FIAT EXCHANGE IN FINITE ECONOMIES

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The state of the art of rendering fiat money valuable is either to impose a boundary condition or to make the boundary condition unimportant through an infinite sequence of markets so as to circumvent backward induction. We show fiat exchange may nevertheless arise in finite economies if agents have incomplete information about their relative position in the trade cycle or when the barter and autarky equilibria of the one-shot trading round support a monetary equilibrium with repeated trades. (*JEL* E0, E5)

I. MODELING FIAT MONEY

A model of the transactions role of money stemming from the absence of a double coincidence of wants requires a dynamic economy with separated markets and no possibility to coordinate transactions via a single market. A sequential economy with pairwise trade gives commodity money an explicit role, because it relaxes the constraints from the bilateral quid pro quo (see, e.g., Ostroy and Starr's [1990] review essay). More difficult is the resolution of the Hahn (1965) problem, which, as formulated by Hellwig (1993), asks: "Why does fiat money have a positive value in exchange against goods and services even though it is not intrinsically useful?" In a dynamic economy, fiat money may have value in a transaction because it is expected to have value in future exchange. Granting the necessity of a sequence economy, the currently known solutions to the Hahn problem can be broadly classified along two lines. Borrowing terminology from the theory of differential equations, the solutions are that fiat money can be rendered valuable by either (1) imposing a boundary condition, or (2) avoiding the need of a boundary condition by pushing it away to infinity. Both are devices to circumvent the unravelling of the monetary equilibrium through backward induction (cf. Cass and Shell [1980]).[1]

The first response (1) is axiomatic and resembles the way in which the budget constraint is handled in micro theory. The Clower (1967) constraint "requires" that transactions are settled in terms of money. Smith (1776) argued that the possibility of paying taxes with money may render fiat money valuable.[2] The legal restrictions theory (see Wallace [1980]) imputes essentially the same requirement. Alternatively, Shubik (1981) imposes a bankruptcy penalty. McCallum (1983) and others stick money directly into the utility function to implicitly account for the transactions technology.

A number of authors have felt that the boundary condition approach is too ad hoc. The second approach (2) assumes that with positive (conditional) probability the sequence of markets extends beyond any given finite number of trades. Hence the

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1. Cass and Shell (1980, 252) argue: "At the end of the last period, money is worthless . . . . Individuals with foresight, not wanting to be stuck with the monetary 'hot potato,' thus drive the price of money to zero in each period."

2. We are grateful to a referee who pointed out the following quote from Smith (1776, book II, ch. II): "A prince who should enact that a certain proportion of his taxes should be paid in paper money of a certain kind might thereby give a certain value to this paper money."

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sequence of markets becomes endless, and the boundary condition is never binding and need not be imposed. A very popular model in this class is Samuelson's (1958) overlapping generations model (see, e.g., the comprehensive treatment in Balasko and Shell [1981]). Even though each agent only lives for two periods, the infinity of periods and generations may support a monetary equilibrium.[3] Alternatively, the assumption of finite but continuous time as in De Vries (1986) and Faust (1989) gets rid of the boundary condition because there is no penultimate instance in time.[4] In the more recent search based approach of Kiyotaki and Wright (1989), both the number of agents and time are assumed to be a continuum. This literature combines and extends the work on pairwise trade as reviewed in Ostroy and Starr (1990) and the random encounters model of Jones (1976) by having agents meet randomly and implement dynamically optimal trading schemes. In these models, money serves to overcome the absence of a double coincidence of wants.[5]

One motive for investigating monetary exchange in a finite economy is that the infinity of agents assumption is easily falsified and that the possibility of an unbounded time axis or a continuum of trades is questionable on the basis of physics. Indeed, experimental tests of infinite agent or time monetary models, such as the overlapping generations experiments of Marimon and Sunder (1993), Aliprantis and Plott (1992), and Duffy and Ochs (1999) all invoke (by necessity) boundary conditions ensuring finiteness.

The second motive is that by pushing away the boundary condition to infinity, one sweeps under the rug the monetary hold-up problem, which has been manifest throughout history in the form of hyperinflations and currency reforms. To understand fiat exchange, it is essential to model explicitly how society is able to cope with the monetary hold-up problem.

As in game theory, where infinitely repeated games are regarded as unsatisfactory for similar reasons, we provide two resolutions to the backward induction unraveling that constitutes the monetary hold-up problem in a finite economy. In both approaches, fiat money may be considered as a bubble because its circulation value is above its intrinsic value. The first approach, introduced in section II, assumes that agents have incomplete information. In finite discrete time the boundary condition coupled with complete information leads to backward unraveling, precluding the possibility of bubbles (see Tirole [1985]). However, Allen et al. (1993) show in a different setting that if agents have private information, bubbles in finite economies are nevertheless possible.

We construct a model of fiat exchange in which agents have incomplete information about exactly where they are located in the chain of (monetary) exchanges. With incomplete information about the sequence of trades, the information sets in the extensive form of the game are no longer singletons. The probabilities reflecting players' beliefs are jointly determined with the strategies, and the number of proper subgames is greatly reduced. Because players' beliefs at these information sets are jointly determined with the strategies, the ability of backward induction to restrict equilibrium behavior is significantly reduced. The incomplete information has the effect of turning the game into a lottery over who will end up with the hot potato.[6] The penultimate agent, who does not know that he is next to last because he does not know how many went before him, may therefore decide to accept money from the agent that goes before him, because he attaches a high enough probability to not being the penultimate agent.

The second resolution relies on the presence of multiple equilibria in the stage game. Benoit and Krishna (1985) show that if there are multiple equilibria in a single-shot game, the finitely repeated analogue can have perfect equilibria that are not repeated one-shot equilibria. The idea is that if the multiple equilibria of the single-shot game are payoff nonequivalent, then it may be possible to

3. Care has to be taken that the (Pareto optimal) monetary equilibrium is within the core; see Kovenock (1984), Esteban and Millán (1990), and Fisher (1997).

4. These models assume that marginal utility is unbounded at zero. This assumption is made to offset a probabilty of trade that converges to zero as time approaches its least upper bound.

5. In Kultti (1995) money is accepted in a discrete finite time continuum of agents economy, as the goods given up by the producer yield no utility. As soon as these do yield some utility, the hold-up problem appears.

6. A well-known hot potato game is the children's card game Old Maid, or *Zwarte Pieten*, as it is called in Dutch.

support a multistage equilibrium that does not consist of a sequence of single-shot equilibria through the threat of playing an equilibrium with a lower payoff for a deviating player in the last round. In the last round any single-shot equilibrium can be played, and hence playing the less desirable equilibrium is a credible threat. Our positive result relies on the existence of an autarkic equilibrium that is dominated by a barter equilibrium in the single-shot version of the game. Both are genuine nonmonetary equilibria, cf. Faust (1989).[7]

II. HELICOPTER MONEY

Consider a very simple economy made up of three agents, labeled 1, 2, and 3. Agents always meet pairwise in the following order: (1, 2), (2, 3), (3, 1), (1, 2), ... The chain of meetings does not necessarily start with agent 1 and 2, that is, the sequence (2, 3), (3, 1), (1, 2) is admissible, but the order of the bilateral markets is the same. To begin with, we only consider the single round of meetings: (1, 2), (2, 3), (3, 1).

During a meeting agents can decide to trade, but there is an absence of a double coincidence of wants. For simplicity we assume that agents are specialized so that the lower-indexed agent has (net) nothing to offer to the higher-indexed agent, except agent 1, whose goods are desired by 3. Thus one can imagine agents being grouped around a triangle with marketplaces at the nodes but no meeting ground in the middle. Each agent values his endowment positively so that autarky is a possible outcome. The agents' payoff to autarky is labeled a. Let an agent's payoff to multilateral trade be m. If all agents could meet together the Walrasian trade equilibrium would yield a payoff vector (m, m, m), where $m > a$, because in the aggregate there is quid pro quo. The friction imposed by the sequential encounters, however, necessitates sequential trade imbalances. As is standard in the literature, we assume that transport costs are sufficiently high to prevent commodity money equilibria from arising.[8] This allows us to focus on fiat exchange or no trade at all. Given this restriction, how are these imbalances to be settled?

Suppose agent 1 is handed some fiat money that can be used to settle temporal trade imbalances in the chain: (1, 2), (2, 3), (3, 1). If fiat currency could support the Walrasian allocation (m, m, m), the decentralized market economy would Pareto improve upon the autarky outcome. Unfortunately, such a sequence of fiat exchanges is not an equilibrium due to the monetary hold-up problem. In the final market, when agent 3 offers currency to 1 in return for goods, agent 1 has an incentive to renege and not honor the notes. By not exchanging part of his goods agent 1 receives a payoff of t, where $t > m$. By accepting the fiat currency he would give up something for nothing. Suppose that the payoff to agent 3 in the case that 1 refuses his money is k, where $k < a$. Due to the fact that agent 3 accepted money from 2 and is unable to spend it on agent 1's commodities, he is worse off than if he had remained autarkic. Realizing this, agent 3 does not engage in monetary exchange with agent 2. Continuing by backward induction, it follows that the monetary equilibrium collapses at the outset. Let the action "rejecting to trade" be denoted by R, and "willingness to trade" be indicated by P. The autarkic equilibria: $[(R, R); (R, R); (R, R)]$, and $[(P, R); (R, R); (R, R)]$, corresponding to the payoff (a, a, a) are the only subgame perfect equilibria in the cycle (1, 2), (2, 3), (3, 1).[9]

The collapse of the monetary equilibrium also occurs if agent 2 or 3 is handed the money, as in the cycles (2, 3), (3, 1), (1, 2) and (3, 1), (1, 2), (2, 3). Nor does it help to randomly initiate the trade cycle, so long as all agents observe who gets the money first. We now introduce incomplete information by assuming that agents are imperfectly informed about the specifics of monetary policy actions. We believe this is plausible in

7. This is of some interest because of the following remark in the conclusion of Faust (1989): "It is difficult to imagine what second equilibruim could exist in the final period of the monetary model besides one in which the value of money is zero." Faust recognizes the Benoit and Krishna analysis as the "only one other relevant solution to the terminal problem," but does not believe it can be used to validate fiat money because there cannot be multiple and payoff nonequivalent nonmonetary equilibria.

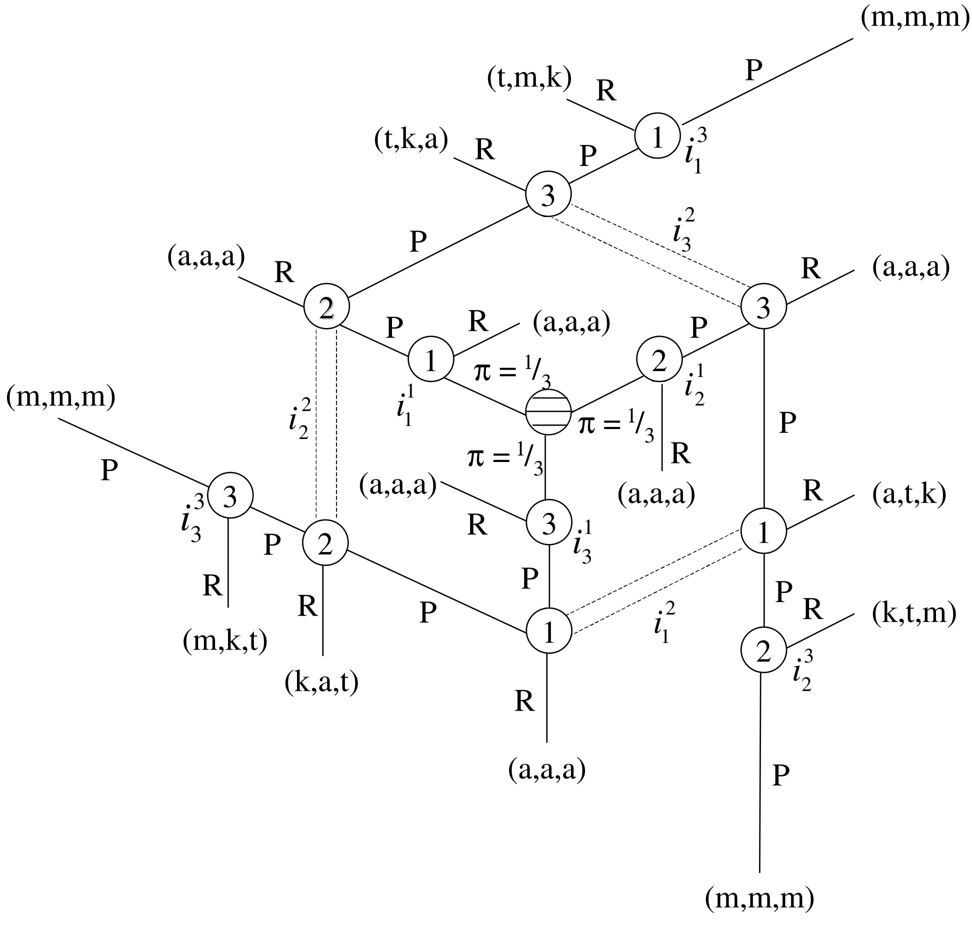
8. If, to the contrary, commodities could be transported costlessly, money would be superfluous.

9. To economize we do not report "empty" equilibrium strategies such as $[(P, R); (P, R); (R, R)]$. Once the chain of monetary exchanges has stopped, strategy P later down the chain has no bite (because the agent has no money to offer).

economies with large and highly interwoven chains of transactions. Assume agents know the level of money supply but are not informed about how money is injected into the economy. Imagine that the legendary monetary helicopter randomly drops one unit of fiat money at one of the agents' doorsteps. Each agent receives the unit with probability $\pi = 1/3$, and each only knows whether or not he has received the money. Thus the agent who receives the money also knows he will be the first and the last in the chain of transactions, but the other two agents remain in the dark over who has received the money.

The random insertion of money is depicted in Figure 1. The dashed lines indicate the uncertainty of an agent about the node at which he is located. Agent j's information sets are denoted by $i_j^h, h = 1, 2, 3$. Thus, for example, information set i_2^1, indicates that agent 2 knows he was distributed the fiat money from the central bank and that he is the first agent in the trade cycle. If agent j decides to trade money for goods, he meets with agent $(j+1)(mod\ 3)$. Agent $(j+1)(mod\ 3)$ must decide whether to accept or reject the money. Because $(j+1)(mod\ 3)$ does not know which of the agents received the money first, he must choose an action at an information set i_{j+1}^2, which is comprised of two nodes (see Figure 1). If agent $j+1$ chooses to accept the money, he then attempts to pass the money to agent

FIGURE 1
Random Insertion of Money



$(j+2)(mod\ 3)$ who, in turn, cannot distinguish which of the nodes in i^2_{j+2} is relevant. If agent $j+2$ accepts the money, he in turn will try to exchange it for goods with agent j. At information set i^3_j agent j finds it optimal to reject the monetary exchange, as it would imply giving up something for nothing.

It is evident from the information structure illustrated in Figure 1 that a strategy for player $j, j = 1, 2, 3$, in the game is a triple (s^1_j, s^2_j, s^3_j) specifying whether to accept or reject money in return for goods at information sets i^1_j, i^2_j, and i^3_j. It is always a conditionally strictly dominant strategy for j to reject money at i^3_j, and it is a weakly dominant strategy to offer monetary exchange at i^1_j. Behavior at $i^2_k, k = 1, 2, 3$, determines the nature of the equilibrium.

PROPOSITION 1. *Let a unit of money be randomly allocated to one of the agents. Each agent receives the money with probability* $\pi = 1/3$. *Suppose payoffs are such that* $k + m \geq 2a$. *Then there exists a symmetric sequential (monetary) equilibrium in which each player j uses a strategy* $(s^1_j, s^2_j, s^3_j) = (P, P, R)$, *and attaches probability* $1/2$ *to each node in the information set* $i^2_j, j = 1, 2, 3$. *Each player's expected payoff in the game is* $(t+m+k)/3$. *Along the branch of the game tree in which player j first receives money, j receives a payoff t, player* $(j+1)(mod\ 3)$ *receives m, and player* $(j+2)(mod\ 3)$ *receives k.*

Proof. We need only verify the sequential rationality and consistency of the assessment specified by the equilibrium. The random action of the central bank is completely revealing to the agent who is issued the money. Thus the agent knows he is the first and the last agent in the chain and that the other two agents are uninformed about his identity. Because it is conditionally strictly dominant for j to reject money at i^3_j and weakly dominant to offer money at i^1_j, the actions specified at these information sets are sequentially rational. Any agent j who must make a choice at i^2_j updates as follows: Because j is offered monetary exchange by agent $(j-1)(mod\ 3)$ at i^2_j, j calculates $[1/2] \cdot 1/([1/2] \cdot 1 + [1/2] \cdot 1) = 1/2$ as the probability of being on either of the two branches (in equilibrium j is offered monetary exchange with probability 1 along either branch). Given the strategies chosen by his rivals, accepting money is a best response, because $(k+m) \geq 2a$. Consistency follows immediately because in equilibrium every decision node is reached with positive probability.

There are other symmetric perfect Bayesian equilibria as well. These are the autarkic equilibria (R, R, R) and (P, R, R) and a mixture between the monetary and autarkic equilibria in which the uninformed players accept money at i^2_j with probability

$$\rho = (a-k)/(m-a). \tag{1}$$

To see this, consider first the autarkic equilibrium. Note that the Bayesian updating process is irrelevant for the decision as to whether to accept the money or not. The Nash property is verified as follows. At any node of the game agents are expected to reject trading in the continuation of the game. Therefore it never pays for the uninformed to accept money, even if an out-of-equilibrium offer of monetary exchange occurs.

To derive the mixed strategy equilibrium we first focus on the Bayesian updating process of agent 2 when he is not the first receiver of money. Along the branch starting with 1, agent 2 is offered the money with probability 1. Along the branch starting with 3, agent 2 is offered to trade with agent 1 with probability $\rho < 1$. Note that this is different from the case of Proposition 1, where $\rho = 1$. The probability of being on either branch is 1/2. Therefore the conditional probability of being on the branch that starts with 1 is given by Bayes's rule: $[1/2] \cdot 1/([1/2] \cdot 1 + [1/2] \cdot \rho) = 1/(1+\rho) > 1/2$. In equilibrium the uniformed agent must be indifferent between refusing to trade, for which he receives a payoff a, and accepting trade:

$$a = \frac{\rho}{1+\rho}k + \frac{1}{1+\rho}[\rho m + (1-\rho)k]. \tag{2}$$

If he accepts trade he is either the penultimate agent and gets k or he is second to last and the next agent in line determines to trade or not to trade through randomization. Solving this equation for ρ yields $(a-k)/(m-a)$. Furthermore, sequential rationality is clearly satisfied.

Also note that printing one's own money does not pay. Suppose, for instance, that along the branch that starts with agent 1, 1 plays R or 2 does not accept the money. Does 2 have an incentive to print his own money? If agent 2's money is distinct from the helicopter money, then 3 infers along which branch he is located and rejects trade with 2. Furthermore, if it is common knowledge that agents can perfectly counterfeit, no monetary exchange would develop either. It is straightforward to extend Proposition 1 to an n-agent economy. Without proof we state:

COROLLARY 2. *With $n > 2$ agents and $\pi = 1/n, (P, \ldots, P, R)$ is a perfect Bayesian monetary equilibrium if*

$$(3) \qquad \frac{n-2}{n-1}m + \frac{1}{n-1}k \geq a.$$

In the equation, the possible loss of being the penultimate agent rapidly becomes a negligible factor as n increases. Thus the question of whether to trade or not to trade is almost completely determined by the difference $m - a$. In fact, for any triple $m > a > k$ and any number of agents $n \geq n^*$, where

$$(4) \qquad n^* = 1 + (m-k)/(m-a),$$

the monetary equilibrium can be supported. Presumably the difference $m - a$ is large in today's society, whereas a and k are not too far apart. Hence, n is much larger than n^*. This is why we believe that our assumption of asymmetric information is both plausible and supportive of the existence of a monetary equilibrium.

Instead of considering a longer trade cycle, another extension is a repetition of the same cycle. Suppose that across the multiple trading rounds, money can be carried over from one round to the next (the money is issued only once). It is straightforward to show the following Corollary.

COROLLARY 3. *For a given initial allocation of the unit of fiat money, with s trading rounds the monetary equilibrium $(P, P, P, \ldots, P, P, R)$ yields the payoff vector*

$$(5) \qquad ([s-1]m + t, sm, [s-1]m + k).$$

Hence, each player's expected payoff is $(s - 1)m/3 + (t + m + k)/3$. The autarkic equilibrium yields the payoff vector (sa, sa, sa).

Note that a single-round monetary equilibrium yields an ex post loss to the penultimate agent in comparison with the autarkic equilibrium. But with $s > 1$ trading rounds, the monetary equilibrium (money being issued only once) also yields an ex post gain to the ultimate loser as long as $(s-1)(m-a) > (a-k)$. If a is close to k, m is sufficiently large compared to a, and s is very large, it seems difficult to argue from a behavioral standpoint that backward induction could render money worthless, because the ultimate loser stands to gain so much from cooperation. Indeed, under our asymmetric information model, the paradox of backward induction does not arise, as it does in the case of complete information.

This section started with a finite and even very small economy of three agents who buy and sell only once. Because the economy is so small, the uncertainty over one's position in the chain needs to be quite large for a monetary equilibrium to exist. This requirement, however, can be rapidly relaxed as the chain of exchanges becomes longer and longer. At the same time, this effect renders plausible the assumption that agents have incomplete information over where exactly their trades are located in the chain of exchanges. In reality, few people have any idea where their money goes one or two transactions after they have spent it.[10]

III. MONEY AS A SANCTIONED EXCHANGE INTERMEDIARY

In this section we show that the Benoit and Krishna (1985) construction, in which multiple payoff nonequivalent equilibria in the single-shot game can support Pareto superior equilibria in the repeated game, is applicable to the problem of valuing fiat money. Consider again three agents who produce and trade in the following dynamic economy: At stage 1 the agents simultaneously decide whether to produce for autarky and not go to the market (action A) or to produce for trade

10. Concerning this uncertainty regarding location, a referee has pointed out the following correspondence between our model and the monetary overlapping generations model. In our model there is a strictly positive subjective probability of not being the last agent in the sequence of trades, and in the overlapping generations model there is a strictly positive probability of there being a succeeding period.

and go to the market (action T). This pre-market production stage is intended to capture the idea that gains from specialization and scale may arise when producing for trade, but that these gains are lost when agents must provide another spectrum of goods at lower volumes in autarky. In particular, we assume that agents are better at producing goods that are desired by other agents than at producing the goods for their own consumption. The impossibility of communication inhibits coordination on the Pareto superior production decisions (cf. Bryant [1983]). The outcome of the first stage is common knowledge before the start of the second stage.

There are six commodities labeled: a, b, c, d, e, and f, respectively. The agents have simple linear utility functions exhibiting different preferences. The commodity weights for the utility functions of agents 1, 2, and 3 are as follows:

$$U_1 = (10, 2, 0, 0, 3, 1) \cdot (a, b, c, d, e, f)^T,$$
$$U_2 = (1, 3, 10, 2, 0, 0) \cdot (a, b, c, d, e, f)^T,$$
$$U_3 = (0, 0, 1, 3, 2, 10) \cdot (a, b, c, d, e, f)^T,$$

where the superscript T indicates the transpose of a vector. Observe that agents do not care about all commodities and that the set of desired goods differs across agents.

The set of production possibilities simply consists of two elements for each agent. The first, element denoted by A, gives production under autarky; the second element, denoted by T, yields production for the market. The respective production possibility sets are as follows:

$$\begin{pmatrix} A_1 \\ T_1 \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 & 1/2 & 0 \\ 0 & 3/2 & 0 & 0 & 1/2 & 3/2 \end{pmatrix},$$
$$\begin{pmatrix} A_2 \\ T_2 \end{pmatrix} = \begin{pmatrix} 0 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 3/2 & 1/2 & 0 & 3/2 & 0 & 0 \end{pmatrix},$$
$$\begin{pmatrix} A_3 \\ T_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1/2 & 1/2 & 1/2 \\ 0 & 0 & 3/2 & 1/2 & 3/2 & 0 \end{pmatrix}.$$

In autarky, each agent can produce the commodity he or she desires most, but the comparative advantage lies in producing commodities that are valued more highly by the other agents. Thus the aggregate transformation schedule expands considerably if agents specialize and produce for trade. The gains from this specialisation have to come from trade, because it is easily checked that $U_i(T_i) = 6 < U_i(A_i) = 7.5$, for $i = 1, 2, 3$.

Subsequently, agents may undertake to trade. Our model of trade is in the spirit of the miniature trade model known from the works of Menger (1892) and Wicksell (1934 and 1935). Agents labeled 1, 2, and 3 are again aligned around a triangle and can meet for bilateral exchange at the nodes. For ease of reference we will refer to the bilateral markets (1, 2), (2, 3), (3, 1) by the roman numerals *I, II, III*, respectively. Meetings between agents take place in the given order (1, 2), (2, 3), (3, 1).[11] If any agent i opts out by choosing autarky in stage 1, all meetings involving agent i are removed from the sequence. If trade takes place, shipping any of the commodities to another agent incurs semivariable costs of transportation. These costs are modeled following the Samuelsonian iceberg model of trade. We assume that per unit shipment or less, one loses half the unit. In addition, it is assumed that there is an indivisibility of 1/2. Given that the maximum output of any commodity by a single agent is 3/2, we only have to consider shipments of 0, 1/2, 1, or 3/2 of the output. Shipping 0 amounts to nothing, but so does shipping 1/2 unit because one forgoes the entire cargo as the cost of transportation. Sending 1 unit gives the receiving party 1/2 unit. Sending 3/2 costs a whole unit and is equivalent to shipping only 1 unit from the receiving party's point of view. It follows that either 1 unit is shipped or nothing. Though it is assumed that exchanging commodities is costly, we assume that transferring fiat money can be accomplished costlessly. This assumption appears plausible given that the defining characteristic of fiat money is its intrinsic worthlessness.

At the second, market stage several possibilities arise. First and trivially, there may be no exchange on any market because no more than one agent chose the action T. The autarky choice yields $U_i = 7.5$, whereas a single agent producing for trade earns a lower payoff, $U_j = 6$, due to the loss in consumption resulting from specialization for trade.

11. The search based literature by Jones (1976) and Kiyotaki and Wright (1989) has generalized this to random meetings. This feature is an important aspect of our daily economic life, but the generalization is not essential for the purpose of modeling the monetary hold-up problem. Hence we use the deterministic scheme to keep the exposition as simple as possible.

We discuss some typical patterns of trade. In the trade pattern (TB, TB, A), the third agent decides to remain autarkic and makes

$$(6) \quad (0,0,1,3,2,10) \cdot (0,0,0,1/2,1/2,1/2)^T = 7.5,$$

and the other two agents have produced for market exchange. Before trading, the value of each player's endowment is $U_1 = U_2 = 6$. The only mutually beneficial and feasible trade is for agent 1 to ship 1 unit of commodity b to agent 2, while agent 2 forwards 1 unit of a in return. In the process only 1/2 unit of b and 1/2 unit of a survive transport. This yields

$$(7) \quad U_1 = (10,2,0,0,3,1) \cdot (1/2,1/2,0,0,1/2,3/2)^T = 9$$

and

$$(8) \quad U_2 = (1,3,10,2,0,0) \cdot (1/2,1,0,3/2,0,0)^T = 6.5.$$

Clearly, both agents gain from trading. If exactly two agents choose to produce for trade, monetary exchange and bilateral exchange are equivalent. Money requires at least three parties. Because it is assumed that the endowments of skills and other factors are such that agent $(j+1)$ produces the good most desired by agent j but that the goods produced by j provide only moderate gains from trade for $(j+1)$, it follows that agent 1 gains more than agent 2. Note that exchanging any of the other commodities would be pure waste because either the endowments are insufficient—that is, less than 1—or the commodity is not valued by the counterparty.

When all three agents produce for the market we must carefully specify the sequence of trade. In the Wicksellian triangular pattern of trade agents are aligned in order 1, 2, 3. Agent 1 first meets agent 2 in market *I*. Then 2 meets 3 in market *II* and, finally, 3 meets 1 in market *III*. Transportation costs and the limited production possibilities are such that it only pays to transport an item once. Hence, no commodity money can exist. On the other hand, the intrinsically worthless fiat money can be transferred costlessly. Apart from economy-wide barter, monetary or fiat exchange can develop as follows. We assume monetary exchange must start in market *I* and always breaks down as soon as the chain of monetary exchanges is broken by a barter trade. Because in trade between j and $(j+1)(mod\,3)$, player j is better off with such a transfer vis-à-vis barter exchange, he never opposes such a transaction. Whether bilateral exchange between j and $(j+1)(mod\,3)$ is barter (*TB*) or monetary (*TM*), is decided by agent $(j+1)$.

The economy-wide barter scheme is (*TB, TB, TB*). It is easily shown that each agent has a strict incentive to trade in each market that he enters. In particular agent 1 first trades $1b$ (including the costs of shipment) for $(1/2)a$ (net) with agent 2, and forwards $1f$ to agent 3, for which he receives net of shipment costs $1/2e$. Moreover, this multilateral trade program generates the highest possible payoff when all exchange takes place on a quid pro quo basis, that is, $U_i = 9.5$ for $i = 1, 2, 3$.

Under monetary exchange, that is, (*TM, TM, TM*), commodity flows are clockwise, whereas fiat money flows the other way. This exchange yields all agents $U_i = 10$. For example, in this scheme agent 1 sends $1f$ (gross) to agent 3 and receives $(1/2)a$ (net) from agent 2 in exchange for fiat money. Monetary exchange Pareto dominates barter because, given our assumption on the cost of resale of goods, barter requires bilateral coincidence of wants, whereas monetary exchange allows the one-way flow of goods around the triangle. We note that this is not the only fiat exchange that is feasible. Consider, for example, the clockwise flow of commodities whereby agent 1 ships 1 unit of b to agent 2, agent 2 sends 1 unit of d to agent 3, and agent 3 forwards 1 unit of e to agent 1, while money flows the other way around. This yields each agent $U_i = 5.5$, which leaves every agent worse off in comparison to the valuation of their endowment, $U_i(T) = 6$. Because such trade patterns are not individually rational, they are not considered to be part of the equilibrium selection.

The last trade configuration we discuss describes the case in which agent 1 does not honor the fiat notes in his exchange with agent 3 at the end of the trade cycle, even though agent 1 has paid with these notes for his trade deficit with agent 2. The third agent is stuck holding the useless paper money and only partly recoups his loss through barter exchange with agent 1. This is a manifestation of the monetary hold-up problem. Thus agent 3 makes significantly less, $U_3 = 8$, than the other two agents. Agent 2 has monetary exchange with both counterparties and makes $U_2 = 10$, as in the case of economy-wide monetary exchange. The first agent, who decides not to honor his IOU, gets an extra bonus and makes $U_1 = 11.5$.

All the possible configurations that emanate from the basic assumptions are summarized in the payoff matrices in Figure 2. We have reduced each player's strategy set to three elements by equating strategies that are payoff equivalent. A player chooses at the first stage between autarky (A), and

FIGURE 2
Payoff Matrices

	A_2	TB_2	TM_2	
A_1	7.5 7.5 7.5	7.5 6 7.5	7.5 6 7.5	
TB_1	6 7.5 7.5	9 6.5 7.5	11 5 7.5	A_3
TM_1	6 7.5 7.5	9 6.5 7.5	11 5 7.5	

	A_2	TB_2	TM_2	
A_1	7.5 7.5 6	7.5 9 6.5	7.5 9 6.5	
TB_1	6.5 7.5 9	9.5 9.5 9.5	11.5 8 9.5	TB_3
TM_1	6.5 7.5 9	9.5 9.5 9.5	11.5 8 9.5	

	A_2	TB_2	TM_2	
A_1	7.5 7.5 6	7.5 9 6.5	7.5 9 6.5	
TB_1	6.5 7.5 9	9.5 9.5 9.5	11.5 10 8	TM_3
TM_1	6.5 7.5 9	9.5 9.5 9.5	10 10 10	

production for trade (T). In the second stage, which is the market phase, the agents who produced for trade choose between barter trade (TB) or monetary exchange (TM). There are three pure strategy equilibria of the game: (A, A, A), (TB, TB, TB), and (TM, TB, TB). The last two are payoff and observationally equivalent and involve a sequence of bilateral barter trades between the three agents. We will refer to both of these equilibria as the barter equilibrium. However, only the equilibria (A, A, A) and (TB, TB, TB) are perfect equilibria in the normal form of the game (see Figure 2). The barter equilibrium strictly payoff dominates the autarkic equilibrium. Of course, the agents would benefit if all three would engage in fiat exchange (TM, TM, TM). Unfortunately, due to the hold-up problem the first agent has an incentive to renege when he is asked to provide goods for worthless fiat money in the last market (see the payoffs to [TB, TM, TM]). Hence, given that all three players choose initially to produce for the market, the only subgame equilibrium involves barter exchange. Fiat exchange is not an equilibrium of the game.

Suppose, however, that the game in Figure 2 is repeated at least once. Playing each single-shot equilibrium twice, certainly represents an equilibrium in the repeated game. In addition, we have the following result.

PROPOSITION 4. *In a once-repeated version of the trade game, the strategies* (TM, TM, TM) *for the first round and* (TB, TB, TB) *for the second round constitute a subgame perfect equilibrium path. Each agent receives a total equilibrium payoff of* 19.5.

Proof. Because second-stage behavior represents an equilibrium of the one-shot game, it clearly is consistent with subgame perfect equilibrium behavior. The first round choice (TM, TM, TM) is supported by the threat of punishment of any deviation through reversion to the dominated autarky equilibrium (A, A, A), which would yield only 19 to the deviator (instead of 19.5).[12]

With some extra notation one can extend this result to an n-agent $2n$-product

12. We do not consider the issue of renegotiation proofness as this seems to violate the spirit of decentralized markets where money plays a transactions role.

economy.[13] Let $U_i = \Sigma_{j=1}^{2n} c_{i,j} q_{i,j}$, where

$$(9) \qquad c_{i,j} = \sum_{h=1}^{4} d_h \delta_{j,(2i-4+h)(mod\, 2n)}$$

$$(10) \qquad \delta_{j,(2i-4+h)(mod\, 2n)} = \begin{cases} 0 & \text{if } j \neq (2i-4+h)(mod\, 2n) \\ 1 & \text{if } j = (2i-4+h)(mod\, 2n) \end{cases}$$

$\{d_1, d_2, d_3, d_4\} = \{1, 3, 10, 2\}$, $h \in \{1, 2, 3, 4\}$, $i \in \{1, \ldots, n\}$, and $j \in \{1, \ldots, 2n\}$. Players are indicated by the index i, products by the index j. The coefficient $c_{i,j}$ is the marginal utility of product j for agent i, and $q_{i,j}$ is i's total consumption of product j. The modulo operator in $(2i-4+h)(mod\, 2n)$ ensures that $2i-4+h$ is strictly positive. Note that for $i = 1$ and $h = 1$, $2i-4+h = -1$, while for $i = 1, h = 2$: $2i-4+h = 0$. The modulo operator replaces these values by $2n-1$ and $2n$, respectively. The production functions are

$$(11) \qquad \begin{pmatrix} A_{i,j} \\ T_{i,j} \end{pmatrix} = \begin{pmatrix} \sum_{h=1}^{4} e_h \delta_{j,(2i-4+h)(mod\, 2n)} \\ \sum_{h=1}^{4} f_h \delta_{j,(2i-4+h)(mod\, 2n)} \end{pmatrix},$$

where $\{e_1, e_2, e_3, e_4\} = \{0, 1/2, 1/2, 1/2\}$ and $\{f_1, f_2, f_3, f_4\} = \{3/2, 1/2, 0, 3/2\}$. In the one-shot game autarky and barter trade are again equilibria, and one can verify that in the repeated game fiat exchange is also an equilibrium, except in the last round.

IV. CONCLUSION

Mainstream monetary economics bases the value of fiat money in exchange on infinities, such as an indefinite future, to circumvent backward induction. Due to the clear benefits from monetary exchange over barter, backward induction arguments against the use of fiat money in finite, but nevertheless large, economies seem implausible. Moreover, models that push the boundary condition to infinity present difficulties for positive economists who want to test these theories. Exchanges in current economies are so numerous and intricate that people easily loose track of who is at the beginning and the end of a chain of exchanges. In our first model this is made explicit by inserting uncertainty over who initiates a chain of monetary exchanges. Nevertheless, agents always know with whom they are engaged in exchange. Because agents do not know where their meetings in the chain take place relative to the beginning or end (i.e., at which node they are located), backward induction breaks down.

We started with a relatively small economy, three agents and exchanges, for which the nonobservability of the initiator of monetary exchange may appear questionable. Nevertheless, this simple set up lends itself easily to experimental investigation. By repeating the chain and making it longer, the nonobservability assumption seems eminently plausible. Future work along these lines would probably benefit from applying more intricate trading patterns, such as those discussed in the pairwise trade and random meeting models.

The other approach articulated in this article does not rely on an incomplete information structure but exploits the multiplicity of equilibria that is present in an appropriately defined single-shot trading game. A priori it appears quite natural that a trading game has autarkic and barter equilibria. The fact that a barter equilibrium Pareto dominates an autarkic equilibrium is also plausible. In this respect, the setup of our model is reminiscent of the coordination failure literature. Given the cost of transporting commodities and the asymmetries in production possibilities and preferences, a unidirectional flow of goods coupled with fiat money (costlessly) flowing the other way constitutes a more efficient outcome than either autarky or barter. As in the finite horizon overlapping generations model, however, this outcome cannot be sustained as an equilibrium in our decentralized market economy. The reason is that the agent who issues the notes is not willing to redeem them. In a twice-repeated setting, though, fiat exchange can be supported in the first round by the threat of reverting from the barter equilibrium to the autarkic equilibrium in the second round. Obviously, this line of analysis can be extended by considering more agents, commodities, trading rounds, and random search.[14] However, our

13. We are grateful to Pieter Offers, who discussed this extension in his undergraduate thesis at Erasmus University Rotterdam.

14. The threat of autarky can be made even more appealing by changing the payoff matrix so that the autarkic equilibrium risk dominates the barter equilibrium.

model already captures the essence of the transactions facilitating role of money, even though money has no intrinsic value.

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