Rigging the Lobbying Process: An Application of the All-Pay Auction

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Why do politicians frequently "announce" that they have narrowed down a set of potential recipients of a "prize" to a slate of finalists?[1] In general, does the slate of finalists comprise the "best" candidates, and does the best candidate always win?[2] This paper provides answers to these questions. Our model of the political process is one of rent-seeking, which takes the (perhaps overly jaded) view that persons with power award political prizes on the basis of self-interest.

In a world where a politician can explicitly auction off a prize to the high bidder, the standard auction literature can be used to analyze political behavior. The justice system, however, precludes politicians from explicitly selling the prize to the highest bidder; thus politicians cannot let it become public knowledge that they are in the business of selling political favors.

An interesting institution has emerged in political markets to overcome this constraint: lobbying. Lobbyists make *implicit* payments to the politician, through campaign contributions or "wining-and-dining." If these up-front payments were rebated to those failing to receive the prize, it would be clear that the politician was selling favors. It is natural, therefore, for a political institution to arise such that lobbyists "ante up" before the prize is awarded, and these up-front payments are not refunded to those failing to win the prize. This view of lobbying has a structure isomorphic to the all-pay auction, which differs from standard auctions in one principal respect: all bids are forfeited by the bidders.

Before we describe our model of the lobbying process, it is useful to provide an overview of the existing literature and to contrast it with the present analysis. The case in which more than two lobbyists value the prize identically was first analyzed by Hervé Moulin (1986), who characterizes the symmetric equilibrium to the all-pay auction. Similar analysis is provided by Arye Hillman (1988), who argues that the equilibrium is unique. It turns out, however, that the symmetric equilibrium is not unique; in fact there is a *continuum* of equilibria (in Baye et al. [1990], we provide a full characterization of the equilibria.)

The case in which some lobbyists value the prize more than others has been analyzed by, among others, Hillman and John Riley (1989), who argue that equilibrium involves only the top two lobbyists. In this *Review*, Tore Ellingsen (1991) has considered the interesting case in which one lobbyist values the prize more than $n-1$ competitors with common valuations (see his proposition 1), and he demonstrates the existence of n equilibria. Baye et al. (1990) have shown, however, that there actually exists a continuum of equilibria in this case. Moreover, the expected revenue earned by the politician differs across this continuum of equilibria; there is *not* revenue equivalence across the equilibria.

The present analysis provides a simple closed-form expression for expected revenues that is valid for *all* equilibria. Our

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[1]The International Olympic Committee, for instance, selected six cities as "finalists" for the 1996 Summer Olympics: Belgrade, Manchester, Toronto, Melbourne, Athens, and Atlanta.

[2]Atlanta won the bid for the 1996 Olympics.

technique does not require an explicit calculation of the Nash-equilibrium mixed strategies. Consequently, our results provide a framework with which one may reexamine the implications of the equilibria missed for previous results, without explicitly calculating the (uncountable infinity) of equilibria.

The objective of the present paper is to take into account the continuum of equilibria and to determine the amount of rents the politician can expect to earn given such a political institution. To answer this question, we model the political process as a two-stage game of complete information. In stage 1 the politician takes the political institution of lobbying as given but is free to constrain the process by "narrowing down" the slate of candidates to a set of "finalists." In stage 2, the finalists compete in an all-pay auction: the lobbyist giving the greatest bribe wins the prize, while the others receive nothing for their payments. We solve this decision problem by backwards induction, solving first for the Nash-equilibrium payments that accrue to the politician in the second-stage lobbying game given an arbitrary set of lobbyists. We then solve for the optimal first-stage decision of the politician, which involves the selection of the set of "finalists" that maximizes expected political rents. We will show that, under plausible circumstances, the politician has a perverse incentive to preclude lobbyists most valuing the prize from participating in the second-stage lobbying game. Intuitively, this precommitment may take the form of announcing prior to any lobbying that "five states have been selected as finalists for the site of a new military base." We will refer to this precommitment as the *exclusion principle.* The exclusion principle has obvious implications for efficiency; states deriving the greatest economic benefit from a military base (and hence having the highest valuation of the prize) may be excluded a priori from the announced set of finalists.

I. The Model

Consider a politician who must determine which of $n > 2$ lobbyists will receive a prize. The value of the prize to lobbyist i is $v_i > 0$, where the v_i's are common knowledge and ordered such that $v_1 \geq v_2 \geq \cdots \geq v_n$.

The politician does not care which lobbyists wins the prize but does care about how much money he has available in his campaign chest. Accordingly, he decides to award the prize to the lobbyist who gives him the greatest up-front, nonrefundable *implicit* bribe. The objective of the politician is to select a set of lobbyists (the set of finalists) that maximizes his expected rents, $W = E\sum_{i=1}^{n} b_i$, where b_i is the bribe paid by player i.

Given a set of "finalists," lobbying is an all-pay auction: the payoff to lobbyist i if he offers a bribe of b_i is $\pi_i = v_i - b_i$ if b_i is the highest of all n bribes. However, if some other lobbyist offers a higher bribe, lobbyist i's payoff is $\pi_i = -b_i$. We assume that when multiple lobbyists submit the highest bribe, the prize is awarded to one of them at random. Thus, the payoff of lobbyist i is given by

$$(1)\quad \pi_i(b) = \begin{cases} v_i - b_i & \text{if } b_i > b_j \ \forall\, i \neq j \\ \dfrac{v_i}{M} - b_i & \text{if } i \text{ ties } M-1 \text{ others for high bid} \\ -\, b_i & \text{if } b_i < b_j \text{ for some } j \neq i. \end{cases}$$

This payoff structure is standard in the lobbying literature (cf. Hillman, 1988). It can also be viewed as the limiting case of an alternative payoff structure suggested by Gordon Tullock (1980) that is also used in this literature (see Baye et al., 1989). In Section II we characterize the expected payments by lobbyists in the second-stage lobbying game. These results are used in Section III to determine the politician's rent-maximizing selection of the set of "finalists."

II. The Lobbying Game

We first sketch a proof of the nonexistence of a pure-strategy Nash equilibrium for the all-pay auction. Consider the two-player case and suppose (b_1, b_2) did comprise a pure-strategy Nash equilibrium (without loss of generality, suppose $b_1 \geq b_2$). If $b_1 < v_2$, player 2 could deviate to earn a higher payoff by increasing b_2 slightly above b_1 to win the prize. If $b_1 \geq v_2$, player 2's best reply to b_1 is zero; but with $b_2 = 0$, it pays player 1 to deviate from b_1 by lowering the bid to (small) $\varepsilon > 0$, contradicting the hypothesis that $b_1 \geq v_2$.

It is known, though, that there does exist an equilibrium in mixed strategies, in which lobbyists randomize their bribes (cf. Partha Dasgupta and Eric Maskin, 1986; Moulin, 1986; Hillman and Riley, 1989; Baye et al., 1990). Moreover, with more than two players, there generally exists a continuum of possible equilibria (Baye et al., 1990). As our focus centers around the politician's rent-maximizing selection of finalists, we need only characterize the expected total bribes that accrue in a given Nash equilibrium of the lobbying game. The innovation is that the techniques employed below do not rely on the algebraic form of the mixed strategies used by the lobbyists in equilibrium and, thus, are valid even in the presence of a continuum of Nash equilibrium mixed strategies.

The following theorem is the key ingredient that enables us to determine the set of finalists that maximizes the politician's rents. The novelty of the result is that it is valid for *each* equilibrium in the continuum of possible Nash equilibria and thus can be used for purposes beyond the present paper. For example, the formula allows one to strengthen the results of Ellingsen (1991), which are based on a finite subset of equilibria.

THEOREM 1: *Let $v_1 \geq v_2 \geq \cdots \geq v_n$ denote the valuations of lobbyists $\{1,2,\ldots,n\}$ in the stage-2 lobbying game. Let $E_1 b_1$ denote the expected bid of a lobbyist with the highest valuation. Then in any Nash equilibrium,*

$$(2) \qquad W = \frac{v_2}{v_1} v_2 + \left[1 - \frac{v_2}{v_1}\right] E_1 b_1 \leq v_2.$$

PROOF:

Let $F_i(b_i)$ denote the cumulative distribution function of lobbyist i in an arbitrary (mixed-strategy) Nash equilibrium, and let S_i denote the support of the distribution. Lobbyist i must earn constant (expected) profits almost everywhere (a.e.) in S_i. For lobbyist 1 this constant must equal $v_1 - v_2$, and for lobbyists $2,3,\ldots,n$, this constant is zero (see Baye et al., 1990).[3] Hence, the following conditions must hold:

$$(3) \qquad \pi_1(b_1) = \prod_{i \neq 1}^{n} F_i(b_1)[v_1 - b_1] + \left[1 - \prod_{i \neq 1}^{n} F_i(b_1)\right][-b_1] = v_1 - v_2 \qquad \text{a.e. on } S_1$$

and

$$(4) \qquad \pi_i(b_i) = \prod_{j \neq i}^{n} F_j(b_i)[v_i - b_i] + \left[1 - \prod_{j \neq i}^{n} F_j(b_i)\right][-b_i] = 0 \qquad \text{a.e. on } S_i,\ i \neq 1.$$

Let $p_i(b_i) \equiv \prod_{j \neq i}^{n} F_j(b_i)$ denote the probability that lobbyist i wins the prize, conditional on his bid and the strategies employed by the other $n-1$ lobbyists in a Nash equilibrium.[4] Then, since equations (3) and (4) hold almost everywhere in their respective supports, taking the expectations of these equations and manipulating reveals that

$$(5) \qquad P_1 v_1 - E_1 b_1 = v_1 - v_2$$

and

$$(6) \qquad P_i v_i - E_i b_i = 0 \qquad \forall\ i \neq 1$$

where E_j denotes the expectation with respect to lobbyist j's (equilibrium) mixed

[3] Note that when lobbyist 2's valuation equals that of lobbyist 1, $v_1 - v_2 = 0$.

[4] We can rule out mass points for any agent at a bid $b > 0$ (see Baye et al., 1990).

strategy and $P_j \equiv E_j p_j(b_j)$. Summing over equations (5) and (6), we then obtain

$$(7) \quad W \equiv \sum_{j=1}^{n} E_j b_j$$

$$= (P_1 - 1) v_1 + \sum_{i \neq 1} P_i v_i + v_2.$$

Applying the fundamental theorem of integral calculus to $\Sigma_{j=1}^{n} P_j$, it follows that $\Sigma_{j=1}^{n} P_j = 1$. Furthermore, if $v_2 > v_i$, $i > 2$, then $P_i = 0$ (see Baye et al., 1989, 1990). Hence,

$$(8) \quad W = (P_1 - 1) v_1 + (2 - P_1) v_2.$$

Rearranging (5), we find

$$P_1 = \frac{v_1 - v_2 + E_1 b_1}{v_1}$$

which, inserted into (8), yields our results.

Two implications of Theorem 1 are worth noting. First, if two or more players most value the prize at some common level, v, the expected rents accruing to the politician equal v; there is full rent dissipation. Secondly, if $v_1 > v_2$, then the expected rents accruing to the politician are strictly *less* than v_2, since $E_1 b_1 < v_2$ in any Nash equilibrium. In other words, regardless of whether there is a unique equilibrium[5] or a continuum of equilibria,[6] in every equilibrium there is *under*dissipation of rents. In the following section, this result will be used to establish when it pays a politician to preclude some lobbyists from competing in the lobbying game. First, however, we state the following lemma from Hillman and Riley (1989).

LEMMA 1: *Suppose that the valuations of the lobbyists in the stage-2 lobbying game are such that $v_1 \geq v_2 > v_3 \geq v_4 \cdots \geq v_n$. Then in the unique Nash equilibrium, $Eb_1 = v_2/2$.*

Theorem 1 and Lemma 1 together imply that, when two players value the prize strictly more than all other players, the expected rents accruing to the politician in the stage-2 lobbying game are

$$(9) \qquad W(v_1, v_2) = \left[1 + \frac{v_2}{v_1}\right] \frac{v_2}{2}.$$

Note that, when $v_1 > v_2 > v_3$, expected rents are increasing in v_2 but decreasing in v_1. Intuitively, as player 1's valuation increases, the playing field becomes more unequal. Hence, player 2 reduces his expected payment to the politician, and total expected rents decline.

It is important to note that the formula in equation (9) is based on specific configurations of valuations and does not hold in general (it does not hold when $v_1 > v_2 = v_3$). The reason is that, when $v_1 > v_2 = v_3$, Eb_1 in equation (2) varies depending upon which of the continuum of equilibria is played, and thus, the politician's expected rents depend upon which equilibrium the lobbyists play. This point has not been addressed in the existing literature, and it plays a crucial role in our analysis.

III. Selecting the Finalists

Since there exists a continuum of expected political rents for some configurations of valuations, our next task is to characterize properties of the maximum expected political rents that can be extracted from the lobbyists.

PROPOSITION 1: *If $\{\hat{1}, \ldots, \hat{m}\}$ is a rent-maximizing set of finalists (with valuations $\hat{v}_1 \geq \cdots \geq \hat{v}_m$), then expected rents are*

$$(10) \qquad W(\hat{v}_1, \hat{v}_2) = \left(1 + \frac{\hat{v}_2}{\hat{v}_1}\right) \frac{\hat{v}_2}{2}.$$

PROOF:

We must show that if $\{\hat{1}, \ldots, \hat{m}\}$ is a set of finalists that maximizes expected rents (and the corresponding valuations are $\hat{v}_1 \geq \cdots \geq$

[5]The equilibrium is unique when $v_2 > v_3$.
[6]There is a continuum of equilibria when $v_2 = v_3$.

$\hat{v}_m$), then expected rents are $W(\hat{v}_1, \hat{v}_2)$. This is clearly true if $m = 2$; hence, suppose $m > 2$. If $\hat{v}_1 = \hat{v}_2 \equiv \hat{v}$, equation (2) reveals that $W = \hat{v} = W(\hat{v}_1, \hat{v}_2)$. If $\hat{v}_1 > \hat{v}_2 > \hat{v}_3$, equation (9) shows again that $W = W(\hat{v}_1, \hat{v}_2)$. Finally, if $\hat{v}_1 > \hat{v}_2 = \hat{v}_3 \equiv \hat{v}$, expected rents increase by excluding player $\hat{1}$, since by Theorem 1 $W(\hat{v}_1, \hat{v}_2) < \hat{v}_2 = W(\hat{v}_2, \hat{v}_3)$. However, this contradicts the hypothesis that the set $\{\hat{1}, \dots, \hat{m}\}$ maximizes expected rents. Hence, we conclude that any rent-maximizing set of finalists generates expected rents of $W(\hat{v}_1, \hat{v}_2)$.

Thus, while equation (9) does not hold for all possible configurations of values, it does hold when the set of finalists is selected so as to maximize expected rents [equation (10)]. This result allows us to determine the set of finalists that maximizes the politician's expected rents. Specifically, since equation (10) is decreasing in the highest valuation and increasing in the second-highest valuation, it never pays to exclude a player with a valuation that lies between the valuations of any two lobbyists who are in the set of finalists. Thus, the expected rent-maximizing set of finalists is determined by considering all pairwise combinations of adjacent lobbyists until lobbyists k and $k+1$ are found such that

$$W(v_k, v_{k+1}) = \max_i W(v_i, v_{i+1}).$$

To realize these rents, the politician must exclude players with valuations greater than v_k from the set of finalists. Formally, we have shown the following:

PROPOSITION 2: *Suppose $v_1 \geq v_2 \geq v_3 \geq \cdots \geq v_n$. Then the politician maximizes expected rents by constructing a set of finalists that excludes lobbyists with valuations strictly greater than v_k, where k is such that*

$$\left(1 + \frac{v_{k+1}}{v_k}\right)\frac{v_{k+1}}{2} \geq \left(1 + \frac{v_{i+1}}{v_i}\right)\frac{v_{i+1}}{2} \quad \forall\, i.$$

In order to highlight the implications of our results, consider the following two corollaries.

COROLLARY 1: *Suppose $v_1 = v_2 \geq v_3 \cdots \geq v_n$. Then the politician does not gain by constructing an agenda that excludes some lobbyists from the lobbying game.*

COROLLARY 2: *Suppose $v_1 > v_2 = v_3 \geq \cdots \geq v_n$. Then the politician maximizes expected rents by excluding the lobbyist with the highest valuation from the set of finalists.*

It may also be optimal for a politician to exclude more than one lobbyist from the stage-2 game. For instance, suppose $v_1 > v_2 > v_3 = v_4 \geq \cdots \geq v_n$. Then the politician maximizes expected rents by constructing an agenda that excludes lobbyists 1 and 2 from the set of finalists whenever

$$\left(1 + \frac{v_2}{v_1}\right)\frac{v_2}{2} < v_3.$$

These results demonstrate the *exclusion principle*: a politician may benefit from precluding the lobbyists valuing the prize the most from participating in the lobbying process.

We conclude with a numerical example to aid in elucidating our findings. Suppose $v_1 = 50$, $v_2 = 40$, and $v_3 = 38$. The theorem and lemma imply that the politician earns $W = 36$ if he does not constrain the lobbying process or limits lobbying to only players 1 and 2. However, if the politician announces that players 2 and 3 are the finalists, then the expected payments to the politician are $W(40, 38) = 37.05$. Thus it pays the politician to exclude lobbyist 1, who values most the prize, from participating in the lobbying game.

IV. Conclusions

This paper has examined an interesting principle arising in all-pay auctions: the exclusion principle. This principle states that a politician wishing to maximize political rents may find it in his best interest to exclude certain lobbyists from participating in the lobbying process—particularly lobbyists valuing most the political prize. In addition to pointing out the exclusion principle, our

Theorem 1 characterizes expected revenue for the entire continuum of equilibria that can arise in $n > 2$-player all-pay auctions with arbitrary valuations of the prize. This is in contrast to the results of Ellingsen (1991), Hillman (1988), Hillman and Riley (1989), and Hillman and Dov Samet (1987), among others, which are valid only for a subset of possible equilibria.

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