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GENERATIONAL ACCOUNTING, SOLIDARITY AND PENSION LOSSES***

BY

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Summary

The stock market collapse led to political tensions between generations due to the fuzzy definition of the property rights over the pension funds' wealth. The problem is best resolved by the introduction of *generational accounts*. Modern consumption and portfolio theory shows that the younger generations should have the higher equity exposure due to their human capital. Stock market losses should be distributed smoothly over lifetime consumption by adjusting both current contributions and future entitlements. We present expressions for the substantial welfare losses involved in various practically relevant deviations from the optimal system.

Key words: Pension funds, generational accounts, portfolio choice, Life cycle Models

JEL Code(s): D91, G11, G23

1 INTRODUCTION

Currently many pension systems are in crisis. The creeping stock market collapse has eroded the wealth of funded pension systems. The seeming opulence of the funded pensions schemes attributable to the bullishness of the stock market in the 1990s, moreover, induced politicians and corporate executive officers to skim the pension funds by giving pension holidays to company stakeholders. Some funds no longer guarantee the defined benefits. Other funds have temporarily stopped the inflation indexation of the pension payouts and contributions have shot up. This raises the fundamental question which generation should pay for the decline in the pension funds' wealth: Retirees, workers or future generations? This paper pleads for the adoption of a system of generational accounts (GA) similar to Auerbach and Kotlikof's (1987) model for the public sector. All contributions paid by one generation would be paid into that generation's account. The account's investments must

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be separately administered, so that each generation's property rights on the return to investments would be clearly defined. When a generation retires, its pensions would be deducted from its account. Ideally, the account would show a zero balance when the last participant of that generation dies.

The lack of transparency as to who should pay for the losses is closely related to the defined benefit (DB) characteristics of many pension systems. This system guarantees pension rights independent of the pension fund's ability to pay. With a fuzzy definition of property rights, investment decisions are likely to be suboptimal. The current DB systems are, moreover, also unreasonably rigid. Any model of the intertemporal consumption planning is flexible regarding both current and future consumption. A decline in total wealth is optimally dealt with by a reduction of consumption along the entire planning horizon. In our interpretation, the primary rationale for pension funds is that they make these intertemporal consumption decisions on our behalf and enable intra-generational risk sharing on the longevity risk. We may suffer from hyperbolic discounting and transaction costs may prohibit individuals from acquiring sufficient knowledge concerning lifetime portfolio decisions. This makes many of us inapt to make long term intertemporal decisions. Hence, our societies have delegated these decisions to pension funds. According to this interpretation, pension funds should decide on their investment and pay-out policies as if they solve our intertemporal consumption planning problem. The pension fund's overall investment policy is simply the sum of optimal saving and investment policies per generation. The optimal generational investment policy follows from the modern lifetime portfolio theory as presented in Bodie et al. (1992). This literature shows that the younger generations should have a higher equity exposure than the older generations due to the fact that youngsters have a higher stock of human capital.

Keeping separate track of each generation's investment policy is a crucial element in a GA system, as the risk profile of optimal investment policies differs between generations. This ensures optimal intertemporal smoothing behavior. For a given shock in stock prices, all current and planned future consumption of a generation should be reduced by the same amount. That is, the percentage point increase in the pension contributions for the remaining working career until the date of retirement is equal to the percentage decrease in expected pensions. When each generation's portfolio contains the optimal amount of risk, the percentage reduction of consumption in response to a particular shock in stock prices will be the same for all generations. We calculate that the optimal decrease in consumption due to the fall in stock prices in the period 2001–2002 is about 14%. Other systems, like the DB system inefficiently put the whole burden of adjustment on the current working generation.

The GA's do not interfere with the options for intra-generational solidarity, e.g., between men and women. This type of solidarity can easily be arranged for within the account of a generation. Hence, we shall not discuss the pros and cons of these options in this paper. The GA, however, do seem to frustrate inter-generational solidarity. We show that this perception is mistaken. Lumping together the pension fund wealth of all generations does not improve the options for inter-generational insurance against stock market fluctuations. We show that there is only one option for improving the trade-off between risk and the expected return on investment. The option is to expose young generations to stock market risk before their entry on the labor market. This improves risk smoothing, because the risk is spread over a longer time period. Though this option may appear unconventional, we argue that this is what pension funds already do in practice, albeit perhaps unwittingly and in a non-transparent way. This type of pre-labor-market-entry exposure to stock market fluctuations is not an argument against GA, since it can be perfectly arranged within such a system. However, this type of policy would yield strong adverse selection problems, since new generations are only willing to enter the fund if these pre-labor-market-entry investments have yielded an above average return. Hence, this type of policy does provide a genuine argument for mandatory participation in pension funds.

In response to the creeping stock market decline some people have argued that the pension funds should be restricted in the share of their capital that they are allowed to invest in stocks. We argue against such restrictions. Pension funds require public supervision regarding their operations. However, supervision should not impose restrictions on the funds' investment policies. Consider the extreme restriction that a fund would not be allowed to hold equity at all. We calculate that real labor income should go up permanently by at least 11% to offset the negative effect on expected utility.

The structure of the rest of the paper is simple. The next section lays out the assumptions of the model that we apply throughout this paper. In the subsequent sections, we work through all topics that are raised in this introduction.

2 THE MODEL OF PORTFOLIO SELECTION

To derive portfolio rules for funded systems we build on the literature which started with Merton's (1969) and Samuelson's (1969) analyses of lifetime portfolio selection for an individual. We take into account the anticipated returns on human capital on the optimal composition of portfolio. More recently this literature has been extended in labor economics by investigating the expected benefits from human capital and by modeling the uncertainty regarding future wage income, see Bodie et al. (1992) and more recently Menoncin (2003). Our focus is on the implications of this framework for the optimal portfolio of pension funds and for their risk management. We focus our analysis completely at these aspects, and neglect any complications that have no direct implication for these issues.

Let t denote the age of the generation. For our purpose, we ignore any other uncertainty than the return on investment. Hence, a generation works from $t=0$ till a fixed retirement age $t=T$ and lives on a pension from $t=T$ till its deterministic age of death at $t=D$. Furthermore, we assume labor income while working to be constant, and we normalize it to unity. The individuals of a generation have a Constant Relative Risk Aversion (CRRA) utility function. Since the labor supply and retirement decision are exogenous in our analysis, we neglect any disutility of working. Hence, consumption is the only argument of the utility function:

$$\begin{aligned} \overline{U}(t) &\equiv \int_t^D e^{-\beta(s-t)} u\left[C(s)\right] ds, \\ u(C) &\equiv \tfrac{1}{1-\theta} C^{1-\theta}, \end{aligned} \tag{1}$$

where $\overline{U}(t)$ is lifetime utility at age t conditional on the path of consumption $C(s)$ during the rest of a generation's life; $u(C)$ is an individual's instantaneous utility, where θ is the coefficient of relative risk aversion; β is the subjective discount rate.

In order to finance its consumption while retired, a generation saves labor income. It has two options for investment: a risk free bond with a return ρ, or risky equity with price $P(t)$. Suppose the $\log P(t)$ evolves according to a random walk with drift μ and normally distributed innovations with variance $\sigma^2 dt$. Thus the price of the risky asset follows a geometric Brownian motion:

$$dP(t) = \bar{\mu} P(t) dt + \sigma P(t) dW(t),$$

where $\bar{\mu} \equiv \mu + \frac{1}{2}\sigma^2$ is the expected return on the equity, where the factor $\sigma^2/2$ is a consequence of Ito's lemma or alternatively follows from the mean of a lognormal distribution, and $W(t)$ is a Wiener process. We assume that there is a trade off between return and risk: $\bar{\mu} > \rho$.

Let $S(t)$ be a generation's wealth and let $f(t)$ be the fraction of its savings that it is willing to invest on the stock market. Then, the law of motion of savings for a generation reads:

$$\begin{aligned} dS(t) = {} & \left[f(t)\,\bar{\mu} + \{1 - f(t)\}\,\rho\right] S(t)\,dt + \left[Y(t) - C(t)\right]dt \\ & + \sigma f(t)\, S(t) dW(t), \end{aligned} \tag{2}$$

where $Y(t)$ denotes labor income; $Y(t)=1$ while working until the time of retirement T, while $Y(t)=0$ from retirement T until death at time D. The first term on right hand side is the expected return on investment, the second

TABLE 1 – PARAMETER CALIBRATION

Population parameters		
T	40	Retirement age
D	55	Time of death
Preference parameters		
θ	5	Relative risk aversion
β	0.02	Time preference
Asset return parameters		
ρ	0.02	Bond return
μ	0.04	Drift
σ	0.2	Standard deviation
$\bar{\mu}\left(=\mu+\frac{1}{2}\sigma^2\right)$	0.06	Mean equity return

term is the change in wealth due to saving or dissaving, and the final term reflects the volatility of stock prices. The budget constraint of a generation can be written as an initial and a terminal condition on $S(t)$:

$$S(0)=0, \quad S(D)\geq 0.$$

Generations start working without any financial wealth and are not allowed to die in debt.

From the literature we calibrate the parameters of the model. We choose an active working life T of 40 and 15 years of retirement, $D-T=15$. The coefficient of relative risk aversion θ plays a crucial role in our analysis. Experimental economics, see, e.g., evidence in Newbery and Stiglitz (1981, ch.7), and real life experiments such as reported in Beetsma and Schotman (2001) suggest values between one (logarithmic utility) and ten. Indirect evidence on the attitudes towards risk from the equity premium literature such as presented in Campbell's review (1999) yield less precise estimates.[1] The real business cycle literature typically experiments with the values in the range between one and ten, see, e.g., Cooley and Prescott (1995), Rouwenhorst (1995), and Backus et al. (1995). We benchmark our calculations using $\theta=5$. The value of the rate of time preference β is set at 2% per year, which is a fair compromise between Cooley and Prescott (1995), Rouwenhorst (1995) and Backus et al. (1995). From Campbell's review and on basis of the revealing study by Dimson et al. (2002) providing the long run statistical properties of different countries' asset market performances, we set the return on riskless bonds ρ equal to 2%, the drift in equity μ is calibrated to 4% and the

1 For example, Table 5 from Campbell for eleven countries implies estimates that range from minus 295 to plus 7215 (with mean value 1010, standard error of the mean 662, and a standard deviation of 2198).

standard deviation of equity returns σ is taken equal to 20% per year, so that equity has an expected return of 6%.

3 A GENERATION'S OPTIMAL POLICY

The derivation of a generation's savings and portfolio choice is along the lines of Bodie et al. (1992) using the continuous time dynamic programming technique. In the appendix, a detailed derivation is provided. Here we outline the results. A generation chooses $C(t)$ and $f(t)$ to maximize the expected utility of its members $E\left[\overline{U}(t)\right]$ subject to the dynamic budget constraint (2). Let $U(t, S)$ denote the optimal expected utility as a function of the state variables of the system, t and S. The first-order conditions for a maximum entail:

$$\begin{aligned} C(t) &= U_S^{-\frac{1}{\theta}}, \\ f(t) &= -\frac{\bar{\mu}-\rho}{\sigma^2}\frac{U_S}{SU_{SS}}, \end{aligned} \tag{3}$$

where we suppress the arguments of U for brevity. In the appendix, we show that these first-order conditions imply:

$$f(t) = \frac{\bar{\mu}-\rho}{\theta\sigma^2}\frac{S(t)+\frac{1}{\rho}\left(1-e^{-\rho(T-\hat{t})}\right)}{S(t)}, \tag{4}$$

where $\hat{t} \equiv \min(T, t)$. The second term in the numerator $\rho^{-1}\left(1-e^{-\rho(T-\hat{t})}\right)$, is a generation's net discounted value of human capital. Hence, the denominator is the sum of its financial and human capital, or equivalently, the net discounted value of all its expected future consumption. The final factor in equation (4) is therefore, the inverse of the ratio of financial to total capital of a generation. When retired, $T \le t \le D$, the human capital is fully depleted and all future consumption of that generation has to be financed from its is financial capital. Thus for all $T \le t \le D$, the proportion of investment in the risky asset follows the static mean variance portfolio choice dictum:

$$f(T) = \frac{\bar{\mu}-\rho}{\theta\sigma^2}.$$

It also is the celebrated rule from Merton (1969) and Samuelson (1969) for dynamic intertemporal asset allocation. The share is increasing in the risk premium $\bar{\mu}-\rho$ and decreasing in the degree of riskiness of the risky asset σ^2 and the coefficient of relative risk aversion θ. During the working life $t < T$, however, when the net discounted value of labor income is still positive, the share of investment in the risky asset is higher than during retirement. It is proportional to the inverse of the financial wealth relative to the total wealth of a generation. This implies that $f(t)$ can even be larger than unity. Hence, $f(t)$ is not really a share. The higher investment in the risky asset at young ages fits the layman's intuition. However, the reason for this investment rule is not the standard argument that young people face a longer time horizon to make up for losses in the value of their investment, compare Merton (1969) and Samuelson (1969) for the refutation of that argument. The real reason is that when a generation enters the labor market, its financial capital is only a small part of its total wealth. The largest part is human capital. Thus going short in bonds while young is not as risky as it appears, since this is counterbalanced by the stock of human capital. The stock of human capital works like a buffer. Since human capital is largest when young, a young generation can take on more risk than an older generation. Hence, the young invest a greater share of their financial capital in equity. The optimal investment policy is to distribute risk exposure "evenly" over the lifetime of a generation. For example, consider two investment policies: investing 50 Euro in stocks and 50 Euro in bonds during 2 years or investing 100 Euro in stocks in the first year and 100 Euro in bonds during the second year. Although the expected return of both policies is equal (approximately, we ignore accumulated interests for simplicity), the riskiness of the second policy is larger. An "even" distribution of risk exposure over the lifetime requires the amount of money invested on the stock market to be a fixed fraction of discounted wealth, or equivalently, the discounted value of expected future consumption. Hence, since the initial stock of savings is low, the "share" of savings invested on the stock market should be high. Using the parameter values calibrated above, one finds that for the pensioners with no remaining human capital, $f(T) = 0.20$.

Multiplication of equation (4) by $S(t)$ yields an expression for the amount invested in equity, $I(t)$, as a function of one's work experience t and financial wealth $S(t)$:

$$I(t) = \frac{\bar{\mu} - \rho}{\theta \sigma^2} \left[S(t) + \frac{1}{\rho} \left(1 - e^{-\rho(T - \hat{t})} \right) \right]. \tag{5}$$

A pension fund operating under a system of GA would just solve for each generation t the optimal amount amount of saving $S(t)$ and the optimal investment in equity $I(t)$. The pensions fund's aggregate wealth and its aggregate investment in equity should be equal to the sum over t of $S(t)$ and $I(t)$, respectively. The value of investment for a generation at the beginning of the working career, $I(0)$, can be calculated easily, using $S(0) = 0$. Taking the benchmark parameter values gives $I(0) = 5.50$, which means that at beginning of one's career a generation should borrow five times one's yearly wage and invest this in the equity markets. Similar numbers can be found in Bodie et al. (1992). The investment in equity has to be financed by simultaneous borrowing of the risk free asset. This is a risky strategy, which seems infeasible in practice since youngsters are credit constrained. Nevertheless, one has only to take into account the loans for college and mortgages to realize that the leverage (in terms of financial wealth) of young and middle aged is often considerable. In particular, the investment in housing paid for by mortgages, contributes to efficient risk taking at younger ages. In fact the young effectively purchase a call option on the macro economic performance of the economy with their human capital as collateral. One interpretation of the rationale for pension funds is that they are a resolution of the credit constraint faced by the young generations. Without mandatory participation, these funds would run a strong adverse selection problem. Young participants would drop out in the case that stock misfortunes during their early career have led to a negative net financial wealth of their generation. Mandatory participation can help to resolve this problem.

4 INTER-GENERATIONAL SOLIDARITY

A frequently raised objection against a system of GA is that it rules out inter-generational solidarity. Judging by the current political and generational stalemate in countries like Italy and Germany one wonders whether inter-generational solidarity is feasible. The problem was formally analyzed within an overlapping generations economy by Hendricks et al. (1980), who showed that the core in such models can be empty due to the impossibility to trade with the deceased, implying that there is no political support for inter-generational transfers and taxes.[2] Trade between active generations and the young who yet have to enter the labor market is, however, feasible. This section investigates the benefits from this type of inter-generational risk sharing.

Inter-generational solidarity is equivalent to optimal insurance of stock market risks across generations. Markets provide sufficient instruments for the distribution of the stock market risk across generations at a particular point in time. A generation that wants to take more risk should buy equity and sell bonds, a generation that wants to obtain more certainty should do the reverse. Financial markets price these assets, and therefore the reward for risk taking. Hence, contrary to the popular intuition, lumping together the wealth of various generations does not add to the amount insurance that can be provided. The only further option we have to improve the trade-off between riskiness and return on investment is to diversify the risk over time. The longer the time period over which a generation invests in equity, the more diversified is its risk exposure, and hence, the lower is its standard deviation. A fundamental insight here is that the opportunities for diversification are

2 The pay-as-you-go systems in many of the EU countries are under pressure exactly due to this problem. These schemes easily received public support at the time when the populations were rapidly expanding. With the current low fertility rates and population aging, however, the younger generations are starting to revolt against the generation tax.

asymmetric. Including uncertainty on future stock prices in a generation's portfolio is theoretically impossible. The information on the realization of P_t becomes available only after the death of the generation. Hence, it can not adjust its consumption to these realizations.

Contrary to future shocks, including past shocks to stock prices is feasible, but only under strict conditions. For this type of solidarity to work, a generation is supposed to have invested in the risky asset before the start of its career. Instead of a deterministic starting value of its financial capital, $S(0) = 0$, a generation borrows to buy stocks at some date $t < 0$ (at birth, say). Hence, $S(0)$ becomes a random variable with a positive expectation due to risk premium on the investment in equity, $\bar{\mu} - \rho > 0$. Though the expectation is positive, the starting value of $S(0)$ will be negative for some realizations. The optimal investment policy maximizes the expected value of the starting utility, $E[U(0, S(0))]$, where we take expectations over $S(0)$. This system exposes pension funds to even stronger adverse selection problems than the high leverage for young generations considered in the previous section. If stock prices have risen during the investment period prior to labor market entry, people will love to participate in the pension fund. However, if stock prices have come down, nobody will voluntary enter the fund upon entering the labor market. Only government regulation to impose mandatory participation can solve this problem.

Mathematically, the problem of entry upon birth into the pension scheme is not very different from the standard problem treated in the previous section. One only has to include an additional term for the time span between the date of birth, $t = -B$, and labor market entry, $t = 0$. During this period, a generation has no labor income: $Y(t) = 0$. For a clean comparison with the case discussed till sofar, we ignore consumption before labor market entry. The initial condition $S(0) = 0$ is now replaced by the condition $S(-B) = 0$. For the rest, the problem remains exactly the same. By a similar derivation as before, a generation's optimal investment in the risky asset satisfies for $-B \leq t \leq 0$:

$$I(t) = \frac{\bar{\mu} - \rho}{\theta \sigma^2} \left[S(t) + \frac{1}{\rho} \left(1 - e^{-\rho T} \right) e^{\rho t} \right],$$

while equation (5) remains valid for $I(t)$ after the entry on the labor market at $t = 0$. For $S(t) = 0$, the investment in equity is lower at $t = -B$ than at $t = 0$, since the net discounted value of labor income is smaller for $t < 0$. However, there is still a strong incentive to invest in stocks, showing the value of diversifying risk to the period before labor market entry. For example, taking $B = 15$, one finds that $I(-15) = 4.08$. This is less than the 5.50 at $I(0)$ if the participation in the pension fund starts only at the inception of ones career, but still constitutes a considerable leverage.

How much can we gain by allowing for this pre-labor-market-entry investment in stocks? Or stated differently, how much of a relative increase in labor income is needed to offset the negative effect of a restriction that does not allow pension funds to assume risk before the start of the labor market career? In the appendix, we derive the following approximate relationship:

$$\frac{1}{2}\left(\frac{\bar{\mu}-\rho}{\sigma}\right)^2 \frac{B}{\theta} + O\left(B^2\right), \tag{6}$$

where $O\left(B^2\right)$ is Landau's symbol indicating that the second term in the expansion is (maximally) of order B^2. For $B = 15$ and for our benchmark parameters, this expression indicates that we need a 6% increase in labor income to offset the negative effect of this restriction on expected utility.

It may appear an unrealistic idea to let a generation invest in stocks even before its entry on the labor market. It is therefore important to realize that in the current systems without GA, pension funds do actually apply this type of inter-generational solidarity, by levying equal contributions for all generations. The fall in stock prices has forced many pension funds to raise the contributions, to compensate for losses in their investment portfolio. Hence, a new generation entering the labor market right now has to carry part of the burden of stock market misfortune from past generations. In the practice of a GA system this is made explicit. A mature fund implements this by borrowing administratively on behalf of a new born generation from the older generations against the market rate of interest ρ, and investing the proceeds in the stock market. An immature fund has to borrow in the market against the collateral of future contributions. For this to work, the social contract must make GA participation mandatory.

5 THE WELFARE LOSS OF RULING OUT INVESTMENT IN STOCKS

Funded pension plans in the Netherlands and the UK do not operate under direct portfolio restrictions. Historically this was quite different. For example, the fund for teachers in Holland is still suffering from the obligation to swallow all loans placed by the government. France and Spain have recently tried to limit equity investments of any EU-based pension fund to EU stocks, thereby foregoing the benefits from investment in high-growth Asian countries. We investigate the social cost of an extreme legal restriction on the investment policy, which rules out any investment of pension funds in equity. This rule would restrict the set of policies available to a generation, and therefore, reduce the maximum achievable expected utility of its members. By how much do we have to increase its labor income to offset this negative effect? We assume that workers do not perform any intertemporal consumption planning, but just consume the income which they receive currently. Without this assumption, workers would just undo any restriction on the funds' investment policy by offsetting capital market transactions. The methodology for answering this question is similar to the methodology, which was used in the previous section for analyzing the welfare gains of investing in equity before labor market entry. In the appendix, we derive the following approximate expression for the required relative wage increase:

$$\frac{1}{4}\left(\frac{\bar{\mu}-\rho}{\sigma}\right)^2 \frac{D}{\theta} + O\left(D^2\right). \tag{7}$$

For the benchmark parameter values the permanent increase in labor income needed to offset the loss due to the obligation to invest all wealth in the risk free asset is 11% (respectively, 9% if one uses the exact expression, see equation (A.8)). This boils down to almost 5 years extra work.

These numbers account for only part of the welfare loss due to the obligation to invest all wealth in the risk free asset. It captures the welfare loss due to inappropriate restrictions on the supply of risky funds, keeping the risk premium constant. However, the lower supply of risk bearing funds raises the risk premium (macro shortage of risk bearing capital), and thereby the cost of not supplying these funds. Alternatively, the procedure discussed above accounts for the Harberger triangle on the supply side, not for that on the demand side.

A similar type of restriction is embedded in a DB system. A DB system fixes pensions relative to wages, and thereby puts the full burden of adjustment on contributions. At present, DB systems are run without keeping separate accounts for each generation, so that current workers pay for the stock market losses on the wealth of current retirees (since their benefits are fixed). However, a DB system can just as well be run within a GA framework. In that case, each generation has to apply an investment rule that fixes the wealth at the date of retirement at a level sufficient to pay out the future benefits, with certainty. Due to this guarantee, the pension's savings should by always be fully invested in risk free bonds. Instead of current workers paying for the losses of current retirees, current workers now have to pay for their own losses and guarantees. However, the basic feature of a DB system, the full burden of adjustment being put on working age income, while retirement income is guaranteed, that feature characterizes both the currently applied DB system and the DB-within-GA system.

Since the limitation of the burden of adjustment to the working age is a deviation from the optimal portfolio rule and optimal consumption plan, a DB-within-GA system yields a welfare loss relative to the optimal GA system. In general, portfolio rules like stop-loss are inefficient from the point of view of an expected utility maximizer, see Dybvig (1988) for a formal argument. Again, we assume that workers do not engage in capital market transaction that offset this constraint. Then, the welfare loss can be calculated in a largely similar way as the welfare loss of a rule that forbids funds to invest in equity.

The standard terminal condition that the financial wealth at the age of death should be greater or equal to zero, $S(D) \geq 0$, has to be replaced by a condition that guarantees that there is sufficient wealth at the age of retirement to pay the guranteed benefits:

$$S(T) \geq \frac{1}{\rho}\left(1 - e^{-\rho(D-T)}\right) C^{\mathrm{DB}},$$

where C^{DB} is the planned fixed consumption level after retirement, or alternatively, the pension promised by the pension fund. We derive an expression for the welfare loss for the case that the pension fund sets C^{DB} to maximize expected utility at the date of labor market entry. The derivation of this expression is relegated to the appendix. In this case, there is no useful first-order expansion available. Evaluating the expression in the appendix for our standard parameter values shows that a DB system yields a welfare loss at the date of labor market entry comparable to a 3% reduction in wages. This is smaller than the loss from a 100% bond portfolio requirement for all savings, but still amounts to about an 1 year increase in the age of retirement.

6 THE CONSEQUENCES OF A FALL IN STOCK PRICES

What happens to the distribution of future consumption when stock prices fall, as happened during the period 2001–2002? Consider equation (5). A fall in stock prices by one yearly standard deviation σ reduces the total wealth of a generation, its financial plus its human capital, by:

$$\frac{I(t)\,\sigma}{S(t) + \frac{1}{\rho}\left(1 - e^{-\rho(T-\hat{t})}\right)} = \frac{\bar{\mu} - \rho}{\theta\sigma} = 4\%. \tag{8}$$

The numerator is the loss in the value of savings, which is taken relative to the total wealth. A one standard deviation fall in stock prices reduces total wealth by 4%. By the martingale property of stock prices, this fall is expected to be permanent. Since all future consumption is paid for from this wealth, it has to decline by 4% on average, too.[3] By the CRRA utility function, this decline should be distributed equally across all periods. Hence, future pensions are reduced by 4%. The rate of saving (or equivalently: pension contribution rate) is increased by 4% point. Note that this expression does not depend on the age of a generation t. Due to the optimal investment policy,

3 Note that consumption equals

$$C(t) = \frac{S + \frac{1}{\rho}\left(1 - e^{-\rho(T-t)}\right) y}{\frac{\theta}{\alpha}\left(1 - e^{-\frac{\alpha}{\theta}(D-t)}\right)}$$

and where y is income.

a fall in stock prices has the same effect on consumption per period for all generations, whether the shock occurs at the start of the career, or just before dying. Equation (8) implies that the contribution rate for each generation follows a random walk. This random walk is the same for all generations, so that the contribution rates for each generation move parallel across time. The contribution rate upon labor market entry starts at the same level for each generation. Current contribution rates differ across generations, though, because each generation has been exposed to a different part of the trajectory of the random walk due to entering the labor market at different points in time.

The holding of equity indeed carries quite some risk. Over a lifetime career equation (8) implies that the standard deviation of the contribution rate is as large as

$$\frac{\bar{\mu}-\rho}{\theta\sigma}\sqrt{T}=25\%.$$

But note that the total expected extra return is also sizable, i.e. $\exp(0.04*40)\approx$ five fold increase of an initial investment. We can also use equation (8) to calculate the consequences for consumption of the fall in stock prices in the period 2001–2002. Stock prices were approximately halved, which is a decline of about 70 log points, or a fall of $0.7/\sigma=3.5$ yearly standard deviations. Hence, the negative effect on future consumption is a sizeable $3.5\times 4\%=14\%$.

What is the effect of a fall in stock prices on the portfolio of a generation? Should it buy or sell stocks? Again, we take for granted our assumptions of a constant return on the risk free asset, a constant risk premium and a constant volatility of stock prices, and again, consider equation (5). Since $(\bar{\mu}-\rho)/(\theta\sigma^2)<1$, a fall in stock prices P_t reduces the right-hand side of this expression less than it reduces the value of the investment, simply because the investment in equity is only a fraction of the sum of financial and human capital. Hence, a fall in stock prices reduces the value of the investment in equity below the desired level. This should be offset by buying additional equity. Thus pension funds should be buying in a falling market. This is what pension funds did during the recent stock market decline, but supervisors aborted this remixing, or even reversed the process thereby adding to the stock market volatility.

7 SHARE OF EQUITY FOR MATURE PENSION FUND

In the previous sections, we have derived the optimal portfolio for a single generation. However, a mature pension fund has participants of all generations, ranging from people who have just started their working career to retirees close to the age of death. Where pension funds do not keep separate track of each generation's account, it might be useful to have some idea whether their share of total investment held in equity nevertheless satisfies the rules for optimal portfolio management per generation. That is: if we add up the optimal portfolios of all generations in the pension fund, what should be the average share of the fund's wealth that is invested in equity? Let $N(t)$ be the size of a generation of age t. Then, the share of investment in stock for the pension funds as a whole, f^*, satisfies:

$$f^* = \frac{\int_0^D f(t)\, S(t)\, N(t)\, dt}{\int_0^D S(t)\, N(t)\, dt} = \frac{\bar{\mu} - \rho}{\theta \sigma^2} \frac{\int_0^D \left[S(t) + \frac{1}{\rho} \left(1 - e^{-\rho(T - \hat{t})} \right) \right] N(t)\, dt}{\int_0^D S(t)\, N(t)\, dt}. \tag{9}$$

Assume that generations grow at a fixed rate λ, so that the size $N(t)$ of the current generation of age t is $N(0)\, e^{-\lambda t}$. The problem of this expression is that we do not have a deterministic expression for $S(t)$, since it depends on past fortunes on the stock market. Suppose that the fund has operated in the past under the portfolio constraint that all investments should be made in the riskfree asset. Furthermore, suppose that this restriction is lifted today. Then, how much of its wealth should the pension fund invest in equity? Such was the situation prevailing till a few years ago for the pension fund of the Dutch government employees, one of the largest pension funds in the world. In the appendix, we show that, as a first-order approximation, f^* satisfies:

$$f^* \cong \frac{\bar{\mu} - \rho}{\theta \sigma^2} \frac{D}{D - T}. \tag{10}$$

The average ratio of total wealth to savings is approximately equal to total life time D divided by the length of the retirement period $D - T$. Note that the rate of population growth λ is only of second-order importance to the value of f^*. Applying this crude approximation to our benchmark parameters suggests that a mature pension fund should invest 73% of its wealth in equity [using the exact solution (19) from the appendix, assuming a 2% population growth, $\lambda = 0.02$, yields $f^* = 0.86$, while in case of a static population, $\lambda = 0.00$, the values is $f^* = 0.66$]. Hence, the optimal portfolio rules imply that mature pension funds should invest quite a substantial part of their wealth in equity.

8 DISCUSSION

The GA administered by pension funds enable society and generations to adhere to the optimal lifetime portfolio rules. This is beneficial since such an institution smoothens shocks from stock price fluctuations, which affect consumption and investment, thereby exercising a dampening effect on the business cycle amplitudes. The GA avoid distributional conflicts between generations which might otherwise arise when the stock market goes down. The implications are equitable since the cost of adjustment are spread evenly over the whole remaining lifetime. Looking at this issue from a generational perspective, this implies that all generations share equally in the fortunes and misfortunes of the economy. DB plans, though, require funds to put the full burden of adjustment on the working generations, since pensions are fixed.

We shortly discuss the uncertainty regarding future labor income as treated in, e.g., Menoncin (2003). This uncertainty makes human capital a risky investment. At first sight, one would expect that the optimal response to a greater riskiness of human capital would reduce the exposure to stock market volatility. This, however, is not necessarily true. In the longer run, the evolution of real wages is likely to be correlated with the evolution of stock prices. Since in practice desired future disposable income is indexed by future real wages, this correlation reduces the volatility of stock prices relative to desired consumption. A lower relative volatility raises the optimal share of wealth to be invested in equity. This conclusion is turned around when desired future income is indexed by consumption prices. A further issue along these lines is the endogeneity of the retirement decision. The flexibility in the retirement age provides an additional insurance device, as is analyzed extensively in Bodie et al. (1992). When a fall stock prices erodes the value of the financial capital, people will adjust by postponing their retirement. The availability of this additional form of insurance induces a higher exposure to stock market volatility.

Finally, the introduction of GA has radical implications for the governance of pension funds. Currently, many European firms are stressed because the new International Accounting Standards require them to put large liabilities due to DB pension systems on their balance sheet. Under GA, since a pension fund's liabilities to a generation must be paid from the available wealth on that generation's account, there is a clear separation between the liabilities of the employer and that of the pension fund. The employer has neither a responsibility nor a say in the pension policies. Pension liabilities do not pop up on the employer's balance sheet. The counterpart of this argument is that pension fund's wealth is just deferred compensation, owned by the participants in the fund and not by the employer. Hence, employers should be removed from the pension fund's board. This would also enhance the free movement of labor across Europe, which is hampered by the current differences in pension systems as exist today.

In summary, we believe that GA allow generations to benefit from optimal investment policies and remedy a number of the current deficiencies in the definition of property rights on pension fund's wealth.

APPENDIX DERIVATIONS

Derivation of Equations (3) *and* (4)

Optimal utility is an implicit function of time and savings. By Ito's Lemma

$$\begin{aligned} dU(t,S) = \{&U_S(t,S)\big([f(t)\bar{\mu} + (1-f(t))\rho]S(t) + [Y(t) - C(t)]\big) \\ &+ U_t(t,S) + U_{SS}(t,S)\frac{1}{2}\sigma^2 f(t)^2 S(t)^2\big]\big\}dt \\ &+ \sigma f(t)S(t)dW(t), \end{aligned} \tag{A.1}$$

where subscripts denote partial derivatives. The individual chooses $C(t)$ and $f(t)$ to maximize her expected utility $E[U]$ subject to the dynamic budget constraint (2). From (1) by differentiation

$$\frac{d\overline{U}}{dt} = -u[C(t)] + \beta U(t). \tag{A.2}$$

To maximize expected utility, take expectations in equation (A.1) and substitute on the left hand side the total differential from (A.2). This yields:

$$\begin{aligned} &-u[C(t)] + \beta U(t,S) \\ &\quad = U_t(t,S) + U_S(t,S)E[dS(t)] + U_{SS}(t,S)\frac{1}{2}\sigma^2 f(t)^2 S(t)^2. \end{aligned}$$

From (2), we have:

$$E[dS(t)] = \big([f(t)\bar{\mu} + (1-f(t))\rho]S(t) + [Y(t) - C(t)]\big)dt.$$

Rearranging terms, using equation (1) and suppressing reference to the state variables yields:

$$\beta U = \frac{1}{1-\theta}C^{1-\theta} + [(f\bar{\mu} + (1-f)\rho)S + Y - C]U_S + \frac{1}{2}\sigma^2 f^2 S^2 U_{SS} + U_t. \tag{A.3}$$

Since U is the optimal expected utility, the first-order conditions for consumption and the investment policy require the derivatives of the right hand side with respect to C and f to be zero. From these first order conditions we obtain (3). Substitution in equation (A.3) yields:

$$\beta U = \frac{-\theta}{\theta-1}U_S^{\frac{\theta-1}{\theta}} + (\rho S + Y)U_S - \frac{1}{2}\left(\frac{\bar{\mu}-\rho}{\sigma}\right)^2 \frac{U_S^2}{U_{SS}} + U_t. \tag{A.4}$$

This is a partial differential equation. Its solution takes a particular simple form for a retired person:

$$U(t,S) = \frac{1}{1-\theta} S^{1-\theta} g(t), \quad g(D) = 0.$$

For working individuals, when $Y = 1$, the solution is somewhat more complicated:

$$U(t,S) = \frac{1}{1-\theta}\left[S + \frac{1}{\rho}\left(1 - e^{-\rho(T-t)}\right)\right]^{1-\theta} g(t). \tag{A.5}$$

The validity of these solutions is checked easily. Twice differentiating equation (A.5) and substitution in equation (A.4) yields a first-order differential equation for $g(t)$:

$$g_t(t) = \alpha g(t) - \theta g(t)^{\frac{\theta-1}{\theta}}, \quad \text{where } g(D) = 0 \tag{A.6}$$

and where

$$\alpha \equiv \beta + \rho(\theta - 1) + \frac{1}{2}\frac{\theta - 1}{\theta}\left(\frac{\bar{\mu} - \rho}{\sigma}\right)^2.$$

Solving this differential equation yields:

$$g(t) = \left[\frac{\theta}{\alpha}\left(1 - e^{-\frac{\alpha}{\theta}(D-t)}\right)\right]^{\theta}. \tag{A.7}$$

Substitution in equation (A.4) and (A.5) yields equation (4).

Derivation of Equations (6), (7) *and the Welfare Loss from DB Promises*

We start with equation (7). Consider the more general case where $Y(t)$ while working is y. Then utility and consumption satisfy:

$$U(t,S) = \frac{1}{1-\theta}\left[S + \frac{1}{\rho}\left(1 - e^{-\rho(T-t)}\right)y\right]^{1-\theta}\left[\frac{\theta}{\alpha}\left(1 - e^{-\frac{\alpha}{\theta}(D-t)}\right)\right]^{\theta},$$

see equation (A.5) and (A.7). The utility at the beginning of the individual's career is $U(0,0)$. In the absence of a risky asset, this utility is equal to the case where the risk premium is equal to zero, $\bar{\mu} - \rho = 0$, for in that case the optimal policy is not to invest in stocks, $f = 0$. Hence, $\alpha^{\text{no risk}} = \beta + \rho(\theta - 1)$ for that special case. Take $y = 1$ and $\alpha = \beta + \rho(\theta - 1) + \frac{1}{2}\frac{\theta-1}{\theta}\left(\frac{\bar{\mu}-\rho}{\sigma}\right)^2$ as the benchmark case. Let y^* be the labor income that yields the same lifetime utility as the benchmark case for $\alpha^{\text{no risk}} = \beta + \rho(\theta - 1)$; y^* can be solved by setting equal the expressions for $U(0, 0)$ in the benchmark and in the special case. Some simplification yields:

$$y^* = \left[\frac{\beta + \rho(\theta - 1)}{\alpha} \frac{1 - e^{-\frac{\alpha}{\theta}D}}{1 - e^{-\frac{\beta + \rho(\theta - 1)}{\theta}D}} \right]^{\frac{\theta}{1-\theta}}. \tag{A.8}$$

The log of y^* is the relative wage increase needed to offset the utility loss of an obligation not to invest in equity. A Taylor expansion of $\ln y^*$ in D yields (7).

A similar argument yields equation (6). Consider the case where workers start investing at $t = -B$ such that $S(-B) = 0$. Since the consumption term drops out for $t < 0$ in equations (A.4) and (A.6) reduces to:

$$g_t(t) = \alpha g(t),$$

so that:

$$g(t) = g(0)\, e^{\alpha t}.$$

Since $S = 0$ at $-B$, expected lifetime utility at that point in time satisfies:

$$U(-B, 0) = \frac{1}{1 - \theta} \left[\frac{1}{\rho} e^{-\rho B} \left(1 - e^{-\rho T} \right) y \right]^{1-\theta} g(0)\, e^{-\alpha B}.$$

When individuals cannot assume risk before labor market entry, this utility is equal to the case where the risk premium is zero, $\bar{\mu} - \rho = 0$. Again, let y^* be the labor income that yields the same lifetime utility as the benchmark case for $\alpha^{\text{no risk}} = \beta + \rho(\theta - 1)$; y^* can be solved by setting equal the expressions for $U(-B, 0)$ in the benchmark and in the special case. Some simplification yields:

$$y^* = e^{\frac{\beta + \rho(\theta - 1) - \alpha}{1 - \theta} B}.$$

The log of y^* is the relative wage increase needed to offset the utility loss of an obligation not to invest in equity. A Taylor expansion of y^* in B yields (6).

We use the same methodology for the welfare loss due to a DB system in which during one's active career sufficient funds are invested in the riskless bonds to guarantee the fixed consumption flow C^{DB} during retirement. Discounted back to the start of one's career, the pension claim amounts to

$\frac{1}{\rho}\left(1-e^{-\rho(D-T)}\right)e^{-\rho T}C^{\mathrm{DB}}$. The utility under such a DB system at $t=0$ and $S=0$ is:

$$U(0,0)=\frac{1}{1-\theta}\left[\frac{1}{\rho}\left(1-e^{-\rho T}\right)y^{*}-\frac{1}{\rho}\left(1-e^{-\rho(D-T)}\right)e^{-\rho T}C^{\mathrm{DB}}\right]^{1-\theta}$$
$$\times\left[\frac{\theta}{\alpha}\left(1-e^{-\frac{\alpha}{\theta}T}\right)\right]^{\theta}+\frac{1}{1-\theta}\frac{1}{\beta}\left(1-e^{-\beta(D-T)}\right)e^{-\beta T}C^{\mathrm{DB}^{1-\theta}}.$$

The last term in this expression is the utility during retirement, the first part is the utility when working. Maximizing this expression with respect to C^{DB} at $t=0$ yields:

$$\frac{y^{*}}{C^{\mathrm{DB}}}=\frac{1-e^{-\rho(D-T)}}{1-e^{-\rho T}}e^{-\rho T}+\rho\frac{\theta}{\alpha}\frac{1-e^{-\frac{\alpha}{\theta}T}}{1-e^{-\rho T}}\left[\frac{\rho}{\beta}\frac{1-e^{-\beta(D-T)}}{1-e^{-\rho(D-T)}}e^{-(\beta-\rho)T}\right]^{-1/\theta}.$$

This can be used to rewrite $U(0,0)$ as a function of y^{*} only. Equate the utility for the DB system $U(0,0)$ with the optimal utility

$$\frac{1}{1-\theta}\left[\frac{1}{\rho}\left(1-e^{-\rho(T-t)}\right)\right]^{1-\theta}\left[\frac{\theta}{\alpha}\left(1-e^{-\frac{\alpha}{\theta}(D-t)}\right)\right]^{\theta}.$$

After some rearrangement this yields:

$$\ln y^{*}=\ln\left(\frac{1-e^{-\rho(D-T)}}{1-e^{-\rho T}}e^{-\rho T}+\rho\frac{\theta}{\alpha}\frac{1-e^{-\frac{\alpha}{\theta}T}}{1-e^{-\rho T}}\right.$$
$$\left.\times\left(\frac{\rho}{\beta}\frac{1-e^{-\beta(D-T)}}{1-e^{-\rho(D-T)}}e^{-(\beta-\rho)T}\right)^{-1/\theta}\right)$$
$$+\frac{1}{\theta-1}\ln\left\{\left(\frac{\rho}{\beta}\frac{1-e^{-\beta(D-T)}}{1-e^{-\rho(D-T)}}e^{-(\beta-\rho)T}\right)^{-(1-\theta)/\theta}\right.$$
$$\times\left(\rho\frac{\theta}{\alpha}\frac{1-e^{-\frac{\alpha}{\theta}T}}{1-e^{-\rho T}}\right)^{1-\theta}\left(\frac{1-e^{-\frac{\alpha}{\theta}T}}{1-e^{-\frac{\alpha}{\theta}D}}\right)^{\theta}$$
$$\left.+\frac{\rho^{1-\theta}}{\beta}\frac{1-e^{-\beta(D-T)}}{\left(1-e^{-\rho T}\right)^{1-\theta}}e^{-\beta T}\left(\frac{\theta}{\alpha}\left(1-e^{-\frac{\alpha}{\theta}D}\right)\right)^{-\theta}\right\}.$$

Evaluating this expression for our benchmark parameters yields $\ln y^{*}=0.032\,35$, which gives the 3% reduction in wages reported in the text.

Derivation of Equation (10)

Total future consumption has to be financed from total wealth, hence:

$$\frac{\theta}{\beta-(1-\theta)\rho}\left(e^{\frac{\rho-\beta}{\theta}t}-e^{\frac{\rho-\beta}{\theta}D-\rho(D-t)}\right)C(0)=\frac{1}{\rho}\left(1-e^{-\rho(T-t)}\right)+S(t).$$

The initial level of consumption follows from setting t equal to zero, using $S(0)=0$:

$$C(0)=\frac{\beta-(1-\theta)\rho}{\theta\rho}\frac{1-e^{-\rho T}}{1-e^{\frac{\rho-\beta}{\theta}D-\rho D}}.$$

Hence, the stock of savings at age t satisfies:

$$S(t)=\frac{1}{\rho}\left(e^{\frac{\rho-\beta}{\theta}t}-e^{\frac{(1-\theta)\rho-\beta}{\theta}D+\rho t}\right)\frac{1-e^{-\rho T}}{1-e^{\frac{\rho-\beta}{\theta}D-\rho D}}-\frac{1}{\rho}\left(1-e^{-\rho(T-\hat{t})}\right).$$

Substitution of these relations in equation (9) yields:

$$f^{*}=\frac{\bar{\mu}-\rho}{\theta\sigma^{2}}\left[1-\frac{\frac{1-e^{\frac{\rho-\beta}{\theta}D-\rho D}}{1-e^{-\rho T}}\int_{0}^{T}\left(e^{-\lambda t}-e^{-\rho T+(\rho-\lambda)t}\right)dt}{\int_{0}^{D}\left(e^{\frac{\rho-\beta}{\theta}t-\lambda t}-e^{\frac{\rho-\beta}{\theta}D-\rho D+(\rho-\lambda)t}\right)dt}\right]^{-1}. \tag{A.9}$$

A Taylor expansion of the integrals yields:

$$\begin{aligned}&\int_{0}^{D}\left(e^{\frac{\rho-\beta}{\theta}t-\lambda t}-e^{\frac{\rho-\beta}{\theta}D-\rho D+(\rho-\lambda)t}\right)dt\\&\quad=\frac{1}{2}\frac{\rho\theta-\rho+\beta}{\theta}\left(D^{2}-\frac{\rho\theta-2\rho+\theta\lambda+2\beta}{3\theta}D^{3}\right)+O\left(D^{4}\right),\\&\frac{1}{1-e^{-\rho T}}\int_{0}^{T}\left(e^{-\lambda t}-e^{-\rho T+(\rho-\lambda)t}\right)dt\\&\quad=\frac{1}{2}T-\frac{1}{12}(2\lambda-\rho)T^{2}+O\left(T^{3}\right)\end{aligned}$$

and

$$\begin{aligned}&1-e^{\frac{\rho-\beta}{\theta}D-\rho D}\\&\quad=\left(\rho-\frac{\rho-\beta}{\theta}\right)D-\frac{1}{2}\left(\rho-\frac{\rho-\beta}{\theta}\right)^{2}D^{2}+O\left(D^{3}\right).\end{aligned}$$

Retaining the first two non-zero terms for each expression yields equation (10).

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