, and large lumpy trades are strictly sub-optimal. Choosing $\theta_0 > \eta$

[12] In the U.S., owners of more than 5% of the equity of a public firm are required to file a report with the SEC, at which point the price adjusts to reflect this information. Collin-Dufresne and Fos (2015) report that the average blockholder holds 7.51% of the target's shares when making its first public disclosure.

is equivalent to acquiring first a stake η in the firm and immediately buying $\theta_0 - \eta > 0$ units of the firm equity in a large lumpy trade. However, this is sub-optimal.

2.9 Ownership Dynamics, Debt, and Investment

In our model, the blockholder enters the firm, exerts effort, and changes the firm's investment and financing policies to improve firm value. Three scenarios regarding entry and subsequent dynamic trading can occur. First, the blockholder does not enter. This scenario prevails when the cost of exerting effort κ, the cost of holding the stock π, or managerial agency conflicts λ are large. Second, the blockholder enters to take advantage of the lower acquisition price at entry but does not find it optimal to remain invested in the firm for long, in that it gradually divests its stake. Third, the blockholder enters and gradually acquires a larger stake in the firm, potentially acquiring the entire firm eventually.

Corollary 1 (Long-run ownership and comparative statics)**.** *The following holds:*

1. *In the limit $t \to \infty$, we have $\theta_\infty := \lim_{t\to\infty} \theta_t \in \{0, \underline{\theta}, \overline{\theta}\}$. When $\theta_0 > \underline{\theta}$, then $\theta_\infty = \overline{\theta}$. When $\theta_0 = \underline{\theta}$, then $\theta_\infty = \underline{\theta}$. When $\theta_0 < \underline{\theta}$, then $\theta_\infty = 0$.*

2. *The blockholder is more likely to buy (and less likely to sell) when μ is larger, or r and κ are lower. Specifically, $\underline{\Theta}$ decreases in μ, and increases in r and κ. When $\overline{\theta} > 0$, then $\overline{\theta}$ increases in μ, and decreases in r and κ.*

3. *When $\mu = 0$, we have that $\underline{\theta} = 0$ and*

$$\overline{\theta} = \left[\frac{\pi^I \widetilde{\theta}}{\pi^I - \left(\frac{(\rho - r)(1-\lambda^2 - \kappa\pi)}{2\kappa(r+\delta)} - \frac{\lambda(1-\lambda)}{\kappa} - \pi \right)} \right] \mathbb{I}\{\overline{\theta} > \widetilde{\theta}\}. \tag{28}$$

4. *When $\mu = \rho - r = 0$, we have $\theta_\infty > 0$ whenever $\lambda \in (\lambda_-, \lambda_+)$ and $\theta_\infty = 0$ otherwise, where $\lambda_\pm = \frac{1 \pm \sqrt{1-4\kappa\pi}}{2}$. Thus, the blockholder engages with the firm over the long term for intermediate levels of agency conflicts λ but exits when the level of agency conflicts λ is too large or too small.*

Corollary 1 shows that the blockholder is more likely to acquire a larger stake in the firm if it has a lower cost of effort or a lower holding cost. Furthermore, it is more likely to

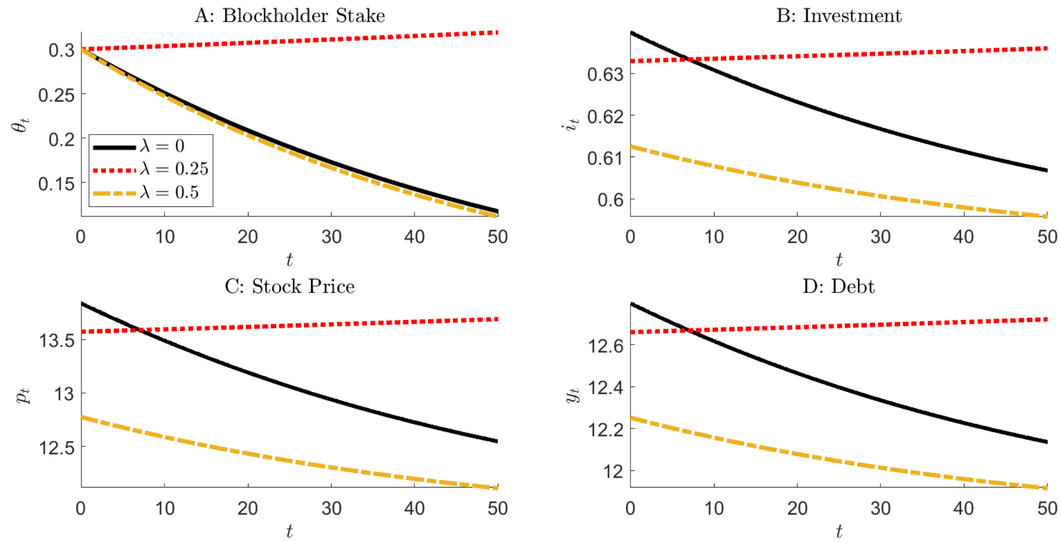

Figure 1: **Managerial Agency Conflicts and Model Dynamics.** We numerically calculate the equilibrium under varying levels of agency conflicts λ. The parameters are $\alpha = \kappa = 1$, $\rho + \delta = 0.2$, $r + \delta = 0.1$, $\lambda = 0.25$, $\mu = 0.05$, and $\pi = 0.5$, as well as $\widetilde{\theta} = 0.99$ and $\pi^I = 1$. We set $\theta_0 = 0.3$.

increase its stake when investment opportunities are good (i.e., μ is high) or the net benefits of debt are large (i.e., r is low). In our baseline model, supply is inelastic and the stake of the blockholder evolves between $\underline{\theta}$ and $\overline{\theta}$, which are stationary points. Interestingly, when $\underline{\theta}$=0, the blockholder may choose to exit the firm after divesting its stake smoothly.

In addition, Corollary 1 shows that the blockholder's trading rate is generally hump-shaped in the level of internal agency conflicts λ. Indeed, as shown by equation (23), when agency conflicts are high or low, the blockholder has no incentives to exert effort as either the benefits of effort are captured by management or the double moral hazard problem is weak. As a result, the blockholder is more likely to exert effort and remain invested in firms with intermediate levels of governance and agency conflicts between management and shareholders, i.e., intermediate λ. The effects are illustrated in Figure 1, which plots the time dynamics of key equilibrium quantities for varying levels of managerial agency conflicts.

Lastly, Section 5.1 shows that the optimal trading strategy still features two thresholds (within which the blockholder trades smoothly) when (11) is not met, but the blockholder acquires the remainder of the firm at once in a lumpy trade at the upper threshold.

Importantly, in our baseline model, parameters are constant, while the state variables

(K, θ) evolve over time. Because all value functions and prices scale with K, the meaningful dynamics in our model stem solely from the blockholder's trading. That is, model dynamics are entirely endogenous, and given the parameters, one can perfectly predict whether the blockholder increases its stake towards a stationary point (potentially acquiring the entire firm) or exits as $t \to \infty$. Section 5.2 extends the model to accommodate stochastic dynamics in the activist's ownership stake.

3 Control Rights over Financing and Investment

In our baseline model, the blockholder determines corporate policies. In this section, we explore the consequences of reallocating control rights from the blockholder to other shareholders. Specifically, we consider governance structures that prevent the blockholder from gaining control or influencing key corporate decisions—such as those featuring anti-takeover or entrenchment provisions (Bebchuk et al., 2009). We analyze how such restrictions affect the blockholder's incentives to acquire a significant ownership stake and to exert effort in monitoring or engaging with management. Our results indicate that while the blockholder and passive shareholders are aligned on optimal managerial compensation and capital structure, they disagree on the firm's investment policy. Restricting the blockholder's control over investment decisions reduces its incentive to acquire a large stake, ultimately lowering firm value. This highlights that governance provisions limiting blockholder influence may diminish overall shareholder value, in line with evidence in Bebchuk et al. (2009).

3.1 Financing and Mangerial Compensation

In the baseline model, the debt level is such that the liability constraint of the blockholder binds. As a result, the amount of debt issued by the firm satisfies $L = K\ell(\theta)$ while book leverage is given by $\ell(\theta) = y(\theta)$, where $\ell(\theta)$ is an increasing function of θ.

Recall that, conditional on no default and ignoring the borrowing constraint (5), both the blockholder's payoff $v(\theta)$ and the stock price $p(\theta)$ increase in ℓ. However, even if passive shareholders had control over debt policy, it would not be possible for them to increase leverage beyond $y(\theta)$. Indeed, $\ell > y(\theta)$ violates (5) and implies a negative private valuation for the blockholder that would trigger its immediate exit. This exit would in turn lead to a downward jump in the stock price (since $p'(\theta) > 0$), leading to negative equity value and making it optimal for passive shareholders to default, as shown in Lemma 1. That is, even if given full control, passive shareholders cannot raise ℓ above $y(\theta)$ due to the blockholder's limited commitment to a particular equity stake and to the threat of immediate exit.

Likewise, the blockholder and passive shareholders are aligned on the optimal managerial contract, as already discussed before. Both parties find it optimal to choose the contract that minimizes agency rents.

3.2 Investment

Similarly, in the baseline model, the blockholder controls investment and chooses the investment rate $i = \mu y(\theta)$. That is, the blockholder's valuation for the firm pins down investment. Given θ and smooth trading, passive shareholder value would be maximized under the investment rate $i = \mu p(\theta) > \mu y(\theta)$.[13] In essence, there is an under-investment problem that reduces passive shareholder value. To address this issue, suppose that passive investors control investment as long as $\theta < 1$. Thus, $i = \mu p(\theta)$, which increases investment and the stock price in a given state, i.e., holding θ fixed. However, the blockholder cannot commit to maintaining a given ownership stake and allocating control rights over investment to passive shareholders reduces its valuation for the firm, causing the blockholder to lower its stake in the firm. As we next show, this effect ultimately reduces the stock price and passive shareholders' value, as well as firm investment in the long-run.

To establish this result, we again solve for a continuous, scaled Markov equilibrium where the equilibrium is defined as in the baseline, except that we impose parameter condition B.1 instead of (11) and the investment rate is chosen by passive investors—put differently, the blockholder is restricted from choosing investment. Appendix B and Proposition 4 demonstrate that a unique continuous, scaled Markov equilibrium in which the blockholder trades smoothly at rate $\dot{\theta}$ exists.[14]

Unlike in the baseline, the value function is no longer available in closed form but instead characterized by an ordinary differential equation since the stock price, and thus investment,

[13] Given θ, passive shareholder value is maximized according to $i = \arg\max_{\hat{i} \geq 0}(\mu \hat{i} p(\theta) - \hat{i}^2/2)$.

[14] Admittedly, this uniqueness result is slightly less general than uniqueness among all continuous, scaled Markov equilibrium (which we established for the baseline). However, although we are unable to prove it, we believe that our equilibrium is also unique among continuous, scaled Markov equilibria.

depend on the derivative of $v(\theta)$. In particular, the value function of the blockholder solves:

$$(\rho+\delta)v(\theta) = \theta\left(\alpha + b + (\rho - r)\ell(\theta) - \frac{(\mu v'(\theta))^2}{2}\right) - \frac{\pi\theta^2}{2} - \frac{\kappa b^2}{2} + \mu^2 v'(\theta)v(\theta), \tag{29}$$

where $p(\theta) = v'(\theta)$, $\ell(\theta) = y(\theta)$, and $b = \frac{\theta(1-\lambda)}{\kappa}$. This ODE is solved subject to the boundary condition

$$\lim_{\theta\to 0}\frac{v(\theta)}{\theta} = \lim_{\theta\to 0}\frac{\hat{v}(\theta)}{\theta}. \tag{30}$$

That is, the solution and the blockholder's value function approach that of the baseline when $\theta \to 0$. We also show that this boundary condition implies $\lim_{\theta\to 0} v'(\theta) = p(0)$ where $p(0)$ is characterized in Proposition 1. The economic interpretation of this boundary condition is that the blockholder's and passive shareholders valuation of the company coincide as $\theta \to 0$, so that control rights over investment do not matter since both parties follow the same objective for investment. Indeed, the baseline also features $\hat{v}'(\theta) = \lim_{\theta\to 0}\frac{\hat{v}(\theta)}{\theta} = p(0)$.

When passive shareholders control investment, the smooth trading rate satisfies

$$\dot{\theta} = \frac{1}{p'(\theta)}\left[\theta(\rho - r)\ell'(\theta) + \frac{\theta\lambda(1-\lambda)}{\kappa} - \pi\theta - \eta(\theta) + \theta\mu i'(\theta)\left(y(\theta) - p(\theta)\right)\right]. \tag{31}$$

Thus, when investment is set by passive shareholders, the trading rate involves the novel term

$$\theta\mu i'(\theta)\left(y(\theta) - p(\theta)\right) = (\rho + \delta - \mu i)\left(\frac{\partial v(\theta)}{\partial i}\frac{\partial i}{\partial \theta}\right) < 0,$$

which is absent in the baseline model where investment solves the first-order condition $\frac{\partial v(\theta)}{\partial i} = 0$. This term is negative, implying that the blockholder is more likely to sell its shares over time when passive shareholders control investment. This reflects the blockholder's lower valuation of the firm, as it internalizes that purchasing additional shares drives up the firm's stock price and investment, further worsening inefficiencies from its perspective. In other words, the blockholder exhibits an increased propensity to acquire more of the firm rather than to exit when it controls investment. We summarize our findings in Proposition 4:

Proposition 4. *When passive investors control investment and $i = \mu p(\theta)$, a scaled, continuous Markov equilibrium with smooth trading exists and is unique. In this equilibrium:*

1. *The blockholder's scaled value function $v(\theta)$ is strictly convex and solves (29) subject*

to (30)*. The stock price satisfies* $p(\theta) = v'(\theta)$ *with* $y(\theta) = \frac{v(\theta)}{\theta} > p(\theta)$*.*

2. *The smooth trading rate* $\dot{\theta}$ *satisfies* (31)*.*

3. *The blockholder's value function is lower than in the baseline where the blockholder controls investment, i.e.,* $v(\theta) < \hat{v}(\theta)$*.*

We can additionally characterize analytically the effects of investor control on the trading rate of the blockholder where there are no benefits of debt ($\rho = r$) and no holding costs or firm-level moral hazard ($\pi = \lambda = 0$). By continuity, the takeaways also apply as long as $\rho - r > 0$, $\pi > 0$, and $\lambda > 0$ are positive but sufficiently close to zero.

Corollary 2. *Suppose that* $\pi = \lambda = 0$ *and* $\rho = r$*. Suppose that, given investor control over investment,* θ_0 *satisfies* $\theta_0 \in (0, \widetilde{\theta})$*. Then, the following holds:*

1. *When passive investors control investment, we have* $\dot{\theta} < 0$ *and the blockholder eventually exits with* $\lim_{t\to\infty} \theta_t = \theta_\infty = 0$ *as well as* $\lim_{t\to\infty} p(\theta_t) = \hat{v}'(0)$*.*

2. *When the blockholder controls investment, we have* $d\theta_t = 0$ *for all* $t \geq 0$*, and* θ_t *and* $p(\theta_t)$ *remain constant at* $\theta_0 > 0$ *and* $p(\theta_0) = \hat{v}'(\theta_0) > \hat{v}'(0)$ *respectively.*

Figure 2 shows an example of equilibrium dynamics under two governance structures: blockholder control (solid black line) and passive investor control (dotted red line), both starting from the same initial stake, θ_0. Shifting control rights from blockholders to passive investors—represented by the transition from the solid black to the dotted red line—lowers the blockholder's valuation of the firm. This, in turn, leads to a gradual reduction in their ownership stake. Panel A shows that when blockholders control investment, they increase their stake over time. Conversely, under passive investor control, blockholders steadily divest. As a result, blockholder control fosters rising levels of effort, firm value, debt, and investment, whereas all of these metrics decline under dispersed investor control. Since blockholders contribute positively to firm value, their reduced ownership under passive investor control diminishes both the firm's stock price and its debt capacity (see Panels C and D). Notably, while passive investor control leads to higher initial investment at a given ownership level, the blockholder's divestment causes investment to decline over time. As shown in Panel B, investment under blockholder control eventually surpasses that under passive investor control due to the blockholder's increasing engagement.

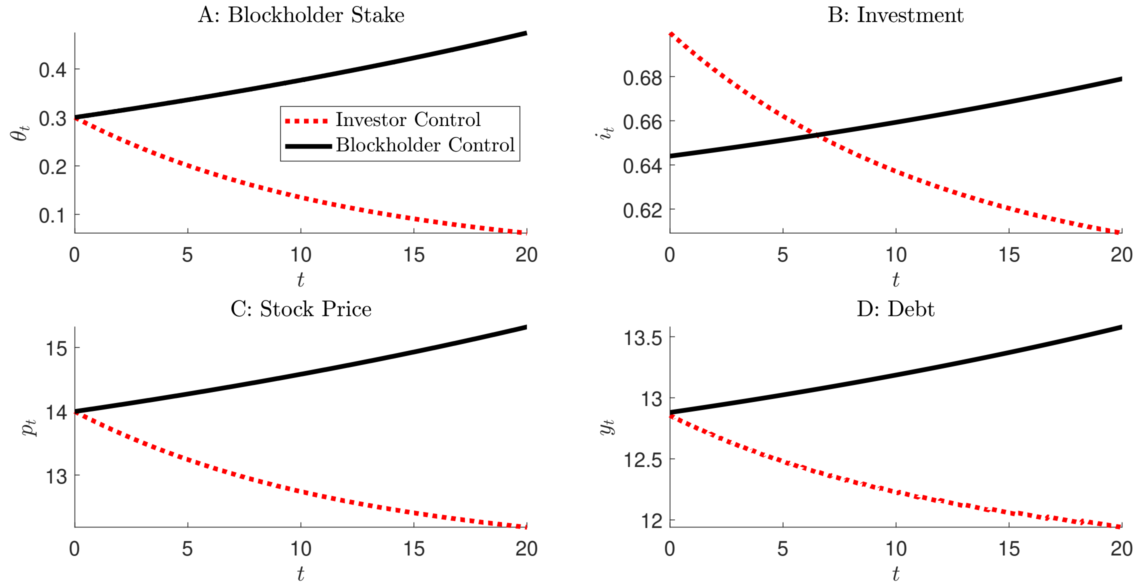

Figure 2: **Model Dynamics and Control.** We numerically calculate the equilibrium both under blockholder control (baseline; solid black line) and investor control (dotted red line). The parameters are $\alpha = \kappa = 1$, $\rho + \delta = 0.2$, $r + \delta = 0.1$, $\lambda = 0.25$, $\mu = 0.05$, and $\pi = 0.4$, as well as $\widetilde{\theta} = 0.99$ and $\pi^I = 1$. We set $\theta_0 = 0.3$.

Our analysis shows that allocating control to passive investors has distinct effects in the short versus long run, as it influences current corporate policies and the blockholder's trading rate, which, in turn, affects future ownership levels and firm decisions. Notably, allocating control to passive shareholders increases investment at a given level of blockholder ownership, but reduces both the blockholder's trading rate and future investment. Thus, as shown in Corollary 2, whether exit can be part of an optimal strategy when there is no holding cost ($\pi = 0$) uniquely depends on the allocation of control rights. Exit is not optimal when the blockholder controls investment and financing. Exit becomes a feature of the optimal strategy when blockholder does not control investment.

4 Large Shocks, Default, and Risky Debt

The specification of cash flows in (1) implies that the firm never defaults on its debt provided that (5) is satisfied, which is always the case in equilibrium. We now extend the model by incorporating large, persistent shocks to the capital shock K as in, e.g., Bolton et al. (2023). Since cash flows scale with K, these shocks can also be interpreted as persistent shocks to cash flows. In particular, we assume that a fraction $1 - S$ of the capital stock is destroyed at a Poisson rate $\Lambda \geq 0$ where, for simplicity, S is uniformly distributed on $[0, 1]$. In the event of default following the shock, the firm is liquidated. In liquidation, creditors recover the liquidation value RSK, and equity value falls to zero. We assume that $R \geq 0$ is sufficiently small that the liquidation value does not fully repay creditors in default under equilibrium debt levels. In particular, $R < p^0$ where p^0 is the scaled stock price under passive ownership from the baseline model. We also assume that the capital shock is not influenced by the manager and is not contractible. As in the baseline, the blockholder chooses debt and investment, and the contract of the manager features a base wage of zero and an exposure $\beta_t = \lambda$ to the Brownian shocks.

We provide a heuristic characterization of the continuous, scaled Markov equilibrium. We start the analysis by clarifying the timing of events and actions within a time interval $[t, t + dt)$, with pre-shock capital $K = K_{t^-} = \lim_{s \uparrow t} K_t$ where use the left-limit notation to emphasize that K_{t^-} is the capital stock *before* realization of the jump shock. First, given θ, the blockholder chooses the managerial contract, the investment rate, and the amount of debt. Second, the blockholder chooses its effort level, the shock dN_t realizes and, observing dN_t, the manager chooses diversion m_t. Then, cash flows dX_t realize, and the manager receives its promised payments. Then, uncertainty regarding the capital shock unfolds: The capital stock remains at K if no shock realizes and drops to SK if a shock realizes. Third, shareholders repay debt or default. In case of no default, cash flows net of managerial compensation, investment cost, and debt repayment are distributed as dividends to sharheolders. As in the baseline model, the blockholder can trade before or after debt is repaid. Finally, the capital stock adjusts for investment and depreciation in the case of no default.

A shock to capital leads to a drop in the blockholder's payoff. When the blockholder's post-shock payoff $K(Sv(\theta) - \theta\ell)$ is negative, the firm defaults. A default occurs either because the blockholder has the authority to force default or because it sells its entire stake (at a positive price), thereby reducing equity value below the level of debt and making default optimal for passive shareholders. Due to $y(\theta) < p(\theta)$, it suffices to look at the blockholder's incentives to default, which are stronger than those of passive shareholders. That is, as in the baseline, the blockholder's valuation of the firm $y = y(\theta)$ pins down default irrespective of whether the blockholder or passive shareholders control default. We can rewrite the default condition as

$$Sy(\theta) < \ell = \ell(\theta) \quad \Longleftrightarrow \quad S < \frac{\ell(\theta)}{y(\theta)} = \frac{\ell}{y}.$$

Conditional on a shock occurring, the probability of default is given by $\Delta := \frac{\ell}{y}$. In default, creditors recover in expectation $\frac{\ell}{2y}RK$ dollars or $\frac{R}{2y}$ dollars per unit of debt. The fair interest rate on risky debt is thus

$$\hat{r} := r + \Lambda\Delta\left(1 - \frac{R}{2y}\right) \tag{32}$$

with a credit spread $\hat{r} - r = \Lambda\Delta\left(1 - \frac{R}{2y}\right)$. The following Proposition, proven in Appendix C, characterizes equilibrium outcomes.

Proposition 5. *In a continuous, scaled Markov equilibrium,the blockholder's valuation of the firm $y(\theta)$ increases in θ and satisfies $y(\theta) < p(\theta)$. Furthermore:*

1. *The trading rate satisfies*

$$\dot{\theta} = \frac{1}{p'(\theta)}\left[\theta(\rho - r)y'(\theta)\Delta + \frac{\theta\lambda(1-\lambda)}{\kappa} - \pi\theta - \eta(\theta)\right].$$

2. *Scaled debt $\ell = \ell(\theta)$ and the probability of default $\Lambda\Delta$ satisfy*

$$\ell = \ell(\theta) = \min\left\{y(\theta), \frac{(\rho - r)y(\theta)^2}{\Lambda\big(y(\theta) - R\big)}\right\} \quad \textit{and} \quad \Lambda\Delta = \min\left\{\Lambda, \frac{(\rho - r)y(\theta)}{y(\theta) - R}\right\}.$$

3. *The default probability $\Lambda\Delta$ and credit spreads $\hat{r}-r$ decrease in θ. Scaled debt ℓ increases in θ whenever $\frac{y(\theta)}{2} \geq R$ and decreases in θ when $\frac{y(\theta)}{2} < R$. When $R \leq \frac{p^0}{2}$, then $\frac{y(\theta)}{2} \geq R$ for all $\theta \geq \underline{\theta}$ and scaled debt increases in θ. When $R \in \left(\frac{p^0}{2}, p^0\right)$, ℓ decreases in a right-neighborhood of $\underline{\theta}$.*

We note that the baseline model and results are obtained when the arrival intensity of capital shocks tends to zero, i.e., $\Lambda \to 0$. Under these circumstances, we have $\ell \to y(\theta)$, $\Delta \to 1$ for any θ, while the default intensity $\Lambda\Delta$ vanishes, rendering debt risk-free.

In this model with large shocks, the choice of debt reflects a standard trade-off, whereby firm leverage balances the benefits of debt with the cost of default. One can interpret RK as the firm's tangible or collateralizable assets, which can be seized by creditors in default. All else equal, higher R increases creditors' recovery in default and therefore lowers the interest rate, making it attractive to issue more debt and raising scaled debt ℓ and default risk.

Blockholder engagement—that is, the value that the blockholder adds to the firm—is an intangible, non-pledgeable asset, whose value increases with θ. As the blockholder's stake in the firm increases, the blockholder adds greater value through engagement, which raises the going-concern value of the firm and thus incentives to avoid default. Therefore, an increase in θ has two effects. First, holding the level of debt fixed, a higher θ reduces default risk and credit spreads. Second, unless R is large, the firm takes advantage of the lower spreads and accordingly raises more debt as θ increases, thereby increasing credit risk and spreads. When R is large, debt is high to begin with, and the firm finds it attractive to mitigate default risk by lowering debt as θ increases and default becomes more costly. Overall, the first effect dominates the second one, in that credit spreads and default risk decrease with θ. Book debt increases in θ, unless R is large; for larger R, book debt is U-shaped in θ.

As in the baseline model, the trading rate $\dot{\theta}$ reflects the gains from trade associated with debt issuance, increasing the blockholder's propensity to acquire a larger stake in the firm. First, when $\Delta = 1$ and debt choice is constrained by the borrowing constraint (5) in that $\ell = y(\theta)$, then $\frac{\partial v(\theta)}{\partial \ell} > 0$ and $\frac{\partial \Delta}{\partial \theta} = 0$. As in the baseline, the gains from trade term associated with debt is proportional to $\frac{\partial v(\theta)}{\partial \ell}\frac{\partial \ell}{\partial \theta} > 0$. Specifically, the blockholder internalizes that by buying additional shares, it increases the firm's debt capacity, which increases firm value.

Second, when $\Delta < 1$, debt choice is not constrained by the borrowing constraint $\ell \leq y(\theta)$ but solves the first-order condition $\frac{\partial v(\theta)}{\partial \ell} = 0$. In this case, the blockholder, acting as a large strategic trader, internalizes that purchasing additional shares reduces credit risk and the cost of default. This channel generates gains from trade, as passive investors do not account for this when valuing the firm's stock, given by $\theta(\rho - r)y'(\theta)\Delta = (\rho + \delta + \Lambda - \mu i)\left(\frac{\partial v(\theta)}{\partial \Delta}\frac{\partial \Delta}{\partial \theta}\right) > 0$.

Finally, note that in a given state θ, the blockholder chooses too high leverage relative to passive investors' preferences. The intuition is that, upon default, the blockholder loses $y(\theta)$ per unit of equity, while passive shareholders lose $p(\theta) > y(\theta)$. Thus, the blockholder associates a lower cost with default than do passive shareholders. Appendix C.7 shows how the allocation of control rights affects blockholder trading in this model variant.

5 Extensions and Other Results

5.1 Large Trades and Public-to-Private Transactions

In our baseline model, the blockholder faces dispersed shareholders who differ in their willingness to sell the firm's stock, with a set of shareholders who are reluctant to sell beyond a certain point, as reflected in condition (11). We now show that when condition (11) is not met, for instance, because $\eta(\theta)$ is small or zero, we can similarly construct a Markov equilibrium whereby quantities scale with K, specifically, $P(K,\theta) = Kp(\theta)$ and $V(K,\theta) = Kv(\theta)$. However, in this Markov equilibrium, the price function $p(\theta)$ is no longer continuous.

Specifically, with (11) not being met, we now characterize an equilibrium that follows the definition from Section 2.3 but without the requirement that $p(\theta)$ is continuous. While in this case, there can be multiple equilibria, we characterize the equilibrium closest to the one from the baseline with a minimum number of price discontinuities. When (11) is not met, a price discontinuity may indeed arise as it may become optimal for the blockholder to initiate a takeover bid for all remaining shares at a fair price when reaching some upper threshold θ^* from below, even if such a bid is not mandatory. Within the class of equilibria where $p(\theta)$ has at most one discontinuity—corresponding to the lumpy trade associated with a successful takeover bid—we obtain uniqueness.[15]

Proposition 6 characterizes the scaled equilibrium that is analogous to our baseline.

Proposition 6 (Equilibrium with takeover bid)**.** *When (11) does not hold, then there exists a scaled Markov equilibrium with state variables (K,θ), so that $P(K,\theta) = Kp(\theta)$ and $V(K,\theta) = Kv(\theta)$. The choice of control variables (ℓ, i, b) is as in the baseline, with $\ell(\theta) = y(\theta)$, $i(\theta) = \mu y(\theta)$, and $b(\theta) = \frac{(1-\lambda)}{\kappa}$. Further, there exists a threshold θ^*—defined in the Appendix in (D.2)—such that:*

1. *For $\theta \in [0, \theta^*)$, the blockholder trades smoothly at rate $\dot{\theta}$ given in (23), where $\dot{\theta} > 0$ ($\dot{\theta} < 0$) for $\theta > \underline{\theta}$ ($\theta < \underline{\theta}$) and $\underline{\theta} = \underline{\Theta}$. Further, we have $v(\theta) = \hat{v}(\theta)$ and $p(\theta) = \hat{v}'(\theta)$.*

2. *For $\theta \in (\theta^*, 1)$, the blockholder conducts an immediate lumpy trade toward one (acquires the entire firm) and thereafter stops trading. Its value function is $v(\theta) = \hat{v}(1) - (1 -$*

[15] A mandatory bid is required by law in many countries, usually when the bidder already owns a certain percentage of shares (often over 30%). There are no provisions for mandatory bids in the United States.

$\theta)\hat{p}(1)$ where $\hat{p}(1)$ is the stock prevailing when the blockholder maintains perpetually full ownership of the firm. The stock price satisfies $p(\theta) = \hat{p}(1)$ where $\hat{p}(1) < \hat{v}'(1)$.

3. *Suppose that θ reaches θ^* from below, i.e., $\lim_{\theta \to \theta*} \dot{\theta}(\theta) > 0$. Then, once $\theta = \theta^*$, the blockholder randomizes between not trading at all or buying the entire firm at once at an endogenous rate $\gamma^* > 0$, given in (D.3). When $\lim_{\theta \to \theta*} \dot{\theta}(\theta) \leq 0$, the blockholder trades smoothly at $\theta = \theta^*$ at rate $\dot{\theta}$ from (23). The blockholder's value function satisfies $\hat{v}(\theta^*) = \hat{v}(1) - (1 - \theta^*)\hat{p}(1)$, which pins down the threshold θ^*.*

Thus, the price exhibits one discontinuity (upward jump) at $\theta = \theta^*$. While there can be multiple equilibria and, in particular, numerous equilibria with price discontinuities, we characterize the equilibrium closest to the one from the baseline with a minimum number of price discontinuities, specifically, a single discontinuity. Within the class of equilibria where $p(\theta)$ has at most one discontinuity, we obtain uniqueness.

5.2 Stochastic Trading and Liquidity Shocks

In the benchmark model, we abstract from elements that would introduce exogenous trading dynamics and are not crucial for our key findings. As a result, given the parameters, one can predict whether the blockholder will maintain its stake or acquire the entire firm or exit as $t \to \infty$. Our framework can be extended by allowing model parameters to evolve over time, leading to stochastic trading and long-run ownership. For instance, preference parameters π, ρ, r, or technological parameters such as μ, κ, could follow a Markov switching process, introducing exogenous randomness into the blockholder's equilibrium trading behavior that would complement the endogenous dynamics highlighted in the baseline framework.

To illustrate this in the simplest possible setting, suppose the holding cost π is initially zero but can jump to a positive value $\pi^+ = \pi$ at a Poisson arrival time with intensity ϕ. We assume that both before and after the jump, parameter condition (11) holds, so that we obtain an equilibrium with smooth trading. We sketch the equilibrium in this setting. The equilibrium after the jump is similar to that described in Proposition 2, with blockholder value function $v(\theta) = \hat{v}(\theta)$, stock price $p(\theta) = \hat{v}'(\theta)$, and smooth trading according to (23). Prior to the jump, we denote the blockholder value function by $v_{Pre}(\theta)$ and the stock price by $p_{Pre}(\theta)$. For $\theta \in [0,1]$, the value function solves:

$$(\rho+\delta+\phi)v_{Pre}(\theta) = \max_{\ell,i}\left[\theta\left(\alpha+b+(\rho-r)\ell-\frac{i^2}{2}\right)-\frac{\kappa b^2}{2}+\mu i v_{Pre}(\theta)+\phi v(\theta)\right], \quad (33)$$

where $\phi(v(\theta)-v_{Pre}(\theta))$ captures the effects of a jump on the blockholder value function and b satisfies (15).

It is clear that for $y_{Pre}(\theta) = \frac{v_{Pre}(\theta)}{\theta}$, we obtain $\ell_{Pre}(\theta) = y_{Pre}(\theta)$, $i_{Pre}(\theta) = \mu y_{Pre}(\theta)$, and $b_{Pre}(\theta) = b(\theta)$. Also, we have that $v_{Pre}(\theta) > v(\theta)$ as an upward jump in π reduces the blockholder's valuation of the firm. Next, note that $p_{Pre}(\theta) = v'_{Pre}(\theta)$ and $v'(\theta) = p(\theta)$. We then can solve for the trading rate as

$$\dot{\theta}_{Pre} = \frac{1}{p'_{Pre}(\theta)}\left[\theta(\rho-r)\ell'_{Pre}(\theta)+\frac{\theta\lambda(1-\lambda)}{\kappa}-\eta(\theta)\right]. \quad (34)$$

Note that $\eta(\theta)=0$ and $\dot{\theta}_{Pre} > 0$ for $\theta < \widetilde{\theta}$. Thus, the upper stationary point $\overline{\theta}_{Pre}$ pre-shock is larger than $\widetilde{\theta}$, while $\underline{\theta}$ is smaller than $\widetilde{\theta}$ (see Lemma 3). Thus, given $\theta_0 \in (0,\widetilde{\theta})$, two scenarios can happen

1. First, $\underline{\theta} < \theta_0 < \overline{\theta}_{Pre}$. Then, the blockholder trades up to $\overline{\theta}_{Pre}$ where it stops trading. Once the shock hits, it will trade toward target level $\overline{\theta}$. Either way, we have $\theta_\infty = \overline{\theta}$.

2. Second, $\theta_0 \leq \underline{\theta} < \overline{\theta}_{Pre}$. Again, the blockholder gradually trades up to $\overline{\theta}_{Pre}$. If the liquidity shock hits early before θ surpasses $\underline{\theta}$, the blockholder will exit and $\theta_\infty = 0$. If, on the other hand, the liquidity shock hits late only after $\theta > \underline{\theta}$, the blockholder will remain invested in this firm and trade to $\overline{\theta}$. Thus, the long-run outcome is stochastic.

Overall, we find that early realizations of negative liquidity shocks can lead to exit. In contrast, late realizations are less impactful and often do not prevent long-run acquisition by the blockholder. This highlights the critical role of timing in liquidity shocks, shaping both the short- and long-run evolution of blockholder ownership.

5.3 Blockholder-Level versus Firm-Level Debt

Blockholders, such as investment funds, often finance deals with debt that is on their own balance sheet, in addition to debt that is on the balance sheet of the firms they invest in.[16] While the baseline assumes that all debt is on the firm-level, we sketch how the model would change if blockholders had leveraged on their own; details are presented in Appendix D.2.

We denote the firm's debt level by $L = K\ell(\theta)$ and the blockholder's own debt by $L^B = K\ell^B(\theta)$. As in the baseline model, debt is subject to a limited liability constraint, preventing immediate default. Debt at the blockholder level is identical to firm-level debt in all aspects, except that it is fully repaid by the blockholder. In contrast, the blockholder repays firm-level debt in proportion to its ownership stake, i.e., it only repays a fraction θ of the firm-level debt. As such, the blockholder's limited liability constraint becomes

$$v(\theta) \geq \theta\ell(\theta) + \ell^B(\theta). \tag{35}$$

Using arguments similar to those in the baseline, it can be shown that this constraint is optimally binding. In what follows, we denote by $\omega = \frac{\ell^B(\theta)}{v(\theta)}$ the fraction of the blockholder's effective debt which is not on the firm-level. The equilibrium remains qualitatively unchanged relative to the baseline.

The blockholder's value function and the stock price are independent of ω, so the blockholder is, in principle, indifferent over choices of ω. The blockholder's value function remains unchanged, and so do its marginal and average valuations. The pricing equation remains (22), but note that $\ell(\theta) = (1-\omega)y(\theta)$. As shown in the following Proposition, the key difference arising with respect to the baseline is that ω—whether a parameter or dynamic choice—increases the blockholder's trading rate.

Proposition 7. *With blockholder-level leverage, total debt equals $y(\theta)$, and the blockholder's payoff is independent of the share of blockholder-level debt to total debt, ω. The trading rate increases in the use of blockholder-level leverage ω in that*

$$\dot{\theta} = \frac{1}{p'(\theta)}\left[\frac{\theta\lambda(1-\lambda)}{\kappa} - \pi\theta - \eta(\theta) + (\rho - r)\big(p(\theta) - (1-\omega)y(\theta)\big)\right]. \tag{36}$$

[16] For instance, private equity funds raise, next to equity capital from limited partners, debt from banks, often in the form of subscription lines of credit or loans based on the net asset value of their portfolio.

Having leverage at the blockholder level reduces, all else being equal, the firm's stock price, since passive shareholders can capture less of the benefits of issuing debt. To neutralize this effect, the blockholder buys more (or sells less) of the firm, exerting upward pressure on the stock price. Taken together, blockholder-level leverage enables the blockholder to purchase a larger stake in the firm. It also implies lower book leverage at the firm level for given θ, but, since $\ell(\theta)$ increases in θ, it can lead to higher firm-level leverage over time.

6 Conclusion and Implications

This paper studies the optimal trading strategy and level of engagement of activist blockholders in a model in which a blockholder can expend costly effort to improve firm productivity, contract with management to limit agency costs, and influence investment and financing decisions. We obtain the equilibrium in closed form, that is, the blockholder's dynamic trading strategy, its entry and exit decisions, and the firm's compensation, financing, and investment policies, assuming no commitment on the part of the blockholder. We then use the model to study firm and ownership dynamics and examine the effects of the allocation of control rights on passive shareholder value.

The trading strategy of the blockholder exhibits features commonly observed in practice. Notably, after acquiring an endogenous toehold, the blockholder's optimal trading policy involves a combination of a double-barrier policy and the continuous management of the ownership stake through dynamic sales and purchases. Depending on the starting level of ownership and firm characteristics, the activist may follow one of several paths: fully exit the firm, gradually converge to a stable target ownership level, initiate a takeover bid, or exhibit a bifurcation pattern where small initial stakes lead to exit while sufficiently large stakes result in continued engagement.

The ownership stake of the blockholder, which is endogenous and evolves dynamically, shapes blockholder engagement and firm policies. Our model shows that blockholder entry leads to an increase in firm value, book leverage, and long-run investment. In addition, both the rate of investment and leverage are predicted to increase with the stake of the blockholder. However, both policies are suboptimal from the perspective of passive investors in any given state, with an investment rate that is too low and a debt level that is too high relative to the levels that would maximize passive shareholder value. Debt financing at the blockholder level mitigates the latter effect by reducing firm-level leverage.

While firm policies do not maximize passive shareholder value when the blockholder is in control, our analysis shows that granting passive shareholders control over investment or financing decisions paradoxically reduces their value. This outcome arises due to the impact on the blockholder's trading behavior and engagement, and the blockholder's inability to commit to a specific stake. Notably, a key aspect of our model is the endogenous and dynamic nature of the blockholder's stake, which leads to divergent effects on shareholder welfare in static versus dynamic settings. Specifically, assigning investment control to passive shareholders boosts investment at a given level of blockholder ownership but diminishes the blockholder's trading activity and future investment potential. Moreover, in our model, the blockholder does not find it optimal to exit when controlling both investment and financing while also enhancing the firm's return on capital through effort. In contrast, exit becomes a part of the optimal strategy if the blockholder's control is limited solely to improving the firm's return on capital via effort. Ultimately, our findings reveal that allocating investment control to passive shareholders reduces their value due to its adverse effects on the blockholder's trading, ownership stake, and engagement.

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Appendix

A Omitted Proofs

A.1 Proof of Lemma 1

We focus on a Markov equilibrium in which all quantities scale with K. To analyze trading, we therefore consider scaled payoffs, using that $V(K,\theta) = Kv(\theta)$, $P(K,\theta)$, and $L(K,\theta) = K\ell(\theta)$.

The "if part" is immediate from the arguments in the main text. It remains to prove the "only if" part for $\theta > 0$. We consider the firm at time $t > 0$ in state $(K_t, \theta_{t^-}) = (K, \theta)$, which, by assumption, is attainable. Attainability of (K,θ) requires that there no profitable deviations in state (K,θ), in that there does not exist a trade toward any other state $(K,\hat{\theta})$ that yields *strictly* higher payoff for the blockholder. That is, any trade toward another state $(K,\hat{\theta})$ must yield weakly lower payoff for the blockholder than state (K,θ).

Suppose to the contrary that there exists a stake $\hat{\theta}$ for which the firm does not default and for which $v(\theta) < v(\hat{\theta}) - (\hat{\theta} - \theta)p(\hat{\theta})$ holds. If this was the case, the blockholder would realize a strictly positive payoff by trading toward state $\hat{\theta}$ just before time t, at time $t^- = \lim_{s \uparrow t} s$. As such, state (K,θ) is not attainable. Thus, $v(\theta) \geq v(\hat{\theta}) - (\hat{\theta} - \theta)p(\hat{\theta})$ for all $\hat{\theta} \in [0,1]$.

Consider now $\ell = \ell(\theta) > \frac{v(\theta)}{\theta}$. Suppose the firm does not default after the blockholder trades toward stake $\hat{\theta}$ just before debt repayment. As default would be optimal post-trade for both types of shareholders when $\ell > p(\hat{\theta}) \geq \frac{v(\hat{\theta})}{\hat{\theta}}$, it suffices to consider $p(\hat{\theta}) \geq \ell$. Then, the gains from trading toward state $\hat{\theta}$ just before debt repayment read

$$G^-(\theta,\hat{\theta}) =: \left[v(\hat{\theta}) - \hat{\theta}\ell\right] - \left[v(\theta) - \theta\ell\right] - (\hat{\theta} - \theta)(p(\hat{\theta}) - \ell) = v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta}) \leq 0 \quad \text{(A.1)}$$

for all $\hat{\theta} \in [0,1]$ that do not lead to default. The last inequality used that by attainability of state (K,θ), we have $v(\theta) \geq v(\hat{\theta}) - (\hat{\theta} - \theta)p(\theta)$ for all $\hat{\theta} \in [0,1]$, as shown above. Thus, conditional on no default occurring, the blockholder's continuation payoff is (weakly) smaller than $v(\theta) - \theta\ell < 0$ and therefore strictly negative.

However, the blockholder can do strictly better than that by selling its entire stake at any positive price, which yields a weakly positive payoff for the blockholder; indeed, due to the option to default, the firm's stock price must be weakly positive at any time. Note that selling the entire stake yields a weakly higher payoff than forcing immediate default, which is possible if the blockholder is in control of the default decision and yields a payoff of zero. Consequently, it suffices to prove the claim assuming that passive shareholders decide on default and default at time t whenever $p_t < \ell_t$.

When the blockholder sells its entire stake θ, it trades toward (post-trade) stake $(K,\hat{\theta})$ with $\hat{\theta} = 0$. Due to $p(0) \leq \frac{v(\theta)}{\theta} < \ell$, the firm optimally defaults post-trade for $\hat{\theta} = 0$.

By the above arguments, any other trade toward $\hat{\theta}$ that does not trigger default yields gains from trade $G^-(\theta,\hat{\theta}) \leq 0$ and therefore strictly negative post-trade payoff for the blockholder. In contrast, selling the entire stake triggers default and yields zero payoff. Thus, when $v(\theta) < \theta\ell$, the blockholder's optimal trading necessarily triggers default. Since the stock price in default is zero, the blockholder cannot do better than selling its entire stake once $v(\theta) < \theta\ell$.

A.2 Proof of Lemma 2

For a Markov equilibrium with states (K, θ), we conjecture and verify that $V(K, \theta) = Kv(\theta)$, $P(K, \theta) = Kp(\theta)$, and $L(K, \theta) = K\ell(\theta)$. Given these conjectures, we can rewrite the borrowing constraints from (5) as $\ell = \ell(\theta) \leq \min\{\frac{v(\theta)}{\theta}, p(\theta)\}$. Moreover, the partial derivatives of the value function then read $V_K(K, \theta) = v(\theta)$ and $V_\theta(K, \theta) = Kv'(\theta)$.

By the dynamic programming principle, the integral representation (6) implies that the blockholder's value function solves (under smooth trading) the HJB equation:

$$\rho V(K,\theta) = \max_{\ell,\dot{\theta},i} \Bigg\{ K \left[\theta \left(\alpha + b + (\rho - r)\ell - \frac{i^2}{2} \right) - \frac{\kappa b^2}{2} - \frac{\pi \theta^2}{2} \right] + V_K(K,\theta) K(\mu i - \delta) + \dot{\theta}\big[V_\theta(K,\theta) - P(K,\theta)\big] \Bigg\},$$

subject to (15) and the borrowing constraint, where we used that, in equilibrium, expected payouts to the manager are zero, i.e., $\mathbb{E}[dC] = \lambda \mathbb{E}[dX - (\alpha + b)dt] = \lambda\sigma\mathbb{E}[dZ] = 0$. Using $V(K, \theta) = Kv(\theta)$, $P(K, \theta) = Kp(\theta)$, $V_K(K, \theta) = v(\theta)$, and $V_\theta(K, \theta) = Kv'(\theta)$ and rearranging and canceling $K > 0$ on both sides, we obtain

$$(\rho + \delta)v(\theta) = \max_{\ell,\dot{\theta},i} \left[\theta \left(\alpha + b + (\rho - r)\ell - \frac{i^2}{2} \right) - \frac{\kappa b^2}{2} - \frac{\pi\theta^2}{2} + \mu i v(\theta) + \dot{\theta}[v'(\theta) - p(\theta)] \right], \tag{A.2}$$

which is (17). This verifies our conjecture $V(K, \theta) = Kv(\theta)$. Since the right-hand side of (17) does not depend on K and the borrowing constraint does not depend on K, it follows that (scaled) control variables do not depend on K either and are functions of θ only. Specifically, we obtain $L = \ell(\theta)K$ in optimum.

Next, note that by (7), a passive shareholder's valuation for the firm's stock, i.e., the firm's stock price, satisfies:

$$\rho P(K,\theta) = K\big(\alpha + b - \frac{i^2}{2} + (\rho - r)\ell + \eta(\theta)\big) + P_K(K,\theta)K(\mu i - \delta) + P_\theta(K,\theta)\dot{\theta},$$

where we used that in optimum $\mathbb{E}[dC] = 0$. Using $P(K, \theta) = Kp(\theta)$, $P_\theta(K, \theta) = Kp'(\theta)$, and $P_K(K, \theta) = p(\theta)$, we obtain after simplifications

$$(\rho + \delta - \mu i)p(\theta) = \alpha + b + (\rho - r)\ell(\theta) - \frac{i^2}{2} + \eta(\theta) + p'(\theta)\dot{\theta},$$

which is (22), which verifies our conjecture $P(K, \theta) = Kp(\theta)$.

To derive closed-form expressions for $v(\theta)$ and $p(\theta)$, first conjecture that $p(\theta) \geq \frac{v(\theta)}{\theta}$, which we verify at the end of the proof. We now solve the optimization in (17). First, since the right-hand-side of (17) increases in ℓ, optimal scaled debt satisfies

$$\ell = \ell(\theta) = \frac{v(\theta)}{\theta}.$$

Thus, the borrowing constraint $\ell = \ell(\theta) \leq \min\{\frac{v(\theta)}{\theta}, p(\theta)\} = \frac{v(\theta)}{\theta}$ binds in optimum. Second, the optimization with respect to the investment rate i yields $i = i(\theta) = \frac{\mu v(\theta)}{\theta}$. Third, notice that the right-hand side of (17) is linear in $\dot{\theta}$. As such, for smooth trading, i.e., $\dot{\theta} \in (-\infty, +\infty)$ to be optimal, it must be that $v'(\theta) = p(\theta)$.

Inserting these relations back into (17) and using (15), we can solve

$$v(\theta) = \hat{v}(\theta) := \frac{\theta\left(\kappa(r+\delta) - \sqrt{\kappa\,\left(\kappa\,(r+\delta)^2 - \mu^2(1-\lambda^2-\kappa\pi)\,\theta - 2\kappa\,\mu^2\alpha\right)}\right)}{\kappa\,\mu^2}. \tag{A.3}$$

Note that (9) implies

$$\kappa\,(r+\delta)^2 - \mu^2(1-\lambda^2-\kappa\pi)\,\theta - 2\kappa\,\mu^2\alpha > 0, \tag{A.4}$$

so the term under square root is positive.

We can then calculate the stock price via

$$p(\theta) = \hat{v}'(\theta) = \frac{\theta\,(1-\lambda^2-\kappa\pi)}{2\sqrt{\kappa\,\left(\kappa\,(r+\delta)^2 - \mu^2(1-\lambda^2-\kappa\pi)\,\theta - 2\kappa\,\mu^2\alpha\right)}} + \frac{\hat{v}(\theta)}{\theta}. \tag{A.5}$$

Due to condition (10), we have that $p(\theta) > \frac{v(\theta)}{\theta} = y(\theta)$.

Finally, calculate

$$\hat{v}''(\theta) = \frac{\kappa\,(1-\lambda^2-\kappa\,\pi)\,(4\,\kappa\,r^2 - 3\,\mu^2\,\theta + 3\,\lambda^2\,\mu^2\,\theta - 8\,\alpha\,\kappa\,\mu^2 + 3\,\kappa\,\mu^2\,\pi\,\theta)}{4\,(\kappa\,(\kappa\,r^2 - \mu^2\,\theta + \lambda^2\,\mu^2\,\theta - 2\,\alpha\,\kappa\,\mu^2 + \kappa\,\mu^2\,\pi\,\theta))^{3/2}}$$

Conditions (9) and (10) imply that $\hat{v}''(\theta) > 0$.

A.3 Proof of Lemma 3

Consider an equilibrium with smooth trading. Under smooth trading, we have $\hat{v}(\theta) = v(\theta)$ and $p(\theta) = v'(\theta)$, and the trading rate $\dot{\theta}$ satisfies (23):

$$\dot{\theta} = \dot{\theta}(\theta) = \frac{\theta}{\hat{v}''(\theta)}\left[\frac{\lambda(1-\lambda)}{\kappa} + (\rho - r)\ell'(\theta) - \pi - \frac{\eta(\theta)}{\theta}\right]$$

In addition, $\ell(\theta) = \frac{\hat{v}(\theta)}{\theta}$ and thus $\ell'(\theta) = \frac{\theta\hat{v}'(\theta) - \hat{v}(\theta)}{\theta^2} = \frac{1}{\theta}\left(\hat{v}'(\theta) - \frac{\hat{v}(\theta)}{\theta}\right)$. Using the last expression together with (A.5), we get:

$$\ell'(\theta) = \frac{(1-\lambda^2-\kappa\pi)}{2\sqrt{\kappa\,\left(\kappa\,(r+\delta)^2 - \mu^2(1-\lambda^2-\kappa\pi)\,\theta - 2\kappa\,\mu^2\alpha\right)}}. \tag{A.6}$$

For $\theta = 0$, we have $\dot{\theta} = 0$. Otherwise, for $\theta > 0$, the sign of $\dot{\theta} = \dot{\theta}(\theta)$ is determined by

$$\mathcal{D}(\theta) := \frac{\lambda(1-\lambda)}{\kappa} + (\rho - r)\ell'(\theta) - \pi - \frac{\eta(\theta)}{\theta} \tag{A.7}$$
$$= \frac{(\rho - r)\left(1-\lambda^2-\kappa\pi\right)}{2\sqrt{\kappa\,\left(\kappa\,(r+\delta)^2 - \mu^2(1-\lambda^2-\kappa\pi)\,\theta - 2\kappa\,\mu^2\alpha\right)}} + \frac{\lambda(1-\lambda)}{\kappa} - \pi - \pi^I\left(1-\frac{\widetilde{\theta}}{\theta}\right)^+,$$

where $\eta(\theta)$ is defined in (8). Observe that by parameter condition (11), $\mathcal{D}(1) < 0$ and therefore $\dot{\theta}(1) < 0$. This implies that there exists a left-neighborhood of one in which we have $\dot{\theta} < 0$, which, in turn, implies $\overline{\theta} < 1$.

For $\theta \neq \widetilde{\theta}$, $\mathcal{D}(\theta)$ is differentiable and we obtain

$$\mathcal{D}'(\theta) = \frac{(\rho - r)\kappa\mu^2\left(1-\lambda^2-\kappa\pi\right)^2}{4\left[\kappa\,\left(\kappa\,(r+\delta)^2 - \mu^2(1-\lambda^2-\kappa\pi)\,\theta - 2\kappa\,\mu^2\alpha\right)\right]^{3/2}} - \left(\frac{\pi^I\widetilde{\theta}}{\theta^2}\right)\mathbb{I}\{\theta > \widetilde{\theta}\}.$$

Thus, when $\theta < \widetilde{\theta}$, we have $\mathcal{D}'(\theta) \geq 0$ with the inequality being strict if $\mu > 0$. Further, for $\theta \neq \widetilde{\theta}$, we have $\mathcal{D}''(\theta) > 0$.

This implies the following. First, for $\theta \in [0, \widetilde{\theta}]$, $\mathcal{D}(\theta)$ has at most one root. Moreover, on the interval $(\widetilde{\theta}, 1]$, the function $\mathcal{D}(\theta)$ also has at most one root, since $\mathcal{D}(1) < 0$. It has precisely one root on this interval if and only if $\mathcal{D}(\tilde{\theta}) > 0$. Taken together, the function $\mathcal{D}(\theta)$ has at most two roots on the interval $[0, 1]$. Since $\dot{\theta} \propto \theta\mathcal{D}(\theta)$, there exist at most three stationary points, i.e., $\dot{\theta}(\theta)$ has (weakly) more than one but (weakly) less than three roots on $[0, 1]$.

There are three cases to distinguish.

1. Suppose $\mathcal{D}(\theta)$ has zero roots on $[0, 1]$. Then, $\mathcal{D}(\theta) < 0$ and $\dot{\theta} < 0$ for all $\theta \in [0, 1]$. Then, the definition (24) implies $\underline{\theta} = \overline{\theta} = 0$ and note that $\dot{\theta}(0) = 0$.

2. Suppose that $\mathcal{D}(\theta)$ has precisely one root, denoted θ^C, on $[0, 1]$. Then, it must be that this root θ^C lies in $[\widetilde{\theta}, 1]$. To see this, suppose to the contrary that $\theta^C \in (0, \widetilde{\theta})$. Then, $\mathcal{D}(\widetilde{\theta}) > 0$; thus, there exists a second root on $(\widetilde{\theta}, 1]$, a contradiction.

   By definition of $\overline{\theta}$ in (24), the root equals $\overline{\theta}$, i.e., $\theta^C = \overline{\theta}$, with $\overline{\theta} \in [\widetilde{\theta}, 1]$.

   We then have $\mathcal{D}(\theta) > 0$ and $\dot{\theta} > 0$ for all $\theta < \overline{\theta}$, as well as $\mathcal{D}(\theta) < 0$ and $\dot{\theta} < 0$ for all $\theta \in (\overline{\theta}, 1)$. The definition of $\underline{\theta}$ implies $\underline{\theta} = 0$. Overall, $\dot{\theta}(\underline{\theta}) = \dot{\theta}(\overline{\theta}) = 0$.

3. Suppose that $\mathcal{D}(\theta)$ has two roots, denoted $\theta_1^C < \theta_2^C$, on $[0, 1]$. Then, it must be that $\mathcal{D}(0), \mathcal{D}(1) < 0 < \mathcal{D}(\widetilde{\theta})$, and the smaller (larger) root satisfies $\theta_1^C \in (0, \widetilde{\theta})$ ($\theta_2^C \in (\widetilde{\theta}, 1)$). By defintion of the thresholds in (24), we then have $\theta_1^C = \underline{\theta}$ and $\theta_2^C = \overline{\theta}$. It follows that $\dot{\theta} < 0$ for $\theta \in (0, \underline{\theta})$ or $\theta \in (\overline{\theta}, 1)$, while $\dot{\theta} > 0$ for $\theta \in (\underline{\theta}, \overline{\theta})$. Moreover, $\dot{\theta}(\underline{\theta}) = \dot{\theta}(\overline{\theta}) = 0$.

   In this scenario, whenever $\mu > 0$, we can solve $\mathcal{D}(\underline{\theta}) = 0$ for $\underline{\theta} < \widetilde{\theta}$, yielding

$$\underline{\theta} = \underline{\Theta} := \frac{\kappa}{\mu^2\left(1-\lambda^2-\kappa\pi\right)}\left[(r+\delta)^2 - 2\mu^2\alpha - \frac{(\rho - r)^2\left(1-\lambda^2-\kappa\pi\right)^2}{4\left(\kappa\pi - \lambda(1-\lambda)\right)^2}\right],$$

which is (26), as desired.

The expression $\underline{\Theta}$ determines the number of roots of $\mathcal{D}(\theta)$. First, when $\underline{\Theta} \geq \widetilde{\theta}$, then $\mathcal{D}(\theta)$ has zero roots, and $\underline{\theta} = \overline{\theta} = 0$. Second, when $\underline{\Theta} \in (0, \widetilde{\theta})$, then $\mathcal{D}(\theta)$ has two roots, i.e., $\underline{\theta}$ and $\overline{\theta}$, with $\underline{\theta} = \underline{\Theta}$ and $\overline{\theta} \in (\widetilde{\theta}, 1)$. Third, when $\underline{\Theta} \leq 0$, then $\mathcal{D}(\theta)$ has a single root $\overline{\theta} \in (\widetilde{\theta}, 1)$, and $\underline{\theta} = 0$.

A.4 Proof of Proposition 2

We prove the existence of a unique scaled, continuous Markov equilibrium in which the price satisfies the scaling property, in that $P(K, \theta) = Kp(\theta)$. The proof proceeds in several parts. Part I presents the blockholder's HJB equation (A.12)— allowing for general trading processes and lumpy trading—and presents general results regarding the blockholder's optimal trading and equilibrium conditions—these conditions must hold in any Markov equilibrium where the price satisfies $P(K, \theta) = Kp(\theta)$ and they are also valid in other model variants. Part II verifies the optimality of the proposed trading strategy: Conjecturing the equilibrium (scaled) price $p(\theta)$ from Proposition 2, it shows that smooth trading is strictly optimal and the (scaled) value function from Proposition 2 solves the HJB equation. Part III argues that the equilibrium is unique. Unlike Part I, which presents general equilibrium conditions, the other parts present proofs that only apply to our baseline.

A.4.1 Part I: Equilibrium Properties

We allow for continuous and lumpy trading by specifying the dynamics of the blockholder's stake as

$$d\theta_t = \dot{\theta}_t dt + dI_t, \tag{A.8}$$

where $\dot{\theta}_t$ is the drift of $d\theta_t$ and dI_t captures solely lumpy trading, in that $I_t = \int_0^t dI_s$ is constant except for a countable number of times t. Since we focus on a Markov equilibrium, we will depress time subscripts in the remainder of the proof.

More specifically, we consider that at an endogenous (state-dependent) intensity $\gamma \in [0, \infty]$, the blockholder conducts a lumpy trade toward state $\hat{\theta} \in [0, 1]$, where $\hat{\theta}$ is optimally chosen by the blockholder and thus endogenous. With a slight abuse of notation, $\gamma = +\infty$ corresponds to a lumpy trade that occurs with some atom of probability (possibly with probability one). That is, we can write

$$d\theta = \dot{\theta} dt + (\hat{\theta} - \theta) dN, \tag{A.9}$$

where $dN \in \{0, 1\}$ is a jump process with $\mathbb{E}[dN] = \gamma dt$. We introduce the convention that whenever $\hat{\theta} = \theta$ in aforementioned process, we set $\gamma = 0$.

Given a price $P(K, \theta) = Kp(\theta)$ for an increasing, continuous function $p(\theta)$, we conjecture and verify that $V(K, \theta) = Kv(\theta)$ in the Markov equilibrium, where $v(\theta)$ and $p(\theta)$ are almost everywhere differentiable (i.e., they are differentiable on $[0, 1]$ except at countably many points). Thus, for any $\theta \in (0, 1)$, the limits $\lim_{x \uparrow \theta} \omega(\theta)$ and $\lim_{x \downarrow \theta} \omega(\theta)$ exist and are well-defined for $\omega \in \{v', p\}$.

We characterize the blockholder's dynamic optimization, given the conjectured equilibrium price $P(K,\theta) = Kp(\theta)$—we note that the scaled price function $p(\theta)$ need not be the one proposed in Proposition 2 and our results of this part of the proof hold in any equilibrium.

To allow for generality and to encompass the model variant of Section 3 or B, we impose that the investment i_t must satisfy a constraint $i_t \in \mathcal{I}_t$. In Section 3, we set $\mathcal{I} = \{\mu p(\theta)\}$, while the baseline has $\mathcal{I}_t = [0,\infty)$. We restrict $i_t = i(\theta)$ to be Markovian.

HJB Equation. Analogous to (6), in state $(K_t, \theta_t) = (K, \theta)$ at time t, the value function equals

$$V(K,\theta) = \max_{(dL_s, d\theta_s, i_s \in \mathcal{I}_s)_{s\geq t}} \mathbb{E}_t\Bigg[\int_0^\infty e^{-\rho(s-t)}\bigg\{\theta_s\big[K_s dX_s - \frac{K_s i_s^2}{2}ds + L_s(\rho - r)ds\big] \tag{A.10}$$
$$- K_s\left(\frac{\pi\theta_s^2}{2} + \frac{\kappa b(\theta_s)^2}{2}\right)ds - d\theta_s K_s p(\theta_s + d\theta_s)\bigg\}\bigg|(K_t,\theta_t) = (K,\theta)\Bigg],$$

where we impose the optimal contract (with $\mathbb{E}[dC] = 0$), the optimal effort choice from (15), $b_s = b(\theta_s)$, the default time of $T = \infty$ and the borrowing constraint $L_s \leq \min\{\frac{V(K_s,\theta_s)}{\theta_s}, P(K_s,\theta_s)\}$.

By the dynamic programming principle and the integral expression (A.10), the blockholder's value function solves the HJB equation:

$$\rho V(K,\theta)dt = \max_{\ell, d\theta, i\in\mathcal{I}}\bigg\{K\left[\theta\left(\alpha + b + (\rho - r)\ell - \frac{i^2}{2}\right)dt - \frac{\pi\theta^2}{2} - \frac{\kappa b^2}{2}dt\right] \tag{A.11}$$
$$+ V_K(K,\theta)K(\mu i - \delta)dt + V(K,\theta + d\theta) - d\theta P(K,\theta + d\theta)\big]\bigg\},$$

where trading $d\theta$ follows the endogenous process (A.9), b is given in (15), and the borrowing constraint becomes $K\ell \leq \min\{\frac{V(K,\theta)}{\theta}, P(K,\theta)\}$.

Using the conjecture $V(K,\theta) = Kv(\theta)$ and $P(K,\theta) = Kp(\theta)$, we note that the gains from trade $V(K,\theta + d\theta) - d\theta P(K,\theta + d\theta) = K\big[v(\theta + d\theta) - d\theta p(\theta + d\theta)\big]$ are linear in K. Specifically, if, in state (K,θ), the blockholder changes its stake by $\hat{\theta} - \theta$, then its total payoff changes by $V(K,\hat{\theta}) - (\hat{\theta} - \theta)P(K,\theta) - V(K,\theta) = K\big[v(\theta) - (\hat{\theta} - \theta)p(\theta)\big]$. It is therefore without loss of generality to work with scaled payoffs when characterizing optimal trading. Using (A.9), one can show—similar to the derivation of (A.2) in Lemma 2—that the scaled value function satisfies (in equilibrium) the following HJB equation:

$$(\rho + \delta)v(\theta) = \max_{\dot{\theta}, \gamma, \hat{\theta}, \ell, i\in\mathcal{I}}\bigg\{\theta\left(\alpha + b + (\rho - r)\ell - \frac{i^2}{2}\right) - \frac{\pi\theta^2}{2} - \frac{\kappa b^2}{2} + \mu i v(\theta)$$
$$+ \dot{\theta}\Big[\mathbb{I}\{\dot{\theta} \geq 0\}\lim_{x\downarrow\theta}\big(v'(x) - p(x)\big) + \mathbb{I}\{\dot{\theta} < 0\}\lim_{x\uparrow\theta}\big(v'(x) - p(x)\big)\Big]$$
$$+ \gamma\big[v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta})\big]\bigg\}, \tag{A.12}$$

subject to incentive constraint (15) for b and the borrowing constraint $\ell \leq \min\{\frac{v(\theta)}{\theta}, p(\theta)\}$.

HJB equation (A.12) generalizes (A.2) to account for the possibility of lumpy trades and randomized lumpy trades. It is well-defined for $\dot{\theta} \in (-\infty, +\infty)$ and $\gamma \in [0, +\infty)$. When

$\gamma = +\infty$, the HJB equation becomes with some abuse of notation: $v(\theta) = \max_{\hat{\theta}\in[0,1]} v(\hat{\theta}) - (\hat{\theta} - \theta)p(\hat{\theta})$. Moreover, we regard an infinite trading rate $\dot{\theta} \in \{-\infty, +\infty\}$ as equivalent to a lumpy trade, $\gamma = +\infty$.

Also note that according to (A.9), the optimization in (A.12) implies that optimal trading solves the equilibrium condition (13).

Gains From Trade and Equilibrium Trading Condition. For an interior solution $\dot{\theta} \in (-\infty, +\infty)$ and $\gamma \in [0, \infty)$ to be optimal, it must be

$$
\begin{aligned}
&\lim_{x\downarrow\theta} \big(v'(x) - p(x)\big) \le 0, \quad \text{and} \quad \lim_{x\uparrow\theta} \big(v'(x) - p(x)\big) \ge 0, \\
&\max_{\hat{\theta}\in[0,1]} \big[v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta})\big] \le 0.
\end{aligned} \tag{A.13}
$$

We define the endogenous region

$$\mathcal{S} = \{\theta \in [0,1] : \dot{\theta} \in (-\infty, \infty) \text{ and } \gamma \in [0, \infty)\}. \tag{A.14}$$

As in the definition of the trading process in (A.9), we follow the convention that whenever $\hat{\theta} = \theta$, we set $\gamma = 0$. That is, whenever there is no lumpy trade in state θ, then $\gamma = \gamma(\theta) = 0$.

Note that for all $\theta \in \mathcal{S}$, the condition (A.13) holds. Moreover, $\mathcal{S}$ encompasses all states that may be attained in equilibrium, in that for any $\theta \in \mathcal{S}$, the blockholder does not immediately trade away from that state.

For $\theta \notin \mathcal{S}$, by definition, an immediate lumpy trade is optimal so that $\max_{\hat{\theta}\in[0,1]} \big[v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta})\big] = 0$. Either way, we have

$$v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta}) \le 0 \quad \text{for all} \quad \theta, \hat{\theta} \in [0,1]. \tag{A.15}$$

Whenever $v(0) = 0$, we can set $\theta = 0$ in (A.15) to obtain $p(\hat{\theta}) \ge \frac{v(\hat{\theta})}{\hat{\theta}}$. That is, when $v(0) = 0$, then $\frac{v(\theta)}{\theta} \le p(\theta)$ holds for $\theta \in [0,1]$.

Next, consider $\theta \in \mathcal{S}$. Clearly, $\max_{\hat{\theta}\in[0,1]} \big[v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta})\big] \le 0$, which implies $\gamma\big[v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta})\big] = 0$ for any $\hat{\theta}$. Likewise, it is optimal to set $\dot{\theta} \le 0$ when $\lim_{x\downarrow\theta} \big(v'(x) - p(x)\big) < 0$ and $\dot{\theta} \ge 0$ when $\lim_{x\uparrow\theta} \big(v'(x) - p(x)\big) > 0$. Overall, whenever $\dot{\theta} \in (-\infty, +\infty)$ and $\gamma \in [0, \infty)$ is optimal:

$$
\begin{aligned}
\max_{\dot{\theta},\gamma,\hat{\theta}} \bigg\{ &\dot{\theta}\Big[\mathbb{I}\{\dot{\theta} \ge 0\} \lim_{x\downarrow\theta} \big(v'(x) - p(x)\big) + \mathbb{I}\{\dot{\theta} < 0\} \lim_{x\uparrow\theta} \big(v'(x) - p(x)\big)\Big] \\
&+ \gamma\big[v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta})\big] \bigg\} = 0.
\end{aligned} \tag{A.16}
$$

Plugging condition (A.16) back into (A.12), we obtain for the blockholder's value function on $\mathcal{S}$:

$$(\rho + \delta)v(\theta) = \max_{\ell, i\in\mathcal{I}} \bigg\{ \theta \left(\alpha + b + (\rho - r)\ell - \frac{i^2}{2} \right) - \frac{\pi\theta^2}{2} - \frac{\kappa b^2}{2} + \mu i v(\theta) \bigg\}, \tag{A.17}$$

with b characterized in (15) and $\ell \leq \min\{\frac{v(\theta)}{\theta}, p(\theta)\}$. In the baseline, the optimal investment is $i = \frac{v(\theta)}{\theta}$.

Baseline solution and $v(0) = 0$. Conjecture that $v(0) = 0$. Then, (A.15) implies $\frac{v(\theta)}{\theta} \leq p(\theta)$. Thus, the optimization in (A.17) implies $\ell = \frac{v(\theta)}{\theta}$, and we can solve (A.17) for $v(\theta) = \hat{v}(\theta)$ for all $\theta \in \mathcal{S}$. We verify that $v(0) = 0$. When $0 \in \mathcal{S}$, then (A.17) immediately implies that $v(0) = 0$. $v(0)$ solves (A.17) and the claim follows. When $0 \notin \mathcal{S}$, then an immediate lumpy trade toward some state $\hat{\theta} \in \mathcal{S}$ is optimal and

$$v(0) = \max_{\hat{\theta} \in [0,1]} G(\hat{\theta}) := \max_{\hat{\theta} \in [0,1]} \left[\hat{v}(\hat{\theta}) - \hat{\theta} p(\hat{\theta}) \right].$$

Note that $p(\hat{\theta}) = \hat{v}'(\hat{\theta})$, and $G'(\hat{\theta}) = -\hat{\theta} v''(\hat{\theta}) < 0$. In addition, $G(0) = 0$ and, because $G(\hat{\theta})$ decreases, we obtain $v(0) = 0$.

A.4.2 Part II: Optimality of Smooth Trading and Trading Strategy

We verify the optimality of the proposed trading strategy under the proposed price function satisfying $p(\theta) = \hat{v}'(\theta)$. That is, we conjecture the equilibrium price $p(\theta) = \hat{v}'(\theta)$ and verify that, given this price, smooth trading is optimal and constitutes an equilibrium strategy. This part then establishes the existence of a continuous, scaled Markov equilibrium.

Optimality of Smooth Trading. Consider state $\theta \in \mathcal{S}$, so that (A.13) holds and $v(\theta) = \hat{v}(\theta)$ (as shown before). Because the price is continuous, (A.13) implies that $v(\theta)$ is differentiable, with $p(\theta) = v'(\theta)$.

We now solve the blockholder's optimal trading and value function given the conjectured price function $p(\theta) = \hat{v}'(\theta)$. For this sake, we solve (13) or, equivalently, the choice of trading in the HJB equation (A.12). We show that smooth trading is optimal—that is,

$$\dot{\theta} dt \in \arg \max_{\Delta \in [-\theta, 1-\theta]} K_t \left\{ v(\theta + \Delta) - \Delta p(\theta + \Delta) \right\} \tag{A.18}$$

for finite trading rate $\dot{\theta} \in (-\infty, +\infty)$. Having established the optimality of smooth trading, the HJB equation (A.12) then implies $v(\theta) = \hat{v}(\theta)$.

Note that for any finite trading rate $d\theta = \dot{\theta} dt$, we have

$$v(\theta + \dot{\theta} dt) - \dot{\theta} dt \cdot p(\theta + \dot{\theta} dt) = \hat{v}(\theta) + (\hat{v}'(\theta) - p(\theta) \dot{\theta} dt = \hat{v}(\theta).$$

after ignoring terms of order $(dt)^2$ or higher (which are negligible in continuous time). Thus, smooth trading at any finite rate $\dot{\theta}$ yields continuation payoff $\hat{v}(\theta)$. The blockholder is indifferent across any finite trading rates and, in particular, $\hat{v}(\theta)$ is also the payoff when the blockholder does not trade at all.

In contrast, consider a lumpy trade toward state $\hat{\theta}$ from any state θ. We assume that $\hat{\theta} \in \mathcal{S}$, as otherwise the lumpy trade would be followed immediately by another lumpy trade and we could consolidate these trades. Thus, the blockholder chooses $d\theta = \hat{\theta} - \theta$ in state θ, which yields (scaled) payoff (given θ):

$$G(\hat{\theta}) := \hat{v}(\hat{\theta}) - (\hat{\theta} - \theta)p(\hat{\theta}) = \hat{v}(\hat{\theta}) - (\hat{\theta} - \theta)\hat{v}'(\hat{\theta}).$$

Note that $G(\theta) = v(\theta)$ and $G'(\hat{\theta}) = -(\hat{\theta} - \theta)\hat{v}''(\hat{\theta})$. Thus, $G(\hat{\theta})$ obtains its maximum for $\theta = \hat{\theta}$. Thus, given the price $p(\theta) = \hat{v}'(\theta)$, a lumpy trade is never optimal. This It follows that smooth trading (including not trading at all) strictly dominates any lumpy trade with $\hat{\theta} \neq \theta$.

By definition, for all $\theta \notin \mathcal{S}$, the blockholder would find it optimal to conduct a lumpy trade toward a point $\hat{\theta} \in \mathcal{S}$. Above argument applies for both $\theta \in \mathcal{S}$ and $\theta \notin \mathcal{S}$. However, we have shown that the blockholder would be better off not trading at all. Thus, the set $[0, 1] - \mathcal{S}$ must be empty or, equivalently, $\mathcal{S} = [0, 1]$.

Thus, given the conjectured price function, smooth trading is optimal for all $\theta \in [0, 1]$ and yields value function $v(\theta) = \hat{v}(\theta)$. In particular, $v(\theta) = \hat{v}(\theta)$ solves the HJB equation (A.12).

Solving the Trading Rate. To show that $p(\theta) = \hat{v}'(\theta)$ is indeed an equilibrium price under smooth trading and to establish that the proposed equilibrium indeed exists, we need to solve for the equilibrium trade that is consistent with the price $p(\theta) = \hat{v}'(\theta)$ for all $\theta \in [0, 1]$.

To determine trading rate, we differentiate the closed-form expression for $\hat{v}(\theta)$ with respect to θ and using $\hat{v}'(\theta) = p(\theta)$, we obtain

$$(\rho + \delta)p(\theta) = \alpha + b + \mu i p(\theta) - \frac{i^2}{2} + (\rho - r)\ell(\theta) - \pi + \frac{\theta\lambda(1 - \lambda)}{\kappa} + \theta(\rho - r)\ell'(\theta). \quad \text{(A.19)}$$

In addition to satisfying (A.19), $p(\theta)$ satisfies the pricing equation of passive shareholders

$$(\rho + \delta - \mu i)p(\theta) = \alpha + b - \frac{i^2}{2} + (\rho - r)\ell(\theta) + \eta(\theta) + p'(\theta)\dot{\theta}, \quad \text{(A.20)}$$

where $p(\theta)$ increases with θ, i.e., $p'(\theta) > 0$. Combining (A.19) and (A.20) yields

$$\dot{\theta} = \frac{1}{p'(\theta)} \left[\frac{\theta\lambda(1 - \lambda)}{\kappa} + \theta(\rho - r)\ell'(\theta) - \pi\theta - \eta(\theta) \right] \quad \text{(A.21)}$$

with $p'(\theta) = \hat{v}''(\theta) > 0$ and $\theta\ell'(\theta) = p(\theta) - \frac{v(\theta)}{\theta} > 0$.

Finally, as shown by Lemma 3, the trading rate $\dot{\theta} = \dot{\theta}(\theta)$ satisfies $\dot{\theta}(0) = 0$ and $\dot{\theta}(1) < 0$. Thus, the smooth trading is such that θ adheres to the "feasibility" constraint $\theta[0, 1]$. Specifically, the process $d\theta = \dot{\theta}dt$ with aforementioned trading rate stays within $[0, 1]$.

A.4.3 Part III: Equilibrium Uniqueness

Having shown that a continuous, scaled Markov equilibrium exists, we now argue that it must be unique.

To see this, first recall that in equilibrium the condition (A.13) must hold for all $\theta \in \mathcal{S}$. As the price function is continuous, then (A.13) implies that in equilibrium, $v(\theta)$ is differentiable at θ with $v'(\theta) = p(\theta)$ for all $\theta \in \mathcal{S}$. Thus, on $\mathcal{S}$, we get that $v(\theta) = \hat{v}(\theta)$ and $p(\theta) = \hat{v}'(\theta)$.

Given this price, we have shown that the blockholder's trading rate is uniquely determined according to (A.21). Thus, on the set $\mathcal{S}$, the price is uniquely determined as $p(\theta) = \hat{v}'(\theta)$, while the value function satisfies $v(\theta) = \hat{v}(\theta)$.

This leaves the possibility open that there might be an equilibrium with continuous scaled price $p(\theta)$ which features $\mathcal{S} \neq [0,1]$. However, repeating arguments made before, one can then consider $\theta \in [0,1] - \mathcal{S}$. In state $\theta \notin \mathcal{S}$, it is then, by definition of the set $\mathcal{S}$, optimal to lumpily trade toward state $\hat{\theta} \in \mathcal{S}$ (where $v(\hat{\theta}) = \hat{v}(\hat{\theta})$ and $p(\hat{\theta}) = \hat{v}'(\hat{\theta})$). This trade yields continuation payoff

$$\hat{v}(\hat{\theta}) - (\hat{\theta} - \theta)\hat{v}'(\hat{\theta}) < \hat{v}(\theta).$$

Thus, the blockholder would prefer not trading at all in $\theta \notin \mathcal{S}$. Thus, the set $[0,1] - \mathcal{S}$ must be empty in any continuous, scaled trading equilibrium, while the equilibrium is uniquely determined on $\mathcal{S}$.

Taken together, the continuous, scaled Markov equilibrium that we have characterized is unique.

A.5 Proof of Corollary 1

We recall for the purpose of this proof $\mathcal{D}(\theta)$ defined in (A.7), satisfying

$$\mathcal{D}(\theta) = \frac{(\rho - r)\,(1 - \lambda^2 - \kappa\pi)}{2\sqrt{\kappa\;(\kappa\,(r+\delta)^2 - \mu^2(1-\lambda^2-\kappa\pi)\,\theta - 2\kappa\,\mu^2\alpha)}} + \frac{\lambda(1-\lambda)}{\kappa} - \pi - \pi^I\left(1 - \frac{\widetilde{\theta}}{\theta}\right)^+.$$

We prove claims (1), (2), and (3) of the Corollary in the respective order.

1. Claim 1 follows directly from Lemma 3. In the long run, as $t \to \infty$, the state θ drifts to one of the stationary points. The dynamics of θ are deterministic, so that θ_∞ is deterministic.

2. We start by proving the claim regarding $\overline{\theta}$. Suppose that $\overline{\theta} > 0$. Then, $\overline{\theta} > \widetilde{\theta}$ satisfies $\mathcal{D}(\overline{\theta}) = 0$. The function $\mathcal{D}(\theta)$ is differentiable at $\overline{\theta} \in (\widetilde{\theta}, 1)$, satisfying $\mathcal{D}'(\overline{\theta}) < 0$. Totally differentiating $\mathcal{D}(\overline{\theta})$ with respect to an arbitrary model parameter x, we get from the implicit function theorem:

$$\frac{\partial\overline{\theta}}{\partial x} = -\frac{1}{\mathcal{D}'(\overline{\theta})}\frac{\partial\mathcal{D}(\overline{\theta})}{\partial x}, \tag{A.22}$$

   which has the same sign as $\frac{\partial\mathcal{D}(\overline{\theta})}{\partial x}$. Clearly, $\mathcal{D}(\theta)$ increases in μ for any θ. Thus, $\overline{\theta}$ increases in μ. Next, note that $\mathcal{D}(\theta)$ decreases in r for any θ, so that $\overline{\theta}$ decreases in r. Finally, $\mathcal{D}(\theta)$ decreases in κ for any θ, so that $\overline{\theta}$ decreases in κ too.

   Next, we prove the claims regarding $\underline{\Theta}$. Note that $\mathcal{D}(\underline{\Theta}) = 0$ with $\mathcal{D}'(\underline{\Theta}) > 0$. Thus,

$$\frac{\partial\underline{\Theta}}{\partial x} = -\frac{1}{\mathcal{D}'(\underline{\theta})}\frac{\partial\mathcal{D}(\underline{\Theta})}{\partial x}, \tag{A.23}$$

which has the opposite sign as $\frac{\partial \mathcal{D}(\underline{\theta})}{\partial x}$. Thus, $\underline{\Theta}$ decreases in μ, increases in r, and increases in κ. Next, note that $\mathcal{D}(\theta)$ decreases in r for any θ, so that $\overline{\theta}$ decreases in r.

3. Finally, consider the case $\mu = 0$. Then, $\mathcal{D}(\theta)$ simplifies to

$$\mathcal{D}(\theta) = \frac{(\rho - r)\left(1 - \lambda^2 - \kappa\pi\right)}{2\kappa(r+\delta)} + \frac{\lambda(1-\lambda)}{\kappa} - \pi - \pi^I \left(1 - \frac{\widetilde{\theta}}{\theta}\right)^+$$

Note that for $\theta < \widetilde{\theta}$, $\mathcal{D}(\theta)$ is a constant and does not change with θ, so that there cannot be a root on $(\underline{\theta}, \overline{\theta})$ (ignoring the knife-edge case that $\mathcal{D}(\theta) \equiv 0$ on this interval). Either way, we have $\underline{\theta} = 0$. Then, we can solve

$$\overline{\theta} = \left[\frac{\pi^I \widetilde{\theta}}{\pi^I - \left(\frac{(\rho-r)(1-\lambda^2-\kappa\pi)}{2\kappa(r+\delta)} - \frac{\lambda(1-\lambda)}{\kappa} - \pi\right)}\right] \mathbb{I}\{\overline{\theta} > \widetilde{\theta}\}.$$

Note that $\overline{\theta} > \widetilde{\theta}$ if and only if $\frac{(\rho-r)(1-\lambda^2-\kappa\pi)}{2\kappa(r+\delta)} - \frac{\lambda(1-\lambda)}{\kappa} - \pi > 0$. In this case, we have $\mathcal{D}(\widetilde{\theta}) > 0$.

A.6 Proof of Proposition 3

For $\theta_0 \in [0, 1]$, the blockholder's scaled entry payoff reads

$$e(\theta_0) := v(\theta_0) - R(\theta_0) = v(\theta_0) - \theta_0 p(\theta_0) + \min\{\eta, \theta_0\}(p(\theta_0) - p(0)), \tag{A.24}$$

where $v(\theta_0) = \hat{v}(\theta_0)$ and $p(\theta_0) = \hat{v}'(\theta_0)$, as well as $p(0) = \hat{v}'(0)$.

Consider $\theta_0 \leq \eta$. Then, $e(\theta_0) = \hat{v}(\theta_0) - \theta_0 \hat{v}'(0)$. Note that $e(0) = 0$ and $e'(\theta_0) = \hat{v}'(\theta_0) - \hat{v}'(0) = 0$, i.e., $e(\theta_0)$ is increasing and positive for $\theta_0 \in (0, \eta)$. This implies $e(\eta) > 0$.

Next, consider $\theta_0 \geq \eta$. Then, $e(\theta_0) = \hat{v}(\theta_0) - \theta_0 \hat{v}'(\theta_0) + \eta\big[\hat{v}'(\theta_0) - \hat{v}'(0)\big]$. Note that $e(\eta) > 0$, while $e'(\theta_0) = -(\theta_0 - \eta)\hat{v}''(\theta_0) < 0$. Thus, $e(\theta_0)$ decreases for $\theta_0 > \eta$.

Therefore, $e(\theta_0)$ is maximized for $\theta_0 = \eta$.

B Investment Set by Passive Shareholders—Solution Details and Proof of Proposition 4

We now provide the solution details for the model variant with passive shareholders controlling investment. Unless otherwise mentioned, all assumptions remain as in the baseline. We assume $\mu > 0$, as otherwise there is no investment and the baseline model applies.

In particular, we assume that instead of the blockholder, the passive shareholders choose investment whenever $\theta < 1$; for $\theta = 1$, the blockholder chooses investment. We look for a scaled, continuous Markov equilibrium with state variables $(K_t, \theta_t) = (K, \theta)$, where $L(K, \theta) = K\ell(\theta)$, $V(K, \theta) = Kv(\theta)$, and $P(K, \theta) = K_t p(\theta)$ with continuous and increasing price function $p(\theta)$. Besides investment choice, all other elements, including debt choice and the contracting with management, remain unchanged relative to the baseline. When writing down the payoffs, we already impose the optimal structure of the optimal contract with management, which is the same as in the baseline. In addition, as in the baseline, it can be shown that $y(\theta) < v'(\theta) = p(\theta)$.

Formally, passive shareholders choose the investment rate to maximize the stock price $P(K,\theta)$, taking the blockholder's trading $d\theta$ and other controls as given. That is, passive shareholders choose investment rate i to maximize

$$\rho P(K,\theta) = \max_{i\geq 0}\left\{K\left(\alpha + b - \frac{i^2}{2} + \eta(\theta)\right) + (\rho - r)L(K,\theta) + \frac{\mathbb{E}[dP(K,\theta)]}{dt}\right\},$$

Due to $P(K,\theta) = Kp(\theta)$ and $\frac{dK}{K} = (\mu i - \delta)dt$, this optimization can be rewritten as

$$(\rho+\delta)p(\theta) = \max_{i\geq 0}\left\{\alpha + b + \mu i p(\theta) - \frac{i^2}{2} + (\rho - r)\ell + \eta(\theta) + \frac{\mathbb{E}[dp(\theta)]}{dt}\right\},$$

where, as we show, equilibrium effort b satisfies $b = \frac{\theta(1-\lambda)}{\kappa}$ and debt ℓ satisfies $\ell = \ell(\theta) = \frac{v(\theta)}{\theta}$. The optimization with respect to i yields

$$i = i(\theta) = \mu p(\theta)$$

for all $\theta \in (0,1)$.

To focus on non-trivial cases and to guarantee the existence of a unique continuous, scaled Markov equilibrium, we make the following assumptions. First, as in the baseline, we assume (9), (10), and $\mu i < r + \delta$. Second, analogously to (11), we assume that

$$\eta(1) = \pi^I(1-\widetilde{\theta}) > (\rho - r)\sqrt{\frac{1-\lambda^2 - \kappa\pi + 2\alpha\kappa}{\kappa\mu^2}} + \frac{\lambda(1-\lambda)}{\kappa} - \pi. \tag{B.1}$$

The condition (B.1) slightly differs from (11), but has similar economic meaning. It will ensure $\dot{\theta}(1) < 0$ and $\theta < 1$, i.e., the blockholder never acquires the entire firm. Likewise, as shown in Lemma 3, this outcome also prevailed in the baseline. Since the blockholder never acquires the entire firm or, more generally, a sufficiently large stake (unless it is born with it), both with and without control over investment, one could assume that the blockholder controls investment for ownership levels θ sufficiently close to one, without changing the key findings and equilibrium properties.

We note that the equilibrium properties of Appendix A.4.1—that is, (A.12), (A.15), or (A.17)—are also valid in this model variant and are used in the following proofs. In particular, Appendix A.4.1 derives equilibrium properties that must hold in any continuous, scaled Markov equilibrium under any Markovian investment policy.

We solve for a continuous, scaled Markov equilibrium where the blockholder trades smoothly at rate $\dot{\theta} = \dot{\theta}(\theta)$ for all $\theta \in [0,1]$, in that the set $\mathcal{S}$—defined in (A.14)—satisfies $\mathcal{S} = [0,1]$. In what follows, we suppose $\mathcal{S} = [0,1]$, solve for the equilibrium, and then verify in Appendix B.6 that $\mathcal{S} = [0,1]$ holds in equilibrium. Appendix B.6 also establishes the uniqueness of the continuous, scaled Markov equilibrium with smooth trading. We note that this uniqueness result is slightly less general than uniqueness among all continuous, scaled

Markov equilibria (which we established for the baseline).

B.1 Optimization and HJB Equation

In a continuous, scaled Markov equilibrium, (A.12) must hold whenever $\dot{\theta} \in (-\infty, \infty)$ and a lumpy trade is not strictly optimal, in that gains from trade are zero in equilibrium. By continuity of the price function, we have $p(\theta) = v'(\theta)$. Analogously to (A.17), the blockholder's scaled value function $v(\theta)$ solves the HJB equation:

$$(\rho + \delta)v(\theta) = \max_{\ell} \left[\theta \left(\alpha + b + (\rho - r)\ell - \frac{i^2}{2} \right) - \frac{\pi\theta^2}{2} - \frac{\kappa b^2}{2} + \mu i v(\theta) \right], \tag{B.2}$$

subject to (5) and $b = \frac{\theta(1-\lambda)}{\kappa}$. We used that the gains from trade are zero, and the blockholder's value function is determined "as if" it could not trade at all (i.e., $\dot{\theta} = 0$). The choice of scaled debt satisfies, as in the baseline, $\ell(\theta) = y(\theta) = \frac{v(\theta)}{\theta}$. The only and key difference to (17) and (A.17) is that the blockholder does not choose investment, which, instead, equals $i = \mu p(\theta) = \mu v'(\theta)$ and statically maximizes equity value $p(\theta)$, i.e., $i = \arg\max_{\hat{i} \geq 0} \left[\mu \hat{i} p(\theta) - \frac{\hat{i}^2}{2} \right] = \mu p(\theta)$.

We can insert optimal debt policy, $\ell(\theta) = y(\theta)$, and the investment policy, $i = \mu p(\theta) = \mu v'(\theta)$, into (B.2) to obtain the following first-order ODE for $v(\theta)$:

$$(r + \delta)v(\theta) = \theta \left(\alpha + b - \frac{(\mu v'(\theta))^2}{2} \right) - \frac{\kappa b^2}{2} - \frac{\pi\theta^2}{2} + \mu^2 v'(\theta) v(\theta). \tag{B.3}$$

Using $y(\theta) = \frac{v(\theta)}{\theta}$ for $\theta \in (0, 1)$, we obtain from (B.3):

$$(r + \delta)y(\theta) = \alpha + \frac{\theta(1 - \lambda^2)}{2\kappa} - \frac{\pi\theta}{2} + \mu^2 v'(\theta) \left(y(\theta) - \frac{1}{2} v'(\theta) \right) \tag{B.4}$$

Note that when passive investors choose investment, the investment choice generally does not maximize the blockholder's scaled value function. As such, the blockholder's scaled value function is lower than that in the baseline, in that $v(\theta) < \hat{v}(\theta)$ and $y(\theta) < \frac{\hat{v}(\theta)}{\theta}$ for all $\theta \in (0, 1)$.

B.2 ODE for Value Function and Stock Price

First note that according to (A.15), we have

$$v(\hat{\theta}) - v(\theta) - (\hat{\theta} - \theta)p(\hat{\theta}) \leq 0 \quad \text{for all} \quad \theta, \hat{\theta} \in [0, 1].$$

We conjecture and verify that $v(0) = 0$ (see below). Setting $\theta = 0$ and using $v(0) = 0$, we have $p(\hat{\theta}) \geq \frac{v(\hat{\theta})}{\hat{\theta}}$. Thus, $y(\theta) = \frac{v(\theta)}{\theta} \leq p(\theta) = v'(\theta)$ holds for all $\theta \in [0, 1]$.

Using $v'(\theta) \geq y(\theta)$ for $\theta \in (0,1)$, we can solve (B.3) for $v'(\theta)$ to obtain:

$$v'(\theta) = \sqrt{\frac{\theta(1-\lambda^2-\kappa\pi) + 2\,\alpha\,\kappa\, + y(\theta)^2\,\kappa\,\mu^2 - 2y(\theta)\,\kappa\,(r+\delta)}{\kappa\mu^2}} + y(\theta). \tag{B.5}$$

The ODE (B.5) is solved on $(0,1)$ subject to the boundary condition

$$\lim_{\theta\to 0}\frac{v(\theta)}{\theta} = \lim_{\theta\to 0} y(\theta) = \lim_{\theta\to 0}\frac{\hat{v}(\theta)}{\theta}, \tag{B.6}$$

where $\hat{v}(\theta)$ is the value function under the baseline (i.e., when blockholder controls investment) given in (A.3). The boundary condition (B.6) implies in particular, $\lim_{\theta\to 0} v(\theta) = \lim_{\theta\to 0}\hat{v}(\theta) = 0$, i.e., $v(0)=0$. The boundary condition reflects that in the limit $\theta \to 0$, the blockholder's value function approaches the value under the baseline.

We now prove that the term under the square root in (B.5) is strictly positive and that $v'(\theta) > y(\theta)$ in the continuous, scaled Markov equilibrium. Since we have already shown $v'(\theta) \geq y(\theta)$, we suppose to the contrary there exists $\theta \in (0,1)$ such that $v'(\theta) = y(\theta)$ for $\theta > 0$. Thus, the baseline case effectively prevails where $i = \mu p(\theta) = \mu y(\theta)$ maximizes the blockholder's value function. Then, (B.3) reduces to (A.17) with $i = \frac{\mu v(\theta)}{\theta}$ and $\ell(\theta) = \frac{v(\theta)}{\theta}$. In this case, we could solve (B.3) or, equivalently, (B.4) for

$$v(\theta) = \hat{v}(\theta) = \frac{\theta\,\left(\kappa(r+\delta) - \sqrt{\kappa\,\left(\kappa\,(r+\delta)^2 - \mu^2(1-\lambda^2-\kappa\pi)\,\theta - 2\kappa\,\mu^2\alpha\right)}\right)}{\kappa\,\mu^2}$$

where above expression for $\hat{v}(\theta)$ is taken from (A.3). Thus, $y(\theta) = \frac{\hat{v}(\theta)}{\theta}$. Note that when $v(\theta) = \hat{v}(\theta)$, the term under the square root in (B.4) becomes zero.

The ODE (B.5) and $\lim_{\theta\to 0} v'(\theta) = \lim_{\theta\to 0} y(\theta)$ imply then $y(\hat{\theta}) = v'(\hat{\theta})$ and $v(\hat{\theta}) = \hat{v}(\hat{\theta})$ for all $\hat{\theta} \in [0,\theta]$. However, $v'(\hat{\theta}) = \hat{v}'(\hat{\theta}) > \frac{\hat{v}(\hat{\theta})}{\hat{\theta}} = y(\hat{\theta})$ (see (A.5) and (A.3)), a contradiction. The claim follows, i.e., $v'(\theta) > y(\theta)$ in a continuous, scaled Markov equilibrium for $\theta \in (0,1)$.

We note that $\lim_{\theta\to 0}\frac{\hat{v}(\theta)}{\theta} = p(0)$ where $p(0) = p^0$ is defined in Proposition 1. For $\theta \to 0$, we obtain that the term under the square root in (B.5) approaches zero, so that $\lim_{\theta\to 0} v'(\theta) = \lim_{\theta\to 0} y(\theta) = p(0)$, i.e., $v'(0) = p^0$.

B.3 Existence and Uniqueness of ODE Solution

Together, the ODE (B.5) and the boundary condition (B.6), as well as $p(\theta) = v'(\theta)$ and the trading strategy (discussed below), characterize the continuous, scaled Markov equilibrium. A solution to (B.5) exists, but may not be unique. We argue now that $v(\theta)$ is uniquely determined.

Note that the right-hand side of (B.5) is not Lipschitz continuous in $v(\theta)$ or $y(\theta)$, as the term under the square root approaches zero as $y(\theta)$ approaches $v'(\theta)$. Thus, standard uniqueness results do not apply, and there could be a solution to (B.5) where $v'(\theta) = y(\theta)$ on an interval. However, such a solution is ruled out, as we have shown $v'(\theta) > y(\theta)$ for all $\theta \in (0,1)$.

That is, $v(\theta)$ is the unique solution to (B.5) subject to (B.6) which satisfies $v'(\theta) > y(\theta)$ for all $\theta \in (0,1)$. This solution can also be obtained by solving an auxiliary ODE and invoking standard uniqueness results. Specifically, for $\varepsilon > 0$ and $y_\varepsilon(\theta) = \frac{v_\varepsilon(\theta)}{\theta}$, we can solve

$$v'_\varepsilon(\theta) = \sqrt{\max\left\{\varepsilon, \frac{\theta(1-\lambda^2-\kappa\pi) + 2\,\alpha\,\kappa\, + y_\varepsilon(\theta)^2\,\kappa\,\mu^2 - 2y_\varepsilon(\theta)\,\kappa\,(r+\delta)}{\kappa\mu^2}\right\}} + y_\varepsilon(\theta)$$

subject to

$$\lim_{\theta\to 0}\frac{v_\varepsilon(\theta)}{\theta} = \lim_{\theta\to 0} y_\varepsilon(\theta) = \lim_{\theta\to 0}\frac{\hat{v}(\theta)}{\theta}.$$

The solution $v_\varepsilon(\theta)$ to this ODE is unique for any $\varepsilon > 0$, satisfying $v'_\varepsilon(\theta) > y_\varepsilon(\theta)$. We then obtain $v(\theta) = \lim_{\varepsilon\to 0} v_\varepsilon(\theta)$ for all $\theta \in [0,1]$, satisfying $v'(\theta) > y(\theta)$ for $\theta \in (0,1)$.

B.4 Strict Convexity of Value Function

We start by differentiating both sides of the ODE (B.3) with respect to θ to obtain

$$\begin{aligned}(r+\delta)v'(\theta) = {} & \left(\alpha + b - \pi\theta - \frac{(\mu v'(\theta))^2}{2}\right) + \frac{\theta\lambda(1-\lambda)}{\kappa} \\ & - \theta\mu^2 v'(\theta)v''(\theta) + \mu^2(v'(\theta))^2 + \mu^2 v(\theta)v''(\theta).\end{aligned}$$

Thus, provided $\theta > 0$, we can solve for

$$v''(\theta) = \frac{1}{\mu^2\theta\big(v'(\theta) - y(\theta)\big)} \underbrace{\left[\alpha + b - \pi\theta + \frac{(\mu v'(\theta))^2}{2} + \frac{\theta\lambda(1-\lambda)}{\kappa} - (r+\delta)v'(\theta)\right]}_{=:\mathcal{A}(\theta)}. \tag{B.7}$$

Note that by (B.5), it follows that $v'(\theta) > y(\theta)$ and $\mu^2\theta\big[v'(\theta) - y(\theta)\big] > 0$. Thus, to show that $v''(\theta) > 0$, we need to show that the term in square brackets in (B.7)—which we denote by $\mathcal{A}(\theta)$—is positive. More generally, (B.7) implies that on $(0,1)$, $v''(\theta)$ has the same sign as $\mathcal{A}(\theta)$.

Recall that, as discussed above, the boundary condition (B.6) implies $v'(0) = p(0)$ where $p(0) = p^0$ is characterized in Proposition 1. Thus, inserting $v'(0) = p(0)$ into our expression for $\mathcal{A}(\theta)$, we get $\mathcal{A}(0) = 0$.

Next, calculate

$$\begin{aligned}\mathcal{A}'(\theta) &= \frac{1-\lambda^2-\kappa\pi}{\kappa} - (r+\delta)v''(\theta) - \mu^2 v'(\theta)v''(\theta) \\ &= \frac{1-\lambda^2-\kappa\pi}{\kappa} - (r+\delta-\mu i)v''(\theta).\end{aligned} \tag{B.8}$$

where we used $i = \mu v'(\theta)$. Note that $r + \delta - \mu i > 0$ (finite valuations requirement ensured by parameter condition (9)) and that $1 - \lambda^2 - \kappa\pi > 0$ due to parameter condition (10).

We now prove that $\mathcal{A}(\theta) > 0$ (i.e., $v''(\theta) > 0$) in a right-neighborhood of zero. Suppose to the contrary that there exists $\varepsilon > 0$ such that $\mathcal{A}(\theta) < 0$ (i.e., $v''(\theta) < 0$) for all $\theta \in (0,\varepsilon)$.

However, $v''(\theta) < 0$ implies $\mathcal{A}'(\theta) > 0$ owing to (B.8). Thus, due to $\mathcal{A}(0) = 0$, we must have $\mathcal{A}(\theta) > 0$ on this interval, a contradiction. Thus, $\mathcal{A}(\theta) > 0$ and $v''(\theta) > 0$ in a right-neighborhood of zero.

Last, suppose to the contrary that there exists $\theta' \in (0, 1)$ such that $v''(\theta') < 0$, i.e., $\mathcal{A}(\theta') < 0$. Then, there exits—by continuity of $v''(\theta)$ and $\mathcal{A}(\theta)$—a value $\theta'' \in (0, \theta')$ where $v''(\theta'') = 0$, i.e., $\mathcal{A}(\theta'') = 0$, as well as $\mathcal{A}'(\theta'') < 0$. That is, because $v''(\theta) > 0$ and $\mathcal{A}(\theta) > 0$ in a right-neighborhood of zero and $v''(\theta'), \mathcal{A}(\theta') < 0$, there must exists a root θ'' of $v''(\theta)$ and $\mathcal{A}(\theta)$, whereby $\mathcal{A}(\theta)$ crosses zero from above with $\mathcal{A}'(\theta'') < 0$. However, inserting $v''(\theta'') = 0$ into (B.8) we obtain $\mathcal{A}'(\theta'') > 0$, a contradiction. It follows that $v''(\theta) > 0$ on $(0, 1)$.

B.5 Smooth Trading Rate

We determine the trading rate $\dot{\theta} = \dot{\theta}(\theta)$. Differentiating both sides of (B.2) with respect to θ and using $p(\theta) = v'(\theta)$, we get

$$\begin{aligned}(\rho + \delta - \mu i)p(\theta) = \alpha + b - \frac{i^2}{2} + (\rho - r)\ell(\theta) - \pi\theta \\ + \frac{\theta\lambda(1-\lambda)}{\kappa} + \theta(\rho - r)\ell'(\theta) + i'(\theta)\theta\mu\left(\frac{v(\theta)}{\theta} - p(\theta)\right).\end{aligned} \tag{B.9}$$

where $\ell = \ell(\theta) = \frac{v(\theta)}{\theta}$ and $i = i(\theta) = \mu p(\theta) = \mu v'(\theta)$.

In analogy to (A.20) in the benchmark model, $p(\theta)$ satisfies the fair pricing equation of passive shareholders:

$$(\rho + \delta - \mu i)p(\theta) = \alpha + b - \frac{i^2}{2} + (\rho - r)\ell(\theta) + \eta(\theta) + p'(\theta)\dot{\theta}.$$

Combining (B.9) with the fair pricing equation, we can solve for the trading rate:

$$\dot{\theta} = \dot{\theta}(\theta) = \frac{1}{p'(\theta)}\left[\frac{\theta\lambda(1-\lambda)}{\kappa} + \theta(\rho - r)\ell'(\theta) - \pi\theta - \eta(\theta) + \theta\mu i'(\theta)\left(y(\theta) - p(\theta)\right)\right]. \tag{B.10}$$

Due to $v''(\theta) > 0$ and $v'(\theta)\theta > v(\theta)$, we get that $\ell'(\theta) = \frac{1}{\theta}\left(p(\theta) - y(\theta)\right) > 0$ as well as $i'(\theta) = \mu v''(\theta) > 0$, so that

$$\theta\mu i'(\theta)\left(\frac{v(\theta)}{\theta} - p(\theta)\right) = \theta\mu^2 v''(\theta)\big(y(\theta) - v'(\theta)\big) < 0.$$

Finally, we verify that the proposed trading is such that θ stays within $[0, 1]$, which boils down to showing $\dot{\theta}(0) \geq 0$ and $\dot{\theta}(1) \leq 0$. First, inserting $\theta = 0$ into (B.10), we note that

$\dot{\theta}(0) = 0$. Second, we can calculate

$$\begin{aligned}\dot{\theta}(1) &= \frac{1}{p'(1)}\left[\frac{\lambda(1-\lambda)}{\kappa} + (\rho - r)(p(\theta) - y(\theta)) - \pi - \eta(1) + \underbrace{\mu i'(1)\,(y(1) - p(1))}_{\leq 0}\right] \\ &\leq \frac{1}{p'(1)}\left[\frac{\lambda(1-\lambda)}{\kappa} + (\rho - r)\sqrt{\frac{(1-\lambda^2-\kappa\pi) + 2\,\alpha\,\kappa}{\kappa\mu^2}} - \pi - \eta(1)\right] < 0,\end{aligned}$$

where we used (B.5) to transition from the first to the second line, and we used (B.1) for the last inequality. In addition, we used that

$$\sqrt{\frac{\theta(1-\lambda^2-\kappa\pi) + 2\,\alpha\,\kappa\, + y(\theta)^2\,\kappa\,\mu^2 - 2y(\theta)\,\kappa\,(r+\delta)}{\kappa\mu^2}} \leq \sqrt{\frac{(1-\lambda^2-\kappa\pi) + 2\,\alpha\,\kappa}{\kappa\mu^2}}.$$

To see this, define

$$\mathcal{B}(y) := \frac{\theta(1-\lambda^2-\kappa\pi) + 2\,\alpha\,\kappa\, + y^2\,\kappa\,\mu^2 - 2y\,\kappa\,(r+\delta)}{\kappa\mu^2}.$$

Observe that $\mathcal{B}(0) > 0$ and $\mathcal{B}'(y) \propto -(r+\delta-\mu^2 y)$. For $y \leq v'(\theta)$, we get $r+\delta-\mu^2 y > r+\delta-\mu i$, which is positive under the assumption of finite valuations (see (9)).

B.6 Optimality of Smooth Trading and Uniqueness

We show that lumpy trades are strictly sub-optimal, in that

$$0 = \arg\max_{\Delta}\,[v(\theta+\Delta) - \Delta p(\theta+\Delta)] \tag{B.11}$$

for any $\Delta \in [-\theta, 1-\theta]$. The derivative with respect to Δ of the term in square brackets reads $\mathcal{G}(\Delta) := v'(\theta+\Delta) - p(\theta+\Delta) - \Delta p'(\theta+\Delta)$. We have $p(\theta+\Delta) = v'(\theta+\Delta)$, so $\mathcal{G}(\Delta) = -\Delta v''(\theta+\Delta)$. Condition (B.11) holds if $v''(\theta) > 0$ holds for all $\theta \in (0,1)$. We have shown that $v(\theta)$ is strictly convex, establishing the optimality of smooth trading. Indeed, given $v''(\theta) > 0$, it follows that smooth trading dominates lumpy trading in any given state, also verifying our conjecture that $\mathcal{S} = [0,1]$ holds in our equilibrium.

Uniqueness of the equilibrium—within the class of continuous, scaled Markov equilibrium with smooth trading—follows from the uniqueness of the solution $v(\theta)$ to (B.5) subject to (B.6) — which must hold in any continuous, scaled Markov equilibrium with $\mathcal{S} = [0,1]$ — combined with the pricing condition $p(\theta) = v'(\theta)$.

Thus, $v(\theta)$ and $p(\theta)$ are uniquely pinned down in continuous, scaled Markov equilibrium with smooth trading (which implies $\mathcal{S} = [0,1]$). As we have shown previously, given $v(\theta)$ and $p(\theta)$, the trading rate is uniquely pinned down too, whereby trading is such that $\dot{\theta}(0) \geq 0$ and $\dot{\theta}(1) < 0$, i.e., the equilibrium trading is consistent the constraint $\theta \in [0,1]$.

Thus, a continuous, scaled Markov equilibrium with smooth trading exists and is unique.

B.7 Proof of Corollary 2

First, the assumptions imply that under investor control over investment, the trading rate satisfies according to (31):

$$\dot{\theta} = \frac{1}{p'(\theta)} \left[\theta \mu i'(\theta) \left(y(\theta) - p(\theta)\right)\right] < 0.$$

Thus, $\lim_{t\to\infty} \theta_t = \hat{\theta}$ as well as $\lim_{t\to\infty} p(\theta_t) = p(\hat{\theta})$ for some value $\hat{\theta} \in [0, \theta_0]$. Suppose, to the contrary, that $\hat{\theta} > 0$. Then, $\lim_{\theta\downarrow\hat{\theta}} \dot{\theta} = 0$ which implies $\lim_{\theta\downarrow\hat{\theta}} y(\theta) = \lim_{\theta\downarrow\hat{\theta}} p(\theta)$, which contradicts that $y(\theta) < p(\theta)$ on $(0, 1)$. Therefore, $\lim_{t\to\infty} \theta_t = 0$, which implies—by virtue of (B.6)—that $\lim_{t\to\infty} p(\theta_t) = \hat{v}'(0)$.

Now, consider the baseline model. that the blockholder obtains the baseline, so the trading rate in the continuous, scaled Markov equilibrium is given in (23). Under our assumptions $\pi = \lambda = \rho - r = 0$, (23) readily implies $\dot{\theta} = 0$, so that θ_t and $p(\theta_t)$ remain constant at $\theta_t = \theta_0$ and $p(\theta_t) = p(\theta_0)$, where $p(\theta_0) > p(0) = p^0$.

C Risky Debt

Suppose that the firm experiences a downward jump in capital stock at Poisson rate Λ, in that K_t follows

$$\frac{dK_t}{K_{t^-}} = (\mu i_t - \delta)dt - (1 - S_t)dN_t^K,$$

where $K_{t^-} = \lim_{s\uparrow t} K_s$ denotes the left-limit of capital, i.e., the capital stock at time t just before the shock to capital is realized. Here, $dN_t^K \in \{0, 1\}$ is the increment of a Poisson process with intensity $\Lambda = \frac{\mathbb{E}[dN_t^K]}{dt} > 0$ and S_t (or equivalently $1 - S_t$) is uniformly distributed over $[0, 1]$ and i.i.d across time t. That is, each time t, there is a new random draw of S_t from $U([0, 1])$ which is not observable unless $dN_t^K = 1$ and in particular not observable at time t^-. Instantaneous cash flows equal $K_{t^-}dX_t$. Intuitively, a capital shock over $[t, t + dt)$ does not affect current cash flows dX_t, but "next-period" cash flows.

Throughout, we assume that the blockholder chooses debt and investment. Furthermore, we assume that the large capital shock, specifically dN_t^K and S_t are not contractible with management; they are also not influenced by management, which only affects the cash flows $K_{t^-}dX_t$. That is, the manager's contract takes the same form as in the baseline, featuring a base wage c_t and an exposure β_t to cash flow realizations. By this assumption, we rule out the possibility that the firm could write an insurance contract with management in which the manager provides a large cash payment to repay debt following the shock—such a contract is counterfactual. Thus, the manager's contract $\mathcal{C}_t = (c_t, \beta_t)$ is only contingent on cash flows $K_{t^-}dX_t$. We already use that the flow wage equals zero, i.e., $c_t = 0$, and that the sensitivity satisfies $\beta_t = \lambda$.

In the following, we omit time subscripts and the left-limit notation, unless necessary. Specifically, fraction $1 - S$ of the capital is destroyed, where S is uniformly distributed on $[0, 1]$. Hence, when pre-shock capital stock equals K, then post-shock capital stock equals SK. In the event that the firm defaults following this shock, creditors recover the firm's liquidation value or RSK, where $R < p(0)$ is sufficiently small to ensure that liquidation is inefficient and creditors cannot be repaid in full in the event of default in equilibrium. See Section C.6 for a characterization of $p(0)$—since $p(0)$ is generally not available in closed-form, there is unfortunately no explicit condition on parameters that ensures $R < p(0)$.

The heuristic timing of the model with large shocks to capital within a time interval $[t, t+dt)$ is as follows. Initially, the blockholder's stake equals θ_t and capital stock equals K_{t^-}. First, at the beginning of $[t, t+dt)$, given a blockholder stake θ_t, shareholders choose the managerial contract $\mathcal{C}_t = (c_t, \beta_t)$, the investment rate $i_t \geq 0$, and the amount of short-term debt L_t, where we assume that the proceeds from debt issuance are distributed as dividends (one can net out these dividends with the debt repayment of the previous instant). Second, the blockholder chooses its effort; then dN_t realized and, observing dN_t, the manager chooses diversion m_t. Then, cash flows dX_t and capital shock $dN_t^K(1-S_t)$ are realized; the manager receives its promised payments. At this point in time, capital stock equals $K_{t^-}\big[1-(1-S_t)dN_t^K\big]$. Third, at the end of $[t, t+dt)$, debt matures and shareholders repay debtholders $(1+r_t dt)L_t$, where r_t is the endogenous interest rate, or default, in which case it is liquidated and the blockholder's continuation payoff and firm value become zero. In case of no default, cash flows net of investment cost, managerial compensation, and debt repayment are distributed as dividends to shareholders. Fourth, after cash flows and capital shock are realized, the blockholder can trade and chooses $d\theta_t$, determining the next-period stake $\theta_{t+dt} = \theta_t + d\theta_t$. Notably, the blockholder can trade just before or just after debt is repaid. Finally, investment and deprecation materialize, so $K_{t+dt} = K_{t^-}\big[1-(1-S_t)dN_t^K\big] + K_{t^-}\big[\mu i_t - \delta\big]dt$.[17]

We characterize a continuous, scaled Markov equilibrium with state variables $(K_{t^-}, \theta_{t^-}) = (K, \theta)$, where $V_{t^-} = K_{t^-}v(\theta_{t^-})$, $P_{t^-} = K_{t^-}p(\theta_t)$, and $L_{t^-} = K_{t^-}\ell(\theta_t)$—we omit time subscripts unless necessary. For brevity, we omit a formal proof of existence and uniqueness and sketch the arguments here. However, one could adapt our baseline arguments to prove the existence and uniqueness of a scaled, continuous Markov equilibrium, as well as to establish the optimality of smooth trading.

Following the same arguments as in the baseline, we obtain $v'(\theta) = p(\theta)$ as well as the strict convexity of $v(\theta)$, i.e., $v''(\theta) > 0$ at points of differentiability, as necessary conditions for the optimality of smooth trading. Moreover, we use that $y = y(\theta) = \frac{v(\theta)}{\theta}$ increases in θ, i.e., $\theta v'(\theta) - v(\theta) > 0$. Likewise, we already use that optimal investment satisfies $\frac{\mu v(\theta)}{\theta}$, as well as that blockholder effort satisfies $b = \frac{\theta(1-\lambda)}{\kappa}$. All these properties can be proven by repeating the same arguments as in the baseline; we omit them for the sake of brevity.

C.1 Default and Credit Spreads

We start by characterizing endogenous default and credit spreads. Consider the firm just after it has issued scaled debt of ℓ in state (K, θ). When a shock of size S realizes, the blockholder's (unscaled) payoff drops from $K(v(\theta) - \ell)$ pre-shock to $K\big[Sv(\theta) - \theta\ell\big]$ post-shock. When the blockholder's post-shock payoff $K\big[Sv(\theta) - \theta\ell\big]$ is negative, i.e., $\frac{Sv(\theta)}{\theta} < \ell$, the

[17] The change in the capital stock due to investment and depreciation is of order dt and its effect on other quantities of order dt therefore negligible. As such, the exact timing of when the capital stock is updated does not matter for trading or default. For convenience and to avoid tedious notation, we assume it occurs at the very end of the period.

firm defaults following the shock. Analogous to the reasoning in Lemma 1, which shows that the firm defaults whenever the blockholder's payoff is strictly negative, default following the shock occurs in one of two ways. Either the blockholder has the authority to force default, or it sells its entire stake (at a positive price), thereby reducing the firm's equity value below the level of outstanding debt and making default optimal for passive shareholders. Otherwise, when $\frac{Sv(\theta)}{\theta} \geq \ell$, the firm does not default following a shock.

Recall the definition (16), that is, $y(\theta) := \frac{v(\theta)}{\theta}$. In the Appendix, we may occasionally suppress the dependence of y on θ. As argued above, following a shock with size S, the firm defaults if and only if

$$Sy(\theta) < \ell = \ell(\theta) \quad \Longleftrightarrow \quad S < \frac{\ell(\theta)}{y(\theta)}.$$

Conditional on a shock occurring, the probability of default is therefore given by $\Delta := \frac{\ell}{y}$—which is a function of θ (dependence suppressed for convenience). Thus, the probability of default over an instant $[t, t + dt)$ is $\Lambda\Delta dt$.

The recovery value in default following a size S-shock is RKS. Conditional on default, i.e., $S < \frac{\ell}{y}$, creditors recover in expectation $\frac{\ell}{2y}RK$ dollars, i.e., $\frac{R}{2y}$ dollars per unit of debt (with face value of one dollar). When risk-neutral creditors with discount rate r lend one dollar, they require an expected repayment at time $t + dt$ of $1 + rdt$ dollars, in that

$$1 + rdt = \Lambda\Delta dt \cdot \frac{R}{2y} + (1 - \Lambda\Delta dt)(1 + \hat{r}dt),$$

where $\hat{r}$ is the endogenous interest rate on debt.

To understand the above equality, note that over $[t, t + dt)$, the creditors are repaid the dollar plus interest $\hat{r}dt$ in case the firm does not default, which happens with probability $1 - \Lambda\Delta dt$. With probability $\Lambda\Delta dt$, the firm defaults and creditors recover in expectation $\frac{R}{2y} < 1$ dollars per dollar lent. We can solve the above expression for the fair interest rate

$$\hat{r} := r + \Lambda\Delta \left(1 - \frac{R}{2y}\right),$$

which coincides with (32). Therefore, the credit spread equals $\hat{r} - r = \Lambda\Delta \left(1 - \frac{R}{2y}\right)$.

C.2 Payoffs

The following derivations turn out to be convenient in characterizing the blockholder's payoff function. Conditional on a shock occurring and no default, the average scaled continuation payoff post-shock equals

$$\frac{v(\theta)}{2} \left(\frac{\ell}{y} + 1\right) - \theta\ell = \frac{v(\theta)}{2}(1 + \Delta) - \theta\ell = \frac{\theta y}{2}(1 + \Delta) - \theta\ell$$

for the blockholder. It equals

$$\frac{p(\theta)}{2}\left(\frac{\ell}{y}+1\right)-\ell=\frac{p(\theta)}{2}(1+\Delta)-\ell$$

for the passive investors. Given a level of debt ℓ, we can calculate the expected continuation payoff for the blockholder conditional on a shock occurring as:

$$\begin{aligned}E^b(\theta) &:= \mathbb{E}_t^S\left[\max\{0, Sv(\theta)-\theta\ell\}\middle|dN_t=1;\theta_t=\theta\right] \\ &= (1-\Delta)\left(\frac{v(\theta)}{2}(1+\Delta)-\theta\ell\right)=\left(\frac{\theta y}{2}\right)\left[1-\Delta^2\right]-\ell(1-\Delta),\end{aligned}$$

where the $\mathbb{E}_t^S$ is taken with respect to the random variable S *conditional* on $dN_t^K=1$. Likewise, the expected continuation payoff for the passive shareholders, i.e., the expected stock price, conditional on a shock occurring becomes

$$\begin{aligned}E^p(\theta) &:= \mathbb{E}_t^S\left[\left(Sp(\theta)-\ell\right)\mathbb{I}\{Sv(\theta)\geq\theta\ell\}\middle|dN_t=1;\theta_t=\theta\right] \\ &= (1-\Delta)\left(\frac{p(\theta)}{2}(1+\Delta)-\ell\right)=\left(\frac{p(\theta)}{2}\right)\left[1-\Delta^2\right]-\ell(1-\Delta),\end{aligned} \tag{C.1}$$

where the indicator $\mathbb{I}\{Sv(\theta)\geq\theta\ell\}$ equals one if and only if the firm does not default following the shock, that is, if and only if $S\geq\frac{\ell}{y}$.

We heuristically derive the blockholder's HJB equation under optimal debt and investment choice, starting in state (K,θ) with zero debt outstanding. Recall that in optimum $i=\mu y$. Importantly, in equilibrium, gains from trade are zero and $v'(\theta)=p(\theta)$, so the blockholder's payoff is "as if" it could not trade and θ remains constant. Thus, consider the hypothetical scenario that θ remains constant over $[t,t+dt)$. First, recall that the firm receives a capital shock with probability Λdt, in which case it defaults with probability Δdt and does not default otherwise. The scaled payoff from issuing $K\ell$ dollars of debt maturing at $t+dt$, with proceeds paid out as dividends, satisfies:

$$\begin{aligned}Kv(\theta) &= \theta K\ell + K\left(\theta\left(\alpha+b-\frac{i^2}{2}\right)-\frac{\pi\theta^2}{2}-\frac{\kappa b^2}{2}\right)dt \\ &\quad + e^{-\rho dt}(1-\Lambda dt)\left\{[K+(\mu i-\delta)dt]v(\theta)-\theta K\ell(1+\hat{r}dt)\right\} \\ &\quad + e^{-\rho dt}\Lambda(1-\Delta)dt\Big\{K[\tilde{S}+(\mu i-\delta)dt]v(\theta)-\theta K\ell(1+\hat{r}dt)\Big\},\end{aligned}$$

where $\tilde{S}:=\mathbb{E}\left[S\middle|S\geq\frac{\ell}{y}\right]=\frac{1}{2}+\frac{\ell}{2y}$ for $y=\frac{v(\theta)}{\theta}$, $\hat{r}=r+\Lambda\Delta\left(1-\frac{R}{2y}\right)$, and $\Delta=\frac{\ell}{y}$.

Taking the limit $dt\to 0$, ignoring terms of order $(dt)^2$, and doing some algebra, we can derive that the blockholder's value function solves

$$(\rho+\delta+\Lambda-\mu i)v(\theta)=\theta\left(\alpha+b-\frac{i^2}{2}+(\rho+\Lambda-\hat{r})\ell\right)-\frac{\pi\theta^2}{2}-\frac{\kappa b^2}{2}+\Lambda E^b(\theta).$$

Note that the shock arrival rate Λ effectively augments the blockholder's discount rate, i.e., the "effective" discount rate becomes $\rho+\Lambda$. Inserting the interest rate $\hat{r}$ from (32) into above equation, we can rearrange to obtain for the blockholder's value function:

$$(\rho+\delta-\mu i+\Lambda)v(\theta) = \theta\left(\alpha+b+(\rho-r)\ell-\frac{i^2}{2}\right)-\frac{\pi\theta^2}{2}-\frac{\kappa b^2}{2}+\frac{\theta\Lambda}{2}\big[y-\Delta^2(y-R)\big], \tag{C.2}$$

given the controls.

C.3 Optimal Debt Choice

Taking the derivative of the right-hand-side of (C.2) with respect to ℓ, we get

$$\frac{\partial v(\theta)}{\partial \ell} \propto \theta(\rho-r)-\frac{\theta\Lambda\Delta(y-R)}{y} \propto \frac{(\rho-r)}{\Lambda}-\frac{\ell(y-R)}{y^2},$$

where the proportionality sign $\propto$ indicates the omission of positive scaling constants. If $\ell<\frac{v(\theta)}{\theta}$, i.e. the borrowing constraint (5) does not bind, the optimal choice of debt ℓ solves $\frac{\partial v(\theta)}{\partial \ell}=0$. As such, we can solve for

$$\ell=\ell(\theta)=\min\left\{y,\frac{(\rho-r)y^2}{\Lambda(y-R)}\right\}, \tag{C.3}$$

where, as we recall, $y=y(\theta)=\frac{v(\theta)}{\theta}$. We also note that $y=y(\theta)\geq p(0)=p^0$, so the denominator of above expression is strictly positive.

As such, the default probability, conditional on a shock occurring, becomes

$$\Delta(\theta)=\frac{\ell(\theta)}{y(\theta)}=\min\left\{1,\frac{(\rho-r)y}{\Lambda(y-R)}\right\}$$

C.4 Stock Price and Trading Rate

The stock price satisfies the pricing equation

$$(\rho+\delta+\Lambda-\mu i)p(\theta) = \alpha+b-\pi\theta-\frac{i^2}{2}+(\rho+\Lambda-\hat{r})\ell-\frac{\kappa b^2}{2}+\Lambda E^p(\theta),$$

where $E^p(\theta)$ is the expected continuation stock price conditional on a shock occurring from (C.1). Here, Λ augments the discount rate of passive shareholders. Upon a shock occurring with probability Λ, the passive shareholders realize expected continuation stock price $E^p(\theta)$. We can rewrite the pricing equation as follows:

$$\begin{aligned}(\rho+\delta+\Lambda)p(\theta) = &\alpha+b+(\rho-r)\ell-\frac{i^2}{2}+\mu i p(\theta)+\eta(\theta)\\ &+\frac{\Lambda\big[p(\theta)-\Delta^2(p(\theta)-R)\big]}{2}+p'(\theta)\dot{\theta}.\end{aligned} \tag{C.4}$$

Next, we derive the blockholder's valuation of an additional unit of stock, i.e., $v'(\theta)$, which, in equilibrium, must equal the stock price $p(\theta)$. We distinguish two cases, (i) $\ell < 1$ and (ii) $\ell = y$.

Borrowing Constraint $\ell \leq y(\theta)$ is slack. First, suppose that $\ell < y = y(\theta)$, which implies $\Delta < 1$. Then, the choice of debt ℓ solves the first-order condition $\frac{\partial v(\theta)}{\partial \ell} = 0$, and so does investment, i.e., $\frac{\partial v(\theta)}{\partial i} = 0$. We now invoke the envelope theorem and differentiate both sides of (C.2) with respect to θ. After some algebra, we obtain that the marginal valuation of an additional unit of stock from the blockholder perspective satisfies $v'(\theta) = p(\theta)$ with

$$(\rho + \delta + \Lambda)p(\theta) = \alpha + b - \pi\theta + (\rho - r)\ell - \frac{i^2}{2} + \mu i p(\theta) + \frac{\theta\lambda(1-\lambda)}{\kappa} + \frac{\Lambda\big[p(\theta) - \Delta^2(p(\theta) - R)\big]}{2} + \theta\Lambda\frac{y'(\theta)\ell^2}{y^3}(y - R)\,.$$

The last term $\theta\Lambda\frac{y'(\theta)\ell^2}{y^3}(y-R)$ is strictly positive due to $y'(\theta) > 0$ and can be rewritten as follows:

$$\theta\Lambda\frac{y'(\theta)\ell^2}{y^3}(y-R) = \theta\Lambda\Delta^2\frac{y'(\theta)[y-R]}{y} = \theta y'(\theta)\left(\frac{(\rho-r)^2 y}{\Lambda(y-R)}\right) = \theta(\rho - r)y'(\theta)\Delta.$$

Borrowing Constraint $\ell \leq y(\theta)$ is binding. Second, consider $\ell = \ell(\theta) = y(\theta)$, i.e., $\Delta = 1$. Then, the choice of debt is constrained by the borrowing constraint $\ell \leq y$ and does not solve a first-order condition. Again, we differentiate both sides of (C.2) with respect to θ. We then obtain that the marginal valuation of an additional unit of stock from the blockholder perspective satisfies $v'(\theta) = p(\theta)$ with

$$(\rho + \delta + \Lambda)p(\theta) = \alpha + b - \pi\theta + (\rho - r)\ell - \frac{i^2}{2} + \mu i p(\theta) + \frac{\theta\lambda(1-\lambda)}{\kappa} + \frac{\Lambda\big[p(\theta) - \Delta^2(p(\theta) - R)\big]}{2} + \theta(\rho - r)\ell'(\theta).$$

Due to $\ell(\theta) = y(\theta)$ whenever $\Delta = 1$, we have $\theta(\rho - r)\ell'(\theta) = \theta(\rho - r)y'(\theta)\Delta = (\rho - r)[p(\theta) - y(\theta)] > 0$.

In both scenarios, we can combine above equations for $p(\theta) = v'(\theta)$ with (C.4) to solve for the trading rate:

$$\dot{\theta} = \frac{1}{p'(\theta)}\left[\frac{\theta\lambda(1-\lambda)}{\kappa} - \pi\theta - \eta(\theta) + \theta(\rho - r)y'(\theta)\Delta\right]. \tag{C.5}$$

Here, $\theta(\rho - r)y'(\theta)\Delta = (\rho - r)\Delta\big[p(\theta) - y(\theta)\big] > 0$ captures the gains from trade associated with risky debt.

C.5 Dynamics of Debt, Default Probabilities, and Credit Spreads

Suppose that in state (K, θ), optimal debt satisfies $\ell = \frac{(\rho-r)y(\theta)^2}{\Lambda(y(\theta)-R)} < y$. Then,

$$\ell'(\theta) = \frac{y'(\theta)(\rho - r)}{\Lambda} \frac{2y(y-R) - y^2}{(y-R)^2} = \frac{y'(\theta)(\rho - r)}{\Lambda} \frac{y(y-2R)}{(y-R)^2}.$$

If $R \leq \frac{p(0)}{2}$, $y - 2R$ is unambiguously positive and debt ℓ increases with θ. Otherwise, when $\frac{p(0)}{2} < R < p(0)$, then there exists a right-neighborhood of 0 in which $\ell'(\theta)$. Then, $\ell(\theta)$ is generally U-shaped in θ, i.e., first decreases and then increases in θ.

Next, the default probability $\Delta(\theta)$ satisfies

$$\Delta'(\theta) = \frac{y'(\theta)}{\Lambda} \frac{(y(\theta) - R)(\rho - r) - (\rho - r)y(\theta)}{(y(\theta) - R)^2} = -R\frac{y'(\theta)}{\Lambda} \frac{(\rho - r)}{(y(\theta) - R)^2} < 0.$$

Thus, the default probability decreases with θ.

Finally, calculate the credit spread

$$\hat{r} - r = \frac{(\rho - r)y}{y - R} - \frac{R(\rho - r)}{2(y - R)} = (\rho - r)\frac{2y - R}{2(y - R)}$$

which decreases with θ.

C.6 Passive Ownership Benchmark

Under (perpetual) passive ownership, the stock price satisfies $Kp(0)$ and investment satisfies $i = i^0 \mu p(0)$, as in the baseline. Debt issuance $\ell = \ell^0$ is subject to the borrowing constraint $\ell \leq p(0)$, which ensures that equity value remains positive and precludes immediate default. Moreover, following a capital shock of size S, the firm defaults under passive ownership whenever $Sp(0) < \ell$, leading to the probability of default $\Delta = \Delta^0 = \frac{\ell}{p(0)}$.

Analogously to the pricing equation (C.4)—which holds for $\theta > 0$—the scaled passive-ownership price satisfies

$$(\rho + \delta + \Lambda)p(0) = \max_{\ell} \left(\alpha + (\rho - r)\ell - \frac{i^2}{2} + \mu i p(0) + \frac{\Lambda\big[p(0) - \Delta^2(p(0) - R)\big]}{2} \right). \quad \text{(C.6)}$$

It can be obtained similar to (C.4) upon setting $\dot{\theta} = 0$, $\eta(\theta) = 0$, and $\theta = 0$, while optimizing the right-hand-side over debt.

Debt choice satisfies under passive ownership solves the first-order condition (if interior):

$$(\rho - r) - \frac{\Lambda\ell(p(0) - R)}{(p(0))^2} = 0,$$

leading to

$$\ell = \ell^0 = \min\left\{p(0), \frac{(\rho - r)(p(0))^2}{\Lambda(p(0) - R)}\right\}$$

One can insert this expression for $\ell = \ell^0$ back into (C.6), but there is no closed-form solution for $p(0)$; one needs to solve $p(0)$ numerically. Overall, $p(0)$ will be a function of R. We impose $p(0) > R$, which leads to an implicit parameter condition on R.

Note that $p(0) > R$ implies that in equilibrium, the recovery value is insufficient to repay creditors in full in the event of default, making debt is risky. To see this, note that the firm default in state $\theta = 0$ whenever $S < \frac{\ell^0}{p(0)}$. Upon default, creditors then recover SRK dollars. Due to $SR < Sp(0) < \ell^0$ — where we used $R < p(0)$ and $S < \frac{\ell^0}{p(0)}$ — this amount is insufficient to repay creditors in full. Likewise, in state θ, the firm defaults whenever $S < \frac{\ell}{y}$, in which case creditors recover SRK dollars. Due to $SR < \frac{p(0)}{y}\ell < \ell$, this amount is insufficient to repay creditors in full.

C.7 Investor Control

We now briefly discuss how the solution changes when passive investors are in control of corporate policies, allowing them to choose both investment and debt. In a continuous, scaled Markov equilibrium with smooth trading, we have $p(\theta) = v'(\theta)$. Under investor control, debt and investment are determined according to:

$$(\rho + \delta + \Lambda)p(\theta) = \max_{i,\ell} \Bigg\{ \alpha + b + \eta(\theta) + (\rho - r)\ell - \frac{i^2}{2} + \mu i p(\theta) + \frac{\Lambda\big[p(\theta) - \Delta^2(p(\theta) - R)\big]}{2} + p'(\theta)\dot{\theta} \Bigg\}. \tag{C.7}$$

where $\Delta = \frac{\ell}{y}$ and $\ell \leq y$. This leads to $i = i^I = \mu p(\theta) = \mu v'(\theta)$. Debt choice satisfies under investor control the first-order condition (if interior)

$$(\rho - r) - \frac{\Lambda \ell [p(\theta) - R]}{y^2} = 0,$$

which we solve for

$$\ell = \ell^I = \min\left\{ y, \frac{(\rho - r)y^2}{\Lambda[p(\theta) - R]} \right\}.$$

Given these policies, the blockholder value function satisfies (C.2), that is:

$$(\rho + \delta - \mu i + \Lambda)v(\theta) = \theta\left(\alpha + b + (\rho - r)\ell - \frac{i^2}{2}\right) - \frac{\pi\theta^2}{2} - \frac{\kappa b^2}{2} + \frac{\theta\Lambda}{2}\big[y - \Delta^2(y - R)\big],$$

where the default probability $\Delta = \Delta^I = \frac{\ell^I}{y}$ is a non-linear function of $v'(\theta) = p(\theta)$, provided $\ell^I < y$; $\ell(\theta) = \ell^I(\theta)$ and $i(\theta) = i^I(\theta)$ are functions of θ too. This ODE is solved subject to $\lim_{\theta \to 0} \frac{v(\theta)}{\theta} = p(0)$ — with $p(0)$ characterized in the passive ownership benchmark.

The determination of the continuous, scaled Markov equilibrium becomes similar to that in Section 3, but the ODE is now significantly more complicated and cannot even be explicitly solved for $v'(\theta)$. We leave the further analysis of this model variant, acknowledging that it would be tedious.

Trading Rate with Investor Control. Nonetheless, we can solve for the trading rate, without further characterizing the ODE for $v(\theta)$ or having a closed-form solution for $v(\theta)$ or $p(\theta)$.

For this sake, we assume that the borrowing constraint does not bind, in that $\ell^I < y(\theta)$. Then, we can differentiate both sides of (C.2) with respect to θ to obtain that $p(\theta) = v'(\theta)$ satisfies:

$$\begin{aligned}(\rho + \delta + \Lambda - \mu i)p(\theta) = \alpha + b - \pi\theta + (\rho - r)\ell - \frac{i^2}{2} + \frac{\theta\lambda(1-\lambda)}{\kappa} \\ + \frac{\Lambda\big[p(\theta) - \Delta^2(p(\theta) - R)\big]}{2} + \theta\Lambda\frac{y'(\theta)\ell^2}{y^3}(y - R) \\ + (\rho + \delta + \Lambda - \mu i)\Bigg(\underbrace{\frac{\partial v(\theta)}{\partial \ell}\frac{\partial \ell^I}{\partial \theta}}_{>0}\Bigg) + \underbrace{i'(\theta)\theta\mu(y(\theta) - p(\theta))}_{<0}.\end{aligned}$$

The term $\theta\Lambda\frac{y'(\theta)\ell^2}{y^3}(y - R)$ satisfies

$$\theta\Lambda\frac{y'(\theta)\ell^2}{y^3}(y - R) = \theta\Lambda\Delta^2\frac{y'(\theta)[y - R]}{y} = \theta y'(\theta)\left(\frac{(\rho - r)^2 y}{\Lambda(y - R)}\right) = \theta(\rho - r)y'(\theta)\Delta.$$

Combining these with (C.7), we can solve for the trading rate through $\dot{\theta} = \frac{\mathcal{A}(\theta)}{p'(\theta)}$ where

$$\begin{aligned}\mathcal{A}(\theta) = \frac{\theta\lambda(1-\lambda)}{\kappa} - \pi\theta - \eta(\theta) + \theta(\rho - r)y'(\theta) \\ + (\rho + \delta + \Lambda - \mu i)\Bigg(\underbrace{\frac{\partial v(\theta)}{\partial \ell}\frac{\partial \ell^I}{\partial \theta}}_{>0}\Bigg) + \underbrace{i'(\theta)\theta\mu(y(\theta) - p(\theta))}_{<0}.\end{aligned}$$

To gain some intuition, note that if passive investors are in control, they choose too high investment $i = i^I = \mu p(\theta)$ and too low debt $\ell = \ell^I$ from the blockholder's perspective. Therefore, a marginal increase in investment (debt) reduces (increases) the blockholder's value function, in that $\frac{\partial v(\theta)}{\partial i} < 0$ and $\frac{\partial v(\theta)}{\partial \ell} > 0$. The blockholder internalizes that when increasing its ownership stake marginally, it induces passive shareholders to choose higher levels of investment and debt, which affects its own payoff. This effect influences the blockholder's valuation of an additional unit of equity, and therefore the gains from trade and trading rate $\dot{\theta}$. Consequently, as also shown in Section 3, delegating investment decisions to passive shareholders reduces the blockholder's propensity to acquire additional shares and encourages exit. In contrast, delegating debt decisions to passive shareholders increases the blockholder's propensity to acquire additional shares.

D Other Results—Details and Proofs

D.1 Large Trades and Proof of Proposition 6

We characterize a scaled Markov equilibrium where $P(K, \theta) = p(\theta)$ and $V(K, \theta) = Kv(\theta)$. The only difference from the baseline is that (11) is not met, while all other elements remain unchanged. We note that when (11), it is not possible to construct a scaled Markov equilibrium where the price function is continuous. Instead, we will construct a Markov equilibrium where $p(\theta)$ exhibits one discontinuity at the endogenous threshold θ^*. Here, we note that the results from Appendix Section A.4.1 also apply in this context, since this Appendix establishes generalized equilibrium properties that hold in any scaled Markov equilibrium, i.e., so long as $P(K, \theta) = Kp(\theta)$ and $V(K, \theta) = Kv(\theta)$.

All state dependent controls $b = b(\theta)$, the managerial contract, $i = i(\theta)$, and $\ell = \ell(\theta)$ are analogous to the baseline.

D.1.1 Preliminaries

In the following, it will be convenient to work with $\hat{p}(\theta)$ defined via

$$\hat{p}(\theta) := \frac{\alpha + b(\theta) + (\rho - r)\ell(\theta) - \frac{i(\theta)^2}{2} + \eta(\theta)}{\rho + \delta - \mu i(\theta)}, \tag{D.1}$$

with $\ell(\theta) = \frac{\hat{v}(\theta)}{\theta}$, $i(\theta) = \frac{\mu \hat{v}(\theta)}{\theta}$, and $b = b(\theta)$ from (15). Note that $\hat{p}(\theta)$ is the hypothetical scaled price that would prevail if the blockholder perpetually maintained ownership θ.

One can show that whenever (11) does not hold, we have $\hat{v}'(1) \geq \hat{p}(1)$, i.e., by construction, the condition (11) is *equivalent* to $\hat{v}'(1) < \hat{p}(1)$.

Next, define

$$J_H(\theta) = \hat{v}(1) - (1 - \theta)\hat{p}(1) - \hat{v}(\theta),$$

where $\hat{p}(1)$ is the hypothetical price that prevails if the blockholder perpetually owns the entire firm with $\theta = 1$. Note that $J_H'(\theta) = \hat{p}(1) - \hat{v}'(\theta)$ and $J_H''(\theta) = -\hat{v}''(\theta) < 0$. Thus, $J_H'(1) < 0$. Since $J_H(1) = 0$, it follows that $J_H(\theta)$ has maximally one root on $(0, 1)$, while $J_H(\theta) < 0$ for $\theta > 1$.

We define

$$\theta^* = \inf\{\theta \geq 0 : J_H(\theta) \geq 0\} \tag{D.2}$$

Due to $J_H''(\theta) < 0$ and $J_H(1) = 0$ as well as $J_H'(1) < 0$, there exist a left-neighborhood of 1 on which $J_H(\theta) > 0$. Thus, $\theta^* \in (0, 1)$. If $J_H(\theta)$ has a root on $(0, 1)$, then θ^* is this root, i.e., $J_H(\theta^*) = 0$. Otherwise, $J_H(\theta) \geq 0$ for all $\theta \in (0, 1)$ and $\theta^* = 0$.

We note that $\theta^* \in (0, 1)$ if and only if $J_H(0) < 0$, i.e., $\hat{v}(1) - \hat{p}(1) < 0$. If $J_H(0) \geq 0$, the equilibrium is trivial, with the blockholder immediately acquiring the entire firm and perpetually maintaining full ownership ($\theta = 1$).

We therefore assume $J_H(0) = \hat{v}(1) - \hat{p}(1) < 0$ in what follows to obtain an equilibrium with meaningful trading dynamics. This implies $\theta^* \in (0, 1)$.

D.1.2 Conjecturing the Equilibrium

We conjecture the following equilibrium, where the blockholder optimally trades smoothly on the interval $[0, \theta^*)$ according to rate $\dot{\theta} = \dot{\theta}(\theta)$ characterized in (23). The trading rate can be positive or negative, as in the baseline, with $\dot{\theta}(0) = 0$. On $[0, \theta^*]$, the (scaled) value function satisfies $v(\theta) = \hat{v}(\theta)$ and the price satisfies $p(\theta) = \hat{v}'(\theta)$.

When $\hat{v}'(\theta^*) > \hat{p}(\theta^*)$, the trading rate $\dot{\theta}(\theta)$ is positive in a left-neighborhood of θ^* and θ reaches θ^* from below, in that $\lim_{\theta \to \theta*} \dot{\theta}(\theta) > 0$. Once θ reaches θ^*, the blockholder is indifferent and randomizes between buying the entire firm at once (i.e., $d\theta = 1 - \theta^*$) and not trading at all (i.e., $d\theta = 0$). The rate at which the blockholder buys the entire is denoted $\gamma^* > 0$ and will be characterized later. The state θ remains at θ^* until the blockholder buys the entire firm.

When $\hat{v}'(\theta^*) \leq \hat{p}(\theta^*)$, the blockholder trades smoothly in state θ^* at rate $\dot{\theta}(\theta^*) \leq 0$. We then have $\lim_{\theta \to \theta*} \dot{\theta}(\theta) \leq 0$, and θ^* is either absorbing or θ drifts away and below θ^*.

The threshold θ^* satisfies $J_H(\theta^*)$, with $\theta^* \in (0, 1)$. At θ^*, we have $v(\theta^*) = \hat{v}(\theta^*)$ and $p(\theta^*) = \hat{v}'(\theta^*)$.

When $\theta = 1$, the blockholder stops trading and maintains perpetually full ownership of the firm, with value function $v(1) = \hat{v}(1)$ and price $p(1) = \hat{p}(1)$.

When $\theta \in (\theta^*, 1)$, the blockholder immediately buys the entire firm at price $\hat{p}(1)$, i.e., $d\theta = 1 - \theta$. The price equals $p(\theta) = \hat{p}(1)$ and the value function equals $v(\theta) = \hat{v}(1) - (1 - \theta)\hat{p}(1) > \hat{v}(\theta)$.

Thus, in this equilibrium, the value function is smooth and strictly convex on $[0, \theta^*)$ with $\hat{v}''(\theta) > 0$, while the price is continuous and increasing on this interval. At θ^*, the price exhibits an upward jump while the value function exhibits a kink.

For the following analysis, we recall from Appendix section A.4.1 that $v(\theta) = \hat{v}(\theta)$ on the set

$$\mathcal{S} = \{\theta \in [0, 1] : \dot{\theta} \in (-\infty, +\infty) \quad \text{and} \quad \gamma \in [0, \infty)\}$$

— defined analogously in (A.14).

In this model variant, the set $\mathcal{S}$ is conjectured as $\mathcal{S} \in [0, \theta^*] \cup \{1\}$ — with θ^* defined in (D.2). For $\theta \notin \mathcal{S}$, the blockholder finds it optimal to immediately conduct a lumpy trade, which, without loss of generality, brings θ into the set $\mathcal{S}$.

D.1.3 Optimality of the Trading Strategy

We verify the optimality of the proposed trading strategy. To do so, we show that, given the conjectured equilibrium price, the blockholder's scaled value function $v(\theta)$ solves the HJB equation (A.12). If $v(\theta)$ solves the HJB equation (A.12), it follows that the proposed trading strategy is optimal, since this strategy yields payoff $v(\theta)$. To show that $v(\theta)$ solves the HJB equation, we first need to characterize the optimal trading in different parts of the state space under the conjectured equilibrium price $p(\theta)$ and continuation payoff $v(\theta)$. Then, one can verify that the HJB equation holds by invoking the optimal trading strategy.

We start with preliminary findings, and distinguish the following cases and characterize the trading in different parts of the state space.

Result: Any Lumpy Trade must be onto the edges of $\mathcal{S}$. As a lumpy trade outside of $\mathcal{S}$ would be immediately followed by another lumpy trade, it is without loss of generality to assume that a lumpy trade brings the state θ into the set $\mathcal{S}$.

In the interior of $\mathcal{S}$, i.e., in $int(\mathcal{S})$, we have $v(\theta) = \hat{v}(\theta)$ and $p(\theta) = \hat{v}'(\theta)$. Consider state θ and suppose to the contrary that the blockholder conducts a trade from state θ toward state $\hat{\theta} \in int(\mathcal{S})$. While not trading at all yields continuation payoff $\hat{v}(\theta)$, the trade toward $\hat{\theta}$ yields continuation payoff

$$G(\hat{\theta}) = \hat{v}(\hat{\theta}) - (\hat{\theta} - \theta)\hat{v}'(\hat{\theta})$$

Note that $G(\theta) = \hat{v}(\theta)$, equal to the payoff of not trading at all. Calculate $G'(\hat{\theta}) = -(\hat{\theta} - \theta)\hat{v}''(\hat{\theta})$. Due to strict convexity of $\hat{v}(\theta)$ it follows that $G(\hat{\theta})$ is optimized for $\hat{\theta} = \theta$. Thus, a lumpy trade toward state $\hat{\theta} \in int(\mathcal{S})$ is strictly dominated by not trading at all. Hence, any lumpy trade must bring θ onto the edges of $\mathcal{S}$, i.e., $\hat{\theta} \in \{0, 1, \theta^*\}$

State $\theta \in (\theta^*, 1)$. Note that $\hat{v}(1) \leq v(1)$, as the blockholder always has the option not to trade at all in state $\theta = 1$ yielding a payoff $\hat{v}(1)$.

Next, suppose to the contrary that $\dot{\theta} \in (-\infty, \infty)$ and $\gamma \in [0, \infty)$ is optimal. Then, $\theta \in \mathcal{S}$ and $v(\theta) = \hat{v}(\theta)$. But, as $\theta \in (\theta^*, 1)$, we have $\hat{v}(\theta) < \hat{v}(1) + (1 - \theta)\hat{p}(1) \leq v(1) + (1 - \theta)\hat{p}(1)$ and the blockholder could attain strictly higher payoff through a lumpy trade toward 1, i.e., $d\theta = 1 - \theta$, a contradiction. As a consequence, $\theta \notin \mathcal{S}$.

Thus, the blockholder conducts a lumpy trade. According to our findings above, this lumpy trade brings θ onto the edge of $\mathcal{S}$, so the blockholder trades toward $\hat{\theta} \in \{0, 1, \theta^*\}$. Trading toward $\hat{\theta} = 1$ yields $v(1) - (1 - \theta)\hat{p}(1) \geq \hat{v}(1) + (1 - \theta)\hat{p}(1)$. Trading toward $\hat{\theta} = \theta^*$ yields

$$\begin{aligned}\hat{v}(\theta^*) + (\theta - \theta^*)p(\theta^*) &= \hat{v}(1) - (1 - \theta^*)\hat{p}(1) + (\theta - \theta^*)p(\theta^*)\\ &< \hat{v}(1) - (1 - \theta^*)\hat{p}(1) + (\theta - \theta^*)\hat{p}(1) \leq v(1) - (1 - \theta)\hat{p}(1),\end{aligned}$$

where we have used that $\hat{v}(\theta^*) = \hat{v}(1) - (1 - \theta^*)\hat{p}(1)$ and $p(\theta^*) < \hat{p}(1)$ for $\theta^* < 1$. Thus, trading toward $\hat{\theta} = 1$ strictly dominates trading toward $\hat{\theta} = \theta^*$.

A trade toward 0 yields payoff $\hat{p}(0)\theta < \hat{v}(\theta)$ and thus is strictly dominated by not trading at all.

In state $\theta \in (\theta^*, 1)$, immediately trading toward $\hat{\theta} = 1$, i.e., $d\theta = (1 - \theta)$, is thus strictly optimal, delivering payoff $v(\theta) = v(1) - (1 - \theta)\hat{p}(1)$. Inserting $\hat{\theta} = 1$ and $\gamma = +\infty$ into (A.12), we can see that (A.12) holds.

State $\theta = 1$. If the blockholder conducts a lumpy trade, then this trade is toward state $\hat{\theta} \in \{0, \theta^*\}$ on the edges of $\mathcal{S}$. Relative to not trading and collecting payoff $\hat{v}(1)$, a trade toward $\theta^* \in (0, 1)$ changes payoff by

$$\hat{v}(\theta^*) + (1 - \theta^*)p(\theta^*) - \hat{v}(1) < \hat{v}(\theta^*) + (1 - \theta^*)\hat{p}(1) - \hat{v}(1) = 0,$$

where we have used that $p(\theta^*) < \hat{p}(1)$ (for $\theta^* < 1$) and that, by definition of θ^*, $\hat{v}(\theta^*) + (1 - \theta^*)\hat{p}(1) - \hat{v}(1) = 0$ for $\theta^* \in (0, 1)$.

A trade toward $\hat{\theta} = 0$ delivers payoff $\theta\hat{p}(0) < \hat{v}(\theta)$, and thus is strictly suboptimal too.

Next, suppose to the contrary that smooth trading in state $\theta = 1$ is optimal. Thus, $\dot{\theta} < 0$. But, as we have shown, in any state $\theta \in (\theta^*, 1)$, it is strictly optimal to trade toward one. As such, smooth trading $\dot{\theta} < 0$ in state $\theta = 1$ cannot be, because it would be immediately followed by a lumpy trade toward one. As a result, state $\theta = 1$ is absorbing, yielding continuation payoff $\hat{v}(1)$, which solves the HJB equation (A.12) under $d\theta = 0$.

State $\theta \in [0, \theta^*)$. Any lumpy trade must bring θ onto the edges of $\mathcal{S}$, as shown before, i.e., onto $\{0, \theta^*, 1\}$. The value function on $[0, \theta^*)$ satisfies $v(\theta) = \hat{v}(\theta)$.

First, a lumpy trade toward zero yields payoff $\theta\hat{p}(0) < \hat{v}(\theta)$ (for $\theta > 0$), and thus is strictly suboptimal.

A lumpy trade toward θ^* yields payoff $\hat{v}(\theta^*) - (\theta^* - \theta)\hat{v}'(\theta^*) < \hat{v}(\theta)$.

By definition of θ^*, we have $\hat{v}(\theta) > \hat{v}(1) - (1-\theta)\hat{p}(1)$ for $\theta \in [0, \theta^*)$, hence a lumpy trade toward one is strictly suboptimal too.

This verifies that any lumpy trade in state $\theta \in [0, \theta^*)$ is strictly suboptimal. Thus, $\frac{d\theta}{dt} = \dot{\theta} \in (-\infty, +\infty)$ and $\gamma = 0$ are optimal, and $v(\theta) = \hat{v}(\theta)$ satisfies (A.12).

One can verify that in state $\theta = 0$, the blockholder stops trading, in that $\dot{\theta}(0)$ from (23) is zero. By our arguments, it is optimal to stop trading in state $\theta = 0$.

State $\theta = \theta^*$. In state $\theta = \theta^*$, the blockholder is indifferent between not trading, yielding payoff $\hat{v}(\theta^*)$, or buying the entire firm at once, yielding payoff $\hat{v}(1) - (1-\theta^*)\hat{p}(1)$. Thus, it is weakly optimal to randomize between these two options. This implies that it is optimal to set $\dot{\theta} \leq 0$, as $\dot{\theta} > 0$ is akin to an immediate lumpy trade. A strictly positive trading rate $\dot{\theta} > 0$ would bring θ into the region $(\theta^*, 1)$ which would trigger an immediate lumpy trade toward one. Thus, in state $\theta = \theta^*$, it is optimal for the blockholder to either (i) randomize between buying the entire firm and not trading (i.e., $\dot{\theta} = 0$ and $\gamma \in [0, \infty)$ with $\hat{\theta} = 1$) or (ii) trade smoothly at negative (finite) rate $\dot{\theta} < 0$ which also delivers payoff $\hat{v}(\theta^*)$. Either way, the blockholder's payoff in state θ^* is $\hat{v}(\theta^*)$, solving (A.12) under $\dot{\theta} = 0$ and $\gamma \geq 0$.

D.1.4 Trading Rate

We now determine trading rate and randomized trading to ensure that the conjectured price $p(\theta)$ under the blockholder's proposed trading is consistent with passive shareholders' valuation of the firm, thereby verifying that $p(\theta)$ is the scaled equilibrium price. Recall that the blockholder stops trading in states 0 and 1, as shown above. Further, it is strictly optimal to trade towards 1 when $\theta \in (\theta^*, 1)$, i.e., $d\theta(\theta) = (1-\theta)$ for $\theta \in (\theta^*, 1]$.

State $\theta \in [0, \theta^*)$. For $\theta \in [0, \theta^*)$, the optimality condition for trading (A.13) implies $v(\theta) = \hat{v}(\theta)$ and $p(\theta) = \hat{v}'(\theta)$. Furthermore, it is optimal to trade smoothly, i.e., $\gamma = 0$. We determine the trading rate as in the baseline to obtain $\dot{\theta}$ from (23); see Appendix A.4.2 for details. The trading rate is uniquely determined in this region under smooth trading.

State $\theta = \theta^*$. Consider state $\theta = \theta^*$. When $\hat{v}'(\theta^*) < \hat{p}(\theta^*)$, then $\dot{\theta}(\theta) < 0$ in a left-neighborhood of θ^* and $\lim_{\theta \to \theta*} \dot{\theta}(\theta) \leq 0$. Then, θ drifts into the interior of $[0, \theta^*]$ and no time is spent in state θ^*. In this case, the price satisfies $p(\theta) = \hat{v}'(\theta)$ and $\dot{\theta}(\theta)$ satisfies (23); there is no randomization over lumpy trading in that $\gamma = 0$. In the knife-edge case $\hat{v}'(\theta^*) = \hat{p}(\theta^*)$, we have $\dot{\theta} = \gamma = 0$ at $\theta = \theta^*$.

Next, consider that $\hat{v}'(\theta^*) > \hat{p}(\theta^*)$. Then, $\dot{\theta} = \dot{\theta}(\theta) > 0$ in a left-neighborhood of θ^*, in that θ reaches θ^* from below and $\lim_{\theta \to \theta*} \dot{\theta}(\theta) > 0$. Because we have in addition that $\dot{\theta} \leq 0$ at θ^* from (D.2), it must be that $\dot{\theta} = 0$ at θ^*. By definition of θ^*, the blockholder is indifferent between not trading at all and buying the entire firm at once in state θ^*. Once θ reaches one, the endogenous stock price becomes $p(1)$. As the blockholder stops trading once

θ reaches one, we have $p(1) = \hat{p}(1)$. Because $p(\theta) = \hat{v}'(\theta)$ on $[0, \theta^*)$ and because θ reaches θ^* from below, we have $p(\theta^*) = \hat{v}'(\theta^*)$.

At $\theta = \theta^*$ where $\dot{\theta} = 0$, the randomization rate γ^* is such that $p(\theta^*) = \hat{v}'(\theta^*)$. Here, $p(\theta^*)$ satisfies passive investors' pricing equation:

$$(\rho + \delta - \mu i(\theta^*))p(\theta^*) = \underbrace{\alpha + b + (\rho - r)\ell(\theta^*) - \frac{i(\theta^*)^2}{2} + \eta(\theta^*)}_{=\hat{p}(\theta^*)} + \gamma^*(\hat{p}(1) - p(\theta^*)),$$

which can be rewritten as $p(\theta^*) = \frac{(\rho+\delta-\mu i(\theta^*))\hat{p}(\theta^*)+\gamma^*\hat{p}(1)}{\rho+\delta-\mu i(\theta^*)+\gamma^*}$. As a consequence,

$$\begin{aligned} \gamma^* &= \frac{[\rho + \delta - \mu i(\theta^*)]\left[\hat{p}(\theta^*) - p(\theta^*)\right]}{p(\theta^*) - \hat{p}(1)} \\ &= \frac{1}{\hat{p}(1) - p(\theta^*)}\left[\frac{\theta^*\lambda(1-\lambda)}{\kappa} + \theta^*(\rho - r)\ell'(\theta^*) - \pi - \eta(\theta^*)\right] > 0. \end{aligned} \tag{D.3}$$

The term in square brackets is precisely the numerator of (23) evaluated at θ^*.

Note that $\dot{\theta} > 0$ for θ close to θ^*, so $\left[\frac{\theta^*\lambda(1-\lambda)}{\kappa} + \theta(\rho - r)\ell'(\theta^*) - \pi - \eta(\theta^*)\right] > 0$. Moreover, $\hat{p}(\theta) < p(\theta)$ in a left-neighborhood of θ^*, so $\hat{p}(\theta^*) < p(\theta^*)$. Because $\hat{v}(\theta)$ is strictly convex and $\hat{v}(\theta^*) = \hat{v}(1) - (1 - \theta^*)\hat{p}(1)$, we have $p(\theta^*) < \hat{p}(1)$. As a result, the randomization rate γ^* from (D.3) is well-defined and strictly positive.

D.2 Blockholder-level Leverage—Solution Details and Proof of Proposition 7

We solve for the smooth trading rate in the model variant with blockholder-level leverage. Following arguments analogous to the baseline analysis, one could easily characterize the unique scaled, continuous Markov equilibrium. However, since we are primarily interested in identifying the effects that generate gains from trade, we restrict our attention to the trading rate, which highlights these effects.

We have, as usual, $p(\theta) = v'(\theta)$. With blockholder-level leverage, $\omega \in [0, 1]$ the blockholder's value function solves

$$(\rho + \delta)v(\theta) = \max_{\ell,\ell^B,i}\left[\theta\left(\alpha + b + (\rho - r)\ell - \frac{\pi\theta}{2} - \frac{i^2}{2}\right) + \ell^B(\rho - r) - \frac{\kappa b^2}{2} + \mu i v(\theta)\right], \tag{D.4}$$

where investment satisfies as usual $i = \frac{\mu v(\theta)}{\theta}$ and (35) applies. Clearly, it is optimal to choose ℓ and ℓ^B, such that (35) binds. Also note that when (35) binds, the blockholder's value function does not depend on ℓ, ℓ^B, or $\omega = \frac{\ell^B}{v(\theta)}$. Intuitively, the blockholder can have effective debt up to $v(\theta)$, and its payoff is the same regardless of whether this debt is on the firm or fund level. Finally, we can solve $v(\theta) = \hat{v}(\theta)$ where $\hat{v}(\theta)$ is from (A.3).

Define the blockholder's total debt per unit of stock ownership as $\hat{\ell}(\theta) = \frac{v(\theta)}{\theta}$. Differentiating both sides of (D.4) under optimal policies with respect to θ, we obtain the blockholder's valuation of an additional unit of stock $v'(\theta) = p(\theta)$:

$$(\rho + \delta)p(\theta) = \alpha + b + \mu i p(\theta) - \frac{i^2}{2} + (\rho - r)\hat{\ell}(\theta) - \pi\theta + \frac{\theta\lambda(1-\lambda)}{\kappa} + \theta(\rho - r)\hat{\ell}'(\theta).$$

Note that $\theta\hat{\ell}'(\theta) = p(\theta) - \frac{v(\theta)}{\theta} > 0$, due to convexity (as in the baseline), with $y(\theta) = \frac{v(\theta)}{\theta}$. Moreover, $\ell(\theta) = (1-\omega)y(\theta)$. The fair pricing equation for the stock price becomes

$$(\rho + \delta - \mu i)p(\theta) = \alpha + b + (\rho - r)\ell(\theta) - \frac{i^2}{2} + \eta(\theta) + p'(\theta)\dot{\theta}.$$

Combining these and solving for $\dot{\theta}$, we obtain

$$\dot{\theta} = \frac{1}{p'(\theta)}\left[\frac{\theta\lambda(1-\lambda)}{\kappa} + (\rho - r)\left[p(\theta) - (1-\omega)y(\theta)\right] - \pi\theta - \eta(\theta)\right].$$

E Micro-Foundation of Utility Benefit

We micro-found the utility benefit and its functional form by modeling passive investors as consisting of a large mass of standard passive investors, who value the firm solely based on the present value of its dividends, and a unit mass of index investors, who apply the same valuation but aim to hold a fixed fraction θ^*_{Index} of the firm's equity. Neither type of investor can short-sell the firm's stock.

All index investors act symmetrically, holding a fraction $\theta_{t,Index}$ of the firm. When deviating from their desired ownership allocation θ^*_{Index}, index investors incur a disutility flow cost $K_t C(\theta_{t,Index})$, where $C(\cdot)$ is convex and differentiable, satisfying $C(\theta^*_{Index}) = C'(\theta^*_{Index}) = 0$. The negative of the disutility flow is a utility benefit. The disutility flow or utility benefit scale with the firm's capital stock. In what follows, we work with scaled quantities (by K_t), as in the remainder of the paper.

Although not explicitly modeled, the interpretation is that index investors allocate a given amount of wealth to the stock market and aim to invest in the firm according to its weight in the market.[18] Deviating from their desired ownership level may be costly for index investors, for example, because it reduces diversification or because these investors represent funds or institutional investors aiming to minimize tracking error.

When the blockholder's stake satisfies $\theta_t + \theta^*_{Index} > 1 \iff \theta_t > 1 - \theta^*_{Index}$, index investors hold at most a fraction $\theta_{t,Index} \leq 1 - \theta_t < \theta^*_{Index}$ of the firm. Thus, they hold strictly less

[18] That is, if the firm represents $\chi_t\%$ of the stock market index (e.g., S&P 500) and index investors allocate W_t dollars to this stock market index, they would aim to hold $\theta^*_{Index} = \chi_t\% \times W_t$ dollars in the firm, corresponding to an ownership share of $\chi_t\% \times \frac{W_t}{P_t}$. Assuming that $\frac{\chi_t W_t}{P_t}$ remains approximately constant over time, index investors would aim for a roughly constant ownership stake in the firm. This case prevails, for instance, when stock market capitalization equals M_t and wealth invested in the stock market is proportional to market capitalization, i.e., $W_t \propto M_t$, and the firm's stock price is also proportional to market capitalization, i.e., $P_t \propto M_t$, we obtain $\chi_t\% = \frac{P_t}{M_t}$ = constant and consequently, $\frac{\chi_t W_t}{P_t}$ = constant. Instead of explicitly modeling the dynamics of $\frac{\chi_t W_t}{P_t}$ and index investors' investment decisions, we simplify the analysis by assuming that they target a specific, constant fraction of ownership, θ^*_{Index}. Our results would remain qualitatively similar if we allowed θ^*_{Index} to vary over time.

than their desired ownership fraction θ^*_{Index} and, accordingly, value an additional unit of stock more than standard financial investors do, since they could reduce their disutility by increasing their ownership stake. Thus, in equilibrium, standard passive investors have zero ownership stake and index investors hold $\theta_{t,Index} = 1-\theta_t$, yielding a disutility of $C(1-\theta_t) > 0$ — the disutility is positive, since index investors deviate from their desired allocation. This disutility increases in θ, in that $\frac{d}{d\theta}C(1-\theta_t) = -C'(1-\theta_t) > 0$ and $C'(1-\theta_t) < 0$.

The blockholder trades against index investors who are marginal: A marginal reduction in the blockholder's stake θ, resulting in a marginal increase of index investors' stake, changes index investors' cost by $-\frac{d}{d\theta}C(1-\theta_t) = C'(1-\theta_t) < 0$. This marginal reduction in blockholder stake thus generates a marginal utility benefit to index investors of $\eta(\theta_t) := -C'(1-\theta_t) > 0$. Hence, $\eta(\theta_t) = -C'(1-\theta_t)$ is passive investors' utility benefit from an additional unit of ownership under these circumstances.

When the blockholder's stake satisfies $\theta_t + \theta^*_{Index} < 1 \iff \theta_t < 1 - \theta^*_{Index}$, then index investors hold their desired allocation $\theta_{t,Index} = \theta^*_{Index}$ and other passive investors hold fraction $1-\theta_t-\theta_{t,Index} > 0$. In this scenario, the blockholder trades against the other passive investors who are marginal and adjust their holdings when θ_t changes, while index investors remain at their desired allocation. Index investors' disutility cost is zero and not affected by small changes in the stake of the blockholder: In this case, passive investors derive zero marginal utility benefit (i.e., zero disutility cost reduction) from increasing their stake, in that $\eta(\theta_t) = 0$.

In sum, the utility benefit is the negative of the marginal disutility flow of passive investors from deviating from their desired allocation, i.e., $\eta(\theta_t) = -C'(1-\theta_t)$ for $\theta_t > 1 - \theta^*_{Index}$, while zero otherwise.

We assume quadratic disutility:

$$C(\theta_{t,Index}) = \frac{\pi^I}{2}(\theta_{t,Index} - \theta^*_{Index})^2 \quad \text{for} \quad \pi^I > 0.$$

For $\theta_t > 1 - \theta^*_{Index}$ where $\theta_{t,Index} = 1 - \theta_t < \theta^*_{Index}$, we obtain

$$C'(\theta_{t,Index}) = C'(1-\theta_t) = \pi^I(\theta_{t,Index} - \theta^*_{Index}) = \pi^I(1 - \theta_t - \theta^*_{Index}) < 0.$$

Then, the utility benefit takes the functional form $\eta(\theta_t) = -C'(1-\theta_t) = \pi^I\big[\theta_t - (1-\theta^*_{Index})\big]$. For $\theta_t \leq 1 - \theta^*_{Index}$, the utility benefit $\eta(\theta_t)$ equals zero.

In combination, we obtain a utility benefit that satisfies:

$$\eta(\theta_t) = \pi^I[\theta_t - \widetilde{\theta}]^+, \tag{E.1}$$

for parameters $\pi^I > 0$ and $\widetilde{\theta} = 1 - \theta^*_{Index} \in [0,1]$. Here, $[\cdot]^+ = \max\{0, \cdot\}$.

Thus, the utility benefit reflects the demand for the stock from certain passive investors who are reluctant to sell beyond a certain point and thus attach a high value to the stock, such as index investors. This utility benefit implies that when θ_t is large, the blockholder must pay a high price because they are buying from investors who are reluctant to sell and place a high valuation on the firm. When π^I is sufficiently large, the blockholder will never acquire the entire firm in that $\theta_t < 1$ in equilibrium, although θ_t may become arbitrarily close to one. Indeed, acquiring the entire firm would be hard or prohibitively costly in practice, as index investors would always hold some of the firm's ownership.[19]

Motivated by our micro-foundation, we interpret $\pi^I > 0$ as sufficiently large and $\widetilde{\theta}$ as close to one, implying $\theta_t < 1$ in equilibrium. In particular, we impose parameter condition (11).

[19]When the blockholder buys the firm's stock, it would push up the price and thus increase the firm's weight in the stock market index, further stimulating demand from index investors, which in turn makes it harder to buy more of the stock.